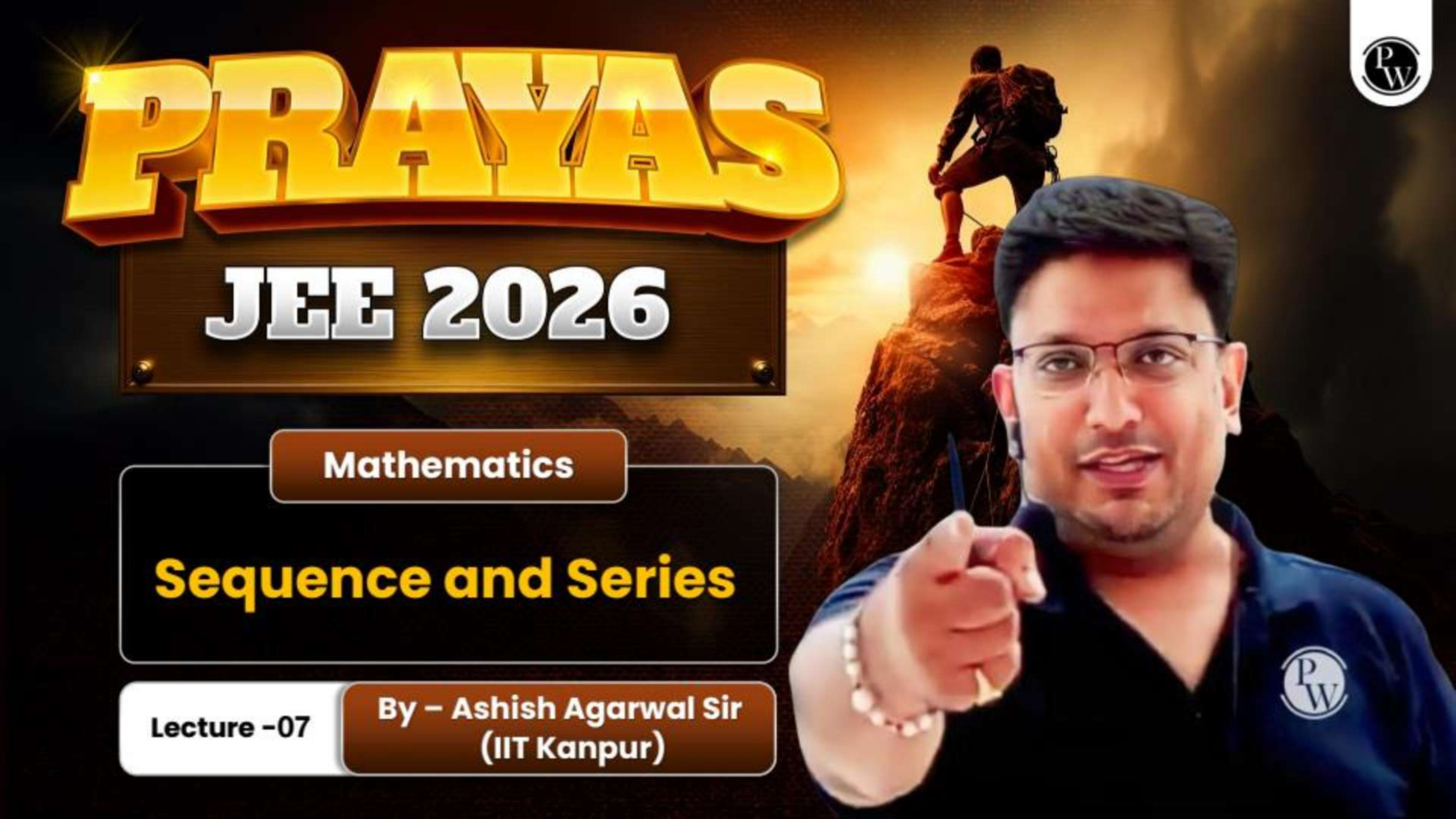
PRAYAS
JEE 2026
Mathematics
Sequence and Series
Lecture -07
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Topics *to be covered*

- **A** Question Practice on Some Special Sequences
- **B** Method of Difference
- **C** Telescoping Series

Aao Machaay Dhamaal
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Special Sequences

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Special Sequences

Recursive Relation Based Sequences

QUESTION [JEE Mains 2025 (28 Jan)]

★★KCLS★★

Let $\langle a_n \rangle$ be a sequence such that $a_0 = 0, a_1 = \frac{1}{2}$ and $2a_{n+2} = 5a_{n+1} - 3a_n$, n = 0, 1, 2, 3, ...

Then $\sum_{k=1}^{100} a_k$ is equal to

(A) $3a_{100} + 100$

(B) $3a_{100} - 100$

(C) $3a_{99} - 100$

(D) $3a_{99} + 100$

M1 $a_0 = 0, \; a_1 = \frac{1}{2}$

$2a_{n+2} = 5a_{n+1} - 3a_n$

$2a_{n+2} = 2a_{n+1} + 3a_{n+1} - 3a_n$

$2(a_{n+2} - a_{n+1}) = 3(a_{n+1} - a_n)$

let $a_{n+1} - a_n = b_n$.

$b_0 = a_1 - a_0$

$b_0 = \frac{1}{2} - 0 = 1/2$

$2b_{n+1} = 3b_n$

$\frac{b_{n+1}}{b_n} = \frac{3}{2} \to \langle b_n \rangle$ is a G.P with CR $= \frac{3}{2}$

Ans. B

$$b_n = a_{n+1} - a_n$$

$$\sum_{r=1}^{n} K = nK$$

$$b_0 = a_1 - a_0$$
$$b_1 = a_2 - a_1$$
$$b_2 = a_3 - a_2$$
$$b_3 = a_4 - a_3$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$b_{n-1} = a_n - a_{n-1}$$

$$b_0 + b_1 + b_2 + \cdots + b_{n-1} = a_n - a_0.$$

$$\frac{1}{2} \cdot \frac{\left(\left(3/2\right)^n - 1\right)}{3/2 - 1} = a_n - 0$$

$$a_n = (3/2)^n - 1$$

$$a_{100} = (3/2)^{100} - 1$$

$$S = \sum_{k=1}^{100} a_k = \sum_{k=1}^{100} \left(\left(\frac{3}{2}\right)^k - 1\right)$$

$$= \frac{\frac{3}{2}\left((3/2)^{100} - 1\right)}{3/2 - 1} - 100$$

$$= 3\left((3/2)^{100} - 1\right) - 100$$

$$S = 3a_{100} - 100$$

QUESTION [JEE Mains 2025 (28 Jan)]

★★KCLS★★

Let $\langle a_n \rangle$ be a sequence such that $a_0 = 0, a_1 = \frac{1}{2}$ and $2a_{n+2} = 5a_{n+1} - 3a_n$, n = 0, 1, 2, 3, ...

Then $\sum_{k=1}^{100} a_k$ is equal to

M2 $a_0 = 0,\ a_1 = \frac{1}{2}$ $2a_{n+2} = 5a_{n+1} - 3a_n$

$n=0,\ 2a_2 = 5a_1 - 3a_0$

$a_2 = \frac{5}{4}$

$n=1$ $2a_3 = 5a_2 - 3a_1 = \frac{25}{4} - \frac{3}{2} \Rightarrow a_3 = \frac{19}{8}$

$n=2$ $2a_4 = 5a_3 - 3a_2 = \frac{95}{8} - \frac{15}{4} = \frac{65}{8}$

$\Downarrow$

$a_4 = \frac{65}{16}$

G.P: $\frac{3}{4}, \frac{9}{8}, \frac{27}{16}$ - - -

$\frac{1}{2}, \frac{5}{4}, \frac{19}{8}, \frac{65}{16}$

(A) $3a_{100} + 100$

(B) $3a_{100} - 100$

(C) $3a_{99} - 100$

(D) $3a_{99} + 100$

Ans. B

$$S = \frac{1}{2} + \frac{5}{4} + \frac{19}{8} + \frac{65}{16} + \cdots + a_n.$$

$$S = \quad \frac{1}{2} + \frac{5}{4} + \frac{19}{16} + \cdots + a_{n-1} + a_n$$

$$0 = \frac{1}{2} + \left(\frac{3}{4} + \frac{9}{8} + \frac{27}{16} + \cdots \text{up to } (n-1) \text{ terms}\right) - a_n$$

$$a_n = \frac{1}{2} + \frac{\frac{3}{4}\left(\left(\frac{3}{2}\right)^{n-1} - 1\right)}{3/2 - 1}$$

$$= \frac{1}{2} + \frac{3}{2}\left((3/2)^{n-1} - 1\right)$$

$$= 1/2 + (3/2)^n - 3/2$$

$$a_n = (3/2)^n - 1$$

QUESTION

★★KCLS★★

Let $a_0, a_1, a_2, \ldots\ldots, a_n \ldots\ldots$ be a sequence of numbers satisfying

$(3 - a_{n+1}) \cdot (6 + a_n) = 18$ and $a_0 = 3$ then $\sum_{i=0}^{n} \frac{1}{a_i} =$

(A) $\frac{1}{3}(2^{n+2} + n - 3)$

(B) $\frac{1}{3}(2^{n+2} - n + 3)$

(C) $\frac{1}{3}(2^{n+2} - n - 3)$

(D) $\frac{1}{3}(2^{n+2} + n + 3)$

$\cancel{18} + 3a_n - 6a_{n+1} - a_n \cdot a_{n+1} = \cancel{18}$

$3a_n = 6a_{n+1} + a_n \cdot a_{n+1}$

$\frac{3}{a_{n+1}} = \frac{6}{a_n} + 1$

$a_0 = 3.$

$\frac{3}{a_{n+1}} = \frac{6}{a_n} + 1$

Add 1 to both sides

$\frac{3}{a_{n+1}} + 1 = \frac{6}{a_n} + 2 = 2\left(\frac{3}{a_n} + 1\right)$

Ans. C

$$\frac{3}{a_{n+1}}+1=2\left(\frac{3}{a_n}+1\right)$$

let $b_n=\frac{3}{a_n}+1$

$b_0=\frac{3}{a_0}+1=\frac{3}{3}+1=2$

$$b_{n+1}=2b_n$$

$$\frac{b_{n+1}}{b_n}=2 \longrightarrow \langle b_n\rangle \text{ is a G.P., CR}=2.$$

$b_n=\frac{3}{a_n}+1$

$b_0=\frac{3}{a_0}+1$

$b_1=\frac{3}{a_1}+1$

$b_2=\frac{3}{a_2}+1$

$b_3=\frac{3}{a_3}+1$

$\vdots$

$b_n=\frac{3}{a_n}+1$

$b_0+b_1+b_2+\cdots+b_n=3\left(1/a_0+1/a_1+1/a_2+\cdots+1/a_n\right)+n+1$

$$2\cdot\frac{(2^{n+1}-1)}{2-1}=3\sum_{i=0}^{n}\frac{1}{a_i}+n+1$$

$$\frac{2(2^{n+1}-1)-n-1}{3}=\sum_{i=0}^{n}\frac{1}{a_i}$$

$$\frac{1}{3}(2^{n+2}-2-n-1)=\sum_{i=0}^{n}\frac{1}{a_i}$$

QUESTION

★★KCLS★★

It is given that the sequence $\{a_n\}$ satisfies $a_1 = 0, a_{n+1} = a_n + 1 + 2\sqrt{1 + a_n}$ for $n \in N$. Then

(A) $a_{100} = 9999$

(B) $a_{2001} = 4004000$

(C) $a_{2001} = 4002000$

(D) $a_{19} = 360$

$a_1 = 0 \quad , a_{n+1} = a_n + 1 + 2\sqrt{1+a_n}$

$a_{n+1} + 1 = (\sqrt{a_n+1})^2 + 2\sqrt{1+a_n} \cdot 1 + 1$

$a_{n+1} + 1 = (\sqrt{a_n+1} + 1)^2$

$\sqrt{a_{n+1}+1} = \sqrt{a_n+1} + 1$

let $b_n = \sqrt{a_n+1}$

$b_1 = \sqrt{a_1+1}$

$b_1 = 1$

$b_{n+1} = b_n + 1$

$b_{n+1} - b_n = 1 \Rightarrow \langle b_n \rangle$ is an A.P

Ans. A, B, D

$b_1 = 1$ $\langle b_n \rangle$ is A.P with cd $= 1$, $b_n = \sqrt{a_n + 1}$

$b_n^2 - 1 = a_n$

$a_n = b_n^2 - 1$

$b_n = 1 + (n-1)\cdot 1$

$b_n = n$

$a_n = n^2 - 1$

QUESTION

Tah 01

$a_1, a_2, a_3, ...$ is a sequence such that $a_1 = \frac{1}{2}$ & $(3 + a_n)(4 - a_{n+1}) = 12$ then

$$\sum_{n=1}^{n} \frac{1}{a_n} = \_\_\_\_\_\_\_\_$$

Ans. $4\left(1 - \left(\frac{3}{4}\right)^n\right) - n$

Problems Based on $S_n = \Sigma T_n$

If for any series we know T_r

then $S_n = T_1 + T_2 + T_3 + \cdots + T_n$

$$S_n = \sum_{r=1}^{n} T_r$$

QUESTION

Evaluate: $1^2 + 3^2 + 5^2 + \ldots$ n terms

$T_r = (2r-1)^2 = 4r^2 - 4r + 1$

$$S_n = \sum_{r=1}^{n} (4r^2 - 4r + 1) = 4\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$$

$$S_n = 4 \cdot \frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2} + n.$$

QUESTION [JEE Mains 2019]

The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + ...$ upto 11^{th} term is :

$S_n = 1 \times 1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + ---$

(A) 916

$T_r = r \times (2r-1) = 2r^2 - r$

(B) 945

$S_{11} = \sum_{r=1}^{11} T_r = \sum_{r=1}^{11} (2r^2 - r) = 2\sum_{r=1}^{11} r^2 - \sum r$

(C) 915

$= 2 \cdot \frac{11 \cdot \cancel{12}^{2} \cdot 23}{\cancel{6}} - \frac{11 \cdot 12}{2}$

(D) 946

$= 44 \times 23 - 66.$

$= 1112 - 66$

$= 946$

QUESTION [JEE Mains 2025 (4 April)]

Tah 02

$1^{2} + 3 + 5^2 + 7 + 9^2 + \ldots$ upto 40 terms is equal to

(A) 40870

(B) 41880

(C) 43890

(D) 33980

Ans. B

QUESTION [JEE Mains 2022 (25 July)]

Let $a_1 = b_1 = 1, a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$.

Then $\sum_{n=1}^{15} a_n . b_n$ is equal to

$a_n - a_{n-1} = 2$

$\downarrow$

$\langle a_n \rangle$ is an A.P, cd $= 2$, $a_1 = 1$

$a_n = 1 + (n-1)2 = 2n-1$

Now $b_n = a_n + b_{n-1}$

$b_n - b_{n-1} = 2n-1$

$n=2 \quad b_2 - b_1 = 3$

$n=3 \quad b_3 - b_2 = 5$

$n=4 \quad b_4 - b_3 = 7$

$n=n \quad b_n - b_{n-1} = 2n-1$

$b_n - b_1 = 3+5+7+ - - +(2n-1) \quad (\because b_1 = 1)$

$b_n = 1+3+5+7+ - - +(2n-1)$

$b_n = n^2$

Ans. 27560

$$\sum_{n=1}^{15} a_n \cdot b_n = \sum_{n=1}^{15} (2n-1) \cdot n^2$$

$$= 2\sum_{n=1}^{15} n^3 - \sum_{n=1}^{15} n^2$$

$$= 2\left(\frac{15 \cdot 16}{2}\right)^2 - \frac{\overset{5}{\cancel{15}} \cdot \overset{8}{\cancel{16}} \cdot 31}{\cancel{6}}$$

$$= 2 \cdot (120)^2 - 40 \times 31$$

$$= 28800 - 1240.$$

$$= 27560$$

QUESTION [JEE Mains 2019]

The sum $\frac{3\times1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}+ \ldots$ upto 10^{th} term, is :

(A) 680

(B) 660

(C) 600

(D) 620

$$T_r=\frac{(2r+1)(1^3+2^3+\cdots+r^3)}{1^2+2^2+3^2+\cdots+r^2}$$

$$=\frac{\cancel{(2r+1)}\,\frac{\cancel{r^2}\cdot\cancel{(r+1)^2}}{4}}{\frac{\cancel{r}\cdot\cancel{(r+1)}\cancel{(2r+1)}}{6}}=\frac{3}{2}\cdot r(r+1)$$

$$S_{10}=\sum_{r=1}^{10}T_r=\frac{3}{2}\sum_{r=1}^{10}(r^2+r)$$

$$=\frac{3}{2}\left(\frac{\overset{5}{\cancel{10}}\cdot11\cdot\overset{7}{\cancel{21}}}{\cancel{6}}+\frac{10\cdot11}{2}\right)$$

$$=3/2\,(385+55)=\frac{3}{2}(440)=660.$$

QUESTION [JEE Mains 2019]

Jah03

The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+\cdots+\frac{1^3+2^3+3^3+\cdots+15^3}{1+2+3+\cdots+15}-\frac{1}{2}(1+2+3+\cdots+15)$ is equal to

(A) 620

(B) 1240

(C) 1860

(D) 660

QUESTION [JEE Mains 2020]

Tan04

The sum $\sum_{k=1}^{20}(1+2+3+\cdots+k)$ is

QUESTION [JEE Mains 2019]

Tah 05

Let $S_k = \frac{1+2+3+\cdots+k}{k}$. If $S_1^2 + S_2^2 + \cdots + S_{10}^2 = \frac{5}{12}A$, then A is equal to:

(A) 303

(B) 156

(C) 283

(D) 301

QUESTION [JEE Advanced 2009]

If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is

(A) $\frac{n(4n^2-1)c^2}{6}$

(B) $\frac{n(4n^2+1)c^2}{3}$

(C) $\frac{n(4n^2-1)c^2}{3}$

(D) $\frac{n(4n^2+1)c^2}{6}$

$S_n = cn^2$
$\downarrow$
$T_1 = c, \quad d = 2c$

$S_n = an^2 + bn$
$\downarrow$
A.P
$T_1 = a+b, \quad cd = 2a$

A.P : $c, 3c, 5c, 7c, \ldots$

$S_n = c^2 + 9c^2 + 25c^2 + 49^2 + \ldots$ up to n terms.

$= c^2(1^2 + 3^2 + 5^2 + 7^2 + \ldots \text{ up to n terms})$

$= c^2 \sum_{r=1}^{n} (2r-1)^2 = c^2 \sum_{r=1}^{n} (4r^2 - 4r + 1)$

$= c^2 \left(4 \cdot \frac{n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n\right)$

$= nc^2 \left(4 \frac{(n+1)}{2} \left(\frac{2n+1}{3} - 1\right) + 1\right)$

$$S_n = nc^2\left(\frac{2(n+1)\cdot(2n-2)}{3}+1\right)$$

$$= nc^2\left(\frac{4}{3}(n+1)(n-1)+1\right)$$

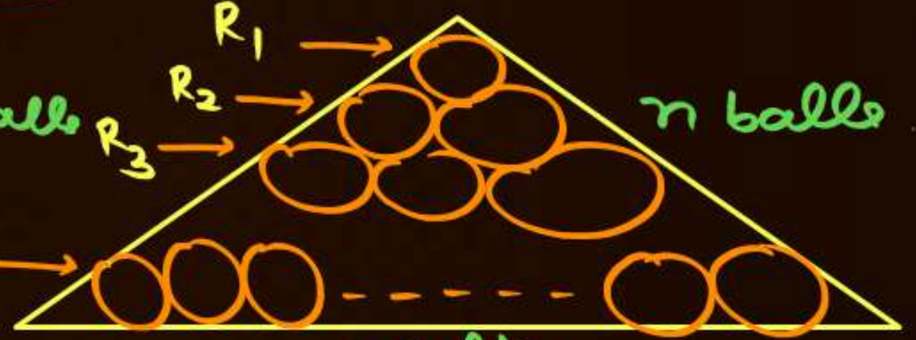
$$= nc^2\left(\frac{4n^2-4+3}{3}\right)$$

$$= \frac{nc^2(4n^2-1)}{3}$$

QUESTION [JEE Mains 2019]

★★ASRQ★★

Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is:



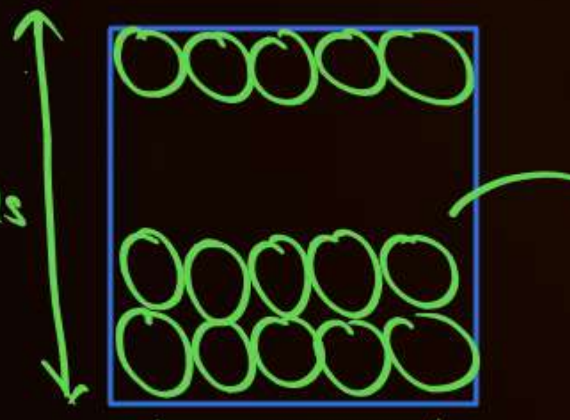
(A) 157

(B) 262

(C) 225

(D) 190

No: of balls in equilateral triangle $= 1+2+3+\cdots+n = \frac{n(n+1)}{2}$



No: of Balls in Square $= (n-2)^2$

$$\frac{n(n+1)}{2} + 99 = (n-2)^2$$

$$n^2+n+198 = 2n^2-8n+8$$

$$n^2 - 9n - 190 = 0$$

$$(n - 19)(n + 10) = 0$$

$$n = 19$$

No: of Balls used to form Eq. Triangle $= \frac{19 \cdot 20}{2} = 190.$

QUESTION [JEE Mains 2024 (30 Jan)]

Tah 06

Let S_n be the sum to n-terms of an arithmetic progression 3, 7, 11, ...

If $40 < \left(\dfrac{6}{n(n+1)}\sum_{k=1}^{n} S_k\right) < 42$, then n equals

Ans. 9

QUESTION [JEE Mains 2023 (13 April)]

Tah07

The sum to 20 terms of the series $2 \cdot 2^2 - 3^2 + 2 \cdot 4^2 - 5^2 + 2 \cdot 6^2 - .....$ is equal to

Ans. 1310

Methods of Difference

Types 1:

(Using method of difference)

If $T_1, T_2, T_3,$ ..... are the terms of a sequence then the terms

$T_2 - T_1, T_3 - T_2, T_4 - T_3$ .....

T_1, T_2, T_3, T_4, T_5 - - - -

$T_2 - T_1, T_3 - T_2, T_4 - T_3, T_5 - T_4$ - - -

A.P or G.P

we use MOD.

Some times are in A.P. and sometimes in G.P. for such series we first compute their n^{th} term and then compute the sum to n terms, using sigma notation.

QUESTION

$6 + 13 + 22 + 33 + \ldots\ldots$ n terms.

7 , 9 , 11 _ _ are in A.P we use MoD

M (1)

$S_n = 6 + 13 + 22 + 33 + - - - - + T_n$

$S_n = \quad 6 + 13 + 22 + - - - + T_{n-1} + T_n$

$0 = 6 + \underbrace{(7 + 9 + 11 + - - - - - - \text{upto})}_{(n-1)\text{ terms}} - T_n$

$$T_n = 6 + \frac{n-1}{2}(14 + (n-2)2)$$
$$= 6 + (n-1)(7 + n - 2)$$
$$= 6 + (n-1)(n+5)$$
$$= 6 + n^2 + 4n - 5$$
$$= n^2 + 4n + 1$$

$T_r = r^2 + 4r + 1$

$$S_n = \sum_{r=1}^{n} T_r = \sum_{r=1}^{n}(r^2 + 4r + 1)$$
$$= \frac{n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} + n$$

$T_1, T_2, T_3, T_4, T_5 - - -$

1st order diff $T_2 - T_1, T_3 - T_2, T_4 - T_3, T_5 - T_4$

2nd order diff — — — — —

3rd order diff. — — — —

$f(n) = an + b$

$f(n-1) = a(n-1) + b$

1st order diff $f(n) - f(n-1) = a$ = constant

$f(n) = an^2 + bn + c$

$f(n-1) = a(n-1)^2 + b(n-1) + c$

$g(n) = f(n) - f(n-1) = 2an - a + b$

$g(n) = 2an - a - b$

$g(n-1) = 2a(n-1) - a - b$

$g(n) - g(n-1) = 2a$ = constant

- 1st order diff of a linear in 'n' is constant
- 2nd order diff of a quad in 'n' is constant
- 3rd order diff of a cubic in 'n' is constant

⋮

- mth order difference of a 'm' degree polynomial in 'n' is constant

QUESTION

$6 + 13 + 22 + 33 + \ldots\ldots$ n terms.

M 2

1st order diff: 7 , 9 , 11 ... are in A.P we use MOD

2nd order diff: 2 , 2 , --- is constant

$T_n = an^2 + bn + c$

$T_1 = a + b + c = 6$

$T_2 = 4a + 2b + c = 13$

$T_3 = 9a + 3b + c = 22$

$3a + b = 7$

$5a + b = 9$

$2a = 2$

$a = 1$

$b = 4$

$c = 1$

$T_n = n^2 + 4n + 1$

QUESTION

$3 + 8 + 15 + 24 + \ldots\ldots$ up to n terms. Tah 08 M ① M ②

5 , 7 , 9 - - - -

QUESTION [JEE Mains 2025 (3 April)]

Jan 09 - M (2)

The sum $1 + 3 + 11 + 25 + 45 + 71 + \ldots$ upto 20 terms, is equal to

2, 8, 14, 20, - - -

2nd order diff 6, 6, 6 - - -

(A) 7240

(B) 8124

(C) 7130

(D) 6982

Ans. A

QUESTION

$5 + 7 + 13 + 31 + 85 + \ldots\ldots$ up to n terms.

2 , 6 , 18 , 54 --- are in G.P

M (1)

$S_n = 5+7+13+31+85+ \cdots + T_n$

$S_n = \quad 5+7+13+31+ \cdots + T_{n-1} + T_n$

$0 = 5 + \left(2+6+18+54+ \cdots \text{ upto } (n-1) \text{ terms}\right) - T_n$

$T_n = 5 + \dfrac{2\cdot(3^{n-1}-1)}{3-1}$

$T_n = 5 + 3^{n-1} - 1 = 3^{n-1} + 4$

$S_n = \sum_{n=1}^{n} T_n = \sum_{n=1}^{n} 3^{n-1} + \sum_{n=1}^{n} 4 = 3^0 \dfrac{(3^n-1)}{3-1} + 4n = \dfrac{3^n-1}{2} + 4n.$

$$f(n) = a\cdot r^{n} + b \text{ — polynomial of degree } 0$$

$$f(n-1) = a\cdot r^{n-1} + b$$

$$g(n) = f(n) - f(n-1) = a\cdot(r^{n} - r^{n-1}) = a(r-1)\cdot r^{n-1} \text{ are in G.P}$$

b'coz : $a(r-1), a(r-1)r, a(r-1)r^2, a(r-1)r^3, \ldots$

lly $f(n) = ar^{n} + (bn + c)$ — 1 degree polynomial

$$f(n-1) = ar^{n-1} + b(n-1) + c$$

$$g(n) = f(n) - f(n-1) = a(r-1)r^{n-1} + b \text{ — 1st order diff.}$$

$$g(n) = a(r-1)r^{n-1} + b$$

$$g(n-1) = a(r-1)r^{n-2} + b.$$

$$g(n) - g(n-1) = a(r-1)^2\cdot r^{n-2} = \text{2nd order diff in G.P}$$

★ If 1st order diff are in G.P with CR = r then

$$f(n) = a\cdot r^{n} + b$$

★ If 2nd order diff are in G.P with CR = r then

$$f(n) = ar^{n} + (bn + c)$$

★ If 3rd order diff are in G.P with CR = r then

$$f(n) = ar^{n} + bn^2 + cn + d$$

★ If m^{th} order differences are in G.P with CR = r then

$$f(n) = ar^{n} + (\text{polynomial in } n \text{ of degree } m-1)$$

QUESTION

$5 + 7 + 13 + 31 + 85 + \ldots\ldots$ up to n terms.

1st order diff $2, 6, 18, 54 - - -$ are in G.P $CR = 3$

$T_n = a\cdot 3^n + b$

$T_1 = 3a + b = 5$

$T_2 = 9a + b = 7$

$6a = 2$

$a = 1/3$

$b = 4$

$\Downarrow$

$T_n = \frac{1}{3}\cdot 3^n + 4 = 3^{n-1} + 4$

QUESTION [JEE Mains 2024 (8 April)]

★★KCLS★★

An arithmetic progression is written in the following way

2, 5, 8, 11, 13 - - -

$R_1 \longrightarrow$ 2

$R_2 \rightarrow$ 5 8

$R_3 \rightarrow$ 11 14 17

$R_4 \rightarrow$ 20 23 26 29

The sum of all the terms of the 10^{th} row is ________

$T_n = an^2 + bn + c$

$T_1 = a + b + c = 2$

$T_2 = 4a + 2b + c = 5$

$T_3 = 9a + 3b + c = 11$

$3a + b = 3$

$5a + b = 6$

$a = 3/2$

$b = -3/2$

$c = 2$

2, 5, 11, 20, 32 - - - -

1st order diff 3 6 9 12 - - -

2nd order diff 3 , 3 , 3 - - - - constant

Ans. 1505

$$T_n = \frac{3}{2}n^2 - \frac{3}{2}n + 2$$

$T_{10} = 150 - 15 + 2 = 137$ — 1st term of 10^{th} Row.

Row 10: 137 140 143 - - - up to 10 terms

$$S = \frac{10}{2}(2 \cdot 137 + 9 \cdot 3)$$
$$= 5(274 + 27)$$
$$= 5(301)$$
$$= 1505 \text{ Ans.}$$

QUESTION [JEE Mains 2022 (26 July)]

Jan 10

★★KCLS★★

The series of positive multiples of 3 is divided into sets:

$\{3\}, \{6, 9, 12\}, \{15, 18, 21, 24, 27\}, \ldots\ldots$

Then the sum of the elements in the 11^{th} set is equal to ________

Ans. 6993

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Today's Shikaars

QUESTION [JEE Mains 2024 (1 Feb)]

S01

Let $3, a, b, c$ be in A.P. and $3, a-1, b+1, c+9$ be in G.P. Then, the arithmetic mean of a, b and c is:

(A) -4

(B) -1

(C) 13

(D) 11

Ans. D

QUESTION [IIT-JEE 2005 Screening]

S02

In the quadratic equation $ax^2 + bx + c = 0$, $\Delta = b^2 - 4ac$ and $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$, are in G.P. where α, β are the root of $ax^2 + bx + c = 0$, then

(A) $\Delta \neq 0$

(B) $b\Delta = 0$

(C) $c\Delta = 0$

(D) $\Delta = 0$

Ans. C

QUESTION [IIT-JEE 2010]

S03

Let $a_1, a_2, a_3 \ldots, a_{11}$ be real numbers satisfying

$a_1 = 15, 27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots 11$.

If $\frac{a_1^2+a_2^2+\cdots+a_{11}^2}{11} = 90$, then the value of $\frac{a_1+a_2+\cdots+a_{11}}{11}$ is equal to:

Ans. 0

QUESTION [JEE Mains 2023]

S04

If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296, respectively, then the sum of common ratio of all such G.P.s is

(A) 9/2

(B) 3

(C) 7

(D) 14

Ans. C

QUESTION [JEE Mains 2024 (8 April)]

S05

In an increasing geometric progression of positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the 4^{th}, 6^{th} and 8^{th} terms is equal to:

(A) 96

(B) 91

(C) 84

(D) 78

Ans. B

Solution to Previous TAH

[Mains-23] Tah-01

$$S=\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2\cdot3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}-\frac{1}{2^2\cdot3}+\frac{1}{2\cdot3^2}-\frac{1}{3^3}\right)+\left(\frac{1}{2^4}-\frac{1}{2^3\cdot3}+\frac{1}{2^2\cdot3^2}-\frac{1}{2\cdot3^3}+\frac{1}{3^4}\right)+\cdots$$

Let $\frac{1}{2}\to x$; $\frac{1}{3}=y$

$$S=(x-y)+(x^2-xy+y^2)+(x^3-x^2y+xy^2-y^3)+(x^4-x^3y+x^2y^2-xy^3+y^4)+\cdots$$

Richathakur

$$S(x+y)=(x^2-y^2)+(x^3+y^3)+(x^4-y^4)+(x^5+y^5)+\cdots$$

$$S(x+y)=(x^2+x^3+x^4+\cdots\infty)-(y^2-y^3+y^4-y^5\cdots\infty)$$

$$S(x+y)=\frac{x^2}{1-x}-\frac{y^2}{1+y}$$

$$S(x+y)=\frac{x^2(1+y)-y^2(1-x)}{(1-x)(1+y)}\Rightarrow\frac{x^2+x^2y-y^2+xy^2}{(1-x)(1+y)}$$

$$S(\cancel{x+y})=\frac{(x-y)\cancel{(x+y)}+xy\cancel{(x+y)}}{(1-x)(1+y)}$$

$$S=\frac{x-y+xy}{(1-x)(1+y)}\Rightarrow\frac{\frac{1}{2}-\frac{1}{3}+\frac{1}{6}}{\left(\frac{1}{2}\right)\left(\frac{4}{3}\right)}\Rightarrow\frac{1/3}{\frac{2}{3}}\Rightarrow\frac{1}{2}$$

$$S=\frac{1}{2}$$

$\alpha=1$; $\beta=2$

$\alpha+3\beta=1+6\Rightarrow7$

Lech 6

Tah 01 | KCLS

$$S=\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2\cdot3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}-\frac{1}{2^2\cdot3}+\frac{1}{2\cdot3^2}-\frac{1}{3^3}\right)+\left(\frac{1}{2^4}-\frac{1}{2^3\cdot3}+\frac{1}{2^2\cdot3^2}-\frac{1}{2\cdot3^3}+\frac{1}{3^4}\right)+\cdots=\frac{\alpha}{\beta}$$

$$=\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots\right)-\left(\frac{1}{3}+\frac{1}{2\cdot3}+\frac{1}{2^2\cdot3}+\cdots\right)+\left(\frac{1}{3^2}+\frac{1}{2\cdot3^2}+\frac{1}{2\cdot3^3}+\cdots\right)-\cdots$$

$$=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n-\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^{n-1}+\sum_{n=1}^{\infty}\frac{1}{3^2}\left(\frac{1}{2}\right)^{n-1}-\sum_{n=1}^{\infty}\frac{1}{3^3}\left(\frac{1}{2}\right)^{n-1}+\cdots$$

$$=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n-\frac{1}{3}\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}+\frac{1}{3^2}\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}\cdots$$

$$=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n+\sum_{n=1}^{\infty}\left(-\frac{1}{3}\right)^n\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}$$

$$=\frac{\frac{1}{2}}{1-\frac{1}{2}}+\left(\frac{-\frac{1}{3}}{1-\left(-\frac{1}{3}\right)}\right)\left(\frac{1}{1-\frac{1}{2}}\right)$$

$$=\frac{\frac{1}{2}}{\frac{1}{2}}+\left(\frac{-\frac{1}{3}}{\frac{4}{3}}\right)\left(\frac{1}{\frac{1}{2}}\right)$$

$$=1+\left(-\frac{1}{4}\right)(2)$$

$$\boxed{S=\frac{1}{2}}$$

$\therefore S=\frac{\alpha}{\beta}=\frac{1}{2}$

$\therefore \alpha=1,\ \beta=2$

$\therefore \alpha+3\beta=1+6=\boxed{7}$ Ans

TAH 01/ KCLS
by Katha
from WB

Lecture-06 Sequence and series

If the sum of the series $\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2.3}+\frac{1}{3^2}\right)+\dots$ is $\frac{\alpha}{\beta}$, where α and β are coprime, then $(\alpha+3\beta)$ is equal to :-

$\Longrightarrow$ Let's say, $x=\frac{1}{2}$ & $y=\left(-\frac{1}{3}\right)$

thus,

$(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+(x^4+x^3y+x^2y^2+\dots$

hence,

$$(x-y)S = x^2-y^2+x^3-y^3+\dots$$
$$= (x^2+x^3+x^4+\dots)-(y^2+y^3+y^4+y^5+\dots)$$

$$(x-y)S = \frac{x^2}{1-x}-\frac{y^2}{1-y}$$

Tah 01

Kritisha (W.B)

putting the values of x & y,

$$\left(\frac{1}{2}+\frac{1}{3}\right)S = \frac{\frac{1}{4}}{1/2}-\frac{\frac{1}{9}}{\frac{1}{3}+1} \Rightarrow \frac{5}{6}S = \frac{1}{2}-\frac{1}{12} = \frac{5}{12}$$

hence, $\frac{5}{6}S=\frac{5}{12}$

$S=\frac{1}{2}$; $S=\frac{\alpha}{\beta}$ where α & β are coprime

on comparing $\alpha=1, \beta=2$ [1 & 2 are also coprime]

Thus, $\alpha+3\beta = 1+6 = 7$ (Ans)

QUESTION [JEE Mains 2023 (27 Jan)]

★★ASRQ★★

Let $a_1, a_2, a_3, \ldots$ be in an arithmetic progression of positive terms.

Let $A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \cdots + a_{2k-1}^2 - a_{2k}^2$.

If $A_3 = -153$, $A_5 = -435$ and $a_1^2 + a_2^2 + a_3^2 = 66$, then $a_{17} - A_7$ is equal to

Ans. 910

Tah-02 : Let; $a_1, a_2, a_3, \ldots \rightarrow$ in A.P.

Let; $A_K = \underbrace{a_1^2 - a_2^2} + \underbrace{a_3^2 - a_4^2} + \ldots + \underbrace{a_{2K-1}^2 - a_{2K}^2}$.

$A_K = (a_1+a_2)\underbrace{(a_1-a_2)}_{-d} + (a_3+a_4)\underbrace{(a_3-a_4)} + \ldots +$
$(a_{2K-1}+a_{2K})\underbrace{(a_{2K-1}-a_{2K})}_{-d}$.

$A_K = -d\,[a_1+a_2+a_3+\ldots+a_{2K-1}+a_{2K}]$.

given: $A_3 = -153$.

$\cancel{-}153 = \cancel{-}d\,[a_1+a_2+a_3+\ldots+a_6]$.

$153 = d\,[6a+15d]$. —— ① $\times 3$

$A_5 = -435$.

$\cancel{-}435 = \cancel{-}d\,[a_1+a_2+\ldots+a_{10}]$.

$435 = d\,[10a+45d]$ —— ② $\times 1$

$\Rightarrow$ $459 = 18ad + \cancel{45d^2}$ —— ③

$\underline{\begin{array}{l} 435 = 10ad + \cancel{45d^2} \text{ —— ④} \\ - \quad\;\; - \quad\;\; - \end{array}}$

$8ad = 24$

$\boxed{ad = 3}$ $\rightarrow$ put in eqn ②

$435 = 30 + 45d^2$

$d^2 = 9$

$\boxed{a = 1}$ $\boxed{d = 3}$, $d = -3$ ✗

(AP is positive term)

then: $a_{17} - A_7$.

$= a + 16d$
$= 1 + 48$
$= 49$.

$A_7 = -3\,[a_1+a_2+\ldots+a_{14}]$.

$A_7 = -3\,[14a+91d]$.

$= -861$.

$\Rightarrow 49 - (-861) =$

$\Rightarrow$ 910 Ans.

Tah 02

$a_1, a_2, a_3 \ldots \rightarrow AP$

$A_K = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \cdots + a_{2K-1}^2 - a_{2K}^2$

$A_3 = -153 = a_1^2 - a_2^2 + a_3^2 - a_4^2 + a_5^2 - a_6^2$

$A_3 = (a_1-a_2)(a_1+a_2) + (a_3-a_4)(a_3+a_4) + (a_5-a_6)(a_5+a_6)$

$A_3 = (-d)(2a_1+d) + (-d)(2a_1+5d) + (-d)(2a_1+9d)$

$A_3 = (-d)(6a_1+15d)$

$-153 = (-d)(6a_1+15d)$

$\{51 = d(2a_1+5d)\}$

$A_5 = -435 = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \cdots + a_9^2 - a_{10}^2$

$A_5 = (a_1-a_2)(a_1+a_2) + (a_3-a_4)(a_3+a_4) + \cdots + (a_9-a_{10})(a_9+a_{10})$

$A_5 = (-d)(2a_1+d) + (-d)(2a_1+5d) + \cdots + (-d)(2a_1+17d)$

$A_5 = -d(10a_1+45d)$

$+435 = +d(10a_1+45d)$

$\{87 = d(2a_1+9d)\}$

$\therefore 87 - 51 = d(2a_1+9d) - d(2a_1+5d)$

$36 = 4d^2$

$\boxed{d = 3}$ [as terms are +ve]

$\therefore 51 = 3(2a_1+15)$

$17 = 2a_1 + 15$

$2a_1 = 2$

$\boxed{a_1 = 1}$

$a_{17} = a_1 + 16d = 1 + 16(3)$

$a_{17} = 49$

$A_7 = -d(14a_1 + 91d)$

$A_7 = -3(14 + 91(3))$

$A_7 = -3(287) = -861$

$\therefore a_{17} - A_7 = 49 + 861 = \boxed{910}$ Ans

TAH 02
by Katha
from WB

Tah-02 $A_K = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \cdots + a_{2K-1}^2 - a_{2K}^2$

$A_K = (a_1+a_2)(a_1-a_2) + (a_3+a_4)(a_3-a_4) + \cdots$

$A_K = -d(a_1 + a_2 + \cdots + a_{2K})$

Kritisha (W.B)

$A_K = -d(K(2a_1 + (2K-1)d))$

$A_3 = -d(3)(2a_1+5d) = -153 \Rightarrow \cancel{A_3 = +d(2a_1+5d) = 51}$

$\boxed{d(2a_1+5d) = 51}$

$A_5 = -5d(2a_1+9d) = -435 \Rightarrow \boxed{d(2a_1+9d) = 87}$

thus, $9d^2 - 5d^2 = 4d^2 = 87 - 51 = 36$

$\boxed{d^2 = 9}$; $d = +3$

Kyuki, $a_1, a_2, a_3 \ldots$ is an A.P of +ve terms
$d = -3$ hota toh kabhi na kabhi -ve ban jaata.

also, $a_1^2 + a_2^2 + a_3^2 = 66$

$\Rightarrow a_1^2 + (a_1+d)^2 + (a_1+2d)^2 = 66$

$\Rightarrow a_1^2 + a_1^2 + 2a_1d + d^2 + a_1^2 + 4a_1d + 4d^2 = 66$

$\Rightarrow 3a_1^2 + 6a_1d + 5d^2 = 66$

$3a_1^2 + 18a_1 - 21 = 0 \Rightarrow a_1^2 + 6a_1 - 7 = 0$

$(a_1+7)(a_1-1) = 0$

$\boxed{a_1 = 1}$ (a_1 is +ve)

thus, $a_{17} = a_1 + 16d = 1 + 16(3) = 49$

$A_7 = -7d(2a_1+13d) = -21(41) = -861$

hence, $a_{17} - A_7 = 49 + 861 = 910$ (Ans)

Tah2

$a_1\ a_2\ a_3 \ldots$ A.P. let $A_K = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \cdots + a_{2K-1}^2 - a_{2K}^2$

if $A_3 = -153$, $A_5 = -435$, $a_1^2 + a_2^2 + a_3^2 = 66$ then $a_{17} - A_7 = ?$

$$A_K = (a_1 - a_2)(a_1 + a_2) + (a_3 + a_4)(a_3 - a_4) \cdots + (a_{2K-1} + a_{2K})(a_{2K-1} - a_{2K})$$

$$= -d\left\{(a_1 + a_2 + a_3 + a_4 + \cdots + a_{2K-1} + a_{2K})\right\}$$

$$= -d\left\{\frac{\cancel{2}K}{\cancel{2}}\left[2a_1 + (2K-1)d\right]\right\}$$

$$A_K = -dK\left[2a_1 + (2K-1)d\right]$$

$$A_3 = -3d\left[2a_1 + 5d\right] = -153$$

$$A_3 = d(2a_1 + 5d) = 51 \quad (1)$$

$$A_5 = -5d\left[2a_1 + 9d\right] = -435$$

$$= d(2a_1 + 9d) = 87 \quad (2)$$

(2) - (1)

$$\begin{array}{r} 2a_1d + 9d^2 = 87 \\ -2a_1d \pm 5d^2 = 51 \\ \hline d = 3 \end{array} \quad \boxed{a = 1}$$

Now, $a_{17} - A_7$

$$a + 16d + 7d\left[2a_1 + 13d\right]$$

$$\Rightarrow 1 + 16d + 7\times 3\left[2\times 1 + 13\times 3\right]$$

$$\Rightarrow 1 + 48 + 21\left[41\right]$$

$$\Rightarrow 49 + 861$$

$$\Rightarrow 910 \text{ Ans}$$

Sakshi

QUESTION

Evaluate

(a) $0.7 + 0.77 + 0.777 + \ldots\ldots$ n terms

(b) $0.9 + 0.99 + 0.999 + \ldots\ldots$ n terms

Tah-03

Evaluate :

(a) $0.7+0.77+0.777+ \dots n$ terms

$7(0.1+0.11+0.111+\dots)$

Richathakur

$\frac{7}{9}(0.9+0.99+0.999\dots n)$

$\frac{7}{9}\left[(1-10^{-1})+(1-10^{-2})+(1-10^{-3})+\dots n \text{ terms}\right]$

$\frac{7}{9}\left[(1+1+1\dots n \text{ terms})-(10^{-1}+10^{-2}+10^{-3}+\dots n \text{ terms})\right]$

$\frac{7}{9}\left[n-(0.1+0.01+0.001+\dots n \text{ terms})\right]$

$\frac{7}{9}\left[n-\frac{0.1\left(1-(0.1)^n\right)}{1-0.1}\right]$

$\frac{7}{9}\left[n-\frac{0.1}{0.9}\left(1-(0.1)^n\right)\right]$

$\frac{7}{9}\left(n-\frac{1}{9}\left(1-(0.1)^n\right)\right)$

$\frac{7n}{9}-\frac{7}{81}\left(1-(0.1)^n\right) \Rightarrow \frac{7}{9}n-\frac{7}{81}\left[1-\left(\frac{1}{10}\right)^n\right]$

Ans

(b) $0.9 + 0.99 + 0.999 + \cdots n$ terms

$(1-0.1) + (1-0.01) + (1-0.001) + \cdots n$ terms

$(1+1+1+\cdots n \text{ terms}) - (0.1 + 0.01 + 0.001 + \cdots n \text{ terms})$

$$n - \frac{0.1\left[1-(0.1)^n\right]}{1-0.1}$$

$$n - \frac{0.1}{0.9}\left[1-(0.1)^n\right]$$

$$n - \frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]$$ Ar

Tah-03

Evaluate a) $0.7 + 0.77 + 0.777 + \dots n$ terms

$\Rightarrow \frac{7}{10} + \frac{77}{100} + \frac{777}{1000} + \dots n$ terms

$= \frac{7}{9}\left(\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots n \text{ terms}\right)$

$= \frac{7}{9}\left(1 - 10^{-1} + 1 - 10^{-2} + 1 - 10^{-3} \dots n \text{ terms}\right)$

$= \frac{7}{9}\left(n - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots n \text{ terms}\right)\right)$

$= \frac{7}{9}\left(n - \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^n\right)}{1 - \frac{1}{10}}\right)$

$= \frac{7}{9}\left(n - \frac{1}{9}\left(1 - \left(\frac{1}{10}\right)^n\right)\right)$

Kritisha

$= \frac{7}{9}\left(n - \frac{1}{9}\left(\frac{10^n - 1}{10^n}\right)\right)$

b) $0.9 + 0.99 + 0.999 + \dots + n$ terms

$\Rightarrow 1 - \frac{1}{10} + 1 - \frac{1}{100} + 1 - \frac{1}{1000} + \dots n$ terms

$= n - \left(\frac{1}{10} + \frac{1}{100} + \dots + \frac{1}{10^n}\right)$

$= n - \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^n\right)}{\left(1 - \frac{1}{10}\right)} = n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)$

Tah-03: Evaluate:

(a) $0.7 + 0.77 + 0.777 + \dots n$ term.

$\Rightarrow 7(0.1 + 0.11 + 0.111 + \dots n)$.

$\Rightarrow \frac{7}{9}(0.9 + 0.99 + 0.999 + \dots)$.

$\Rightarrow \frac{7}{9}(1 - 10^{-1} + 1 - 10^{-2} + 1 - 10^{-3} + \dots)$

$\Rightarrow \frac{7}{9}\left[(1+1+1+\dots) + (-1)(10^{-1} + 10^{-2} + 10^{-3} + \dots)\right]$

$\Rightarrow \frac{7}{9}\left[n - (10^{-1})\frac{((10^{-1})^n - 1)}{(10^{-1} - 1)}\right]$.

$\Rightarrow \frac{7}{9}\left[n - \frac{1}{10}\left(\frac{(10^{-n}) - 1}{-9/10}\right)\right]$

krish

$\Rightarrow \frac{7}{9}\left[n + \left(\frac{10^{-n} - 1}{9}\right)\right]$ Ans.

$\Rightarrow \frac{7}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$ Ans.

(b) $0.9 + 0.99 + 0.999 + \dots n$ terms.

$\Rightarrow (1 - 10^{-1} + 1 - 10^{-2} + 1 - 10^{-3} + \dots)$.

$\Rightarrow \left[(1+1+1+\dots) + (-1)(10^{-1} + 10^{-2} + \dots)\right]$.

$\Rightarrow \left[n - (10^{-1})\frac{((10^{-1})^n - 1)}{(10^{-1} - 1)}\right]$.

$\Rightarrow \left[n + \frac{1}{10}\left(\frac{(1 - 10^{-n})}{\frac{-9}{10}}\right)\right]$.

$\Rightarrow \left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$ Ans.

krish

QUESTION

Find the sum of the following series

(i) $5 + 55 + 555 + \ldots\ldots$ to n terms

(ii) $0.3 + 0.33 + 0.333 + \ldots\ldots$ to n terms

Ans. (i) $\frac{5}{9}\left[10\left(\frac{10^n-1}{9}\right)n\right]$

(ii) $\frac{1}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$

Tah-04 Sum of the Series

(a) $5+55+555+ \cdots n$ terms

$$S_n = \frac{5}{9}\left[9+99+999+ \cdots n \text{ terms}\right]$$

$$\Rightarrow \frac{5}{9}\left[(10+100+1000+ \cdots n) - (1+1+1+ \cdots n \text{ terms})\right]$$

$$S_n = \frac{5}{9}\left[\left(\frac{10(10^n-1)}{10-1} - n\right)\right]$$

Richathakur

$$\boxed{S_n = \frac{5}{9}\left[\frac{10}{9}(10^n-1) - n\right]}$$ Ans

(b) $0.3+0.33+0.333+ \cdots$ to n terms

$$S_n = \frac{3}{9}\left[0.9+0.99+0.999+ \cdots \text{to } n \text{ terms}\right]$$

$$\frac{3}{9}\left[1-0.1+1-0.01+1-0.001+ \cdots n\right]$$

$$\frac{3}{9}\left[n - \left(\frac{0.1(1-0.1^n)}{1-0.1}\right)\right]$$

$$\frac{1}{3}\left[n - \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)\right] \Rightarrow \boxed{\frac{1}{3}\left[n - \frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]}$$ Ans

Tah-04 Find the sum of the following series

i) $5+55+555+ \cdots$ to n terms

$$\Rightarrow \frac{5}{9}(9+99+999+ \cdots n \text{ terms})$$

$$= \frac{5}{9}(10-1+100-1+1000-1+ \cdots n \text{ terms})$$

$$= \cancel{\frac{5}{9}(-100+1)} \Rightarrow \frac{5}{9}(10+100+1000+ \cdots n \text{ terms} - n)$$

$$\Rightarrow \frac{5}{9}\left(\frac{10(10^n-1)}{9} - n\right)$$ (Ans)

ii) $0.3+0.33+0.333+ \cdots$ to n terms

$$\Rightarrow \frac{3}{10}+\frac{33}{100}+\frac{333}{1000}+ \cdots n \text{ terms}$$

$$= 3\left(\frac{1}{10}+\frac{11}{100}+\frac{111}{1000}+ \cdots n \text{ terms}\right)$$

$$= \frac{3}{9}\left(\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+ \cdots n \text{ terms}\right)$$

$$= \frac{3}{9}\left(1-\frac{1}{10}+1-\frac{1}{100}+1-\frac{1}{1000}+ \cdots n \text{ terms}\right)$$

$$= \frac{3}{9}\left(n - \left(\frac{1}{10}+\frac{1}{100}+ \cdots n \text{ terms}\right)\right)$$

$$= \frac{3}{9}\left(n - \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{9/10}\right) = \frac{3}{9}\left(n - \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)\right)$$

Kritisha

$$= \frac{1}{3}\left(n - \frac{1}{9}\left(1-\frac{1}{10^n}\right)\right)$$

Ans

Tah 04

i) $5 + 55 + 555 + \cdots n$ terms

$S_n = 5(1 + 11 + 111 + \cdots n \text{ terms})$

$S_n = \frac{5}{9}(9 + 99 + 999 + \cdots n \text{ terms})$

$S_n = \frac{5}{9}[(10-1) + (10^2-1) + (10^3-1) + \cdots n \text{ terms}]$

$S_n = \frac{5}{9}[(10 + 10^2 + 10^3 + \cdots n \text{ terms}) - (1 + 1 + \cdots n \text{ terms})]$

$S_n = \frac{5}{9}\left[\frac{10(1-10^n)}{1-10} - n\right]$

$\boxed{S_n = \frac{5}{9}\left[\frac{10(10^n-1)}{9} - n\right]}$ Ans

TAH 04
By Katha
from WB

ii) $S_n = 0.3 + 0.33 + 0.333 + \cdots n \text{ terms}$

$= 3[0.1 + 0.11 + 0.111 + \cdots n \text{ terms}]$

$S_n = 3[0.1 + 0.11 + 0.111 + \cdots n \text{ terms}]$

$= \frac{3}{9}[0.9 + 0.99 + 0.999 + \cdots n \text{ terms}]$

$= \frac{3}{9}[(1-0.1) + (1-0.01) + \cdots n \text{ terms}]$

$= \frac{3}{9}[(1+1+\cdots n \text{ terms}) - (0.1 + 0.01 + \cdots n \text{ terms})]$

$= \frac{\cancel{3}1}{\cancel{9}3}\left\{n - 0.1\left[\frac{1-(0.1)^n}{1-0.1}\right]\right\}$

$$\boxed{S_n = \frac{1}{3}\left\{n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right\}}$$ Ans

Tah-04: Find the sum of the following series:

(i) $5 + 55 + 555 + \cdots$ to n terms.

$\Rightarrow 5(1 + 11 + 111 + \cdots)$

krish

$\Rightarrow \frac{5}{9}(9 + 99 + 999 + \cdots)$

$\Rightarrow \frac{5}{9}(10-1 + 10^2-1 + 10^3-1 + \cdots)$

$\Rightarrow \frac{5}{9}\left[(10 + 10^2 + 10^3 + \cdots + 10^n) - n\right]$

$\Rightarrow \frac{5}{9}\left[10\left(\frac{(10^n-1)}{9}\right) - n\right]$ Ans.

(ii) $0.3 + 0.33 + 0.333 + \cdots$ to n term.

$\Rightarrow \frac{1}{3}(0.9 + 0.99 + 0.999 + \cdots)$

$\Rightarrow \frac{1}{3}(1-10^{-1} + 1-10^{-2} + 1-10^{-3} + \cdots)$

$\Rightarrow \frac{1}{3}\left[n + (-1)(10^{-1} + 10^{-2} + 10^{-3} + \cdots)\right]$

$\Rightarrow \frac{1}{3}\left[n - (10^{-1})\frac{((10^{-1})^n - 1)}{10^{-1}-1}\right]$

$\Rightarrow \frac{1}{3}\left[n - \frac{1}{\cancel{10}}\left(\frac{10^{-n}-1}{\frac{-9}{\cancel{10}}}\right)\right]$

krish

$\Rightarrow \frac{1}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$ Ans.

QUESTION [JEE Advanced 2022 (Paper 1)]

★★KCLS★★

Let $l_1, l_2, \ldots, l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1, and let $w_1, w_2, \ldots, w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1 d_2 = 10$. For each $i = 1, 2, \ldots, 100$, let R_i be a rectangle with length l_i, width w_i and area A_i. If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is ________

Ans. 18900

Tah 05 [Jee Adv 2022 (Paper 1)]

$L_i = L_1 + (i-1)d_1$

$W_i = w_1 + (i-1)d_2$

$A_i = L_i W_i = (L_1 + (i-1)d_1)(w_1 + (i-1)d_2)$

TAH 05
By Katha
from WB

$A_{51} = (L_1 + 50d_1)(w_1 + 50d_2)$

$A_{50} = (L_1 + 49d_1)(w_1 + 49d_2)$

$A_{51} - A_{50} = (L_1 + 50d_1)(w_1 + 50d_2) - (L_1 + 49d_1)(w_1 + 50d_2)$

$= L_1w_1 + 50L_1d_2 + 50w_1d_1 + 2500d_1d_2$
$- L_1w_1 - 49L_1d_2 - 49w_1d_1 - 2401d_1d_2$

$A_{51} - A_{50} = L_1d_2 + w_1d_1 + 99d_1d_2$

$1000 = L_1d_2 + w_1d_1 + 990 \quad [\because d_1d_2 = 10]$

$L_1d_2 + w_1d_1 = 10 \quad [\because A_{51} - A_{50} = 1000]$

$A_{100} = (L_1 + 99d_1)(w_1 + 99d_2)$

$A_{90} = (L_1 + 89d_1)(w_1 + 89d_2)$

$A_{100} - A_{90} = (L_1 + 99d_1)(w_1 + 99d_2) - (L_1 + 89d_1)(w_1 + 89d_2)$

$= L_1w_1 + 99L_1d_2 + 99w_1d_1 + 9801d_1d_2$
$- L_1w_1 - 89L_1d_2 - 89w_1d_1 - 7921d_1d_2$

$= 10L_1d_2 + 10w_1d_1 + 1880d_1d_2$

$A_{100} - A_{90} = 10(L_1d_2 + w_1d_1) + 18800$

$= 10 \times 10 + 18800$

$\boxed{A_{100} - A_{90} = 18900}$ Ans

Tah-05 $l_1, l_2, \dots, l_{100} \Rightarrow$ (common diff $= d_1$)

$w_1, w_2, \dots, w_{100} \Rightarrow$ (" " $= d_2$)

$R_i \Rightarrow$ Rectangle with length l_i & width w_i

area $= l_iw_i = A_i$ & $\boxed{d_1d_2 = 10}$

$A_{51} - A_{50} = l_{51}w_{51} - l_{50}w_{50} = 1000$

$= (l_1 + 50d_1)(w_1 + 50d_2) - (l_1 + 49d_1)(w_1 + 49d_2)$

$= \cancel{l_1w_1} + 50d_2l_1 + 50d_1w_1 + 2500d_1d_2$
$- \cancel{l_1w_1} - 49l_1d_2 - 49d_1w_1$
$- (49)^2d_1d_2$

$\Rightarrow l_1d_2 + d_1w_1 + 25000 - 24010 = 1000$

$\Rightarrow \boxed{l_1d_2 + d_1w_1 = 10}$

Thus, $A_{100} - A_{90} = l_{100}w_{100} - l_{90}w_{90}$

$= (l_1 + 99d_1)(w_1 + 99d_2) - (l_1 + 89d_1)(w_1 + 89d_2)$

$= \cancel{l_1w_1} + 99d_2l_1 + 99d_1w_1 + (99)^2d_1d_2$
$- \cancel{l_1w_1} - 89l_1d_2 - 89d_1w_1$
$- (89)^2d_1d_2$

$= 10(l_1d_2 + d_1w_1) + (99^2 - 89^2) \times 10$

$= 100 + (99 + 89)100 = 100(188 + 1)$

$= 18900$ (Ans)

Kritisha (W.B)

Tah-05. Solⁿ: Let; $l_1, l_2, l_3, \dots, l_{100} \longrightarrow$ in A.P.

$\boxed{D = d_1}$

Let; $w_1, w_2, w_3, \dots, w_{100} \longrightarrow$ in A.P.

$\boxed{D = d_2}$

For each $i = 1, 2, \dots, 10$.

where: $d_1 \cdot d_2 = 10$.

$l_i \rightarrow$ length.
$w_i \rightarrow$ width.
$A_i \rightarrow$ Area.

given: $A_{51} - A_{50} = 1000$.

Area = length × width

$\Rightarrow l_{51} \cdot w_{51} - l_{50} \cdot w_{50} = 1000$.

$\Rightarrow (l_1 + 50d_1) \cdot (w_1 + 50d_2) - (l_1 + 49d_1) \cdot (w_1 + 49d_2) = 1000$.

$\Rightarrow [\cancel{l_1 w_1} + 50 l_1 d_2 + 50 d_1 w_1 + 2500 d_1 d_2] - [\cancel{l_1 w_1} + 49 l_1 d_2 + 49 d_1 w_1 + 2401 d_1 \cdot d_2] = 1000$.

$\Rightarrow [50(l_1 d_2 + d_1 w_1) + 2500 d_1 d_2] - [49(l_1 d_2 + d_1 w_1) + 2401 d_1 d_2] = 1000$

$\Rightarrow (l_1 d_2 + d_1 w_1) + 99 \underbrace{d_1 d_2}_{10} = 1000$.

$\Rightarrow l_1 d_2 + d_1 w_1 = 10$

Similarly: find $A_{100} - A_{90} = ?$

$\Rightarrow l_{100} \cdot w_{100} - l_{90} \cdot w_{90}$

$\Rightarrow (l_1 + 99d_1)(w_1 + 99d_2) - (l_1 + 89d_1)(w_1 + 89d_2)$.

$\Rightarrow 10(l_1 d_2 + d_1 w_1) + [(99 + 89)(99 - 89) d_1 . d_2]$

$\Rightarrow 100 + (188)(100)$

$\Rightarrow 100 + 18800 \Rightarrow \boxed{18900}$ Ans.

krish

Ad-2022

Jan-05

$A_{51} - A_{50} = 1000$

$l_1, l_2, l_3 \ldots l_{100} \longrightarrow$ AP with CD $\rightarrow d_1$

$w_1, w_2, w_3 - - - w_{100} \longrightarrow$ AP with CD $\rightarrow d_2$

$d_1 \cdot d_2 = 10$

$l_{51}w_{51} - l_{50}w_{50} = 1000$

$(l_1 + 50d_1)(w_1 + 50d_2) - (l_1 + 49d_1)(w_1 + 49d_2)$

$(l_1w_1 + l_1 50d_2 + 50d_2w_1 + 2500d_1d_2) - (l_1w_1 + 49d_2l_1 + 49d_1w_1 + 2401d_1d_2)$

$\cancel{l_1w_1} + 50(l_1d_2 + w_1d_1) + 25000 - \cancel{l_1w_1} - 49(d_2l_1 + d_1w_1) - 24010$

$l_1d_2 + w_1d_1 + 990 = 1000$

$l_1d_2 + w_1d_1 = 10$ —— (1)

$A_{100} - A_{90} \Rightarrow (l_1 + 99d_1)(w_1 + 99d_2) - (l_1 + 89d_2)(w_1 + 89d_2)$

$\cancel{l_1w_1} + 99d_2l_1 + 99d_1w_1 + 98010 - \cancel{l_1w_1} - 79210 - l_1 89d_2 - 89d_1w_1$

$l_1d_2(99 - 89) + w_1d_2(99 - 89) + 18800$

Richathakur

$10(l_1d_1 + w_1d_1) + 18800$

$100 + 18800$

$A_{100} - A_{90} \Rightarrow 18900$ Ans

QUESTION

★★KCLS★★

Show that $\underbrace{444\ldots4}_{n}\ \underbrace{888\ldots89}_{n-1}$ is always a perfect square.

$768 = 7\times10^2 + 6\times10^1 + 8\times10^0$

$44\ldots4 \times 10^n + 88\ldots8 \times 10 + 9$

$4\,(\underbrace{11\ldots1}_{n\text{ times}}) \times 10^n + 80\cdot(\underbrace{111\ldots1}_{(n-1)\text{ times}}) + 9$

$\frac{4}{9}\cdot(99\ldots9) \times 10^n + \frac{80}{9}(\underbrace{99\ldots9}_{(n-1)\text{ times}}) + 9$

$\frac{4}{9}(10^n - 1) \times 10^n + \frac{80}{9}(10^{n-1} - 1) + 9$

Tán-6 Show that $\underbrace{444\ldots4}_{n}\ \underbrace{888\ldots89}_{n-1}$ is always a perfect sq.

Richathakur

$444\ldots4 \times 10^n + 888\ldots8 \times 10 + 9$

$4(111\ldots1) \times 10^n + 8(111\ldots1) \times 10 + 9$

$\frac{4}{9}[999\ldots9] \times 10^n + \frac{8}{9}[999\ldots9] \times 10 + 9$

$\frac{4}{9}[10^n - 1] \times 10^n + \frac{8}{9}[10^{n-1} - 1] \times 10 + 9$

$\frac{4}{9}10^{2n} - \frac{4}{9}10^n + \frac{8}{9}10^n - \frac{80}{9} + 9$

$\frac{4}{9} \times 10^{2n} + \frac{4}{9} \times 10^n + \frac{1}{9}$

$\left[\frac{2}{3}10^n + \frac{1}{3}\right]^2$

Now its proved that It's always a perfect square.

Q. Show that $\underbrace{444\ldots4}_{n}\ \underbrace{888\ldots8}_{n-1}9$ is always a perfect sq.

$$\Rightarrow 4\times\underbrace{111\ldots1}_{n}\times10^{n}+8\times\underbrace{111\ldots1}_{n-1}\times10+9.$$

$$\Rightarrow 4\times\frac{10^{n}-1}{9}\times10^{n}+8\times\frac{10^{n-1}-1}{9}\times10+9.$$

$$\Rightarrow \frac{4}{9}10^{2n}-\frac{4}{9}10^{n}+\frac{8}{9}10^{n}-\frac{80}{9}+9.$$

$$\Rightarrow \frac{4}{9}10^{2n}+\frac{4}{9}10^{n}+\frac{1}{9}.$$

$$\boxed{\underbrace{111\ldots1}_{n}\Rightarrow\frac{1}{9}(\underbrace{999\ldots9}_{n}) \Rightarrow\frac{1}{9}(10^{n}-1)}$$

$$\Rightarrow \frac{4\times10^{2n}+4\times10^{n}+1}{9}$$

$$\Rightarrow \frac{(2\times10^{n}+1)^{2}}{3^{2}}\Rightarrow\left(\frac{2\times10^{n}+1}{3}\right)^{2}$$

Hence this term is always a perfect square.

krish

Solution to Previous Shikaars

QUESTION [BITSAT 2024]

There are four numbers of which the first three are in G.P. and the last three are in A.P., whose common difference is 6. If the first and the last numbers are equal, then two other numbers are

(A) $-2, 4$

(B) $-4, 2$

(C) $2, 6$

(D) None of the above

Ans. B

Q. S-01: Solⁿ:

Let; $a_1, a_2, a_3, a_4 \rightsquigarrow$ four no's.

$a_1, a_2, a_3 \longrightarrow$ G.P

given: $a_1 = a_4$

$\boxed{a_1 = a + 18}$.

$a_2, a_3, a_4 \longrightarrow$ A.P

$\boxed{d=6}$

$a+d,\ a+2d,\ a+3d$

$\Rightarrow a+6,\ a+12,\ a+18$.

G.P $\Rightarrow a+18,\ a+6,\ a+12$.

$(a+6)^2 = (a+18)(a+12)$

$\Rightarrow \cancel{a^2} + 12a + 36 = \cancel{a^2} + 30a + 216$.

$\Rightarrow 18a + 180 = 0$

$\Rightarrow \boxed{a = -10}$

krish

four no's $\Rightarrow a+18d,\ a+6,\ a+12,\ a+18$.

$\Rightarrow 8, -4, 2, 8$.

given: first and last no. are equal.

$\Rightarrow$ other two $= -4, 2$ Ans.

$\boxed{S-01}$ $\quad a_1, a_2, a_3, a_4 \rightarrow$ in A.P ; common diff $= 6$

$a_1, a_2, a_3 \rightarrow$ in G.P

$\boxed{a_1 = a_4}$ (given) $\quad ; a_1 a_3 = a_2^2 \; ; \boxed{a_3 a_4 = a_2^2}$

$a_2 + a_4 = 2a_3$

$\boxed{a_2 = 2a_3 - a_4}$

S.B.S

$a_2^2 = 4a_3^2 + a_4^2 - 4a_3 a_4$

Kritisha

$\Rightarrow 4a_3^2 + a_4^2 - 4a_3 a_4 - a_3 a_4 = 0$

$\Rightarrow 4a_3(a_3 - a_4) + a_4(a_4 - a_3) = 0$

$= 4a_3(a_3 - a_4) - a_4(a_3 - a_4) = 0 \Rightarrow \boxed{4a_3 = a_4}$

$a_4 - a_3 = a_4 - \frac{a_4}{4} = \frac{3a_4}{4} = 6$

$a_4 = 8 \; ; \boxed{a_3 = 2, a_2 = -4}$

(option B)

$\boxed{a_3 = a_4} \rightarrow$ N.P

they are in A.P with C.d $= 6$

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Sikaars A.P. common diff. = 6

Q1) $\overset{\text{A.P.}}{a, b, c, d}$ $\quad c - b = 6$, $d - c = 6$

$\underbrace{a, b, c}_{\text{G.P.}}$ $\quad c = b + 6 \quad d = 6 + c \quad d = 6 + 6 + b$

$b^2 = ac \qquad b, b+6, b+12$

$b^2 = dc \qquad \boxed{a = d}$ (given)

$b^2 = (b+12)(b+6)$

$\cancel{b^2} = \cancel{b^2} + 18b + 72$

$18b = -72$

$\boxed{b = -4} \qquad -4, 2$ Ans

$\boxed{c = 2}$

Sakshi

Gyan

QUESTION [BITSAT 2023]

Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}, \frac{1}{b}$, 6 be in A.P., where $a, b > 0$. Then, $72(a + b)$ is equal to

(A) 12

(B) 14

(C) 16

(D) 18

Ans. B

$\boxed{S-02}$ $\frac{1}{16}, a, b$ are in G.P

$\frac{1}{a}, \frac{1}{b}, 6$ are in A.P $\quad [a, b > 0]$

$\Rightarrow \boxed{a^2 = \frac{b}{16}}$

$\frac{1}{a} + 6 = \frac{2}{b}$

$\frac{1}{a} = \frac{2}{b} - 6 = \frac{2-6b}{b}$

$\boxed{a = \frac{b}{2-6b}}$; $a^2 = \frac{b^2}{4-24b+36b^2} = \frac{b}{16}$

thus, $\frac{b}{1-6b+9b^2} = \frac{1}{4}$

$4b = 1-6b+9b^2$

$9b^2 - 10b + 1 = 0$

$9b^2 - 9b - b + 1 = 0$

$9b(b-1) - (b-1) = 0$

$\boxed{b = 1/9}$

$a^2 = \frac{1}{9 \times 16}$

$\boxed{a = \frac{1}{12}}$

$72(a+b)$

$= 72\left(\frac{1}{9} + \frac{1}{12}\right)$

$= \cancel{72}^{2} \frac{\times 7}{\cancel{36}} = 14$

$\boxed{b = 1}$

$\boxed{a = \frac{1}{4}}$

$72\left(1 + \frac{1}{4}\right)$

$= \cancel{72}^{18}\left(\frac{5}{\cancel{4}}\right)$

$= 90$ (not in the option)

Hence,

$72(a+b) = 14$

(B)

Ans.

Kritisha

82

$\frac{1}{16}, a, b \to GP$

$16a^2 = b$

$\cancel{16}\times\frac{1}{\cancel{144}\,9} = b$

$\boxed{b=\frac{1}{9}}$

$\boxed{a=\frac{1}{12}}$

$72(a+b)$

$72\left(\frac{1}{9}+\frac{1}{12}\right)$

$\cancel{72}^{\,2}\left(\frac{7}{\cancel{36}}\right)$

$\Rightarrow 14$ Ans

$\frac{1}{a}, \frac{1}{b}, 6 \to AP$

$\frac{2}{b} = 6+\frac{1}{a}$

$\frac{\cancel{2}}{\cancel{16}\,8\,a^2} = 6+\frac{1}{a}$

$6a+1 = \frac{\cancel{a}}{8a^{\cancel{2}}}$ $\quad a\neq 0$

$48a^2+8a-1=0$

$48a^2+12a-4a-1=0$

$(12a-1)(4a+1)=0$

$a=\frac{1}{12}$ ✓, $a=\frac{-1}{4}$ ✗

Q. S-02 : Solⁿ : Let; $\frac{1}{16}$, a & b $\rightsquigarrow$ G.P

$\Rightarrow \frac{1}{a}, \frac{1}{b}, 6 \longrightarrow$ A.P

$\Rightarrow a^2 = \frac{1}{16} \cdot b$

$\Rightarrow b = 16a^2$ — (1)

given : $\boxed{a, b > 0}$

$\Rightarrow \frac{2}{b} = \frac{1}{a} + 6$

$\Rightarrow \frac{\cancel{2}}{8\,\cancel{16}a^{\cancel{2}}} = \frac{1+6a}{\cancel{a}}$

$\Rightarrow 8a + 48a^2 - 1 = 0$

$\Rightarrow 48a^2 + 12a - 4a - 1 = 0$

$\Rightarrow 12a(4a+1) - 1(4a+1) = 0$

$\Rightarrow (12a-1)(4a+1)$

$\Rightarrow a = \frac{1}{12}$ *, $-\frac{1}{4}$ ×

$\Rightarrow b = \cancel{16} \times \frac{1}{\cancel{144}\,9}$

$\Rightarrow b = \frac{1}{9}$ *

find : $72(a+b)$

$\Rightarrow 72\left(\frac{1}{12} + \frac{1}{9}\right)$

$\Rightarrow \overset{2}{\cancel{72}} \times \frac{7}{\cancel{36}} \Rightarrow 14$ Ans.

krish

QUESTION [BITSAT 2022]

In a sequence of 21 terms, the first 11 terms are in A.P. with common difference 2 and the last 11 terms are in G.P. with common ratio 2. If the middle term of A.P. be equal to the middle term of the G.P., then the middle term of the entire sequence is

(A) $-\frac{10}{31}$

(B) $\frac{10}{31}$

(C) $\frac{32}{31}$

(D) $-\frac{31}{32}$

Ans. A

S-03 $n = 21$

1st (11) terms are in A.P ; common diff = 2

last (11) " " " G.P ; common ratio = 2

the 11^{th} term of the sequence is present in both

middle term of the A.P ⇒ $t_6 = t_{11} - 5d$

" " " " G.P ⇒ $t_{16} = t_{11}(r^5)$

$$t_{11} - 5(2) = t_{11}(2^5)$$

Kritisha

$$t_{11} - 10 = 32t_{11}$$

$$-31t_{11} = 10$$

$$t_{11} = \frac{-10}{31}$$ (A) (Ans)

middle term of the sequence

S3

$\underbrace{a_1, a_2, a_3 ---- a_{11}}_{A.P}, \underbrace{a_{12}, a_{13}, a_{14} ---- a_{20}, a_{21}}_{G.P}$

$d=2$ $r=2$ $a_{11} = a+10d$

middle term $a_6 = a_{16}$

$a+5d = a_{11}r^5$

$a+10 = (a+10d)(2)^5$

Sakshi

$a+10 = 32a+320d$

$a+10 = 32a+320\times2$

$31a+640-10=0$

$\boxed{a = \frac{-630}{31}}$

$a_{11} = a+10d$

$= \frac{-630}{31}+20$

$= \frac{-10}{31}$

$\boxed{a_{11} = \frac{-10}{31}}$ Ans

sikaar3

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Gyan

Q S-03 Solⁿ: $a_1, a_2, a_3 ---- a_{21} \rightsquigarrow$ sequence

$a_1, a_2, a_3 --- a_{11} \rightsquigarrow AP$ & $a_{11}, a_{12}, a_{13} ---, a_{21} \rightsquigarrow GP$

$\boxed{D = 2}$ $\boxed{r = 2}$

given: middle term of A.P = middle term of G.P.

$\Rightarrow a_6 = a_{16}$

$\Rightarrow a + 5d = a_{11}(r^5)$

$\Rightarrow a + 5d = (a + 10d)(32)$

$\Rightarrow a + 10 = 32a + 640$

$\Rightarrow 31a = -630 \Rightarrow a = \frac{-630}{31}$ *

find: middle term of entire seq.

$\# \; a_{11} = a + 10d$

$= -\frac{630}{31} + 20$

$\Rightarrow \frac{-630 + 620}{31}$

$\Rightarrow \frac{-10}{31}$ Ans.

krish

QUESTION [WB JEE 2019]

Let x_1, x_2 be the roots of $x^2 - 3x + a = 0$ and x_3, x_4 be the roots of $x^2 - 12x + b = 0$. If $x_1 < x_2 < x_3 < x_4$ and x_1, x_2, x_3, x_4 are in G.P., then ab equals

(A) 24/5

(B) 64

(C) 16

(D) 8

Ans. B

S-04

$\boxed{x^2 - 3x + a = 0} \to x_1, x_2$ $\quad$ $\boxed{x^2 - 12x + b = 0} \to x_3, x_4$

$x_1 x_2 = a/1 = a$ $\quad$ $x_3 x_4 = b$

$x_1 + x_2 = 3$ $\quad$ $x_3 + x_4 = 12$

$\boxed{x_1 x_2 x_3 x_4 = ab}$

x_1, x_2, x_3, x_4 are in G.P

thus, $x_1 x_4 = x_2 x_3$

or, $x_1(12 - x_3) = (3 - x_1) x_3$

or, $12x_1 - \cancel{x_1 x_3} = 3x_3 - \cancel{x_1 x_3}$

or; $\boxed{4x_1 = x_3}$

$x_3 = x_1 r^2$ $\quad$ [as, $x_1 < x_2 < x_3 < x_4$

hence, $\boxed{r = 2}$ $\quad$ r can't be -2]

common ratio

Kritisha

hence, $2x_1 = x_2$

thus, $x_1 + x_2 = 3$

$3x_1 = 3$

$x_1 = 1$, $x_2 = 2$, $x_3 = 4$, $x_4 = 8$

$ab = x_1 x_2 x_3 x_4 = 64$ (B)

S4

$x^2 - 3x + a = 0 \to x_1, x_2$ $\quad$ $x^2 - 12x + b = 0 \to x_3, x_4$ $\quad$ $x_1 < x_2 < x_3 < x_4$

$x_1, x_2, x_3, x_4 \to$ GP

$a \quad ar \quad ar^2 \quad ar^3$ $\quad \to ar^2 + ar^3 = 12$

Sakshi

$a + ar = 3 \to a(1+r) = 3$ ① $\quad$ $ar^2(1+r) = 12$ ②

divide ② by ① $\quad \dfrac{\cancel{a}r^2\cancel{(1+r)}}{\cancel{a}\cancel{(1+r)}} = \dfrac{\cancel{12}\,4}{\cancel{3}}$

$a^2 r = a = 2$

$a^2 r^5 = b = 32$ $\quad \boxed{a=1} \quad \boxed{r=2}$

then $ab = 64$ Ans

Q-04 : Soln: $x^2 - 3x + a = 0 \begin{cases} \to x_1 \\ \to x_2 \end{cases}$ $\left.\begin{aligned} x_1 + x_2 &= 3 \\ x_1 \cdot x_2 &= a \end{aligned}\right\}$

$x^2 - 12x + b = 0 \begin{cases} \to x_3 \\ \to x_4 \end{cases}$ $\left.\begin{aligned} x_3 + x_4 &= 12 \\ x_3 \cdot x_4 &= b \end{aligned}\right\}$

given : $x_1 < x_2 < x_3 < x_4$.

& $x_1, x_2, x_3, x_4 \rightsquigarrow$ in G.P.

$x_1 + x_2 = 3$

$a + ar = 3$

$a(1+r) = 3$

$1 + r = 3/a$

$\cancel{3} = \frac{\cancel{3}}{a}$

$\Rightarrow \boxed{a = 1}$

$x_3 + x_4 = 12$

$ar^2 + ar^3 = 12$

$ar^2(1+r) = 12$

$\cancel{a}r^2 \times \frac{\cancel{3}}{\cancel{a}} = \cancel{12}\,4$

$\Rightarrow r^2 = 4 \Rightarrow \boxed{r = 2}$

$r \neq -2$ because G.P is increasing.

$x_1 \cdot x_2 \cdot x_3 \cdot x_4 = ab$ (required).

$\Rightarrow a \cdot ar \cdot ar^2 \cdot ar^3 = ab$

$\Rightarrow ab = a^4 \cdot r^6$

$= 1 \cdot 2^6$

$= 64$ Ans.

krish

QUESTION [JEE Mains 2025 (7 April)]

Let a_n be the n^{th} term of an A.P. If $S_n = a_1 + a_2 + a_3 + \cdots + a_n = 700$, $a_6 = 7$ and $S_7 = 7$, then a_n is equal to:

(A) 65

(B) 56

(C) 70

(D) 64

Ans. D

$\boxed{S-05}$ $a_n \longrightarrow n^{th}$ term

$S_n = a_1 + a_2 + \dots + a_n = \frac{n}{2}(2a_1 + (n-1)d)$
$= 700$

~~$a_6 = 5+$~~ $a_6 = a_1 + 5d = 7$

$S_7 = \frac{7}{2}(2a_1 + 6d) = 7$

$a_1 + 3d = 1$ thus, $2d = 6$

$d = 3$

hence, $a_1 = -8$

$\frac{n}{2}(-16 + (n-1)3) = 700$

$-8n + \frac{n(n-1)3}{2} = 700$

~~-16~~ $-8n + \frac{3n^2}{2} - \frac{3n}{2} = 700$

$\frac{3n^2}{2} - \frac{19n}{2} - 700 = 0$

$3n^2 - 19n - 1400 = 0$

$3n^2 - 75n + 56n - 1400 = 0$

$3n(n-25) + 56(n-25) = 0$

$1400 \times 3 = 70 \times 20 = 5 \times 7 \times 2 \times 2 \times 5 \times 2 \times 3$

$3n = -56$ (N.P) | $\boxed{n = 25}$

hence, $a_n = a_{25} = a_1 + 24d$

$a_{25} = -8 + (24)(3) = 72 - 8 = 64$ (Ans)

Kritisha

S5 if $S_n = a_1 + a_2 + a_3 + \dots + a_n = 700$ $a_6 = 7$ and $S_7 = 7$

$a + 5d = 7$ $\quad \frac{7}{2}(2a + 6d) = 7$

$a + 3d = 1$

$\boxed{d = 3}$

$\boxed{a = -8}$

$S_n = 700$

$\frac{n}{2}[2a + (n-1)d] = 700$

$3n^2 - 19n - 1400 = 0$

$n = \frac{19 \pm \sqrt{361 + 16800}}{6}$ ≈ 131

$n = \frac{19 \pm 131}{6} = \oplus \frac{150}{6}$ ✓ $\quad \ominus \frac{-112}{6}$ ✗

$\boxed{n = 25}$

Now, $a_n = a + (n-1)d$

$= -8 + 24 \times 3$

$= -8 + 72$

$= 64$ Ans

Sakshi

Q S-05: Solⁿ: # $S_n = a_1 + a_2 + a_3 + \dots + a_n = 700$

$a_6 = 7$ # $S_7 = 7$

$\Rightarrow a + 5d = 7$ —— ① $\times 3$ $\Rightarrow \frac{\cancel{7}}{\cancel{2}}[\cancel{2}a + \cancel{6}^{3}d] = \cancel{7}$

$\Rightarrow a + 3d = 1$ —— ② $\times 5$

$\Rightarrow 3a + 15d = 21$ —— ③

$\Rightarrow 5a + 15d = 5$ —— ④

$-2a = 16$

$\Rightarrow \boxed{a = -8}$ put in eq ② : $-8 + 3d = 1$

$3d = 9$

$\boxed{d = 3}$.

$\frac{n}{2}[2a + (n-1)d] = 700$

$\Rightarrow \frac{n}{2}[-16 + 3n - 3] = 700$

$\Rightarrow n[-19 + 3n] = 1400$

$\Rightarrow 3n^2 - 19n - 1400 = 0$

$\Rightarrow 3n^2 - 75n + 56n - 1400 = 0$

$\Rightarrow 3n(n - 25) + 56(n - 25) = 0$

$\Rightarrow (3n + 56)(n - 25) = 0$

$\Rightarrow n = -\frac{56}{3}$ ✗, 25 ✓

find : $a_{25} = a + 24d = -8 + 24(3)$

$= -8 + 72$

$= 64$ Ans.

krish

Solution to Previous RPPs

QUESTION

The expression $y = ax^2 + bx + c (a, b, c \in R$ and $a \neq 0)$ represents a parabola which cuts the x-axis at the points which are roots of the equation $ax^2 + bx + c = 0$. Column-II contains values which correspond to the nature of roots mentioned in Column-I.

Column-I		Column-II	
(a)	For $a = 1, c = 4$, if both roots are greater than 2 then b can be equal to $x^2 + bx + 4 = 0 <^{\alpha}_{\beta}$ $\alpha\beta > 4$ But $\alpha\beta = 4$	(p)	4
(b)	For $a = -1, b = 5$, if roots lie on either side of -1 then c can be equal to $-x^2 + 5x + c = 0 \Rightarrow x^2 - 5x - c = 0$	(q)	8
(c)	For $b = 6, c = 1$, if one root is less than -1 and the other root greater than $\frac{-1}{2}$ then a can be equal to	(r)	10
		(s)	No real value

RPP-01 $y = ax^2+bx+c$ ($a,b,c \in R$ and $a \neq 0$) represents a parabola which cuts the x axis at the points which are roots of $ax^2+bx+c=0$

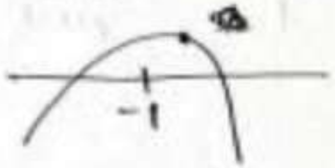
a) for $a=1$, $c=4$; $x^2+bx+4=0$ → if both roots are greater than (2)

P.O.R > 4

but here P.O.R = 4 ; **hence, no real value of 'b' exists (s)**

b) for $a=-1$, $b=5$ if both the roots lie on either side of (-1)

$-x^2+5x+c=0$; x co-ordinate of vertex $\Rightarrow \frac{-b}{2a} = \frac{5}{2}$



$f(-1)>0$

$-1-5+c>0$

$c-6>0$

$c>6$ **Thus, 'c' can be (8), (10) (q) (r)**

$ax^2+6x+1=0$

c) for $b=6$, $c=1$, if one root is less than (-1) and another one is greater than $-1/2$, 'a' can be equal to :-

$a\,f(-\frac{1}{2})<0$ also, $a\,f(-1)<0$

$a(\frac{a}{4}-3+1)<0$	$a(f(-1))<0$
$a(\frac{a}{4}-2)<0$	$a(a-6+1)<0$
$a(a-8)<0$	$a(a-5)<0$
$a \in (0,8)$	$a \in (0,5)$

$\boxed{a \in (0,5)}$

Thus, a can be (4) (p)

Kritisha (W.B)

RPP-1.

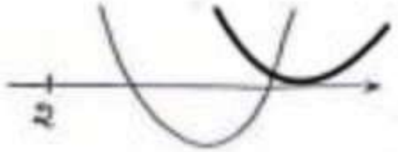
krish

(a) $y=ax^2+bx+c$ ($a,b,c \in R$ and $a \neq 0$)

$a=1$, $c=4$

If both roots are greater than 2.

→ $y = x^2+bx+4$.



a) $D>0$.
b) $f(2)>0$.
c) $-\frac{b}{2a}>2$.

\# $b^2-4(1)(4) \geq 0$

$\Rightarrow b^2-16 \geq 0$

$\Rightarrow (b+4)(b-4) \geq 0$.

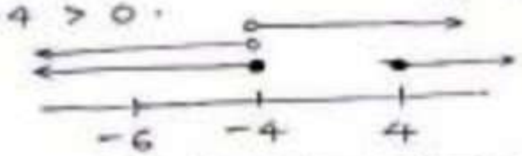
\# $(2)^2+2b+4>0$

$\Rightarrow 2b+8>0$

$\Rightarrow b+4>0$.

\# $-\frac{b}{2}>2$

$\Rightarrow -b>4 \Rightarrow b<-4$.

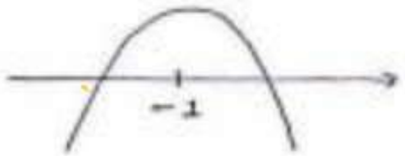
$\Rightarrow$ 

$\Rightarrow$ b = No real value exist Ans.

(b) $a=-1$, $b=5$

If roots lie on either side of -1

→ $y=-x^2+5x+c$



a) $D>0$ → no need

b) $a\,f(-1)<0$.

\# $-1(-1-5+c)<0$

$\Rightarrow 6-c<0$

$\Rightarrow c>6 \Rightarrow$ $c=8$ & 10.

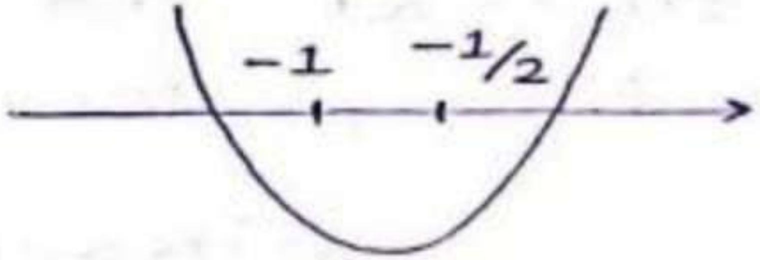
\# then option (q) & (r) both correct.

Ans.

(C) $b = 6, \; c = 1$

if one root is less than -1 & the other root greater than $-\frac{1}{2}$.

$\Rightarrow y = ax^2 + 6x + 1$.



a) $D > 0 \rightsquigarrow$ no need.

b) $af(-1) < 0$.

c) $af(-\frac{1}{2}) < 0$.

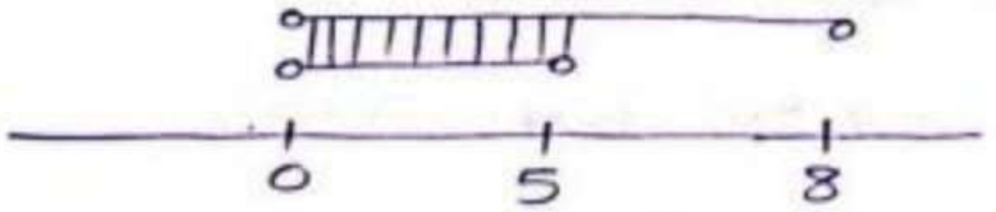
\# $a(a - 6 + 1) < 0$

$\Rightarrow a(a-5) < 0$

\# $a\left(\frac{a}{4} - 3 + 1\right) < 0$

$\Rightarrow a\left(\frac{a}{4} - 2\right) < 0$

$\Rightarrow a(a-8) < 0$.

$\Rightarrow$ 

$\Rightarrow a \in (0, 5) \Rightarrow \boxed{a = 4}$ Ans.

THANK YOU