PRAYAS

JEE 2026

Mathematics

Trigonometric Functions

Lecture -02

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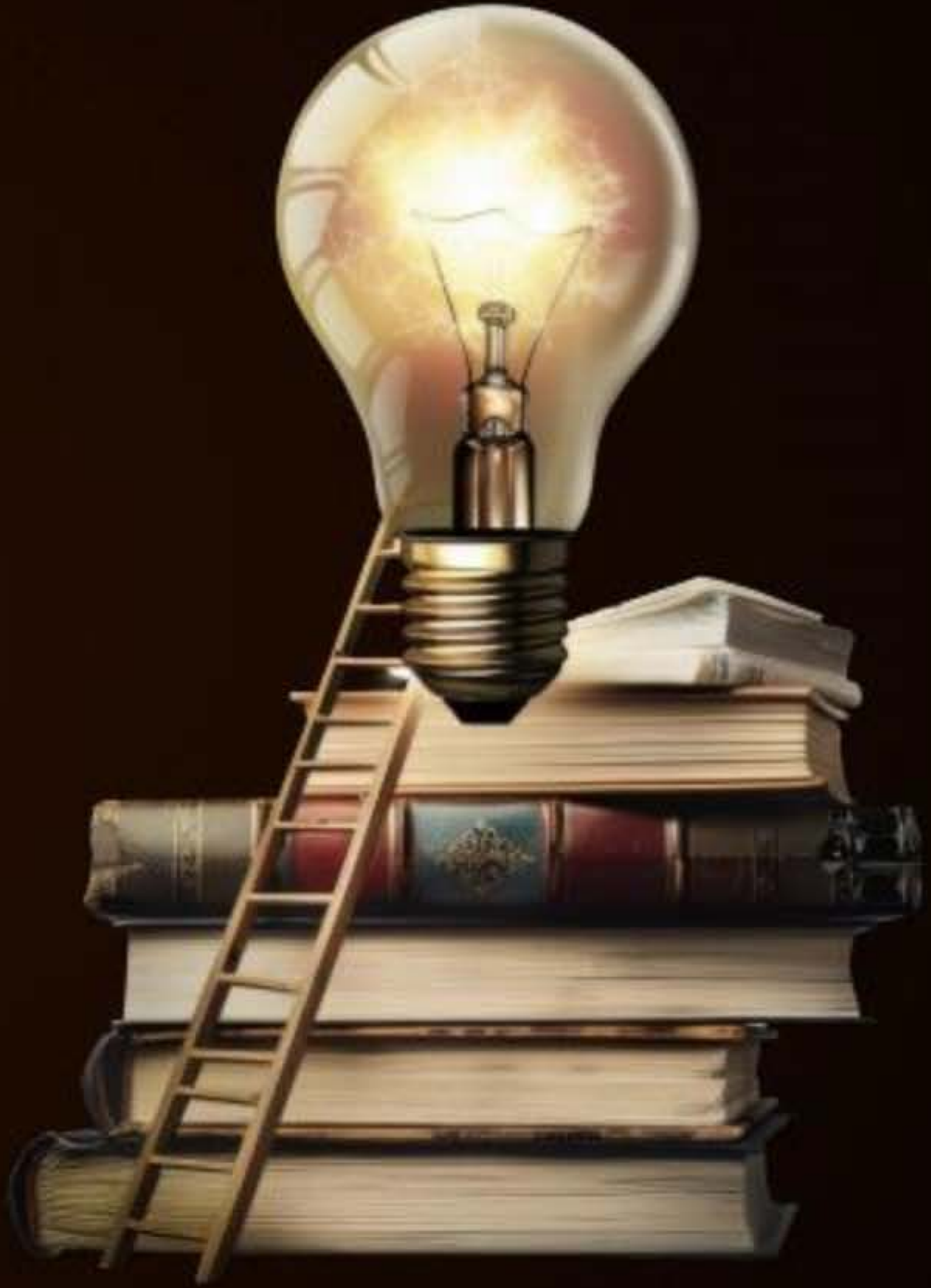
Topics *to be covered*

- **A** Reduction Formula

[redacted]



Recap *of previous lecture*

1. If for two positive numbers a & b we have

   a, $A_1, A_2, \dots A_n$, b are in A.P.

   a, $G_1, G_2, \dots G_n$, b are in G.P.

   a, $H_1, H_2, \dots H_n$, b ore in H.P.

   then $A_1 \geq G_1 \geq H_1$, $A_2 \geq G_2 \geq H_2$, - - - $A_n \geq G_n \geq H_n$

   also $A_1 H_n = A_2 H_{n-1} = A_3 H_{n-2} = \dots\dots = A_n H_1 = ab$

2. $2x + 3y = 5$ then minimum value of $x^2 + y^2 =$ 25/13.

   ② 2, 3, x, y ∈ R

   $\underbrace{\text{By CS ineq}}$

   $(2x+3y)^2 \leq (2^2+3^2)(x^2+y^2)$

   $25 \leq 13(x^2+y^2)$

   $x^2+y^2 \geq \frac{25}{13}.$

3. $a + b + c + d = 1$ then $\sqrt{a} + \sqrt{b} + \sqrt{c} + \sqrt{d} \leq$ 2

$a+b+c+d=1$

$1, 1, 1, 1, \sqrt{a}, \sqrt{b}, \sqrt{c}, \sqrt{d} \in R$

M①

$$\left.\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d}\right|_{max} = 2$$

By CS inequality

$$(1\cdot\sqrt{a}+1\cdot\sqrt{b}+1\cdot\sqrt{c}+1\cdot\sqrt{d})^2 \leq (1^2+1^2+1^2+1^2)(\sqrt{a}^2+\sqrt{b}^2+\sqrt{c}^2+\sqrt{d}^2)$$

$$(\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d})^2 \leq 4\cdot 1$$

$$\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d} \leq 2.$$

M②

Using RMS $\geq$ A.M

$$\sqrt{\frac{\sqrt{a}^2+\sqrt{b}^2+\sqrt{c}^2+\sqrt{d}^2}{4}} \geq \frac{\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d}}{4}$$

$$\frac{1}{2} = \sqrt{\frac{a+b+c+d}{4}} \geq \frac{\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d}}{4}$$

$\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d} \leq 2$ Ans

Recap of previous lecture

Tan1@

4. If $a + b + c = 1$ & $a, b, c > 0$ then the minimum value of $a^2 + 2b^2 + c^2$ is ________

5. $\sec x - \tan x$ & $\sec x + \tan x$ are ____reciprocal____ of each other.

6. $\operatorname{cosec} 30^\circ + \sec 60^\circ + \tan\frac{\pi}{4} + \cot^2\frac{\pi}{3} =$ ____$2+2+1+\frac{1}{3} = 16/3$____

7. $\tan 90^\circ$ is ____N.D____ $\cot 90^\circ$ is ____0____

   $\operatorname{cosec} 0^\circ$ is ____N.D____ $\cot 0^\circ$ is ____N.D____

   $\sec 90^\circ$ is ____N.D____

Homework Discussion

QUESTION

(RPP 1)

Paragraph

Consider the quadratic equation $2x^2 - (4m + 2)x + m^2 + m = 0, m \in R$

1. The number of positive integer values of 'm' such that the equation has exactly one root in (2, 3) is

(A) 3 (B) 4 (C) 5 (D) 6

2. The number of negative integral values of 'm' such that $m > -10$ and atleast one root of the equation is smaller than '2' is

(A) 8 (B) 9 (C) 6 (D) 4

case(i)

$-\frac{b}{2a} < 2$, $D \geq 0$

$f(2) \geq 0$

case(ii)

$f(2) < 0$

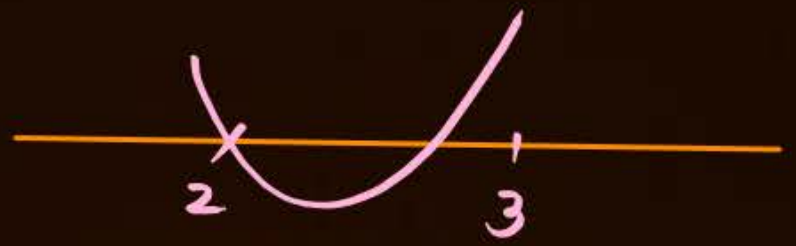
Ans. (1) B, (2) B

Q1.

case ①

$f(2) \cdot f(3) < 0$

case (ii)



case (iii)

QUESTION

(Challenger Problem (Answers Sahi hai)

If $ax^2 + bx + c = 0, a \neq 0, a, b, c \in R$ has two distinct real roots in $(1, 2)$ then

RPP 02

A (a) $(5a + 2b + c) > 0$

B (a) $(5a + 2b + c) < 0$

C $2a + b > 0$

D (a) $(4a + 2b + c) > 0$

$af(1) > 0$

$af(2) > 0 - a(4a+2b+c) > 0$

Now $a(5a+2b+c)$

$= a(a+4a+2b+c)$

$= \underbrace{a^2}_{>0} + \underbrace{a(4a+2b+c)}_{>0} > 0$

ⓒ $1 < -\frac{b}{2a} < 2$

$1 + \frac{b}{2a} < 0$

$\frac{2a+b}{2a} < 0$.

$a > 0$: $2a+b < 0$

$a < 0$: $2a+b > 0$

Ans. A, D

$1 < \alpha, \beta < 2$

$\alpha + \beta > 2$ $\qquad$ $\alpha + \beta < 4$

$\downarrow$

$\frac{-b}{a} > 2$

$2 + \frac{b}{a} < 0$

$\frac{2a+b}{a} < 0$

$a < 0$ $\qquad$ $a > 0$

$2a+b > 0$ $\qquad$ $2a+b < 0$.

Trigonometry

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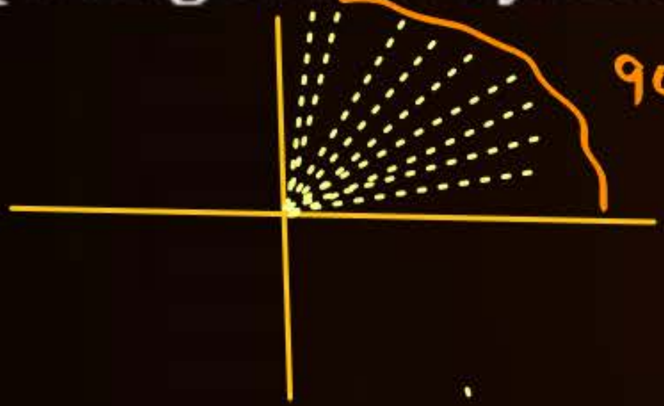
Trigonometry

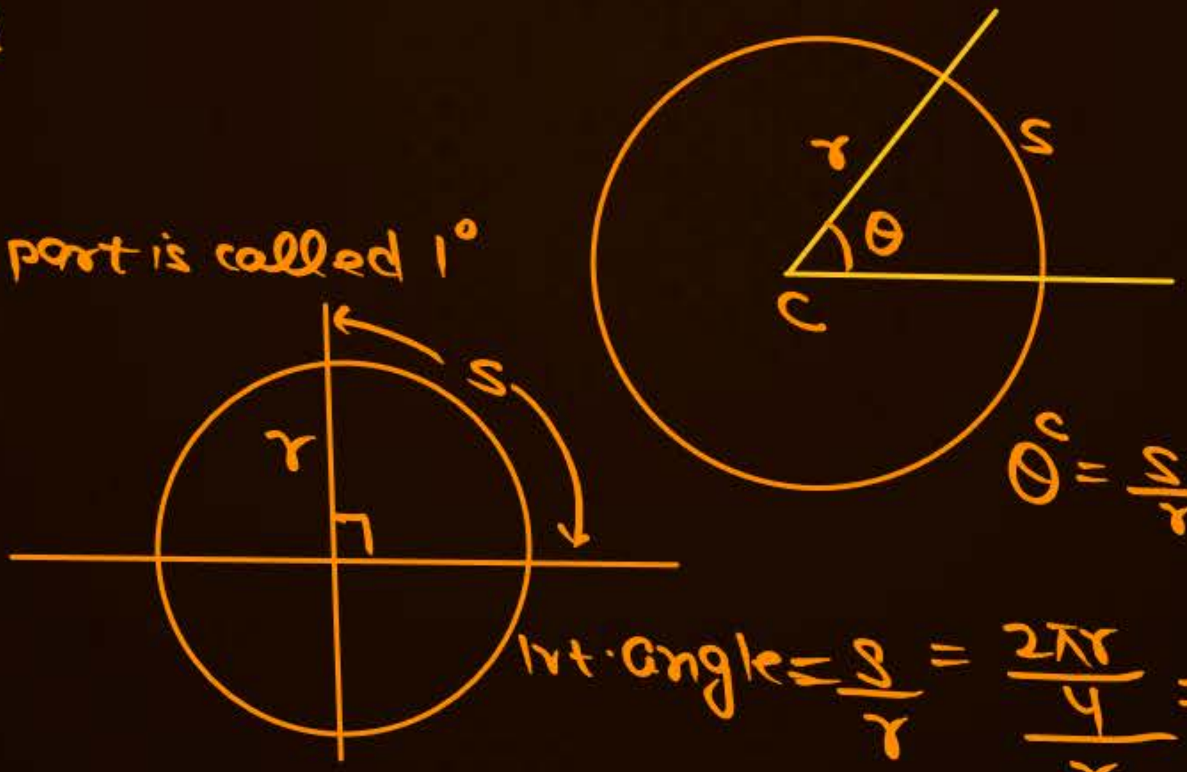
System of measurement of angle

Degree
(Sexagesimal system)



90 equal parts
↓
each part is called 1°

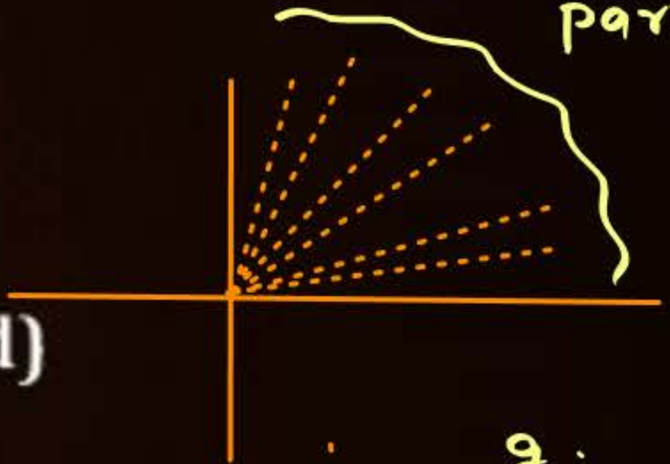
Radian
(Circular system)



$\theta^c = \frac{s}{r}$ > dimensionless.

1rt·angle $= \frac{s}{r} = \frac{\frac{2\pi r}{4}}{r} = \pi/2$

French
(not in used)



each part is called 1 grade

100 equal parts

1rt·angle $= 100^g$

1rt angle $= 90^\circ = \left(\frac{\pi}{2}\right)^c = 100^g$.

$$90^\circ = \left(\frac{\pi}{2}\right)^c$$

$$1^\circ = \left(\frac{\pi}{180}\right)^c = 0.0175^c$$

$$1^c = \left(\frac{180}{\pi}\right)^\circ = 57.3^\circ.$$

★ $\theta^\circ = \left(\frac{\pi}{180} \cdot \theta\right)^c$

★ $\theta^c = \left(\frac{180}{\pi} \theta\right)^\circ$

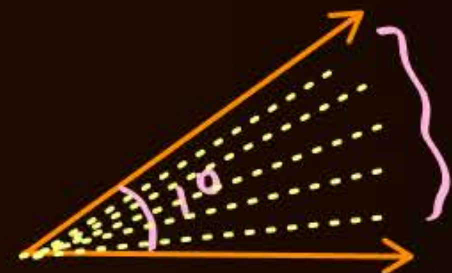
Yaad Rakho

- $\frac{\pi}{6} = 30^\circ$
- $\frac{\pi}{4} = 45^\circ$
- $\frac{\pi}{12} = 15^\circ$
- $\frac{\pi}{15} = 12^\circ$
- $\frac{\pi}{10} = 18^\circ$
- $\pi/8 = 22.5^\circ$

- $\frac{\pi}{3} = 60^\circ$
- $\frac{\pi}{2} = 90^\circ$
- $\frac{2\pi}{3} = 120^\circ$
- $\frac{3\pi}{4} = 135^\circ$
- $\frac{5\pi}{6} = 150^\circ$
- $\frac{3\pi}{8} = 67.5^\circ$

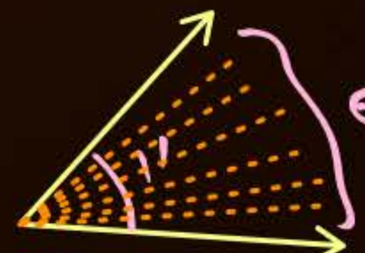
- $\frac{7\pi}{6} = 210^\circ$
- $\frac{4\pi}{3} = 240^\circ$
- $\frac{5\pi}{4} = 225^\circ$
- $\frac{3\pi}{2} = 270^\circ$
- $\frac{11\pi}{6} = 330^\circ$
- $\frac{5\pi}{12} = 75^\circ$

- $\frac{5\pi}{3} = 300^\circ$
- $\frac{7\pi}{4} = 315^\circ$
- $2\pi = 360^\circ$



60° equal parts each part is called one minute (1′)

$1^\circ = 60'$

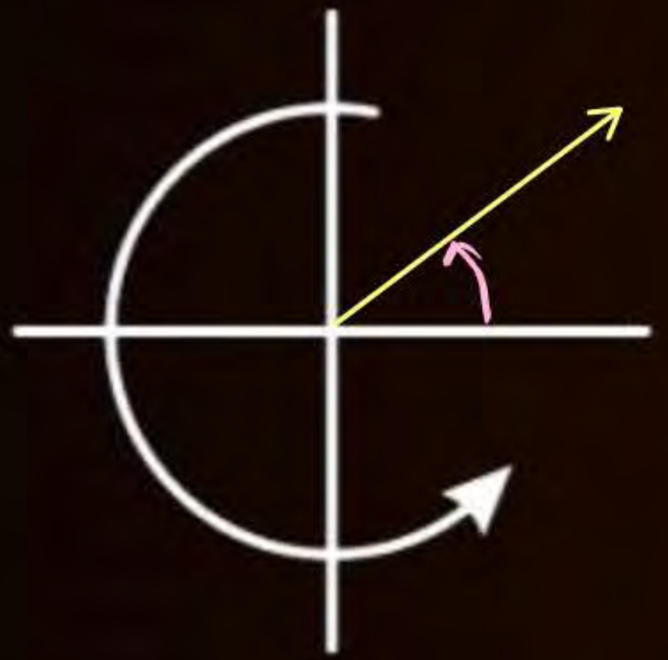


60 equal parts each part is called One second (1″)

$1' = 60''$

Measurement of Angle

Sign Convention:



Anticlockwise direction
will indicate +ve angle

Clockwise direction
will indicate –ve angle

Angles at Boundaries of Quadrants

$5+(n-1)4=4n+1$

$$---\overset{\overset{T_{-2}}{\downarrow}}{-\frac{7\pi}{2}},\ \overset{\overset{T_{-1}}{\downarrow}}{-\frac{3\pi}{2}},\ \overset{\overset{T_{0}}{\downarrow}}{\pi/2},\ \overset{\overset{T_{1}}{\downarrow}}{\frac{5\pi}{2}},\ \overset{\overset{T_{2}}{\downarrow}}{\frac{9\pi}{2}}\ ---\ (4n+1)\frac{\pi}{2}$$

$(2n+1)\pi$

$$--{-3\pi}, -\pi, \pi, 3\pi, 5\pi ---$$

(odd π)

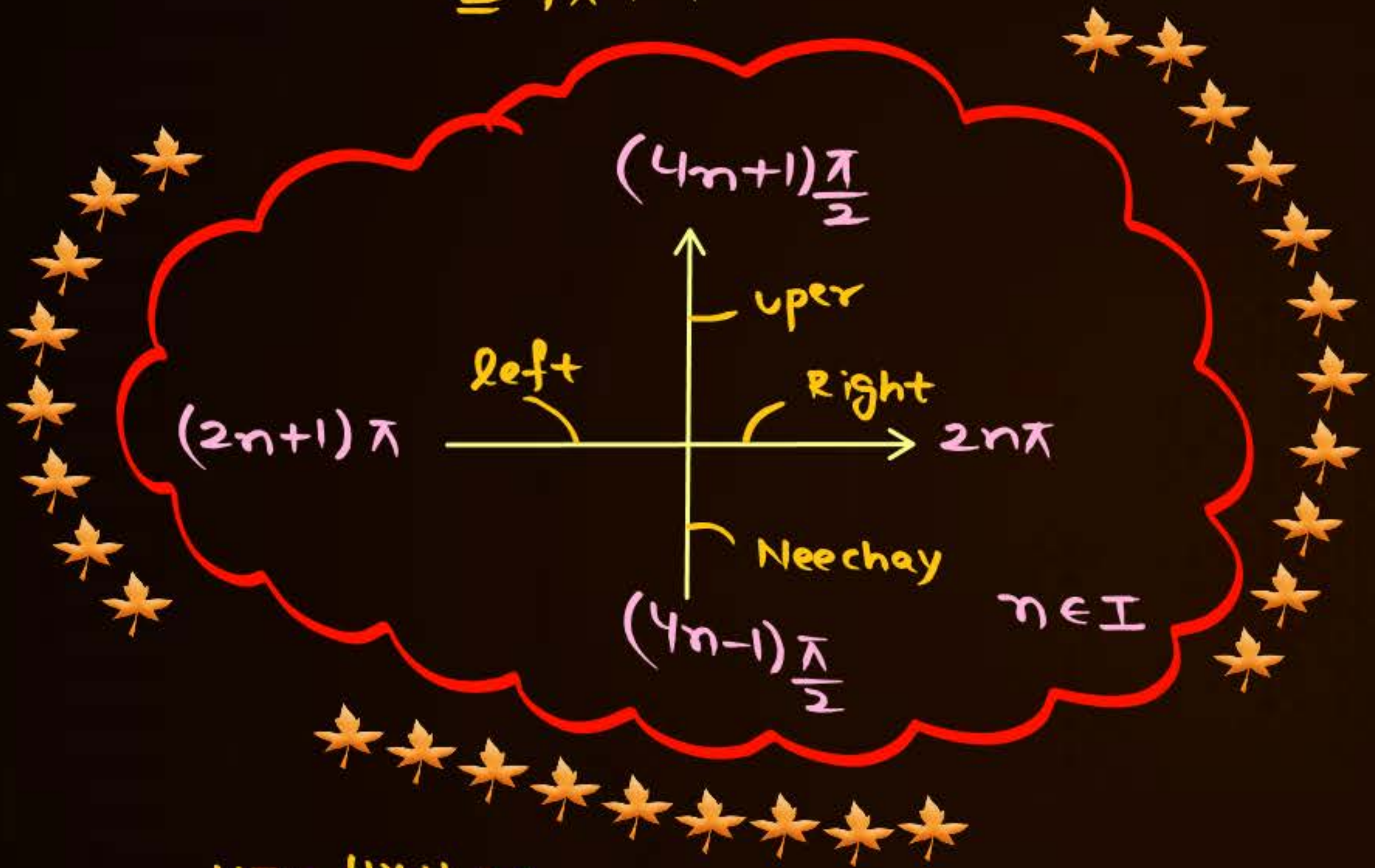
$$--{-4\pi}, -2\pi, 0, 2\pi, 4\pi, 6\pi ----\ (2n\pi)$$

(Even pi)

$$---\frac{-5\pi}{2}, \frac{-\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}\ ---\ (4n-1)\pi/2$$

$n \in I$

$27 = 6\times4+3$
$= 6\times4+4-1$
$= 7\times4-1$



$47 = 4\times\overset{\frown}{11+3}$
$= 4\times11+4-1$
$= 4\times(11+1)-1$
$= 4\times12-1$

$65 = 4\times16+1$

$4\overline{)25}$ quotient 6
24
1

★ $2025\frac{\pi}{2}$ — $(4n+1)\frac{\pi}{2}$ type
↓
uper

★ $2027\frac{\pi}{2}$ → $(4n-1)\frac{\pi}{2}$ type
↓
Neechay.

$4\overline{)27}$ quotient 6
24
3

$27 = 4\cdot7-1$

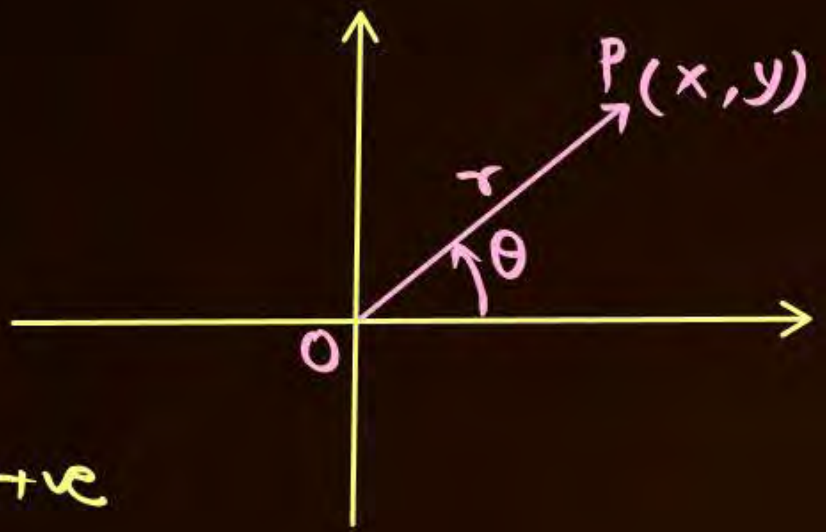
$47 = 4\times12-1$

★ $5047\frac{\pi}{2}$ — $(4n-1)\frac{\pi}{2}$ (Neechay)

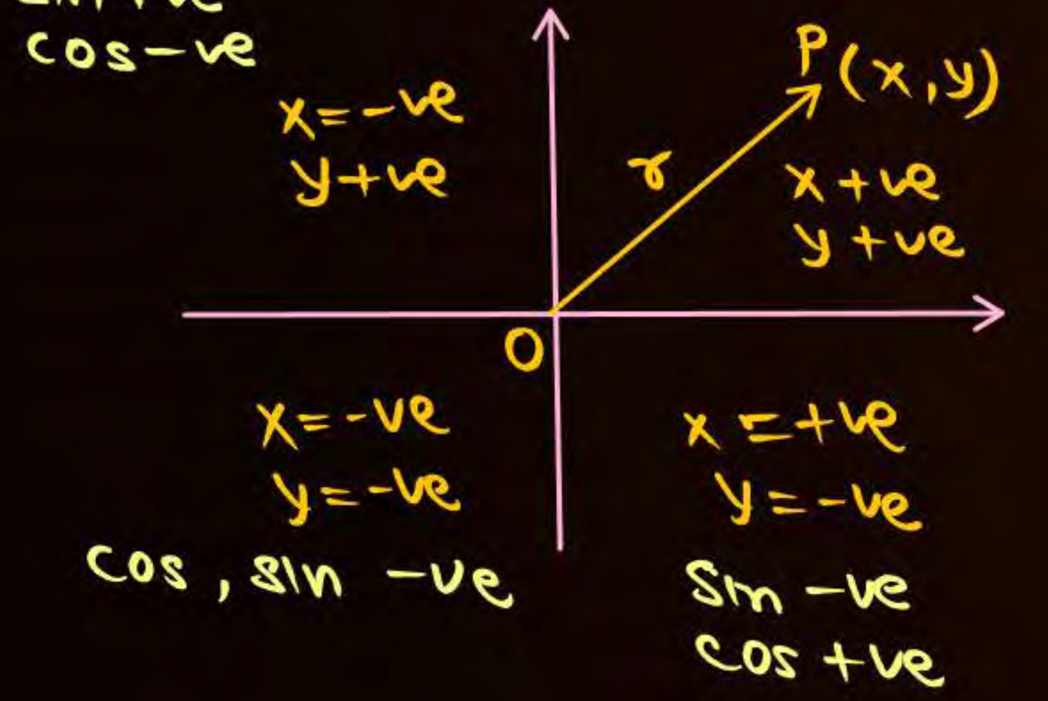
★ 2025π → left

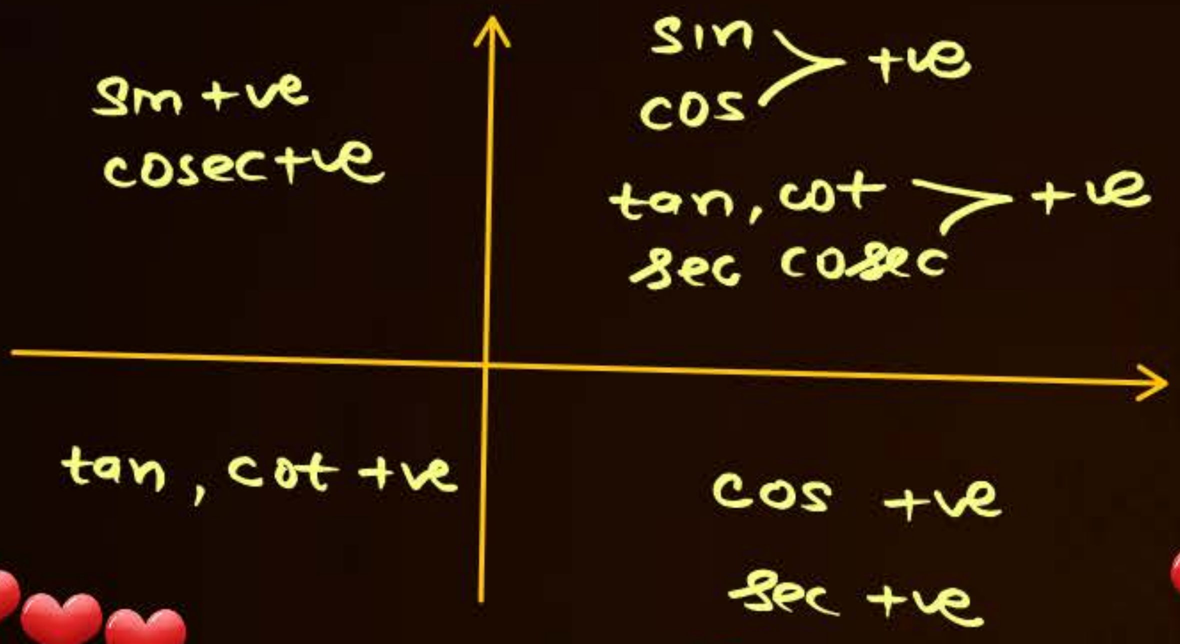
★ $7065\frac{\pi}{2}$ — $(4n+1)\frac{\pi}{2}$ (uper)

Real definition of 2 basic functions (Sine & Cosine)



- $\sin\theta = \frac{y \text{ coord of } P}{\text{length of } OP} = \frac{y}{r}$
- $\cos\theta = \frac{x \text{ coord of } P}{\text{length of } OP} = \frac{x}{r}$
- $\tan\theta = \frac{y}{x}$
- $\cot\theta = \frac{x}{y}$
- $\sec\theta = \frac{r}{x}$
- $\text{cosec}\,\theta = \frac{r}{y}$



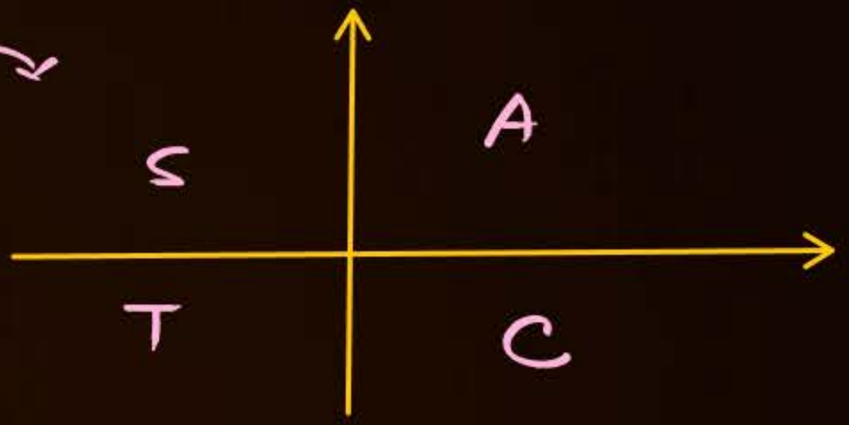


sin +ve
cosec +ve

sin, cos > +ve
tan, cot > +ve
sec cosec

tan, cot +ve

cos +ve
sec +ve



S
A
T
C

$n\pi \cup (2n+1)\pi/2$
$\parallel$
$\frac{n\pi}{2}$

★ $2n\pi \cup (2n+1)\pi = n\pi$

★ $(4n+1)\frac{\pi}{2} \cup (4n-1)\frac{\pi}{2} = (2n+1)\pi/2$

$--- -\frac{3\pi}{2}, -\frac{\pi}{2}, \pi/2, 3\pi/2, 5\pi/2, 7\pi/2 ---$

$-- -\frac{7\pi}{2}, -\frac{3\pi}{2}, \pi/2, 5\pi/2, 9\pi/2 -- \cup --, -\pi/2, 3\pi/2, 7\pi/2 ---$

Value of T-Ratios at Boundaries of Quadrants

$\sin(2n+1)\pi = 0$

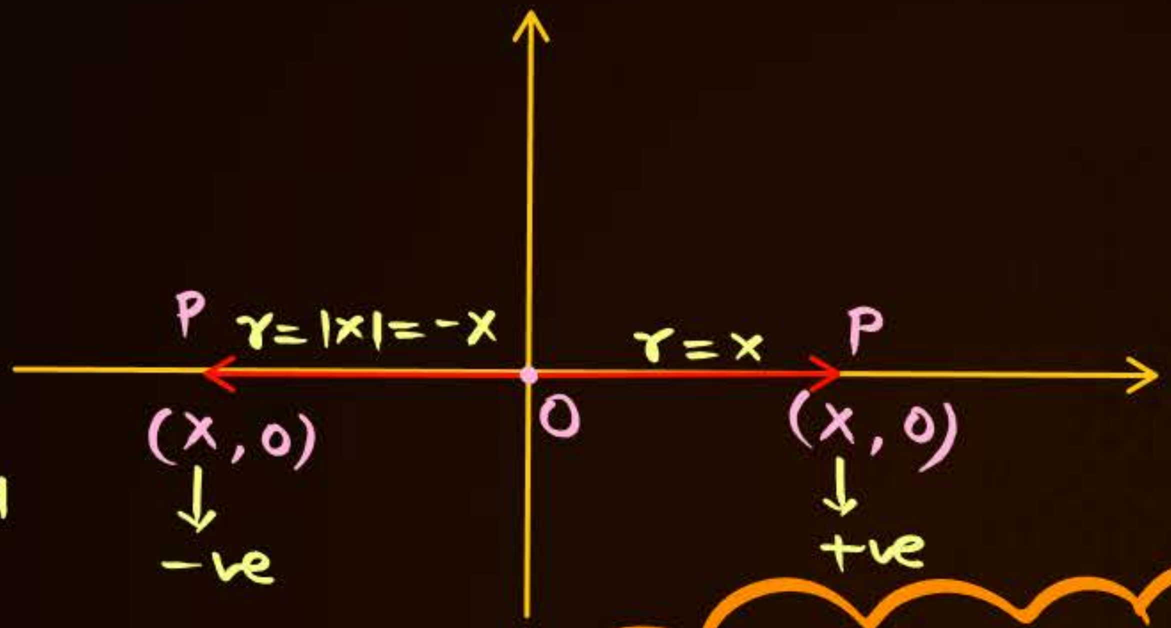
$\cos(2n+1)\pi = \frac{x}{r} = -1$

$\tan(2n+1)\pi = 0$

$\sec(2n+1)\pi = -1$

$\cot(2n+1)\pi$ N.D

$\text{cosec}(2n+1)\pi$ N.D.



$\sin 2n\pi = \frac{0}{r} = 0$

$\cos 2n\pi = \frac{x}{r} = 1$

$\tan 2n\pi = 0$

$\sec 2n\pi = 1$

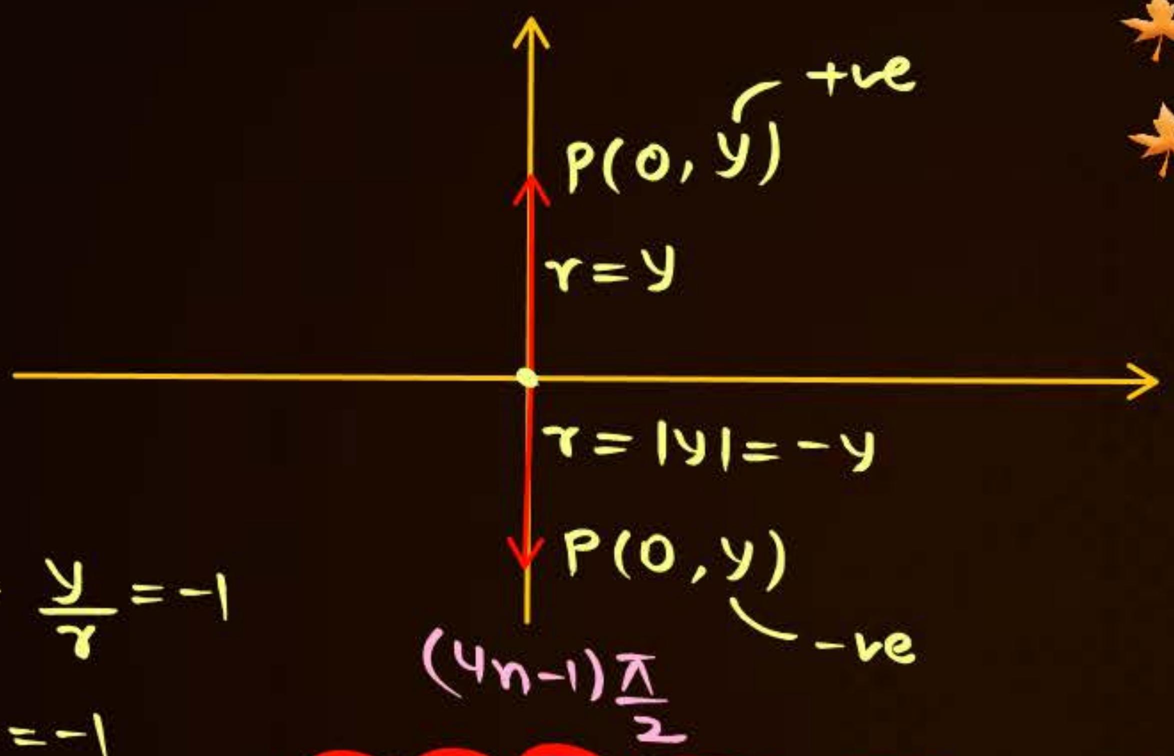
$\text{cosec}\, 2n\pi$ N.D

$\cot 2n\pi$ N.D.

- $\sin\theta = 0 \Leftrightarrow \theta = n\pi$
- $\tan\theta = 0 \Leftrightarrow \theta = n\pi$
- $\sec\theta, \cos\theta = 1 \Leftrightarrow \theta = 2n\pi$
- $\sec\theta, \cos\theta = -1 \Leftrightarrow \theta = (2n+1)\pi$

- $\cot\theta$, $\text{cosec}\,\theta$ are not defined for $n\pi$

Value of T-Ratios at Boundaries of Quadrants



★ $\sin(4n+1)\frac{\pi}{2} = \frac{y}{r} = 1$

★ $\cos(4n+1)\frac{\pi}{2} = \frac{0}{r} = 0$

$\tan(4n+1)\frac{\pi}{2} = N\cdot D$

$\cot(4n+1)\pi/2 = 0$

$\sec(4n+1)\pi/2 = N\cdot D$

$\text{cosec}(4n+1)\pi/2 = 1$

$\sin(4n-1)\frac{\pi}{2} = \frac{y}{r} = -1$

$\text{cosec}(4n-1)\pi/2 = -1$

$\cos(4n-1)\pi/2 = \frac{0}{r} = 0$

$\sec(4n-1)\pi/2 = N\cdot D.$

$\tan(4n-1)\pi/2 = N\cdot D.$

$\cot(4n-1)\pi/2 = 0$

★ $\text{cosec}\theta, \sin\theta = 1 \Leftrightarrow \theta = (4n+1)\pi/2$

★ $\text{cosec}\theta, \sin\theta = -1 \Leftrightarrow \theta = (4n-1)\pi/2$

★ $\cos\theta = 0 \Leftrightarrow \theta = (2n+1)\pi/2$

★ $\tan\theta, \sec\theta$ are N·D at $\theta = (2n+1)\pi/2$

★ $\cot\theta = 0 \Leftrightarrow \theta = (2n+1)\pi/2$

- $\tan\theta, \sin\theta = 0 \iff \theta = n\pi$
- $\sec\theta, \cos\theta = 1 \iff \theta = 2n\pi$
- $\sec\theta, \cos\theta = -1 \iff \theta = (2n+1)\pi$

$\cos\theta = \pm 1 \iff \theta = n\pi$

- $\cot\theta, \operatorname{cosec}\theta$ are not defined at $\theta = n\pi$

- $\sin\theta, \operatorname{cosec}\theta = 1 \iff \theta = (4n+1)\frac{\pi}{2}$
- $\sin\theta, \operatorname{cosec}\theta = -1 \iff \theta = (4n-1)\frac{\pi}{2}$
- $\cos\theta = 0 \iff \theta = (2n+1)\pi/2$
- $\tan\theta, \sec\theta$ are not defined at $\theta = (2n+1)\pi/2$
- $\cot\theta = 0 \iff \theta = (2n+1)\pi/2$

QUESTION

If $a = \cos(2012\pi)$, $b = \sec(2013\pi)$ and $c = \tan(2014\pi)$ then

A $a < b < c$

[✓ B] $b < c < a$

C $c < b < a$

D $a = b < c$

$a = \cos(2012\pi) = 1$

$b = \sec(2013\pi) = -1$

$c = \tan(2014\pi) = 0$

$a > c > b.$

QUESTION

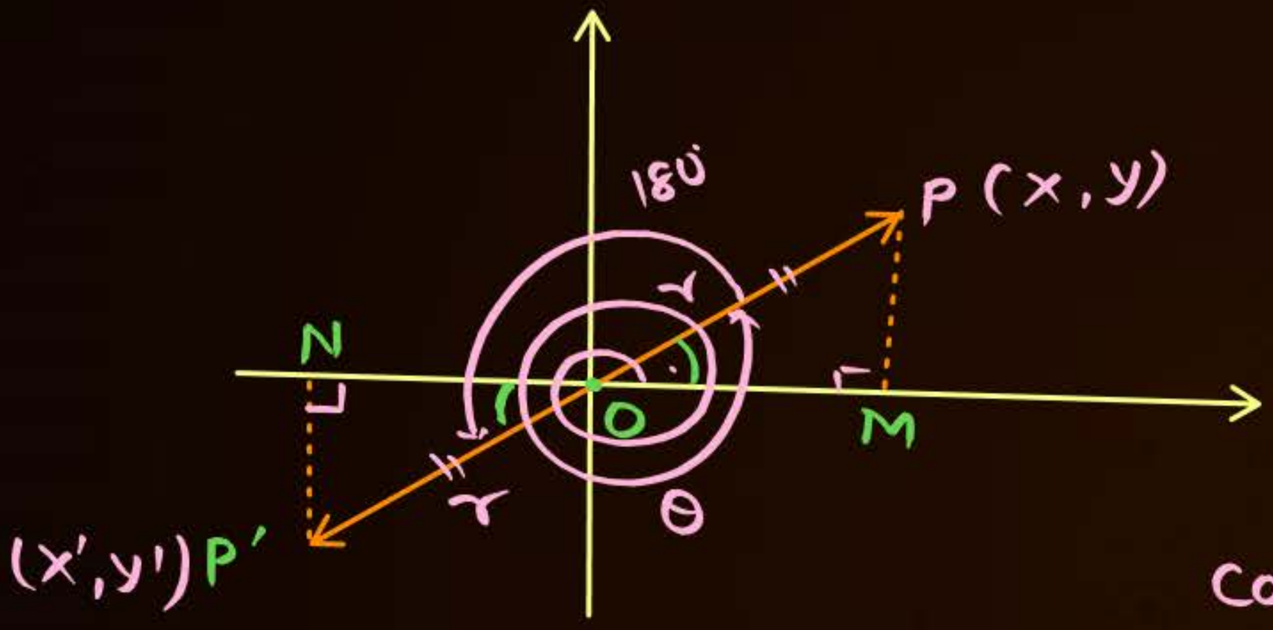
$\dfrac{\sin 4}{\sin 6}$ is negative. T/[✓ F]

$\sin 4 < 0$

$\sin 6 < 0$

Reduction Formulae

— used for finding T-ratios of larger Angles



$\Delta POM \cong \Delta P'ON$

$NP' = MP$
$ON = OM$ (cpct)

$$\sin(180^\circ+\theta) = \frac{y'}{r} = -\frac{NP'}{r} = -\frac{MP}{r} = \frac{-y}{r}$$

$$\sin(180^\circ+\theta) = -\frac{y}{r} = -\sin\theta$$

$$\cos(180^\circ+\theta) = \frac{x'}{r} = -\frac{ON}{r} = -\frac{OM}{r} = -\frac{x}{r} = -\cos\theta$$

- $\sin(180^\circ+\theta) = -\sin\theta$
- $\cos(180^\circ+\theta) = -\cos\theta$
- $\tan(180^\circ+\theta) = \tan\theta$
- $\cot(180^\circ+\theta) = \cot\theta$
- $\operatorname{cosec}(180^\circ+\theta) = -\operatorname{cosec}\theta$
- $\sec(180^\circ+\theta) = -\sec\theta$

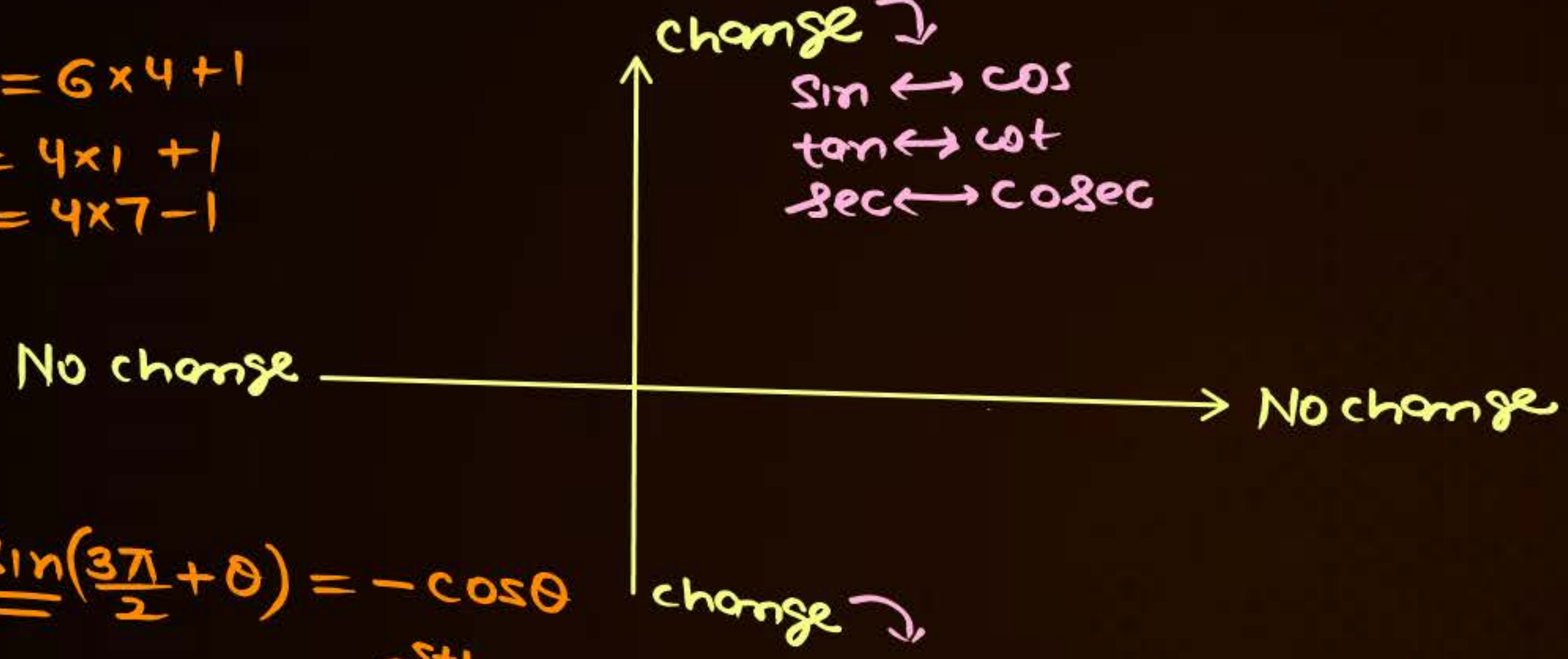
Reduction Formulae

used for finding T-ratios of larger Angles

$25 = 6 \times 4 + 1$

$5 = 4 \times 1 + 1$

$27 = 4 \times 7 - 1$



sin ↔ cos

tan ↔ cot

sec ↔ cosec

sin ⟷ cos

tan ⟷ cot

sec ⟷ cosec.

★ $\underline{\underline{\sin}}\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta$

★ $\underline{\underline{\cos}}\left(2025\frac{\pi}{2}+\overset{5th}{\theta}\right) = -\sin\theta$

$(4m+1)\frac{\pi}{2}$ type

★ $\tan(5\pi/2-\theta) = \cot\theta$

★ $\cot\left(2027\frac{\pi}{2}+\theta\right) = -\tan\theta$

$\sin(5000\pi+\theta) = +\sin\theta$

$\cos(2521\pi-\theta) = -\cos\theta$

★ $\underline{\underline{\sin}}(\pi/2+\overset{5th}{\theta}) = +\cos\theta$

★ $\underline{\underline{\tan}}(\pi/2+\theta) = -\cot\theta$

★ $\sin(\pi-\overset{5th}{\theta}) = +\sin\theta$

★ $\sin(2\pi-\theta) = -\sin\theta$

Reduction Formulae

x	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$
$\sin x$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$
$\cos x$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos\alpha$
$\tan x$	$\cot\alpha$	$-\cot\alpha$	$-\tan\alpha$	$\tan\alpha$	$\cot\alpha$	$-\cot\alpha$	$-\tan\alpha$
$\cot x$	$\tan\alpha$	$-\tan\alpha$	$-\cot\alpha$	$\cot\alpha$	$\tan\alpha$	$-\tan\alpha$	$-\cot\alpha$

Reduction Formulae for −θ

$$\cos(-\theta) = \cos(2\pi + (-\theta))$$
$$= \cos(2\pi - \theta)$$
$$= \cos\theta$$

$$\sin(-\theta) = \sin(2\pi + (-\theta))$$
$$= \sin(2\pi - \theta)$$
$$= -\sin\theta$$

$$\sin(2\pi + \theta) = \sin\theta$$
$$\cos(2\pi + \theta) = \cos\theta$$
$$\tan(2\pi + \theta) = \tan\theta$$
$$\vdots$$

- ★ $\sin(-\theta) = -\sin\theta$
- ★ $\cos(-\theta) = \cos\theta$
- ★ $\tan(-\theta) = -\tan\theta$
- ★ $\cot(-\theta) = -\cot\theta$
- ★ $\mathrm{cosec}(-\theta) = -\mathrm{cosec}\,\theta$
- ★ $\sec(-\theta) = \sec\theta$

QUESTION

$$\begin{array}{r} 8 \\ 3\overline{)25} \\ \underline{24} \\ 1 \end{array}$$

Find the value of

(i) $\sin\left(\frac{25\pi}{3}\right)$ (ii) $\cos\left(\frac{41\pi}{4}\right)$ (iii) $\tan\left(\frac{-16\pi}{3}\right)$ (iv) $\cot\left(\frac{29\pi}{4}\right)$

$\sin\left(8\pi+\frac{\pi}{3}\right) = +\sin\pi/3 = \frac{\sqrt{3}}{2}$

$\cos\left(10\pi+\frac{\pi}{4}\right)$

$\shortparallel$

$\cos\pi/4 = \frac{1}{\sqrt{2}}$

$-\tan\frac{16\pi}{3}$

$= -\tan\left(5\pi+\frac{\pi}{3}\right)$

$= -\tan\pi/3 = -\sqrt{3}$

$\cot\left(7\pi+\frac{\pi}{4}\right)$

$\shortparallel$

$\cot\pi/4$

$\shortparallel$

1

QUESTION

360°

Find the value of

$\sin(360° + 45°) = \sin 45° = \frac{1}{\sqrt{2}}$

(i) sin(405°)

(ii) sec (–1470°)

(iii) tan (-300°)

(iv) cot (585°)

TAH 1A

$\sec(1470°) = \sec(\underset{\substack{\downarrow \\ 2\pi}}{4 \times 360°} + 30°)$

$= \sec(8\pi + 30°)$

$= \sec 30° = \frac{2}{\sqrt{3}}.$

✓

$$\begin{array}{r|l} & 4 \\ \hline 360° & 1470° \\ & 1440 \\ \hline & 30° \end{array}$$

QUESTION

Express in terms of ratios of a positive angle, which is less than 45°, the quantities.

(i) $\cos 1410^\circ$

(ii) $\cot(-1054^\circ)$

(iii) $\operatorname{cosec}(-756^\circ)$

TAH 1B

$$360^\circ \,\overline{)\,1410\,}\ \ 3$$

$$\begin{array}{r} 1410 \\ \underline{1080} \\ \underline{330} \end{array}$$

$\cos(360^\circ \times 3 + 330^\circ)$

$=$

$\cos(6\pi + 330^\circ)$

$=$

$\cos 330^\circ$

$\cos(360^\circ - 30^\circ)$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\cos(4 \times 360^\circ - 30^\circ)$

$\cos(8\pi - 30^\circ)$

$= \cos 30^\circ = \frac{\sqrt{3}}{2}$

QUESTION

TAH 2

Fill in the rest of the blanks yourself.

∠A	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1												
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0												
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND												
cosec A	ND	2	$\sqrt{2}$	$2\sqrt{3}$	1												
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND												
cot A	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0												

$\theta \in (0, \pi/4)$

- $\sin\theta < \cos\theta$
- $\tan\theta < \cot\theta$

$\theta \in (\pi/4, \pi/2)$

- $\cos\theta < \sin\theta$
- $\cot\theta < \tan\theta$

QUESTION

What sign has sin A – cos A for the following values of A?

(i) 215°

(ii) –634°

$$\begin{array}{r} 720 \\ 634 \\ \hline 86 \end{array}$$

+ ✓

$\sin 215^\circ - \cos 215^\circ = \sin(180^\circ + 35^\circ) - \cos(180^\circ + 35^\circ)$

$= -\sin 35^\circ + \cos 35^\circ$

$= \cos 35^\circ - \sin 35^\circ = +ve$

$\cos 35^\circ > \sin 35^\circ$

$\sin(-634^\circ) - \cos(-634^\circ)$

$= -\sin 634^\circ - \cos 634^\circ$

$= -\sin(\underset{\overset{\downarrow}{4\pi}}{720^\circ} - 86^\circ) - \cos(\underset{\overset{\downarrow}{4\pi}}{720^\circ} - 86^\circ)$

$= -\sin(4\pi - 86^\circ) - \cos(4\pi - 86^\circ)$

$= -(-\sin 86^\circ) - (+\cos 86^\circ)$

$= \sin 86^\circ - \cos 86^\circ = +ve$

QUESTION

TAH 3

What sign has sin A + cos A for the following values of A?

(i) 278°

(ii) –1125°

QUESTION

TAH 4A

Find the value of

(i) $\operatorname{cosec}(-750^\circ)$

(ii) $\cos(-2220^\circ)$

QUESTION

TAH 4B

$\sin 420^\circ \cos 390^\circ + \cos(-300^\circ) \sin(-330^\circ)$ equals

- **A** 0
- **B** 1
- **C** -1
- **D** 2

QUESTION

TAH 4C

Find the value of

(i) $\sec\left(-\frac{19\pi}{3}\right)$

(ii) $\operatorname{cosec}\left(-\frac{33\pi}{4}\right)$

ASNC (Nayi Soch)

$\sin(\pi - \theta) = \sin\theta$ $\quad$ $\tan(\pi - \theta) = -\tan\theta$

$\cos(\pi - \theta) = -\cos\theta$ $\quad$ $\cot(\pi - \theta) = -\cot\theta$

$\text{cosec}(\pi - \theta) = \text{cosec}\,\theta$ $\quad$ $\sec(\pi - \theta) = -\sec\theta$

$$\sin(180^\circ - \overset{5^{th}}{5000^\circ})$$

$$\| $$

$$+\sin 5000$$

$=$ 

$\alpha + \beta = \pi$ i.e α, β are supplementory

★ $\sin\alpha = \sin\beta$, $\text{cosec}\,\alpha = \text{cosec}\,\beta$

★ $\cos\alpha = -\cos\beta$ ★ $\cot\alpha = -\cot\beta$

★ $\tan\alpha = -\tan\beta$ ★ $\sec\alpha = -\sec\beta$

b'coz $\sin\alpha = \sin(\pi - \beta) = \sin\beta$

ASNC (Nayi Soch)

if $\alpha+\beta=\frac{\pi}{2}$

- $\sin\alpha=\cos\beta$
- $\tan\alpha=\cot\beta$
- $\sec\alpha=\operatorname{cosec}\beta$

QUESTION

$$\frac{11\pi}{17} + \frac{6\pi}{17} = \pi$$

Evaluate :

(i) $$\frac{\overset{\checkmark}{\sin\frac{11\pi}{17}}\,\overset{-1}{\cos\frac{10\pi}{13}}\,\overset{-1}{\tan\frac{\pi}{7}}}{\cos\frac{3\pi}{13}\,\overset{\checkmark}{\sin\frac{6\pi}{17}}\,\tan\frac{6\pi}{7}} = 1$$

(ii) $$\frac{\overset{1}{\sin\frac{\pi}{5}}\,\overset{-1}{\cancel{\cos\frac{7\pi}{9}}}\,\overset{-1}{\cancel{\tan\frac{6\pi}{11}}}}{\cancel{\cos\frac{2\pi}{9}}\,\sin\frac{4\pi}{5}\,\cancel{\tan\frac{5\pi}{11}}} = 1$$

$$\frac{10\pi}{13} + \frac{3\pi}{13} = \pi \qquad \pi/7 + \frac{6\pi}{7} = \pi$$

$$\cos\frac{10\pi}{13} = -\cos\frac{3\pi}{13}$$

QUESTION

Evaluate :

(iii) $$\frac{\overset{1}{\sin\frac{\pi}{7}}\ \overset{1}{\cos\frac{5\pi}{11}}\ \overset{-1}{\tan\frac{3\pi}{7}}}{\cos\frac{5\pi}{14}\ \sin\frac{\pi}{22}\ \tan\frac{4\pi}{7}} = -1$$

$$\frac{\pi}{7}+\frac{5\pi}{14}=\frac{7\pi}{14}=\pi/2$$

$$\frac{5\pi}{11}+\frac{\pi}{22}=\frac{11\pi}{22}=\pi/2$$

$$\frac{3\pi}{7}+\frac{4\pi}{7}=\pi$$

QUESTION

$\tan\frac{\pi}{11}+\tan\frac{2\pi}{11}+\tan\frac{4\pi}{11}+\tan\frac{7\pi}{11}+\tan\frac{9\pi}{11}+\tan\frac{10\pi}{11}$ equals

$\frac{\pi}{11}+\frac{10\pi}{11}=\pi$

(A) 1

(B) -1

[✓ (C) 0]

(D) 2

QUESTION

$\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9}$ equals

$$\cos^2\frac{\pi}{9} + \cos^2\frac{\pi}{18}$$

$$\frac{\pi}{18} + \frac{4\pi}{9} = \frac{9\pi}{18} = \frac{\pi}{2}$$

$$\frac{\pi}{9} + \frac{7\pi}{18} = \frac{2\pi + 7\pi}{18} = \frac{\pi}{2}$$

- A 1
- B - 1
- C 0
- [✓ D] 2

QUESTION [JEE Mains 2024 (8 April)]

If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to

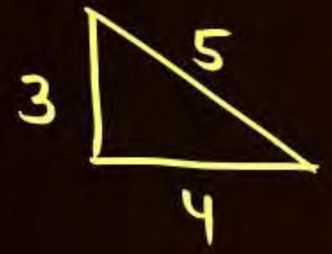
[✓ A] 109

(B) 108

(C) 19

(D) 18

$\sin x = -\frac{3}{5}$ — sign bhool jao



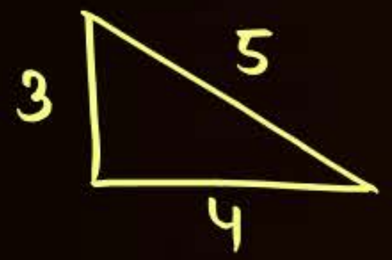
$\tan x = +\frac{3}{4}$ — $\tan^2 x = \frac{9}{16}$

$\cos x = -\frac{4}{5}$ — $-\cos x = \frac{4}{5}$

$\tan^2 x - \cos x = \frac{9}{16} + \frac{4}{5} = \frac{45+64}{80}$

$80(\tan^2 x - \cos x) = 109.$

$\cos x = -\frac{4}{5}$ — $x \in$ quad II or quad III



$\tan x = -\frac{3}{4}$ or $\tan x = \frac{3}{4}$

$\sin x = \frac{3}{5}$ $\sin x = -\frac{3}{5}$

↓   ↓

$x \in$ quad II $x \in$ quad III

$\cos\theta = -\frac{4}{5} = \frac{x \text{ coor}}{r}$.

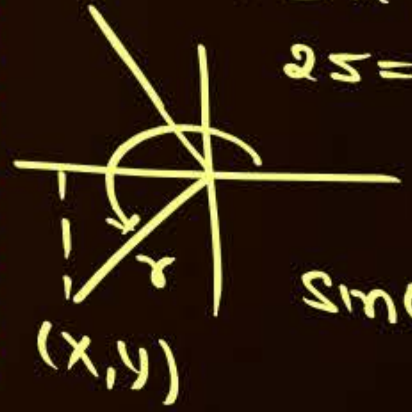
$x = -4$ $\theta \in \text{II}, \text{III}$

$r = 5$

$r^2 = x^2 + y^2$

$25 = 16 + y^2$

$y = \pm 3$



$\sin\theta = \frac{y}{r} = \frac{3}{5}$ or $-\frac{3}{5}$.

$\tan\theta = \frac{y}{x} = \frac{3}{-4}, \frac{-3}{-4}$

quad II   quad III

Sabse Important Baat

Sabhi Class illustrations Retry Karni hai

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

KTK 1

$$\frac{\tan(90°-\theta)\sec(180°-\theta)\sin(-\theta)}{\sin(180°+\theta)\cot(360°-\theta)\operatorname{cosc}(90°-\theta)} \text{ equals}$$

A 1

B –1

C 0

D 2

Ans. A

QUESTION

KTK 2

If ABCD is a quadrilateral, then show that

(a) $\cos\frac{B+C}{2}+\cos\frac{A+D}{2}=0$

(b) $\tan\frac{A+C}{4}=\cot\frac{B+D}{4}$

QUESTION

KTK 3

The value of $\cos^2\frac{\pi}{16}+\cos^2\frac{3\pi}{16}+\cos^2\frac{5\pi}{16}+\cos^2\frac{7\pi}{16}$ is

(A) 2

(B) 1

(C) 0

(D) None of these

Ans. A

QUESTION [JEE Mains 2019]

KTK 4

Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for $k = 1,2,3, \ldots..$

Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to

- **A** $\frac{5}{12}$
- **B** $\frac{-1}{12}$
- **C** $\frac{1}{4}$
- **D** $\frac{1}{12}$

Ans. D

QUESTION

KTK 5

If θ is the each interior angle of a regular dodecagon then the value of $\sin\theta + \cos\theta + \tan\theta + \cot\theta + \sec\theta + \text{cosec}\,\theta$, is

- **A** positive
- **B** negative and less than (–1)
- **C** zero
- **D** negative and less than (–2)

Ans. B

QUESTION

KTK 6

The value of $\sum_{n=0}^{1947} \frac{1}{2^n + \sqrt{2^{1947}}}$ is equal to

Ⓐ $\frac{487}{\sqrt{2^{1945}}}$

Ⓑ $\frac{1946}{\sqrt{2^{1947}}}$

Ⓒ $\frac{1947}{\sqrt{2^{1947}}}$

Ⓓ $\frac{1948}{\sqrt{2^{1947}}}$

Ans. A

QUESTION

KTK 7

If a, b, c $\in R^+$ then the minimum value of $a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2)$ is equal to

(A) abc

(B) 2abc

(C) 3abc

(D) 6abc

Ans. D

QUESTION

KTK 8

The line $x + y = 1$ meets x-axis at A and y-axis at B, P is the mid-point of AB;

P is the foot of die perpendicular from P to OA;

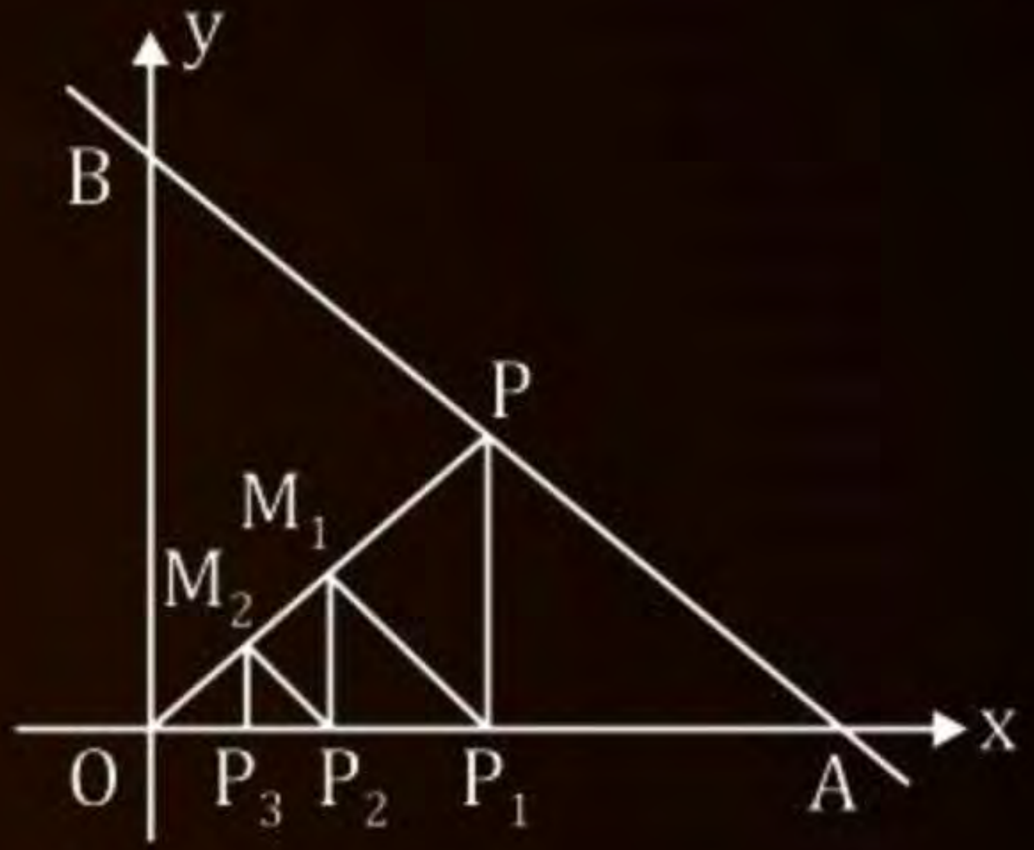
M_1 is that of P_1 from OP;

P_2 is that of M_1 from OA;

M_3 is that of P_2 from OP;

P_3 is that of M_2. from OA; and so on.

If P_n denotes the n^{th} foot of the perpendicular on OA; then OP is



Ⓐ $\left(\frac{1}{2}\right)^{n-1}$

Ⓑ $\left(\frac{1}{2}\right)^{n}$

Ⓒ $\left(\frac{1}{2}\right)^{n+1}$

Ⓓ None of these

Ans. B

Homework From Module

Sequence Series:

Prarambh (Topicwise) : Complete

Prabal (JEE Main Level) : Complete

Parikshit (JEE Advanced Level) : Complete

Revision Practice Problems (RPP)

QUESTION

RPP 01

If a, b are the roots of $x^2 + px + 1 = 0$ and c, d are the roots of $x^2 + qx + 1 = 0$. Then find the value of $(a - c)(b - c)\,(a + d)(b + d)/(q^2 - p^2)$.

Ans. 1

QUESTION

RPP 02

The least prime integral value of '2a' such that the roots α, β of the equation $2x^2 + 6x + a = 0$ satisfy the inequality $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} < 2$ is

Ans. 11

Solution to Previous TAH

QUESTION [JEE Mains 2021 (Aug)]

Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3 \cdot (n-3) + \cdots + (n-1) \cdot 1, n \geq 4.$

The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ is equal to:

- (A) $\frac{e-1}{3}$
- (B) $\frac{e-2}{6}$
- (C) $\frac{e}{3}$
- (D) $\frac{e}{6}$

Tah-01: Let $S_n = 1\cdot(n-1) + 2\cdot(n-2) + 3\cdot(n-3) + \cdots + (n-1)\cdot 1$, $n \ge 4$.

The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ is equal.

$\Rightarrow T_r = r(n-r) = rn - r^2.$

$\Rightarrow S_n = \sum_{r=1}^{n} rn - \sum_{r=1}^{n} r^2.$

$= n\cdot\frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}.$

$= \frac{n(n+1)}{2}\left[n - \frac{(2n+1)}{3}\right]$

$= \frac{n(n+1)}{2}\left(\frac{n-1}{3}\right) = \frac{n(n+1)(n-1)}{6}.$

$= \frac{n^3-n}{6}.$

$\sum_{n=4}^{\infty} \frac{2S_n}{n!} - \frac{1}{(n-2)!}.$

$\Rightarrow \sum_{n=4}^{\infty} \frac{n^3-n}{3(n)!} - \frac{1}{(n-2)!} \Rightarrow \sum_{n=4}^{\infty} \frac{n^2-1^2}{3(n-1)!} - \frac{1}{(n-2)!}$

$\Rightarrow \sum_{n=4}^{\infty} \frac{(n+1)}{3(n-2)!} - \frac{1}{(n-2)!} \Rightarrow \sum_{n=4}^{\infty} \frac{(n-2)}{3(n-2)!}$

$\Rightarrow \sum_{n=4}^{\infty} \frac{1}{3(n-3)!}$

$\Rightarrow \frac{1}{3}\left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \infty\right)$

$\Rightarrow \frac{1}{3}(e-1) \Rightarrow \frac{e-1}{3}$ Ans.

krish

TAH01

Soln: $S_n = 1.(n-1) + 2(n-2) + 3(n-3) + \ldots (n-1).1 \geqslant 4$

$T_K = K(n-K)$

$= nK - K^2$

$S_n = \sum_{K=1}^{n} T_K = n\sum_{K=1}^{n} K - \sum_{K=1}^{n} K^2$

$S_n = \frac{n(n)(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow S_n = \frac{n(n+1)\{3n - 2n-1\}}{6}$

$\Rightarrow S_n = \frac{n(n+1)(n-1)}{6}$

RASIDUL

$\sum_{n=4}^{\infty}\left\{\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right\}$

$= \sum_{n=4}^{\infty}\left\{\frac{2n(n+1)(n-1)}{6\,n!} - \frac{1}{(n-2)!}\right\}$

$= \sum_{n=4}^{\infty}\left\{\frac{2(n+1)}{6(n-2)!} - \frac{1}{(n-2)!}\right\}$

$= \sum_{n=4}^{\infty}\left\{\frac{n+1-3}{3(n-2)!}\right\}.$

$= \sum_{n=4}^{\infty}\left\{\frac{1(n-2)}{3(n-2)!}\right\}$

$= \frac{1}{3}\sum_{n=4}^{\infty} \frac{1}{(n-3)!}$

$= \frac{1}{3}\left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \infty\right)$

$= \frac{1}{3}\left(\left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \infty\right) - 1\right)$

$= \frac{e-1}{3}.$

Ans: Ⓐ $\frac{e-1}{3}$

TAH-01 $S_n = 1\cdot(n-1) + 2\cdot(n-2) + 3\cdot(n-3) + \cdots (n-1)\cdot 1$

$$\Rightarrow S_n = 1(n-1) + 2(n-2) + 3\cdot(n-3) + \cdots + (n-1)(n-(n-1))$$

$$\Rightarrow S_n = [n + 2n + 3n + \cdots + (n-1)n] - [1 + 4 + 9 + 16 + \cdots + (n-1)^2]$$

$$\Rightarrow S_n = n\left[\frac{(n-1)n}{2}\right] - \frac{(n-1)n(2n-1)}{6}$$

Kritisha

$$\Rightarrow 2S_n = n^2(n-1) - \frac{n(n-1)(2n-1)}{3}$$

$$\Rightarrow \sum_{n=4}^{\infty}\left(\frac{n^2(n-1) - \frac{n(n-1)(2n-1)}{3}}{n!}\right) - \sum_{n=4}^{\infty}\frac{1}{(n-2)!}$$

$$\Rightarrow \sum_{n=4}^{\infty}\frac{n(n-1)}{(n-1)!} - \sum_{n=4}^{\infty}\frac{n(n-1)(2n-1)}{3n(n-1)!} - \sum_{n=4}^{\infty}\frac{1}{(n-2)!}$$

$$\Rightarrow \sum_{n=4}^{\infty}\frac{n}{(n-2)!} - \frac{1}{3}\sum_{n=4}^{\infty}\frac{2n-1}{(n-2)!} - \sum_{n=4}^{\infty}\frac{1}{(n-2)!}$$

$$\Rightarrow \sum_{n=4}^{\infty}\frac{n-2+2}{(n-2)!} - \frac{1}{3}\sum_{n=4}^{\infty}\frac{2n-4+3}{(n-2)!} - \sum_{n=4}^{\infty}\frac{1}{(n-2)!}$$

$$\Rightarrow \sum_{n=4}^{\infty}\frac{1}{(n-3)!} + \cancel{2\sum_{n=4}^{\infty}\frac{1}{(n-2)!}} - \frac{2}{3}\sum_{n=4}^{\infty}\frac{1}{(n-3)!} - \cancel{\sum_{n=4}^{\infty}\frac{1}{(n-2)!}}$$

$$- \cancel{\sum_{n=4}^{\infty}\frac{1}{(n-2)!}}$$

$$\Rightarrow \frac{1}{3}\sum_{n=4}^{\infty}\frac{1}{(n-3)!} = \frac{1}{3}\left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\right)$$

$$= \frac{1}{3}(e-1)$$

$$= \frac{e-1}{3} \text{ (A) Ans.}$$

QUESTION

Let $f(x) = (a^2 + b^2 - 4a - 6b + 13)(2x^2 - 4x + 5)$, $a, b, x \in R$ such that $f(0) = f(1) = f(2)$. If $a, A_1, A_2, \ldots \ldots A_{10}, b$ is an arithmetic progression and $a, H_1, H_2, \ldots \ldots, H_{10}, b$ is harmonic progression then the value of

$$\frac{1}{10}\left(\sum_{i=4}^{8} A_i H_{11-i}\right) \text{ is equal to}$$

Ans. 3

Tah-02 .

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$$f(x) = (a^2+b^2-4a-6b+13)(2x^2-4x+5)$$

$$f(0) = f(1)$$

$$(a^2+b^2-4a-6b+13)(5) = (a^2+b^2-4a-6b+13)(3)$$

$$5a^2+5b^2-20a-30b+65 = 3a^2+3b^2-12a-18b+39$$

$$2a^2+2b^2-8a-12b+26=0$$

$$a^2+b^2-4a-6b+13=0$$

$$(a^2-4a+4)+(b^2-6b+9)=0$$

$$\underset{\geq 0}{(a-2)^2}+\underset{\geq 0}{(b-3)^2}=0$$

$$a=2 \quad \& \quad b=3.$$

$a, A_1, A_2, \text{-------}, A_{10}, b \}AP$

$a, H_1, H_2, \text{--------}, H_{10}, b \}HP$

So, $ab = A_1H_{10} = A_2H_9 = \text{.......} = A_{10}H_1$.

$$\frac{1}{10}\sum_{i=4}^{8} A_iH_{11-i} = \frac{1}{10}\left(\underbrace{A_4H_7}_{ab}+\underbrace{A_5H_6}_{ab}+\underbrace{A_6H_5}_{ab}+\underbrace{A_7H_4}_{ab}+\underbrace{A_8H_3}_{ab}\right)$$

$$= \frac{ab\times 5}{10}$$

$$= \frac{2(3)(5)}{10} = 3.$$

Solution to Previous KTKs

QUESTION

KTK 01

The dimensions of a Cuboid are $a > b > c$. The volume = 216 and the total outer surface area = 252. If a, b, c are in G.P., then c =

- A 3
- B 1
- C 5
- D 2

Ans. A

$\boxed{\text{KTK-1}}$ $\quad a > b > c \longrightarrow a, b, c$ are in G.P

$$\text{vol.} \Rightarrow \boxed{abc = 216}$$

$$2(ab + bc + ca) = 252$$

$$\boxed{ab + bc + ca = 126}$$

Let $a = \frac{p}{r}$

$b = p$

$c = pr$

$$abc = p^3 = 216$$

$$\boxed{p = 6}$$

Kritisha (W.B)

$$ab + bc + ca = \frac{p^2}{r} + p^2 r + p^2$$

$$= p^2\left(1 + r + \frac{1}{r}\right) = 126$$

$$1 + r + \frac{1}{r} = \frac{\cancel{126}\ \cancel{21}\ 7}{\cancel{36}\ \cancel{6}\ 2}$$

$$r + \frac{1}{r} = \frac{7-2}{2} = \frac{5}{2}$$

$$r + \frac{1}{r} = \frac{5}{2}$$

$$\Rightarrow r^2 + 1 - \frac{5}{2}r = 0 \Rightarrow \boxed{2r^2 - 5r + 2 = 0}$$

$$2r^2 - 5r + 2 = 0$$

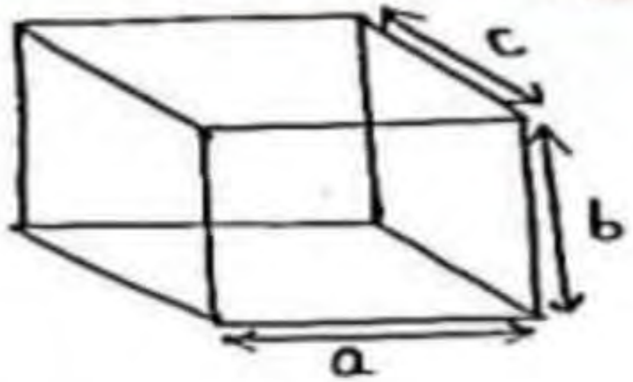
$$2r^2 - 4r - r + 2 = 0$$

$$\boxed{(2r-1)(r-2) = 0} \longrightarrow r = 1/2 \text{ as, } a > b > c$$

Thus, $c = pr = \frac{6}{2} = 3$ (A)

Ans.

KTK-01



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⇒ Volume = 216

$a \times b \times c = 216$

$b(b^2) = 216$

$\boxed{b = 6}$

$\left.a, b, c\right\}$ GP

$b^2 = ac$

$\boxed{ac = 36}$

⇒ Total outer S.A = 252

$2(ab+bc+ca) = 252$

$(ab+bc+b^2) = 126$

$(6a+6c+36) = 126$

$6(a+c) = 90$

$a+c = 15$

$a + \frac{36}{a} = 15$

$a^2 + 36 = 15a$

$a^2 - 12a - 3a + 36 = 0$

$(a-12)(a-3) = 0$

$\boxed{\begin{matrix} a = 12 \\ c = 3 \end{matrix}}$ $\begin{matrix} \& \ a = 3 \\ \& \ c = 12 \end{matrix}$ $\left(\text{But } \begin{matrix} a > b > c \\ \Rightarrow a > c \end{matrix} \right)$

Hence, $\boxed{c = 3}$

KTK-01

$a > b > c$,

Volume of a cuboid $= abc = 216$

outer surface area of a cuboid
$= 2(ab+bc+ca) = 252$

$\Rightarrow ab+bc+ca = 126$

$a, b, c \longrightarrow$ in G.P

Let C.R $= r$

$\Rightarrow 0 < r < 1$ because, $a > b > c$ (given)

[$r < 0$ not possible because sides are never negative]

$b = ar$
$c = ar^2$

$\therefore abc = 216$

$a(ar)(ar^2) = 216$

$a^3r^3 = 216$

$\boxed{ar = 6} \longrightarrow a = 6/r$

$ab+bc+ca = 126$

$a(ar) + ar(ar^2) + ar^2 \cdot a = 126$

$a^2(r+r^3+r^2) = 126$

$\frac{36}{r^2}(r^3+r^2+r) = 126$

$\frac{\cancel{36}^{6\,2}}{r}(r^2+r+1) = \cancel{126}^{21\ 7}$

$2r^2+2r+2 = 7r$

$2r^2-5r+2 = 0$

$2r^2-4r-r+2 = 0$

$(2r-1)(r-2) = 0$

$r = 1/2$, $r = 2$ × rejected [$0 < r < 1$]

$\therefore a = 6/r = 12$ $\quad \therefore c = ar^2 = 12 \times 1/4 = 3$ Ans Ⓐ

★ **KTK-1.** The dimension of a cuboid are $a > b > c$. The volume $= 216$. and the total outer surface area $= 252$. If a, b, c are in G.P, then $c =$

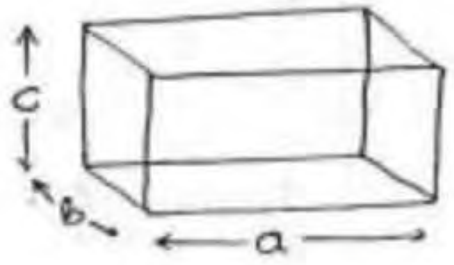
\# volume $= 216$

$abc = 216$ — ①

\# T.S.A $= 252$

$\cancel{2}(ab+bc+ca) = 2\cancel{5}2$

$ab+bc+ca = 126$ — ②



Cuboid.

\# If a, b, c are in G.P.

then: $\boxed{b^2 = ac} \Rightarrow a = \frac{b^2}{c} = \frac{36}{c}$.

\# Put in eq ①

$b^3 = 216$

$\boxed{b = 6}$

\# $6a + 6c + 36 = 126$

$6(a+c) = 90$

$\boxed{a + c = 15}$

$\Rightarrow \frac{36}{c} + c = 15$

$\Rightarrow 36 + c^2 = 15c$

$\Rightarrow c^2 - 15c + 36 = 0$

$\Rightarrow (c-12)(c-3) = 0$

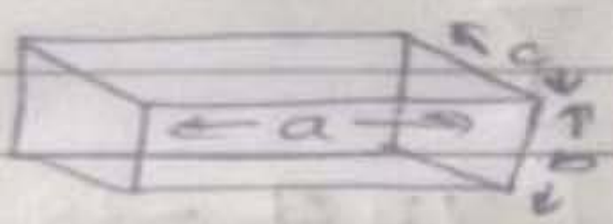
$\Rightarrow c = 12$ ×, $c = 3$ ✓

$\Rightarrow$ bcz given: $a > b > c$ in Question & $\boxed{b = 6}$ then $\boxed{c = 3}$ only possible.

— × —

KTK01:

Solⁿ:-



$\therefore$ $a, b, c \Rightarrow GP$

$b = ar, \quad c = ar^2$

Then, $a \cdot b \cdot c = 216$

$a \cdot ar \cdot ar^2 = 216$

$a^3 r^3 = 216$

$\boxed{ar = 6}$

$\therefore$ $ab + bc + ca = 126$

$a \cdot ar + ar \cdot ar^2 + ar^2 \cdot a = 126$

$a^2 r + a^2 r^3 + a^2 r^2 = 126$

$a^2 r + a^2 r^3 = 126 - 36$

$\boxed{a^2 r + a^2 r^3 = 90}$

Volume $= 216$

$a \cdot b \cdot c = 216$ —— (i)

Total outer surface area $= 252$

$2(ab + bc + ca) = 252$

$ab + bc + ca = 126$ —— (ii)

Now, $\dfrac{a^2 r + a^2 r^3}{ar} = \dfrac{90}{6}$

$a + ar^2 = 15$

$a(1 + r^2) = 15 \Rightarrow \frac{6}{r}(1 + r^2) = 15$

$6 + 6r^2 = 15r \Rightarrow 2r^2 - 5r + 2 = 0$

$(2r - 1)(r - 2) \Rightarrow r = \frac{1}{2}, r \neq 2 \therefore 0 < r < 1$

$\therefore$ a, b, c are GP

$b^2 = ac$

$c = \dfrac{b^2}{a} = \dfrac{a^2 r^2}{a} = \dfrac{36 \times r}{6} = 6 \times \frac{1}{2}$

Spaint $= 3$

$\boxed{c = 3}$

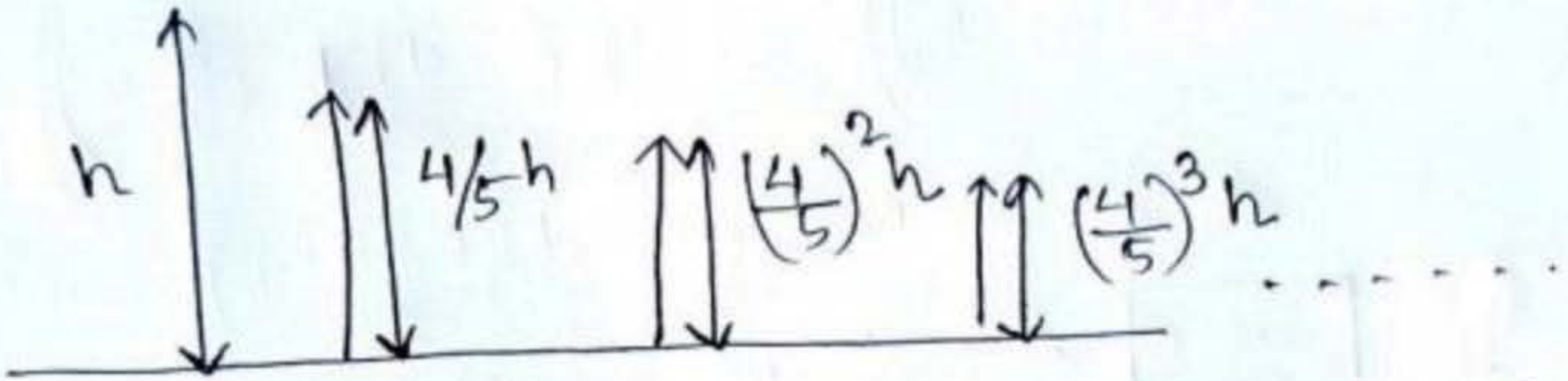
QUESTION

KTK 02

A ball falls from a height of 100 m on a floor. If in each rebound, it describes $(4/5)^{\text{th}}$ height of the previous falling height, then the total distance travelled by the ball before it comes to rest is?

Ans. 900 m

KTK-02



total distance travelled $\left(h + 2\left(\frac{4}{5}h + \left(\frac{4}{5}\right)^2 h + \left(\frac{4}{5}\right)^3 + \cdots\cdots\right)\right)$

$$= h + 2h\left(\frac{\frac{4}{5}}{1-\frac{4}{5}}\right)$$

$$= h + 2h\left(\frac{4}{1}\right)$$

$$= 9h = \boxed{900\,m}\ (\text{Ans})$$

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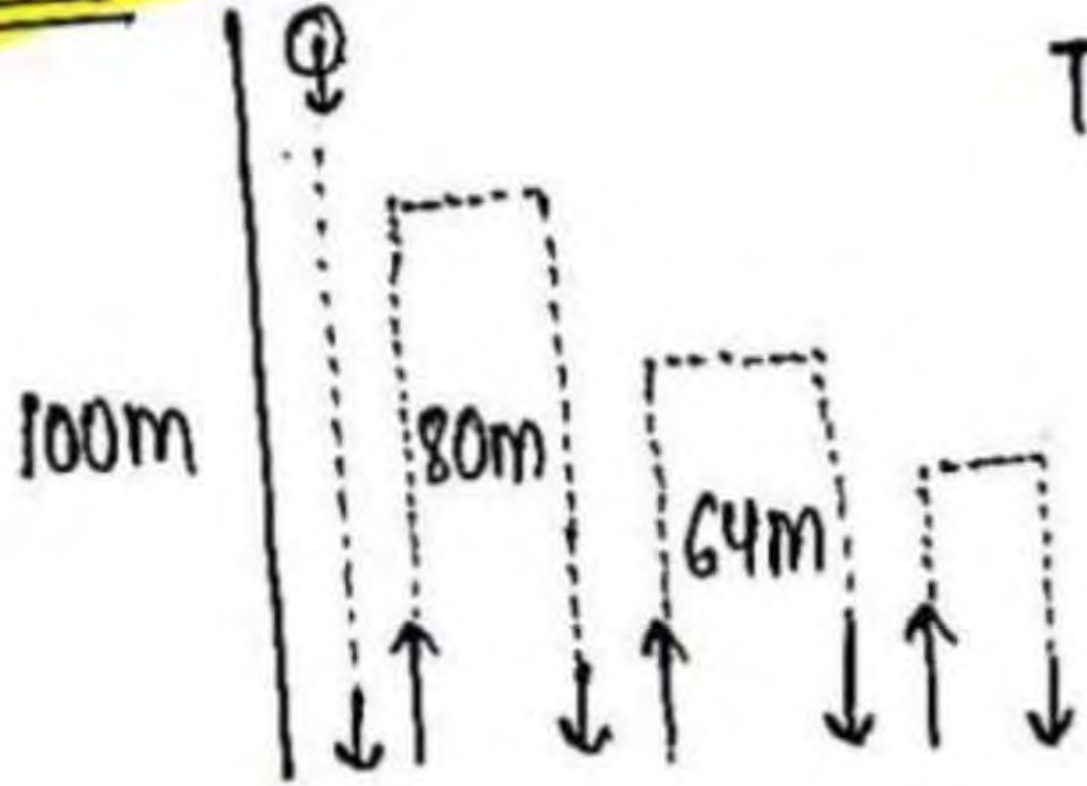
KIK-02



Total dist. travelled

$$= 100 + 2\left(\frac{4}{5}(100) + \frac{4}{5}\left(\frac{4}{5}\right)(100) + \ldots\ldots\infty\right)$$

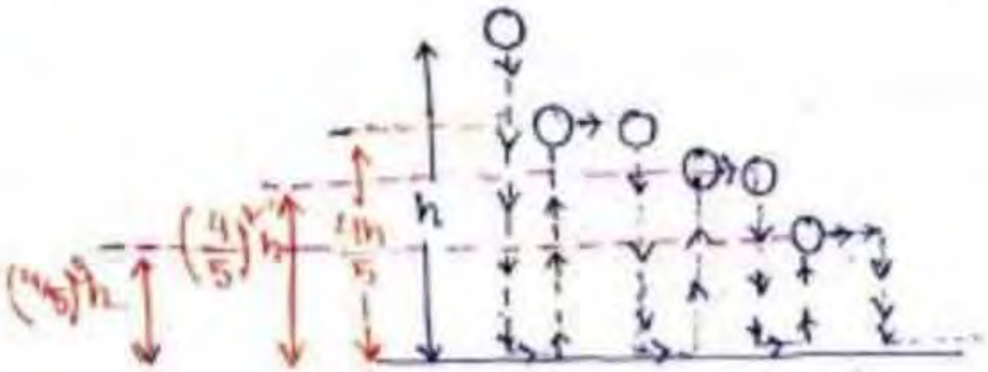
$$= 100 + 2(100)\left(\frac{4}{5} + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + \ldots\ldots\infty\right)$$

$$= 100 + 200\left(\frac{4/5}{1 - 4/5}\right)$$

$$= 100 + 200(4)$$

$$= (100 + 800)\text{m} = 900\text{ m}.$$

KTK-02



Total distance travelled by the ball

$$= h + \left(\frac{4h}{5} + \frac{4h}{5}\right) + \left(\left(\frac{4}{5}\right)^2 h + \left(\frac{4}{5}\right)^2 h\right) + \left(\left(\frac{4}{5}\right)^3 h + \left(\frac{4}{5}\right)^3 h\right) + \ldots\ldots \infty$$

$$= h + 2\left[\frac{4h}{5} + \left(\frac{4}{5}\right)^2 h + \left(\frac{4}{5}\right)^3 h + \left(\frac{4}{5}\right)^4 h + \ldots \infty\right]$$

$$= h + 2h\left[\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + \left(\frac{4}{5}\right)^4 + \ldots \infty\right]$$

$$= h + 2h\left(\frac{4/5}{1-4/5}\right)$$

$$= h + 2h\,\frac{4/\cancel{5}}{1/\cancel{5}}$$

$$= h + 8h$$

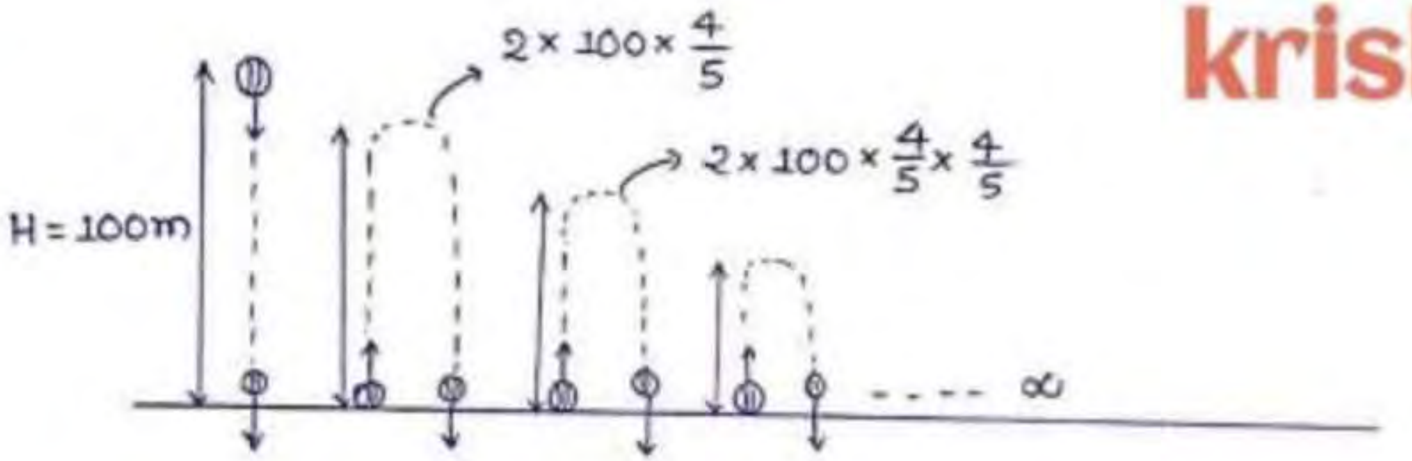
$$= 9h \quad [h = 100\text{m given}]$$

$$= 9 \times 100$$

$$= 900\text{m}$$

Ans

KTK-2. A ball falls from a height 100m on a floor. If in each rebound, it describes $(4/5)^{th}$ height of previous falling height, then the total distance travelled by the ball before it comes to rest is?



$$S = 100 + 2 \times 100 \times \frac{4}{5} + 2 \times 100\left(\frac{4}{5}\right)^2 + \ldots \infty$$

$$= 100 + 2 \times \overset{20}{\cancel{100}} \times \frac{4}{\cancel{5}}\left[1 + \frac{4}{5} + \left(\frac{4}{5}\right)^2 + \ldots \infty\right]$$

$$= 100 + 160\left[\frac{1}{1-4/5}\right]$$

$$= 100 + 160 \times 5$$

$$= 100 + 800 = 900\text{ m} \quad \text{Ans.}$$

QUESTION [JEE Mains 2020 (Jan)]

KTK 03

The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}}$ ...... to ∞ is equal to

(A) $2^{\frac{1}{4}}$

(B) $2^{\frac{1}{2}}$

(C) 1

(D) 2

Ans. B

KTK-03

The product $2^{1/4}\cdot 4^{1/16}\cdot 8^{1/48}\cdot 16^{\frac{1}{128}}$ ..... to ∞ is equal to

$$\Rightarrow 2^{\frac{1}{4}}\cdot 2^{\frac{2}{16}}\cdot 2^{\frac{3}{48}}\cdot 2^{\frac{4}{128}}\cdots\cdots\infty$$

$$= 2^{\frac{1}{4}}\cdot 2^{\frac{1}{8}}\cdot 2^{\frac{1}{16}}\cdot 2^{\frac{1}{32}}\cdots\cdots$$

$$= 2^{\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\cdots\right)}$$

$$= \boxed{2^{1/2}}\ \text{(B)}$$

Ans.

$$\left[\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\cdots = \frac{\frac{1}{4}}{1-\frac{1}{2}} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}\right]$$

KTK-03.

Lucky kumari

$$2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdots\cdots \infty$$

$$\Rightarrow 2^{1/4} \cdot 2^{\frac{2}{16}} \cdot 2^{\frac{3}{48}} \cdot 2^{\frac{4}{128}} \cdots\cdots \infty$$

$$\Rightarrow 2^{\frac{1}{4}} \cdot 2^{\frac{1}{8}} \cdot 2^{\frac{1}{16}} \cdot 2^{\frac{1}{32}} \cdots\cdots \infty$$

$$\Rightarrow 2^{\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots \infty\right)}$$

$$\Rightarrow 2^{\left(\frac{\frac{1}{4}}{1-1/2}\right)} = 2^{\left(\frac{1}{4}\right)\left(\frac{2}{1}\right)} = 2^{1/2}.$$

KTK-3. The product of $2^{1/4} \cdot 4^{1/16} \cdot 8^{1/48} \cdot 16^{1/128} \dots$ to ∞ is equal to :

$$\Rightarrow (2)^{1/4} \cdot (2)^{2 \times 1/16} \cdot (2)^{3 \times 1/48} \cdot (2)^{4 \times 1/128} \cdot \dots \infty.$$

$$\Rightarrow 2^{1/4} \cdot 2^{1/8} \, 2^{1/16} \cdot 2^{1/32} \cdot \dots \infty$$

$$\Rightarrow 2^{\left(1/4 + 1/8 + 1/16 + 1/32 + \dots \infty\right)}.$$

$$\Rightarrow 2^{\left(\frac{1/4}{1-1/2}\right)} \Rightarrow 2^{\left(\frac{1/4}{1/2}\right)} \Rightarrow 2^{1/2}$$ Ans.

QUESTION [JEE Mains 2020 (Jan)]

KTK 04

If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$, for $0 < \theta < \frac{\pi}{4}$, then:

A $x(1 + y) = 1$

B $y(1 - x) = 1$

C $y(1 + x) = 1$

D $x(1 - y) = 1$

Ans. B

$\boxed{\text{KTK-04}}$

$$x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$$

$$= 1 - \tan^2\theta + \tan^4\theta - \tan^6\theta + \tan^8\theta - \tan^{10}\theta + \tan^{12}\theta - \tan^{14}\theta + \cdots\cdots$$

$$= (1-\tan^2\theta) + \tan^4\theta(1-\tan^2\theta) + \tan^8\theta(1-\tan^2\theta) + \tan^{12}\theta(1-\tan^2\theta) + \cdots\cdots$$

$$= (1-\tan^2\theta)(1+\tan^4\theta+\tan^8\theta+\tan^{12}\theta+\cdots\cdots)$$

$$= (1-\tan^2\theta)\left(\frac{1}{1-\tan^4\theta}\right)$$

$$x = \frac{1}{1+\tan^2\theta} = \frac{1}{\sec^2\theta} = \cos^2\theta$$

$$y = \sum_{n=0}^{\infty} \cos^{2n}\theta = 1+\cos^2\theta+\cos^4\theta+\cos^6\theta+\cdots\cdots$$

$$= \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}$$

thus, $x(1+y) = \cos^2\theta\left(1+\frac{1}{\sin^2\theta}\right) \neq 1$

$$y(1-x) = \frac{1}{\sin^2\theta}(1-\cos^2\theta) = \frac{\sin^2\theta}{\sin^2\theta} = 1 \checkmark$$

(B)

Ans.

Kritisha (W.B)

KTK-04.

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$$x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta \quad \text{and } y = \sum_{n=0}^{\infty} \cos^{2n}\theta$$

$$\Rightarrow x = 1 + (-1)\tan^2\theta + (1)\tan^4\theta + (-1)\tan^6\theta + (1)\tan^8\theta + \cdots\cdots \infty$$

$$\Rightarrow x = 1 - \tan^2\theta + \tan^4\theta - \tan^6\theta + \tan^8\theta + \cdots\cdots \infty$$

$$\Rightarrow x = \frac{1}{1-(-\tan^2\theta)} = \frac{1}{1+\tan^2\theta} = \frac{1}{\sec^2\theta} = \cos^2\theta.$$

$$\Rightarrow y = 1 + \cos^2\theta + \cos^4\theta + \cos^6\theta + \cdots\cdots \infty$$

$$\Rightarrow y = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta} = \operatorname{cosec}^2\theta.$$

option Ⓐ. $x(1+y) = \cos^2\theta(1+\operatorname{cosec}^2\theta)$

$$= \frac{\cos^2\theta}{\sin^2\theta}(\sin^2\theta+1)$$

$$= \cot^2\theta(\sin^2\theta+1)$$

$$\neq \text{RHS}.$$

option Ⓑ $y(1-x) = \operatorname{cosec}^2\theta(1-\cos^2\theta)$

$$= \operatorname{cosec}^2\theta(\sin^2\theta)$$

$$= \frac{1}{\sin^2\theta}\cdot\sin^2\theta$$

$$= 1 = \text{RHS}.$$

KTK-4: If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ & $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$,

for $0 < \theta < \frac{\pi}{4}$ then :

$x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$

$$x = 1 - \tan^2\theta + \tan^4\theta \quad \tan^6\theta + \cdots \infty$$

$$= \frac{1}{1-(-\tan^2\theta)} = \frac{1}{1+\tan^2\theta}$$

$$= \frac{1}{\sec^2\theta} = \cos^2\theta .$$

$y = \sum_{n=0}^{\infty} \cos^{2n}\theta$

$$y = 1 + \cos^2\theta + \cos\theta + ----+ \infty$$

$$= \frac{1}{1-\cos^2\theta}$$

$$\Rightarrow \quad y = \frac{1}{1-x} \quad \because \boxed{x = \cos^2\theta}$$

$$\Rightarrow \quad y - xy = 1$$

$$\Rightarrow \quad y(1-x) = 1 \quad \text{Ans.}$$

QUESTION [JEE Mains 2024 (1 Feb)]

KTK 05

Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to:

A 800

B 890

C 790

D 690

Ans. C

$\boxed{\text{KTK-05}}$

$$S_{10} = 5(2a+9d) = 390$$

$$\boxed{2a+9d = 78}$$

Kritisha (W.B)

$$\frac{a_{10}}{a_5} = \frac{15}{7}$$

$$\frac{a+9d}{a+4d} = \frac{15}{7} \Rightarrow \left(\frac{a+9d+a+4d}{5d} = \frac{22}{8}\right)$$

$$\Rightarrow \frac{78+4d}{5d} = \frac{11}{4}$$

$$\Rightarrow 4(78+4d) = 55d$$

$$\Rightarrow 312 = (55-16)d$$

$$\Rightarrow \boxed{d = \frac{312}{39} = 8} \; ; \; 2a = 78-72 \quad \boxed{a=3}$$

$$S_{15} - S_5 = \frac{15}{2}(2a+14d) - \frac{5}{2}(2a+4d)$$

$$= \frac{15}{2}(6+112) - \frac{5}{2}(6+32)$$

$$= \frac{15}{2}(118) - \frac{5}{2}(38)$$

$$= (15\times 59) - (5\times 19)$$

$$= 885 - 95 = 790(c)$$

KTK-05

$$S_{10} = 390$$

$$\Rightarrow \cancel{5}\,\frac{\cancel{10}}{\cancel{2}}\left[2a+9d\right] = \cancel{390}\ 78$$

$$\Rightarrow 2a+9d = 78 \quad \text{———} ①$$

$$\frac{T_{10}}{T_5} = \frac{15}{7}$$

$$\Rightarrow \frac{a+9d}{a+4d} = \frac{15}{7}$$

$$\Rightarrow 7a+63d = 15a+60d$$

$$\Rightarrow 3d = 8a$$

$$\Rightarrow \boxed{2a = \frac{3d}{4}} \quad \text{———} ⑪$$

from ① & ⑪

$$\frac{3d}{4} + 9d = 78$$

$$\cancel{39}d = \overset{2}{\cancel{78}}(4)$$

$$\boxed{d=8}$$

$$\boxed{a=3}$$

Now, $S_{15} - S_5$

$$\frac{15}{2}\left[2a+14d\right] - \frac{5}{2}\left[2a+4d\right]$$

$$\Rightarrow 15(a+7d) - 5(a+2d)$$

$$\Rightarrow 10a + 95d$$

$$\Rightarrow 10(3) + 95(8)$$

$$= 30 + 760 = 790.$$

KTK-5. Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{10} = 390$ & the ratio of the tenth and the fifth term is $15:7$, then $S_{15} - S_5$ is equal to :

$\Rightarrow \quad S_{10} = 390$

$\Rightarrow \cancel{5}\,\frac{\cancel{10}}{2}[2a+9d] = \overset{78}{\cancel{390}}$

$\Rightarrow \quad 2a+9d = 78$ — ①

$\Rightarrow \quad \cancel{2}\left(\frac{3d}{\cancel{8}_{4}}\right) + 9d = 78$

$\Rightarrow \quad 3d + 36d = 78 \times 4$

$\Rightarrow \quad 39d = \overset{2}{\cancel{78}} \times 4$

$\boxed{d = 8}$

$\Rightarrow \quad \frac{a_{10}}{a_5} = \frac{15}{7}$

$\Rightarrow \quad \frac{a+9d}{a+4d} = \frac{15}{7}$

$\Rightarrow \quad 7a + 63d = 15a + 60d$

$\Rightarrow \quad 8a = 3d \Rightarrow a = \frac{3d}{8}.$

$\Rightarrow \quad a = \frac{3\times\cancel{8}}{\cancel{8}} \Rightarrow \boxed{a = 3}$

\# find : $S_{15} - S_5$

$\Rightarrow \frac{15}{\cancel{2}}[\cancel{2}a + \overset{7}{\cancel{14}}d] - \frac{5}{\cancel{2}}[\cancel{2}a + \overset{2}{\cancel{4}}d]$

$\Rightarrow 15a + 105d - 5a - 10d$

$\Rightarrow 10a + 95d$

$\Rightarrow 30 + 760$

$\Rightarrow 790$ Ans.

Solution to Previous RPPs

QUESTION

(RPP 1)

Paragraph

Consider the quadratic equation $2x^2 - (4m+2)x + m^2 + m = 0, m \in R$

1. The number of positive integer values of 'm' such that the equation has exactly one root in $(2, 3)$ is

(A) 3 (B) 4 (C) 5 (D) 6

2. The number of negative integral values of 'm' such that $m > -10$ and atleast one root of the equation is smaller than '2' is

(A) 8 (B) 9 (C) 6 (D) 4

Ans. (1) B, (2) B

QUESTION

(Challenger Problem (Answers Sahi hai)

If $ax^2 + bx + c = 0, a \neq 0, a, b, c \in R$ has two distinct real roots in $(1, 2)$ then

RPP 02

A (a) $(5a + 2\,b + c) > 0$

B (a) $(5a + 2b + c) < 0$

C $2a + b > 0$

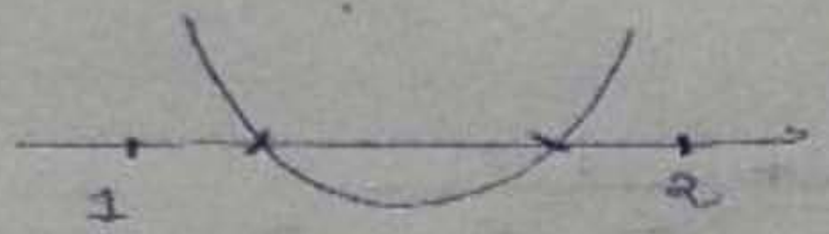
D (a) $(4a + 2b + c) > 0$

Ans. A, D

RPP-1

If $ax^2+bx+c=0$, $a \neq 0$, $a, b, c \in R$ has two distinct roots in $(1, 2)$ then

Sol^n



$1 < -\frac{b}{2a} < 2$ — (iii)

$a \cdot f(1) > 0$ — (i)

$a \cdot f(2) > 0$ — (ii)

$a \cdot f(2) > 0$

$a \cdot (4a+2b+c) > 0 \longrightarrow$ (D) True

$1 < \frac{-b}{2a} < 2$

$\Rightarrow 1 < -\frac{b}{2a}$ | $\frac{-b}{2a} < 2$

$2a+b < 0$ | $0 < 4a+b$

In option - A

(a) $(a+4a+2b+c)$

$\underbrace{a^2}_{\geq 0} + \underbrace{a(4a+2b+c)}_{> 0}$ → always ≥ 0

(A) & (D)

ADRISH SIL FROM WEST BENGAL HOOGH

THANK YOU