PRAYAS

JEE 2026

Mathematics

Sequence and Series

Lecture - 6

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Topics *to be covered*

- **A** Question Practice
- **B** Some Special Sequences

Recap *of previous lecture*

1. If a, A_1, A_2, A_3, ... A_n, b are in A.P. then A_1, A_2, A_3, ... A_n are called n A.M.s between ___a___ & ___b___

   also $\sum_{i=1}^{n} A_i$ = $\frac{n(a+b)}{2}$ = $n \cdot \left(\text{single A.M b/w } a \,\&\, b\right)$ also A.P. = $A_1 = a+d$, $A_2 = a+2d$, $A_3 = a+3d$, ⋮ $A_n = a+nd$.

2. If a, b are two positive number a, G_1, G_2, ..., G_n, b are in G.P. then G_1, G_2, G_3, ... G_n are called ___n. G.Ms___ between ___a___ & ___b___

   also $\prod_{i=1}^{n} G_i$ = $(\sqrt{ab})^n$ = $(\text{single G.M})^n$ also G.P. = $G_1 = ar$, $G_2 = ar^2$, $G_3 = ar^3$, ⋮ $G_n = ar^n$

Recap *of previous lecture*

3. If $a_1, a_2, a_3, a_4, \ldots\ldots, a_n$ are in G.P. then

   $a_{n-1} = a_n \cdot \frac{1}{r}$, $a_{n-2} = a_n \cdot \frac{1}{r^2}$, $a_{n-3} = a_n \cdot \frac{1}{r^3}$

4. If $a_1, a_2, a_3, a_4, \ldots\ldots, a_n$ are in A.P. then

   $a_{n-1} = a_n - d$, $a_{n-2} = a_n - 2d$, $a_{n-3} = a_n - 3d$

Recap *of previous lecture*

5. $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, 5, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}$ if the sequence is

give Ans in terms of 5 & d, r

(a) A.P. then $a_6 = $ $5-3d$

$a_7 = $ $5-2d$

$a_{10} = $ $5+2d$

$a_{11} = $ $5+3d$

(b) G.P. then $a_6 = $ $5\cdot(1/r)^3$

$a_7 = $ $5\cdot(1/r)^2$

$a_{10} = $ $5r^2$

$a_{11} = $ $5r^3$.

Recap *of previous lecture*

6. If $\log a_1, \log a_2, \log a_3, \ldots\ldots$ are in A.P. then $a_1, a_2, a_3, \ldots\ldots$ are in ___G.P___

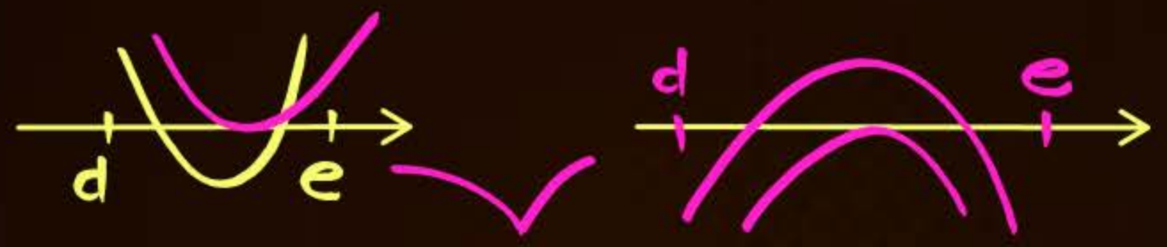
$10^{\log a_1}, 10^{\log a_2}, 10^{\log a_3} \ldots$ is a G.P $\Rightarrow a_1, a_2, a_3 \ldots$ are in G.P

7. If $\log a_1, \log a_2, \log a_3, \ldots\ldots$ are in A.P. then $P^{a_1}, P^{a_2}, P^{a_3}, \ldots\ldots$ is a ___G.P___

8. If $a_1, a_2, a_3, \ldots\ldots$ is a G.P. of positive terms then $\log a_1, \log a_2, \log a_3 \ldots\ldots$ are in ___A.P___

Recap *of previous lecture*

9. Conditions that should be applied so that roots of the quadratic $f(x) = ax^2 + bx + c$ both lies in (d, e) are :



(i) $d < \frac{-b}{2a} < e$ (ii) $D \geq 0$ (iii) $af(d) > 0$ (iv) $af(e) > 0$

10. Condition for $a_1x^2 + b_1x + c_1 = 0$ & $a_2x^2 + b_2x + c_2 = 0$ to have a common root

$a_2x^2 + b_2x + c_2 = 0$

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}^2$$

Home Work Discussion

QUESTION

Tan07

The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots\ldots$ is equal to :

(ADBST)

(A) $\frac{9}{4}$

(B) $\frac{13}{4}$

(C) $\frac{15}{4}$

(D) $\frac{11}{4}$

QUESTION [JEE Mains 2023 (11 April)]

Tan 10

Let $S = \frac{109}{5^0} + \frac{108}{5} + \frac{107}{5^2} + \cdots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is equal to

$$\frac{S}{5} = \frac{109}{5} + \frac{108}{5^2} + \cdots + \frac{3}{5^{107}} + \frac{2}{5^{108}} + \frac{1}{5^{109}}$$

$$\frac{4}{5}S = 109 - \left(\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \cdots + \frac{1}{5^{108}}\right) - \frac{1}{5^{109}}$$

$$\frac{4}{5}S = 109 - \frac{\frac{1}{5}\left(1 - \left(\frac{1}{5}\right)^{108}\right)}{1 - 1/5} - \frac{1}{5^{109}}$$

$$\frac{4S}{5} = 109 - \frac{1}{4}\left(1 - \frac{1}{5^{108}}\right) - \frac{1}{5^{109}}$$

$$4S = 545 - \frac{5}{4}\left(1 - \frac{1}{5^{108}}\right) - \frac{1}{5^{108}}$$

$$= 545 - 5/4 + \frac{5}{4} \cdot \frac{1}{5^{108}} - \frac{1}{5^{108}} = 545 - \frac{5}{4} + \frac{1}{4} \cdot \frac{1}{5^{108}} = \frac{2180 - 5}{4} + 1/4 \cdot (25)^{-54}$$

Ans. 2175

$16S = 2175 + 25^{-54}$

$16S - (25)^{-54} = 2175$ Ans

QUESTION [JEE Mains 2021]

Tah 12 $2^7 = 128$ $2^8 = 256$.

Let $A_1, A_2, A_3, \ldots$ be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is

Diagonal of square = side $\sqrt{2}$

$A_1 \to a_1 = 12$

A_2 kaa diagonal = side of A_1

$a_2\sqrt{2} = a_1 = 12$

$a_2 = \frac{12}{\sqrt{2}}$

A_3 kaa diagonal = side of a_2

$a_3\sqrt{2} = \frac{12}{\sqrt{2}}$

$a_3 = \frac{12}{(\sqrt{2})^2}$

sides. $12, \frac{12}{\sqrt{2}}, \frac{12}{(\sqrt{2})^2}, \frac{12}{(\sqrt{2})^3} \ldots$

$a_n = 12 \cdot \left(\frac{1}{\sqrt{2}}\right)^{n-1}$

$\downarrow$

$A_n = a_n^2 = 144 \cdot \left(\frac{1}{2}\right)^{n-1} < 1$

$144 < 2^{n-1}$

$n-1 > 7 \Rightarrow n > 8 \Rightarrow n|_{min} = 9$

Ans. 9

QUESTION [JEE Mains 2022]

Tah13

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ all $n \geq 0$.

Then $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to: $= \sum_{n=0}^{\infty} \frac{a_n}{7^n}$

(A) $\frac{6}{343}$

(B) $\frac{7}{216}$

(C) $\frac{8}{343}$

(D) $\frac{49}{216}$

$S_1 = S - \frac{a_0}{7^0} - \frac{a_1}{7^1} = S$

$S_2 = S - \frac{a_0}{7^0} = S$

$a_0, a_1 = 0$

$a_{n+2} = 2a_{n+1} - a_n + 1$

$\frac{a_{n+2}}{7^n} = 2\frac{a_{n+1}}{7^n} - \frac{a_n}{7^n} + \frac{1}{7^n}$

$7^2 \frac{a_{n+2}}{7^{n+2}} = 14 \frac{a_{n+1}}{7^{n+1}} - \frac{a_n}{7^n} + \frac{1}{7^n}$

$7^2 \sum_{n=0}^{\infty} \frac{a_{n+2}}{7^{n+2}} = 14 \sum_{n=0}^{\infty} \frac{a_{n+1}}{7^{n+1}} - \sum_{n=0}^{\infty} \frac{a_n}{7^n} + \sum_{n=0}^{\infty} \frac{1}{7^n}$

$\downarrow S_1 \qquad \downarrow S_2 \qquad \downarrow S$

$49S = 14S - S + \frac{1}{1 - 1/7}$

$36S = \frac{7}{6} \Rightarrow S = \frac{7}{216}$.

Question Practice

$$S = a^n + a^{n-1}\cdot b + a^{n-2}b^2 + a^{n-3}b^3 + \cdots + ab^{n-1} + b^n = \frac{a^{n+1} - b^{n+1}}{a-b}.$$

$$\Rightarrow (a-b)(a^n + a^{n-1}b + a^{n-2}b^2 + a^{n-3}b^3 + \cdots + ab^{n-1} + b^n) = a^{n+1} - b^{n+1}$$

$$S = \frac{a^n\left(1 - \left(\frac{b}{a}\right)^{n+1}\right)}{1 - \frac{b}{a}} = \frac{1}{(a-b)}\, a^{n+1}\left(1 - \frac{b^{n+1}}{a^{n+1}}\right)$$

$$= \frac{1}{a-b}\left(a^{n+1} - b^{n+1}\right)$$

$$a^n - a^{n-1}b + a^{n-2}\cdot b^2 - a^{n-3}b^3 + \cdots + (-1)^n b^n = \frac{a^n\left(1 - (-b/a)^{n+1}\right)}{1 + \frac{b}{a}} = \frac{a^{n+1} - (-b)^{n+1}}{a+b}.$$

QUESTION [JEE Mains 2020]

$-1 < x < 1 \rightarrow x^2 \in [0,1)$

If $|x| < 1, |y| < 1$ and $x \neq y$, then the sum to infinity of the following series

$S = (x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots$ is

(A) $\frac{x + y + xy}{(1 - x)(1 - y)}$

(B) $\frac{x + y - xy}{(1 + x)(1 + y)}$

(C) $\frac{x + y + xy}{(1 + x)(1 + y)}$

(D) $\frac{x + y - xy}{(1 - x)(1 - y)}$

$S(x-y) = (x^2-y^2) + (x^3-y^3) + (x^4-y^4) + x^5-y^5 + \cdots \infty.$

$(x-y)S = (x^2+x^3+x^4+\cdots\infty) - (y^2+y^3+\cdots\infty)$

$= \frac{x^2}{1-x} - \frac{y^2}{1-y} = \frac{x^2-x^2y-y^2+xy^2}{(1-x)(1-y)}$

$(x-y)\cdot S = \frac{(x-y)(x+y) - xy(x-y)}{(1-x)(1-y)}$

$S = \frac{x+y-xy}{(1-x)(1-y)}$

QUESTION [JEE Mains 2023 (15 April)]

Tan 02

⭐⭐⭐ KC LS ⭐⭐⭐

If the sum of the series

$$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2\cdot3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}-\frac{1}{2^2\cdot3}+\frac{1}{2\cdot3^2}-\frac{1}{3^3}\right)+\left(\frac{1}{2^4}-\frac{1}{2^3\cdot3}+\frac{1}{2^2\cdot3^2}-\frac{1}{2\cdot3^3}+\frac{1}{3^4}\right)+\cdots$$

is $\frac{\alpha}{\beta}$, where α and β are co-prime, then $\alpha+3\beta$ is equal to

$x=\frac{1}{2}$ $\quad S=(x-y)+(x^2-xy+y^2)+(x^3-x^2y+xy^2-y^3)+(x^4-x^3y+x^2y^2-xy^3+y^4)+\cdots$

$y=1/3$

Ans. 7

QUESTION

KCLS

If m, n are relatively prime positive integers such that $\sum_{\substack{i,j\geq 0 \\ i+j=\text{odd}}} \frac{1}{3^i \cdot 5^j} = \frac{m}{n}$, then $(5n-4m)$ is equal to

$S =$

$i=0 \quad \frac{1}{3^0}\left(\frac{1}{5^1}+\frac{1}{5^3}+\frac{1}{5^5}+\cdots\infty\right)$

$(i=1) + \frac{1}{3^1}\left(\frac{1}{5^0}+\frac{1}{5^2}+\frac{1}{5^4}+\cdots\infty\right)$

$(i=2) + \frac{1}{3^2}\left(\frac{1}{5^1}+\frac{1}{5^3}+\frac{1}{5^5}+\cdots\infty\right)$

$(i=3) + \frac{1}{3^3}\left(\frac{1}{5^0}+\frac{1}{5^2}+\frac{1}{5^4}+\cdots\infty\right)$

$S=\left(\frac{1}{5^1}+\frac{1}{5^3}+\cdots\infty\right)\left(\frac{1}{3^0}+\frac{1}{3^2}+\frac{1}{3^4}+\cdots\infty\right)$

$+$

$\left(\frac{1}{5^0}+\frac{1}{5^2}+\frac{1}{5^4}+\cdots\infty\right)\left(\frac{1}{3}+\frac{1}{3^3}+\frac{1}{3^5}+\cdots\infty\right)$

$= \frac{1/5}{1-\frac{1}{25}} \times \frac{1}{1-1/9} + \frac{1}{1-\frac{1}{25}} \times \frac{1/3}{1-\frac{1}{9}}$

$= \frac{5}{24}\cdot\frac{9}{8} + \frac{25}{24}\cdot\frac{3}{8}$

$= \frac{15}{64} + \frac{25}{64} = \frac{40}{64} = 5/8.$

$$5n - 4m = 40 - 20 = \underline{20 \text{ Ans}}$$

$m = 5$

$n = 8$

QUESTION [JEE Mains 2023 (6 April)] ASRQ

If $\gcd(m, n) = 1$ and $1^2 - 2^2 + 3^2 - 4^2 + \cdots + (2021)^2 - (2022)^2 + (2023)^2 = 1012m^2n$, then $m^2 - n^2$ is equal to :

(A) 220

(B) 200

(C) 240

(D) 180

$(1^2 - 2^2) + (3^2 - 4^2) + \cdots + ((2021)^2 - (2022)^2) + 2023^2 = 1012m^2n$.

$-(3 + 7 + 11 + 15 + \cdots + 4043) + 2023^2 = 1012m^2n$. (1011 terms.)

$\Rightarrow -\frac{1011}{2}(2\cdot 3 + 1010\cdot 4) + 2023^2 = 1012m^2n$

$(2023)^2 - 1011\cdot(2023) = 1012m^2n$

$(2023)(2023 - 1011) = 1012m^2n$

$2023 = m^2n$.

Another Possibility $m = 1, n = 2023$

$m^2n = 7 \times 289 = 7 \times 17^2$

$m = 17$, $n = 7$ $\Rightarrow m^2 - n^2 = 289 - 49 = 240$

Ans. C

QUESTION [JEE Mains 2023 (27 Jan)]

Tah02

Let $a_1, a_2, a_3, \ldots$ be in an arithmetic progression of positive terms.

Let $A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \cdots + a_{2k-1}^2 - a_{2k}^2$.

If $A_3 = -153$, $A_5 = -435$ and $a_1^2 + a_2^2 + a_3^2 = 66$, then $a_{17} - A_7$ is equal to

Ans. 910

QUESTION [JEE Mains 2023 (31 Jan)]

Let $a_1, a_2, a_3, \ldots$ be an A.P. If $a_7 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to :

(A) 24

(B) $\frac{381}{4}$

(C) 9

(D) $\frac{33}{4}$

$a_7 = 3$

$a + 6d = 3$

$a = 3 - 6d$

$a_1 \cdot a_4 = a(a+3d)$

$a_1 \cdot a_4 = (3-6d)(3-3d)$

$= 9((1-2d)(1-d))$

$= 9(1-d-2d+2d^2)$

$= 9(2d^2-3d+1)$

$= 9\left(2\left(d^2-\frac{3d}{2}+\frac{9}{16}-\frac{9}{16}\right)+1\right)$

$= 9\left(2(d-3/4)^2-\frac{9}{8}+1\right) = 18(d-3/4)^2 - \frac{9}{8}$, $(d-3/4)^2 \geq 0$

$a_1 a_4\big|_{min} = -\frac{9}{8}$ @ $d = 3/4$, $a = -3/2$

$S_n = \frac{n}{2}(2a + (n-1)\cdot 3/4) = 0$

$\Rightarrow \frac{n}{2}\left(-3 + (n-1)\cdot\frac{3}{4}\right) = 0$

$\frac{3}{4}(n-1) = 3$

$n = 5$

Ans. A

$$n! - 4 \cdot a_{n(n+2)} = 5! - 4a_{35}.$$

$$120 - 4 \cdot \left(-\frac{3}{2} + 34 \cdot \frac{3}{4}\right)$$

$$= 120 - (-6 + 102)$$

$$= 120 - 96$$

$$= 24 \text{ Ans}$$

Problems Involving Number series

QUESTION

Evaluate : $S = 9 + 99 + 999 + \cdots + \underbrace{999 \ldots 9}_{\text{n times}}$

$9 = 10^1 - 1$

$99 = 10^2 - 1$

$\vdots$

$\underbrace{999 \ldots 9}_{n\text{ times}} = 10^n - 1$

$S = 10 - 1 + 10^2 - 1 + 10^3 - 1 + \cdots + 10^n - 1$

$= 10 + 10^2 + \cdots + 10^n - n$

$S = 10\dfrac{(10^n - 1)}{9} - n$

QUESTION

Evaluate : $\frac{3}{19}+\frac{33}{19^2}+\frac{333}{19^3}+\frac{3333}{19^4}+\cdots\infty$

$$S = 3\left(\frac{1}{19}+\frac{11}{19^2}+\frac{111}{19^3}+\frac{1111}{19^4}+---\infty\right)$$

$$= \frac{3}{9}\left(\frac{9}{19}+\frac{99}{19^2}+\frac{999}{19^3}+\frac{9999}{19^4}+---\infty\right)$$

$$= \frac{3}{9}\left(\frac{10-1}{19}+\frac{10^2-1}{19^2}+\frac{10^3-1}{19^3}+\frac{10^4-1}{19^4}+---\infty\right)$$

$$= \frac{3}{9}\left(\left(\frac{10}{19}+\left(\frac{10}{19}\right)^2+\left(\frac{10}{19}\right)^3+---\infty\right)-\left(\frac{1}{19}+\frac{1}{19^2}+\frac{1}{19^3}+---\infty\right)\right)$$

$$= \frac{3}{9}\left(\frac{\frac{10}{19}}{1-\frac{10}{19}}-\frac{1/19}{1-1/19}\right)=\frac{3}{9}\left(\frac{10}{9}-\frac{1}{18}\right)=\frac{3}{9}\cdot\frac{19}{18}=19/54 \text{ Ans}$$

QUESTION

Tan 03

Evaluate

(a) $0.7 + 0.77 + 0.777 + \ldots$ n terms $\to 7(0.1 + 0.11 + 0.111 + \ldots)$

(b) $0.9 + 0.99 + 0.999 + \ldots$ n terms $\quad \frac{7}{9}(0.9 + 0.99 + 0.999 + \ldots)$

$0.9 = 1 - 10^{-1}$

$0.99 = 1 - 10^{-2}$

$0.999 = 1 - 10^{-3}$

$\vdots$

$\underbrace{0.999\ldots9}_{n\text{ times}} = 1 - 10^{-n}$

QUESTION

Tanoy

Find the sum of the following series

(i) $5 + 55 + 555 + \ldots\ldots\ldots$ to n terms

(ii) $0.3 + 0.33 + 0.333 + \ldots\ldots\ldots$ to n terms

Ans. (i) $\frac{5}{9}\left[10\left(\frac{10^n-1}{9}\right) n\right]$

(ii) $\frac{1}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$

$$675 = 6\times 10^2 + 7\times 10^1 + 5\times 10^0$$

$$72764 = 7\times 10^4 + 2\times 10^3 + 7\times 10^2 + 6\times 10^1 + 4\times 10^0$$

QUESTION [JEE Advanced 2023 (Paper 1)]

Let $7\overbrace{5\cdots5}^{r}7$ denote the (r + 2) digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum $S = 77 + 757 + 7557 + \cdots + 7\overbrace{5\cdots5}^{98}7$. If $S = \frac{7\overbrace{5\ldots5}^{99}7+m}{n}$, where m and n are natural numbers less than 3000, then the value of m + n is M(1)

$S = 77 + 757 + 7557 + 75557 + \cdots + 7\overbrace{555\ldots5}^{98\text{ times}}7$

$= (7\times10^1 + 7\times10^2 + 7\times10^3 + \cdots + 7\times10^{99}) + (5 + 55 + 555 + \cdots + \overbrace{555\ldots5}^{98\text{ times}})\times10 + \underbrace{(7 + 7 + \cdots + 7)}_{99\text{ times}}$

$= 70\frac{(10^{99}-1)}{9} + 50(1 + 11 + 111 + 1111 + \cdots + 111\ldots1) + 7\times99$

$= \frac{70}{9}(10^{99}-1) + \frac{50}{9}(9 + 99 + 999 + \cdots + \overbrace{99\ldots9}^{98\text{ times}}) + 693$

Ans. 1219

$$S=\frac{70}{9}(10^{99}-1)+\frac{50}{9}(10-1+10^2-1+--+10^{98}-1)+693.$$

$$=\frac{70}{9}(10^{99}-1)+\frac{50}{9}(10+10^2+--+10^{98}-98)+693$$

$$=\frac{70}{9}(10^{99}-1)+\frac{50}{9}\left(\frac{10\cdot(10^{98}-1)}{9}-98\right)+693.$$

$$=\frac{70}{9}(10^{99}-1)+\frac{50}{9}\left(\frac{10^{99}-10}{9}-98\right)+693.$$

$$=\frac{70}{9}(10^{99}-1)+\frac{50}{9}\left(\frac{10^{99}-1-9}{9}-98\right)+693.$$

$$=\frac{70}{9}(10^{99}-1)+\frac{50}{9}\left(\frac{99--9}{9}-1-98\right)+693.$$

$$=\frac{7\times10^{100}}{9}-\frac{70}{9}+\frac{50}{9}(\overbrace{111--1}^{99}-99)+693$$

$$S=\frac{7\times 10^{100}}{9}+\frac{\overbrace{55\ldots5}^{99}\times 10^{1}}{9}-\frac{70}{9}-550+693$$

$$=\frac{7\times 10^{100}+\overbrace{55\ldots50}^{100}}{9}-\frac{70}{9}+143$$

$$=\frac{7\overbrace{55\ldots50}^{99}-70+1287}{9}$$

$$=\frac{755\ldots50+1217}{9}$$

$$=\frac{7\overbrace{55\ldots50}+7+1210}{9}$$

$$=\frac{7\overbrace{55\ldots57}^{99}+1210}{9}$$

$m=1210$ $\quad$ $m+n=1219.$

$n=9$

QUESTION [JEE Advanced 2023 (Paper 1)]

Let $7\overbrace{5\cdots5}^{r}7$ denote the (r + 2) digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum $S = 77 + 757 + 7557 + \cdots + 7\overbrace{5\cdots5}^{98}7$. If $S = \dfrac{7\overbrace{5\ldots5}^{99}7+m}{n}$, where m and n are natural numbers less than 3000, then the value of m + n is

M(2) $S = 77 + 757 + 7557 + 75557 + \cdots + 7\overbrace{555\cdots5}^{98\text{ times}}7$

$10S = \quad 770 + 7570 + 75570 + \cdots + 7\overbrace{55\cdots5}^{97}70 + 7\overbrace{55\cdots5}^{98}70$

$-9S = 77 - (\underbrace{13 + 13 + 13 + \cdots + 13}_{98\text{ terms}}) - 755\cdots570$

$-9S = 77 - 98\times13 - 755\cdots570$

$9S = 98\times13 + 755\cdots570 - 77$

$9S = 1274 - 77 + 7\overbrace{55\cdots5}70 = 1197 + 7\overbrace{55\cdots5}^{98\text{ times}}70$

Ans. 1219

$$9S = 7\overbrace{55\ldots5}^{98}70 + 1197$$

$$S = \frac{755\ldots557 + 13 + 1197}{9}$$

$$S = \frac{7\overbrace{55\ldots5}^{99}7 + 1210}{9}$$

$m + n = 1219.$

QUESTION [JEE Advanced 2022 (Paper 1)]

★★KCLS★★ Jan 04

Let $l_1, l_2, \ldots, l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1, and let $w_1, w_2, \ldots, w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1 d_2 = 10$. For each $i = 1, 2, \ldots, 100$, let R_i be a rectangle with length l_i, width w_i and area A_i. If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is ________

Ans. 18900

QUESTION

Tah05

Show that $\underbrace{444\ldots4}_{n}\ \underbrace{888\ldots8}_{n-1}9$ is always a perfect square.

Special Sequences

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Special Sequences

Special Sequences

Sigma Notations (Σ) :

(a) $\sum_{r=1}^{n}(a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$ — Σ distributes over sum & diff of terms

* $\sum_{r=1}^{n}(a_r + b_r) = (a_1+b_1) + (a_2+b_2) + \ldots + (a_n+b_n)$
$= (a_1+a_2+\ldots+a_n) + (b_1+b_2+\ldots+b_n)$
$= \sum_{r=1}^{n} a_r + \sum_{r=1}^{n} b_r$

(b) $\sum_{r=1}^{n} k a_r = k \sum_{r=1}^{n} a_r$ (k does not depend on r)

* $\sum_{r=1}^{n} n \cdot a_r = na_1 + na_2 + \ldots + na_n$
$= n(a_1+a_2+\ldots+a_n) = n\sum_{r=1}^{n} a_r$

* $\sum_{r=1}^{n} 3a_r = 3\sum_{r=1}^{n} a_r$ * $\sum_{r=1}^{n} x \cdot a_r = x\sum_{r=1}^{n} a_r$.

(c) $\sum_{r=1}^{n} k = nk$; where k is a constant (k does not depend on r.)

$\underbrace{k+k+\ldots+k}_{n\text{ times}} = nk$.

- $\sum_{r=0}^{n}$ ⟶ No: of terms $= n+1$
- $\sum_{r=0}^{n-1}$ ⟶ No: of terms $= n$
- $\sum_{r=1}^{n}$ ⟶ No: of terms $= n$

Types 1: Sequence dealing with Σn; Σn^2; Σn^3

(a) $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ (sum of the first n natural numbers)

(b) $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ (sum of the squares of the first n natural numbers)

(c) $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)2}{4} = \left[\sum_{r=1}^{n} r\right]^2$

(sum of the squares of the first n natural numbers)

(d) $\sum_{r=1}^{n}(2r-1)=n^2$ (sum of first n odd natural numbers)

(e) $\sum_{r=1}^{n} r = n(n+1)$ (sum of first n even natural numbers)

$$1^2+2^2+3^2+\dots+n^2=\frac{n(n+1)(2n+1)}{6}$$

Proof $\quad K^3-(K-1)^3=1+3K^2-3K$

$k=1 \quad 1^3-0^3=1+3\cdot1^2-3\cdot1$

$k=2 \quad 2^3-1^3=1+3\cdot2^2-3\cdot2$

$k=3 \quad 3^3-2^3=1+3\cdot3^2-3\cdot3$

$\vdots$

$k=n \quad n^3-(n-1)^3=1+3\cdot n^2-3\cdot n$

$$n^3=(1+1+\dots+1)+3(1^2+2^2+3^2+\dots+n^2)-3(1+2+\dots+n)$$

$$n^3=n+3\sum_{r=1}^{n}r^2-3\frac{n(n+1)}{2}$$

3. $\sum_{r=1}^{n} r^2 = n^3 + \frac{3n(n+1)}{2} - n$

$= n\left(n^2 + \frac{3(n+1)}{2} - 1\right)$

$\sum_{r=1}^{n} r^2 = n\frac{(2n^2+3n+1)}{2\cdot 3} = \frac{n(2n+1)(n+1)}{6}$

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Proof: $k^4 - (k-1)^4 = 4k^3 - 6k^2 + 4k - 1$

put $k = 1, 2, \ldots, n$ & add.

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Today's Shikaars

QUESTION [BITSAT 2024]

Sol

There are four numbers of which the first three are in G.P. and the last three are in A.P., whose common difference is 6. If the first and the last numbers are equal, then two other numbers are

(A) $-2, 4$

(B) $-4, 2$

(C) $2, 6$

(D) None of the above

Ans. B

QUESTION [BITSAT 2023]

S02

Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}, \frac{1}{b}$, 6 be in A.P., where $a, b > 0$. Then, $72(a+b)$ is equal to

(A) 12

(B) 14

(C) 16

(D) 18

Ans. B

QUESTION [BITSAT 2022]

S03

In a sequence of 21 terms, the first 11 terms are in A.P. with common difference 2 and the last 11 terms are in G.P. with common ratio 2. If the middle term of A.P. be equal to the middle term of the G.P., then the middle term of the entire sequence is

(A) $-\frac{10}{31}$

(B) $\frac{10}{31}$

(C) $\frac{32}{31}$

(D) $-\frac{31}{32}$

Ans. A

QUESTION [WB JEE 2019]

S04

Let x_1, x_2 be the roots of $x^2 - 3x + a = 0$ and x_3, x_4 be the roots of $x^2 - 12x + b = 0$. If $x_1 < x_2 < x_3 < x_4$ and x_1, x_2, x_3, x_4 are in G.P., then ab equals

(A) 24/5

(B) 64

(C) 16

(D) 8

Ans. B

QUESTION [JEE Mains 2025 (7 April)]

SOS

Let a_n be the n^{th} term of an A.P. If $S_n = a_1 + a_2 + a_3 + \cdots + a_n = 700$, $a_6 = 7$ and $S_7 = 7$, then a_n is equal to:

(A) 65

(B) 56

(C) 70

(D) 64

Ans. D

Revision Practice Problems (RPP)

QUESTION

RPP01

The expression $y = ax^2 + bx + c (a, b, c \in R \text{ and } a \neq 0)$ represents a parabola which cuts the x-axis at the points which are roots of the equation $ax^2 + bx + c = 0$. Column-II contains values which correspond to the nature of roots mentioned in Column-I.

Column-I		Column-II	
(a)	For $a = 1, c = 4$, if both roots are greater than 2 then b can be equal to	(p)	4
(b)	For $a = -1, b = 5$, if roots lie on either side of -1 then c can be equal to	(q)	8
(c)	For $b = 6, c = 1$, if one root is less than -1 and the other root greater than $\frac{-1}{2}$ then a can be equal to	(r)	10
		(s)	No real value

Solution to Previous TAH

QUESTION [JEE Mains 2023 (25 Jan)]

TAH 01

For the two positive numbers a, b, if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}$, 10 and $\frac{1}{b}$ are in an arithmetic progression, then $16a + 12b$ is equal to

Ans. 3

TAH-01.

$a, b, \frac{1}{18} \longrightarrow G.P \quad (a, b > 0)$

& $\frac{1}{a}, 10, \frac{1}{b} \longrightarrow A.P.$

$b^2 = \frac{a}{18} \Rightarrow 18b^2 = a \quad \text{—— (i)}$

& $20 = \frac{a+b}{ab} \Rightarrow 20ab = a+b \quad \text{— (ii)}$

$20 \times 18b^3 = 18b^2 + b$

$360b^3 - 18b^2 - b = 0$

$360b^2 - 18b - 1 = 0$

$360b^2 - 30b + 12b - 1 = 0$

$30b(12b - 1) + (12b - 1) = 0$

$b = \frac{1}{12}$ & , $-\frac{1}{30}$ ✗

So, $a = \frac{1}{8}$

Now, $16a + 12b = \frac{16}{8} + \frac{12}{12} = 2 + 1 = 3$

QUESTION

TAH 02

If m is the A.M. of two distinct real numbers l and n (l, n > 1) and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals

(A) $4\,l\,m\,n^2$

(B) $4\,l^2\,m^2\,n^2$

(C) $4\,l^2\,m\,n$

(D) $4\,l\,m^2\,n$

Ans. D

Date

Tak 02:-

l, A_1, A_2, n AP $\Rightarrow$ $A_1 + A_2 = l + n \Rightarrow 2m = l + n$

l, G_1, G_2, G_3, n GP

$G_1^2 = lG_2$, $G_3^2 = G_2 . n$ $G_2^2 = G_1 G_3 = ln$

Then: $G_1^4 + 2G_2^4 + G_3^4$

$\Rightarrow l^2 G_2^2 + 2l^2n^2 + G_2^2 n^2$

$\Rightarrow l^2 . ln + 2l^2n^2 + ln . n^2$

$\Rightarrow ln(l^2 + 2ln + n^2)$

$\Rightarrow ln(l+n)^2$

$\Rightarrow ln(2m)^2 = ln 4m^2$

Ans $= 4lm^2n$

TAH - 02.

$l, m, n \longrightarrow A.P. \quad (l, n > 1)$

$l, G_1, G_2, G_3, n \longrightarrow G.P.$

$G_1^2 = lG_2 \; ; \; G_2^2 = G_1 G_3 \quad G_3^2 = G_2 n$

& $2m = l + n$

Now, $G_2 n = G_3^2$

$G_2 n = \frac{G_2^4}{G_1^2}$

$G_2 n = \frac{G_2^4}{l G_2} \Rightarrow G_2^2 = nl.$

Now, $G_1^4 + 2G_2^4 + G_3^4$

$l^2 G_2^2 + 2n^2 l^2 + G_2^2 n^2$

$l^2 nl + 2n^2 l^2 + nl n^2$

$nl^3 + 2n^2 l^2 + l n^3$

$ln(l^2 + 2nl + n^2)$

$ln(l + n)^2$

$ln \times 4m^2 \Rightarrow 4lm^2 n$

Rajkanya
From Bihar

Q2. Solⁿ: ⇒ $l, A_1, A_2, A_3, n \longrightarrow$ A.P.

AM $m = \frac{l+n}{2}$

$2m = l+n$ —— (1)

⇒ $l, G_1, G_2, G_3, n \longrightarrow$ G.P

$G_1^2 = l \cdot G_2$ $\quad$ $G_3^2 = n \cdot G_2$

$G_2^2 = G_1 \cdot G_2 = l \cdot n$

find: $G_1^4 + 2G_2^4 + G_3^4$

$\Rightarrow l^2 \cdot G_2^2 + 2G_2^4 + n^2 G_2^2$

$\Rightarrow G_2^2 (l^2 + 2(G_2^2) + n^2)$

$\Rightarrow ln (l^2 + 2(ln) + n^2)$

$\Rightarrow ln (l+n)^2$

$\Rightarrow ln (2m)^2 \longrightarrow [l+n = 2m]$ Eqⁿ (1).

$\Rightarrow 4lm^2n$ Ans.

krish

QUESTION [JEE Mains 2021 (26 Aug)]

TAH 03

Let $a_1, a_2, \ldots\ldots, a_{10}$ be an A.P. with common difference –3 and $b_1, b_2, \ldots\ldots b_{10}$ be a G.P. with common ratio 2. Let $c_k = a_k + b_k$, $k = 1,2, \ldots\ldots, 10$. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to

Ans. 2021

Date ______

TAH 03:

$a_1, a_2, \ldots a_{10}$ be A.P $(d=-3)$

$b_1, b_2, \ldots b_{10}$ be GP $(r=2)$

$a_k + b_k = c_k \quad k = 1, 2, \ldots 10$

$c_2 = 12 \Rightarrow a_2 + b_2$

$12 = a + d + br$

$12 = a - 3 + 2b$

$\boxed{15 = a + 2b}$

$c_3 = 13 \Rightarrow a_3 + b_3$

$13 = a + 2d + br^2$

$13 = a + 2(-3) + 4b$

$\boxed{19 = a + 4b}$

$4 = 2b$

$\boxed{b = 2}, \boxed{a = 11}$

Then: $\sum_{k=1}^{10} c_k = a_k + b_k$

$$\sum_{k=1}^{10} c_k = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k$$

$= (a_1 + a_2 + \ldots a_{10}) + (b_1 + b_2 + \ldots b_{10})$

$= 5[a_1 + a_{10}] + \frac{3[2^{10} - 1]}{2-1}$

$= 5[11 + 11 + 9(-3)] + 3(1023)$

$= 5(-5) + 2046$

$= 2021$

$\boxed{\sum_{k=1}^{10} c_k = 2021}$ Ans

TAH-03

$a_1, a_2, \ldots, a_{10} \longrightarrow A.P\ (d = -3)$; $c_2 = 12$

$b_1, b_2, \ldots, b_{10} \longrightarrow G.P\ (r = 2)$; $c_3 = 13$

$c_k = a_k + b_k, \quad k = 1, 2, \ldots, 10.$

$c_2 = 12$ & $c_3 = 13$

$a + d + br = 12$ $\quad a + 4b = 19$ — (ii) $\quad$ from (i) & (ii) $b = 2$ & $a = 11$

$2b + a = 15$, — (i)

$$\sum_{k=1}^{10} c_k = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k \Rightarrow (a_1 + a_2 + \cdots + a_{10}) + (b_1 + b_2 + \cdots + b_{10})$$

$= -25 + 2046$

$= 2021$

Rajkanya From Bihar

QUESTION

TAH 04

If $|x| < 1$, then compute S_∞:

(a) $1 + 2x + 3x^2 + 4x^3 + ...$

(b) $1 + 3x + 6x^2 + 10x^3 + ...$

(c) $1^2 + 2^2x + 3^2x^2 + 4^2x^2 + ...$

Tah 04 (a):

$$S = 1^2 + 2^2x + 3^2x^2 + 4^2x^3 + \cdots \infty$$

$$xS = x + 2^2x^2 + 3^2x^3 + \cdots \infty$$

$$(1-x)S = 1 + 3x + 5x^2 + 7x^3 + \cdots \infty$$

$$xS(1-x) = x + 3x^2 + 5x^3 + \cdots \infty$$

$$S(1-x)^2 = 1 + 2x + 2x^2 + 2x^3 + \cdots \infty$$

$$= 1 + 2[x + x^2 + x^3 + \cdots \infty]$$

$$= 1 + 2\left[\frac{x}{1-x}\right] = 1 + \frac{2x}{1-x}$$

$$S(1-x)^2 = 1 + \frac{2x}{1-x}$$

$$S(1-x)^2 = \frac{1+x}{1-x}$$

$$\boxed{S = \frac{1+x}{(1-x)^3}}$$

Tah-04 If $|x|<1$, then compute S_∞ :

(c) $1^2+2^2x+3^2x^2+4^2x^3+\ldots$

$S=1^2+2^2x+3^2x^2+4^2x^3+\ldots\infty$

$S=1+4x+9x^2+16x^3+\ldots\infty$

$xS=\quad x+4x^2+9x^3+16x^4+\ldots\infty$

$S(1-x)=1+3x+5x^2+7x^3+9x^4+\ldots\infty$

$x(1-x)S=\quad x+3x^2+5x^3+7x^4+\ldots\infty$

$(1-x)^2S=1+2x+2x^2+2x^3+2x^4+\ldots\infty$

$(1-x)^2S=1+2\left(\frac{x}{1-x}\right) \Rightarrow \frac{1-x+2x}{1-x} \Rightarrow \frac{x+1}{1-x}$

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$S=\frac{1+x}{(1-x)^2}$ Ans

QUESTION [JEE Mains 2024 (27 Jan)]

TAH 05

If $8 = 3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) + \ldots\ldots \infty$, then the value of p is

Tah-05 If $8 = 3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \frac{1}{4^3}(3+3p) + \cdots \infty$, then $p = ?$ [M-2024]

$S = 8$

$$S = 3 + (3+p)\frac{1}{4} + (3+2p)\frac{1}{4^2} + (3+3p)\frac{1}{4^3} + \cdots \infty = 8$$

$$\frac{S}{4} = \frac{3}{4} + (3+p)\frac{1}{4^2} + (3+2p)\frac{1}{4^3} + \cdots \infty$$

$$\frac{3}{4}S = 3 + \frac{p}{4} + \frac{p}{4^2} + \frac{p}{4^3} + \cdots \infty$$

$$\frac{3}{4} \times 8 = 3 + p\left(\frac{1/4}{1 - 1/4}\right)$$

$$6 - 3 = p\left(\frac{1}{4} \times \frac{4}{3}\right)$$

$$3 = \frac{p}{3}$$

$$\boxed{p = 9}$$

Richathakur

Tah-05 (b) If $8 = 3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \dots \infty$. then the value of p is

$$\Rightarrow 5 = \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \dots \infty$$

$$\frac{5}{4} = \qquad + \frac{3+p}{4^2} + \frac{3+2p}{4^3} + \dots \infty$$

$$\left(5 - \frac{5}{4}\right) = \frac{1}{4}(3+p) + \frac{3+2p-3-p}{4^2} + \frac{3+3p-3-2p}{4^3} + \dots \infty$$

$$\frac{15}{4} = \frac{3+p}{4} + \frac{p}{4^2} + \frac{p}{4^3} + \dots \infty$$

Kritisha

$$\frac{15}{4} = \frac{3}{4} + p\left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots \infty\right)$$

$$\frac{12}{4} = p\left(\frac{1/4}{1-\frac{1}{4}}\right) \Rightarrow 3 = p\left(\frac{1}{3}\right)$$

$p = 9$ (Ans)

Q 5. Sol^n: $\boxed{S = 8}$.

$$S = 3 + (3+P)\frac{1}{4} + (3+2P)\cdot\frac{1}{4^2} + \cdots \infty.$$

$$\frac{1}{4}S = \frac{3}{4} + (3+P)\frac{1}{4^2} + \cdots \infty.$$

$$\frac{3S}{4} = 3 + \frac{P}{4} + \frac{P}{4^2} + \cdots \infty.$$

$$\Rightarrow \frac{3S}{4} = 3 + P\left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots \infty\right).$$

$$\Rightarrow \frac{3S}{4} = 3 + P\left(\frac{1/4}{1 - 1/4}\right) \Rightarrow \frac{3S}{4} = 3 + \frac{P}{3}$$

(8)

$$\Rightarrow P = 3 \times 3$$

$$\Rightarrow \boxed{P = 9} \text{ Ans.}$$

krish

QUESTION [JEE Mains 2014]

TAH 06

If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \cdots + 10(11)^9 = k(10)^9$, then k is equal to

(A) 121/10

(B) 441/100

(C) 100

(D) 110

Ans. C

Date ____

Tah ob:

$$S = 1(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \cdots + 10(11)^9$$

$$\frac{11}{10}S = \quad + 1\cdot 11(10)^8 + 2\cdot(11)^2(10)^7 + \cdots 9(11)^9 + 11^{10}$$

$$-\frac{S}{10} = \left(10^9 + 11(10)^8 + (11)^2(10)^7 + \cdots + (11)^9\right) - 11^{10}$$

$$= \frac{10^9\left(\left(\frac{11}{10}\right)^{10} - 1\right)}{\frac{11}{10} - 1} - 11^{10} = \frac{10^9\left(\frac{11^{10}}{10^{10}} - 1\right)}{\frac{1}{10}} - 11^{10}$$

$$= 10^{10}\left(\frac{11^{10}}{10^{10}} - 1\right) - 11^{10} = 10^{10}\left(\frac{11^{10} - 10^{10}}{10^{10}}\right) - 11^{10}$$

$$= 11^{10} - 10^{10} - 11^{10} = -10^{10}$$

$$-\frac{S}{10} = -10^{10} \Rightarrow S = 10^{11} = K\,10^9$$

$$\boxed{K = 100}$$

Jan-06

$$S = (10)^9 + 2(11)(10)^8 + 3(11)^2(10)^7 + \dots + 10(11)^9$$

$\rightarrow CR = \frac{11}{10}$

$$\frac{11S}{10} = 0 + 11(10)^8 + 2(11)^2(10)^7 + \dots 9(11)^9 + 10(11)^9 \frac{11}{10}$$

$$S\left(1 - \frac{11}{10}\right) = 10^9 + 11(10)^8 + (11)^2(10)^7 + \dots + (11)^9 - 10(11)^9 \times \frac{11}{10}$$

$$S\left(\frac{-1}{10}\right) = 10^9 \left[\frac{(11/10)^{10} - 1}{11/10 - 1}\right] - \left[10(11)^9 \times \frac{11}{10}\right]$$

$$S\left(\frac{-1}{10}\right) = \frac{10^9}{10^9}\left(\frac{11^{10} - 10^{10}}{1}\right) - (11)^{10}$$

$$S(-1/10) = 11^{10} - 10^{10} - (11)^{10}$$

$$S\left(\frac{1}{10}\right) = 10^{10}$$

$$S = K(10)^9$$

$$S = 10^{11}$$

$$S = 10^2 (10)^9$$

$$K = 10^2 \Rightarrow 100$$

$$\boxed{K = 100} \quad (C)$$

Richathakur

Q 6. **Sol^n:** $S = (10)^9 + 2(11)(10)^8 + 3(11)^2(10)^7 + \dots + 10(11)^9$

$$\Rightarrow \frac{11}{10}S = \quad + 1(11)\times(10)^8 + 2(11)^2(10)^7 + \dots + 9(11)^9 + (11)^{10}.$$

$$\Rightarrow -\frac{S}{10} = (10)^9 + (11)\times(10)^8 + (11)^2(10)^7 + \dots + (11)^9(10)^0 - (11)^{10}$$

$$\Rightarrow -\frac{S}{10} = \frac{(10)^9\left(\left(\frac{11}{10}\right)^{10} - 1\right)}{\left(\frac{11}{10} - 1\right)} - (11)^{10}.$$

$$\Rightarrow -\frac{S}{10} = \frac{(10)^9\left(\frac{(11)^{10} - (10)^{10}}{(10)^{10}}\right)}{\frac{1}{10}} - (11)^{10}.$$

$$\Rightarrow -\frac{S}{10} = \cancel{(10)^{10}} \times \frac{\cancel{(11)^{10}} - (10)^{10}}{\cancel{(10)^{10}}} - \cancel{(11)^{10}}.$$

$$\Rightarrow \cancel{-}S = \cancel{-}(10)^{11}$$

krish

$$\Rightarrow K(10)^9 = (10)^{11} \Rightarrow \boxed{K = 100} \text{ Ans.}$$

QUESTION

TAH 07

The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots\ldots$ is equal to :

(A) $\frac{9}{4}$

(B) $\frac{13}{4}$

(C) $\frac{15}{4}$

(D) $\frac{11}{4}$

Tah-07

$$S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \cdots \infty$$

$$\frac{S}{3} = 0 + \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} + \cdots \infty$$

$$\frac{2}{3}S \Rightarrow 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \frac{5}{3^4} + \cdots \infty$$

$$\frac{2}{3}S = \frac{4}{3} + 5\left(\frac{1/3^2}{1 - 1/3}\right)$$

$$\frac{2}{3}S = \frac{4}{3} + 5\left(\frac{1}{9_3} \times \frac{3}{2}\right)$$

$$\frac{2}{3}S = \frac{4}{3} + \frac{5}{6} \Rightarrow \frac{8+5}{6} \Rightarrow$$

$$S = \frac{13}{6_2} \times \frac{3}{2}$$

Richathakur

(B) $S = \frac{13}{4}$ Ans

Tah-07 The sum of the infinite series

$1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+\cdots$ is equal to

$\Rightarrow S = 1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\cdots\infty$

$(S-1) = \frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\cdots\infty$

$\frac{S-1}{3} = +\frac{2}{3^2}+\frac{7}{3^3}+\cdots\infty$

$\frac{2(S-1)}{3} = \frac{2}{3}+\frac{5}{3^2}+\frac{5}{3^3}+\cdots\infty$

$\frac{2}{3}(S-1-1) = 5\left(\frac{1}{3^2}+\frac{1}{3^3}+\cdots\infty\right)$

$\frac{2}{3}(S-2) = 5\left(\frac{\frac{1}{3^2}}{1-\frac{1}{3}}\right) = 5\left(\frac{\frac{1}{3^2}}{\frac{2}{3}}\right)$

$\frac{2}{3}(S-2) = \frac{5}{6\,2}$

Kritisha (W.B)

$S-2 = \frac{5}{4}$

$S = 2+\frac{5}{4} = \frac{13}{4}$ (B) (Ans)

Q7. Sol^n: $S = 1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+\cdots\infty.$

$\frac{1}{3}S = +\frac{1}{3}+\frac{2}{3^2}+\frac{7}{3^3}+\frac{12}{3^4}+\frac{17}{3^5}+\cdots\infty.$

$\Rightarrow \frac{2S}{3} = 1+\frac{1}{3}+\frac{5}{3^2}+\frac{5}{3^3}+\frac{5}{3^4}+\frac{5}{3^5}+\cdots\infty.$

$\Rightarrow \frac{2S}{3} = \frac{4}{3}+5\left(\frac{1/3^2}{1-1/3}\right)$

krish

$\Rightarrow \frac{2S}{3} = \frac{4}{3}+\frac{5}{6} \Rightarrow \frac{2S}{3} = \frac{13}{6\,2} \Rightarrow \boxed{S = \frac{13}{4}}$ Ans.

QUESTION [JEE Mains 2023 (11 April)]

TAH 08

For $k \in \mathbb{N}$, if the sum of the series $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots\ldots$ is 10, then the value of k is

Ans. 2

Tah-08

$10 \Rightarrow 1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \dots \infty$

$9 = \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \dots \infty$

$\frac{9}{k} = \frac{4}{k^2} + \frac{8}{k^3} + \frac{13}{k^4} + \frac{19}{k^5} + \dots \infty$

$9\left(1-\frac{1}{k}\right) = \frac{4}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} + \dots \infty$

$9\left(1-\frac{1}{k}\right)\frac{1}{k} \Rightarrow \frac{4}{k^2} + \frac{4}{k^3} + \frac{5}{k^4} + \frac{6}{k^5} + \dots \infty$

$9\left(1-\frac{1}{k}\right)\left(1-\frac{1}{k}\right) \Rightarrow \frac{4}{k} + \frac{1}{k^3} + \frac{1}{k^4} + \frac{1}{k^5} + \dots \infty$

$9\left(1-\frac{1}{k}\right)\left(1-\frac{1}{k}\right) \Rightarrow \frac{4}{k} + \left(\frac{1/k^3}{1-1/k}\right)$

$9\left(1+\frac{1}{k^2}-\frac{2}{k}\right) = \frac{4}{k} + \left(\frac{1}{k^2(k-1)}\right)$

$9 + \frac{9}{k^2} - \frac{18}{k} = \frac{4}{k} + \frac{1}{(k^3-k^2)}$

Richathakur

$\frac{9k^2+9-18k}{k^2} = \frac{1}{k}\left(4 + \frac{1}{k^2-k}\right)$

$\frac{9k^2+9-18k}{k} = \frac{4k^2-4k+1}{k(k-1)}$

$9k^2+9-18k(k-1) = 4k^2-4k+1$

$9k^3-9k^2+9k-9-18k^2+18k = 4k^2-4k+1$

$9k^3-31k^2+31k-10=0$

$9k^2(k-2)-13k(k-2)+5(k-2)=0$

$(9k^2-13k+5)(k-2)=0$

$D<0$ no real roots

$k=2$

Tah-08

$10 = 1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \dots$

$9 = \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \dots$

$\frac{9}{k} = \frac{4}{k^2} + \frac{8}{k^3} + \dots$

$9 - \frac{9}{k} = \frac{4}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} \dots$

$9\left(1-\frac{1}{k}\right)\left(\frac{1}{k}\right) = \frac{4}{k^2} + \frac{4}{k^3} + \frac{5}{k^4} + \dots$

$9\left(1-\frac{1}{k}\right)\left(1-\frac{1}{k}\right) = \frac{4}{k} + \frac{1}{k^3} + \frac{1}{k^4} + \frac{1}{k^5} + \dots \infty$

$= \frac{4}{k} + \frac{1/k^3}{1-\frac{1}{k}} = \frac{4}{k} + \frac{1}{k^2(k-1)}$

$9\left(1-\frac{2}{k}+\frac{1}{k^2}\right) = \frac{4}{k} - \frac{1}{k^2(k-1)}$

Kritisha (W.B)

$9 - \frac{22}{k} + \frac{9}{k^2} = \frac{1}{k^2(k-1)}$

$9k^2-22k+9 = \frac{1}{(k-1)}$

$9k^3-9k^2-22k+22k+9k-9-1=0$

$9k^3-31k^2+31k-10=0$

putting $(k=2)$ expression becomes '0'

$9k^2(k-2)-13k(k-2)+5(k-2)=0 \Rightarrow (9k^2-13k+5)(k-2)=0$

$D<0$ no real root

$k=2$ Ans.

8. Sol^n,

$$\boxed{S = 10} = 1 + \frac{4}{K} + \frac{8}{K^2} + \frac{13}{K^3} + \frac{19}{K^4} + \dots \infty$$

$$\Rightarrow 9 = \frac{4}{K} + \frac{8}{K^2} + \frac{13}{K^3} + \frac{19}{K^4} + \dots \infty$$

$$\Rightarrow \frac{9}{K} = \frac{4}{K^2} + \frac{8}{K^3} + \frac{13}{K^4} + \dots \infty$$

$$\Rightarrow 9\left(1 - \frac{1}{K}\right) = \frac{4}{K} + \frac{4}{K^2} + \frac{5}{K^3} + \frac{6}{K^4} \dots \infty$$

$$\Rightarrow 9\left(1 - \frac{1}{K}\right)\frac{1}{K} = \frac{4}{K^2} + \frac{4}{K^3} + \frac{5}{K^4} \dots \infty$$

$$\Rightarrow 9\left(1 - \frac{1}{K}\right)^2 = \frac{4}{K} + \frac{1}{K^3} + \frac{1}{K^4} + \dots \infty$$

$$\Rightarrow 9\left(1 - \frac{2}{K} + \frac{1}{K^2}\right) = \frac{4}{K} + \left(\frac{1/K^3}{1 - 1/K}\right)$$

$$\Rightarrow 9 - \frac{18}{K} + \frac{9}{K^2} = \frac{4}{K} + \left(\frac{1}{K^3 - K^2}\right)$$

$$\Rightarrow \frac{9K^2 - 18K + 9}{K^2} = \frac{1}{K} + \left(4 + \frac{1}{K^2 - K}\right)$$

$$\Rightarrow \frac{9K^2 + 9 - 18K}{K} = \frac{1}{K}\left(\frac{4K^2 - 4K + 1}{K - 1}\right)$$

krish

$$\Rightarrow (9K^2 + 9 - 18K)(K - 1) = 4K^2 - 4K + 1$$

$$\Rightarrow 9K^3 - 9K^2 + 9K - 9 - 18K^2 + 18 - 4K^2 + 4K - 1 = 0$$

$$\Rightarrow 9K^3 - 31K^2 + 31K - 10 = 0 \longrightarrow$$ ($K = 2$ is a factor.)

$$\Rightarrow 9K^2(K-2) - 13K(K-2) + 5(K-2) = 0$$

$$\Rightarrow (9K^2 - 13K + 5)(K - 2) = 0$$

$D < 0$, $a > 0$ always +ve.

$$\Rightarrow \boxed{K = 2}$$ Ans.

QUESTION [JEE Mains 2022]

TAH 09

Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \cdots$. Then 4S is equal to

(A) $\left(\frac{7}{3}\right)^2$

(B) $\frac{7^3}{3^2}$

(C) $\left(\frac{7}{3}\right)^3$

(D) $\frac{7^2}{3^3}$

Tah-09

$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \cdots$

$\frac{S}{7} = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + \cdots$

$\frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \frac{10}{7^4} + \cdots \infty$

$\frac{6S}{7^2} = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \frac{8}{7^4} + \cdots$

$\frac{6S}{7}\left(1 - \frac{1}{7}\right) = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \cdots$

$\frac{36S}{49} = 2 + \left(\frac{2/7}{1 - 1/7}\right)$

$\frac{36S}{49} = 2 + \frac{2}{6} \frac{1}{3} \Rightarrow \frac{7}{3}$

$36S = \frac{49}{3} \times 7$

Richathakur

$S = \frac{49}{3} \times \frac{7}{36}$

$4S = \frac{49}{36_9} \times \frac{7}{3} \times 4$

$4S = \left(\frac{7}{3}\right)^3$ (c)

Tah-09 Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} \cdots$ then 4S is equal to

$\Rightarrow S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} \cdots$

$\frac{S}{7} = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \cdots$

$\frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \cdots$

$\frac{6S}{(7)^2} = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \cdots$

$\frac{6S}{7} \times \frac{6}{7} = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \cdots$

$\frac{(6)^2 S}{(7)^2} = 2\left(1 + \frac{1}{7} + \frac{1}{7^2} + \cdots \infty\right)$

$= 2\left(\frac{1}{1 - \frac{1}{7}}\right) = 2\left(\frac{1}{6/7}\right)$

Kritisha (W.B)

$\frac{(6)^2 S}{(7)^2} = \frac{7}{6_3} \times 2$

$S = \frac{(7)^3}{3 \times 3^2 \times 4}$

hence, $4S = \frac{7^3}{3^3} = \left(\frac{7}{3}\right)^3$ (c)

Ans.

Q 9. Solⁿ: $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \cdots \infty$

$\frac{1}{7}S = \quad + \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + \cdots \infty$

$\Rightarrow \frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \frac{10}{7^4} + \cdots \infty$

$\Rightarrow \frac{6S}{7^2} = \quad + \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \frac{8}{7^4} + \cdots \infty$

$\Rightarrow \frac{6S}{7}\left(1 - \frac{1}{7}\right) = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \frac{2}{7^4} + \cdots \infty$

$\Rightarrow \frac{36}{49}S = 2 + 2\left(\frac{1/7}{1 - 1/7}\right)$

$\Rightarrow \frac{36}{49}S = 2 + 2 \times \frac{1}{6_3}$

$\Rightarrow S = \frac{7}{3} \times \frac{49}{36} = \frac{7^3}{3^3 \times 4}$

$\Rightarrow \boxed{4S = \left(\frac{7}{3}\right)^3}$ Ans.

krish

QUESTION [JEE Mains 2023 (11 April)]

TAH 10

Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \cdots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is equal to

Ans. 2175

Tah-10 Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \cdots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$

Then value of $\left(16S - (25)^{-54}\right)$ is equal to

$$\Rightarrow \quad S = 109 + \frac{108}{5} + \frac{107}{5^2} + \cdots + \frac{1}{5^{108}}$$

$$\frac{S}{5} = \frac{109}{5} + \frac{108}{5^2} + \cdots + \frac{2}{5^{108}} + \frac{1}{5^{109}}$$

$$-\frac{4S}{5} = -109 + \left(\frac{1}{5} + \frac{1}{5^2} + \cdots + \frac{1}{5^{109}}\right)$$

$$-\frac{4S}{5} = -109 + \frac{\frac{1}{5}\left(\left(\frac{1}{5}\right)^{109} - 1\right)}{\left(\frac{1}{5} - 1\right)} = -109 + \frac{\frac{1}{5}\left(1 - \left(\frac{1}{5}\right)^{109}\right)}{1 - \frac{1}{5}}$$

$$-\frac{4S}{5} = -109 + \frac{1}{5}\left(\frac{5^{109} - 1}{\frac{4}{5}(5^{109})}\right) = \left(-109 + \frac{5^{109} - 1}{4(5^{109})}\right)$$

$$-4S = -545 + \frac{5^{109} - 1}{4(5^{108})}$$

$$-16S = (-545 \times 4) + \frac{5^{109} - 1}{5^{108}}$$

$$-16S = -2180 + 5 - 5^{-108}$$

$$16S = +2175 + 5^{-108}$$

$$= 2175 + (5^2)^{-54}$$

$$16S - (25)^{-54} = 2175 \quad \text{Ans}$$

Kritisha (W.B)

Q 10. Solⁿ: $S = \frac{109}{5^0} + \frac{108}{5} + \frac{107}{5^2} + \cdots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$.

$\frac{1}{5}S = \quad + \frac{109}{5} + \frac{108}{5^2} + \cdots + \frac{3}{5^{107}} + \frac{2}{5^{108}} + \frac{1}{5^{109}}$

$\Rightarrow \frac{4S}{5} = 109 - \frac{1}{5} - \frac{1}{5^2} - \frac{1}{5^3} - \cdots - \frac{1}{5^{107}} - \frac{1}{5^{108}} - \frac{1}{5^{109}}$.

$\Rightarrow \frac{4S}{5} = 109 + (-1)\left(\frac{1/5\left((1/5)^{109} - 1\right)}{1/5 - 1}\right)$.

$\Rightarrow \frac{4S}{5} = 109 + 1/\cancel{5} \times \cancel{5}/4 \left[\left(\frac{1}{5}\right)^{109} - 1\right]$.

$\Rightarrow \frac{4S}{5} = 109 + \frac{1}{4}\left[(5)^{(-109)} - 1\right]$.

$\Rightarrow 4S = 545 + \frac{5}{4}\left[(5)^{(-109)} - 1\right]$.

$\Rightarrow$ (multiply by 4)

$\Rightarrow 16S = 2180 + (5)^{(-108)} - 5$.

$\Rightarrow 16S - (25)^{(-54)} = 2180 - 5$

$= \boxed{2175}$ Ans.

krish

QUESTION [JEE Mains 2024 (1 Feb)]

TAH 11

If three successive terms of a G.P. with common ratio $r (r > 1)$ are the lengths of the sides of a triangle and $[r]$ denotes the greatest integer less than or equal to r, then $3[r] + [-r]$ is equal to ________

Ans. 1

Tah 11

$a, ar, ar^2 \to$ be G.P

$\Rightarrow a + ar > ar^2$ [Acc to Δ inequality]

$\Rightarrow a + ar^2 > ar$

$\Rightarrow ar + ar^2 > a$

[$\because a > 0$]

$\Rightarrow 1 + r > r^2$

$\Rightarrow r + r^2 > 1$

$\Rightarrow r^2 - r - 1 < 0$

$\Rightarrow r = \frac{1 \pm \sqrt{5}}{2}$

as $r > 1$,

$r = \frac{1+\sqrt{5}}{2} \approx 1.6\ldots$

$r^2 - r - 1 < 0$

$\frac{1-\sqrt{5}}{2} < r < \frac{1+\sqrt{5}}{2}$

[$\because r > 1$]

$1 < r < \frac{1+\sqrt{5}}{2}$

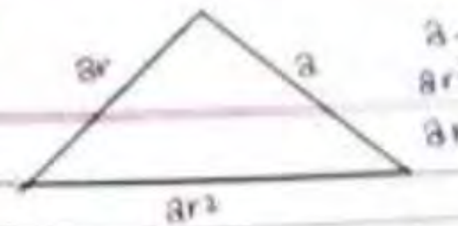
$[r] = 1$

$[-r] = -2$

$\therefore 3[r] + [-r] = 3 - 2 = \boxed{1}$ Ans

TAH 11
by Katha
from WB

Tah-11



$r > 1$

$a + ar > ar^2$

$ar^2 + a > ar$

$ar + ar^2 > a$

(i) $ar + ar^2 > a$

$ar^2 + ar - a > 0$

$ax^2 + ax - a > 0 \to$ also written as $r^2 + r - 1 > 0$

$r \Rightarrow \frac{-1 \pm \sqrt{1^2 - 4(-1)1}}{2}$

Richathakur

$r = \frac{-1 \pm \sqrt{1+4}}{2} \Rightarrow \frac{-1 \pm \sqrt{5}}{2}$

(ii) $ar^2 + a > ar$

$ar^2 - ar + a > 0$

$r^2 - r + 1 > 0$

$r = \frac{+1 \pm \sqrt{-3}}{2}$

no real roots

(iii) $a - ar^2 + ar > 0$

$ar^2 - ar - a < 0$

$r^2 - r - 1 < 0$

$r = \frac{1 \pm \sqrt{5}}{2}$

$+ \quad \frac{-1-\sqrt{5}}{2} \quad + \quad \frac{1-\sqrt{5}}{2} \quad - \quad \frac{1+\sqrt{5}}{2} \quad +$

$r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$

$r \in (0.6, 1.6)$

$[r] = [0.6] = 0$

$[r] = [1.6] = 1$

$[-r] = [-1.6] = -2$

$3[r] + [-r]$

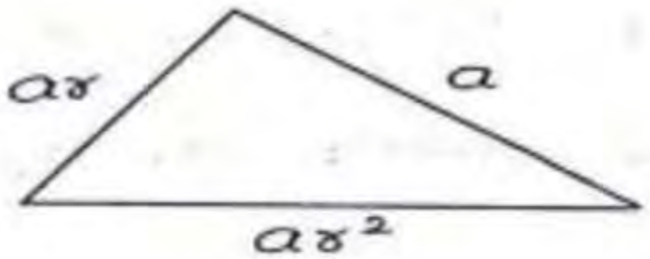
$3(1) + (-2)$

$3 - 2 \Rightarrow 1$

$3[r] + [-r] = 1$

Q 11. Solⁿ: a, ar, ar^2.

figure:



$a + ar > ar^2$.

$\Rightarrow$ $\boxed{1 + r > r^2}$

$\Rightarrow r^2 - r - 1 > 0$

$\Rightarrow r = \frac{1 \pm \sqrt{5}}{2}$

$ar + ar^2 > a$.

$\Rightarrow \boxed{r + r^2 > 1}$

$\Rightarrow r^2 + r - 1 > 0$

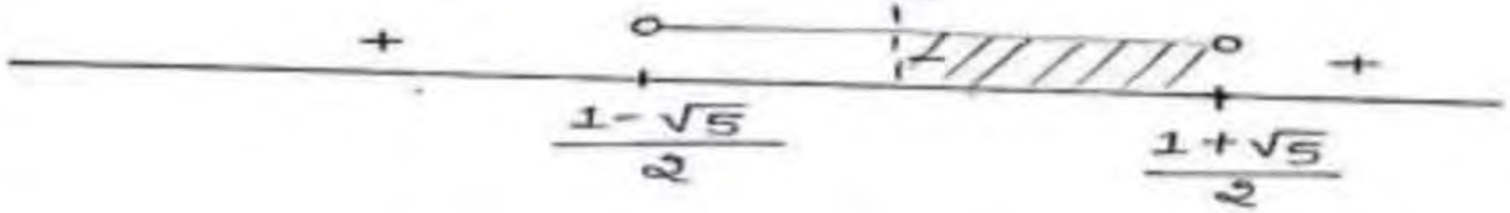
$\Rightarrow r = \frac{-1 \pm \sqrt{5}}{2}$.

$a + ar^2 > ar$.

$\Rightarrow \boxed{1 + r^2 > r}$

$\Rightarrow r^2 - r + 1 > 0$.

$\Rightarrow D < 0,\ a > 0$ always +ve.



krish

$\Rightarrow r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$.

$\Rightarrow r \in (0.6, 1.6)$. # $r > 1$ (given)

find: $3[r] + [-r]$

$\Rightarrow 3(1) + (-2)$

$\Rightarrow 3 - 2 = ⓵$ Ans.

$[r] = [0.6] = 0$ ✗

$= [1.6] = 1$ ✓

$[-r] = [-1.6] = -2$ ✓

QUESTION [JEE Mains 2021]

TAH 12

Let A_1, A_2, A_3, ... be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is

Ans. 9

Tan 12

Let S_n be side length of A_n

$S_1 = 12$

diagonal $A_2 = S_1$

$A_2 = S_2\sqrt{2}$

$S_2\sqrt{2} = S_1$

$S_2 = \frac{S_1}{\sqrt{2}} = \frac{12}{\sqrt{2}} = 6\sqrt{2}$

$S_n = \frac{S_{n-1}}{\sqrt{2}}$

$S_n = S_1\left(\frac{1}{\sqrt{2}}\right)^{n-1} = 12\left(\frac{1}{\sqrt{2}}\right)^{n-1}$

Area of $A_n \rightarrow a_n = S_n^2$

$a_n = \left[12\left(\frac{1}{\sqrt{2}}\right)^{n-1}\right]^2 = 144\left(\frac{1}{2}\right)^{n-1}$

$\therefore$ smallest n, $a_n < 1$

$144\left(\frac{1}{2}\right)^{n-1} < 1$

$\left(\frac{1}{2}\right)^{n-1} < \frac{1}{144}$

$2^{n-1} > 144$

$n-1 \geq 8$ $\{2^7 = 128$ or $2^8 = 256\}$

$n \geq 9$

$\therefore$ smallest value of n is less than 1 is $\boxed{9}$ Ans

TAH 12
by Katha
from WB

Tah-12

length of side $A_n \Rightarrow l_n$

length of side $A_{n+1} \Rightarrow l_{n+1}$

$l_n = (l_{n+1})\sqrt{2}$

$\frac{l_{n+1}}{l_n} = \frac{1}{\sqrt{2}}$ = common ratio

$(l_n)^2 < 1$

$2(l_{n+1})^2 < 1$

$\left(l_1\left(\frac{1}{\sqrt{2}}\right)^n\right)^2 < \frac{1}{2} \Rightarrow l_1^2\left(\frac{1}{2}\right)^n < \frac{1}{2}$ $[l_1 = 12\text{ cm}]$

$\left(\frac{1}{2}\right)^n < \frac{1}{288}$

Kritisha (W.B)

$288 < 2^n$

~~$2^8 \Rightarrow 288$~~

$[2^8 = 256$

$2^9 = 512]$

hence, $n = 9$ smallest value of 'n' for which Area is less than one.

Mann KI BAAT

Basic Math

Class notes pdfs se Phadho

- Jaha samajh na aayay watch corresponding video for clarity
- Class Notes Questions Rough copy mai dekh dekhkor solve karo.

* class illustration
* KTK / TAH / SHIKAAR
* RPP

without selection impossible

* DPP
* Module.

99.97+ percentile in Jee mains. + Good JEE Adv RANK.

THANK YOU