PRAYAS

JEE 2026

Mathematics

Sequence and Series

Lecture - 05

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Topics *to be covered*

- **A** Properties of G.P
- **B** Question Practice

Recap *of previous lecture*

1. For the sequence a, ar, ar^2, ar^3, ... $ar^{n-1} = \ell$ $\xrightarrow{r}$ $\xleftarrow{1/r}$

   T_n = ar^{n-1}, S_n = $\frac{a(r^n-1)}{r-1}$ or $\frac{a(1-r^n)}{1-r}$ $r \neq 1$, If $r=1$, $S = na$.

   (From end) T_k = $\ell \cdot (1/r)^{k-1}$ = $(n-k+1)^{th}$ term from beginning

2. For G.P. $a_1, a_2, a_3, a_4, a_5, ...$

   $a_2 \cdot a_7 = a_3 \cdot$ a_6 $= a_5 \cdot$ a_4

   $a_9 \cdot a_7 = a_8 \cdot$ a_8 $= a_6 \cdot$ a_{10}

$m+n = p+q$, $m, n, p, q \in N$

$T_m \cdot T_n = T_p \cdot T_q$

Recap of previous lecture

3. In a G.P., $T_{r+k} \cdot T_{r-k} =$ $\underline{\quad T_r^2 \quad}$

4. If we multiply each term of G.P. by 'k' then it is still a $\underline{\quad G.P \quad}$ with common ratio $\underline{\quad}$ same as initial G.P

5. If we divide each term of a G.P. by k, $k \neq 0$ then it is still a $\underline{\quad G.P \quad}$ with common ratio $\underline{\quad}$ same as intial G.P

Recap *of previous lecture*

6. If we multiply corresponding terms of two G.P.s we get a G.P with common ratio = $r_1 \cdot r_2$

7. If divide corresponding terms of two G.P.s we get a G.P with common ratio = r_1/r_2

8. Product of equidistant terms from beginning & ends is Constant & is equal to product of first & last term.

Recap *of previous lecture*

9. $S_\infty = a + ar + ar^2 + \ldots\ldots + \infty =$ $\frac{a}{1-r}$ provided $r \in$ $(-1, 1)$

10. If $r > 1$ then $a + ar + ar^2 + \ldots + \infty$, $a \neq 0$ is Divergent

11. $S_n = a + ar + ar^2 + \ldots\ldots + ar^{n-1}$

$= \frac{a(r^n - 1)}{r-1}$ if $r \neq 1$

$= na$ if $r = 1$

Recap *of previous lecture*

12. If each term of a G.P. is raised to same power k then resulting sequence is G.P with common ratio r^k

13. If in a G.P. the 3rd term is 4 then the value of product of first five terms is ________

$T_3 = ar^2 = 4$

$P = a_1 \cdot a_2 a_3 a_4 a_5$

$= a \cdot ar \cdot ar^2 \cdot ar^3 \, ar^4$

$= a^5 \cdot r^{10}$

$= (ar^2)^5 = 4^5 = 1024$

Recap of previous lecture

State True or False

1. No term of a G.P. can be 0 (T)

sahi

$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3.$

2. Four numbers in G.P. can be taken as $\frac{a}{r^3}, \frac{a}{r}, a, ar^3$ (F)

3. If $r \geq 1$ then the series $a + ar + ar^2 + ar^3 + \ldots\ldots\ \infty$, $(a \neq 0)$ is divergent. (T)

4. If $r \leq 1$ then the series $a + ar + ar^2 + ar^3 + \ldots\ldots\ \infty$, $(a \neq 0)$ is convergent. (F)

Oscillatory

HOMEWORK Discussion.

QUESTION [JEE Mains 2023 (29 Jan)]

KTK 03

Let $\{a_k\}$ and $\{b_k\}$, $k \in N$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k$, $k \in N$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to

G.P$_1$ $4, 4r_1, 4r_1^2, 4r_1^3 - - -$ $(r_1 < r_2)$

G.P$_2$ $4, 4r_2, 4r_2^2, 4r_2^3 - - -$

M③

By observation

$4(r_1+r_2) = 5$

$4r_1 + 4r_2 = 5$

$(4r_1)^2 + (4r_2)^2 = 13$

$4r_1 = 2,\ 4r_2 = 3$

$r_1 = 1/2 \quad r_2 = 3/4$

$c_3 = a_3 + b_3 = 13/4$

$4r_1^2 + 4r_2^2 = 13/4$

$r_1^2 + r_2^2 = 13/16$.

$c_k = a_k + b_k$

$c_2 = a + b_2 = 5$

$4(r_1+r_2) = 5$.

$r_1 + r_2 = 5/4$

$r_1^2 + r_2^2 + 2r_1r_2 = 25/16$

$\frac{13}{16} + 2r_1r_2 = 25/16$

$2r_1r_2 = \frac{3}{4} \Rightarrow r_1r_2 = 3/8$

Ans. 9

$$r^2 - \frac{5}{4}r + \frac{3}{8} = 0 \left\langle \begin{matrix} r_1 \\ r_2 \end{matrix} \right.$$

$$8r^2 - 10r + 3 = 0$$

$$8r^2 - 6r - 4r + 3 = 0$$

$$(2r-1)(4r-3) = 0$$

$$r = \frac{1}{2}, \ 3/4$$

$$\downarrow \qquad \downarrow$$

$$r_1 \qquad r_2$$

M②

$$r_1 + r_2 = \frac{5}{4}, \quad r_1 r_2 = \frac{3}{8}$$

$$r_1 + \frac{3}{8r_1} = \frac{5}{4}$$

$$8r_1^2 - 10r_1 + 3 = 0$$

$$r_1 = \frac{1}{2}, \cancel{\frac{3}{4}}$$

$$r_2 = \frac{3}{4}, \cancel{\frac{1}{2}}$$

$$\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4) = \sum_{k=1}^{\infty}(a_k + b_k) - (12 \cdot 4r_1^5 + 8 \cdot 4 \cdot r_2^4)$$

$$= \sum_{k=1}^{\infty} a_k + \sum_{k=1}^{\infty} b_k - (48 \cdot r_1^5 + 32 r_2^4)$$

QUESTION [JEE Mains 2025 (22 Jan)]

S 03

Suppose that the number of terms in an A.P. is 2k, k ∈ N. If the sum of all odd terms of the A.P. is 40, the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27, then k is equal to:

$2k$ < k odd number, k even number

$a_2 + a_4 + a_6 + \dots + a_{2k} = 55$

$a_1 + a_3 + a_5 + \dots + a_{2k-1} = 40$

$(a_2 - a_1) + (a_4 - a_3) + (a_6 - a_5) + \dots + (a_{2k} - a_{2k-1}) = 15.$

(A) 8

(B) 6

(C) 4

(D) 5

$d + d + \dots + d = 15$

$kd = 15$ — (1)

$a_{2k} - a_1 = 27$

$\cancel{a} + (2k-1)d - \cancel{a} = 27$

$2kd - d = 27$

$30 - d = 27$

$d = 3.$

$k = 5.$

Ans. D

QUESTION [JEE Mains 2021 (27 July)]

If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to

$\downarrow$

$x = \cancel{2}, 3$

$\downarrow$

$2^x - 5 = 2^2 - 5 = -1$ $\log(2^x - 5)$ becomes undefined in Reals.

Ans. 3

QUESTION [JEE Mains 2025 (4 April)]

Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of A.P.'s in A and B respectively such that $D = d + 3, d > 0$. If $\frac{p+q}{p-q} = \frac{19}{5}$, then $p - q$ is equal to

(A) 540

(B) 450

(C) 600

(D) 630

$A = \{a-d, a, a+d\}$ — $a-d+a+a+d = 36 \Rightarrow a = 12$

$B = \{A-D, A, A+D\}$ — $A-d+A+A+d = 36 \Rightarrow A = 12.$

$p = (a-d) \cdot a(a+d) = (12-d) \cdot 12 \cdot (12+d)$

$q = (A-D) \cdot A \cdot (A+D) = (12-D) \cdot 12 \cdot (12+D) = (9-d) \cdot 12 \cdot (15+d)$

$D = d+3.$

$\frac{p+q}{p-q} = \frac{19}{5}$

$5p + 5q = 19p - 19q$

$14p = 24q \Rightarrow 7p = 12q.$

$7 \cdot (12-d) \cdot 12 \cdot (12+d) = 12 \cdot (9-d) \cdot 12 \cdot (15+d)$

Ans. A

QUESTION [JEE Mains 2024 (31 Jan)]

Let 2^{nd}, 8^{th} and 44^{th} terms of a non-constant A.P. be respectively the 1^{st}, 2^{nd} and 3^{rd} terms of a G.P. If the first term of the A.P. is 1, then the sum of its first 20 terms is equal to -

(A) 990

(B) 980

(C) 960

(D) 970

$$\left.\begin{array}{l} T_2 = 1 + d \text{ — 1st term} \\ T_8 = 1 + 7d \text{ — 2nd term} \\ T_{44} = 1 + 43d \text{ — 3rd term} \end{array}\right\} \text{ of G.P}$$

$$(1+7d)^2 = (1+d)(1+43d)$$

$$49d^2 + 14d + \cancel{1} = 43d^2 + 44d + \cancel{1}$$

$$49d + 14 = 43d + 44.$$

$$6d = 30$$

$$d = 5.$$

$$S_{20} = \frac{20}{2}(2 \cdot 1 + (20-1)5) = 10 \cdot (2 + 95) = 970.$$

Ans. D

QUESTION

★★ASRQ★★

If a, b, c, d, e be 5 numbers such that a, b, c are in A.P.; b, c, d are G.P. & $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P. then:

(i) Prove that a, c, e are in G.P.

(ii) Prove that $e = (2b-a)^2/a$

(iii) If a = 2 & e = 18, find all possible values of b, c, d.

a, b, c, d, e

G.P

A.P $\quad \frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ in A P

(ii) $2b = a + c$

$c = 2b - a$

from ① $c^2 = ae$

$(2b-a)^2 = ae$

$e = \frac{(2b-a)^2}{a}$

(iii) $a = 2$

$e = 18$

$c^2 = ae = 36$

$c = \pm 6$

$\underbrace{\text{If } c = 6}$

$2b = a + c$

$b = 4$

$d = \frac{2 \cdot 6 \cdot 18}{6+18}$

$d = 9$

(i) $2b = a + c$

$c^2 = bd$

$\frac{2}{d} = \frac{1}{c} + \frac{1}{e} \Rightarrow \frac{2}{d} = \frac{c+e}{ce}$

$d = \frac{2ce}{c+e}$

$c^2 = \frac{a+c}{2} \cdot \frac{2ce}{c+e}$

$c^3 + c^2e = ace + c^2e$

$c^2 = ae \Rightarrow a, c, e$ are in G.P

$\underbrace{\text{If } c=-6}$

$$2b = a + c$$

$$b = -2$$

$$d = \frac{2 \cdot (-6) \cdot 18}{-6+18}$$

$$d = -18.$$

QUESTION [JEE Mains 2023 (25 Jan)]

Ta h01

For the two positive numbers a, b, if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}$, 10 and $\frac{1}{b}$ are in an arithmetic progression, then $16a + 12b$ is equal to

$a, b \in R^+$ $\quad a, b, \frac{1}{18}$ G.P $\rightsquigarrow b^2 = a \cdot \frac{1}{18}$

$\frac{1}{a}, 10, \frac{1}{b}$ A.P $\sim 20 = \frac{1}{a} + \frac{1}{b}$.

Ans. 3

QUESTION [JEE Mains 2023 (15 April)]

★★ASRQ★★

Let A_1 and A_2 be two arithmetic means and G_1, G_2, G_3 be three geometric means of two distinct positive numbers. Then $G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2$ is equal to :

(A) ~~$(A_1 + A_2)^2 G_1 G_3$~~

(B) $(A_1 + A_2) G_1^2 G_3^2$

(C) $2(A_1 + A_2) G_1^2 G_3^2$

(D) $2(A_1 + A_2) G_1 G_3$

let the distinct +ve no:s be a & b

$a, A_1, A_2, b \sim A.P \longrightarrow A_1 + A_2 = a + b$

$a, G_1, G_2, G_3, b \sim G.P$

Mentos Zindagi

$a, G_1, G_2, G_3, b.$

$G_1^2 = aG_2 \quad G_3^2 = G_2 b. \quad , G_2^2 = G_1 G_3 = ab.$

$G_1^2 G_3^2 = ab G_2^2$

$G_1 G_3 (A_1 + A_2)^2 = G_1 G_3 (a+b)^2$

$G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2$

$= a^2 G_2^2 + G_2^4 + G_2^2 b^2 + ab G_2^2 = G_2^2 (a^2 + b^2 + ab + G_2^2) = G_1 G_3 (a^2 + b^2 + ab + ab)$

QUESTION

★★ASRQ★★

Jan 02

If m is the A.M. of two distinct real numbers l and n (l, n > 1) and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals

(A) $4\,l\,m\,n^2$

l, A_1, A_2, m — A.P

l, G_1, G_2, G_3, m — G.P

(B) $4\,l^2\,m^2\,n^2$

(C) $4\,l^2\,m\,n$

(D) $4\,l\,m^2\,n$

Ans. D

QUESTION [JEE Mains 2021 (26 Aug)]

Ta h03

Let $a_1, a_2, \ldots\ldots, a_{10}$ be an A.P. with common difference -3 and $b_1, b_2, \ldots\ldots b_{10}$ be a G.P. with common ratio 2. Let $c_k = a_k + b_k$, $k = 1,2, \ldots\ldots, 10$. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to

Ans. 2021

QUESTION

Let $a, b, c > 1$, a^3, b^3 and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444, then abc is equal to :

a^3, b^3, c^3 A.P $\Rightarrow 2b^3 = a^3 + c^3$ — (II)

$\log_a b, \log_c a, \log_b c$ are in G.P

$(\log_c a)^2 = \log_a b \cdot \log_b c = \log_a c$

$(\log_c a)^2 = \frac{1}{\log_c a}$

$(\log_c a)^3 = 1$

$\log_c a = 1 \Rightarrow a = c$ — (I)

from (I) & (II)

$2b^3 = 2a^3$

$b = a$

$a = b = c$

(A) 343

(B) 216

(C) $\frac{343}{8}$

(D) $\frac{125}{8}$

Ans. B

for an A.P

$$T_1 = \frac{a+4b+c}{3} = 2a$$

$$\text{common diff} = \frac{a-8b+c}{10} = \frac{-6a}{10} = -\frac{3}{5}a$$

$$S_{20} = -444$$

$$\frac{20}{2}\left(4a + 19\cdot\left(-\frac{3}{5}\right)a\right) = -444$$

$$5\left(4a - \frac{57}{5}a\right) = -222$$

$$5\cdot\frac{-37a}{5} = -222$$

$$a = 6.$$

Problems Involving AGP

QUESTION

If $|x| < 1$, then compute S_∞:

(a) $1 + 2x + 3x^2 + 4x^3 + ...$

(b) $1 + 3x + 6x^2 + 10x^3 + ...$

(c) $1^2 + 2^2x + 3^2x^2 + 4^2x^2 + ...$

AP

(1) $S = 1 + 2x + 3x^2 + 4x^3 + --- \infty$

G.P C.R = x

$xS = \quad x + 2x^2 + 3x^3 + --- \infty$

$(1-x)S = 1 + x + x^2 + x^3 + --- \infty$

$(1-x)S = \frac{1}{(1-x)} \Rightarrow S = \frac{1}{(1-x)^2}$

diff = 2 , 3 , 4

(2) $S = 1 + 3x + 6x^2 + 10x^3 + ---- \infty$

G.P

$xS = \quad x + 3x^2 + 6x^3 + --- \infty$

$(1-x)S = 1 + 2x + 3x^2 + 4x^3 + --- \infty \longrightarrow$ A.G.P

$x(1-x)S = \quad x + 2x^2 + 3x^3 + --- \infty$

$(1-x)S - x(1-x)S = 1 + x + x^2 + x^3 + -- \infty = \frac{1}{1-x}$

$$(1-x)S(1-x) = \frac{1}{1-x}$$

$$S = \frac{1}{(1-x)^3}.$$

(3) $S = 1^2 + 2^2x + 3^2x^2 + 4^2x^3 + 5^2x^4 + \cdots \infty$ → Jan 04 @

QUESTION [JEE Mains 2024 (27 Jan)]

Jan 5 (5)

If $8 = 3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) + \ldots\ldots \infty$, then the value of p is

$$S = 3 + (3+p)\cdot\frac{1}{4} + (3+2p)\cdot\frac{1}{4^2} + \frac{1}{4^3}(3+3p) + --\infty \quad , \quad S = 8$$

QUESTION [JEE Mains 2023]

If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \dots + 20(21)^{19} = k(20)^{19}$, then k is equal to

$$S = 1.(20)^{19} + 2.21.(20)^{18} + 3.(21)^2.(20)^{17} + \dots + 19\,20^1.(21)^{18} + 20.(21)^{19}$$

G.P C.R $= \frac{21}{20}$

$$\frac{21}{20}S = 1.21.(20)^{18} + 2.21^2.20^{17} + \dots + 18.20.21^{18} + 19.21^{19} + 21^{20}$$

$$-\frac{S}{20} = \left(20^{19} + 21.20^{18} + 21^2.20^{17} + \dots + 20.21^{18} + 21^{19}\right) - 21^{20}.$$

G.P

$$= 20^{19}\frac{\left(\left(\frac{21}{20}\right)^{20} - 1\right)}{\frac{21}{20} - 1 = \frac{1}{20}} - 21^{20} = 20^{20}\left(\frac{21^{20}}{20^{20}} - 1\right) - 21^{20}$$

$$= \cancel{21^{20}} - 20^{20} - \cancel{21^{20}} = -20^{20}$$

$S = 20^{21}$

$S = 20^{2} \cdot 20^{19}$

$S = 400 \cdot 20^{19}$

$K = 400$

QUESTION [JEE Mains 2014]

Tan06

If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \cdots + 10(11)^9 = k(10)^9$, then k is equal to

(A) $121/10$

(B) $441/100$

(C) 100

(D) 110

Ans. C

QUESTION

Jan 07

The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots\ldots.$ is equal to :

(A) $\frac{9}{4}$

(B) $\frac{13}{4}$

(C) $\frac{15}{4}$

(D) $\frac{11}{4}$

QUESTION [JEE Mains 2023 (11 April)]

Jan 08

For $k \in \mathbb{N}$, if the sum of the series $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots\ldots$ is 10, then the value of k is

Ans. 2

QUESTION [JEE Mains 2022]

Jan 09

Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \cdots$. Then 4S is equal to

(A) $\left(\frac{7}{3}\right)^2$

(B) $\frac{7^3}{3^2}$

(C) $\left(\frac{7}{3}\right)^3$

(D) $\frac{7^2}{3^3}$

QUESTION

$$S = 1 + \left(1 + \frac{1}{5}\right)\cdot\frac{1}{2} + \left(1 + \frac{1}{5} + \frac{1}{5^2}\right)\cdot\frac{1}{2^2} + \left(1 + \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3}\right)\cdot\frac{1}{2^3} + \cdots \infty$$

$$\frac{S}{2} = \frac{1}{2} + \left(1 + \frac{1}{5}\right)\cdot\frac{1}{2^2} + \left(1 + \frac{1}{5} + \frac{1}{5^2}\right)\cdot\frac{1}{2^3} + \cdots \infty$$

$$\frac{S}{2} = 1 + \frac{1}{5}\cdot\frac{1}{2} + \frac{1}{5^2}\cdot\frac{1}{2^2} + \frac{1}{5^3}\cdot\frac{1}{2^3} + \cdots \infty$$

$$\frac{S}{2} = 1 + \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \cdots \infty$$

$$\frac{S}{2} = 1\cdot\left(\frac{1}{1-\frac{1}{10}}\right) = \frac{10}{9}$$

$$S = 20/9$$

QUESTION [JEE Mains 2023 (11 April)]

Tan10

A.P

Let $S = \frac{109}{5^0} + \frac{108}{5} + \frac{107}{5^2} + \cdots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is equal to

G.P

Ans. 2175

Geometrical Problems

QUESTION [JEE Mains 2024 (1 Feb)]

Tan!!

If three successive terms of a G.P. with common ratio $r(r > 1)$ are the lengths of the sides of a triangle and $[r]$ denotes the greatest integer less than or equal to r, then $3[r] + [-r]$ is equal to ________

a, ar, ar^2

$a + ar > ar^2$

$ar + ar^2 > a$

$a + ar^2 > ar$

$\cap$

$r \in$

Ans. 1

QUESTION [JEE Mains 2024 (8 April)]

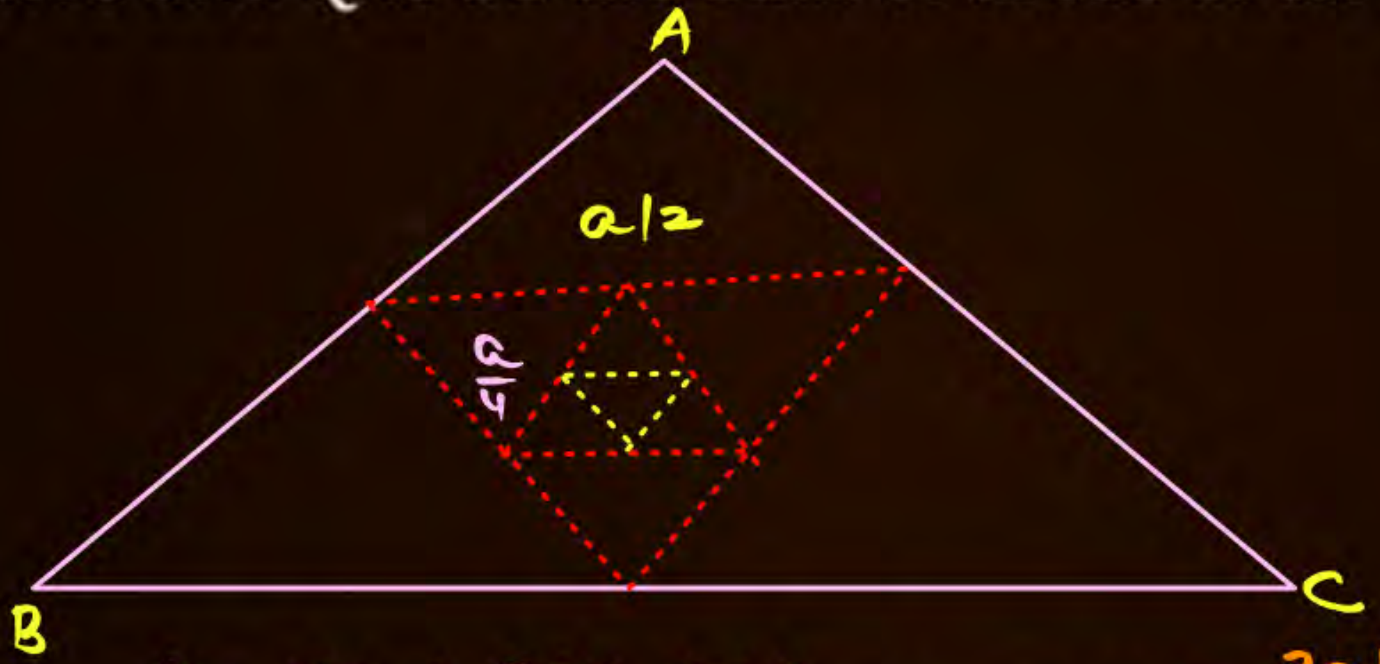
Let ABC be an equilateral triangle. A new triangle is formed by joining the middle points of all sides of the triangle ABC and the same process is repeated infinitely many times. If P is the sum of perimeters and Q is be the sum of areas of all the triangles formed in this process, then:

(A) $P^2 = 72\sqrt{3}Q$

(B) $P^2 = 36\sqrt{3}Q$

(C) $P = 36\sqrt{3}Q^2$

(D) $P^2 = 6\sqrt{3}Q$



Let side of ABC = a.

$P = \frac{3a}{2} + 3a/4 + \frac{3a}{8} + \cdots \infty = \frac{3a/2}{1-1/2} = 3a$

$A = \frac{\sqrt{3}}{4}(a/2)^2 + \sqrt{3}/4\,(a/4)^2 + \frac{\sqrt{3}}{4}(a/8)^2 + \cdots \infty = \frac{\frac{\sqrt{3}}{4}\cdot a^2/4}{1-1/4} = \frac{\sqrt{3}a^2}{12} \Rightarrow a^2 = \frac{12A}{\sqrt{3}}$

$P^2 = 9a^2 = 9\cdot\frac{12A}{\sqrt{3}} = 36\sqrt{3}A.$

Ans. B

QUESTION [JEE Mains 2021]

Tan 12

Let $A_1, A_2, A_3, ...$ be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is

Ans. 9

QUESTION [JEE Mains 2024 (31 Jan)]

$\overbrace{m + m + m + \cdots + m}^{17 \text{ days.}}$

A software company sets up m number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment. The value of m is equal to:

(A) 125

(B) 160

(C) 150

(D) 180

$17m = (m \cdot 1 + (m-4) + m - 8 + \cdots \text{ upto } 25 \text{ terms})$

$17m = \frac{25}{2}(2m + 24 \cdot (-4))$

$34m = 50m - 25 \times 96.$

$16m = 96 \times 25$

$m = 150.$

Ans. C

Some Golden Problems

QUESTION [JEE Mains 2021]

Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1, a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$.

Then the value of $47 \sum_{n=1}^{\infty} \frac{a_n}{8^n}$ is equal to

$$a_{n+2} = 2a_{n+1} + a_n.$$

$$\frac{a_{n+2}}{8^n} = 2 \cdot \frac{a_{n+1}}{8^n} + \frac{a_n}{8^n}$$

$$\sum_{n=1}^{\infty} \frac{a_{n+2}}{8^n} = 2\sum_{n=1}^{\infty} \frac{a_{n+1}}{8^n} + \sum_{n=1}^{\infty} \frac{a_n}{8^n}$$

$$64 \underbrace{\sum_{n=1}^{\infty} \frac{a_{n+2}}{8^{n+2}}}_{S_1} = 16 \underbrace{\sum_{n=1}^{\infty} \frac{a_{n+1}}{8^{n+1}}}_{S_2} + \underbrace{\sum_{n=1}^{\infty} \frac{a_n}{8^n}}_{S.}$$

$$S = \frac{a_1}{8} + \frac{a_2}{8^2} + \frac{a_3}{8^3} + \cdots \infty$$

$$S_1 = \frac{a_3}{8^3} + \frac{a_4}{8^4} + \cdots \infty$$

$$S_1 = S - \frac{a_1}{8} - \frac{a_2}{8^2}$$

Ily $$S_2 = S - \frac{a_1}{8}$$

$$64\left(S-\frac{a_1}{8}-\frac{a_2}{64}\right)=16\left(S-\frac{a_1}{8}\right)+S.$$

$$64S-8a_1-a_2=16S-2a_1+S$$

$$47S.=6a_1+a_2=7.$$

$$S=\frac{7}{47}$$

$$47\sum_{n=1}^{n}\frac{a_n}{n}=47S=7\ \underline{\text{Ans}}$$

QUESTION [JEE Mains 2022]

Tah 13

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ all $n \geq 0$.

Then $\displaystyle\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to:

(A) $\dfrac{6}{343}$

(B) $\dfrac{7}{216}$

(C) $\dfrac{8}{343}$

(D) $\dfrac{49}{216}$

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Solution to Previous TAH

QUESTION [JEE Mains 2025 (3 April)]

TAH

Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive numbers. If $a_3 a_5 = 729$ and $a_2 + a_4 = \frac{111}{4}$, then $24(a_1 + a_2 + a_3)$ is equal to

A 128

B 129

C 131

D 130

Ans. B

TAH 01

Lec-04

(01)

:HW Solutions:

$a_1, a_2, a_3 \ldots$ increasing positive numbers G.P.

$a_3 a_5 = 729$

$(a_1 r^2)(a_1 r^4) = 729$

$a_1^2 r^6 = 729$

$a_1 r^3 = 27$ (for increasing GP (−ve) not be include)

$a_2 + a_4 = \frac{111}{4}$

$a_1 r + a_1 r^3 = \frac{111}{4}$

$a_1(r + r^3) = \frac{111}{4}$

$\div$

$\frac{r + r^3}{r^3} = \frac{\cancel{111}^{37}}{4 \times \cancel{27}_{9}}$

$\frac{1 + r^2}{r^2} = \frac{37}{36}$

$1/r^2 + 1 = 37/36$

$1/r^2 = 1/36$

$r = \pm 6$

$r = 6$ { −ve not possible for positive number G.P.

$a_1 6^3 = 27$

$a_1 = \frac{1}{8}$

$24 \, (a_1 + a_2 + a_3)$

$24 \, (a_1 + a_1 r + a_1 r^2)$

$= 24 \times 1/8 \, (1 + r + r^2)$

$= 3(1 + 6 + 6^2) = 3 \times 43 = 129$

Ans (B)

Tah by Ankush

Tah01:-

$a_3 a_5 = 729$

$ar^2 ar^4 = 729$

$a^2 r^6 = 729$

$(ar^3 = 27)$

$\frac{ar^3}{ar} = \frac{27}{3/4}$

$r^2 = 36$

$r = \pm 6$

$\boxed{r = 6}$ $\boxed{r = -6}$ - reject

$a_2 + a_4 = \frac{111}{4}$

$ar + ar^3 = \frac{111}{4}$

$ar + 27 = \frac{111}{4}$

$\left(ar = \frac{3}{4}\right)$

$\Downarrow$

$\boxed{a = \frac{1}{8}}$

$r = 6, \, a = \frac{1}{8}$

Then: $\Rightarrow 24(a_1 + a_2 + a_3)$

$\Rightarrow 24\left[\frac{1}{8} + \frac{1}{8} \times 6 + \frac{1}{8} \times 36\right]$

$\Rightarrow \frac{24}{8}\left[1 + 6 + 36\right] = 3[43]$

$\Rightarrow 129$

$\boxed{\text{Ans} = 129}$

26 May 2025

TAH-01

$$a_3 \cdot a_5 = 729$$
$$ar^2 \cdot ar^4 = 729$$
$$a^2 \cdot r^6 = 729 \Rightarrow \boxed{ar^3 = 27} = a_4$$

$$a_2 + a_4 = \frac{111}{4}$$
$$ar + ar^3 = \frac{111}{4}$$
$$ar(1+r^2) = \frac{111}{4}$$
$$ar + 27 = \frac{111}{4}$$
$$ar = 3/4$$
$$\boxed{a_2 = 3/4}$$

$$\frac{a_4}{a_2} = \frac{ar^3}{ar}$$
$$\frac{a_4}{a_2} = r^2$$
$$a_4 = a_2 \times r^2$$
$$a_4 = \frac{3}{4} \times r^2$$
$$27 = \frac{3}{4} \times r^2$$
$$r^2 = \frac{\cancel{27}^{9} \times 4}{\cancel{3}}$$
$$\boxed{r = 6}$$

$$a \times r = \frac{3}{4}$$
$$a = \frac{3}{4r} = \frac{\cancel{3}}{4 \times \cancel{6}_2}$$
$$\boxed{a = 1/8}$$

$$24(a_1 + a_2 + a_3) = 24(a + ar + ar^2)$$
$$= 24a(1 + r + r^2)$$
$$= \cancel{24}^{3} \times \frac{1}{\cancel{8}}(1 + 6 + 36)$$
$$= 3(43)$$
$$= 129 \quad (B)$$

QUESTION [JEE Mains 2023 (29 Jan)]

Let a_1, a_2, a_3, ... be a G.P. of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then

$a_1 a_9 + a_2 a_4 a_9 + a_5 + a_7$ is equal to

Ans. 60

Tahoe :-

$a_4 a_6 = 9$

$ar^3 ar^5 = 9$

$a^2 r^8 = 9$

$[ar^4 = 3]$

$a_5 + a_7 = 24$

$ar^4 + ar^6 = 24$

$ar^6 = 21 \Rightarrow a = \frac{21}{343} = \frac{3}{49}$

$\frac{ar^6}{ar^4} = \frac{21}{3} \Rightarrow \boxed{r^2 = 7}$

$$a = \frac{3}{49}, \quad r^2 = 7$$

Then: $\Rightarrow a_1 a_9 + a_2 a_4 a_9 + a_5 + a_7$

$\Rightarrow a \cdot ar^8 + ar \cdot ar^3 \cdot ar^8 + ar^4 + ar^6$

$\Rightarrow a^2 r^8 + a^3 r^{12} + a^2 r^{10} + ar^6$

$\Rightarrow a^2 (r^2)^4 + a^3 (r^2)^6 + \cancel{a^4 (r^2)^5} + a(r^2)^2 + a(r^2)^3$

$\Rightarrow \left(\frac{3}{49}\right)^2 (7)^4 + \left(\frac{3}{49}\right)^3 (7)^6 + \left(\frac{3}{49}\right)^2 (7)^5 + \frac{3}{49} \times 7^2$

$\Rightarrow 9 + 27 + 3 + 81$

$\Rightarrow 60$

$\boxed{\text{Ans} = 60}$

TAH-02

$a_4 \cdot a_6 = 9$

$ar^3 \cdot ar^5 = 9$

$a^2 r^8 = 9$

$\boxed{ar^4 = 3}$

$a_5 + a_7 = 24$

$ar^4 + ar^6 = 24$

$ar^4(1+r^2) = 24$

$3(1+r^2) = 24$

$1+r^2 = 8$

$r^2 = 7$

$\boxed{r = \sqrt{7}}$

$a \times 7 \times 7 = 3$

$\boxed{a = \frac{3}{49}}$

$a_1 a_9 + a_2 a_4 a_9 + a_5 + a_7$

$24 + a \cdot ar^8 + ar \cdot ar^3 \cdot ar^8$

$24 + a^2 r^8 (1 + ar^4)$

$24 + 9(1+3)$

$24 + 9 \times 4 = 24 + 36 = \boxed{60}$

{Jee mains 2023}

Date
Page

Q. Let a_1, a_2, a_3 — be a G.P of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then.
$a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to

Tahira

solving:-

$a_4 \cdot a_6 = 9$

$ar^3 \cdot ar^5 = 9$

$a^2r^8 = 9$

$(ar^4)^2 = 9$

$ar^4 = 3^2$

$ar^4 = 3$ —(i)

$a_5 + a_7 = 24$

$a_4r + a_6r = 24$

$a_4 + a_6 = \frac{24}{r}$

$ar^3 + ar^5 = \frac{24}{r}$

$ar^4 + ar^6 = 24$

$ar^4(1+r^2) = 24$ —(ii)

divide equation (i) and (ii)

$$\frac{ar^4(1+r^2)}{ar^4} = \frac{24}{3}$$

$1 + r^2 = \frac{24}{3}$

$r^2 = \frac{24}{3} - 3$

$r^2 = \frac{21}{3} \Rightarrow 7$

$\boxed{r^2 = 7}$

$a(r^2)^2 = 3$

$a(7)^2 = 3$

$a = \frac{3}{49}$

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$a_1 a_9 + a_2 a_4 a_9 + a_5 + a_7$

$\Rightarrow a \cdot ar^8 + ar \cdot ar^3 \cdot ar^8 + ar^4 + ar^6$

$\Rightarrow a^2 (r^2)^4 + a^3 (r^2)^6 + a(r^2)^2 + a(r^2)^3$

$\Rightarrow \left(\frac{3}{49}\right)^2 (7)^4 + \left(\frac{3}{49}\right)^3 (7)^6 + \frac{3(7)^2}{49} + \frac{3(7)^3}{49}$

$\Rightarrow \frac{9}{\cancel{7^4}} \times \cancel{7^4} + \frac{27}{\cancel{7^6}} \times \cancel{7^6} + \frac{3}{\cancel{7^2}} \times \cancel{7^2} + \frac{3}{\cancel{7^2}} (7)^{\cancel{3}}$

$\Rightarrow 9 + 27 + 3 + 21$

$\Rightarrow$ 60 Ans

QUESTION [JEE Mains 2024 (8 April)]

TAH

In an increasing geometric progression of positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the 4^{th}, 6^{th} and 8^{th} terms is equal to:

(A) 78

(B) 96

(C) 91

(D) 84

Ans. C

TAH-03 (03)

$a_1, a_2 \cdots a_n \rightarrow$ +ve increasing G.P

$a_2 + a_6 = 70/3$.

$a_3 a_5 = 49$

$a_4 + a_6 + a_8 = ?$

$a_2 + a_6 = 70/3$ $\qquad ar^2 \cdot ar^4 = 49$

$ar + ar^5 = 70/3$ $\qquad a^2 r^6 = 49$

$ar(1+r^4) = 70/3$ $\qquad ar^3 = 7$

$$\frac{1+r^4}{r^2} = \frac{70}{3\times 7} = 10/3$$

$3 + 3r^4 = 10r^2$

$3r^4 - 10r^2 + 3 = 0$

$3r^4 - 9r^2 - r^2 + 3 = 0$

$(3r^2 - 1)(r^2 - 3) = 0$

$r^2 = 1/3 \quad r^2 = 3$ ✓

✗ (for inc G.P)

$a_4 + a_6 + a_8$

$= ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 7(1 + 3 + 3^2)$

$= 7 \times 13$

$= 91$ Ans (C)

Tah-03

$a_2 + a_6 = \frac{70}{3}$ $\qquad a_3 \cdot a_5 = 49$

$ar + ar^5 = \frac{70}{3}$ $\qquad ar^2 \cdot ar^4 = 49$

$3ar(1+r^4) = 70$ $\qquad a^2 r^6 = 49$

$ar^3 = 7$

$$\frac{ar^3}{3ar(1+r^4)} = \frac{7}{70} = \frac{1}{10}$$

$a_4 + a_6 + a_8$

$ar^3 + ar^5 + ar^7$

$ar^3(1 + r^2 + r^4)$

$7(1+3+9)$

91 (C)

$$\frac{r^2}{3(1+r^4)} = \frac{1}{10}$$

$10r^2 = 3 + 3r^4$

$3r^4 - 10r^2 + 3 = 0$

$r^2(3r^2-1) - 3(3r^2-1) = 0$

$(r^2-3)(3r^2-1) = 0$

$r^2 = 3$ ✓

$r^2 = \frac{1}{3}$ ✗

Richathakur

QUESTION [IIT-JEE 1987]

★★ASRQ★★

If a, b, c, d and p are distinct real numbers such that $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)\le 0$, then a, b, c, d

(A) are in A.P.

(B) are in G.P.

(C) are in H.P.

(D) satisfy $ab = cd$

$a^2p^2-2apb+b^2+b^2p^2-2bcp+c^2+c^2p^2-2cdp+d^2\le 0$

$(ap-b)^2+(bp-c)^2+(cp-d)^2\le 0$

But $\underbrace{(ap-b)^2}_{\ge 0}+\underbrace{(bp-c)^2}_{\ge 0}+\underbrace{(cp-d)^2}_{\ge 0}\ge 0$

$\Downarrow$

$(ap-b)^2+(bp-c)^2+(cp-d)^2=0.$

$ap-b=bp-c=cp-d=0$

$p=\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$

a, b, c, d are in G.P

Ans. B

TAH-04

(04)

$(a^2+b^2+c^2)p^2 - 2(ab+bc+cd)p + (b^2+c^2+d^2) \le 0$

$a^2p^2 + b^2p^2 + c^2p^2 - 2abp - 2bcp - 2cdp + b^2+c^2+d^2 \le 0$

$(a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) \le 0$

$(ap-b)^2 + (bp-c)^2 + (cp-d)^2 \le 0$

$ap - b = 0$ / $bp - c = 0$ / $cp - d = 0$

$a/b = 1/p$ $\quad$ $b/c = 1/p$ $\quad$ $c/d = 1/p$

$b/a = p$ $\quad$ $c/b = p$ $\quad$ $d/c = p$

$b/a = c/b = d/c = p$

$\Rightarrow$ a, b, c, d are in G.P

(ANS B)

ASRQ

Jah-04

$$(a^2+b^2+c^2)p^2 - 2(ab+bc+cd)p + (b^2+c^2+d^2) \leq 0$$

$$a^2p^2+b^2p^2+c^2p^2 - 2abp - 2bcp - 2cdp + b^2+c^2+d^2 \leq 0$$

$$(a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) \leq 0$$

$$(ap-b)^2 + (bp-c)^2 + (cp-d)^2 \leq 0$$

$ap - b = 0$; $ap = b$

$p = b/a$

$bp - c = 0$

$cp - d = 0 \longrightarrow p = \frac{d}{c}$

$p = \frac{c}{b}$

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

a, b, c, d are in GP

TAH-07

$$(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+b^2+c^2+d^2\le 0$$

$$(a^2p^2+b^2-2abp)+(b^2p^2+c^2-2bcp)+(c^2p^2+d^2-2cdp)\le 0$$

$$(ap-b)^2+(bp-c)^2+(cp-d)^2\le 0$$

$$ap-b=bp-c=cp-d=0$$

$$ap=b \qquad bp=c \qquad cp=d$$

$$p=\frac{b}{a} \qquad p=\frac{c}{b} \qquad p=\frac{d}{c}$$

$$\frac{b}{a}=\frac{c}{b}=d/c$$

$$b^2=ac \qquad c^2=bd$$

$a,b,c\to GP \qquad b,c,d\to GP$

$a,b,c,d\longrightarrow GP$ (B)

QUESTION [JEE Mains 2023 (24 Jan)]

★★ASRQ★★

The 4^{th} term of G.P. is 500 and its common ratio is $\frac{1}{m}$, $m \in \mathbb{N}$. Let S_n denote the sum of the first n terms of this G.P. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is

Gadhe / Gadhiyaa

$$\frac{a\cdot(r^6-1)}{r-1} > \frac{a(r^5-1)}{r-1}+1 \quad , \quad \frac{a(r^7-1)}{r-1} < \frac{a(r^6-1)}{r-1}+1/2$$

$S_6 > S_5 + 1$

$S_6 - S_5 > 1$

$T_6 > 1$

$a\cdot(1/m)^5 > 1$

$\frac{500}{m^2} > 1 \Rightarrow m^2 < 500$

$S_7 < S_6 + 1/2$

$S_7 - S_6 < 1/2$

$T_7 < 1/2$

$a\cdot(1/m)^6 < 1/2$

$\frac{500}{m^3} < 1/2 \Rightarrow m^3 > 1000$

$T_4 = a\cdot(1/m)^3 = 500$

$a = 500m^3.$

Ans. 12

$n^2 < 500$, $n^3 > 1000$

$2 3^2 = 529$

$n < 23$ $\quad$ $n > 10$.

$n = 11, 12, \dots 22$

12 numbers.

TAH-05

05

Common Ratio $1/m$, $m \in N$

Let first term of G.P $\rightarrow a$

$T_4 = 500$

$a(1/m)^3 = 500$

$a/m^3 = 500 \Rightarrow a = 500m^3$

$S_6 > S_5 + 1$

$S_6 - S_5 > 1$

$T_6 > 1$

$a/m^5 > 1$

$\frac{500}{m^2} > 1$

$500 > m^2$

$m^2 < 500$

$S_7 < S_6 + 1/2$

$S_7 - S_6 < 1/2$

$T_7 < 1/2$

$a/m^6 < 1/2$

$\frac{500}{m^3} < 1/2$

$1000 < m^3$

$10 < m$

$m = \{1, 2, \dots 22\} \cap m = \{11, 12, \dots\}$

$\boxed{\text{Ans } 12}$ $m \in \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22\}$

TAH-05

$$T_4 = 500$$
$$r = \frac{1}{m}$$
$$a \times \frac{1}{m^3} = 500$$
$$a = 500m^3$$

$$S_6 > S_5 + 1$$
$$S_6 - S_5 > 1$$
$$\frac{a(r^6-1)}{(r-1)} - \frac{a(r^5-1)}{(r-1)} > 1$$
$$\frac{ar^6 - \cancel{a} - ar^5 + \cancel{a}}{(r-1)} > 1$$
$$\frac{ar^5(r-1)}{r-1} > 1$$
$$\boxed{ar^5 > 1}$$
$$500m^3 \times \frac{1}{m^5} > 1$$
$$\frac{500}{m^2} > 1$$
$$\boxed{m^2 < 500}$$

$$S_7 < S_6 + 1/2$$
$$S_7 - S_6 < 1/2$$
$$\frac{a(r^7-1)}{(r-1)} - \frac{a(r^6-1)}{(r-1)} < 1/2$$
$$\frac{ar^7 - \cancel{a} - ar^6 + \cancel{a}}{(r-1)} < 1/2$$
$$\frac{ar^6(r-1)}{(r-1)} < 1/2$$
$$\boxed{ar^6 < 1/2}$$
$$500m^3 \times \frac{1}{m^6} < \frac{1}{2}$$
$$\frac{500}{m^3} < \frac{1}{2}$$
$$\boxed{m^3 > 1000}$$

$m = 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22$

no. of values of $m = 12$ Ans

Solution to Previous KTKs

QUESTION

KTK 01

The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is

Ans. 14

KTK-01

AP_1 : 3, 7, 11, 15, 19, 23, ... 407 $\rightarrow d_1 = 4$

AP_2 : 2, 9, 16, 23 .... 709 $\rightarrow d_2 = 7$

common term 23

" diff LCM (d_1 & d_2)

$= 28$

$$23 + (n-1)28 \leq 407$$

$$(n-1)\,28 \leq 384$$

$$n-1 \leq \frac{384}{28}$$

$$n-1 \leq 13.7$$

$$n \leq 14.7$$

$n = 14$

→ Number of common terms is 14

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KTK-02

$A = \{n \in [100, 700] \cap N$

multiple of 3 :

$102, 105, 108, 111, 114 - - - - 699 \rightarrow a_1 = 102$

$\rightarrow d_1 = 3$

$a_n = a + (n-1)d$

$699 = 102 + 3d - 3$

$n = \frac{600}{2} \Rightarrow 300$

multiple of 4 :

$100, 104, 108, 112 - - - - 700 \rightarrow a_2 = 100$

$\rightarrow d_2 = 4$

$a_n = a + (n-1)d$

$700 = 100 + 4n - 4$

$n = \frac{604}{4} \Rightarrow 151$

$d = 12$

multiple of 12 :

$108, 120, 132 - - - 696$

$696 = 108 + (n-1)12$

$n = \frac{600}{12} \Rightarrow 50$

Divisible by 3 & 4 $\Rightarrow 300 + 151 - 50 \Rightarrow 301$

not divisible by 3 & 4 $\Rightarrow 600 - 301 + 1$

$\Rightarrow 300$ (c)

QUESTION [JEE Mains 2024 (6 April)]

KTK 02

For $A = \{n \in [100, 700] \cap \mathbb{N} : n \text{ is neither a multiple of 3 nor a multiple of 4}\}$. Then the number of elements in A is

(A) 290

(B) 280

(C) 300

(D) 310

Ans. C

KTK-02 (12)

$n \in [100, 700]$, where $n \in N$

Total natural number in this interval is $= 600+1$
$= 601$

* multiple of 3

$102, 105, \ldots 699$

$102 + (n-1)3 = 699$

$n-1 = \frac{597}{3} = 199$

$n = 200$

* multiple of 4

$100, 104 \ldots 700$

$\Rightarrow 100 + (n-1)4 = 700$

$n-1 = \frac{600}{4} = 150$

$n = 151$

multiple of 12

$108, 120 \ldots 696$

$108 + 12(n-1) = 696$

$n-1 = \frac{588}{12} = 49$

$n = 50$

No's of Total multiple of 3 or 4.

$= 200 + 151 - 50$

$= 301$

No's of Neither multiple of 3 nor 4

$= 601 - 301 = 300$ Ans (C)

KTK-02

$A = \{n \in [100, 700] \cap N$

multiple of 3 :

$102, 105, 108, 111, 114 \ldots 699 \rightarrow a_1 = 102$

$\rightarrow d_1 = 3$

$a_n = a + (n-1)d$

$699 = 102 + 3d - 3$

$n = \frac{600}{2} \Rightarrow 300$

$d = 12$

multiple of 4 :

$100, 104, 108, 112 \ldots 700 \rightarrow a_2 = 100$

$a_n = a + (n-1)d \rightarrow d_2 = 4$

$700 = 100 + 4n - 4$

$n = \frac{604}{4} \Rightarrow 151$

Richathakur

multiple of 12 :

$108, 120, 132 \ldots 696$

$696 = 108 + (n-1)12$

$n = \frac{600}{12} \Rightarrow 50$

Divisible by 3 & 4 $\Rightarrow 300 + 151 - 50 \Rightarrow 301$

not divisible by 3 & 4 $\Rightarrow 600 - 301 + 1$

$\Rightarrow 300$ (C)

QUESTION [JEE Mains 2023 (29 Jan)]

KTK 03

Let $\{a_k\}$ and $\{b_k\}, k \in N$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k, k \in N$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then

$$\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4) \text{ is equal to}$$

Ans. 9

KTK-03

(13)

$GP_1 \rightarrow a_1, a_2 \dots a_k \rightarrow r_1$

$GP_2 \rightarrow b_1, b_2 \dots b_k \rightarrow r_2$

$a_1 = b_1 = 4$

$r_1 < r_2$

$c_k = a_k + b_k$

$c_2 = a_2 + b_2 = 5$ $\qquad c_3 = a_3 + b_3 = 13/4$

$4r_1^2 + 4r_2^2 = 13/4$

$r_1^2 + r_2^2 = 13/16$

$\Rightarrow 4r_1 + 4r_2 = 5$

$4(r_1 + r_2) = 5$

$r_1 + r_2 = 5/4$

$r_1^2 + (5/4 - r_1)^2 = 13/16$

$16r_1^2 + (5 - 4r_1)^2 = 13$

$16r_1^2 + 25 - 40r_1 + 16r_1^2 = 13$

$32r_1^2 + 12 - 40r_1 = 0$

$8r_1^2 - 10r_1 + 3 = 0$

$8r_1^2 - 6r_1 - 4r_1 + 3 = 0$

$(2r_1 - 1)(4r_1 - 3) = 0$

$r_1 = 1/2 \qquad r_1 = 3/4$ ✗ $[\because r_2 > r_1]$

$r_2 = 5/4 - 1/2 \qquad r_2 = 1/2$

$= 3/4$

(14)

$a_1 = 4 \qquad b_1 = 4$

$r_1 = 1/2 \qquad r_2 = 3/4$

$\left(\sum_{k=1}^{\infty} c_k\right) - (12a_6 + 8b_4)$

$= \left(\sum_{k=1}^{\infty} (a_k + b_k)\right) - (12a_6 + 8b_4)$

$= \sum_{k=1}^{\infty} a_k + \sum_{k=1}^{\infty} b_k - (12a_6 + 8b_4)$

$= \frac{a_1}{1-r_1} + \frac{b_1}{1-r_2} - \left(12(a_1r_1^5) + 8(b_1r_2^3)\right)$

$= \frac{4}{1-1/2} + \frac{4}{1-3/4} - \left(12(4\times(1/2)^5) + 8(4\times(3/4)^3)\right)$

$= 8 + 16 - \left(12 \times 1/8 + \cancel{8} \times \frac{27}{\cancel{16}2}\right)$

$= 24 - (3/2 + 27/2)$

$= 24 - (30/2)$

$= 24 - 15$

$= 9$ Ans : 9

QUESTION [JEE Mains 2024 (6 April)]

KTK 04

Let α, β be roots of $x^2 + \sqrt{2}x - 8 = 0$. If $U_n = \alpha^n + \beta^n$, then $\frac{U_{10} + \sqrt{2}U_9}{2U_8}$ is equal to

Ans. 4

KTK-04

(15)

$$x^2+\sqrt{2}x-8=0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$$

By Newton's formula,

$$U_{n+2}+\sqrt{2}\,U_{n+1}-8U_n=0$$

Put $n=8$

$$U_{10}+\sqrt{2}U_9-8U_8=0$$

$$U_{10}+\sqrt{2}U_9=8U_8$$

$$\frac{U_{10}+\sqrt{2}U_9}{2U_8}=4$$

Ans (4)

KTK-04

$x^2+\sqrt{2}x-8=0$ $\qquad$ $U_n=\alpha^n+\beta^n$

$\alpha+\beta=-\sqrt{2}$

$\alpha\beta=-8$

Richathakur

By using N-f :

$$U_{n+2}+\sqrt{2}U_{n+1}-8U_n=0 \qquad [\text{Put } n=8]$$

$$U_{10}+\sqrt{2}U_9-8U_8=0$$

$$U_{10}+\sqrt{2}U_9=8U_8$$

$$\frac{U_{10}+\sqrt{2}U_9}{2U_8} \Rightarrow \frac{8\cancel{U_8}}{2\cancel{U_8}} \Rightarrow 4 \quad \underline{\text{Ans}}$$

QUESTION

KTK 05

If m^{th} term of an A.P. is n & n^{th} term is m, then show that its $(m + n)^{th}$ term is zero.

KTK - 05

Let, first term & common difference of AP is a & d

$$\therefore T_m = a + (m-1)d = n$$

$$T_n = a + (n-1)d = m$$

$$\ominus \qquad \ominus$$

$$(m-1)d - (n-1)d = n - m$$

$$(m-1-n+1)d = (n-m)$$

$$a = n + m - 1$$

$$-(n-m)d = (n-m)$$

$$d = -1$$

$$\therefore T_{m+n} = a + (m+n-1)d$$

$$= a + a(-1)$$

$$= a - a = 0 \text{ (proved)}$$

Solution to Previous Shikaars

QUESTION [JEE Mains 2025 (24 Jan)]

S 01

In an arithmetic progression, if $S_{40} = 1030$ and $S_{12} = 57$, then $S_{30} - S_{10}$ is equal to:

(A) 525

(B) 505

(C) 510

(D) 515

Ans. D

SHIKAAR

S-1

$S_{40} = 1030 \qquad S_{12} = 57$

$\frac{40}{2}(2a+39d) = 1030$ | $\frac{12}{2}(2a+11d) = 57$

$4a+78d = 103$ | $12a + 66d = 57$

$12a + 66d = 57$
$-12a - 234d = -309$

$-168d = -252$

$d = \frac{252}{168} \quad \frac{63}{42} = \frac{9}{6} = \frac{3}{2}$

$4 \times a + 78 \times \frac{3}{2} = 103$

$4a = 103 - 39 \times 3$
$4a = 103 - 117$
$a = -\frac{7}{2}$

$S_{30} - S_{10} = \frac{30}{2}(2a+29d) - \frac{10}{2}(2a+9d)$

$= 15(2a+29d) - 5(2a+9d)$
$= 30a + 435d - 10a - 45d$
$= 20a + 390d$
$= 20(-\frac{7}{2}) + 390(\frac{3}{2})$

$= -70 + 195 \times 3 = 515$

S-01

$\Rightarrow S_{40} = 1030 \qquad \Rightarrow S_{12} = 57$

$\Rightarrow \cancel{20}\frac{40}{2}[2a+39d] = 1030 \qquad \Rightarrow {}^{6}\frac{12}{2}[2a+11d] = 57$

$\Rightarrow 4a + 78d = 103 \quad -① \times 3 \quad \Rightarrow 12a + 66d = 57 \quad -②$

$\Rightarrow \cancel{12a} + 234d = 309 \quad -③$
$-\cancel{12a} + 66d = 57 \quad -②$

$\Rightarrow 168d = 252 \quad \Rightarrow d = \frac{252}{168} = \frac{3}{2}$

$\Rightarrow$ Put in eq ① $\Rightarrow 4a + 78 \cdot \frac{3}{2} = 103$

$\Rightarrow 4a = 103 - 117 \Rightarrow 4a = -14$

$\Rightarrow a = -\frac{7}{2}$ ☆

find : $S_{30} - S_{10} = ?$

$\Rightarrow {}_{15}\frac{30}{2}[2a+29d] - {}_{5}\frac{10}{2}[2a+9d]$

$\Rightarrow 30a + 435d - 10a - 45d$

$\Rightarrow 20a + 390d$

$\Rightarrow \overset{10}{\cancel{20}}\left(\frac{-7}{\cancel{2}}\right) + \overset{195}{\cancel{390}}\left(\frac{3}{\cancel{2}}\right)$

$\Rightarrow -70 + 585 \Rightarrow 515$ Ans.

krish

S-01 In an A.P. If $S_{40} = 1030$ and $S_{12} = 57$ then $S_{30} - S_{10} = ?$

$\Rightarrow \frac{40}{2}(2a + 39d) = 1030$

$20(2a + 39d) = 1030$

$\boxed{2(2a + 39d) = 103}$

$4a + 78d = 103$

$\times 3$

$12a + 234d = 309$

$\cancel{12a} + 66d = 57$

$168d = 252$

$\boxed{d = 1/2}$

$\frac{12}{2}(2a + 11d) = 57$

$6(2a + 11d) = 57$

$\boxed{12a + 66d = 57}$

$12a + 33 = 57$

$12a = 57 - 33$

$12a = 24$

$\boxed{a = 2}$

QUESTION [JEE Mains 2025 (23 Jan)]

S 02

The roots of the quadratic equation $3x^2 - px + q = 0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88, then $q - 2p$ is equal to

Ans. 474

Q. [S-2] (JEE mains - 2025)

Sol $\boxed{d = \frac{3}{2}}$ — (given)

$3x^2 - Px + q = 0$ { T_{10}, T_{11}

$S_{11} = 88$

$\Rightarrow \frac{11}{2}[2a + (10)d] = 88$

$\Rightarrow \frac{11}{2}[2a + 5\times 3] = 88$

$\Rightarrow \frac{11}{2}[2a + 15] = 88$

$\Rightarrow 22a + 165 = 176$

$\Rightarrow 22a = 176 - 165$

$\Rightarrow a = \frac{\cancel{11}\,1}{\cancel{22}\,2}$

$\Rightarrow \boxed{a = \frac{1}{2}}$

$T_{10} = a + 9d$

$= \frac{1}{2} + 9 \times \frac{3}{2}$

$= \frac{1}{2} + \frac{27}{2}$

$= \frac{28}{2} \Rightarrow \boxed{14}$

$T_{11} = a + 10d$

$= \frac{1}{2} + 10 \times \frac{3}{2}$

$= \boxed{\frac{31}{2}}$

Sum of the roots $= \frac{P}{3}$

$\Rightarrow 14 + \frac{31}{2} = \frac{P}{3}$

$\Rightarrow \frac{28 + 31}{2} = \frac{P}{3}$

$\Rightarrow \frac{59}{2} = \frac{P}{3}$

$\Rightarrow 2P = 177$

$\Rightarrow \boxed{P = \frac{177}{2}}$

Product of roots $= \frac{q}{3}$

$\Rightarrow \cancel{14}^{7} \cdot \frac{31}{\cancel{2}} = \frac{q}{3}$

$\Rightarrow 217 = \frac{q}{3}$

$\Rightarrow \boxed{q = 651}$

Now, $q - 2P$

$\Rightarrow 651 - \frac{177 \times \cancel{2}}{\cancel{2}}$

$\Rightarrow 474$ Ans

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Aniket raj
From patna

S-02

$\Rightarrow 3x^2 - Px + q = 0$ → 10^{th} term. → 11^{th} term.

$d = \frac{3}{2}$

$\Rightarrow a_{10} + a_{11} = P/3$ # $a_{10} \cdot a_{11} = q/3$

$\Rightarrow a + 9d + a + 10d = P/3$

$\Rightarrow 2a + 19d = P/3$

$\Rightarrow 2a = \frac{P}{3} - \frac{57}{2}$

$\Rightarrow 2a = \frac{2P - 171}{6}$ — (1)

$\Rightarrow \frac{2P - 171}{6} = \cancel{2} \times \frac{1}{\cancel{2}}$

$\Rightarrow 2P = 177$

$\Rightarrow P = \frac{177}{2}$

$S_{11} = 88$

$\Rightarrow \frac{\cancel{11}}{\cancel{2}}[\cancel{2}a + \cancel{10}^{5}d] = \cancel{88}\,8$

$\Rightarrow a + 5d = 8$

$\Rightarrow a = 8 - \frac{15}{2}$

$a = 1/2$

$a_{10} \cdot a_{11} = q/3$

$\Rightarrow (a + 9d) \cdot (a + 10d) = \frac{q}{3}$

$\Rightarrow 3\,(1/2 + 27/2)\,(1/2 + 30/2) = q$

$\Rightarrow q = 3 \times \frac{\cancel{28}^{7}}{\cancel{2}} \times \frac{31}{\cancel{2}}$

$\Rightarrow q = 31 \times 21 = 651$

find $\Rightarrow q - 2P$

$\Rightarrow 651 - \cancel{2} \times \frac{177}{\cancel{2}}$

$\Rightarrow 474$ Ans.

krish

S-02. Roots of Q.E $3x^2-px+q=0$ are 10^{th} & 11^{th} of AP with $d=3/2$
If $S_{11}=88$ then $q-2p$ is equals to;

$\frac{11}{2}(2a+10d)=88$

$11(a+5d)=88$

$a+5d=8$

$a+5\times\frac{3}{2}=8$

$a=8-\frac{15}{2}$

$\boxed{a=\frac{1}{2}}$

$3x^2-px+q \to T_{10},\ T_{11}$

$T_{10}=a+9d$

$=\frac{1}{2}+9\times\frac{3}{2}$

$=\frac{1}{2}+\frac{27}{2}=\frac{28}{2}=14$

$T_{11}=a+10d$

$=\frac{1}{2}+10\times\frac{3}{2}$

$\frac{1}{2}+15=\frac{31}{2}$

Product of Roots =

$=14\cdot\frac{31}{2}=q/3$

$7\times31\times3=q$

$q=651$

Sum of Root $=14+\frac{31}{2}=\frac{p}{3}$

$=\frac{28+31}{2}=\frac{p}{3}$

$\frac{59\times3}{2}=p$ $=\frac{177}{2}$

$\because q-2p$

$651-2\times\frac{177}{2}$

$651-177=474$ Ans

QUESTION [JEE Mains 2025 (22 Jan)]

S 03

Suppose that the number of terms in an A.P. is $2k$, $k \in N$. If the sum of all odd terms of the A.P. is 40, the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27, then k is equal to:

(A) 8

(B) 6

(C) 4

(D) 5

Ans. D

S-03

\# $a_1 + a_3 + a_5 + \dots + a_{2k-1} = 40$

$\Rightarrow 40 = \frac{k}{\cancel{2}}[\cancel{2}a + (k-1)\cancel{2}d]$

$\Rightarrow 40 = k(a + (k-1)d)$.

\# $a_2 + a_4 + a_6 + \dots + a_{2k} = 55$.

$\Rightarrow 55 = \frac{k}{\cancel{2}}[\cancel{2}a_2 + (k-1)\cancel{2}d]$

$\Rightarrow 55 = k[a + \cancel{d} + kd - \cancel{d}]$

$\Rightarrow 55 = k(a + kd)$.

$\Rightarrow 40 = k(a + kd) - kd$.

$\Rightarrow 40 = 55 - kd$

$\Rightarrow kd = 15$ *

krish

\# $a_{2k} - a = 27$

$\Rightarrow \cancel{a} + (2k-1)d - \cancel{a} = 27$

$\Rightarrow 2kd - d = 27$

$\Rightarrow 30 - d = 27 \Rightarrow d = 3$ *

\# $k = \frac{15}{3} \Rightarrow 5$ Ans.

QUESTION [JEE Mains 2025 (23 Jan)]

S 04

If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

A. -120

B. -1200

C. -1080

D. -1020

Ans. C

$\Rightarrow \quad \boxed{a = 3}$

S-04

& $\quad a_1 + a_2 + a_3 + a_4 = \frac{1}{5}(a_5 + a_6 + a_7 + a_8)$

$\Rightarrow a + a + d + a + 2d + a + 3d = \frac{1}{5}(a + 4d + a + 5d + a + 6d + a + 7d)$

$\Rightarrow \quad 4a + 6d = \frac{1}{5}(4a + 22d)$

krish

$\Rightarrow \quad 4a - \frac{4}{5}a = \frac{22d}{5} - 6d$

$\Rightarrow \quad \frac{\cancel{16}^{2}}{\cancel{5}}a = -\frac{\cancel{8}}{\cancel{5}}d$

$\Rightarrow \quad \boxed{d = -6}$

$S_{20} = \frac{\cancel{20}^{10}}{\cancel{2}}[2(3) + 19(-6)]$

$= 10[6 - 114]$

$= 10 \times (-108)$

$= -1080$ Ans.

Soy If first terms of A.P is 3 & sum of first four terms is equal to one-fifth of the sum of next four terms then the sum of first 20 terms is equal to;

$\boxed{a=3}$

To find $S_{20} = \frac{20}{2}(2a+19d)$

$S_4 = \frac{1}{5}(S_8 - S_4)$

$2(2a+3d) = \frac{1}{5}\left(\frac{8}{2}(2a+7d) - \frac{4}{2}(2a+3d)\right)$

$2(2a+3d) = \frac{1}{5}(4(2a+7d) - 2(2a+3d))$

$10(2a+3d) = (8a+28d-4a-6d)$

$10(2a+3d) = (4a+22d)$

$5(2a+3d) = 2a+11d$

$10a+15d-2a-11d=0$

$8a+4d=0$

$2a+d=0$

$\boxed{-2a=d}$

$S_{20} = 10(2a+19(-2a))$

$= 10(2a-38a)$

$= -10 \times 36 \times 3$

$= -1080$ Ans

QUESTION [JEE Mains 2024 (30 Jan)]

S 05

Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{20} = 790$ and $S_{10} = 145$, then $S_{15} - S_5$ is:

(A) 405

(B) 390

(C) 410

(D) 395

Ans. D

S-05

$\Rightarrow S_{20} = 790$

$\Rightarrow \cancel{10}\frac{\cancel{20}}{\cancel{2}}[2a + 19d] = 79\cancel{0}$

$\Rightarrow \cancel{2}a + 19d = 79$ — (1)

$\Rightarrow S_{10} = 145$

$\Rightarrow \overset{5}{\frac{\cancel{10}}{\cancel{2}}}[2a + 9d] = \overset{29}{\cancel{145}}$

$\Rightarrow \cancel{2}a + 9d = 29$ — (2)

$\Rightarrow$ (1) − (2) $\Rightarrow 1\cancel{0}d = 5\cancel{0} \Rightarrow \boxed{d = 5}$

$\Rightarrow$ Put in Eq (2) $\Rightarrow 2a = -16 \Rightarrow \boxed{a = -8}$

find : $S_{15} - S_5 = ?$

$\Rightarrow \frac{15}{\cancel{2}}[\cancel{2}a + \overset{7}{\cancel{14}}d] - \frac{5}{\cancel{2}}[\cancel{2}a + \overset{2}{\cancel{4}}d]$

$\Rightarrow 15a + 105d - 5a - 10d$

$\Rightarrow 10a + 95d$

$\Rightarrow -80 + 475. \Rightarrow 395$ Ans.

krish

S05 Let S_n denotes sum of first n terms of AP.
If $S_{20} = 790$ & $S_{10} = 145$ then $S_{15} - S_5$ is :

$\boxed{S_{20} = 790}$

$$\frac{20}{2}(2a + 19d) = 790$$

$$(2a + 19d) = 79$$

$$2a + 19d = 79 \quad \text{—(1)}$$

$$\frac{10}{2}(2a + 9d) = 145$$

$$(2a + 9d) = 29 \quad \text{—(2)}$$

$$\ominus \Rightarrow 10d = 50$$

$$\boxed{d = 5}$$

$$\boxed{a = -8}$$

Now;

$$S_{15} - S_5$$

$$= \frac{15}{2}(2a + 14d) - \frac{5}{2}(2a + 4d)$$

$$= 405 - 10$$

$$= 395 \quad \text{Ans}$$

THANK YOU