PRAYAS

JEE 2026

Mathematics

Sequence and Series

Lecture -09

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Topics *to be covered*

- **A** Problems on AM-GM-HM Inequaity
- **B** Some special Telescoping series
- **C** Exponential & Logarithmic Series

Homework Discussion

QUESTION [JEE Mains 2024 (8 April)]

★★KCLS★★

Let the positive integers be written in the form :

sequence 1, 2, 4, 7 - - -
of 1st Terms

$\downarrow$
2nd order diff const.

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & & 3 & & \\ & 4 & & 5 & & 6 & \\ 7 & & 8 & & 9 & & 10 \end{array}$$

$T_n = an^2 + bn + c$

$a = \frac{1}{2}, b = -\frac{1}{2}, c = 1$

$T_n = \frac{n^2 - n + 2}{2}$

If the k^{th} row contains exactly k numbers for every natural number k, then the row in which the number 5310 will be, is ________

Let 5310 occur in n^{th} Row.

$\Rightarrow 5310 \geq T_n, \quad T_n + (103-1)\cdot 1 \geq 5310$

$5310 \geq \frac{n^2 - n + 2}{2} \quad \frac{n^2 - n + 2}{2} \geq 5310 - 102$

Ans. 103

$$n^2 - n + 2 \le 2 \cdot 5310$$

$$n^2 - n + 2 \le 10620$$

$$n^2 - n - 10618 \le 0$$

$$\downarrow$$

$$n = \frac{1 \pm \sqrt{1 + 4 \cdot 10618}}{2}$$

$$n = \frac{1 \pm \sqrt{42473}}{2}$$

$$n = \frac{1 \pm 206\cdots}{2} = \frac{207\cdots}{2}, \; \frac{-205\cdots}{2}$$

$$\frac{-205\cdots}{2} \le n \le \frac{207\cdots}{2} = 103\cdots$$

$$-102\cdots \le n \le 103\cdots$$

$$102\cdots \le n \le 103\cdots$$

$$n = 103$$

$$n^2 - n + 2 \ge 2(5310 - 102)$$

$$= 2 \cdot (5208)$$

$$= 10416$$

$$n^2 - n - 10414 \ge 0$$

$$n = \frac{1 \pm \sqrt{1 + 4 \cdot 10414}}{2}$$

$$n = \frac{1 \pm \sqrt{41657}}{2}$$

$$\begin{array}{r|l} & 204\cdots \\ 2 & \overline{4}\,\overline{16}\,\overline{57} \\ & 4 \\ 40 & 16 \\ & 0 \\ 404 & 1657 \\ & 1616 \end{array}$$

$$n = \frac{1 \pm 204\cdots}{2}$$

$$n = \frac{205\cdots}{2}, \quad n = \frac{-203\cdots}{2}$$

$$n \ge 102\cdots \qquad n \le -101\cdots$$

$$n \ge 102\cdots$$

$$\text{or } n \le -101\cdots$$

$\Rightarrow$ 5310 occurs in n^{th} row where $n \leq 103$.

1st term of row 103 $\quad T_{103} = \frac{103^2 - 103 + 2}{2} = \frac{103 \times 102 + 2}{2}$

$= 51 \times 103 + 1$

$= 5253 + 1 = 5254$

Last term of Row 103 $= 5254 + (103-1) \cdot 1$

$= 5254 + 102$

$= 5356$

Clearly $5254 < 5310 < 5356$

$\Downarrow$

5310 occurs in 103^{th} row.

QUESTION

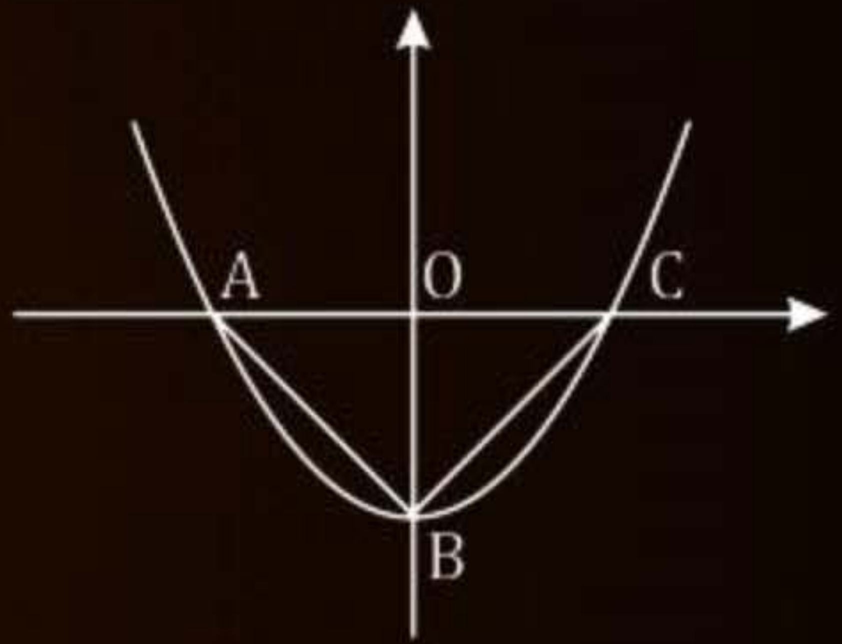
Retry

Rpp 1A

Paragraph

In the given figure vertices of ΔABC lie on $y = f(x) = ax^2 + bx + c$. The ΔABC is right angled isosceles triangle whose hypotenuse $AC = 4\sqrt{2}$ units, then

$y = f(x)$ is given by



(A) $y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$

(B) $y = \frac{x^2}{2} - 2$

(C) $y = x^2 - 8$

(D) $y = x^2 - 2\sqrt{2}$

Ans. A

QUESTION

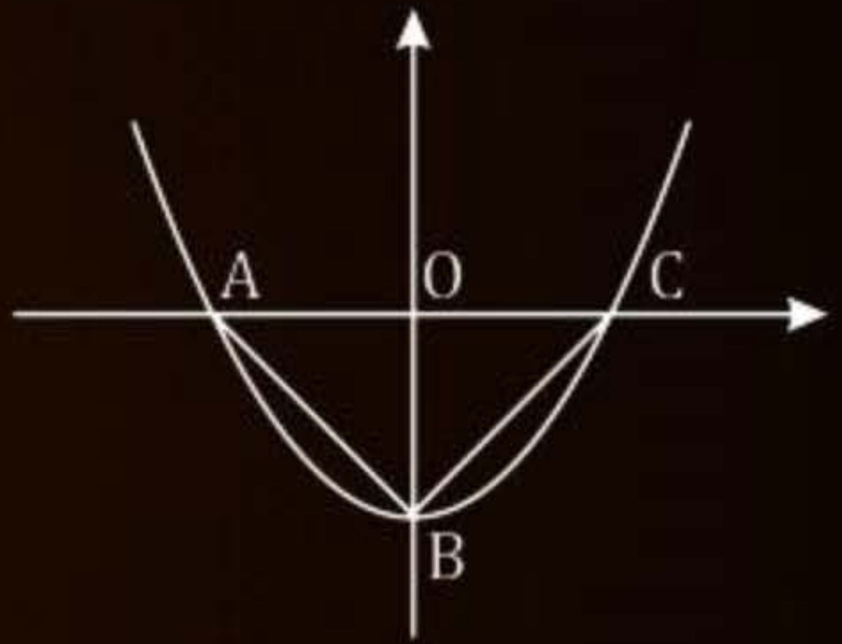
Retry

Rpp 1B

Paragraph

In the given figure vertices of ΔABC lie on $y = f(x) = ax^2 + bx + c$. The ΔABC is right angled isosceles triangle whose hypotenuse $AC = 4\sqrt{2}$ units, then

Minimum value of $y = f(x)$ is



(A) $2\sqrt{2}$

(B) $-2\sqrt{2}$

(C) 2

(D) -2

Ans. B

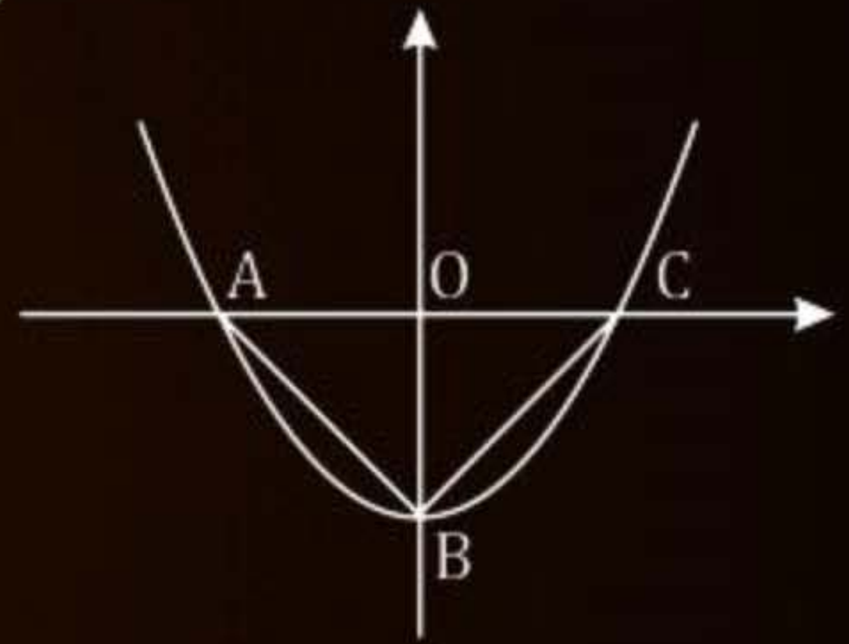
QUESTION

Retry

Rpp 1C

Paragraph

In the given figure vertices of ΔABC lie on $y = f(x) = ax^2 + bx + c$. The ΔABC is right angled isosceles triangle whose hypotenuse $AC = 4\sqrt{2}$ units, then



Number of integral value of k for which $\frac{k}{2}$ lies between the roots of $f(x) = 0$, is

(A) 9

(B) 10

(C) 11

(D) 12

Ans. C

Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal

QUESTION

Find S_n and S_∞ for $\frac{1}{2\cdot 4}+\frac{1\cdot 3}{2\cdot 4\cdot 6}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}+\cdots$

$4, 6, 8, 10 \ldots$

$T_n = 4+(n-1)2 = 2n+2.$

$$T_n = \frac{1\cdot 3\cdot 5\cdot 7\ldots(2n-1)}{2\cdot 4\cdot 6\cdot 8\ldots 2n\cdot(2n+2)}\left(2n+2-(2n+1)\right)$$

$$= \frac{1\cdot 3\cdot 5\cdot 7\ldots(2n-1)}{2\cdot 4\cdot 6\cdot 8\ldots 2n} - \frac{1\cdot 3\cdot 5\ldots(2n-1)(2n+1)}{2\cdot 4\cdot 6\ldots 2n\cdot(2n+2)}$$

$n \to n+1$

$$T_1 = \frac{1}{2} - \frac{1\cdot 3}{2\cdot 4}$$

$$T_2 = \frac{1\cdot 3}{2\cdot 4} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}$$

$$T_3 = \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6} - \frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}$$

$$T_n = \frac{1\cdot 3\cdot 5\ldots(2n-1)}{2\cdot 4\cdot 6\ldots 2n} - \frac{1\cdot 3\cdot 5\ldots(2n+1)}{2\cdot 4\cdot 6\ldots(2n+2)}$$

$$S_n = \frac{1}{2} - \frac{1\cdot 3\cdot 5 \cdots (2n+1)}{2\cdot 4\cdot 6 \cdots (2n+2)}$$

$$S_\infty = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{1}{2} - \underbrace{\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdots\frac{2n+1}{2n+2}}_{\text{diminshing fraction.}}\right) = \frac{1}{2}$$

$\to 0$

QUESTION

Given $a + b = 50, a, b \in R^+$. If A, G and H are, respectively, the AM, GM and HM between the numbers a and b, such that the GM exceeds HM by 4, then (where $A > 1, G > 1, H > 1$)

(A) $A + G = 30H$

(B) $G + H = A + 11$

(C) $4(G + H) = A - 1$

(D) $A + G = 3(H - 1)$

A, G, H are A.M, G.M & H.M resp b/w a & b.

$a+b=50$ (given) $\quad A=\frac{a+b}{2}=25.$

$\underbrace{\text{Also}}$ $G^2 = AH.$ $\quad$ Also $G = H+4$ (given)

$G^2 = 25H$

$G^2 = 25(G-4)$

$G^2 - 25G + 100 = 0$

$(G-20)(G-5)=0$

$G = 20, \cancel{5}.$

$H = 16, \cancel{1}$ $(\because H>1)$

(A) $A+G = 25+20 = 45 \neq 30H$

(B) $G+H = 36 = A+11$

(C) $4(G+H) = 144 \neq A-1$

(D) $A+G = 45 = 3(H-1)$

QUESTION

If $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ are in H.P. then show that $a_1a_2 + a_2a_3 + a_3a_4 + \ldots\ldots + a_{n-1}a_n = (n-1)\, a_1a_n$

$\frac{1}{a_1}, \frac{1}{a_2}, \ldots \frac{1}{a_n}$ are in A.P say with common diff $= d$

$$\frac{1}{a_2} - \frac{1}{a_1} = d \Rightarrow a_1 - \cancel{a_2} = d a_1 a_2$$

$$\frac{1}{a_3} - \frac{1}{a_2} = d \Rightarrow \cancel{a_2} - \cancel{a_3} = d a_2 a_3$$

$$\vdots$$

$$\frac{1}{a_n} - \frac{1}{a_{n-1}} = d \Rightarrow \cancel{a_{n-1}} - a_n = d\, a_{n-1} a_n$$

$$a_1 - a_n = d\,(a_1a_2 + a_2a_3 + \ldots + a_n a_{n-1})$$

Now $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$

$$\frac{1}{a_n} - \frac{1}{a_1} = (n-1)d$$

$$a_1 - a_n = (n-1)d\, a_1 a_n$$

$$(n-1)\cancel{d}\, a_1 a_n = \cancel{d}\,(a_1a_2 + a_2a_3 + \ldots + a_n a_{n-1})$$

(H.P)

Using AM -GM-HM Inequality

applicable to +ve reals

If $a_1, a_2, \ldots a_n \in R^+$ then $\dfrac{a_1+a_2+\cdots+a_n}{n} \geq (a_1 \cdot a_2 \cdots a_n)^{1/n} \geq \dfrac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}}$.

QUESTION

If a, b, c are positive numbers & $a + 2b + 3c = 7$ then find maximum value of :

(i) $(ab^2 c^3)$ (ii) $(a^2b^3c^4)$ (iii) (ab^2c)

(i) $a+2b+3c=7$

$a,b,c \in R^+$
By AM-GM Ineq $\Rightarrow \dfrac{a+b+b+c+c+c}{6} \geq (a\,b\cdot b\cdot c\cdot c\cdot c)^{\frac{1}{6}}$

$\dfrac{a+2b+3c}{6} \geq (ab^2c^3)^{1/6}$

$ab^2c^3 \leq \left(\dfrac{7}{6}\right)^6$.

$ab^2c^3\big|_{MAX} = \left(\dfrac{7}{6}\right)^6$ at $a=b=c$

$a+2a+3a=7$

$a=7/6$

$a=b=c=7/6$

QUESTION

If a, b, c are positive numbers & a + 2b + 3c = 7 then find maximum value of :

(i) $(ab^2 c^3)$ (ii) $(a^2b^3c^4)$ — $a\cdot a\cdot b\cdot b\cdot b\cdot c\cdot c\cdot c\cdot c$ (iii) (ab^2c)

(iii)

$a+2b+3c=7$

$\frac{a+b+b+3c}{4} \geq (ab^2 3c)^{\frac{1}{4}}$

$\frac{7}{4} \geq (3ab^2c)^{1/4}$

$ab^2c \leq (7/4)^4 \cdot \frac{1}{3}$

$ab^2c|_{MAX} \leq \frac{1}{3}(7/4)^4$

(ii) $a+2b+3c=7$

$$\frac{\frac{a}{2}+\frac{a}{2}+\frac{2b}{3}+\frac{2b}{3}+\frac{2b}{3}+\frac{3c}{4}+\frac{3c}{4}+\frac{3c}{4}+\frac{3c}{4}}{9} \geq \left(\frac{a}{2}\cdot\frac{a}{2}\cdot\frac{2b}{3}\cdot\frac{2b}{3}\cdot\frac{2b}{3}\left(\frac{3c}{4}\right)^4\right)^{\frac{1}{9}}$$

$\frac{7}{9} \geq \left(\frac{3}{128}\cdot a^2b^3c^4\right)^{\frac{1}{9}}$

$a^2b^3c^4 \leq \left(\frac{7}{9}\right)^9 \cdot \frac{128}{3}$

$a^2b^3c^4|_{MAX} = \left(\frac{7}{9}\right)^9 \cdot \frac{128}{3}$

$\frac{a}{2}=\frac{2b}{3}=\frac{3c}{4}=6\lambda = 6\cdot\frac{7}{54}=\frac{7}{9}$

$a=12\lambda,\ b=9\lambda,\ c=8\lambda$

$a+2b+3c=7$

$12\lambda+18\lambda+24\lambda=7$

$\lambda=7/54$

QUESTION [JEE Mains 2023 (11 April)]

Jan 01 on Tuesday

Let a, b, c and d be positive real numbers such that $a + b + c + d = 11$. If the maximum value of $a^5 b^3 c^2 d$ is 3750β, then the value of β is

(A) 110

(B) 108

(C) 90

(D) 55

Ans. C

Cauchy Schwarz Inequality

$$|\bar{V}_1 \cdot \bar{V}_2| = \big||\bar{V}_1||\bar{V}_2|\cos\theta\big| = |\bar{V}_1|\,|\bar{V}_2|\,\underbrace{|\cos\theta|}_{\le 1} \le |\bar{V}_1|\,|\bar{V}_2|$$

Also $|\bar{V}_1 \cdot \bar{V}_2| = |\bar{V}_1|\,|\bar{V}_2|$ @ $|\cos\theta| = 1$

$\theta = 0, \pi$

$\downarrow$

$\bar{V}_1 \parallel \bar{V}_2$

$\bar{V}_1 = a_1 i + a_2 j + a_3 k$

$\bar{V}_2 = b_1 i + b_2 j + b_3 k$

$$|a_1b_1 + a_2b_2 + a_3b_3| \le \sqrt{a_1^2 + a_2^2 + a_3^2}\,\sqrt{b_1^2 + b_2^2 + b_3^2}$$

CS inequality: $(a_1b_1 + a_2b_2 + a_3b_3)^2 \le (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$

Equality holds if $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3}$

If $a_1, a_2, a_3 - - a_n$ & $b_1, b_2, - - b_n$ are some Real numbers

By CS Inequality

$$(a_1 b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_n b_n)^2 \le (a_1^2 + a_2^2 + \cdots + a_n^2)(b_1^2 + b_2^2 + \cdots + b_n^2)$$

where equality holds if $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \cdots = \frac{a_n}{b_n}$.

$\bar{V}_1 = 2i + 3j + 4\hat{k}$

$\bar{V}_2 = 4i + 6j + 8k$ $\quad \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$

$2\bar{V}_1 = \bar{V}_2$

$\longrightarrow \bar{V}_2$

$\longrightarrow \bar{V}_1$

$$\boxed{\text{Root Mean Square} \geqslant \text{AM} \geqslant \text{GM} \geqslant \text{HM}}$$

RMS = squares ke mean kaa roots

if $a_1, a_2, \ldots a_n$ are n +ve Reals

also $b_1 = 1, b_2 = 1, \ldots b_n = 1$

By CS Ineq.

$$(a_1b_1 + a_2b_2 + \ldots + a_nb_n)^2 \leq (a_1^2 + a_2^2 + \ldots + a_n^2)(b_1^2 + b_2^2 + \ldots + b_n^2)$$

$$(a_1 + a_2 + \ldots + a_n)^2 \leq (a_1^2 + a_2^2 + \ldots + a_n^2)(1^2 + 1^2 + \ldots + 1^2)$$

$$(a_1 + a_2 + \ldots + a_n)^2 \leq (a_1^2 + a_2^2 + \ldots + a_n^2) \cdot n$$

Divide both sides by n^2

$$\left(\frac{a_1 + a_2 + \ldots + a_n}{n}\right)^2 \leq \frac{a_1^2 + a_2^2 + \ldots + a_n^2}{n}$$

$$\frac{a_1+a_2+\cdots+a_n}{n} \le \sqrt{\frac{a_1^2+a_2^2+\cdots+a_n^2}{n}}$$

$$A \cdot M \le RMS$$

$$\Rightarrow \boxed{RMS \ge A \cdot M \ge G \cdot M \ge HM}$$

Equality holds if $a_1 = a_2 = \cdots = a_n$.

$R \cdot MS \ge A \cdot M$ can be applied any set of reals.

$2, -3, 4$

$$\frac{2-3+4}{3} \le \sqrt{\frac{2^2+(-3)^2+4^2}{3}}$$

$$1 \le \sqrt{\frac{29}{3}}$$

Two No: a, b

$$A = \frac{a+b}{2}$$

$$G = \sqrt{ab}$$

$$H = \frac{2}{\frac{1}{a}+\frac{1}{b}}$$

$\downarrow$

Generalized to n Reals.

$a_1, a_2, a_3 - - a_n \in R^+$

$$A = \frac{a_1 + a_2 + - - + a_n}{n}$$

$$G = (a_1 \cdot a_2 - - a_n)^{1/n}$$

$$H = \frac{n}{1/a_1 + 1/a_2 + - + 1/a_n}$$

n A.M b/w a & b — $a, A_1, A_2, - - A_n, b$ are in A.P

n G.M b/w a & b — $a, G_1, G_2, - - G_n, b$ are G.P

n H.M b/w a & b, $a, H_1, H_2 - - H_n, b$ are in H.P

QUESTION

(a) If $x, y, z > 0$ and $x + y + z = 1$, prove that:

(i) $x^2yz \leq \frac{1}{64}$

Tahoz

(ii) $x^2 + y^2 + z^2 \geq \frac{1}{3}$

$$\sqrt{\frac{x^2+y^2+z^2}{3}} \geq \frac{x+y+z}{3}$$

$$\frac{x^2+y^2+z^2}{3} \geq \left(\frac{1}{3}\right)^2$$

$$x^2+y^2+z^2 \geq 1/3 \text{ (H.P)}$$

QUESTION

Tan 03

(b) If $a + b + c = 3$ and a, b, c are positive, then prove that $a^2b^3c^2 \le \dfrac{3^{10} \cdot 2^4}{7^7}$.

QUESTION

If a, b, c are positive then prove that $\frac{a^3}{4b}+\frac{b}{8c^2}+\frac{1+c}{2a}\geq\frac{5}{4}$

$\underline{LHS}=\frac{a^3}{4b}+\frac{b}{8c^2}+\frac{1}{2a}+\frac{c}{2a}$

$=\frac{a^3}{4b}+\frac{b}{8c^2}+\frac{1}{2a}+\frac{c}{4a}+\frac{c}{4a}$

Now By AM-GM Ineq.

$$\frac{\frac{a^3}{4b}+\frac{b}{8c^2}+\frac{1}{2a}+\frac{c}{4a}+\frac{c}{4a}}{5}\geq\left(\frac{a^3}{4b}\cdot\frac{b}{8c^2}\cdot\frac{1}{2a}\cdot\frac{c}{4a}\cdot\frac{c}{4a}\right)^{1/5}=\left(\frac{1}{2^{10}}\right)^{\frac{1}{5}}=\frac{1}{2^2}=\frac{1}{4}$$

$$\frac{a^3}{4b}+\frac{b}{8c^2}+\frac{1+c}{2a}\geq 5/4 \ (QED)$$

QUESTION

If $a_i < 0$ for all $i = 1, 2, \ldots, n$ prove that

(i) $(a_1 + a_2 + \cdots + a_n)\left(\underbrace{\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}}\right) \geq n^2.$

$a_i < 0 \Rightarrow -a_i > 0$

By AM-HM Ineq

$$\frac{(-a_1) + (-a_2) + \cdots + (-a_n)}{n} \geq \frac{n}{\frac{1}{-a_1} + \frac{1}{-a_2} + \cdots + \frac{1}{-a_n}}.$$

$$-(a_1 + a_2 + \cdots + a_n) \cdot -\left(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}\right) \geq n^2$$

$$(a_1 + a_2 + \cdots + a_n)\left(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}\right) \geq n^2.$$

QUESTION

Tan04

If $a_i < 0$ for all $i = 1, 2, \dots, n$ prove that

(ii) $(1 - a_1 + a_1^2)(1 - a_2 + a_2^2) \cdots (1 - a_n + a_n^2) \geq 3^n (a_1 a_2 \dots a_n)$ (where n is even)

QUESTION

If $x > 0, y > 0, z > 0$, then prove that $(x + y)(y + z)(z + x) \geq 8xyz$.

$$\frac{x+y}{2} \geq \sqrt{xy}$$

$$\frac{y+z}{2} \geq \sqrt{yz}$$

$$\frac{z+x}{2} \geq \sqrt{zx}$$

$$\frac{(x+y)(y+z)(z+x)}{8} \geq \sqrt{x^2y^2z^2} = xyz$$

$$(x+y)(y+z)(z+x) \geq 8xyz.$$

$2 \geq 1$

$3 \geq 2$

$4 \geq 1$

$24 \geq 2$

QUESTION

If $\alpha_i, i \in \{1, 2, 3, \ldots 6\}$ are positive roots of the equation

$x^6 - 12x^5 + ax^4 - bx^3 - dx + 64 = 0$, then value of b is

$x^6 - 12x^5 + ax^4 - bx^3 - 0 \cdot x^2 - dx + 64 = 0$ — $\alpha_1, \alpha_2, \ldots, \alpha_6 \Rightarrow R^+$

$S_1 = \alpha_1 + \alpha_2 + \cdots + \alpha_6 = 12$

$S_6 = \alpha_1 \cdot \alpha_2 \cdots \alpha_6 = 64$

$A = \dfrac{\alpha_1 + \alpha_2 + \cdots + \alpha_6}{6} = 2$

$G = (\alpha_1 \alpha_2 \cdots \alpha_6)^{1/6} = (64)^{1/6} = 2$

$A = G \Rightarrow \alpha_1 = \alpha_2 = \cdots = \alpha_6 = 2.$

$S_3 = \underbrace{\alpha_1\alpha_2\alpha_3 + \alpha_1\alpha_2\alpha_4 + \cdots}_{\text{No. of terms} = {}^6C_3 = \frac{6 \cdot 5 \cdot 4}{3!} = 20} = -(-b) = b \Rightarrow 20 \times 8 = b$

$b = 160$

Properties of $\prod$

- $$\prod_{i=1}^{n} a_i b_i = \prod_{i=1}^{n} a_i \prod_{i=1}^{n} b_i$$
- $$\prod_{i=1}^{n} \frac{a_i}{b_i} = \frac{\prod_{i=1}^{n} a_i}{\prod_{i=1}^{n} b_i}$$
- $$\prod_{i=1}^{n} (k a_i) = k^n \prod_{i=1}^{n} a_i$$
- $$\prod_{i=1}^{n} k = k^n.$$

QUESTION [JEE Advanced 2020 (Paper 1)]

Let m be the minimum possible value of $\log_3(3^{y_1}+3^{y_2}+3^{y_3})$, where y_1, y_2, y_3 are real numbers for which $y_1+y_2+y_3=9$. Let M be the maximum possible value of $(\log_3 x_1+\log_3 x_2+\log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1+x_2+x_3=9$. Then the value of $\log_2(m^3)+\log_3(M^2)$ is

$m = \text{min of } \log_3(3^{y_1}+3^{y_2}+3^{y_3})$

$\frac{3^{y_1}+3^{y_2}+3^{y_3}}{3} \geq (3^{y_1}\cdot 3^{y_2}\cdot 3^{y_3})^{\frac{1}{3}}$

$\frac{3^{y_1}+3^{y_2}+3^{y_3}}{3} \geq (3^{y_1+y_2+y_3})^{1/3} = (3^9)^{\frac{1}{3}} = 27$

$3^{y_1}+3^{y_2}+3^{y_2} \geq 81$

$\log_3(3^{y_1}+3^{y_2}+3^{y_3}) \geq \log_3 81 = 4$

$m=4$

$M = \text{max of } \log_3 x_1+\log_3 x_2+\log_3 x_3$

$\frac{x_1+x_2+x_3}{3} \geq (x_1x_2x_3)^{1/3}$

$\frac{9}{3} \geq (x_1x_2x_3)^{1/3}$

$x_1x_2x_3 \leq 3^3 = 27.$

$\log_3(x_1x_2x_3) \leq \log_3 27 \Rightarrow \log_3 x_1+\log_3 x_2+\log_3 x_3 \leq 3 \Rightarrow M=3$

Ans. 8

$\log_2 m^3 + \log_3 M^2$

$= \log_2 4^3 + \log_3 3^2$

$= 6 + 2 = 8$ Ans

QUESTION

Show that if a, b, c, d be four positive unequal quantities and s = a + b + c + d, then (s – a) (s – b) (s – c) (s – d) > 81 abcd.

$s-a=b+c+d$ $\qquad \dfrac{b+c+d}{3} > \sqrt[3]{bcd}$

$s-b=a+c+d$ $\qquad \dfrac{a+c+d}{3} > \sqrt[3]{acd}$

$s-c=a+b+d$ $\qquad \dfrac{a+b+d}{3} > \sqrt[3]{abd}$

$s-d=a+b+c$ $\qquad \dfrac{a+b+c}{3} > \sqrt[3]{abc}$

$$\frac{(s-a)(s-b)(s-c)(s-d)}{81} > abcd$$

$(s-a)(s-b)(s-c)(s-d) > 81abcd.$

Exponential & Logarithmic Series

Exponential Series

$e^x = \sum_{r=0}^{\infty} \frac{x^r}{r!} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \infty$

$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots \infty = \sum_{r=0}^{\infty} (-1)^r \frac{x^r}{r!}$

$\frac{e^x + e^{-x}}{2} = \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots \infty\right)$

$\frac{e^x - e^{-x}}{2} = \left(\frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \infty\right)$

* $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \infty = \sum_{r=0}^{\infty} \frac{x^r}{r!}$

* $e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \infty = \sum_{r=0}^{\infty} \frac{1}{r!}$

* $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots \infty = \sum_{r=0}^{\infty} \frac{(-x)^r}{r!}$

$= \sum_{r=0}^{\infty} \frac{(-1)^r \cdot x^r}{r!}$

$$\sum_{n=1}^{\infty} \frac{1}{(n-1)!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \cdots \infty = e$$

$$\sum_{n=r}^{\infty} \frac{1}{(n-r)!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \cdots \infty = e.$$

QUESTION

Find the sum of following series :

$* \ (n+1)n! = (n+1)!$

$Ex: 5 \cdot 4! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$

$* \ \frac{n!}{n} = (n-1)!$

$Ex: \frac{5!}{5} = \frac{\cancel{5} \cdot 4 \cdot 3 \cdot 2 \cdot 1}{\cancel{5}} = 4!$

(i) $S = \sum_{n=1}^{100} n(n!)$

$T_n = n \cdot n!$

$= (n+1-1)n!$

$= (n+1) \cdot n! - n!$

$= (n+1)! - n!$

$T_1 = 2! - 1!$

$T_2 = 3! - 2!$

$T_3 = 4! - 3!$

$\vdots$

$T_{100} = 101! - 100!$

$\underline{101! - 1 = S}$

(ii) $S = \sum_{n=1}^{50} \frac{n}{(n+1)!}$

$T_n = \frac{n+1-1}{(n+1)!}$

$T_n = \frac{(n+1)}{(n+1) \cdot n!} - \frac{1}{(n+1)!} = \frac{1}{n!} - \frac{1}{(n+1)!}$

$T_1 = \frac{1}{1!} - \frac{1}{2!}$

$T_2 = \frac{1}{2!} - \frac{1}{3!}$

$T_3 = 1/3! - 1/4!$

$\vdots$

$T_{50} = \frac{1}{50!} - \frac{1}{51!} \Rightarrow S = 1 - \frac{1}{51!}$

(iii) $S = \sum_{r=1}^{100} (r^2 + 1) \cdot r!$

TAH05

QUESTION [JEE Mains 2023 (30 Jan)]

Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$.

Then $a^2 - b + c$ is equal to ________

$$T_n = \frac{n^3 \cdot (2n)! + n!\,(2n-1)}{n!\,(2n)!} = \frac{n^3}{n!} + \frac{2n-1}{(2n)!} = \frac{n^2}{(n-1)!} + \frac{2n}{(2n)!} - \frac{1}{(2n)!}$$

$$= \frac{n^2-1+1}{(n-1)!} + \frac{1}{(2n-1)!} - \frac{1}{(2n)!}$$

$$= \frac{(n+1)(n-1)}{(n-1)!} + \frac{1}{(n-1)!} + \frac{1}{(2n-1)!} - \frac{1}{(2n)!}$$

$$= \frac{n+1}{(n-2)!} + \frac{1}{(n-1)!} + \frac{1}{(2n-1)!} - \frac{1}{(2n)!}$$

$$\frac{n-2+3}{(n-2)!} = \frac{1}{(n-3)!} + \frac{3}{(n-2)!}$$

Ans. 26

$$T_n = \frac{1}{(n-3)!} + \frac{3}{(n-2)!} + \frac{1}{(n-1)!} + \frac{1}{(2n-1)!} - \frac{1}{(2n)!}$$

$$\sum_{n=0}^{\infty} T_n = \sum_{n=3}^{\infty} \frac{1}{(n-3)!} + \sum_{n=2}^{\infty} \frac{3}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

$$= e + 3e + e + \left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \cdots \infty\right) - \left(\frac{1}{0!} + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \cdots \infty\right)$$

$$= 5e + \frac{e - e^{-1}}{2} - \underbrace{\left(1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \cdots \infty\right)}$$

$$= 5e + \frac{e}{2} - \frac{1}{2e} - \left(\frac{e + e^{-1}}{2}\right)$$

$$= 5e + \frac{e}{2} - \frac{e}{2} - \frac{1}{2e} - \frac{1}{2e}$$

$$= 5e - \frac{1}{e}$$

$a = 5,\ b = -1,\ c = 0$

$a^2 - b + c = 25 + 1 + 0 = 26.$

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Solution to Previous TAH

QUESTION [JEE Mains 2024 (8 April)]

★★KCLS★★

Let the positive integers be written in the form :

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 2 & & 3 & & \\
 & 4 & & 5 & & 6 & \\
7 & & 8 & & 9 & & 10 \\
\end{array}
$$

If the k^{th} row contains exactly k numbers for every natural number k, then the row in which the number 5310 will be, is ______

Ans. 103

Jan-01

$R_1 \to 1$

$R_2 \to 2 \quad 3$

$R_3 \to 4 \quad 5 \quad 6$

$7 \quad 8 \quad 9 \quad 10$

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$an^2+bn+c = T_n$

$a+b+c=1$

$4a+2b+c=2$

$9a+3b+c=4$

$3a+b=1$

$5a+b=2$

$2a=1$

$\boxed{a=1/2}$ $\boxed{b=-1/2}$ $\boxed{c=1}$

$T_n = \frac{n^2}{2} - \frac{n}{2} + 1$

$T_n = \frac{n^2-n+2}{2}$; $T_n \le 5310$

$\frac{n^2-n+2}{2} \le 5310$

$n^2-n+2 \le 5310\times 2$

$n^2 - n + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 2 \le 5310\times 2$

Richathakur

$(n-1/2)^2 + 3/2 \le 5310\times 2$

$(n-1/2)^2 \le 10620 - 3/2$

$(n-1/2)^2 \le 10618.5$

$n-1/2 \le 103\ldots$

$n \le 103\ldots + 0.5$

$\boxed{n=103}$

$T_{103} \Rightarrow \frac{(103)^2-(103)+2}{2}$

$T_{103} = \frac{10609-103+2}{2}$

$T_{103} = 5254$

last term of 103^{rd} row $\Rightarrow 5254+102 \Rightarrow 5356$

$\therefore$ 5310 lies in the 103^{rd} row

QUESTION

$2 + 5 + 14 + 41 + 122 + \ldots\ldots$ up to n terms.

Tah-02

$\underbrace{2+5}_{3}\underbrace{+14}_{9}\underbrace{+41}_{27}\underbrace{+122}_{81}+ \ldots$ upto n terms

$3 \quad 9 \quad 27 \quad 81 \longrightarrow CR \to 3$

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$$T_n = ar^n + b$$

$$T_n = ar^n + b$$

$$T_1 = 3a + b = 2$$

$$T_2 = 9a + b = 5$$

$$T_3 = 27a + b = 14$$

$$6a = 3$$

$$\boxed{a = \frac{1}{2}}$$

$$\boxed{b = \frac{1}{2}}$$

Richathakur

$$T_n = \frac{1}{2}3^r + \frac{1}{2}$$

$$\frac{1}{2}\left[\sum_{r=1}^{n} 3^r + \sum_{r=1}^{n} 1\right] \Rightarrow \frac{1}{2}\left[\frac{3(3^n-1)}{(3-1)} + n\right]$$

$$\frac{n}{2} + \frac{3(3^n-1)}{4} \quad \text{Ans}$$

$\boxed{\text{TAH-02}}$ $2+5+14+41+122+\cdots$ up to n terms

$2+5+14+41+122+\cdots$

$3, 9, 27 \cdots$ in G.P with CR = 3

$$\boxed{T_n = ar^n + b}$$

$$T_1 = a(3)^1 + b = 2 \Rightarrow 3a+b=2$$
$$T_2 = a(3)^2 + b = 5 \Rightarrow 9a+b=5$$
$$T_3 = a(3)^3 + b = 14 \Rightarrow 27a+b=14$$

$a = \frac{1}{2}$, $b = \frac{1}{2}$

$$T_n = \frac{1}{2}3^n + \frac{1}{2}$$

$$\sum_{n=1}^{n} \frac{1}{2}3^n + \frac{1}{2}$$

$$\frac{1}{2}\sum_{n=1}^{n} 3^n + \frac{1}{2}n$$

$$\frac{1}{2}(3^1+3^2+3^3+\cdots 3^n) + \frac{1}{2}n$$

$$\frac{1}{2}\left[3\left(\frac{3^n-1}{3-1}\right)\right] + \frac{1}{2}n$$

$$\frac{1}{2}\left[\frac{3(3^n-1)}{2}\right] + \frac{1}{2}n$$

$$\frac{3}{4}(3^n-1) + \frac{1}{2}n$$

QUESTION

The sum of the first 20 terms of the series $5 + 11 + 19 + 29 + 41 + \ldots\ldots$ is

(A) 3420

(B) 3450

(C) 3250

(D) 3520

Ans. D

2 4 A1

Tah.03 Sum of 20 terms of the series $5+11+19+29+41+\ldots$ is

(A) 3420 (C) 3250

(B) 3450 (D) 3520

6 8 10 12 $\longrightarrow$ AP

$T_n = an^2+bn+c$

Richathakur

$T_1 = a+b+c = 5$

$T_2 = 4a+2b+c = 11$

$T_3 = 9a+3b+c = 19$

$3a+b=6$

$5a+b=8$

$2a=2$

$\boxed{a=1}$ $\boxed{b=3}$ $\boxed{c=1}$

$T_n = n^2+3n+1$

$$\sum_{n=1}^{20} n^2+3n+1 \Rightarrow \frac{20\cdot 21\cdot 41}{6} + \frac{3\times 20\times 21}{2} + 20$$

$\Rightarrow$ 3520 (D)

L-8

Sum of first 20 terms $5+11+19+29+41+\ldots$

1st order diff. 6 8 10 12

2nd order diff. 2 2 2

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TAH3

$$T_n' = an^2+bn+c$$

$$T_1' = a+b+c = 5$$

$$T_2' = 4a+2b+c = 11$$

$$T_3' = 9a+3b+c = 19$$

$$3a+b=6$$

$$5a+b=8$$

$a=1$, $b=3$, $c=1$

$$T_n = n^2+3n+1$$

$$\sum_{n=1}^{20} T_n = \sum_{n=1}^{20} n^2 + 3\sum_{n=1}^{20} n + 20$$

$$= \frac{\cancel{20}^{10}\cdot\cancel{21}^{7}\cdot 41}{\cancel{6}_{\cancel{3}}} + \frac{3\cdot\cancel{20}^{10}\cdot 21}{\cancel{2}} + 20$$

sakshi

$$= 2870+630+20$$

$$= 3520 \text{ Ans}$$

QUESTION [JEE Mains 2023 (8 April)]

Let a_n be the n^{th} term of the series $5 + 8 + 14 + 23 + 35 + 50 + \ldots$ and $S_n = \sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to :

(A) 11280

(B) 11290

(C) 11310

(D) 11260

Ans. B

Tah04 $5+8+14+23+35+50+\ldots$

$3 \quad 6 \quad 9 \quad 12 \quad 15 \rightarrow AP$

$T_n = an^2+bn+c$

$T_1 = a+b+c = 5$

$T_2 = 4a+2b+c = 8$ $\quad 3a+b=3$

$T_3 = 9a+3b+c = 14$ $\quad 5a+b=6$ $\quad 2a=3$

$\boxed{a=\frac{3}{2}}$

Richathakur

$\boxed{b=-\frac{3}{2}}$ $\quad \boxed{c=5}$

$T_n = \frac{3}{2}n^2 - \frac{3}{2}n+5$

$\frac{3}{2}\sum n^2 - \frac{3}{2}\sum n + \sum 5 \Rightarrow \frac{3}{2}\left(\frac{30\cdot 31\cdot 61}{6}\right) - \frac{3}{2}\left(\frac{30\cdot 31}{2}\right)+150$

$\Rightarrow \frac{56730}{4} - \frac{2790}{4} + 150$

$S_{30} \Rightarrow \frac{53940}{4}+150 \Rightarrow 13485+150$

$\boxed{S_{30} = 13635}$

$a_{40} = an^2+bn+c$

$= \frac{3}{2}(40)^2+(-\frac{3}{2})(40)+5$

$= \frac{3}{2}\times 1600 - 3\times 20+5$

$= 2405-60$

$\boxed{a_{40} = 2345}$

$S_{30} - a_{40} \Rightarrow 13635-2345 = 11290$ (B)

$5 + 8 + 14 + 23 + 35 + 50 + \cdots$

$3 \quad 6 \quad 9 \quad 12 \quad 15$

$3 \quad 3 \quad 3 \quad 3$

Tah4

$a_n = T_n = \frac{3}{2}n^2 - \frac{3}{2}n + 5$

$T_n' = an^2 + bn + c$

$a_K = \frac{3}{2}K^2 - \frac{3}{2}K + 5$

$T_1 = a + b + c = 5$

$T_2 = 4a + 2b + c = 8$

$T_3 = 9a + 3b + c = 14$

$3a + b = 3$

$5a + b = 6$

$a = \frac{3}{2}$

$b = -\frac{3}{2}$

$c = 5$

$a_{40} = \frac{3}{2}(40)^2 - \frac{3}{2} \times 40 +$

$\boxed{a_{40} = 2345}$

$S_n = \sum_{K=1}^{n} a_K = \frac{3}{2}\sum_{K=1}^{n} K^2 + \sum_{K=1}^{n}\left(-\frac{3}{2}\right)K + \sum_{K=1}^{n} 5$

$S_n = \frac{3}{2}\frac{n(n+1)(2n+1)}{\cancel{6}\,2} - \frac{3}{2}\left(\frac{n(n+1)}{2}\right) + 5n$

$S_{30} = \frac{1}{\cancel{4}\,2}(\overset{15}{\cancel{30}} \cdot 31 \cdot 61) - \frac{3}{\cancel{4}\,2}(\overset{15}{\cancel{30}} \cdot 31) + 5 \cdot 30$

$= \frac{1}{2}[28365 - 1395] + 150$

sakshi

$= \frac{27270}{2} \Rightarrow 13635$

Now, $S_{30} - a_{40} \Rightarrow 13635 - 2345 \Rightarrow 11290$ Ans

Tah-04: Let a_n be the n^{th} term of the series :

$$5 + 8 + 14 + 23 + 35 + 50 + \dots$$

$$3, 6, 9, 12, 15, \dots$$

$$3, 3, 3, 3 \dots$$ 2^{nd} order diff is const.

$T_n = an^2 + bn + c$

$$\left.\begin{aligned} a+b+c &= 5 \\ 4a+2b+c &= 8 \\ 9a+3b+c &= 14 \end{aligned}\right\} \rightarrow \begin{aligned} 3a+b &= 3 \\ 5a+b &= 6 \end{aligned} \rightarrow \begin{aligned} a &= 3/2 \\ b &= -3/2 \\ c &= 5. \end{aligned}$$

$\Rightarrow T_n = \frac{3}{2}n^2 - \frac{3}{2}n + 5$

$= \frac{1}{2}(3n^2 - 3n + 10).$

$$\begin{aligned} T_{40} &= \frac{1}{2}(4800 - 120 + 10) \\ &= \frac{1}{2} \times 4690. \\ &= 2345. \end{aligned}$$

$S_{30} = \frac{1}{2}\sum_{n=1}^{30} 3n^2 - 3n + 10.$

$= \frac{1}{2}\left[\cancel{3} \times \frac{30\times 31\times 61}{\cancel{6}\ 2} - 3\times\frac{30\times 31}{2} + 10\times 30\right]$

$= \frac{1}{2}\left[\frac{30}{2}\left[31\times 61 - 3\times 31 + 20\right]\right]$

$= \frac{1}{2}\left[15\left[1891 - 93 + 20\right]\right]$

$= \frac{1}{2}\left[15\left[1911 - 93\right]\right] \Rightarrow \left[15 \times 1818\right]\times\frac{1}{2}$

$\Rightarrow [27270]\times\frac{1}{2}$

$\Rightarrow \boxed{13,625}$

find : $S_{30} - a_{40}$

$= 13625 - 2345$

$= 11280$ Ans.

krish

QUESTION [JEE Mains 2023 (10 April)]

If $S_n = 4 + 11 + 21 + 34 + 50 + \ldots$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to :

(A) 227

(B) 226

(C) 220

(D) 223

Ans. D

Tah-05 $S_n = 4 + 11 + 21 + 34 + 50 + \ldots$ to n terms

$7 \quad 10 \quad 13 \quad 16$

$T_n = an^2 + bn + c$

$T_1 = a + b + c = 4$

$T_2 = 4a + 2b + c = 11$ $\quad 3a + b = 7$

$T_3 = 9a + 3b + c = 21$ $\quad 5a + b = 10$

$2a = 3$

$\boxed{a = \frac{3}{2}}$ $\quad \boxed{b = \frac{5}{2}}$ $\quad \boxed{c = 0}$

$T_n = \frac{3}{2}n^2 + \frac{5}{2}n$

$S_{29} = \frac{3}{2} \cdot \frac{29 \cdot 30 \cdot 59}{6} + \frac{5}{2} \cdot \frac{29 \cdot 30}{2}$

$\frac{29 \cdot 30}{4}[59 + 5] \Rightarrow \frac{29 \times 30}{\cancel{4}}[\cancel{64}]^{16}$

Richathakur

$\boxed{S_{29} \Rightarrow 29 \times 30 \times 16}$

$S_9 \Rightarrow \frac{3 \cdot 9 \cdot 10 \cdot 19}{2 \cdot 6} + \frac{5}{2} \times \frac{9 \times 10}{2}$

$S_9 = \frac{9 \cdot 10}{2}[19 + 5] \Rightarrow 9 \times 10 \times 6$

$\frac{1}{60}(S_{29} - S_9) \Rightarrow \frac{1}{\cancel{60}}[29 \times \cancel{30} \times 8 - \cancel{60} \times 9]$

$\frac{1}{60}(S_{29} - S_9)$ $\quad 232 - 9 = 223$ $\quad$ (D)

$S_n = 4 + 11 + 21 + 34 + 50 + \cdots$ n terms

7 10 13 16 ...

3 3 3 ...

Tah5

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$T_n' = an^2 + bn + c \longrightarrow T_n = \frac{3}{2}n^2 + \frac{5}{2}n + 0$

$T_1 = a + b + c = 4$

$T_2 = 4a + 2b + c = 11$

$T_3 = 9a + 3b + c = 21$

$3a + b = 7$

$5a + b = 10$

$a = 3/2$

$b = 5/2$

$c = 0$

$$S_n = \sum_{n=1}^{n} T_n \Rightarrow \frac{3}{2}\sum_{n=1}^{n} n^2 + \frac{5}{2}\sum_{n=1}^{n} n$$

$$S_n \Rightarrow \frac{3}{2}\cdot\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{5}{2}\left(\frac{n(n+1)}{2}\right)$$

sakshi

$$\therefore S_{29} \Rightarrow \frac{3}{2}\left(\frac{29\times30\times59}{6}\right) + \frac{5}{2}\left(\frac{29\times30}{2}\right)$$

$$S_{29} \Rightarrow \frac{1}{2}\left[3(145\times59) + 5(435)\right]$$

$$\boxed{S_{29} = 13920}$$

$$S_9 = \frac{1}{2}\left[\left[\frac{3\times9\times\cancel{10}^{5}\times19}{\cancel{6}_{2}}\right] + \left[\frac{5\times9\times\cancel{10}^{5}}{\cancel{2}}\right]\right]$$

$$\frac{1}{2}(855 + 225) \Rightarrow \frac{1080}{2}$$

$$\boxed{S_9 = 540}$$

Now, $\frac{1}{60}(S_{29} - S_9) \Rightarrow \frac{1}{60}(13920 - 540) \Rightarrow \frac{1338\cancel{0}}{6\cancel{0}} = 223$ Ans

Tah-05 : $S_n = 4 + 11 + 21 + 34 + 50 + \dots n$ term.

$7, 10, 13, 16 \dots$

$3, 3, 3, \dots$ 2nd order diff is const.

$\boxed{T_n = an^2 + bn + c}$

$a + b + c = 4$

$4a + 2b + c = 11$

$9a + 3b + c = 21$

$\to 3a + b = 7$

$\to 5a + b = 10$

$\to a = 3/2$

$b = 5/2$

$c = 0$.

$\Rightarrow T_n = \frac{3}{2}n^2 + \frac{5}{2}n$

$= \frac{1}{2}(3n^2 + 5n)$.

krish

$S_n = \sum_{n=1}^{n} T_n \Rightarrow \frac{1}{2}\sum_{n=1}^{n} 3n^2 + 5n$.

$S_{29} = \frac{1}{2}\left[\cancel{3} \times \frac{29 \times 30 \times 59}{\cancel{6}\,2} + 5 \times \frac{29 \times 30}{2}\right]$

$= \frac{1}{2} \times \frac{29 \times 30}{2}[59 + 5]$

$= \frac{1}{\cancel{2}} \times \frac{29 \times 30 \times \cancel{64}^{16}}{\cancel{2}} = \boxed{29 \times 30 \times 16}$.

$S_9 = \frac{1}{2}\left[\cancel{3} \times \frac{9 \times 10 \times 19}{\cancel{6}\,2} + 5 \times \frac{9 \times 10}{2}\right]$.

$= \frac{1}{\cancel{2}} \times \frac{9 \times 10}{\cancel{2}} \times \cancel{24}^{6} = \boxed{9 \times 10 \times 6}$

find : $\frac{1}{60}(S_{29} - S_9) = \frac{1}{\cancel{60}}(29 \times \cancel{30} \times \cancel{16}^{8} - 9 \times \cancel{10} \times \cancel{6})$

$= 232 - 9$

$= 223$ Ans.

QUESTION

$$\frac{1}{1\cdot3\cdot5}+\frac{1}{3\cdot5\cdot7}+\frac{1}{5\cdot7\cdot9}+\ldots\ldots$$

Tah-06 $\frac{1}{1\cdot 3\cdot 5}+\frac{1}{3\cdot 5\cdot 7}+\frac{1}{5\cdot 7\cdot 9}+\cdots$

$$T_n = \frac{1}{(2n-1)(2n+1)(2n+3)}$$

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Richathakur

$$T_n = \frac{(2n+3)-(2n-1)}{4(2n-1)(2n+1)(2n+3)}$$

$$T_n = \frac{1}{4}\left[\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right]$$

$$T_1 = \frac{1}{4}\left[\frac{1}{1\cdot 3} - \cancel{\frac{1}{3\cdot 5}}\right]$$

$$T_2 = 1/4\left[\cancel{\frac{1}{3\cdot 5}} - \cancel{\frac{1}{5\cdot 7}}\right]$$

$$T_3 = \frac{1}{4}\left[\cancel{\frac{1}{5\cdot 7}} - \cancel{\frac{1}{7\cdot 9}}\right]$$

$$\vdots$$

$$T_n = \frac{1}{4}\left[\cancel{\frac{1}{(2n-1)(2n+1)}} - \frac{1}{(2n+1)(2n+3)}\right]$$

$$= \frac{1}{4}\left[\frac{1}{1\cdot 3} - \frac{1}{(2n+1)(2n+3)}\right]$$

$$S_n = \frac{1}{4}\left[\frac{1}{3} - \frac{1}{(2n+1)(2n+3)}\right]$$

$$\boxed{S_\infty = \frac{1}{12}}$$ Ans

Tah-06 $\frac{1}{1\cdot3\cdot5}+\frac{1}{3\cdot5\cdot7}+\frac{1}{5\cdot7\cdot9}+\cdots$

$$T_n = \frac{1}{(1+(n-1)2)(3+(n-1)2)(5+(n-1)2)}$$

$$T_n = \frac{1}{(2n-1)(2n+1)(2n+3)}$$

$$T_n = \frac{(2n+3)-(2n-1)}{4(2n-1)(2n+1)(2n+3)}$$

$$T_n = \frac{1}{4}\left(\frac{1}{(2n-1)(2n+1)}-\frac{1}{(2n+1)(2n+3)}\right)$$

$$T_1 = \frac{1}{4}\left(\frac{1}{3}-\frac{1}{3\cdot5}\right)$$

$$T_2 = \frac{1}{4}\left(\frac{1}{3\cdot5}-\frac{1}{5\cdot7}\right)$$

$\vdots$

$$T_n = \frac{1}{4}\left(\frac{1}{(2n-1)(2n+1)}-\frac{1}{(2n+1)(2n+3)}\right)$$

$$\sum_{n=1}^{n} T_n = S_n = \frac{1}{4}\left(\frac{1}{3}-\frac{1}{(2n+1)(2n+3)}\right)$$

$$S_\infty = \frac{1}{12} \quad \left[\text{as; } S_\infty = \lim_{n\to\infty} S_n\right]$$

Ans.

Tah-06: $\frac{1}{1\cdot3\cdot5}+\frac{1}{3\cdot5\cdot7}+\frac{1}{5\cdot7\cdot9}+\cdots$

$$\Rightarrow T_n = \frac{1}{(2n-1)(2n+1)(2n+3)}.$$

$$T_n = \frac{(2n+3)-(2n-1)}{4(2n-1)(2n+1)(2n+3)}.$$

$$T_n = \frac{1}{4}\left[\frac{1}{(2n-1)(2n+1)}-\frac{1}{(2n+1)(2n+3)}\right]$$

\# $T_1 = \frac{1}{4}\left[\frac{1}{1\cdot3}-\frac{1}{3\cdot5}\right]$

$$T_2 = \frac{1}{4}\left[\frac{1}{3\cdot5}-\frac{1}{5\cdot7}\right]$$

$$T_3 = \frac{1}{4}\left[\frac{1}{5\cdot7}-\frac{1}{7\cdot9}\right]$$

$\vdots$

$$T_n = \frac{1}{4}\left[\frac{1}{(2n-1)(2n+1)}-\frac{1}{(2n+1)(2n+3}\right]$$

$$S_n = \frac{1}{4}\left[\frac{1}{3}-\frac{1}{(2n+1)(2n+3)}\right].$$

$$S_\infty = \lim_{n\to\infty} S_n = \frac{1}{4}\left(\frac{1}{3}-0\right)$$

$$= \frac{1}{12} \text{ Ans.}$$

krish

QUESTION [JEE Mains 2024 (9 April)]

If the sum of the series $\frac{1}{1\cdot(1+d)}+\frac{1}{(1+d)(1+2d)}+\cdots+\frac{1}{(1+9d)(1+10d)}$ is equal to 5, then 50d is equal to

(A) 5

(B) 10

(C) 15

(D) 20

Ans. A

Tah-07: $\frac{1}{1\cdot(1+d)} + \frac{1}{(1+d)(1+2d)} + \dots + \frac{1}{(1+9d)(1+10d)}$ is equal to 5, then $50d$ is :

$$\Rightarrow T_n = \frac{1}{(1+(n-1)d)(1+nd)}.$$

$$T_n = \frac{1+nd-(1+(n-1)d)}{d(1+(n-1)d)(1+nd)}.$$

$$\# \quad T_n = \frac{1}{d}\left[\frac{1}{(1+(n-1)d)} - \frac{1}{(1+nd)}\right].$$

$$\# \quad T_1 = \frac{1}{d}\left[1 - \frac{1}{1+d}\right].$$

$$T_2 = \frac{1}{d}\left[\frac{1}{1+d} - \frac{1}{1+2d}\right]$$

$$T_3 = \frac{1}{d}\left[\frac{1}{1+2d} - \frac{1}{1+3d}\right]$$

$$\vdots$$

$$T_n = \frac{1}{d}\left[\frac{1}{1+(n-1)d} - \frac{1}{(1+nd)}\right]$$

$$S_n = \frac{1}{d}\left[1 - \frac{1}{(1+nd)}\right]$$

5

$$\Rightarrow 5d = \frac{\cancel{1}+nd-\cancel{1}}{1+nd} \qquad n = 10$$

$$\Rightarrow \cancel{5}d = \frac{\overset{2}{\cancel{10}}d}{1+10d}$$

$$\Rightarrow 10d = 1$$

$$\Rightarrow \boxed{d = \frac{1}{10}} \Rightarrow \text{then } 50d = 5 \text{ Ans.}$$

krish

QUESTION

$$\frac{3}{1\cdot 2\cdot 4}+\frac{4}{2\cdot 3\cdot 5}+\frac{5}{3\cdot 4\cdot 6}+\ldots\ldots$$

Tah-7 (b) : $\frac{3}{1\cdot2\cdot4}+\frac{4}{2\cdot3\cdot5}+\frac{5}{3\cdot4\cdot6}+\cdots$

krish

$$\Rightarrow T_n = \frac{(n+2)}{n(n+1)(n+3)}$$

$$= \frac{(n+2)^2}{n(n+1)(n+2)(n+3)}$$

$$= \frac{n^2+4n+4}{n(n+1)(n+2)(n+3)} = \frac{(n^2+3n)+(n)+4}{n(n+1)(n+2)(n+3)}$$

$$= \frac{\cancel{n(n+3)}}{\cancel{n}(n+1)(n+2)\cancel{(n+3)}} + \frac{\cancel{n}}{\cancel{n}(n+1)(n+2)(n+3)} + \frac{4}{n(n+1)(n+2)(n+3)}$$

$$\# \; T_n = \frac{(n+2)-(n+1)}{(n+1)(n+2)} + \frac{(n+3)-(n+1)}{2(n+1)(n+2)(n+3)} + \frac{4}{3}\,\frac{(n+3)-n}{n(n+1)(n+2)(n+3)}$$

$$\Rightarrow T_n = \left(\frac{1}{n+1}-\frac{1}{n+2}\right) + \frac{1}{2}\left(\frac{1}{(n+1)(n+2)} - \frac{1}{(n+2)(n+3)}\right) + \frac{4}{3}\left(\frac{1}{n(n+1)(n+2)} - \frac{1}{(n+1)(n+2)(n+3)}\right)$$

$$\# \; T_1 = \left(\frac{1}{2}-\frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{2\cdot3}-\frac{1}{3\cdot4}\right) + \frac{4}{3}\left(\frac{1}{1\cdot2\cdot3}-\frac{1}{2\cdot3\cdot4}\right).$$

$$T_2 = \left(\frac{1}{3}-\frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{3\cdot4}-\frac{1}{4\cdot5}\right) + \frac{4}{3}\left(\frac{1}{2\cdot3\cdot4}-\frac{1}{3\cdot4\cdot5}\right).$$

$$T_3 = \left(\frac{1}{4}-\frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{4\cdot5}-\frac{1}{5\cdot6}\right) + \frac{4}{3}\left(\frac{1}{3\cdot4\cdot5}-\frac{1}{4\cdot5\cdot6}\right).$$

$\vdots$

$$T_n = \left(\frac{1}{n+1}-\frac{1}{n+2}\right) + \frac{1}{2}\left(\frac{1}{(n+1)(n+2)} - \frac{1}{(n+2)(n+3)}\right) + \frac{4}{3}\left(\frac{1}{n(n+1)(n+2)} - \frac{1}{(n+1)(n+2)(n+3)}\right)$$

$$S_n = \left(\frac{1}{2}-\frac{1}{n+2}\right) + \frac{1}{2}\left(\frac{1}{6}-\frac{1}{(n+2)(n+3)}\right) + \frac{4}{3}\left(\frac{1}{6}-\frac{1}{(n+1)(n+2)(n+3)}\right).$$

QUESTION [JEE Mains 2021]

$$\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+\cdots+\frac{1}{(201)^2-1} \text{ is equal to}$$

(A) 101/404

(B) 25/101

(C) 4

(D) 6

Tah-08 $\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+---+\frac{1}{(201)^2-1}$ is equal to

$$T_n=\frac{1}{(2n+1)^2-1^2} \Rightarrow \frac{1}{(2n+1-1)(2n+1+1)} \Rightarrow \frac{1}{(2n)(2n+2)}$$

$$T_n=\frac{1}{2}\frac{(2n+2)-(2n)}{(2n)(2n+2)}$$

$$\frac{1}{2}\left[\frac{\cancel{2n+2}}{2n(\cancel{2n+2})}-\frac{\cancel{2n}}{\cancel{2n}(2n+2)}\right]$$

$$T_n=\frac{1}{2}\left[\frac{1}{2n}-\frac{1}{2n+2}\right]$$

$$T_1=\frac{1}{2}\left[\frac{1}{2}-\cancel{\frac{1}{4}}\right]$$

Richathakur

$$T_2=\frac{1}{2}\left[\cancel{\frac{1}{4}}-\cancel{\frac{1}{6}}\right]$$

$$T_3=\frac{1}{2}\left[\cancel{\frac{1}{6}}-\frac{1}{8}\right]$$

$$S_n=\frac{1}{2}\left[\frac{1}{2}-\frac{1}{2n+2}\right]$$

$$S_n=\frac{1}{2}\left[\frac{1}{2}-\frac{1}{2n+2}\right]$$

$$S_{100}=\frac{1}{2}\left[\frac{1}{2}-\frac{1}{202}\right]$$

$$S_{100}=\frac{1}{2}\left[\frac{101-1}{202}\right]$$

$$S_{100}=\frac{\cancel{100}\ 25}{\cancel{404}\ 101}$$

$$\boxed{S_{100}=\frac{25}{101}}\ (B)$$

TAH-08

$$\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+\cdots\frac{1}{(201)^2-1}$$

$$T_n=\frac{1}{(2n+1)^2-1^2}=\frac{1}{2n(2n+2)}$$

$$=\frac{1}{4n(n+1)}$$

$$=\frac{1}{4}\left[\frac{1}{n}-\frac{1}{n+1}\right]$$

$$T_1=\frac{1}{4}\left[\frac{1}{1}-\cancel{\frac{1}{2}}\right]$$

$$T_2=\frac{1}{4}\left[\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right]$$

$$T_3=\frac{1}{4}\left[\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right]$$

$\vdots$

$$T_n=\frac{1}{4}\left[\cancel{\frac{1}{n}}-\frac{1}{n+1}\right]$$

$$S_n=\frac{1}{4}\left[1-\frac{1}{n+1}\right]$$

$$=\frac{1}{4}\left[\frac{n+1-1}{n+1}\right]$$

$$S_n=\frac{1}{4}\left[\frac{n}{n+1}\right]$$

$$S_{100}=\frac{1}{4}\left[\frac{100}{100+1}\right]$$

$$=\frac{25}{101}$$ Ans (B)

Tah-08 $\quad \frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+\cdots+\frac{1}{(201)^2-1}$

$$\Rightarrow T_n=\frac{1}{(2n+1)^2-1}=\frac{1}{2n(2n+2)}$$

$$T_n=\frac{(2n+2)-(2n)}{2\cdot 2n\cdot(2n+2)}$$

[now, $(201)=2(100)+1$ thus, $\boxed{n=100}$]

$$T_n=\frac{1}{2\cdot 2n}-\frac{1}{2(2n+2)}$$

$$T_n=\frac{1}{4}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$S_n=\sum_{n=1}^{100}T_n$$

$$T_1=\frac{1}{4}\left(1-\frac{1}{2}\right)$$

$$T_2=\frac{1}{4}\left(\frac{1}{2}-\frac{1}{3}\right)$$

$$T_3=\frac{1}{4}\left(\frac{1}{3}-\frac{1}{4}\right)$$

$\vdots$

$$T_n=\frac{1}{4}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$S_n=\frac{1}{4}\left(1-\frac{1}{n+1}\right)$$

$$S_{100}=\frac{1}{4}\left(1-\frac{1}{101}\right)=\frac{1}{4}\left(\frac{100}{101}\right)=\frac{25}{101}\ (B)$$

Ans.

Kritisha

QUESTION [JEE Mains 2021]

The sum of 10 terms of the series $\frac{3}{1^2\times2^2}+\frac{5}{2^2\times3^2}+\frac{7}{3^2\times4^2}+\cdots$ is:

(A) 1

(B) 120/121

(C) 99/100

(D) 143/144

Tāh-09

$$S_{10} = \frac{3}{1^2\times 2^2} + \frac{5}{2^2\times 3^2} + \frac{7}{3^2\times 4^2} + \cdots$$

$$\boxed{T_n = \frac{2n+1}{n^2(n+1)^2}} \Rightarrow \frac{(n+1)^2 - n^2}{n^2(n+1)^2}$$

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$$T_n = \left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

$$T_1 = \left[1 - \cancel{\frac{1}{4}}\right]$$

$$T_2 = \left[\cancel{\frac{1}{4}} - \cancel{\frac{1}{9}}\right]$$

$$T_3 = \left[\cancel{\frac{1}{9}} - \cancel{\frac{1}{16}}\right]$$

$$T_n = \left[\cancel{\frac{1}{n^2}} - \frac{1}{(n+1)^2}\right]$$

$$\boxed{S_{10} = 120/121}$$

Richathakur

$$S_n = \left[1 - \frac{1}{(n+1)^2}\right]$$

$$S_{10} = \left[1 - \frac{1}{121}\right] = \frac{120}{121} \text{ (B)}$$

TAH - 09

$$S = \frac{3}{1^2\times 2^2} + \frac{5}{2^2\times 3^2} + \frac{7}{3^2\times 4^2} + \cdots$$

$$T_n = \frac{(2n+1)}{\{n(n+1)\}^2}$$

$$T_n = \frac{2n+1}{n^2(n^2+2n+1)}$$

$$= \frac{(n^2+2n+1)-n^2}{n^2(n^2+2n+1)}$$

$$= \frac{1}{n^2} - \frac{1}{n^2+2n+1}$$

$$= \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$T_1 = \frac{1}{1^2} - \frac{1}{2^2}$$

$$T_2 = \frac{1}{2^2} - \frac{1}{3^2}$$

$$T_3 = \frac{1}{3^2} - \frac{1}{4^2}$$

$$\vdots$$

$$T_n = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$S_n = 1 - \frac{1}{(n+1)^2}$$

$$S_{10} = 1 - \frac{1}{(10+1)^2}$$

$$= \frac{121-1}{121} = \frac{120}{121}$$

Ans (B)

Tah - 09 The sum of 10 terms of the series

$$\frac{3}{1^2\times 2^2} + \frac{5}{2^2\times 3^2} + \frac{7}{3^2\times 4^2} + \cdots \text{ is}$$

$$\Rightarrow T_n = \frac{\{3+(n-1)2\}}{n^2\cdot(n+1)^2}$$

$$T_n = \frac{(2n+1)}{n^2\cdot(n+1)^2}$$

$$T_n = \frac{(n+1)^2 - n^2}{n^2\cdot(n+1)^2}$$

$$T_n = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$T_1 = \frac{1}{1} - \frac{1}{2^2}$$

$$T_2 = \frac{1}{2^2} - \frac{1}{3^2}$$

$$T_3 = \frac{1}{3^2} - \frac{1}{4^2}$$

$$\vdots$$

$$T_n = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$S_n = 1 - \frac{1}{(n+1)^2}$$

$$S_{10} = 1 - \frac{1}{(11)^2} = \frac{121-1}{121} = \frac{120}{121} \text{ (B)}$$

Kritisha

QUESTION [JEE Mains 2023 (1 Feb)]

★★ASRQ★★

The sum of 10 terms of the series $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\ldots\ldots$ is

(A) $\frac{58}{111}$

(B) $\frac{56}{111}$

(C) $\frac{55}{111}$

(D) $\frac{59}{111}$

Ans. C

Jan-10 $S_{10} = \frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \cdots$ is

$$\boxed{T_n = \frac{n}{1+n^2+n^4}} \Rightarrow \frac{n}{n^4+2n^2+1-n^2} \Rightarrow \frac{n}{(n^2+1)^2-n^2}$$

$$T_n \Rightarrow \frac{n}{(n^2+n+1)(n^2-n+1)} \Rightarrow \frac{1}{2}\left[\frac{(n^2+n+1)-(n^2-n+1)}{(n^2+n+1)(n^2-n+1)}\right]$$

$$T_n = \frac{1}{2}\left[\frac{1}{n^2-n+1} - \frac{1}{n^2+n+1}\right]$$

$$T_1 = \frac{1}{2}\left[\frac{1}{1-1+1} + \cancel{\frac{1}{1+1+1}}\right]$$

$$T_2 = \frac{1}{2}\left[\cancel{\frac{1}{3}} - \cancel{\frac{1}{7}}\right]$$

$$\vdots$$

$$T_{10} = \frac{1}{2}\left[\cancel{\frac{1}{100-10+1}} - \frac{1}{100+10+1}\right]$$

$$S_{10} = \frac{1}{2}\left[1 - \frac{1}{111}\right]$$

$$\boxed{S_{10} = 55/111} \quad (C)$$

$$S_{10} = \frac{110}{222} \Rightarrow \frac{55}{111}$$

Richathakur

Tah-10 ASRQ★★★

The sum of 10 terms of the series

$$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\cdots\cdots \text{ is}$$

$$\Rightarrow t_n=\frac{n}{1+n^2+n^4}=\frac{n}{n^4+2n^2+1-n^2}$$

$$t_n=\frac{n}{(n^2+1)^2-n^2}$$

$$t_n=\frac{n}{(n^2+n+1)(n^2-n+1)}$$

$$t_n=\frac{(n^2+n+1)-(n^2-n+1)}{2(n^2+n+1)(n^2-n+1)}=\frac{1}{2}\left(\frac{1}{n^2-n+1}-\frac{1}{n^2+n+1}\right)$$

$$t_1=\frac{1}{2}\left(\frac{1}{1}-\frac{1}{3}\right)$$

$$t_2=\frac{1}{2}\left(\frac{1}{3}-\frac{1}{7}\right)$$

$$t_3=\frac{1}{2}\left(\frac{1}{7}-\frac{1}{13}\right)$$

$$\vdots$$

$$t_n=\frac{1}{2}\left(\frac{1}{n^2-n+1}-\frac{1}{n^2+n+1}\right)$$

$$S_n=\frac{1}{2}\left(1-\frac{1}{n^2+n+1}\right)$$

$$S_{10}=\frac{1}{2}\left(1-\frac{1}{100+10+1}\right)=\frac{1}{2}\left(1-\frac{1}{111}\right)$$

$$=\frac{110}{2\times 111}=\frac{55}{111}\ \text{(c)}$$

Ans.

Kritisha

Tah-10: $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\cdots$ 10 terms.

$$\Rightarrow T_n=\frac{n}{1+n^2+n^4}\Rightarrow T_n=\frac{n}{n^4+2n^2+1-n^2}$$

$$\Rightarrow T_n=\frac{n}{(n^2+1)^2-n^2}$$

$$\Rightarrow T_n=\frac{n}{(n^2+n+1)(n^2-n+1)}$$

$$\Rightarrow T_n=\frac{(n^2+n+1)-(n^2-n+1)}{2(n^2+n+1)(n^2-n+1)}$$

$$\Rightarrow T_n=\frac{1}{2}\left[\frac{1}{(n^2-n+1)}-\frac{1}{(n^2+n+1)}\right]$$

$$\# \quad T_1=\frac{1}{2}\left[1-\frac{1}{3}\right]$$

$$T_2=\frac{1}{2}\left[\frac{1}{3}-\frac{1}{7}\right]$$

$$T_3=\frac{1}{2}\left[\frac{1}{7}-\frac{1}{13}\right]$$

$$\vdots$$

$$T_n=\frac{1}{2}\left[\frac{1}{(n^2-n+1)}-\frac{1}{(n^2+n+1)}\right]$$

$$S_n=\frac{1}{2}\left[1-\frac{1}{n^2+n+1}\right]$$

$$S_{10}=\frac{1}{2}\left[1-\frac{1}{111}\right]$$

$$=\frac{1}{2}\times\frac{110}{111}\quad 55$$

$$=\frac{55}{111}\ \text{Ans.}$$

krish

QUESTION [JEE Mains 2025 (4 April)]

★★ASRQ★★

If the sum of the first 20 terms of the series

$$\frac{4\cdot 1}{4+3\cdot 1^2+1^4}+\frac{4\cdot 2}{4+3\cdot 2^2+2^4}+\frac{4\cdot 3}{4+3\cdot 3^2+3^4}+\frac{4\cdot 4}{4+3\cdot 4^2+4^4}+\cdots \text{ is } \frac{m}{n},$$

where m and n are coprime, then m + n is equal to:

(A) 423

(B) 421

(C) 422

(D) 420

Ans. B

Tah-11 $\frac{1\cdot 1}{4+3\cdot 1^2+1^4}+\frac{1\cdot 2}{4+3\cdot 2^2+2^4}+\frac{1\cdot 3}{4+3\cdot 3^2+3^4}+\frac{1\cdot 4}{4+3\cdot 4^2+4^4}+\ldots$ is $\frac{m}{n}$

$$\frac{1n}{4+3n^2+n^4} \Rightarrow \frac{1n}{(n^2+2)^2-n^2} \Rightarrow \frac{1n}{(n^2+2+n)(n^2-n+2)}$$

$$T_n = \frac{1n}{2n}\left[\frac{(n^2+2+n)-(n^2-n+2)}{(n^2+2+n)(n^2-n+2)}\right]$$

$$T_n = 2\left[\frac{1}{n^2-n+2}-\frac{1}{n^2+n+2}\right]$$

Richathakur

$$T_1 = 2\left[\frac{1}{2}-\cancel{\frac{1}{4}}\right]$$

$$T_2 = 2\left[\cancel{\frac{1}{4}}-\cancel{\frac{1}{8}}\right]$$

$$T_{20} = 2\left[\cancel{\frac{1}{400-20+2}}-\frac{1}{400+20+2}\right]$$

$$2\left[\frac{1}{2}-\frac{1}{422}\right] \Rightarrow \boxed{\frac{210}{211}=\frac{m}{n}}$$

$$m+n = 210+211 \Rightarrow 421 \text{ (B)}$$

Tah-11: $\frac{4\cdot 1}{4+3\cdot 1^2+1^4}+\frac{4\cdot 2}{4+3\cdot 2^2+2^4}+\frac{4\cdot 3}{4+3\cdot 3^2+3^4}+\cdots+20$ terms

$$\Rightarrow T_n=\frac{4n}{4+3n^2+n^4}=\frac{4n}{n^4+4n^2+4-n^2}$$

$$\Rightarrow T_n=\frac{4n}{(n^2+2)^2-n^2}$$

$$\Rightarrow T_n=\frac{4n}{(n^2-n+2)(n^2+n+2)}$$

$$\Rightarrow T_n=\frac{2\left[(n^2+n+2)-(n^2-n+2)\right]}{(n^2-n+2)(n^2+n+2)}$$

$$\Rightarrow T_n=2\left[\frac{1}{n^2-n+2}-\frac{1}{n^2+n+2}\right].$$

#

$$T_1=2\left[\frac{1}{2}-\frac{1}{4}\right]$$

$$T_2=2\left[\frac{1}{4}-\frac{1}{8}\right]$$

$$T_3=2\left[\frac{1}{8}-\frac{1}{14}\right]$$

$$\vdots$$

$$T_n=2\left[\frac{1}{n^2-n+2}-\frac{1}{n^2+n+2}\right]$$

$$S_n=2\left[\frac{1}{2}-\frac{1}{n^2+n+2}\right]$$

$$\Rightarrow S_{20}=2\left[\frac{1}{2}-\frac{1}{422}\right]=2\times\frac{1}{2}\times\frac{210}{211}=\frac{210}{211}.$$

$$\Rightarrow \frac{210}{211}=\frac{m}{n}$$

$$\Rightarrow m=210,\ n=211 \Big\}\rightarrow m+n=421 \text{ Ans.}$$

krish

QUESTION [JEE Mains 2025 (2 April)]

★★ASRQ★★

If the sum of the first 10 terms of the series

$$\frac{4\cdot 1}{1+4\cdot 1^4}+\frac{4\cdot 2}{1+4\cdot 2^4}+\frac{4\cdot 3}{1+4\cdot 3^4}+\cdots \text{ is } \frac{m}{n}, \text{ where } \gcd(m,n)=1,$$

then $m+n$ is equal to

Ans. 441

Tah-12 $S_{10} = \frac{4\cdot 1}{1+4\cdot 1^4} + \frac{4\cdot 2}{1+4\cdot 2^4} + \frac{4\cdot 3}{1+4+3^4} + \dots$ is $\frac{m}{n}$

$$\frac{4n}{1+4n^4} \Rightarrow \frac{4n}{(1+2n^2-4n^2+4n^2)} \Rightarrow \frac{4n}{(2n^2+1)^2-4n^2}$$

$$\frac{4n}{(2n^2+1-2n)(2n^2+1+2n)} \qquad T_n = \frac{1}{(2n^2+1-2n)} - \frac{1}{2n^2+2n+1}$$

$$T_1 = 1 - \frac{1}{5}$$

$m = 220$ $\qquad T_2 = \frac{1}{5} - \frac{1}{13}$

$n = 221$ $\qquad T_3 = \frac{1}{13} - \dots$

$\boxed{m+n = 441}$

$$T_{10} = \frac{1}{200+1-20} - \frac{1}{200+1+20}$$

Richathakur

$$T_{10} = 1 - \frac{1}{221}$$

$$\boxed{T_{10} = \frac{220 \to m}{221 \to n}}$$

Tah-12: $\frac{4\cdot 1}{1+4\cdot 1^4} + \frac{4\cdot 2}{1+4\cdot 2^4} + \frac{4\cdot 3}{1+4\cdot 3^4} + \dots + 10$ term.

$$\Rightarrow T_n = \frac{4n}{1+4n^4} = \frac{4n}{(2n^2)^2+1^2}$$

$$\frac{4n}{((2n^2)^2+1^2+4n^2)-4n^2} \Rightarrow T_n = \frac{4n}{(2n^2+1)^2-(2n)^2} = \frac{4n}{(2n^2+1-2n)(2n^2+1+2n)}$$

$$\Rightarrow T_n = \frac{(2n^2+1+2n)-(2n^2+1-2n)}{(2n^2+1-2n)(2n^2+1+2n)}$$

$$\Rightarrow T_n = \frac{1}{(2n^2+1-2n)} - \frac{1}{(2n^2+1+2n)}$$

$$T_1 = 1 - \frac{1}{5}$$

$$T_2 = \frac{1}{5} - \frac{1}{13}$$

$$T_3 = \frac{1}{13} - \frac{1}{25}$$

$\vdots$

$$T_n = \frac{1}{2n^2-2n+1} - \frac{1}{(2n^2+2n+1)}$$

$$S_n = 1 - \frac{1}{(2n^2+2n+1)} \quad n = 10$$

$$S_{10} = 1 - \frac{1}{221}$$

$$= \frac{220}{221} = \frac{m}{n} \Rightarrow m = 220,\ n = 221$$

$$\Rightarrow m+n = 441 \text{ Ans.}$$

krish

Tah-012.

TAH 12

Lucky kumari
Bihar

$$\frac{4\cdot 1}{1+4\cdot 1^4}+\frac{4\cdot 2}{1+4\cdot 2^4}+\frac{4\cdot 3}{1+4\cdot 3^4}+\cdots \text{ upto 10 terms.}$$

$$T_n=\frac{4n}{1+4n^4}=\frac{4n}{1+(2n^2)^2}=\frac{4n}{[(2n^2)^2+1+4n^2]-4n^2}=\frac{4n}{(2n^2+1)^2-(2n)^2}$$

$$T_n=\frac{4n}{(2n^2+1+2n)(2n^2+1-2n)}=\frac{(2n^2+1+2n)-(2n^2+1-2n)}{(2n^2+1+2n)(2n^2+1-2n)}$$

$$T_n=\frac{1}{2n^2-2n+1}-\frac{1}{2n^2+2n+1}$$

$$T_1=\frac{1}{1}-\cancel{\frac{1}{5}}$$

$$T_2=\cancel{\frac{1}{5}}-\cancel{\frac{1}{13}}$$

$$T_3=\cancel{\frac{1}{13}}-\cancel{\frac{1}{25}}$$

$$\vdots$$

$$T_{10}=\cancel{\frac{1}{2(10)^2-2(10)+1}}-\frac{1}{221}$$

$$S_{10}=1-\frac{1}{221}=\frac{220}{221} \quad \text{— (I)}$$

$$S_{10}=\frac{m}{n} \quad \text{— (II)}$$

from (I) & (II) $\quad \frac{m}{n}=\frac{220}{221}$

Now, $\quad m+n=220+221$

$$\boxed{m+n=441}$$

QUESTION

$1 \cdot 3 \cdot 5 \cdot 7 + 3 \cdot 5 \cdot 7 \cdot 9 + 5 \cdot 7 \cdot 9 \cdot 11 + \ldots\ldots$ up to n terms.

Qn-13: $1\cdot3\cdot5\cdot7 + 3\cdot5\cdot7\cdot9 + 5\cdot7\cdot9\cdot11 + \cdots$ up to n terms.

$$\Rightarrow T_n = (2n-1)(2n+1)(2n+3)(2n+5)$$

krish

$$= \frac{(2n+7)-(2n-3)}{10}(2n-1)(2n+1)(2n+3)(2n+5) \quad (1)$$

$T_n = \frac{1}{10}\Big((2n-1)(2n+1)(2n+3)(2n+5)(2n+7) - (2n-3)(2n-1)(2n+1)(2n+3)(2n+5)\Big)$

$T_1 = \frac{1}{10}\left[1\cdot3\cdot5\cdot7\cdot9 - (-1)\cdot3\cdot5\cdot7\right]$

$T_2 = \frac{1}{10}\left[3\cdot5\cdot7\cdot9\cdot11 - 1\cdot3\cdot5\cdot7\cdot9\right]$

$T_3 = \frac{1}{10}\left[5\cdot7\cdot9\cdot11\cdot13 - 3\cdot5\cdot7\cdot9\cdot11\right]$

$\vdots$

$T_n = \frac{1}{10}\left[(2n-1)\cdots(2n+7) - (2n-3)\cdots(2n+5)\right]$

$$S_n = \frac{1}{10}\left[(2n-1)(2n+1)(2n+3)(2n+5)(2n+7) + 1\cdot3\cdot5\cdot7\right]$$

QUESTION

(a) The m^{th} term of a H.P. is n and the nth term is m. Prove that the p^{th} term is $\frac{mn}{p}$.

(b) If m^{th} term of an H.P. is n, and n^{th} term is equal to m then prove that $(m + n)^{th}$ term is $\frac{mn}{m+n}$.

Tah-013.

TAH 13

LUCKY KUMARI
BIHAR

$T_m = n$ (m^{th} term of HP) $\qquad T_n = m$ (n^{th} term of HP)

m^{th} term of AP.

$$T_m = \frac{1}{n}$$

$$a+(m-1)d = \frac{1}{n}$$

$$a = \frac{1}{n} - (m-1)d$$

$$a = \frac{1}{n} - (m-1)\left(\frac{1}{mn}\right)$$

$$= \frac{1}{n} - \frac{(m-1)}{mn}$$

$$= \frac{m-m+1}{mn}$$

$$= \frac{1}{mn}$$

n^{th} term of AP

$$T_n = \frac{1}{m}$$

$$a+(n-1)d = \frac{1}{m}$$

$$\left(\frac{1}{n} - (m-1)d\right) + (n-1)d = \frac{1}{m}$$

$$\frac{1}{n} - md + \cancel{d} + nd - \cancel{d} = \frac{1}{m}$$

$$\frac{1}{n} - \frac{1}{m} = md - nd$$

$$\frac{\cancel{m-n}}{mn} = \cancel{(m-n)}d$$

$$d = \frac{1}{mn}$$

Now, T_{m+n}^{th} term of AP.

$$\cancel{T_{m+n} = a+(n-1)d}$$

$$\cancel{= \frac{1}{mn} - (n-1)\left(\frac{1}{mn}\right)}$$

$$\cancel{= \frac{1-n}{mn}}$$

$$(T_{m+n})_{AP} = a+(m+n-1)d$$

$$= \frac{1}{mn} + \frac{(m+n-1)}{mn}$$

$$= \frac{\cancel{1}+(m+n-\cancel{1})}{mn}$$

$$= \frac{m+n}{mn}$$

$$\boxed{(T_{m+n}^{th})_{HP} = \frac{1}{(T_{m+n}^{th})_{AP}} = \frac{mn}{m+n}}$$

TQH - 15 (b) If m^{th} term of an H.P is n, and n^{th} term is equal to m then prove that $(m+n)^{th}$ term is $\frac{mn}{m+n}$

m^{th} term $= n$

n^{th} term $= m$

$$\frac{1}{a+(m-1)d} = n$$

$$\frac{1}{a+(n-1)d} = m$$

$$\cancel{a} + (m-1)d = 1/n$$

$$\cancel{a} + (n-1)d = 1/m$$

$$(m-n)d = \frac{m-n}{mn}$$

$$\boxed{\begin{aligned} d &= \frac{1}{mn} \\ a &= \frac{1}{mn} \end{aligned}}$$

$$T_{m+n} = \frac{1}{a+(m+n-1)d}$$

$$= \frac{1}{\frac{1}{mn}+(m+n-1)\frac{1}{mn}}$$

$$= \frac{1}{\frac{1}{mn}+\frac{(m+n)}{mn}-\frac{1}{mn}}$$

$$T_{m+n} = \frac{mn}{m+n}$$ Hence Proved

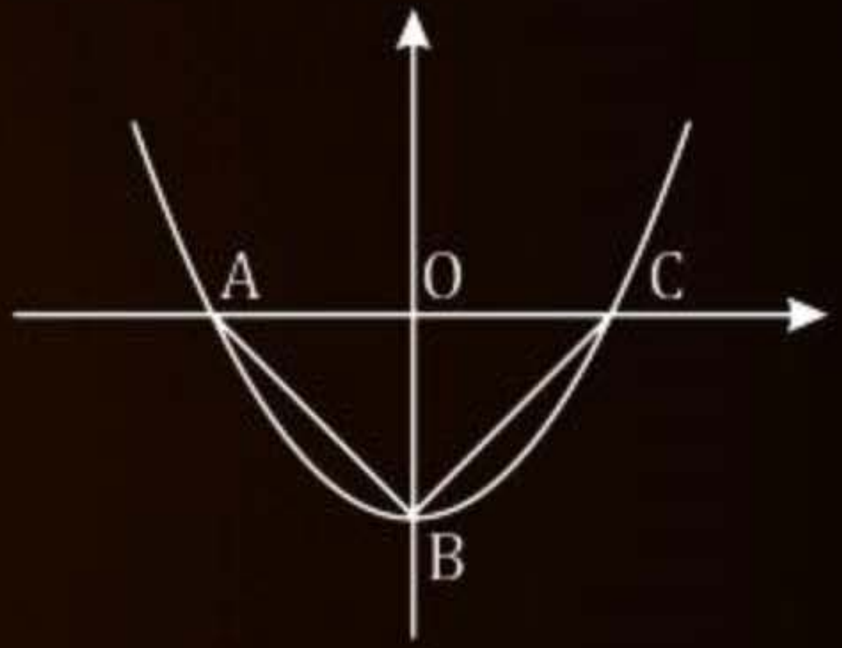
Solution to Previous RPPs

QUESTION

Rpp 1A

Paragraph

In the given figure vertices of ΔABC lie on $y = f(x) = ax^2 + bx + c$. The ΔABC is right angled isosceles triangle whose hypotenuse $AC = 4\sqrt{2}$ units, then $y = f(x)$ is given by



(A) $y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$

(B) $y = \frac{x^2}{2} - 2$

(C) $y = x^2 - 8$

(D) $y = x^2 - 2\sqrt{2}$

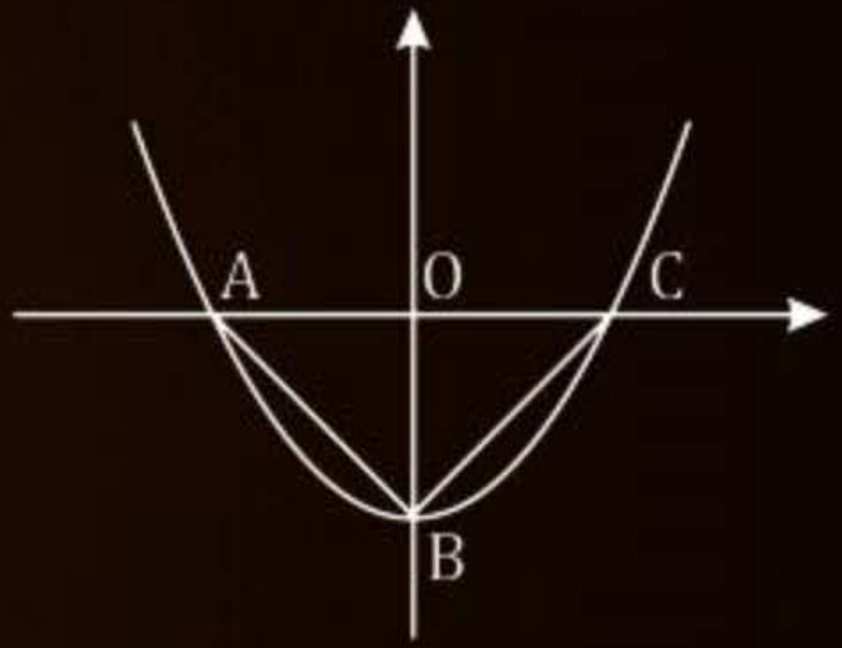
Ans. A

QUESTION

Rpp 1B

Paragraph

In the given figure vertices of ΔABC lie on $y = f(x) = ax^2 + bx + c$. The ΔABC is right angled isosceles triangle whose hypotenuse $AC = 4\sqrt{2}$ units, then



Minimum value of $y = f(x)$ is

(A) $2\sqrt{2}$

(B) $-2\sqrt{2}$

(C) 2

(D) -2

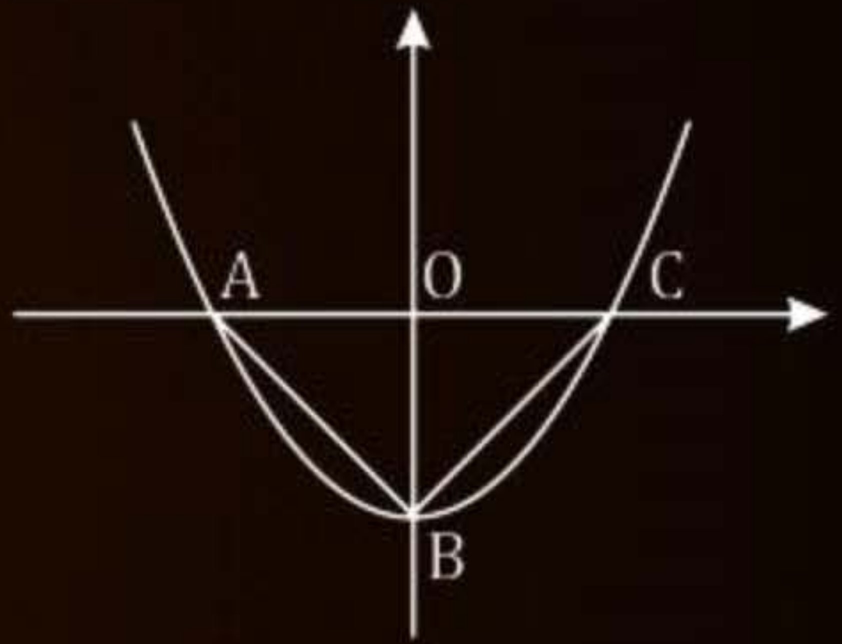
Ans. B

QUESTION

Rpp 1C

Paragraph

In the given figure vertices of ΔABC lie on $y = f(x) = ax^2 + bx + c$. The ΔABC is right angled isosceles triangle whose hypotenuse $AC = 4\sqrt{2}$ units, then



Number of integral value of k for which $\frac{k}{2}$ lies between the roots of $f(x) = 0$, is

(A) 9

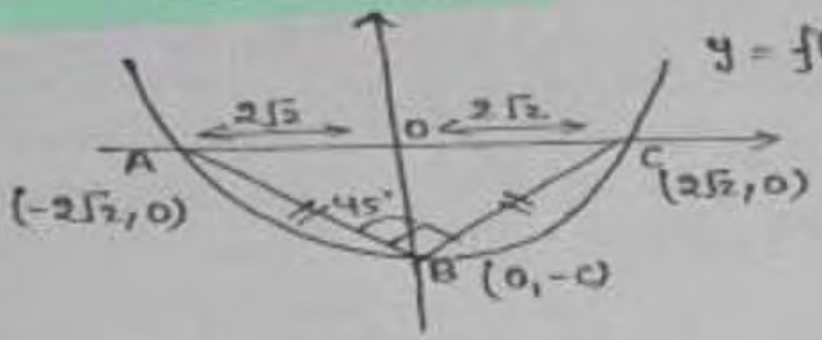
(B) 10

(C) 11

(D) 12

Ans. C

RPP-01.A.



$y = f(x) = ax^2+bx+c$

$\frac{y}{a} = x^2+\frac{b}{a}x+c$

$\frac{y}{a} = \left(x+\frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}+c$

minimal at $x = \frac{-b}{2a}$

$0 = \frac{-b}{2a}$

$\boxed{b=0}$

$\tan 45^\circ = \frac{2\sqrt{2}}{-c}$

$(c = -2\sqrt{2})$

Quadratic Eq^n:

$y = ax^2 - 2\sqrt{2}$

$ax^2 - 2\sqrt{2} = 0 \to 2\sqrt{2},\ -2\sqrt{2}$

$a(2\sqrt{2})^2 - 2\sqrt{2} = 0$

$a(2\sqrt{2})^2 = (2\sqrt{2})$

$\left[a = \frac{1}{2\sqrt{2}}\right]$

Hence, Quadratic polynomial,

$y = f(x)$

$y = \left(\frac{1}{2\sqrt{2}}\right)x^2 - 2\sqrt{2}$

$y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$

RPP-01.B.

y_{min} at B,

$y = -c$

$y = -2\sqrt{2}$.

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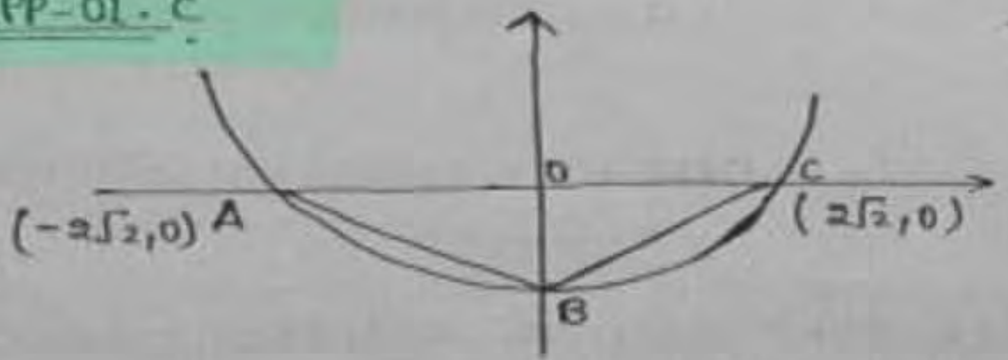
RPP-01.C.



$-2\sqrt{2} < \frac{K}{2} < 2\sqrt{2}$

$-4\sqrt{2} < K < 4\sqrt{2}$

$-4(1.4) < K < 4(1.4)$

$-5.6 < K < 5.6$

integral value of K = $-5,-4,-3,-2,-1,0,1,2,3,4,5$.

$\boxed{\text{no. of integral value of } K = 11}$

THANK YOU