PRAYAS

JEE 2026

Mathematics

Trigonometric Functions

Lecture - 01

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Topics *to be covered*

A Exponential & Logarithmic Series

B Some special Telescoping series

C Introduction to Trigonometry

Recap *of previous lecture*

1. If $a_1, a_2, \ldots a_n \in R$ then RMS = $\sqrt{\frac{a_1^2+a_2^2+\cdots+a_n^2}{n}}$ & AM $\frac{a_1+a_2+\cdots+a_n}{n}$ also we have RMS $\geq$ A.M where equality holds if $a_1=a_2=\cdots=a_n$

2. If $a_1, a_2, a_3, \ldots a_n \in R^+$ then arrange their arithmetic, geometric, hormonic means and RMS respectively, A, G, H, RMS in descending order RMS $\geq$ A $\geq$ G $\geq$ H

3. $8\cos^2 x + 2\sec^2 x$ has minimum value of 8

4. $x^2 + \frac{1}{x^2+1}$ has minimum value = 1

$$\frac{8\cos^2 x + 2\sec^2 x}{2} \geq \sqrt{8\cos^2 x \cdot 2\sec^2 x}$$

$$8\cos^2 x + 2\sec^2 x \geq 8$$

at $8\cos^2 x = 2\sec^2 x$

$\cos^4 x = 1/4$

$\cos^2 x = 1/2$

$\cos x = \pm\frac{1}{\sqrt{2}}$

@ $x = \pi/4, 3\pi/4 \ldots$

$$y = x^2 + \frac{1}{x^2+1} = \underbrace{\overset{R^+}{x^2+1} + \frac{1}{x^2+1}}_{\geq 2} - 1$$

$$y_{min} = 2 - 1 = 1$$

@ $x^2+1=1$

$x^2=0$

$x=0$

If $x \in R^+$

$x + \frac{1}{x}\Big|_{min} = 2$

@ $x=1$

Recap of previous lecture

5. If $a_1, a_2, \dots a_n, b_1, b_2, \dots b_n \in R$ then by C-S inequality we have ______

$(a_1b_1+a_2b_2+\dots+a_nb_n)^2 \le (a_1^2+a_2^2+\dots+a_n^2)\cdot(b_1^2+b_2^2+\dots+b_n^2)$

6. $\sum_{n=r}^{\infty} \frac{1}{(n-r)!} =$ e

7. $e^x =$ $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots\infty$

8. $e^{-x} =$ $1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\dots\infty$

9. $e =$ $1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\dots\infty$

10. $e^{-1} =$ $1-1+1/2!-1/3!+1/4!-1/5!+\dots\infty$

Recap *of previous lecture*

11. $\left(1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\cdots\infty\right)=$ $\frac{e+e^{-1}}{2}$

12. $\left(1+\frac{1}{3!}+\frac{1}{5!}+\cdots\infty\right)=$ $\frac{e-e^{-1}}{2}$

13. $\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}=\left(\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+---\infty\right)=\frac{e-e^{-1}}{2}$

Homework Discussion

Exponential & Logarithmic Series

QUESTION [JEE Mains 2023 (1 Feb)]

The sum $\sum_{n=1}^{\infty} \frac{2n^2+3n+4}{(2n)!}$ is equal to :

(A) $\frac{11e}{2}+\frac{7}{2e}$

(B) $\frac{13e}{4}+\frac{5}{4e}-4$

(C) $\frac{11e}{2}+\frac{7}{2e}-4$

(D) $\frac{13e}{4}+\frac{5}{4e}$

$(2n)! = 2n\cdot(2n-1)!$

$(2n-1)! = (2n-1)\,(2n-2)!$

$$T_n = \frac{2n^2+3n+4}{(2n)!} = \frac{2n\cdot n}{(2n)!} + \frac{3}{2}\cdot\frac{2n}{(2n)!} + \frac{4}{(2n)!}$$

$$= \frac{n}{(2n-1)!} + \frac{3}{2}\,\frac{1}{(2n-1)!} + \frac{4}{(2n)!}$$

$$= \frac{2n-1+1}{2(2n-1)!} + \frac{3}{2(2n-1)!} + \frac{4}{(2n)!}$$

$$= \frac{1}{2(2n-2)!} + \frac{1}{2(2n-1)!} + \frac{3}{2(2n-1)!} + \frac{4}{(2n)!}$$

$$= \frac{1}{2(2n-2)!} + 2\cdot\frac{1}{(2n-1)!} + \frac{4}{(2n)!}$$

$$S = \sum_{n=1}^{\infty}\frac{1}{2}\frac{1}{(2n-2)!} + 2\sum_{n=1}^{\infty}\frac{1}{(2n-1)!} + 4\sum_{n=1}^{\infty}\frac{1}{(2n)!}$$

$$S = \frac{1}{2}\left(\frac{1}{0!} + \frac{1}{2!} + \frac{1}{4!} + \cdots \infty\right) + 2\left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \cdots \infty\right) + 4\left(\frac{1}{2!} + \frac{1}{4!} + \cdots \infty\right)$$

$$= \frac{1}{2}\left(1 + \frac{1}{2!} + \frac{1}{4!} + \cdots \infty\right) + 2\left(1 + \frac{1}{3!} + \frac{1}{5!} + \cdots \infty\right) + 4\left(\frac{1}{2!} + \frac{1}{4!} + \cdots \infty\right)$$

$$= \frac{e + e^{-1}}{4} + \frac{\cancel{2}(e - e^{-1})}{\cancel{2}} + 4\left(\frac{e + e^{-1}}{2} - 1\right)$$

$$= \frac{e}{4} + \frac{e^{-1}}{4} + e - e^{-1} + 2(e + e^{-1}) - 4$$

$$= \frac{13e}{4} + \frac{5e^{-1}}{4} - 4$$

$$= \frac{13e}{4} + \frac{5}{4e} - 4$$

QUESTION [JEE Mains 2021 (Aug)]

Tah01

Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3 \cdot (n-3) + \cdots + (n-1) \cdot 1, n \geq 4$.

The sum $\sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!} \right)$ is equal to:

(A) $\frac{e-1}{3}$

(B) $\frac{e-2}{6}$

(C) $\frac{e}{3}$

(D) $\frac{e}{6}$

Logarithm Series

- $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \infty, \quad -1 < x \le 1$

- $\log_e(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots \infty\right) \quad -1 \le x < 1$

- $\log_e(1-x^2) = -\left(x^2 + \frac{x^4}{2} + \frac{x^6}{3} + \frac{x^8}{4} + \cdots \infty\right) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \frac{x^8}{8} + \cdots \infty\right)$

- $\log_e\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots \infty\right)$ $\quad (-1 < x < 1)$

$$\log_e 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - - - \infty$$

QUESTION [JEE Mains 2021 (Aug)]

If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 \ldots$, is equal to :

(A) $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

(B) ~~$x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$~~

(C) $\frac{1-x}{1+x} + \log_e(1-x)$

(D) $\frac{1+x}{1-x} + \log_e(1-x)$

$$T_n = \frac{(2n+1)}{n+1}x^{n+1}$$

$$= \frac{(2n+2-1)}{n+1}x^{n+1}$$

$$= \left(2 - \frac{1}{n+1}\right)\cdot x^{n+1} = 2\cdot x^{n+1} - \frac{x^{n+1}}{n+1}$$

$$S = 2\sum_{n=1}^{\infty} x^{n+1} - \sum_{n=1}^{\infty}\frac{x^{n+1}}{n+1}$$

$$= 2(x^2 + x^3 + x^4 + \cdots \infty) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots \infty\right)$$

$$= 2\frac{x^2}{1-x} - \left(\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots \infty\right) - x\right)$$

$$= \frac{2x^2}{1-x} - (-\log_e(1-x) - x)$$

$$= \frac{2x^2}{1-x} + x + \log_e(1-x)$$

$$= \frac{2x^2 + x - x^2}{1-x} + \log_e(1-x)$$

$$= \frac{x^2 + x}{1-x} + \log_e(1-x)$$

$$= x\frac{(1+x)}{1-x} + \log_e(1-x)$$

QUESTION

Evaluate the sum of the series $\frac{1}{1\cdot 2}-\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}-\frac{1}{4\cdot 5}+\cdots\infty$

$$T_n=\frac{(-1)^{n+1}}{n(n+1)}$$

$$T_n=(-1)^{n+1}\left(\frac{n+1-n}{(n+1)n}\right)$$

$$T_n=(-1)^{n+1}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$T_n=\frac{(-1)^{n+1}}{n}-\frac{(-1)^{n+1}}{n+1}$$

$$S=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}-\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n+1}$$

$$=\left(1-\frac{1}{2}+\frac{1}{3}-1/4+--\infty\right)-\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{4}+--\infty\right)$$

$$=\log_e 2+\left(-1/2+1/3-1/4+1/5-1/6+--\infty\right)$$

$$S = \log_e 2 + \left(\left(\underbrace{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots \infty}\right) - 1\right)$$

$$S = \log_e 2 + \log_e 2 - 1$$

$$S = 2\log_e 2 - 1$$

$$S = \log_e 4 - 1 = \log_e 4 - \log_e e$$

$$S = \log_e\left(\frac{4}{e}\right)$$

QUESTION [JEE Mains 2024 (5 April)]

$$(\sqrt{3}-\sqrt{2})^3 = 3\sqrt{3}-2\sqrt{2}-3\cdot 3\cdot\sqrt{2}+3\cdot\sqrt{3}\cdot 2$$
$$= 9\sqrt{3}-11\sqrt{2}$$

If $1+\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}}+\frac{5-2\sqrt{6}}{18}+\frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}}+\frac{49-20\sqrt{6}}{180}+\cdots$ upto $\infty = 2+\left(\sqrt{\frac{b}{a}}+1\right)\log_e\left(\frac{a}{b}\right)$, where a and b are integers with $\gcd(a,b)=1$, then $11a+18b$ is equal to

$$S = 1+\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}}+\frac{(\sqrt{3}-\sqrt{2})^2}{18}+\frac{(\sqrt{3}-\sqrt{2})^3}{36\sqrt{3}}+\frac{(\sqrt{3}-\sqrt{2})^4}{180}+\cdots\infty.$$

$$= 1+\frac{\sqrt{3}\left(1-\sqrt{\frac{2}{3}}\right)}{2\sqrt{3}}+\frac{3\left(1-\sqrt{\frac{2}{3}}\right)^2}{18}+\frac{3\sqrt{3}\left(1-\sqrt{\frac{2}{3}}\right)^3}{36\sqrt{3}}+\frac{9\left(1-\sqrt{2/3}\right)^4}{180}+\cdots\infty$$

let $1-\sqrt{\frac{2}{3}}=x \Rightarrow 1-x=\sqrt{\frac{2}{3}}$

$$S = 1+\frac{x}{2}+\frac{x^2}{6}+\frac{x^3}{12}+\frac{x^4}{20}+\cdots\infty$$
$$= 1+\left(\frac{x}{1\cdot 2}+\frac{x^2}{2\cdot 3}+\frac{x^3}{3\cdot 4}+\frac{x^4}{4\cdot 5}+\cdots\infty\right)$$
$$= 1+S'$$

$$T_n = \frac{x^n}{n(n+1)}$$

Ans. 76

$$T_n = \frac{x^n}{n(n+1)} = x^n\left(\frac{n+1-n}{n(n+1)}\right)$$

$$T_n = x^n\left(\frac{1}{n} - \frac{1}{n+1}\right) = \frac{x^n}{n} - \frac{x^n}{n+1}$$

$$S' = \sum_{n=1}^{\infty}\frac{x^n}{n} - \sum_{n=1}^{\infty}\frac{x^n}{n+1}$$

$$= \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots \infty\right) - \left(\frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + \cdots \infty\right)$$

$$= -\log_e(1-x) - \frac{1}{x}\left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots \infty\right)$$

$$= -\log_e(1-x) - \frac{1}{x}\left(\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots \infty\right) - x\right)$$

$$= -\log_e(1-x) - \frac{1}{x}\left(-\log_e(1-x) - x\right)$$

$$S' = -\log_e(1-x) + \frac{1}{x}\cdot\log_e(1-x) + 1.$$

$$= \left(\log_e(1-x)\right)\cdot\left(\frac{1}{x}-1\right) + 1$$

$$S = 1 + S'$$

$$S = 2 + \left(\frac{1}{x}-1\right)\log_e(1-x)$$

$$S = 2 + \frac{1-x}{x}\log_e(1-x)$$

$$= 2 + \frac{\sqrt{\frac{2}{3}}}{1-\sqrt{\frac{2}{3}}}\cdot\log_e\left(\sqrt{\frac{2}{3}}\right)$$

$$= 2 + \frac{1}{2}\left(\sqrt{\frac{2}{3}}\right)\left(\frac{1+\sqrt{2/3}}{1-2/3}\right)\log_e(2/3)$$

$$S = 2 + \frac{3}{2}\left(\frac{2}{3} + \sqrt{\frac{2}{3}}\right)\log_e\frac{2}{3}$$

$$= 2 + \left(1 + \sqrt{\frac{3}{2}}\right)\log_e(2/3)$$

$a=2$, $b=3$

$11a + 18b$

$22 + 54 = 76$ Ans

Some More Problem Practice

Kaam ki Baatien

let a, b be two +ve Reals

$a, A_1, A_2 - - - A_n, b$ are in A.P

$a, G_1, G_2 - - - G_n, b$ are in G.P then ① $A_1 \geq G_1 \geq H_1$

$a, H_1, H_2 - - - H_n, b$ are in H.P.

$$A_2 \geq G_2 \geq H_2$$
$$\vdots$$
$$A_n \geq G_n \geq H_n$$

$A_p = a + pd$ $\quad T_{n+2} = b.$

$a + (n+1)d = b.$

$d = \frac{b-a}{n+1}$

$$A_p = a + \frac{(b-a)p}{n+1}$$

$$A_p = \frac{a(n+1) + (b-a)p}{(n+1)} = \frac{a(n+1-p) + pb}{n+1} \quad \text{(1)}$$

② $ab = A_1 H_n = A_2 H_{n-1} = - - = A_1 H_n$

$a, H_1, H_2 \dots, H_n, b$ are in H.P

$\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \dots \frac{1}{H_n}, \frac{1}{b}$ are in A.P

$$T_{n+2} = \frac{1}{b} = \frac{1}{a} + (n+1)d'$$

$$\frac{1}{b} - \frac{1}{a} = (n+1)d'$$

$$\frac{a-b}{ab(n+1)} = d'$$

$$\frac{1}{H_q} = \frac{1}{a} + q \cdot d'$$

$$= \frac{1}{a} + \frac{q \cdot (a-b)}{ab(n+1)}$$

$$= \frac{b(n+1) + qa - qb}{ab(n+1)}$$

$$\frac{1}{H_q} = \frac{b(n+1-q) + qa}{ab(n+1)}$$

$$H_q = \frac{ab(n+1)}{b(n+1-q)+qa}$$

$$A_p H_q = \frac{(n+1-p)a + pb}{(n+1)} \cdot \frac{ab(n+1)}{(n+1-q)b + qa}$$

$$A_1 H_n = \cancel{\frac{na+b}{n+1}} \cdot \cancel{\frac{ab(n+1)}{b+na}} = ab$$

QUESTION

Let n harmonic means $H_1, H_2, H_3, \ldots . H_n$ and n arithmetic means $A_1, A_2, \ldots . A_n$ be inserted between 25 & 40 such that

$\sum_{i=1}^{n} \log_{10}(A_i H_i) = 2019$. Find the value of n.

$25 \times 40 = A_1 H_n = A_2 H_{n-1} = \ldots = A_n H_1$

$25, A_1, A_2, \ldots, A_n, 40$ are in A.P

$25, H_1, H_2, \ldots, H_n, 40$ are in H.P

$$\sum_{i=1}^{n} \log_{10}(A_i H_i) = 2019$$

$$\log_{10}(A_1 H_1) + \log_{10}(A_2 H_2) + \ldots + \log_{10}(A_n H_n) = 2019$$

$$\log_{10}(A_1 H_1 A_2 H_2 \ldots A_n H_n) = 2019$$

$$(\underbrace{A_1 H_n}_{1000} \underbrace{A_2 H_{n-1}}_{1000} \ldots \underbrace{A_n H_1}_{1000}) = 10^{2019} \Rightarrow (1000)^n = 10^{2019} \Rightarrow 3n = 2019$$

$$n = 673.$$

Ans. 673

QUESTION

Tan 02

Let $f(x) = (a^2 + b^2 - 4a - 6b + 13)(2x^2 - 4x + 5)$, $a, b, x \in R$ such that $f(0) = f(1) = f(2)$. If $a, A_1, A_2, \ldots\ldots A_{10}, b$ is an arithmetic progression and $a, H_1, H_2, \ldots\ldots, H_{10}, b$ is harmonic progression then the value of

$$\frac{1}{10}\left(\sum_{i=4}^{8} A_i H_{11-i}\right) \text{ is equal to}$$

Ans. 3

QUESTION [JEE Advanced 2016 (Paper 2)]

$\log_e b_2 - \log_e b_1 = \log_e 2$

$\log_e \frac{b_2}{b_1} = \log_e 2 \Rightarrow \frac{b_2}{b_1} = 2$ lly $\frac{b_3}{b_2} = 2$

Let $b_i > 1$ for $i = 1, 2, \ldots, 101$. Suppose $\log_e b_1, \log_e b_2, \ldots, \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, \ldots, a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + \cdots + b_{51}$ and $s = a_1 + a_2 + \cdots + a_{51}$, then

(A) $s > t$ and $a_{101} > b_{101}$

(B) $s > t$ and $a_{101} < b_{101}$

(C) $s < t$ and $a_{101} > b_{101}$

(D) $s < t$ and $a_{101} < b_{101}$

$\boxed{a_1}, a_2, \ldots, a_{50}, \boxed{a_{51}}, a_{52}, \ldots, a_{101}$ A.P

$\boxed{b_1}, b_2, b_3, \ldots, b_{50}, \boxed{b_{51}}, b_{52}, \ldots, b_{101}$ G.P CR = 2

49 G.Ms.

$t = b_1 + b_2 + \cdots + b_{50} + b_{51}$

$s = a_1 + a_2 + \cdots + a_{50} + a_{51}$

49 A.M

$a_1 + a_{101} = a_{51} + a_{51} = 2a_{51}$

$b_1 \cdot b_{101} = b_{51} \cdot b_{51} = b_{51}^2$

$b_{51} = $ G.M b_1, b_{101}

$a_{51} = $ A.M a_1, a_{101}

$a_2 > b_2$

$a_3 > b_3$

$\vdots$

$a_{50} > b_{50}$

$\Rightarrow s > t$.

Ans. B

QUESTION [IIT-JEE 2007]

a, b are +ve no:s
$G^2 = AH$

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$. Let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

Which one of the following statements is correct?

(A) $A_1 > A_2 > A_3 > \cdots$

A.P a, A_1, b
G.P a, G_1, b
H.P a, H_1, b

A_2, G_2, H_2 are A.M, G.M, H.M of A_1 & H_1

A_3, G_3, H_3 are A.M G.M & H.M of A_2, H_2

A_4, G_4, H_4 are A.M, G.M & H.M of A_3, H_3

(B) $A_1 < A_2 < A_3 < \cdots$

(C) $A_1 > A_2 > A_3 > \cdots$ and $A_1 < A_2 < A_3 < \cdots$

(D) $A_1 < A_2 < A_3 < \cdots$ and $A_1 > A_2 > A_3 > \cdots$

Ans. A

$a < b$

$a < H_1 < G_1 < A_1 < b$

$H_1 < H_2 < G_2 < A_2 < A_1$

$H_2 < H_3 < G_3 < A_3 < A_2$

$H_3 < H_4 < G_4 < A_4 < A_3$

$a > b$

$a > A_1 > G_1 > H_1 > b$

$A_1 > A_2 > G_2 > H_2 > H_1$

$A_2 > A_3 > G_3 > H_3 > H_2$

$A_3 > A_4 > G_4 > H_4 > H_3$

Easily seen

$A_1 > A_2 > A_3 \text{ ----}$

$H_1 < H_2 < H_3 \text{ ----}$

a, G_1, b $\quad G_1^2 = ab$

a, A_1, b $\quad G_1^2 = A_1 H_1$ $\quad G_1^2 = A_1 H_1 = ab.$

a, H_1, b

A_1, G_2, H_1 $\quad G_2^2 = A_1 H_1$

A_1, A_2, H_1 $\quad G_2^2 = A_2 H_2$

A_1, H_2, H_1 $\quad G_2^2 = A_1 H_1 = A_2 H_2 = ab.$

A_2, G_3, H_2 $\quad G_3^2 = A_2 H_2$

$A_2, A_3\ H_2$ $\quad G_3^2 = A_2 H_2 = ab = A_3 H_3$

A_2, H_3, H_2 $\quad G_3^2 = A_3 H_3$

QUESTION [IIT-JEE 2007]

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$. Let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

Which one of the following statements is correct?

(A) $G_1 > G_2 > G_3 > \cdots$

(B) $G_1 < G_2 < G_3 < \cdots$

(C) $G_1 = G_2 = G_3 = \cdots$

(D) $G_1 < G_2 < G_3 < \cdots$ and $G_1 > G_2 > G_3 > \cdots$

a, G_1, b

$a, A_1 < b$

$a, H_1 < b$

$G_1^2 = A_1 H_1 = ab.$

A_1, G_2, H_1

A_1, A_2, H_1

A_1, H_2, H_1

$G_2^2 = A_1 H_1 = G_1^2 = ab.$

$G_3^2 = A_2 H_2$

A_2, G_3, H_2

A_2, A_3, H_2

$A_2, H_3 < H_2$

$G_3^2 = A_2 H_2 = G_2^2$

$G_1 = G_2 = G_3 = ---$

Ans. C

QUESTION [IIT-JEE 2007]

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$. LetA_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

Which one of the following statements is correct?

(A) $H_1 > H_2 > H_3 > \cdots$

(B) $H_1 < H_2 < H_3 < \cdots$

(C) $H_1 > H_2 > H_3 > \cdots$ and $H_1 < H_2 < H_3 < \cdots$

(D) $H_1 < H_2 < H_3 < \cdots$ and $H_1 > H_2 > H_3 > \cdots$

Ans. B

Trigonometry

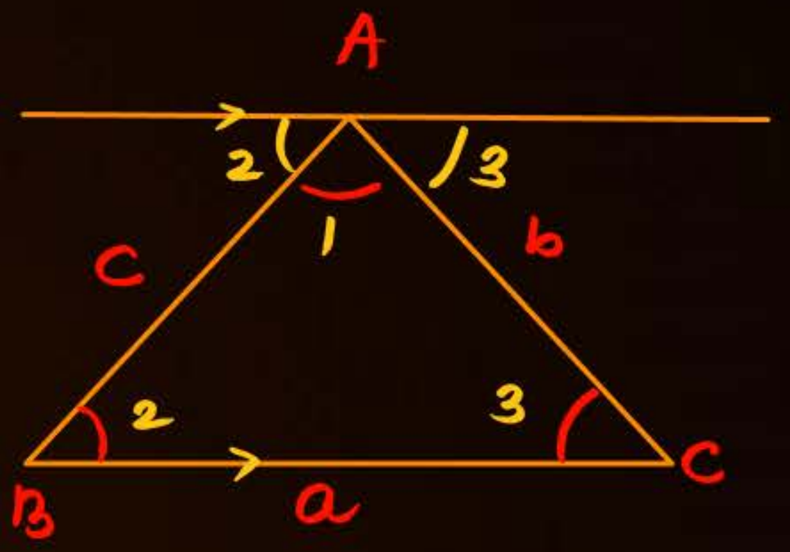
आरंभ

Trigonometry

Trigonometry

TRIGONOMETRY

Tri	Gonos	Metron
(Three)	(Sides)	(Measure)

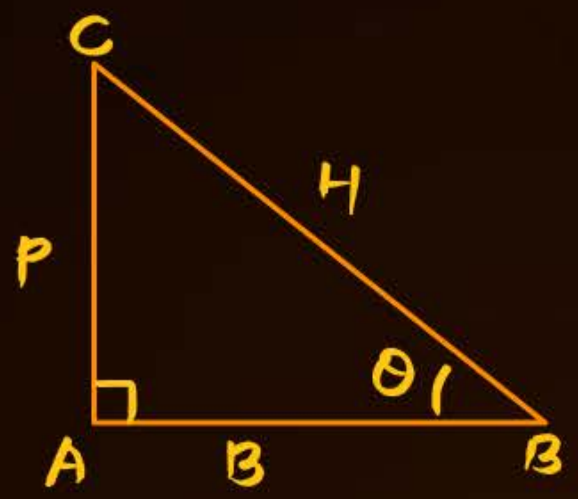


$\angle A + \angle B + \angle C = 180^\circ$

Proof: $\angle 1 + \angle 2 + \angle 3 = 180^\circ$ (st. line)

Trigonometry is the branch of mathematics in which we study about triangle. Basically these are six parameters in a triangle (three sides & angles)

Bachpan KI Yaadien



S	C	T
$\frac{P}{H}$	$\frac{B}{H}$	$\frac{P}{B}$

Basic T-Ratios

$\sin\theta = \frac{P}{H}$

$\cos\theta = \frac{B}{H}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{P}{B}$

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{B}{P}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{H}{B}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{H}{P}$

Derived T-Ratios

* $\sin^2\theta + \cos^2\theta = \frac{P^2+B^2}{H^2} = \frac{H^2}{H^2} = 1$

* $1 + \tan^2\theta = \sec^2\theta$

* $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$$1+\tan^2\theta=\sec^2\theta$$

$$\Rightarrow \sec^2\theta-\tan^2\theta=1$$

$$(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)=1$$

$\Rightarrow$ $\sec\theta-\tan\theta$ & $\sec\theta+\tan\theta$ are reciprocal of each other.

lly $\operatorname{cosec}\theta-\cot\theta$ & $\operatorname{cosec}\theta+\cot\theta$ are reciprocal of each other

QUESTION

Asking : Which of the following reduces to $\overset{1}{\text{unity}}$ for $0 < A < 90°$

(A) $(\overbrace{\sec^2 A - 1}^{\tan^2 A})\cot^2 A$

(B) $\cos A \operatorname{cosec} A \sqrt{\sec^2 A - 1} = \cos A \cdot \frac{1}{\sin A} \cdot \tan A = \cos A \cdot \frac{1}{\sin A} \cdot \frac{\sin A}{\cos A}$

(C) $(\overbrace{\operatorname{cosec}^2 A - 1}^{\cot^2 A})\tan^2 A$

(D) $(1 - \cos^2 A)(1 + \cot^2 A)$

$\sin^2 A \cdot \operatorname{cosec}^2 A$

QUESTION

If $(\sec\alpha + \tan\alpha)(\sec\beta + \tan\beta)(\sec\gamma + \tan\gamma) = \tan\alpha\tan\beta\tan\gamma$. Then value of $(\sec\alpha - \tan\alpha)(\sec\beta - \tan\beta)(\sec\gamma - \tan\gamma)$ equals to

$$= \frac{1}{(\sec\alpha + \tan\alpha)} \cdot \frac{1}{(\sec\beta + \tan\beta)} \cdot \frac{1}{\sec\gamma + \tan\gamma}$$

$$= \frac{1}{\tan\alpha \cdot \tan\beta \tan\gamma}$$

$$= \cot\alpha \cdot \cot\beta \cdot \cot\gamma .$$

(A) $\tan\alpha\tan\beta\tan\gamma$

(B) $\cot\alpha\cot\beta\cot\gamma$

(C) $\tan\alpha + \tan\beta + \tan\gamma$

(D) $\cot\alpha + \cot\beta + \cot\gamma$

QUESTION

The value of $\log_{\sin^2 x + \cos^4 x + 2}(\cos^2 x + \sin^4 x + 2)$ is equal to

(A) 1

(B) -1

(C) 0

(D) 2

$\log_{\sin^2 x+\cos^4 x+2}\left(1-\sin^2 x + (1-\cos^2 x)^2 + 2\right)$

$= \log_{\sin^2 x+\cos^4 x+2}\left(1-\sin^2 x + 1 + \cos^4 x - 2\cos^2 x + 2\right)$

$= \log_{\sin^2 x+\cos^4 x+2}\left(4 + \cos^4 x - \sin^2 x - 2\overset{2(1-\sin^2 x)}{\cos^2 x}\right)$

$= \log_{\sin^2 x+\cos^4 x+2}\left(2 + \cos^4 x + \sin^2 x\right)$

$= 1$

Values of T – Rations of some standard angles

Yaad Rakho!!

	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not Defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec}\theta$	Not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

QUESTION

Let $N = \dfrac{\underbrace{999....9}_{16\text{ digits}} - \underbrace{5555....5}_{8\text{ digits}}}{\underbrace{9000....04}_{9\text{ digits}}}$, then sum of digits of N is

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

KTK 01

The dimensions of a Cuboid are $a > b > c$. The volume = 216 and the total outer surface area = 252. If a, b, c are in G.P., then c =

(A) 3

(B) 1

(C) 5

(D) 2

Ans. A

QUESTION

KTK 02

A ball falls from a height of 100 m on a floor. If in each rebound, it describes $(4/5)^{\text{th}}$ height of the previous falling height, then the total distance travelled by the ball before it comes to rest is?

Ans. 900 m

QUESTION [JEE Mains 2020 (Jan)]

KTK 03

The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}}$ ...... to ∞ is equal to

(A) $2^{\frac{1}{4}}$

(B) $2^{\frac{1}{2}}$

(C) 1

(D) 2

Ans. B

QUESTION [JEE Mains 2020 (Jan)]

KTK 04

If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$, for $0 < \theta < \frac{\pi}{4}$, then:

(A) $x(1+y) = 1$

(B) $y(1-x) = 1$

(C) $y(1+x) = 1$

(D) $x(1-y) = 1$

Ans. B

QUESTION [JEE Mains 2024 (1 Feb)]

KTK 05

Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to:

(A) 800

(B) 890

(C) 790

(D) 690

Ans. C

Revision Practice Problems (RPP)

QUESTION

(RPP 1)

Paragraph

Consider the quadratic equation $2x^2-(4m+2)x+m^2+m=0, m \in R$

1. The number of positive integer values of 'm' such that the equation has exactly one root in $(2, 3)$ is

   (A) 3 (B) 4 (C) 5 (D) 6

2. The number of negative integral values of 'm' such that $m>-10$ and atleast one root of the equation is smaller than '2' is

   (A) 8 (B) 9 (C) 6 (D) 4

Ans. (1) B, (2) B

QUESTION

(Challenger Problem (Answers Sahi hai)

If $ax^2 + bx + c = 0, a \neq 0, a, b, c \in R$ has two distinct real roots in $(1, 2)$ then

RPP 02

A (a) $(5a + 2b + c) > 0$

B (a) $(5a + 2b + c) < 0$

C $2a + b > 0$

D (a) $(4a + 2b + c) > 0$

Ans. A, D

Solution to Previous TAH

QUESTION [JEE Mains 2023 (11 April)]

Let a, b, c and d be positive real numbers such that $a + b + c + d = 11$. If the maximum value of $a^5 b^3 c^2 d$ is 3750β, then the value of β is

(A) 110

(B) 108

(C) 90

(D) 55

Ans. C

Jan-01
[Mains-23]

$a+b+c+d=11$

$a^5b^3c^2d = 3750\beta$

AM: $\dfrac{\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}+d}{11}$

$$\underbrace{\frac{a}{5}\cdot\frac{a}{5}\cdot\frac{a}{5}\cdot\frac{a}{5}\cdot\frac{a}{5}\cdot\frac{b}{3}\cdot\frac{b}{3}\cdot\frac{b}{3}\cdot\frac{c}{2}\cdot\frac{c}{2}\cdot d}_{GM}$$

$$AM \Rightarrow \frac{\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}+d}{11} \Rightarrow \frac{\cancel{11}}{\cancel{11}} \Rightarrow 1$$

$$GM \Rightarrow \frac{a^5b^3c^2d}{(5)^5(3)^3(2)^2}$$

$AM \geq GM$

$$1 \geq \frac{a^5b^3c^2d}{(5)^5(3)^3(2)^2}$$

$$(5^5)(3)^3(2)^2 \geq 3750\beta$$

$$5^2\cdot\cancel{5^3}(3)^3(4) \geq \cancel{3750}^{30}\beta$$

$$\beta \leq \frac{25\times27\times4\,2}{30\,10\,2}$$

$$\boxed{\beta \leq 90}\ (c)$$

Richathakur

Tah-01.

Lucky Kumari

$a+b+c+d=11$

$(a^5b^3c^2d)_{max.} = 3750\beta$

AM $\geq$ GM

$$\Rightarrow \frac{\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}+d}{11} \geq \left(\frac{a^5b^3c^2d}{5^5 3^3 2^2}\right)^{1/11}$$

$$\Rightarrow \frac{a+b+c+d}{11} \geq \left(\frac{a^5b^3c^2d}{5^5 3^3 2^2}\right)^{1/11}$$

$$\Rightarrow \frac{11}{11} \geq \left(\frac{a^5b^3c^2d}{5^5 3^3 2^2}\right)^{1/11}$$

$$\Rightarrow 1 \geq \frac{a^5b^3c^2d}{5^5 3^3 2^2}$$

$$\Rightarrow a^5b^3c^2d \leq 5^5 3^3 2^2$$

Now,

$(a^5b^3c^2d)_{max} = 3750\beta$

$\Rightarrow 5^{\cancel{5}}(3^{\cancel{3}})(\cancel{2^2}) = \cancel{5^4}\times\cancel{3}\times\cancel{2}\,\beta$

$\Rightarrow \beta = 3^2\times 2\times 5$

$\Rightarrow \beta = 90.$

QUESTION

(a) If $x, y, z > 0$ and $x + y + z = 1$, prove that:

(i) $x^2yz \leq \frac{1}{64}$

(ii) $x^2 + y^2 + z^2 \geq \frac{1}{3}$

Tah-02

Prove that $x^2yz \le \dfrac{1}{64}$

$x, y, z > 0$

$x+y+z = 1$

$\xrightarrow{GM} \left(\dfrac{x \cdot x \cdot y \cdot z}{2^2}\right)^{\frac{1}{4}} \Rightarrow \left(\dfrac{x^2yz}{4}\right)^{\frac{1}{4}}$

$$\frac{\frac{x}{2}+\frac{x}{2}+y+z}{4} = \frac{1}{4}$$

$$\boxed{AM = 1/4}$$

$AM \geq GM$

$\dfrac{1}{4} \geq \left(\dfrac{x^2yz}{4}\right)^{1/4}$

$\left(\dfrac{1}{4}\right)^4 \geq \dfrac{x^2yz}{4}$

$\dfrac{1}{256} \geq \dfrac{x^2yz}{4}$

$\boxed{\dfrac{1}{64} \geq x^2yz}$ proved

Tah-02.

$x, y, z > 0 \qquad x+y+z=1$

(i) $x^2yz \leqslant 1/64$

$\cancel{\frac{x}{2}}$ AM $\geqslant$ GM

$$\Rightarrow \frac{\frac{x}{2}+\frac{x}{2}+y+z}{4} \geqslant \left(\frac{x}{2}\cdot\frac{x}{2}\cdot y\cdot z\right)^{1/4}$$

$$\Rightarrow \frac{x+y+z}{4} \geqslant \left(\frac{x^2yz}{4}\right)^{1/4}$$

$$\Rightarrow \left(\frac{1}{4}\right)^4 \geqslant \left(\frac{x^2yz}{4}\right)^{1/4 \times 4}$$

$$\Rightarrow \frac{1}{4^4} \geqslant \frac{x^2yz}{4}$$

$$\Rightarrow x^2yz \leqslant \frac{1}{4^3}$$

$$\Rightarrow x^2yz \leqslant \frac{1}{64}.$$

Lucky kumari

TAH → 02

$x, y, z > 0$ & $x + y + z = 1$

AM $\geq$ GM

$$\frac{\frac{x}{2} + \frac{x}{2} + y + z}{4} \geq \left(\frac{x}{2} \cdot \frac{x}{2} \cdot y \cdot z\right)^{1/4}$$

$$\frac{1}{4} \geq \left(\frac{x^2}{4} yz\right)^{1/4}$$

$$\frac{x^2 yz}{4} \leq \frac{1}{4^4}$$

$$x^2 yz \leq \frac{1}{4^3}$$

$$x^2 yz \leq \frac{1}{64}$$ (Proved)

Tah-02: $x, y, z > 0$, $x+y+z=1$, Prove that:

$$x^2yz \le \frac{1}{64}.$$

$$\Rightarrow \frac{\frac{x}{2}+\frac{x}{2}+y+z}{4} \geqslant \left(\frac{x}{2}\cdot\frac{x}{2}\cdot y\cdot z\right)^{1/4}$$

$$\Rightarrow \frac{1}{4} \geqslant \left(\frac{x^2yz}{4}\right)^{1/4}$$

$$\Rightarrow x^2yz \le \left(\frac{1}{4}\right)^4\cdot 4$$

$$\Rightarrow x^2yz \le \frac{1}{4^{\cancel{4}\,3}}\cdot \cancel{4}$$

$$\Rightarrow x^2yz \le \frac{1}{64}$$ Hence proved.

QUESTION

(b) If $a + b + c = 3$ and a, b, c are positive, then prove that $a^2b^3c^2 \frac{3^{10}\cdot 2^4}{7^7}$.

Tah-03 $a+b+c=3$

a, b, c are (+)ve

$$a^2b^3c^2 \le \frac{3^{10}\cdot 2^4}{7^7}$$

$$\frac{\frac{a}{2}+\frac{a}{2}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}=3}{7}$$

$$GM = \left[\frac{a^2b^3c^2}{2^2\cdot 3^3\cdot 2^2}\right]^{1/7}$$

$$AM = \frac{3}{7}$$

$AM \geq GM$

$$\frac{3}{7} \geq \left[\frac{a^2b^3c^2}{2^2\cdot 3^3\cdot 2^4}\right]^{\frac{1}{7}}$$

$$\left(\frac{3}{7}\right)^7 \geq \left[\frac{a^2b^3c^2}{2^4\cdot 3^3}\right]$$

$$\frac{3^7\cdot 2^4\cdot 3^3}{7^7} \geq a^2b^3c^3$$

$$\frac{3^{10}\cdot 2^4}{7^7} \geq a^2b^3c^3$$

proved

Richathakur

TAH ⇒ 03

$a, b, c > 0 \qquad a+b+c=3$

AM $\geq$ GM

$$\frac{\frac{a}{2}+\frac{a}{2}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}}{7} \geq \left(\frac{a^2}{2^2}\frac{b^3}{3^3}\frac{c^2}{2^2}\right)^{1/7}$$

$$\frac{3}{7} \geq \left(\frac{a^2b^3c^2}{2^4 3^3}\right)^{1/7}$$

$$\frac{3^7}{7^7} \geq \frac{a^2b^3c^2}{2^4 3^3}$$

$$a^2b^3c^2 \leq \frac{3^{10}2^4}{7^7}$$

(Proved)

Tah-03.

$$a+b+c=3 \quad ; (a,b,c)>0$$

To prove, $a^2b^3c^2 \leq \frac{3^{10}\cdot 2^4}{7^7}$

$$\Rightarrow \frac{\frac{a}{2}+\frac{a}{2}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}}{7} \geq \left(\frac{a^2b^3c^2}{2^4 3^2}\right)^{1/7}$$

$$\Rightarrow \left(\frac{a+b+c}{7}\right)^7 \geq \left(\frac{a^2b^3c^2}{2^4\,3^3}\right)^{1/7\times 7}$$

$$\Rightarrow \left(\frac{3}{7}\right)^7 \geq \frac{a^2b^3c^2}{2^4 3^3}$$

$$\Rightarrow a^2b^3c^2 \leq \frac{3^9\cdot 2\cdot 3}{7^7}$$

$$\Rightarrow a^2b^3c^2 \leq \frac{3^{10}\cdot 2}{7}$$

QUESTION

If $a_i < 0$ for all $i = 1, 2, \dots, n$ prove that

(ii) $(1 - a_1 + a_1^2)(1 - a_2 + a_2^2) \cdots (1 - a_n + a_n^2) \geq 3^n (a_1 a_2 \dots a_n)$ (where n is even)

Tan-04

$a_i < 0$; all $i = 1, 2 \cdots n$ $\qquad n \to$ even

Prove that

$$(1-a_1+a_1^2)(1-a_2+a_2^2)\cdots(1-a_n+a_n^2) \geq 3^n (a_1 a_2 \cdots a_n)$$

$$\frac{1+(-a_1)+a_1^2}{3} \geq \left[(1(-a_1)(a_1)^2\right]^{1/3} \to \left[-a_1^3\right]^{1/3}$$

$$\boxed{\frac{1-a_1+a_1^2}{3} \geq -a_1}$$

$$i=1 \Rightarrow \frac{1-a_1+a_1^2}{3} \geq -a_1$$

$$i=2 \Rightarrow \frac{1-a_2+a_2^2}{3} \geq -a_2$$

$$i=3 \Rightarrow \frac{1-a_3+a_3^2}{3} \geq -a_3$$

$$\vdots$$

$$i=n \Rightarrow \frac{1-a_n+a_n^2}{3} \geq -a_n$$

$$\frac{(1-a_1+a_1^2)(1-a_2+a_2^2)\cdots(1-a_n+a_n^2)}{3} \geq (-1)^n (a_1 a_2 \cdots a_n)$$

$$(1-a_1+a_1^2)(1-a_2+a_2^2)\cdots(1-a_n+a_n^2) \geq 3^n (a_1 a_2 \cdots a_n)$$

Proved

Tah-04: If $a_i < 0$ for all $i = 1, 2, \dots n$ prove that

$$(1-a_1+a_1^2)(1-a_2+a_2^2)\dots(1-a_n+a_n^2) \geq 3^n(a_1a_2\dots a_n)$$

(where n is even.)

$$\Rightarrow \frac{1+(-a_i)+a_i^2}{3} \geq (1\cdot(-a_i)\cdot a_i^2)^{1/3}$$

$$\geq ((-a_i)^3)^{1/3} = -a_i$$

$$\Rightarrow i=1; \quad \frac{1-a_1+a_1^2}{3} \geq -a_1$$

krish

$$\Rightarrow i=2; \quad \frac{1-a_2+a_2^2}{3} \geq -a_2$$

$\vdots$

n is even

$$\Rightarrow i=n; \quad \frac{1-a_n+a_n}{3} \geq -a_n$$

$$\Rightarrow (1-a_1+a_1^2)(1-a_2+a_2^2)\dots(1-a_n+a_n^2) \geq (-1)^n 3^n(a_1a_2\dots a_n).$$

$$\Rightarrow (1-a_1+a_1^2)(1-a_2+a_2^2)\dots(1-a_n+a_n^2) \geq 3^n(a_1a_2\dots a_n)$$

—Hence proved.

Tah-04.

$a_i<0 \Rightarrow -a_i>0$

$AM \geqslant GM$

$\Rightarrow \frac{1+(-a_1)+(-a_1)^2}{3} \geqslant \left(1.(-a_1)(-a_1)^2\right)^{1/3}$

$\Rightarrow 1-a_1+a_1^2 \geqslant 3\left(-a_1\right)^{3\times 1/3}$

$\Rightarrow 1-a_1+a_1^2 \geqslant -3a_1$ — (I)

Similarly,

$\Rightarrow 1-a_2+a_2^2 \geqslant -3a_2$ — (II)

$\Rightarrow 1-a_3+a_3^2 \geqslant -3a_3$ — (III)

$\vdots \qquad \vdots$

$\Rightarrow 1-a_n+a_n^2 \geqslant -3a_n$ — (IV)

Multiply all the equations:-

$\Rightarrow (1-a_1+a_1^2).(1-a_2+a_2^2).(1-a_3+a_3^2)\ldots\ldots(1-a_n+a_n^2)$

$\geqslant +3^n(a_1.a_2.a_3\ldots\ldots a_n)$

(where, n is even).

$\Rightarrow (1-a_1+a_1^2).(1-a_2+a_2^2).(1-a_3+a_3^2)\ldots\ldots(1-a_n+a_n^2)$

$\geqslant 3^n(a_1.a_2.a_3,\ldots\ldots a_n)$

Proved.

Lucky kumari

QUESTION

Find the sum of following series :

(i) $S = \sum_{n=1}^{100} n(n!)$

(ii) $S = \sum_{n=1}^{50} \frac{n}{(n+1)!}$

(iii) $S = \sum_{r=1}^{100} (r^2 + 1) \cdot r!$

Tah-05

$$S = \sum_{r=1}^{100} (r^2+1)\cdot r!$$

$$\boxed{[r(r+1)-(r-1)]\cdot r!}$$

$$r(r+1)! - (r-1)r!$$

$$r(r+1)! - (r-1)r!$$

$$T_1 = 1\cdot\cancel{2!} - 0\cdot 1!$$

$$T_2 = 2\cdot\cancel{3!} - \cancel{1\cdot 2!}$$

$$T_3 = \cancel{3\cdot 4!} - \cancel{2\cdot 3!}$$

$$\vdots$$

$$T_{100} = 100\cdot 101! - \cancel{99!}\,100$$

$$S = 100\cdot 10\phi! - 0$$

$$\boxed{S = 100\cdot 101!}$$

Richathakur

TAH-05

(iii) $S = \sum_{r=1}^{100} (r^2+1)\cdot r!$

$= \sum_{r=1}^{100} (r^2+2r+1-2r)\, r!$

$= \sum_{r=1}^{100} \{(r+1)^2 - 2r\}\, r!$

$= \sum_{r=1}^{100} \{(r+1)^2 r! - 2r\,r!\}$

$= \sum_{r=1}^{100} \left[\{(r+1)\,(r+1)!\} - 2\{(r+1-1)\, r!\}\right]$

$= \sum_{r=1}^{100} \left(\{(r+2-1)\,(r+1)! - 2\{(r+1)! - r!\}\right)$

$= \sum_{r=1}^{100} \left(\{(r+2)! - (r+1)!\} - 2\{(r+1)! - r!\}\right)$

$= \sum_{r=1}^{100} \left((r+2)! - (r+1)!\right) - \sum_{r=1}^{100} 2\left((r+1)! - r!\right)$

$= \begin{array}{l} (3! - 2!) \\ +(4! - 3!) \\ +(5! - 4!) \\ \vdots \\ +(102! - 101!) \end{array} - 2\left(\begin{array}{l} (2! - 1!) \\ +(3! - 2!) \\ +(4! - 3!) \\ \vdots \\ +(101! - 100!) \end{array}\right)$

$= (102! - 2!) - 2(101! - 1!) = 100(101!)$

$= 102! - \cancel{2!} - 2(101)! + \cancel{2!} = 102! - 2(101)!$ Ans

Tah-05.

Lucky kumari

$S = \sum_{r=1}^{100} (r^2+1)(r!)$

$\Rightarrow T_r = (r^2+1)(r!)$

$\Rightarrow T_r = ((r+1)^2 - 2r)(r!)$

$\Rightarrow T_r = (r+1)^2 r! - 2r\,r!$

$\Rightarrow T_r = (r+1)(r+1)(r!) - r\,r! - r\,r!$

$\Rightarrow T_r = (r+1)(r+1)! - r(r!) - ((r+1)-1)r!$

$\Rightarrow T_r = \{(r+1)(r+1)! - r(r!)\} - \{(r+1)r! - r!\}$

$\Rightarrow T_r = \{(r+1)(r+1)! - r(r!)\} - \{(r+1)! - r!\}$

$T_1 = \{\cancel{2\cdot2!} - 1\cdot1!\} - \{\cancel{2!} - 1!\}$

$T_2 = \{\cancel{3\cdot3!} - \cancel{2\cdot2!}\} - \{\cancel{3!} - \cancel{2!}\}$

$T_3 = \{\cancel{4\cdot4!} - \cancel{3\cdot3!}\} - \{\cancel{4!} - \cancel{3!}\}$

$\vdots$

$T_r = \{(r+1)(r+1)! - \cancel{r\,r!}\} - \{(r+1)! - \cancel{r!}\}$

$\Rightarrow T_1+T_2+T_3+\dots+T_r = \{(r+1)(r+1)! - \cancel{1}\} - \{(r+1)! - \cancel{1}\}$

$\Rightarrow T_1+T_2+T_3+\dots+T_{100} = \{(101)(101)! - (101)!\}$

$\Rightarrow S = (101)!\,(101-1)$

$\Rightarrow S = (101)!\,(100)$

$\Rightarrow \boxed{S = 100\,(101!)}$

मन की बात Ashish Sir के साथ

Revision

↓

RPP

Read handwritten notes

Question solve nahi hotay??

- Class karayay hue swaal.
- KTK / TAH solutions se HINT lekar solve.

Backlog kaa soln??

- Study from class notes.
- Don't miss live lectures
- kisi chapter mai beech mai mat kudo!!

THANK YOU