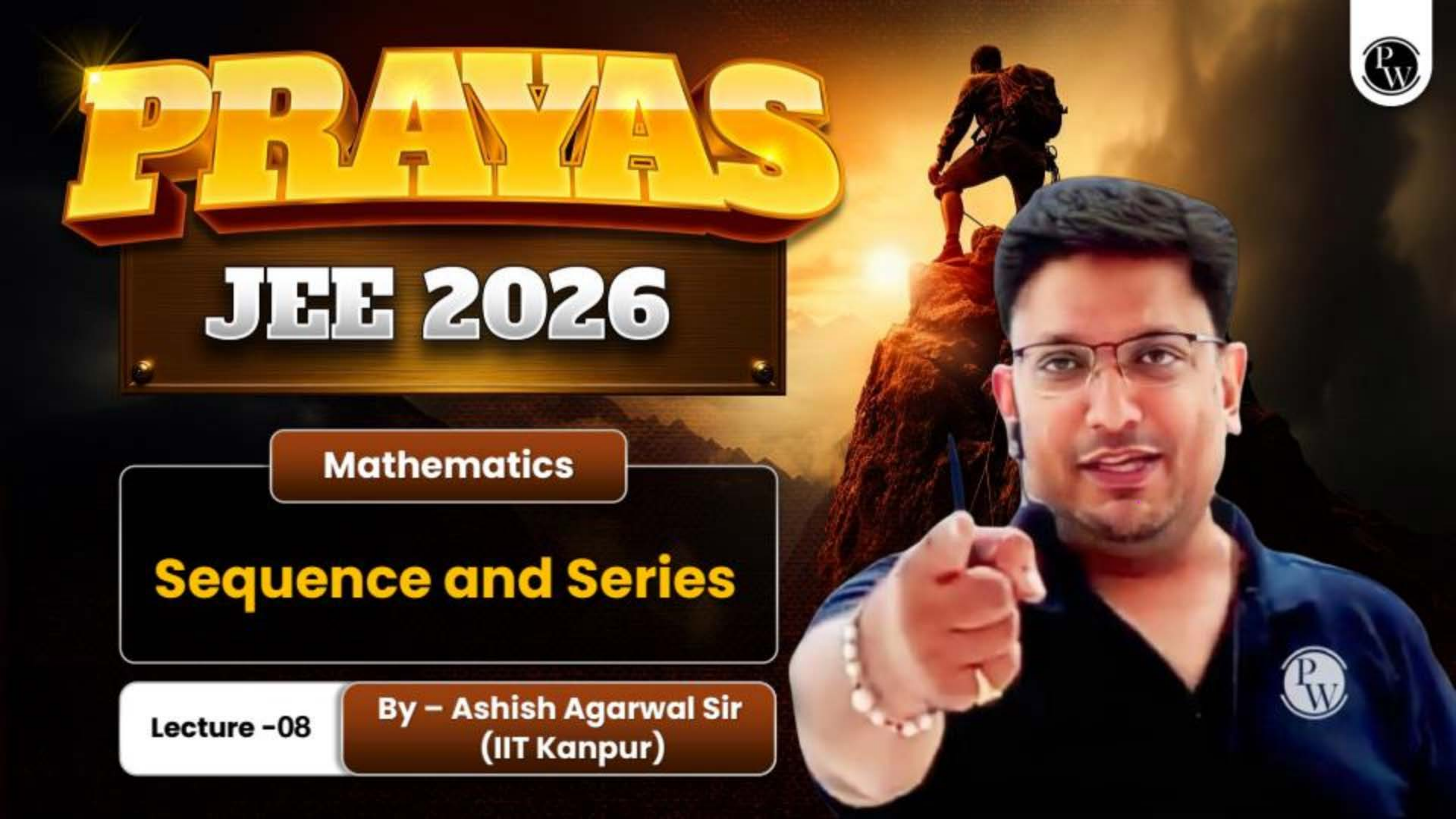
PRAYAS
JEE 2026
Mathematics
Sequence and Series
Lecture -08
By – Ashish Agarwal Sir
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Topics *to be covered*

- **A** Telescoping Series
- **B** Harmonic Progression
- **C** AM-GM-HM Inequaity

Recap of previous lecture

1. $\log_{0.3}(x-2) < 0$ then $x \in$ $(3, \infty)$

$x-2 > (0.3)^0$ & $x-2>0$

$x > 3$ $\downarrow$ No Need

2. Method of difference is used if difference between successive terms are in A.P or G.P

3. Second order difference is constant for two degree polynomial while third order difference is constant for three degree polynomial.

Recap *of previous lecture*

4. For a sequence let T_r be its r^{th} term then the sum of firs 12 terms of series is given by $S_{12} = \sum_{r=1}^{12} T_r$

5. $\sum n^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

6. $\sum n^3 = 1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

$(1+2+3+\cdots+n)^2 = 1^3+2^3+\cdots+n^3$

$(\cancel{T} | F)$

7. $\sum n = 1 + 2 + 3 + n = \frac{n(n+1)}{2}$

Recap of previous lecture

8. If α, β are roots of $ax^2 + bx + c = 0$ then quadratic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$ = $\frac{a}{y^2} + \frac{b}{y} + c = 0$

$y = \frac{1}{x} \rightarrow x = 1/y$

$cy^2 + by + a = 0$

$cx^2 + bx + a = 0$ Ans

9. If $x = 0$ is a root of quadratic $ax^2 + bx + c = 0$ then $P.O.R = 0 \Rightarrow c = 0$

10. If both roots of quadratic $ax^2 + bx + c = 0$ are 0 then $P.O.R = 0$, $c = 0$ & $S.O.R = 0 \Rightarrow b = 0$

Homework Discussion

QUESTION [IIT-JEE 2005 Screening]

S02

In the quadratic equation $ax^2 + bx + c = 0$, $\overset{D}{\Delta} = b^2 - 4ac$ and $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$, are in G.P. where α, β are the root of $ax^2 + bx + c = 0$, then

(A) $\Delta \neq 0$

(B) $b\Delta = 0$

(C) $c\Delta = 0$

(D) $\Delta = 0$

$ax^2+bx+c=0$

$(\alpha^2+\beta^2)^2 = (\alpha+\beta)\cdot(\alpha^3+\beta^3)$; $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$\cancel{\alpha^4}+\cancel{\beta^4}+2\alpha^2\beta^2 = \cancel{\alpha^4}+\alpha\beta^3+\alpha^3\beta+\cancel{\beta^4}$

$\alpha\beta(\alpha^2+\beta^2-2\alpha\beta)=0$

$\frac{c}{a}\cdot(\alpha-\beta)^2=0$

$\frac{c}{a}\cdot\frac{\Delta}{a^2}=0.$

$c\Delta=0$

Ans. C

QUESTION [JEE Mains 2023]

S04

If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296, respectively, then the sum of common ratio of all such G.P.s is

(A) 9/2

(B) 3

(C) 7

(D) 14

$a \cdot ar \cdot ar^2 \cdot ar^3 = 1296$

$a^4 r^6 = 1296$

$a^2 r^3 = 36$ — (I)

$a = \frac{6}{r\sqrt{r}}$

$a(1+r+r^2+r^3) = 126$ — (II)

$\frac{6}{\sqrt{r}}\left(\frac{1}{r}+1+r+r^2\right) = 126.$

$6\left(\frac{1}{r^{3/2}}+\frac{1}{r^{1/2}}+r^{1/2}+r^{3/2}\right)=126.$

$r^{1/2}+\frac{1}{r^{1/2}}=t$

CBS $r^{3/2}+1/r^{3/2}+3\cdot r^{1/2}\cdot 1/r^{1/2}\left(r^{1/2}+\frac{1}{r^{1/2}}\right)=t^3.$

$r^{3/2}+1/r^{3/2}=t^3-3t$

$6(t+t^3-3t)=126$

$t^3-2t=21$

$t^3-2t-21=0$

Ans. C

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

Special Sequences

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Special Sequences

Problems Based on $S_n = \Sigma T_n$

QUESTION

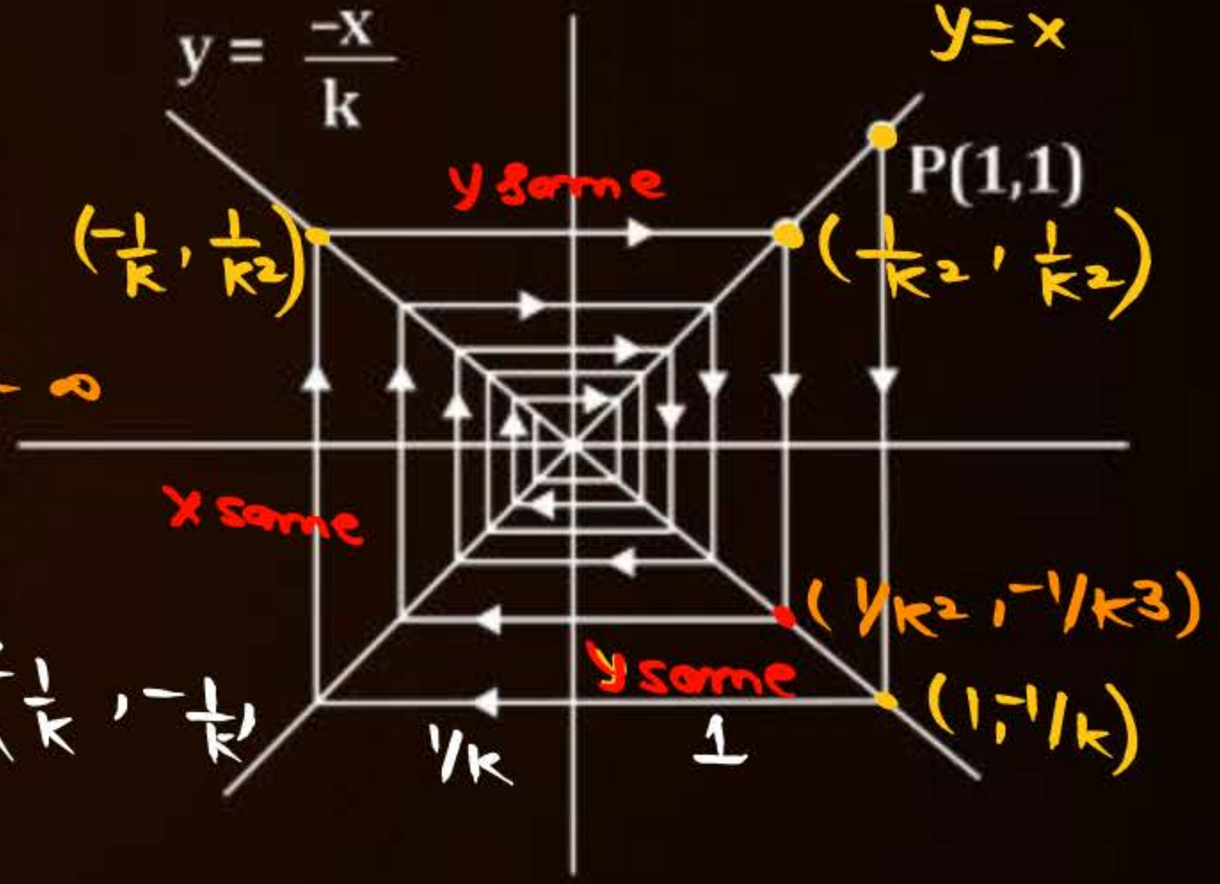
$(\alpha_1, \beta) \qquad (\alpha_2, \beta)$

★★KCLS★★

(α, β_2)

(α, β_1)

Consider two fixed lines y – x = 0 and ky + x = 0, k > 1. A particle P starts from (1, 1) to reach origin in the manner depicted as shown in figure. If the total distance travelled by particle is 3, then find the value of k.



$$S = \left(1+\frac{1}{k}\right) + \left(1+\frac{1}{k}\right) + \left(\frac{1}{k}+\frac{1}{k^2}\right) + \left(\frac{1}{k}+\frac{1}{k^2}\right) + \left(\frac{1}{k^2}+\frac{1}{k^3}\right) + \left(\frac{1}{k^2}+\frac{1}{k^3}\right) + \cdots \infty$$

$$= 2\left[\left(1+\frac{1}{k}\right) + \frac{1}{k}\left(1+\frac{1}{k}\right) + \frac{1}{k^2}\left(1+\frac{1}{k}\right) + \cdots \infty\right]$$

$$S = 2\left(1+\frac{1}{k}\right)\left[1 + \frac{1}{k} + \frac{1}{k^2} + \cdots \infty\right] = 3.$$

$$\frac{2(k+1)}{k} \cdot \frac{1}{1-1/k} = 3$$

$$\frac{2(k+1)}{k-1} = 3 \Rightarrow 2k+2 = 3k-3 \Rightarrow k=5$$

Ans. 0005

QUESTION [JEE Mains 2024 (30 Jan)]

★★KCLS★★

Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \cdots$ upto 10 terms and $\beta = \sum_{n=1}^{10} n^4$.

If $4\alpha - \beta = 55k + 40$, then k is equal to ______

$\alpha' = 1 + 4 + 8 + 13 + 19 + 26 + \cdots$ for $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + \cdots$

1st order diff: 3, 4, 5, 6, 7

2nd order diff: 1, 1, 1, 1

$T_n' = an^2 + bn + c$

$T_n = (an^2 + bn + c)^2$

$T_1' = a + b + c = 1$

$T_2' = 4a + 2b + c = 4$

$T_3' = 9a + 3b + c = 8$

$3a + b = 3$

$5a + b = 4$

$a = 1/2$

$b = 3/2$

$c = -1$

$$T_n = \frac{(n^2 + 3n - 2)^2}{4}$$

Ans. 353

$$\alpha = \sum_{n=1}^{10} T_n = \frac{\sum_{n=1}^{10} \left(n^2+3n-2\right)^2}{4}$$

$$4\alpha = \sum_{n=1}^{10} \left(n^4 + 9n^2 + 4 + 6n^3 - 12n - 4n^2\right)$$

$$4\alpha = \overbrace{\sum_{n=1}^{10} n^4}^{\beta} + 5\sum n^2 + 6\sum n^3 - 12\sum n + 40$$

$$4\alpha - \beta = 5 \cdot \frac{10 \cdot 11 \cdot 21}{6} + 6 \cdot \frac{10^2 \cdot 11^2}{4} - 12 \cdot \frac{10 \cdot 11}{2} + 40$$

QUESTION [JEE Mains 2024 (8 April)]

★★KCLS★★

Let the positive integers be written in the form :

1, 2, 3, - - -

Tahol

$R_1 \rightarrow$ 1

$R_2 \rightarrow$ 2 3

$R_3 \rightarrow$ 4 5 6

$R_4 \rightarrow$ 7 8 9 10

If the k^{th} row contains exactly k numbers for every natural number k, then the row in which the number 5310 will be, is ________

Ans. 103

QUESTION

Tah 02

$2 + 5 + 14 + 41 + 122 + \ldots\ldots$ up to n terms.

$3 \quad 9 \quad 27 \quad 81$

1st order diff in G.P

$T_n = ar^n + b$

QUESTION

Jan 03

The sum of the first 20 terms of the series $5 + 11 + 19 + 29 + 41 + \dots\dots$ is

(A) 3420

(B) 3450

(C) 3250

(D) 3520

Ans. D

QUESTION [JEE Mains 2023 (8 April)]

Jan 04

Let a_n be the n^{th} term of the series $5 + 8 + 14 + 23 + 35 + 50 + \ldots$ and

$S_n = \sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to :

(A) 11280

(B) 11290

(C) 11310

(D) 11260

Ans. B

QUESTION [JEE Mains 2023 (10 April)]

Tah 05

If $S_n = 4 + 11 + 21 + 34 + 50 + \ldots$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to :

(A) 227

(B) 226

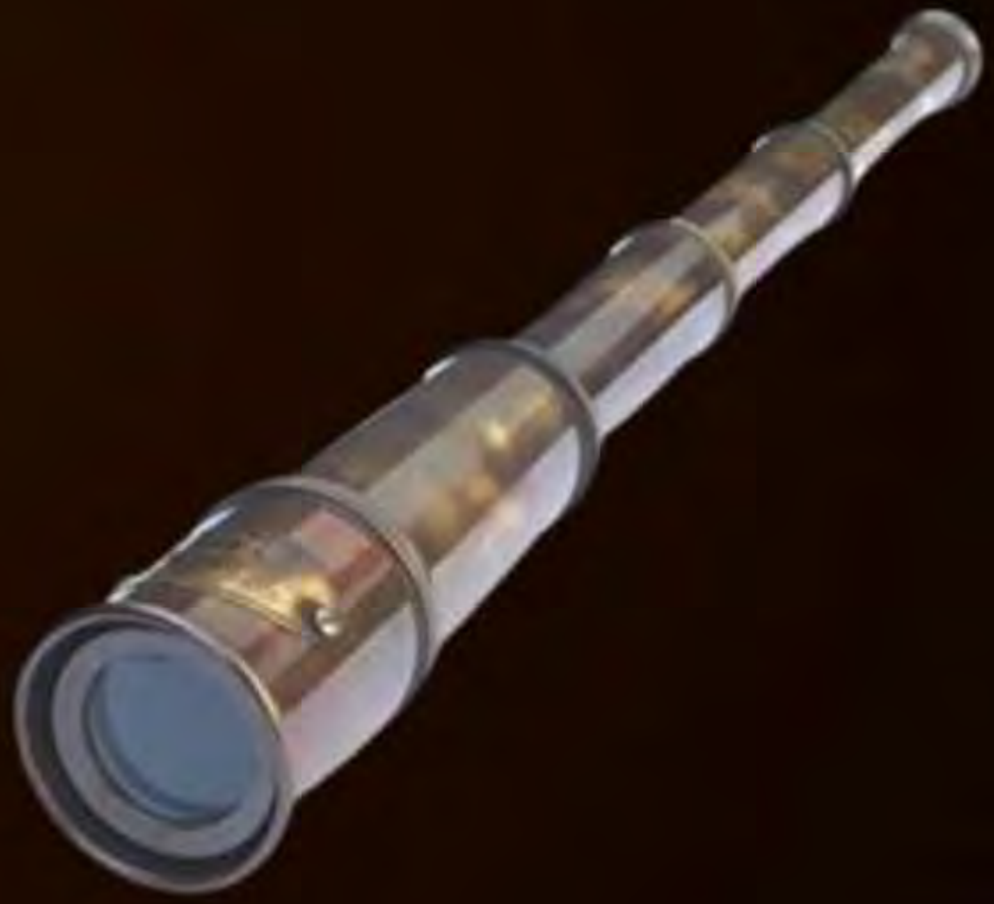
(C) 220

(D) 223

Ans. D

Telescoping Series

A telescoping series is any series with terms that cancel out with other terms. It's called "telescopic" because part of each term is canceled out by a later term, collapsing the series like a folding telescope.



Types 2:

(Splitting the n^{th} terms as a difference of two)

Here is a series in which each term in composed of the reciprocal of the product of r factors in A.P., the first factor of the several terms beings in the same A.P.

QUESTION

A.P: 1, 2, 3, 4, 5, 6, 7 - - -

$$\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{2\cdot3\cdot4\cdot5}+\frac{1}{3\cdot4\cdot5\cdot6}+\ldots\ldots \text{ up to n terms.}$$

$$T_n = \frac{1}{n(n+1)(n+2)(n+3)} \longrightarrow \text{continued product}$$

$$T_n = \frac{(n+3)-n}{3n(n+1)(n+2)(n+3)} = \frac{1}{3}\left(\frac{1}{n(n+1)(n+2)} - \frac{1}{(n+1)(n+2)(n+3)}\right)$$

$n \to n+1$

$$T_1 = \frac{1}{3}\left(\frac{1}{1\cdot2\cdot3} - \frac{1}{2\cdot3\cdot4}\right)$$

$$T_2 = \frac{1}{3}\left(\frac{1}{2\cdot3\cdot4} - \frac{1}{3\cdot4\cdot5}\right)$$

$$T_3 = 1/3\left(\frac{1}{3\cdot4\cdot5} - \frac{1}{4\cdot5\cdot6}\right)$$

$$T_n = 1/3\left(\frac{1}{n(n+1)(n+2)} - \frac{1}{(n+1)(n+2)(n+3)}\right]$$

$$S_n = \frac{1}{3}\left(\frac{1}{6} - \frac{1}{(n+1)(n+2)(n+3)}\right)$$

$$S_\infty = \lim_{n\to\infty} S_n = \frac{1}{3}\cdot(1/6 - 0) = 1/18$$

QUESTION

$$\frac{1}{\underline{\underline{1}}\cdot 3\cdot 5}+\frac{1}{\underline{\underline{3}}\cdot 5\cdot 7}+\frac{1}{\underline{\underline{5}}\cdot 7\cdot 9}+\ldots\ldots$$

Jah 06

$$T_n=\frac{1}{(2n-1)(2n+1)(2n+3)}$$

$$T_n=\frac{(\cancel{2n}+3)-(\cancel{2n}-1)}{4(2n-1)(2n+1)(2n+3)}$$

$$T_n=\frac{1}{4}\left(\frac{1}{(2n-1)(2n+1)}-\frac{1}{(2n+1)(2n+3)}\right)$$

QUESTION [JEE Mains 2024 (9 April)]

Tan07

If the sum of the series $\frac{1}{1\cdot(1+d)}+\frac{1}{(1+d)(1+2d)}+\cdots+\frac{1}{(1+9d)(1+10d)}$ is equal to 5, then 50d is equal to

(A) 5

(B) 10

(C) 15

(D) 20

$$T_n=\frac{1}{(1+(n-1)d)(1+nd)}$$

$$T_n=\frac{1+nd-(1+(n-1)d)}{d(1+(n-1)d)(1+nd)}$$

Ans. A

In case a factor is missing in continued product in denominator we multiply and divide by that factor so that the in denominator we get a complete continued product...

QUESTION

Tah07

$$\frac{3}{1\cdot 2\cdot 4}+\frac{4}{2\cdot 3\cdot 5}+\frac{5}{3\cdot 4\cdot 6}+\ldots\ldots$$

$$T_n=\frac{n+2}{n(n+1)(n+3)} \longrightarrow \text{No continued product}$$

$$=\frac{(n+2)^2}{n(n+1)(n+2)(n+3)}$$

$$=\frac{n^2+4n+4}{n(n+1)(n+2)(n+3)}$$

$$=\frac{(n^2+3n)+(n)+4}{n(n+1)(n+2)(n+3)}$$

$$=\frac{\cancel{n(n+3)}}{\cancel{n}(n+1)(n+2)\cancel{(n+3)}}+\frac{1}{(n+1)(n+2)(n+3)}+\frac{4}{n(n+1)(n+2)(n+3)}$$

$$T_n = \frac{1}{(n+1)(n+2)} + \frac{1}{(n+1)(n+2)(n+3)} + \frac{4}{n(n+1)(n+2)(n+3)}$$

$$T_n = \frac{(n+2)-(n+1)}{(n+1)(n+2)} + \frac{n+3-(n+1)}{2(n+1)(n+2)(n+3)}$$

QUESTION

$$\frac{1}{1\cdot 3}+\frac{2}{1\cdot 3\cdot 5}+\frac{3}{1\cdot 3\cdot 5\cdot 7}+\frac{4}{1\cdot 3\cdot 5\cdot 7\cdot 9}+\ldots\ldots$$

$$T_n = \frac{n}{1\cdot 3\cdot 5 \cdots\cdots (2n+1)}$$

$$= \frac{2n+1-1}{2\cdot 1\cdot 3\cdot 5\cdots (2n-1)(2n+1)} = \frac{1}{2}\left(\frac{1}{1\cdot 3\cdot 5\cdots (2n-1)} - \frac{1}{1\cdot 3\cdot 5\cdots (2n+1)}\right)$$

$$T_1 = \frac{1}{2}\left(1 - \frac{1}{1\cdot 3}\right)$$

$$T_2 = \frac{1}{2}\left(\frac{1}{1\cdot 3} - \frac{1}{1\cdot 3\cdot 5}\right)$$

$$T_3 = \frac{1}{2}\left(\frac{1}{1\cdot 3\cdot 5} - \frac{1}{1\cdot 3\cdot 5\cdot 7}\right)$$

$$\vdots$$

$$T_n = \frac{1}{2}\left(\frac{1}{1\cdot 3\cdot 5\cdots (2n-1)} - \frac{1}{1\cdot 3\cdot 5\cdots (2n+1)}\right)$$

$$S_n = \frac{1}{2}\left(1 - \frac{1}{1\cdot 3\cdot 5\cdots (2n+1)}\right)$$

$$S_\infty = \frac{1}{2}.$$

QUESTION [IIT-JEE 2010]

★★KCLS★★

Let $S_k = 1, 2, ..., 100$, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of

$$\frac{100^2}{100!}+\sum_{k=1}^{100}\left|\left(k^2-3k+1\right)S_k\right| \text{ is}$$

S_k = sum of infinite G.P $\quad a=\frac{k-1}{k!} \quad r=\frac{1}{k}$

$S_1 = 0$ & $S_k = \dfrac{\frac{k-1}{k!}}{1-\frac{1}{k}} = \frac{k}{k!} = \frac{1}{(k-1)!} \quad k = 2, 3, 4 -- , 100$

$$S=\sum_{k=1}^{100}\left|(k^2-3k+1)S_k\right| = \sum_{k=2}^{100}\left|(k^2-3k+1)S_k\right| = \sum_{k=2}^{100}\left|\frac{k^2-3k+1}{(k-1)!}\right|$$

$$S=\sum_{k=2}^{100}\left|\frac{k^2-3k+2-1}{(k-1)!}\right| = \sum_{k=2}^{100}\left|\frac{(k-1)(k-2)}{(k-1)!} - \frac{1}{(k-1)!}\right|$$

Ans. 3

$$S=\sum_{K=2}^{100}\left|\frac{(K-1)(K-2)}{(K-1)!}-\frac{1}{(K-1)!}\right|$$

@ K=2 it is 0.

$$=\sum_{K=2}^{100}\left|\frac{1}{(K-3)!}-\frac{1}{(K-1)!}\right|$$

K = 3, 4 - - -

$$=\left|0-\frac{1}{1!}\right|+\left|\frac{1}{0!}-\frac{1}{2!}\right|+\left|\frac{1}{1!}-\frac{1}{3!}\right|+\left|\frac{1}{2!}-\frac{1}{3!}\right|+\cdots+\left|\frac{1}{97!}-\frac{1}{99!}\right|$$

$$=\frac{1}{1!}$$
$$+1-\frac{1}{2!}$$
$$+\frac{1}{1!}-\frac{1}{3!}$$
$$+1/2!-1/4!$$
$$+1/3!-1/5!$$
$$\vdots$$
$$+\frac{1}{96!}-1/98!$$
$$+\frac{1}{97!}-\frac{1}{99!}$$

$$=1+1+1-\frac{1}{98!}-\frac{1}{99!}=3-\frac{99+1}{99!}=3-\frac{100}{99!}$$

$$S = 3 - \frac{100}{99!} = 3 - \frac{100^2}{100!}$$

$$S + \frac{100^2}{100!} = \underline{3 \text{ Ans}}$$

QUESTION [JEE Mains 2021]

Tah08

$$\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+\cdots+\frac{1}{(201)^2-1}$$ is equal to

(A) 101/404

(B) 25/101

(C) 4

(D) 6

$$T_n = \frac{1}{(2n+1)^2 - 1^2} = \frac{1}{2n\cdot(2n+2)}$$

$$T_n = \frac{2n+2-2n}{2\cdot 2n\cdot(2n+2)}$$

QUESTION [JEE Mains 2021]

Tano9

The sum of 10 terms of the series $\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \cdots$ is:

(A) 1

(B) 120/121

(C) 99/100

(D) 143/144

QUESTION [JEE Mains 2024 (31 Jan)]

★★ASRQ★★

The sum of the series $\frac{1}{1-3\cdot1^2+1^4}+\frac{2}{1-3\cdot2^2+2^4}+\frac{3}{1-3\cdot3^2+3^4}+$ ..... up to 10-terms is

(A) $\frac{45}{109}$

(B) $-\frac{55}{109}$

(C) $\frac{55}{109}$

(D) $-\frac{45}{109}$

$$T_n=\frac{n}{1-3\cdot n^2+n^4}=\frac{n}{n^4-2n^2+1-n^2}=\frac{n}{(n^2-1)^2-n^2}$$

$$=\frac{n}{(n^2-n-1)(n^2+n-1)}=\frac{n^2+n-1-(n^2-n-1)}{2(n^2-n-1)(n^2+n-1)}$$

$$=\frac{1}{2}\left(\frac{1}{n^2-n-1}-\frac{1}{n^2+n-1}\right)$$

$$T_1=\frac{1}{2}\left(\frac{1}{1^2-1-1}-\frac{1}{1^2+1-1}\right)=\frac{1}{2}\left(-1-\cancel{\frac{1}{1}}\right)$$

$$T_2=\frac{1}{2}\left(\frac{1}{4-2-1}-\frac{1}{4+2-1}\right)=\frac{1}{2}\left(\cancel{\frac{1}{1}}-\cancel{\frac{1}{5}}\right)$$

$$T_3=\frac{1}{2}\left(\frac{1}{9-3-1}-\frac{1}{9+3-1}\right)=\frac{1}{2}\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{11}}\right)$$

$\vdots$

$$T_{10}=\frac{1}{2}\left(\frac{1}{100-10-1}-\frac{1}{100+10-1}\right)=\frac{1}{2}\left(\cancel{\frac{1}{89}}-\frac{1}{109}\right)$$

$$S_{10}=\frac{1}{2}\left(-1-\frac{1}{109}\right)$$

$$=\frac{1}{2}\left(\frac{-110}{109}\right)$$

$$=\frac{-55}{109}$$

Ans. B

QUESTION [JEE Mains 2023 (1 Feb)]

Jan 10

★★ASRQ★★

The sum of 10 terms of the series $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\ldots\ldots$ is

$$T_n=\frac{n}{1+n^2+n^4}=\frac{n}{n^4+2n^2+1-n^2}=\frac{n}{(n^2+1)^2-n^2}$$

$$T_n=\frac{n}{(n^2+n+1)(n^2-n+1)}$$

(A) $\frac{58}{111}$

(B) $\frac{56}{111}$

(C) $\frac{55}{111}$

(D) $\frac{59}{111}$

Ans. C

QUESTION [JEE Mains 2025 (4 April)]

Tan!!

★★ASRQ★★

If the sum of the first 20 terms of the series

$$\frac{4\cdot 1}{4+3\cdot 1^2+1^4}+\frac{4\cdot 2}{4+3\cdot 2^2+2^4}+\frac{4\cdot 3}{4+3\cdot 3^2+3^4}+\frac{4\cdot 4}{4+3\cdot 4^2+4^4}+\cdots \text{ is } \frac{m}{n},$$

where m and n are coprime, then $m + n$ is equal to:

(A) 423

(B) 421

(C) 422

(D) 420

Ans. B

QUESTION [JEE Mains 2025 (2 April)]

Tan12 ★★ASRQ★★

If the sum of the first 10 terms of the series

$$\frac{4\cdot 1}{1+4\cdot 1^4}+\frac{4\cdot 2}{1+4\cdot 2^4}+\frac{4\cdot 3}{1+4\cdot 3^4}+\cdots \text{ is } \frac{m}{n}, \text{ where } \gcd(m,n)=1,$$

then $m+n$ is equal to

Ans. 441

Types 3:

Here is a series in which each term in composed of r factors in A.P., the first factor of the several terms beings in the same A.P.

QUESTION

$\bar{1}\cdot 2\cdot 3\cdot 4+\bar{2}\cdot 3\cdot 4\cdot 5+\bar{3}\cdot 4\cdot 5\cdot 6+\ldots\ldots$ up to n terms

$T_n = n\cdot(n+1)(n+2)(n+3)$ ~ (continued)

$= \frac{(n+4-(n-1))}{5} n(n+1)(n+2)(n+3)$

$= \frac{1}{5}\left(n(n+1)(n+2)(n+3)(n+4) - (n-1)n(n+1)(n+2)(n+3)\right)$

$n \to n+1$

$T_1 = \frac{1}{5}(1\cdot 2\cdot 3\cdot 4\cdot 5 - 0)$

$T_2 = \frac{1}{5}(2\cdot 3\cdot 4\cdot 5\cdot 6 - 1\cdot 2\cdot 3\cdot 4\cdot 5)$

$T_3 = 1/5\,(3\cdot 4\cdot 5\cdot 6\cdot 7 - 2\cdot 3\cdot 4\cdot 5\cdot 6)$

$\vdots$

$T_n = 1/5\,(n(n+1)(n+2)(n+3)(n+4) - (n-1)n(n+1)(n+2)(n+3))$

$S_n = \frac{1}{5} n(n+1)(n+2)(n+3)(n+4)$

QUESTION

Tah 13

$1 \cdot 3 \cdot 5 \cdot 7 + 3 \cdot 5 \cdot 7 \cdot 9 + 5 \cdot 7 \cdot 9 \cdot 11 + \ldots\ldots$ up to n terms.

QUESTION

$1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + 7 \cdot 9 + \ldots\ldots$ up to n terms

Harmonic Progression (H.P.)

Definition :

A non-zero sequence is said to be in H.P. if the reciprocals of its terms are in A.P.

e.g. if $a_1, a_2, a_3, \ldots.$ are in H.P., then $1/a_1, 1/a_2, 1/a_3 \ldots.$ are in A.P

A standard H.P. is

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots, \frac{1}{a+(n-1)d}$$

$$T_n = \frac{1}{a+(n-1)d}.$$

Note:

(i) There is no general formula for finding the sum to n term of H.P.

(ii) If a, b, c are in H.P. $\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

(iii) No term of H.P. can be zero.

If a, b, c are H.P then 'b' is called single H.M b/w a & c

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

A.M b/w $1/a$ & $\frac{1}{c}$

$$\frac{1}{b} = \frac{1/a + 1/c}{2}$$

$$\boxed{b = \frac{2}{1/a + 1/c}}$$

$$\boxed{b = \frac{2ac}{a+c}}$$

If $a, \underbrace{H_1, H_2, H_3 --, H_n}_{\downarrow}, b$ are in H.P

n H.Ms b/w a & b

$$\Rightarrow \frac{1}{a}, \underbrace{\frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, --, \frac{1}{H_n}}_{\downarrow}, \frac{1}{b} \text{ are in A.P}$$

n A.Ms b/w $1/a$ & $\frac{1}{b}$.

we know sum of n A.Ms b/w 2 no:s = n (single A.M b/w the two No:s)

$$\frac{1}{H_1}+\frac{1}{H_2}+---+\frac{1}{H_n}=n\left(\frac{1/a+1/b}{2}\right)$$

$$\sum_{i=1}^{n}\frac{1}{H_i}=n\left(\frac{1}{\frac{2}{1/a+1/b}}\right)$$

Sum of reciprocals of n H.Ms b/w two no:s = n times the Reciprocal of single H.m b/w the two No!s.

* If a, b are two No:s

$$A = \frac{a+b}{2}$$

$$G^2 = ab$$

$$H = \frac{2}{\frac{1}{a}+\frac{1}{b}} = \frac{2ab}{a+b} = \frac{ab}{\frac{a+b}{2}} = \frac{G^2}{A}$$

$$G^2 = AH$$

* If a & b are two +ve Numbers with $A \cdot M = A$, $G \cdot M = G$, $H \cdot M = H$ where $G^2 = AH$

$$\frac{G}{A} = \frac{H}{G}.$$

then $(\sqrt{a} - \sqrt{b})^2 \geq 0.$

$$a+b-2\sqrt{ab} \geq 0$$

$$\frac{a+b}{2} \geq \sqrt{ab} \Rightarrow A \geq G \Rightarrow \frac{G}{A} \leq 1 \qquad \frac{H}{G} \leq 1 \Rightarrow H \leq G.$$

⇒ for any two +ve no:s a, b.

$$A \geq G \geq H$$

$$\frac{a+b}{2} \geq (ab)^{1/2} \geq \frac{2}{\frac{1}{a}+\frac{1}{b}}.$$

where equality holds if $a=b$.

A.M – G.M – H.M Ineq:

If $a_1, a_2, a_3, \ldots, a_n \in R^+$

$$A = \frac{a_1+a_2+\cdots+a_n}{n}$$

$$G = (a_1 \cdot a_2 \cdots a_n)^{1/n}$$

$$H = \frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}}$$

$$A \geq G \geq H$$

i.e. $\frac{a_1+a_2+\cdots+a_n}{n} \geq (a_1 \cdot a_2 \cdots a_n)^{1/n} \geq \frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}}$

Equality holds if $a_1 = a_2 = \cdots = a_n$.

QUESTION

(a) The m^{th} term of a H.P. is n and the nth term is m. Prove that the p^{th} term is $\frac{mn}{p}$.

(b) If m^{th} term of an H.P. is n, and n^{th} term is equal to m then prove that $(m+n)^{th}$ term is $\frac{mn}{m+n}$.

Tah 13

$T_m = n \qquad T_n = m$

$\frac{1}{a+(m-1)d} = n \qquad \frac{1}{a+(n-1)d} = m$

$a+(m-1)d = \frac{1}{n}$

$a+(n-1)d = 1/m$

$(m-n)d = \frac{m-n}{mn}$

$d = 1/mn.$

$a + \frac{1}{mn}(m-1) = \frac{1}{n}$

$a + \frac{1}{n} - \frac{1}{mn} = 1/n$

$a = \frac{1}{mn}$

$\Rightarrow T_p = \frac{1}{a+(p-1)d}$

$= \frac{1}{\frac{1}{mn} + \frac{p-1}{mn}} = \frac{mn}{1+p-1}$

$= \frac{mn}{p}$

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

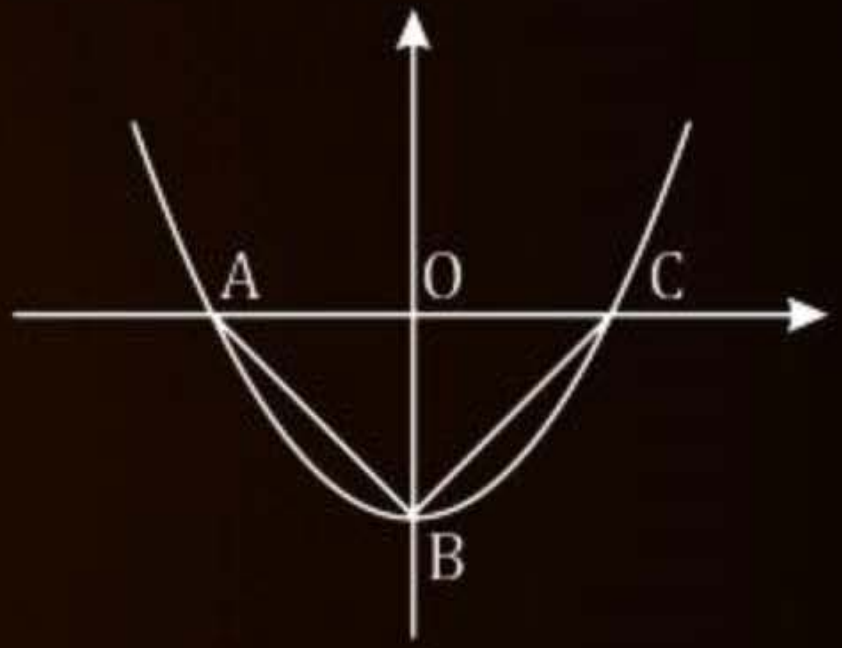
Revision Practice Problems (RPP)

QUESTION

Rpp 1A

Paragraph

In the given figure vertices of ΔABC lie on $y = f(x) = ax^2 + bx + c$. The ΔABC is right angled isosceles triangle whose hypotenuse $AC = 4\sqrt{2}$ units, then $y = f(x)$ is given by



(A) $y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$

(B) $y = \frac{x^2}{2} - 2$

(C) $y = x^2 - 8$

(D) $y = x^2 - 2\sqrt{2}$

Ans. A

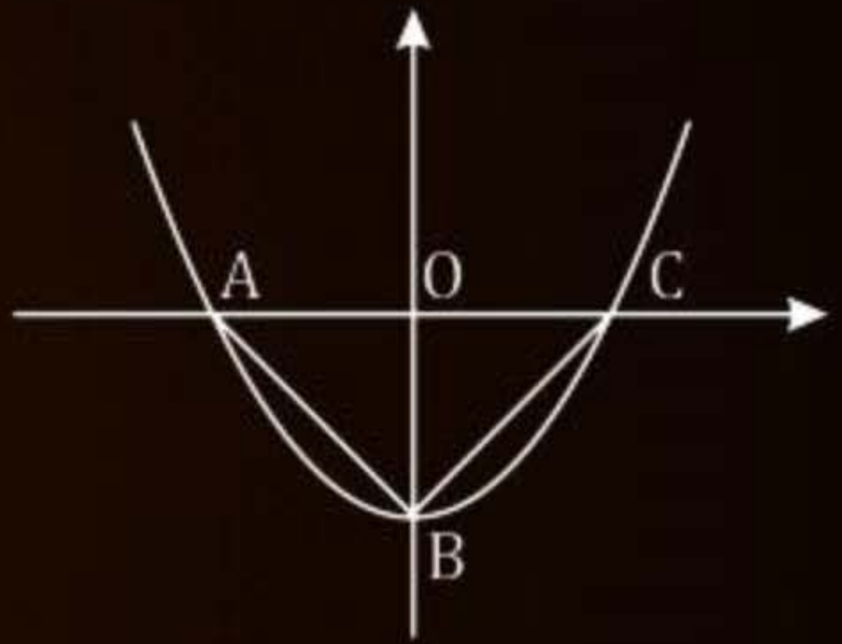
QUESTION

Rpp 1B

Paragraph

In the given figure vertices of ΔABC lie on $y = f(x) = ax^2 + bx + c$. The ΔABC is right angled isosceles triangle whose hypotenuse $AC = 4\sqrt{2}$ units, then

Minimum value of $y = f(x)$ is



(A) $2\sqrt{2}$

(B) $-2\sqrt{2}$

(C) 2

(D) -2

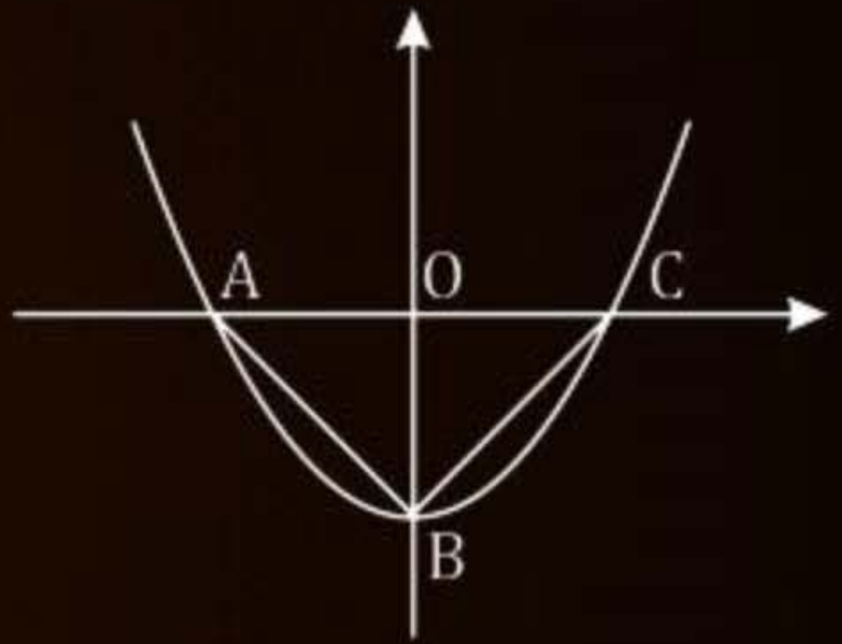
Ans. B

QUESTION

Rpp 1C

Paragraph

In the given figure vertices of ΔABC lie on $y = f(x) = ax^2 + bx + c$. The ΔABC is right angled isosceles triangle whose hypotenuse $AC = 4\sqrt{2}$ units, then



Number of integral value of k for which $\frac{k}{2}$ lies between the roots of $f(x) = 0$, is

(A) 9

(B) 10

(C) 11

(D) 12

Ans. C

Solution to Previous TAH

QUESTION

$a_1, a_2, a_3, ...$ is a sequence such that $a_1 = \frac{1}{2}$ & $(3 + a_n)(4 - a_{n+1}) = 12$ then

$$\sum_{n=1}^{n} \frac{1}{a_n} = \_\_\_\_\_\_\_\_$$

$$4a_n = 3a_{n+1} + a_n \cdot a_{n+1}$$

$$\frac{4}{a_{n+1}} = \frac{3}{a_n} + 1.$$

$$\frac{4}{a_{n+1}} - 4 = \frac{3}{a_n} - 3$$

$$4\left(\frac{1}{a_{n+1}} - 1\right) = 3\left(\frac{1}{a_n} - 1\right) \quad \text{let } \frac{1}{a_n} - 1 = b_n$$

$$4b_{n+1} = 3b_n$$

$$\frac{b_{n+1}}{b_n} = 3/4$$

Ans. $4\left(1 - \left(\frac{3}{4}\right)^n\right) + n$

#TAH-01

Q. $a_1 = \frac{1}{2}$

$(3 + a_n)(4 - a_{n+1}) = 12$

$\cancel{12} - 3a_{n+1} + 4a_n - a_n \cdot a_{n+1} = \cancel{12}$

$4a_n = 3a_{n+1} + a_n \cdot a_{n+1}$

$\frac{4}{a_{n+1}} = \frac{3}{a_n} + 1$

$\frac{4}{a_{n+1}} - 4 = \frac{3}{a_n} - 3$

$4\left(\frac{1}{a_{n+1}} - 1\right) = 3\left(\frac{1}{a_n} - 1\right)$

let $b_n = \frac{1}{a_n} - 1$

$b_{n+1} = \frac{3}{4} b_n$

$4 b_{n+1} = 3 b_n$

$\frac{b_{n+1}}{b_n} = \frac{3}{4}$ $\longrightarrow \langle b_n \rangle$ is in GP, $CR = \frac{3}{4}$

$b_n = \frac{1}{a_n} - 1$

$b_1 = \frac{1}{a_1} - 1$

$b_2 = \frac{1}{a_2} - 1$

$b_3 = \frac{1}{a_3} - 1$

$\vdots$

$b_n = \frac{1}{a_n} - 1$

$b_1 = \frac{1}{a_1} - 1$

$= \frac{1}{1/2} - 1$

$b_1 = 2 - 1$

$b_1 = 1$

$b_1 + b_2 + b_3 + b_4 + \ldots b_n = \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots \frac{1}{a_n}\right) - n$

$\frac{1\left(\left(\frac{3}{4}\right)^n - 1\right)}{\frac{3}{4} - 1} = \sum_{n=1}^{n} \frac{1}{a_n} - n$

$\frac{4\left(\left(\frac{3}{4}\right)^n - 1\right)}{-1} = \sum_{n=1}^{n} \frac{1}{a_n} - n$

$\boxed{\frac{4\left(1 - \left(\frac{3}{4}\right)^n\right)}{1} + n = \sum_{n=1}^{n} \frac{1}{a_n}}$

Ans

QUESTION [JEE Mains 2025 (4 April)]

$1 + 3 + 5^2 + 7 + 9^2 + \ldots\ldots$ upto 40 terms is equal to

(A) 40870

(B) 41880

(C) 43890

(D) 33980

Ans. B

TAH 2

(2) $1^2+3+5^2+7+9^2 - - -$ upto 40 terms

$\Rightarrow \left(3+7+11 - - -\text{ upto 20 terms}\right) + \left(1^2+5^2+9^2+ - - \text{ upto 20 terms}\right)$

$\Rightarrow \underline{4n-1} + \underline{(4n-3)^2} = T_n$

$4n-1+16n^2+9-24n = T_n$

$16n^2-20n+8 = T_n$

$\sum_{n=1}^{20} T_n = 16\sum_{n=1}^{20} n^2 - 20\sum_{n=1}^{20} n + \sum_{n=1}^{20} 8$

$= \frac{16\cdot 20\cdot 21\cdot 41}{\cancel{6}\,2} - 20\cdot\frac{20\cdot 21}{2} + 20\times 8$

$= \frac{91840 - 8400}{2} + 160$

$= 41720 + 160$

$= 41880$ — (B)

TAH - 02 $\quad 1 + 3 + 5^2 + 7 + 9^2 + \cdots$ upto 40 terms

$$(1^2 + 5^2 + 9^2 + \cdots \text{ 20 terms}) + (3 + 7 + 11 \cdots \text{ 20 terms})$$

$$a_n = [1 + (n-1)(4)]^2$$

$$(4n - 4 + 1)^2$$

$$\boxed{a_n = (4n-3)^2}$$

$$a_n = 3 + (n-1)4$$

$$= 3 + 4n - 4$$

$$\boxed{a_n = 4n - 1}$$

$$\rightsquigarrow \sum_{n=1}^{20} (4n-3)^2 + \sum_{n=1}^{20} 4n - 1$$

$$\rightsquigarrow \sum_{n=1}^{20} (16n^2 + 9 - 24n) + \sum_{n=1}^{20} (4n-1)$$

$$\rightsquigarrow 16\sum_{n=1}^{20} n^2 - 24\sum_{n=1}^{20} n + \sum_{n=1}^{20} 9 + 4\sum_{n=1}^{20} n - \sum_{n=1}^{20} 1$$

$$\rightsquigarrow 16\sum_{n=1}^{20} n^2 - 20\sum_{n=1}^{20} n + \sum_{n=1}^{20} 8$$

$$\rightsquigarrow \overset{8}{\cancel{16}}\left(\frac{(20)(\overset{7}{\cancel{21}})(41)}{\cancel{6}\,\cancel{3}}\right) - 20\left(\frac{(\overset{10}{\cancel{20}})(21)}{\cancel{2}}\right) + 8(20)$$

$$\rightsquigarrow 45920 - 4200 + 160 = \boxed{41880} \text{ Ans}$$

[Mains-2025]

Tah-02 $\underbrace{1^2+\overbrace{3+5^2+7}^{S_2}+9^2}_{S_1}+\dots$ upto 40 terms is equal to

20 terms 20 terms

$S_1 = 1^2+5^2+9^2+13^2+\dots 20 \text{ terms}$

$S_2 = 3+7+11+15+\dots 20 \text{ terms}$

$S_2 = \frac{20}{2}[6+(20-1)4]$

Richathakur

$S_2 = 20[3+38] \Rightarrow 821$ —— (i)

$S_1 = \sum_{k=1}^{20}(4k-3)^2 \Rightarrow \sum_{k=1}^{20}(16k^2+9-24k)$

$\sum_{k=1}^{20}16k^2 - \sum_{k=1}^{20}9 + \sum_{k=1}^{20}24k$

$16\left[\frac{\cancel{20}^{10}\cdot\cancel{21}^{7}\cdot 41}{\cancel{6}_{3}}\right] + 20\times 9 - 24\left[\frac{\cancel{20}^{10}\cdot 21}{\cancel{2}}\right]$

$S_1 = 45920+180-2880 \Rightarrow 41060$ —— (ii)

$S = S_1+S_2$ (i + ii)

$S = 41060+821$

$\boxed{S = 41880 \ (B)}$

QUESTION [JEE Mains 2019]

The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+\cdots+\frac{1^3+2^3+3^3+\cdots+15^3}{1+2+3+\cdots+15}-\frac{1}{2}(1+2+3+\cdots+15)$ is equal to

(A) 620

(B) 1240

(C) 1860

(D) 660

$\boxed{\text{TAH-03}}$ The sum $1 + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + \cdots + \frac{1^3+2^3+3^3+\cdots+15^3}{1+2+3+\cdots+15} - \frac{1}{2}(1+2+3+\cdots+15)$ is equal to:

Soln: $\cancel{1+3}$ $T_r = \frac{1^3+2^3+3^3+\cdots+r^3}{1+2+3+4+\cdots+r}$

$$T_r = \frac{\left(\frac{r(r+1)}{2}\right)^2}{\left(\frac{r(r+1)}{2}\right)} \Rightarrow T_r = \frac{r(r+1)}{2} = \frac{1}{2}(r^2+r)$$

$$\sum T_r = \sum_{r=1}^{15} \frac{1}{2}(r^2+r)$$

$$= \frac{1}{2}\left(\sum_{r=1}^{15} r^2 + \sum_{r=1}^{15} r\right)$$

$$\frac{1}{2}\left(\frac{15\times16\times31}{6} + \frac{15\times16}{2}\right) = 680$$

$$680 - \frac{1}{2}\times\frac{15\times16}{2} \Rightarrow 680-60 = \boxed{620 \text{ Ans}}$$

TAH-03
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Prayagraj UP

Tan-03 $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+\cdots+\frac{1^3+2^3+3^3+\cdots+15^3}{1+2+3+\cdots+15}-\frac{1}{2}(1+2+3+\cdots+15)$

$\underbrace{\qquad}_{S_1}$ $\underbrace{\qquad}_{S_2}$

$$S_1=\frac{1^3+2^3+\cdots+r^3}{1+2+3+\cdots+r} \quad \frac{1^3+\cdots+r^3}{} \to \left[\frac{r(r+1)}{2}\right]^2 \quad 1+2+\cdots+r \to \left[\frac{r(r+1)}{2}\right]$$

$$S_1=\frac{[r(r+1)]^2}{4_2}\times\frac{2}{r(r+1)}$$

$$\boxed{S_1=\frac{r^2+r}{2}}$$

Richathakur

$$\sum_{r=1}^{15}\left(\frac{r^2+r}{2}\right)\Rightarrow\frac{1}{2}\left[\sum r^2+\sum r\right]$$

$$\Rightarrow\frac{1}{2}\left[\frac{15\cdot16\cdot31}{6}+\frac{15\cdot16}{2}\right]$$

$$\Rightarrow\frac{1}{2}[1240+120]\Rightarrow\frac{1360}{2}$$

$$\boxed{S_1=680}$$

$$=\frac{1}{2}(1+2+3\cdots15)$$

$$S_2=\frac{1}{2}\left[\frac{15(16)}{2}\right]\Rightarrow\boxed{S_2=60}$$

$$S=S_1-S_2$$

$$S=680-60$$

$$\boxed{S=620}\ \text{Ans}$$

QUESTION [JEE Mains 2020]

The sum $\sum_{k=1}^{20}(1+2+3+\cdots+k)$ is

TAH 4

(4) $$\sum_{k=1}^{20} \frac{k(k+1)}{2} = \sum_{k=1}^{20} \frac{k^2}{2} + \frac{k}{2}$$

$$\frac{1}{2}\sum_{k=1}^{20} k^2 + \frac{1}{2}\sum_{k=1}^{20} k$$

$$5\,\cancel{10}\ \frac{\cancel{20}\cdot \overset{7}{\cancel{21}}\cdot 41}{\cancel{2}\,\cancel{6}\times\cancel{2}} + \frac{1}{\cancel{2}}\cdot\frac{\overset{5}{\cancel{20}}\cdot 21}{\cancel{2}}$$

$$35\times 41 + 105$$

$$1435 + 105 = 1540$$

Q. The sum of $\sum_{k=1}^{20}(1+2+3+\cdots+k)$.

4.

$$\Rightarrow T = 1+2+3+\cdots+K \Rightarrow \frac{K(K+1)}{2}.$$

$$\# \sum_{K=1}^{20} T_K \Rightarrow \frac{1}{2}\left[\sum K^2 + \sum K\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{20\times 21\times 41}{6} + \frac{20\times 21}{2}\right]$$

$$\Rightarrow \frac{1}{\cancel{2}}\times \overset{10}{\cancel{20}}\times 21\left[\frac{41}{6}+\frac{1}{2}\right]$$

$$\Rightarrow \overset{70}{\cancel{210}}\times \frac{\overset{22}{\cancel{44}}}{\underset{3}{\cancel{6}}}$$

$$\Rightarrow \boxed{1540} \text{ Ans.}$$

krish

TAH-04 The Sum $\sum_{k=1}^{20}(1+2+3+\cdots+k)$ is

$$\sum_{k=1}^{20}\frac{k(k+1)}{2} \Rightarrow \frac{1}{2}\sum_{k=1}^{20}(k^2+k)$$

TAH-04
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Prayagraj UP

$$\Rightarrow \frac{1}{2}\left[\frac{20(21)(41)}{6}+\frac{20(21)}{2}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{420\times 41}{6}+\frac{20\times 21}{2}\right] \Rightarrow \frac{1}{2}\left[2870+210\right]$$

$$= \boxed{1540 \text{ Ans}}$$

QUESTION [JEE Mains 2019]

Let $S_k = \frac{1+2+3+\cdots+k}{k}$. If $S_1^2 + S_2^2 + \cdots + S_{10}^2 = \frac{5}{12}A$, then A is equal to:

(A) 303

(B) 156

(C) 283

(D) 301

★ Tah 05:-

Ankush

Sol^n:- $S_n = \dfrac{1+2+3+\dots+n}{n}$

$S_n = \dfrac{\cancel{n}(n+1)}{2\cancel{n}} = \dfrac{n+1}{2}$

$\boxed{S_n = \dfrac{n+1}{2}}$

$S_n = \sum_{n=1}^{10} \left(\dfrac{n+1}{2}\right)^2 = \dfrac{1}{4}\left[\sum_{n=1}^{10} (n+1)^2\right]$

$S_n = \dfrac{1}{4}\left[\sum_{n=1}^{10} (n^2+1+2n)\right]$

$= \dfrac{1}{4}\left[\sum_{n=1}^{10} n^2 + \sum_{n=1}^{10} 2n + \sum_{n=1}^{10} 1\right]$

$= \dfrac{1}{4}\left[\dfrac{10\cdot 11\cdot 21}{6} + 2\cdot\dfrac{10\cdot 11}{2} + 10\right]$

$= \dfrac{1}{4}\left[5\cdot 11\cdot 7 + 10\cdot 11 + 10\right] = \dfrac{1}{4}\left[385 + 110 + 10\right]$

$S_n = \dfrac{\cancel{495}^{505}}{4} = \dfrac{5}{12}A$

$\Rightarrow \dfrac{\cancel{495}^{101} \times \cancel{12}^{3}}{\cancel{4} \times \cancel{5}} = A \Rightarrow 101 \times 3 = 303$

$\boxed{A = 303}$ Ans

(5.) $S_K = \dfrac{1+2+3+\dots+K}{K}$

$S_K = \dfrac{\cancel{K}(K+1)}{2\cancel{K}}$

$S_K^2 = (K^2+1+2K)/4$

$\sum_{K=1}^{10} S_K = \dfrac{1}{4}\left(\sum_{K=1}^{10} K^2 + 2\sum_{K=1}^{10} K + \sum_{K=1}^{10} 1\right)$

$\dfrac{5A}{\cancel{12}3} = \dfrac{1}{\cancel{4}}\left(\dfrac{\cancel{10}^{5}\cdot 11\cdot \cancel{21}^{7}}{\cancel{6}\cancel{2}} + \cancel{2}\dfrac{10\cdot 11}{\cancel{2}} + 10\right)$

$\dfrac{5A}{12} = (35\times 11 + 110 + 10)$

$= (385 + 120 = 505)$

$\cancel{5}A = \cancel{12}3 \times \cancel{505}^{101}$

$A = 303$ //

Q. **5.** Let $S_K = \dfrac{1+2+3+\dots+K}{K} \Rightarrow \dfrac{1}{\cancel{K}}\left(\dfrac{\cancel{K}(K+1)}{2}\right)$

If $S_1^2 + S_2^2 + \dots + S_{10}^2 = \dfrac{5}{12} A$.

$$\Rightarrow S_K = \frac{K+1}{2}.$$

$A = ?$

$$\Rightarrow \sum_{K=1}^{10} \left(\frac{K+1}{2}\right)^2 = \frac{5}{12} A.$$

$$\Rightarrow \sum_{K=1}^{10} \frac{K^2+2K+1}{\cancel{4}} = \frac{5}{\cancel{12}_{3}} A$$

$$\Rightarrow \frac{10 \times 11 \times \cancel{21}^{7}}{\cancel{6}_{2}} + \cancel{2} \times \frac{10 \times 11}{\cancel{2}} + 10 = \frac{5}{3} A$$

$$\Rightarrow \cancel{10}^{2}\left(\frac{77}{2} + 11 + 1\right) = \frac{\cancel{5}}{3} A$$

$$\Rightarrow A = {}^{3}\cancel{6}\left(\frac{77+22+2}{\cancel{2}}\right)$$

krish

$$= 3 \times 101 = \boxed{303} \text{ Ans.}$$

QUESTION [JEE Mains 2024 (30 Jan)]

Let S_n be the sum to n-terms of an arithmetic progression 3, 7, 11, ...

If $40 < \left(\frac{6}{n(n+1)}\sum_{k=1}^{n} S_k\right) < 42$, then n equals

Ans. 9

$3, 7, 11 - - -$

$S_n = 3 + 7 + 11 \dots n \text{ terms}$

$\frac{n}{2}[6 + (n-1)4]$

$S_n = n[3 + 2n - 2] = n[2n+1)$

$\sum_{k=1}^{n} S_n = \sum_{k=1}^{n} 2n^2 + n$

$= 2\sum_{k=1}^{n} n^2 + \sum_{k=1}^{n} n$

$= \cancel{2} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2 \cdot 3}$

$= n(n+1)\left[\frac{2(2n+1)+3}{6}\right]$

$= n(n+1)[4n+5]/6$

$40 < \frac{6}{\cancel{n(n+1)}} \cdot \frac{\cancel{n(n+1)}(4n+5)}{\cancel{6}} < 42$

TAH6

$40 < \frac{6}{n(n+1)} \cdot \frac{n(n+1)(4n+5)}{6} < 42$

$40 < 4n + 5 < 42$

$35 < 4n < 37$

$\frac{35}{4} < n < \frac{37}{4}$

$8.\dots < n < 9.\dots$

$n = 9$ //

[TAH 06] Let S_n be the Sum to n-terms of an A.P. $3, 7, 11, \dots$ If $40 < \left(\frac{6}{n(n+1)} \sum_{k=1}^{n} S_k\right) < 42$, then n equal to:

Given, AP: $3, 7, 11, 15, \dots$

$a_n = 3 + (n-1)(4)$

$a_n = 4n - 1$

$S_n = \frac{n}{2}[6 + 4n - 4] = \frac{n \times 2(2n+1)}{2}$

$S_n = 2n^2 + n$

$\sum_{k=1}^{n} S_k = 2\sum_{n=1}^{n} n^2 + \sum_{n=1}^{n} n$

$= \frac{n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2}\left[1 + \frac{4n+2}{3}\right]$

Now $40 < \frac{6}{n(n+1)} \sum_{k=1}^{n} S_k < 42$

$40 < \frac{6}{n(n+1)} \times \frac{n(n+1)}{2}\left[\frac{5+4n}{3}\right] < 42$

$40 < 4n + 5 < 42$

$35 < 4n < 37$

$\boxed{n = 9 \text{ Ans}}$

TAH-06
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Prayagraj UP

Q.
6.

A.P: 3, 7, 11, ---- $\Rightarrow a=3$, $d=4$.

If $40 < \left(\frac{6}{n(n+1)} \sum_{k=1}^{n} S_k\right) < 42$

$S_n = \frac{n}{2}[6+(n-1)4]$

$= \frac{n}{2}[6+4n-4]$

$= \frac{n}{\cancel{2}}[\cancel{2}+\overset{2}{\cancel{4}}n] \Rightarrow n+2n^2$.

$\sum_{k=1}^{n} S_k = \sum_{k=1}^{n} n+2n^2$

$\Rightarrow 2\cdot\frac{n\times(n+1)(2n+1)}{6} + \frac{3[n(n+1)]}{3\times 2}$

$\Rightarrow \frac{n(n+1)}{6}[4n+2+3] \Rightarrow \frac{n(n+1)}{6}[4n+5]$

$40 < \frac{\cancel{6}}{\cancel{n(n+1)}} \times \frac{\cancel{n(n+1)}}{\cancel{6}}[4n+5] < 42$

$\Rightarrow 40-5 < 4n < 42-5$

$\Rightarrow 36 < 4n < 37$

$\Rightarrow 8.75 < n < 9.25$

$\Rightarrow \boxed{n=9}$ Ans.

krish

QUESTION [JEE Mains 2023 (13 April)]

The sum to 20 terms of the series $2 \cdot 2^2 - 3^2 + 2 \cdot 4^2 - 5^2 + 2 \cdot 6^2 - .....$ is equal to

Ans. 1310

(7.) $2\cdot2^2-3^2+2\cdot4^2-5^2-\cdots$

$= 2(2^2+4^2+6^2+\cdots \text{ upto 10 terms}) - (3^2+5^2+7^2-\cdots \text{ upto 10 terms})$

$T_r = 2(2n)^2 - \{(2n+1)^2\}$

$T_r = 8n^2 - (4n^2+1+4n)$

$T_n = 4n^2-4n-1$

TAH7

$$\sum T_n = 4\sum_{n=1}^{10} n^2 - 4\sum_{n=1}^{10} n - \sum_{n=1}^{10} 1$$

$$= \frac{4\cdot 10\cdot 11\cdot 21}{6} - \frac{4\cdot 10\cdot 11}{2} - 10$$

$140\cdot 11 - 220 - 10$

$1540 - 230$

1310

Q. The sum of 20 term of the series :

7. $2\cdot 2^2 - 3^2 + 2\cdot 4^2 - 5^2 + 2\cdot 6^2 - \cdots$

$\Rightarrow (2\cdot 2^2 + 2\cdot 4^2 + 2\cdot 6^2 + \cdots 10\text{ term}) + (-1)(3^2 + 5^2 + 7^2 + \cdots 10\text{ term})$

$\Rightarrow 2\cdot 2^2 (1^2 + 2^2 + 3^2 + \cdots + 10^2) - 1(3^2 + 5^2 + 7^2 + \cdots 10\text{ term})$

$\Rightarrow 8 \sum_{r=1}^{10} r^2 - \sum_{r=1}^{10} (2r+1)^2 \qquad 4r^2 + 4r + 1$

$\Rightarrow \overset{4}{\cancel{8}} \times \frac{10 \times 11 \times \overset{7}{\cancel{21}}}{\cancel{6}\ \cancel{3}} - \left[\overset{2}{\cancel{4}} \times \frac{10 \times 11 \times \overset{7}{\cancel{21}}}{\cancel{6}\ \cancel{2}} + \overset{2}{\cancel{4}} \times \frac{10 \times 11}{\cancel{2}} + 10 \right].$

$\Rightarrow 3080 - [1540 + 220 + 10].$

$\Rightarrow 3080 - 177$

$\Rightarrow \boxed{1310}$ Ans.

krish

TAH-07 The sum to 20 terms of the series $2\cdot 2^2 - 3^2 + 2\cdot 4^2 - 5^2 + 2\cdot 6^2 - \cdots$ is equal

$$T_r = [2\cdot(2r)^2 - (2r+1)^2]$$

$$\sum_{r=1}^{10} (2\cdot(2r)^2 - (2r+1)^2)$$

$$\sum_{r=1}^{10} (8r^2 - 4r^2 - 4r - 1)$$

$$\sum_{r=1}^{10} (4r^2 - 4r - 1)$$

$$4\sum_{r=1}^{10} r^2 - 4\sum_{r=1}^{10} r - \sum_{r=1}^{10} 1$$

$$\frac{4\cdot 10\cdot 11\cdot 21}{6} - 4\,\frac{10\cdot 11}{2} - 10$$

$$= 44\cdot 35 - 220 - 10$$

$$= 1540 - 230 \Rightarrow \boxed{1310 \text{ Ans.}}$$

QUESTION

$3 + 8 + 15 + 24 + \ldots\ldots$ up to n terms.

TAH8

$5, 7, 9$

(8) $S = 3+8+15+24+\cdots$ n term

(M1) $S = 3+8+15 \cdots T_{n-1} + T_n$

$0 = (\underbrace{3+5+7+9 \cdots}_{n\ terms}) - T_n$

$T_n = 3+5+7+9 \cdots$ upto n terms

$T_n = \frac{n}{2}[6+(n-1)2] = n(3+n-1)$

$\boxed{T_n = n(n+2) = n^2+2n}$

$\sum_{n=1}^{n} T_n = \sum n^2 + 2\sum n$

$S_n = \frac{n(n+1)(2n+1)}{6} + \cancel{2}\left(\frac{n(n+1)}{\cancel{2}}\right)$

$S_n = n(n+1)\left[\frac{2n+1}{6}+1\right]$

$S_n = \frac{n(n+1)(2n+7)}{6}$

(M-2) $3+8+15+24+\cdots$

$5 \quad 7 \quad 9 \cdots \rightarrow$ are in A.P, c.d $=2$

$2 \quad 2 \rightarrow$ is const.

$T_n = an^2+bn+c$

$T_1 = a+b+c = 3$

$T_2 = 4a+2b+c = 8$

$T_3 = 9a+3b+c = 15$

$\Rightarrow 3a+b=5$

$\Rightarrow 5a+b=7$

$2a=2$

$a=1, b=2$

$c=0$

$\Rightarrow \boxed{T_n = n^2+2n}$

similarly $S_n = \frac{n(n+1)(2n+7)}{6}$

Q. $3+8+15+24+\cdots$ up to n term.

M-I: $5, 7, 9 \cdots$ are in A.P.

8. $2, 2, \cdots$ const.

krish

$T_n = an^2+bn+c$

$a+b+c=3$

$4a+2b+c=8$

$9a+3b+c=15$

$\rightarrow 3a+b=5$

$\rightarrow 5a+b=7$

$\rightarrow a=1, b=2, c=0$

$T_n = n^2+2n \Rightarrow S_n = \sum_{n=1}^{n} T_n = n^2+2n$

$\Rightarrow \frac{n(n+1)(2n+1)}{6} + \cancel{2}\cdot\frac{n(n+1)}{\cancel{2}}$

$\Rightarrow \frac{n(n+1)}{6}[2n+1+6]$

$\Rightarrow \frac{n(n+1)(2n+7)}{6}$ Ans.

M-II $S_n = 3+8+15+24+\cdots+T_n$

$S_n = \quad 3+8+15+\cdots+T_{n-1}+T_n$

$0 = 3+(5+7+9+\cdots+\text{up to } (n-1)\text{ term}) - T_n$

$T_n = 3+\frac{n-1}{\cancel{2}}[\overset{5}{\cancel{10}}+(n-2)\cancel{2}]$

$= 3+(n-1)(5+n-2)$

$= 3+(n-1)(n+3)$

$= \cancel{3}+n^2+2n-\cancel{3}$

$= n^2+2n$

$S_n = \sum_{n=1}^{n} n^2+2n$

$\Rightarrow \frac{n(n+1)(2n+7)}{6}$

Ans.

[TAH-08] $3+8+15+24+\cdots$ up to n terms

(M1) $3+8+15+24+\cdots$ up to n term

$5 \quad 7 \quad 9 \cdots$

$S_n = 3+8+15+24+\cdots+T_n$

$S_n = \quad 3+8+15+\cdots+T_{n-1}+T_n$

$0 = 3+5+7+9\cdots$ up to $(n-1)$ Term $- T_n$

$T_n = \frac{n-1}{2}(6+(n-2)2)$

$= \frac{n-1}{2}(6+2n-4)$

$= \frac{n-1}{2}(2+2n) = \frac{(n-1)2(n+1)}{2}$

$T_n = (n^2-1)$ $\quad \boxed{T_r = n^2-1} \Rightarrow T_r = r^2-1$

(M2) $3+8+15+24+\cdots$ up to n term

1st order $\quad 5 \quad 7 \quad 9$

2nd order $\quad 2 \quad 2$ is constant

$T_n = an^2+bn+c$

$T_1 = a+b+c = 3$

$T_2 = 4a+2b+c = 8$

$T_3 = 9a+3b+c = 15$

$3a+b = 5$

$5a+b = 7$

$2a = 2$

$\boxed{a=1}$

$b = 2$

$\boxed{c=0}$

$T_n = n^2+2n$

$T_n = n^2+2n \Rightarrow \boxed{T_r = r^2+2r}$

QUESTION [JEE Mains 2025 (3 April)]

The sum $1 + 3 + 11 + 25 + 45 + 71 + \ldots..$ upto 20 terms, is equal to

(A) 7240

(B) 8124

(C) 7130

(D) 6982

Ans. A

Q) The sum, $1+3+11+25+45+71+\cdots$ upto 20 terms,

Jeh-Q9

$2 \quad 8 \quad 14 \quad 20 \cdots\cdots$ first order diff (AP)

$6 \quad 6 \quad 6 \cdots\cdots$ 2nd order diff (const)

Since, 2nd order diff is const, $T_r \to$ quad

$$T_r = ar^2 + br + c$$

$$T_1 = a+b+c = 1$$
$$T_2 = 4a+2b+c = 3$$
$$T_3 = 9a+3b+c = 11$$

$$3a+b = 2$$
$$5a+b = 8$$
$$2a = 6$$
$$a = 3$$
$$\therefore b = -7$$
$$c = 5$$

$$T_r = 3r^2 - 7r + 5$$

$$S_n = \sum_{r=1}^{n} T_r \Rightarrow S_n = \sum_{r=1}^{n} 3r^2 - 7r + 5$$

$$S_n = 3 \cdot \sum_{r=1}^{n} r^2 - 7\sum_{r=1}^{n} r + \sum_{r=1}^{n} 5$$

$$S_n = \cancel{3} \cdot \frac{n(n+1)(2n+1)}{\cancel{6}\, 2} - 7\left(\frac{n(n+1)}{2}\right) + 5n$$

$$S_n = \frac{n(n+1)(2n+1)}{2} - \frac{7\, n \cdot (n+1)}{2} + 5n$$

$$S_{20} = \frac{\overset{10}{\cancel{20}} \times 21 \times 41}{\cancel{2}} - \frac{7 \times \overset{10}{\cancel{20}} \times 21}{\cancel{2}} + 5 \times 20$$

$$= 8610 - 1470 + 100$$

$$= 7240$$

Q. The sum of $1+3+11+25+45+71+\ldots$ upto 20 term.

9.

$2, 8, 14, 20, \ldots$

$6, 6, 6, \ldots$ const.

\# $T_n = an^2+bn+c$

$a+b+c=1$

$4a+2b+c=3$

$9a+3b+c=11$

$\rightarrow 3a+b=2$

$\rightarrow 5a+b=8$

$\rightarrow a=3$, $b=-7$, $c=5$

\# $T_n = 3n^2-7n+5$

\# $S_{20} = \sum_{n=1}^{20} 3n^2-7n+5$

$= \cancel{3}\times\frac{\overset{10}{\cancel{20}}\times 21\times 41}{\cancel{6}\,\cancel{2}} - 7\times\frac{\overset{10}{\cancel{20}}\times 21}{\cancel{2}} + 5\times 20$

$= 8610 - 1470 + 100$

$= 8710 - 1470$

$= \boxed{7240}$ Ans:

krish

TAH9

(9.) $1+3+11+25+45+71+ \text{---} 20 \text{ terms}$

$2, 8, 14, 20 \rightarrow A.P, \; c.d = 6$

$6 \quad 6 \quad 6 \rightarrow \text{is const.}$

$T_n = an^2+bn+c$

$T_1 = a+b+c = 1$

$T_2 = 4a+2b+c = 3$

$T_3 = 9a+3b+c = 11$

$\Rightarrow 3a+b = 2$

$\Rightarrow 5a+b = 8$

$2a = 6$

$\boxed{a=3}, \boxed{b=-7}$

$\boxed{c=5}$

So, $T_n = 3n^2-7n+5$

$$\sum_{n=1}^{n} T_n = 3\sum_{n=1}^{n} n^2 - 7\sum_{n=1}^{n} n + \sum_{n=1}^{n} 5$$

$$= \cancel{3}\frac{n(n+1)(2n+1)}{\cancel{6}\,2} - 7\cdot\frac{n(n+1)}{2} + 5n$$

$$= \frac{n(n+1)}{2}\left[2n+1-7\right] + 5n$$

$$= \frac{n(n+1)}{2}(2n-6) + 5n$$

$$= n(n+1)(n-3) + 5n$$

$n = 20$

$= 20\cdot 21\cdot 17 + 100$

$= 7140 + 100 = 7240$

QUESTION [JEE Mains 2022 (26 July)]

★★KCLS★★

The series of positive multiples of 3 is divided into sets:

$\{3\}, \{6, 9, 12\}, \{15, 18, 21, 24, 27\}, \ldots\ldots$

Then the sum of the elements in the 11^{th} set is equal to________

Ans. 6993

TAH 10

$\{3\}, \{6,9,12\}, \{15,18 \ldots 27\} \ldots$ (1, 3, 5)

General form for no. of element in n^{th} set $= 1+(n-1)2 = 2n-1$

$3, 6, 15, 30, 48 \ldots$

$3, 9, 15, \ldots$ (18) → are in AP, c.d = 6

$6, 6 \ldots$ → const.

So, in 11^{th} set, no. of elements $= 22-1 = 21$

$T_n = an^2 + bn + c$

$T_1 = a+b+c = 3$

$T_2 = 4a+2b+c = 6$

$T_3 = 9a+3b+c = 15$

$\Rightarrow 3a+b=3$

$\Rightarrow 5a+b=9$

$2a = 6$

$\boxed{a=3}$, $\boxed{b=-6}$, $\boxed{c=+6}$

$T_n = 3n^2 - 6n + 6$

$T_{11} = 121 \cdot 3 - 66 + 6$

$= 363 - 60$

$T_{11} = 303$

Now 11^{th} set $= 303, 306 \ldots$ 21 terms

$= \frac{21}{2}[2 \times 303 + 20 \cdot 3]$

$= 21[303 + 30)$

$= 333 \times 21$

$= 6993$

Q $\underbrace{\{3\}}_{1}, \underbrace{\{6, 9, 12\}}_{3}, \underbrace{\{15, 18, 21, 24, 27\}}_{5}, \ldots$ 11^{th} set.

no. of term $= (2n-1) \rightarrow$ bracket.

$\Rightarrow 3, 6, 15, 30, \ldots T_n$

$3, 9, 15, \ldots$

$6, 6, \ldots$ const.

krish

$T_n = an^2 + bn + c$.

$a+b+c = 3$, $4a+2b+c = 6$ → $3a+b=3$

$4a+2b+c=6$, $9a+3b+c=15$ → $5a+b=9$

→ $a=3$, $b=-6$, $c=6$.

$\boxed{T_n = 3n^2 - 6n + 6}$

$\Rightarrow 1^{st}$ of 11^{th} bracket $= T_{11} = 3(121) - 66 + 6$

$= 363 - 60$

$= 303$.

$\Rightarrow$ No. of term of bracket $= 2(11) - 1 = 21$

$\Rightarrow 303 + 306 + 309 + \ldots + 21$ term

$\Rightarrow S_{21} = \frac{21}{\cancel{2}}[\cancel{2}(303) + \cancel{20}^{10}(3)]$

$= 21 \times 333$

$= \boxed{6993}$ Ans.

Solution to Previous Shikaars

QUESTION [JEE Mains 2024 (1 Feb)]

S01

Let $3, a, b, c$ be in A.P. and $3, a-1, b+1, c+9$ be in G.P. Then, the arithmetic mean of a, b and c is:

(A) -4

(B) -1

(C) 13

(D) 11

Ans. D

S-01

$3, a, b, c \rightarrow A.P$

$2a = 3+b \Rightarrow b = 2a-3$

$2b = a+c$

$2(2a-3) = a+c$

$4a-6 = a+c$

$3a = 6+c$

for $a=7$

$21 = 6+c$

$\boxed{c=15}$

$b = 14-3$

$\boxed{b=11}$

$A.M = \frac{a+b+c}{3} = \frac{7+15+11}{3}$

$= \frac{33}{3}$

$\boxed{A.M = 11}$

$3, a-1, b+1, c+9 \rightarrow G.P$

$\frac{c+9}{b+1} = \frac{b+1}{a-1} = \frac{a-1}{3}$ $\qquad \frac{a+b+c}{3} = ?$

$3(b+1) = (a-1)^2$

$3(2a-3+1) = (a-1)^2$

$\cancel{3}\,6a-6 = a^2-2a+1$

$a^2-8a+7=0$

$a^2-7a-a+7=0$

$a(a-7)-1(a-7)=0$

$a=7, a=1$

for $a=1$

$b = 2-3 = -1$

$c = 3-6 = -3$

$\frac{a+b+c}{3} = \frac{\cancel{1}-\cancel{1}-3}{3} = -1$

N.P

S(1) Let 3, a, b, c be in A.P and 3, a-1, b+1, c+9 be in G.P. Then, the arithmetic mean of a, b and c is

$\therefore$ 3, a, b, c are in A.P.

$\Rightarrow$ C.D, $d = a-3$

Now, 3, a-1, b+1, c+9 are in G.P.

$\Rightarrow 3(c+9) = (b+1)(a-1)$

$\Rightarrow 3(3a-6+9) = (2a-3+1)(a-1)$

$\Rightarrow 3(3a+3) = (2a-2)(a-1) \Rightarrow 9a+9 = 2a^2-4a+2$

$\Rightarrow 2a^2-13a-7=0 \Rightarrow c = 7, \cancel{-1/2}$

$b = 2\times7-3 = 11 \Rightarrow c = 3\times7-6 = 15$

$\therefore$ A.M of a, b, c $= \frac{7+11+15}{3} = \frac{33}{3} \Rightarrow 11$ Ans

$\boxed{S-01}$ Let 3, a, b, c be in A.P. and 3, a−1, b+1, c+9 be in G.P. Then, the Arithmetic mean of a, b and c is:

3, a, b, c are in A.P.

Common difference $d = a-3$

Now 3, a−1, b+1, c+9 are in G.P.

$$3(c+9) = (b+1)(a-1)$$
$$3(3a-6+9) = (2a-3+1)(a-1)$$
$$3(3a+3) = (2a-2)(a-1)$$
$$9a+9 = 2a^2-4a+2$$
$$2a^2-13a-7=0$$
$$a = 7, \cancel{-\frac{1}{2}}$$
$$b = 2\times7-3 = 11$$
$$c = 3\times7-6 = 15$$

A.M of a, b, c : $\frac{7+11+15}{3} = \frac{33}{3} = \boxed{11 \text{ Ans}}$

S-01
Ayush Patel
Prayagraj UP.

QUESTION [IIT-JEE 2005 Screening]

S02

In the quadratic equation $ax^2 + bx + c = 0$, $\Delta = b^2 - 4ac$ and $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$, are in G.P. where α, β are the root of $ax^2 + bx + c = 0$, then

(A) $\Delta \neq 0$

(B) $b\Delta = 0$

(C) $c\Delta = 0$

(D) $\Delta = 0$

Ans. C

QUESTION [IIT-JEE 2010]

S03

Let $a_1, a_2, a_3 \ldots, a_{11}$ be real numbers satisfying

$a_1 = 15, 27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots 11$.

If $\frac{a_1^2+a_2^2+\cdots+a_{11}^2}{11} = 90$, then the value of $\frac{a_1+a_2+\cdots+a_{11}}{11}$ is equal to:

$a_k - a_{k-1} = a_{k-1} - a_{k-2}$

$2a_{k-1} = a_k + a_{k-2}$

a_{k-2}, a_{k-1}, a_k are in A.P

$a_1, a_2, a_3, a_4, a_5 \ldots$ A.P.

$d \quad d \quad d$

$a_2 = a_1 + d < \frac{27}{2}$

$15 + d < \frac{27}{2}$

$d < -3/2$

$990 = a_1^2 + (a_1+d)^2 + (a_1+2d)^2 + \ldots (a_1+10d)^2$

$= 11a_1^2 + d^2(1^2+2^2+ \ldots + 10^2)$

$+ 2a_1 d(1+2+3+\ldots+10)$

Ans. 0

[S-03] Let $a_1, a_2, a_3, ---, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, ----, 11$.

If $\frac{a_1^2 + a_2^2 + --- + a_{11}^2}{11} = 90$, then $\frac{a_1 + a_2 + --- + a_{11}}{11} = ?$

$a_k = 2a_{k-1} - a_{k-2}$

$a_1, a_2, a_3 . ----, a_{11}$ are in A.P.

$\therefore \frac{a_1^2 + a_2^2 + ---+ a_{11}^2}{11} = 90$

$= \frac{11a^2 + 35 \times 11 d^2 + 110ad}{11} = 90$

$35d^2 + 225 + 150d = 90$

$35d^2 + 150d + 225 - 90 = 0$

$35d^2 + 150d + 135 = 0$

$d = -3, -\frac{9}{7}$

$\because a_2 < \frac{27}{2} \quad \therefore d = -3$ and $d = -\frac{9}{7}$

$\Rightarrow \frac{a_1 + a_2 + a_3 + --- + a_{11}}{11}$

$\frac{11}{2}[30 - 10 \times 3] = \boxed{0 \text{ Ans}}$

QUESTION [JEE Mains 2023]

S04

If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296, respectively, then the sum of common ratio of all such G.P.s is

(A) 9/2

(B) 3

(C) 7

(D) 14

Ans. C

[S-04] If the sum and product of four +ve consecutive terms of a G.P., are 126 and 1296 resp., then sum of common ratio of all such G.P.s is

Let four consecutive +ve terms of a G.P. are

$a, ar, ar^2, ar^3 \quad (a, r > 0)$

Product : $a^4 r^6 = 1296$

$a^2 r^3 = 36$

$\boxed{a = \frac{6}{r^{3/2}}}$

Sum: $a + ar + ar^2 + ar^3 = 126$

$\frac{1}{r^{3/2}} + \frac{r}{r^{3/2}} + \frac{r^2}{r^{3/2}} + \frac{r^3}{r^{3/2}} = \frac{126}{6}$

$(r^{-3/2} + r^{3/2}) + (r^{1/2} + r^{-1/2}) = 21$

Let $r^{1/2} + r^{-1/2} = T$

$r^{-3/2} + r^{3/2} = (r^{1/2} + r^{-1/2})^3 - 3(r^{1/2} + r^{-1/2})$

$T^3 - 3T$

$T^3 - 3T + T = 21$

$T^3 - 2T = 21$ $\quad \boxed{T = 3}$

$\therefore \sqrt{r} + \frac{1}{\sqrt{r}} = 3$

$r + 1 = 3\sqrt{r}$ SBS

$r^2 + 2r + 1 = 9r$

$r^2 + 2r + 1 = 9r$

$r^2 - 7r + 1 = 0$

S.O.R = $\boxed{7 \text{ Ans}}$

QUESTION [JEE Mains 2024 (8 April)]

S05

In an increasing geometric progression of positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the 4^{th}, 6^{th} and 8^{th} terms is equal to:

(A) 96

(B) 91

(C) 84

(D) 78

Ans. B

Sol

$T_2 + T_6 = \frac{70}{3}$

$\Rightarrow ar + ar^5 = \frac{70}{3}$

$ar^3\left(\frac{1}{r^2} + r^2\right) = \frac{70}{3}$

$7\left(\frac{1}{r^2} + r^2\right) = \frac{\cancel{70}^{10}}{3}$

$\therefore \frac{1}{r^2} + r^2 = 3 + \frac{1}{3}$

$\therefore \boxed{r^2 = 3}$

$T_3 \cdot T_5 = 49$

$ar^2 \cdot ar^4 = (7)^2$

$ar^3 = 7$

$\therefore T_4 + T_6 + T_8$

$= ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 7(1 + 3 + 9)$

$= 7(13)$

$= 91$ (B) Ans

Anushka Chaurasia
from Delhi

[S-05] In an Inc↑ G.P. of +ve terms, the sum of the second and sixth terms is 70/3 and the product of the third and fifth terms is 49. then the sum of the 4th, 6th and 8th terms is:

$T_2 + T_6 = \frac{70}{3} \Rightarrow ar + ar^5 = \frac{70}{3}$

$T_3 \cdot T_5 = 49 \Rightarrow ar^2 \cdot ar^4 = 49$

$a^2r^6 = 49$

$ar^3 = 7$

$a = \frac{7}{r^3}$

$ar + ar^5 = \frac{70}{3}$

$ar(1 + r^4) = \frac{70}{3}$

$\frac{7}{r^2}(1 + r^4) = \frac{70}{3}$ Let $r^2 = t$

$\frac{1}{t}(1 + t^2) = \frac{10}{3}$ $3t^2 - 10t + 3 = 0$

$t = 3, \frac{1}{3}$

$r = \sqrt{3}, \cancel{\frac{1}{\sqrt{3}}}$ $(\because r > 1)$

Now: $T_4 + T_6 + T_8$

$ar^3 + ar^5 + ar^7$

$ar^3(1 + r^2 + r^4)$

$7(1 + 3 + 9) = 7(13)$

$= \boxed{91 \text{ Ans}}$

S-05
Ayush Patel
Prayagraj UP

THANK YOU