PRAYAS

JEE 2026

Mathematics

Sequence and Series

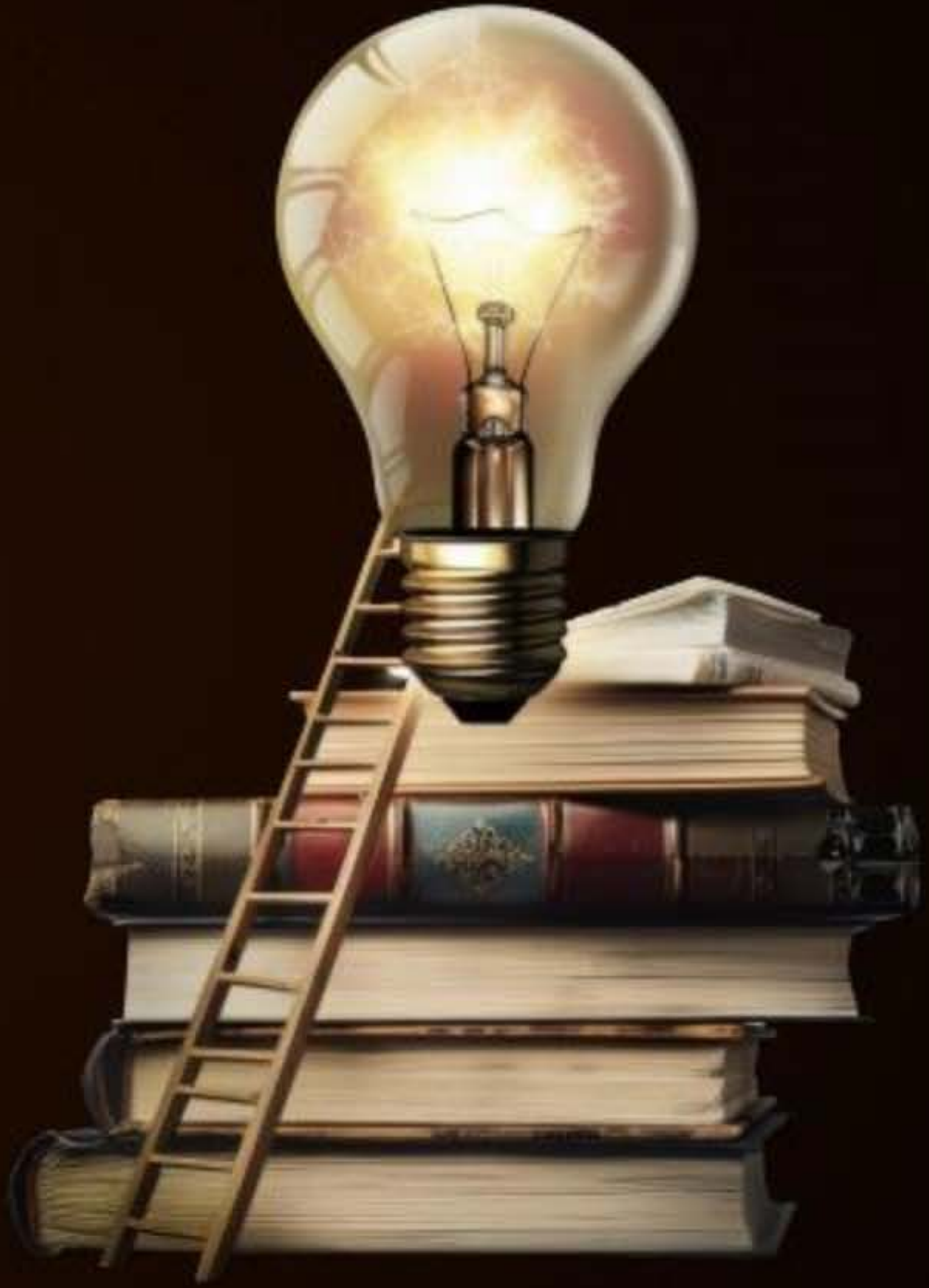
Lecture - 04

By – Ashish Agarwal Sir
(IIT Kanpur)

Topics *to be covered*

A Properties of G.P

B Question Practice



QUESTION

Tah 01

If $a_1 a_2 a_3 \ldots\ldots a_n$ are n arithmetic means inserted between 7 and 2015 whose sum is 56616 then

(A) n is 56

(B) n is 28

(C) $a_{19} = 2029/3$

(D) $a_{19} = 36543/57$

$d = \frac{2008}{57}$

$a_{19} = 7 + 19d.$

$= 7 + \frac{2008}{3}$

$= \frac{2029}{3}$

QUESTION [JEE Advanced 2018]

Tan02

The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

Ans. 6

QUESTION [JEE Mains 2025 (4 April)]

Tah03

Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of A.P.'s in A and B respectively such that $D = d + 3, d > 0$. If $\frac{p+q}{p-q} = \frac{19}{5}$, then $p - q$ is equal to

(A) 540

(B) 450

(C) 600

(D) 630

Ans. A

QUESTION [JEE Mains 2021 (August)]

Tah 04

Let $a_1, a_2, \ldots\ldots a_{21}$ be an A.P. such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$.

If the sum of this A.P. is 189, then $a_6 \cdot a_{16}$ is equal to:

(A) 57

(B) 72

(C) 48

(D) 36

$$\frac{d}{d}\left(\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\cdots+\frac{1}{a_{20}\cdot a_{21}}\right)=\frac{4}{9}$$

$$=\frac{1}{d}\left(\frac{a_2-a_1}{a_1a_2}+\frac{a_3-a_2}{a_2a_3}+\cdots+\frac{a_{21}-a_{20}}{a_{21}\cdot a_{20}}\right)$$

$$=\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+\frac{1}{a_3}-\frac{1}{a_4}+\cdots+\frac{1}{a_{20}}-\frac{1}{a_{21}}\right)$$

$$=\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_{21}}\right)=\frac{a_{21}-a_1}{d\,a_1a_{21}}=\frac{a_1+20d-a_1}{d\,a_1\cdot a_{21}}=\frac{20}{a_1\cdot a_{21}}=\frac{4}{9}$$

$$S=\frac{21}{2}(a_1+a_{21})=189$$

$a_1+a_{21}=18.$

$a_1=3$
$a_{21}=15$
OR
$a_1=15$
$a_{21}=3$

$a_1\cdot a_{21}=45.$

QUESTION [JEE Mains 2021 (27 July)]

Jan 05

If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to

Ans. 3

properties of a G. p

Property 3:

If each term of a G.P. be raised to the same power, then resulting series is also a G.P.

Ex: $2, 4, 8, 16, 32, 64 \ldots$ $CR = 2$

$2^k, 4^k, 8^k, 16^k, 32^k, 64^k \ldots$ $C.R = 2^k$

$CR = \frac{4^k}{2^k} = 2^k$

Property 4:

If each term of a G.P. be multiplied or divided by the same non-zero quantity, then the resulting sequence is also a G.P. with same C.R

Ex: $2, 4, 8, 16, 32 \ldots$ $C.R = 2$

Each term multiplied by 3

$6, 12, 24, 48, 96 \ldots$ $C.R = 2$

$\div$ by 3.

$2/3, 4/3, 8/3, 16/3, 32/3 \ldots$ $C.R = 2$

Property 5:

If a_1, a_2, a_3 ...... and b_1, b_2, b_3 ..... be two G.P.s of common ratio r_1 and r_2 respectively, then a_1b_1, a_2b_2 ..... and $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}$ .... will also form a G.P. common ratio will be r_1r_2 and r_1/r_2 respectively.

$G.P_1 : 2, 2^2, 2^3, 2^4, 2^5 - - - -$ $C.R = 2 = r_1$

$G.P_2 : 3, 3^2, 3^3, 3^4, 3^5 - - - -$ $C.R = 3 = r_2$

$A.P_1 \pm A.P_2 = A.P$

$d_1 \quad d_2 \quad d = d_1 \pm d_2$

$G.P_1 \times G.P_2 : 6, 6^2, 6^3, 6^4, 6^5 - - -$ $C.R = 6 = r_1 r_2$

$\frac{G.P_1}{G.P_2} : \frac{2}{3}, (2/3)^2, (2/3)^3, (2/3)^4 - - -$ $C.R = 2/3 = \frac{r_1}{r_2}$

Property 6:

[redacted] In any GP square of any Term is equal [redacted] product of its two terms which are equidistant from it.

i.e., $T_r^2 =$ [redacted] $T_{r-k}T_{r+k}, k < r$

$$T_{r-k} = \frac{T_r}{r^k}$$

$$T_{r+k} = T_r \cdot r^k$$

$$T_{r-k} \cdot T_{r+k} = T_r^2$$

$T_2 \xleftarrow{1/r} T_5 \xrightarrow{r} T_8$

$2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 \ldots$

$$4 = \frac{32}{2^3}$$

$$256 = 32 \cdot 2^3$$

$$4 \cdot 256 = 32^2$$

$$1024 = 1024$$

Property 7:

+ve terms.

If $a_1, a_2, a_3, \ldots\ldots, a_n$ is a G.P. of non-zero, non-negative terms, then $\log a_1, \log a_2, \ldots\ldots, \log a_n$ is an A.P. and vice versa.

Proof: $a_1, a_2, a_3 - - - -$ G.P of +ve terms

$\downarrow \quad \downarrow \quad \downarrow$

$a, ar, ar^2 - - - -$

$\log a, \log a + \log r, \log a + 2\log r, - - - -$ A.P. common diff $= \log r$.

Now $A_1, A_2, A_3 - - -$ is an A.P

$e^{A_1}, e^{A_2}, e^{A_3} - - -$ is a G.P

$\downarrow$

$e^A, e^{A+d}, e^{A+2d} - - -$ $\rightarrow$ $e^A, e^A \cdot e^d, e^A \cdot e^{2d}, e^A \cdot e^{3d} - - -$ G.P $r = e^d$

G.P : $a_1, a_2, a_3, a_4, \ldots a_{k-3}, a_{k-1}, a_k, a_{k+1}, a_{k+2}, a_{k+3} \ldots$

* $a_3^2 = a_2 \cdot a_4$
* $a_k^2 = a_{k-1} \cdot a_{k+1}$
* $a_{k+1}^2 = a_k \cdot a_{k+2}$
* $a_{k+3}^2 = a_{k+2} \cdot a_{k+4}$

Ex: $2, -4, 8, -16, 32, -64, +128 \ldots$

$(-16)^2 = 8 \cdot 32$

$(32)^2 = -16 \cdot -64$

$(-64)^2 = 32 \cdot 128$.

Ex: $-3, -6, -12, -24, -48 \ldots$

$(-12)^2 = -6 \cdot (-24)$

Geometrical Mean

Definition :

If a, b, c are three positive number in G.P., then b is called the geometrical mean between a and c and $b^2 = ac$. If a and b are two positive real and G is the G.M. between them, then

$$G^2 = ab$$

a, b, c. → three +ve no:s in G.P

↓

Single Geometric mean b/w a & c

$$\frac{b}{a} = \frac{c}{b}$$

a, b, c are in G.P ⟺ $b^2 = ac$

Ex: −2, −4, −8

$(-4)^2 = (-2)\cdot(-8)$

Ex: −2, 4, −8

$4^2 = (-2)\cdot(-8)$

To insert 'n' G.M.'s between a and b

T_1 $\quad$ T_{n+2}

$a, \underbrace{G_1, G_2, G_3, \ldots, G_n}, b$ are in G.P $\quad$ a, b are +ve numbers.

$G_1, G_2, G_3 \ldots, G_n$ are called n G.Ms b/w a & b.

$$T_{n+2} = ar^{n+2-1} = ar^{n+1}$$

$$b = ar^{n+1}$$

$$r^{n+1} = \frac{b}{a}$$

$$T_{n+2} = b = ar^{n+1} \Rightarrow r^{n+1} = \frac{b}{a}$$

$G_1 = ar$

$G_2 = ar^2$

$G_3 = ar^3$

$\vdots$

$G_n = ar^n$

$$G_1 \cdot G_2 \cdot G_3 \ldots G_n = a^n \cdot r^{1+2+3+\ldots+n}$$
$$= a^n \cdot r^{\frac{n(n+1)}{2}}$$
$$= a^n \cdot (r^{n+1})^{\frac{n}{2}}$$
$$= a^n \cdot \left(\frac{b}{a}\right)^{n/2}$$
$$= a^{n/2} \cdot b^{n/2}$$

$$\prod_{i=1}^{n} G_i = (\sqrt{ab})^n$$

product of n G.Ms b/w two +ve no. is equal to n^{th} power of single G.M b/w them.

$\sum \rightarrow$ sum

$\prod \rightarrow$ Product.

Arithmetic Geometric Progression (AGP)

Standard appearance of an AGP is

$$S = a, (a+d)r, (a+2d)r^2, (a+3d)r^3 \ldots\ldots$$

Here each term is the product of corresponding terms in a arithmetic and geometric series.

A.P : $a, \ a+d, \ a+2d, \ a+3d, \ a+4d, \ - - - - -$

G.P : $1, \ r, \ r^2, \ r^3, \ r^4 \ - - - - -$

AGP: $a, \ (a+d)r, \ (a+2d)r^2, \ (a+3d)r^3 - - - - -$

$$S = a + (a+d)r + (a+2d)r^2 + (a+3d)r^3 + \cdots + (a+(n-1)d)r^{n-1}$$

$$rS = ar + (a+d)r^2 + (a+2d)r^3 + \cdots + (a+(n-2)d)r^{n-1} + (a+(n-1)d)r^n$$

$$(1-r)S = a + (\underbrace{dr^1 + dr^2 + dr^3 + \cdots + dr^{n-1}}_{(n-1) \text{ terms.}}) - (a+(n-1)d)r^n$$

$$= a + d\,(r + r^2 + \cdots + r^{n-1}) - (a+(n-1)d)r^n$$

$$(1-r)S = a + rd\,\frac{(r^{n-1}-1)}{(r-1)} - (a+(n-1)d)r^n$$

$$S = \frac{a}{1-r} + \frac{rd}{(1-r)^2}(1-r^{n-1}) - \frac{(a+(n-1)d)r^n}{(1-r)}$$

QUESTION [JEE Mains 2025 (22 Jan)]

Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive terms. If $a_1 a_5 = 28$ and $a_2 + a_4 = 29$, then a_6 is equal to:

M (1)

$a_1 a_5 = 28$, $a_2 + a_4 = 29$

$a_6 = ar^5$

$a \cdot ar^4 = 28$ $\quad ar + ar^3 = 29$

$a^2 r^4 = 28$ $\quad ar(1+r^2) = 29$

$a^2 r^2 (1+r^2)^2 = 29^2$

$\frac{(1+r^2)^2}{r^2} = \frac{29^2}{28}$

$\frac{r^4 + 2r^2 + 1}{r^2} = \frac{841}{28}$

$r^2 + \frac{1}{r^2} + 2 = \frac{841}{28}$

$r^2 + 1/r^2 = \frac{841-56}{28} = \frac{785}{28}$

$\frac{r^4+1}{r^2} = \frac{785}{28}$

$28r^4 - 785r^2 + 28 = 0$

$28r^4 - 784r^2 - r^2 + 28 = 0$

(A) 812

(B) 784

(C) 628

(D) 526

Ans. B

$$28r^2(r^2-28)-1(r^2+28)=0$$

$$(28r^2-1)(r^2-28)=0$$

$$r^2=28,\ \cancel{\frac{1}{28}}$$ — b'wz inc G.P

$$a^2=\frac{1}{28},\ 28^3$$

$$(ar^5)^2=a^2r^{10}$$

$$=\frac{1}{28}\cdot 28^5=28^4$$

$$ar^5=28^2=784$$

QUESTION [JEE Mains 2025 (22 Jan)]

Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive terms. If $a_1 a_5 = 28$ and $a_2 + a_4 = 29$, then a_6 is equal to:

M2 $a_1 \cdot a_5 = 28$, $a_2 + a_4 = 29$ $\qquad a_6 = ar^5$

$a_2 \cdot a_4 = 28$, $a_2 + a_4 = 29$.

$a_2 = 1$ — $ar = 1$
$a_4 = 28$ — $ar^3 = 28$ $\;\; r^2 = 28$

$a_6 = a_4 \cdot r^2 = 28 \cdot 28 = 784$

$a_6 = 784$.

(A) 812

(B) 784

(C) 628

(D) 526

Ans. B

QUESTION [JEE Mains 2025 (3 April)]

TAH 01

Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive numbers. If $a_3 a_5 = 729$ and $a_2 + a_4 = \frac{111}{4}$, then $24(a_1 + a_2 + a_3)$ is equal to

(A) 128

(B) 129

(C) 131

(D) 130

Ans. B

QUESTION [JEE Mains 2023 (29 Jan)]

Ta402

Let $a_1, a_2, a_3, \ldots\ldots$ be a G.P. of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then $a_1 a_9 + a_2 a_4 a_9 + a_5 + a_7$ is equal to

$a_4 \cdot a_6 = 9$ $\quad$ $a_5 + a_7 = 24$

$a_4 \cdot r + a_6 \cdot r = 24$

$a_4 + a_6 = \frac{24}{r}$

Ans. 60

QUESTION [JEE Mains 2024 (8 April)]

TAH 03

In an increasing geometric progression of positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the 4^{th}, 6^{th} and 8^{th} terms is equal to:

(A) 78

(B) 96

(C) 91

(D) 84

Ans. C

QUESTION [JEE Mains 2025 (7 April)]

If the sum of the second, fourth and sixth terms of a G.P. of positive terms is 21 and the sum of its eighth, tenth and twelfth terms is 15309, then the sum of its first nine terms is:

(A) 757

(B) 755

(C) 750

(D) 760

$a_2 + a_4 + a_6 = 21$

$a_8 + a_{10} + a_{12} = 15309 - a_2 \cdot r^6 + a_4 \cdot r^6 + a_6 \cdot r^6 = 15309$

$r^6 (a_2 + a_4 + a_6) = 15309$

$r^6 = \frac{\cancel{15309}^{729}}{\cancel{21}}$

$r^6 = 3^6 \Rightarrow r = 3$

$a_2 + a_4 + a_6 = 21$

$a(r + r^3 + r^5) = 21$

$a(3 + 3^3 + 3^5) = 21$

$a = \frac{21}{273} = \frac{1}{13}.$

$S_9 = a + ar + ar^2 + ar^3 + \dots + ar^8$

$= \frac{a(r^9 - 1)}{r-1} = \frac{1}{13} \frac{(3^9 - 1)}{2} = 757$

Ans. A

QUESTION [JEE Mains 2024 (29 Jan)]

If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P., then the common ratio of the G.P. is equal to

$$a_1 + a_2 + \cdots + a_{64} = 7\left(a_1 + a_3 + a_5 + \cdots + a_{63}\right)$$

(A) 7

$$\frac{\cancel{a(r^{64}-1)}}{r-1} = 7\left(\cancel{a} \cdot \frac{\cancel{((r^2)^{32}-1)}}{r^2-1}\right)$$

(B) 6

$$\frac{1}{r-1} = \frac{7}{(r-1)(r+1)}$$

(C) 5

$$r+1 = 7$$

$$r = 6.$$

(D) 4

QUESTION [JEE Mains 2024 (9 April)]

Let a, ar, ar^2, ... be an infinite G.P.

$a^3 + a^3r^3 + a^3r^6 + a^3r^9 + \cdots \infty$

If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a^3r^{3n} = 9747$, then $a + 18r$ is equal to

(A) 27

(B) 38

(C) 31

(D) 46

$\frac{a}{1-r} = 57, \quad \frac{a^3}{1-r^3} = 9747$

$\frac{a^3}{(1-r)^3} = 57^3$

$\frac{(1-r)^3}{1-r^3} = \frac{\cancel{9747}}{\cancel{57}\cdot\cancel{57}\cdot 57} = \frac{1}{19}$

$\frac{(1-r)^2}{1+r+r^2} = \frac{1}{19}$

$19 + 19r^2 - 38r = 1 + r^2 + r$

$18r^2 - 39r + 18 = 0$

Ans. C

$$18r^2 - 39r + 18 = 0$$

$$6r^2 - 13r + 6 = 0$$

$$6r^2 - 9r - 4r + 6 = 0$$

$$(3r-2)(2r-3) = 0$$

$$r = 2/3, \cancel{3/2} \quad -1 < r < 1, \; r \neq 0$$

$$r = 2/3$$

$$\frac{a}{1-r} = 57$$

$$3a = 57$$

$$a = 19$$

$$a + 18r$$

$$= 19 + 12$$

$$= 31$$

QUESTION [JEE Mains 2017]

★★KCLS★★

For any three positive real numbers a, b and c, if

$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$, then :

(A) b, c and a are in G.P.

(B) b, c and a are in A.P.

(C) a, b and c are in A.P.

(D) a, b and c are in G.P.

$225a^2 + 9b^2 + 25c^2 - 75ac - 45ab - 15bc = 0$

$(15a)^2 + (3b)^2 + (5c)^2 - 15a \cdot 3b - 3a \cdot 5c - 15a \cdot 5c = 0$

$15a = 3b = 5c = 45\lambda.$

$a = 3\lambda$

$b = 15\lambda$

$c = 9\lambda.$

$a + b = 18\lambda = 2c$

b, c, a are in A.P

a, c, b are in A.P

QUESTION [IIT-JEE 1987]

Tah 04

★★ ASRQ ★★

If a, b, c, d and p are distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$, then a, b, c, d

(A) are in A.P.

(B) are in G.P.

(C) are in H.P.

(D) satisfy $ab = cd$

Ans. B

QUESTION [JEE Mains 2023 (10 April)]

$(2\lambda)^2$ – form $4K$
$(2\lambda+1)^2$ – form $4K+1$

Let the first term α and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three terms is 33033, then the sum of these three terms is equal to

(A) 241

(B) 231

(C) 220

(D) 210

$\alpha, r \in I^+$

$\alpha^2 + \alpha^2 r^2 + \alpha^2 r^4 = 33033$

$\alpha^2(1+r^2+r^4) = 11 \times 3003 = 11 \times 3 \times 1001 = 11 \times 3 \times 11 \times 91$

$= 11^2 \times 13 \times 7 \times 3.$

Integer sq. of an integer

Integer

$\alpha^2 = 11^2$

$\alpha = 11$

$1+r^2+r^4 = 91 \times 3 = 273$

$r^4 + r^2 - 272 = 0$

$r^4 + 17r^2 - 16r^2 - 272 = 0$

$(r^2-16)(r^2+17) = 0$

$r = 4.$

$S_3 = \dfrac{\alpha(r^3-1)}{r-1} = 11 \times \dfrac{63}{3} = 11 \times 21 = 231$

$\alpha = 1$

$r^4 + r^2 + 1 = 33033$

$r^4 + r^2 - 33032 = 0$

$D = 1 + 4 \cdot 33032$

$\downarrow$

Not a perfect square

$r^2 \notin I$

QUESTION [JEE Mains 2023 (24 Jan)]

★★ASRQ★★

The 4^{th} term of G.P. is 500 and its common ratio is $\frac{1}{m}$, $m \in \mathbb{N}$. Let S_n denote the sum of the first n terms of this G.P. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is

$S_6 > S_5 + 1$, $S_7 < S_6 + 1/2$

$T_4 = 500$

$r = \frac{1}{m}$, $m \in N$.

Tah 05

Ans. 12

QUESTION [JEE Advanced 2014 (Paper 1)]

★★KCLS★★

If a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of $\frac{a^2+a-14}{a+1}$ is

$a, b, c \in I^+$, $\frac{b}{a} \in I^+$

$b^2 = ac$.

$c = \frac{b^2}{a}$

$\frac{a+b+c}{3} = b+2$.

$a+c = 2b+6$

$a + \frac{b^2}{a} = 2b+6$

$1 + \frac{b^2}{a^2} = \frac{2b}{a} + \frac{6}{a}$

$\frac{b^2}{a^2} - \frac{2b}{a} + 1 = \frac{6}{a}$

$\left(\frac{b}{a} - 1\right)^2 = 6/a$

Ans. 4

$\left(\frac{b}{a}-1\right)^2 = \frac{6}{a} - I^+$ should be square of an Integer

square of an Integer.

$a = \cancel{1}, \cancel{2}, \cancel{3}, 6.$ ✓

$a = 6.$

$\frac{a^2+a-14}{a+1} = \frac{42-14}{7} = 4$ Ans.

QUESTION [JEE Mains 2024 (30 Jan)]

Let a and b be two distinct positive real numbers. Let 11th term of a G.P., whose first term is a and third term is b, is equal to pth term of another G.P., whose first term is a and fifth term is b. Then p is equal to

(A) 20

(B) 24

(C) 21

(D) 25

$a, b \in R^+$

$T_1 = a$

$T_1 = a,\ T_3 = b \longrightarrow T_{11} = T_p$

$T_5 = b. = ar'^4$

$r'^4 = \frac{b}{a}$

$ar^2 = b$

$r^2 = \left(\frac{b}{a}\right)$

$ar^{10} = a\, r'^{p-1}$

$a \cdot (r^2)^5 = a\, r'^{p-1}$

$a\left(\frac{b}{a}\right)^5 = a \cdot (r'^4)^{\frac{p-1}{4}}$

$\cancel{a} \cdot (b/a)^5 = \cancel{a} \cdot (b/a)^{\frac{p-1}{4}}$

$\left(\frac{b}{a}\right)^5 = \left(\frac{b}{a}\right)^{\frac{p-1}{4}}$

$\frac{p-1}{4} = 5.$

$p = 21$

Problems Involving both A.P & G.P.

QUESTION [JEE Mains 2025 (7 April)]

Let x_1, x_2, x_3, x_4 be in a geometric progression. If 2, 7, 9, 5 are subtracted respectively from x_1, x_2, x_3, x_4, then the resulting numbers are in an arithmetic progression. Then the value of $\frac{1}{24}(x_1x_2x_3x_4)$ is: $\frac{1}{24} \cdot a \cdot ar \cdot ar^2 \cdot ar^3 = \frac{1}{24} a^4 r^6 = \frac{(-3)^4 \cdot 2^6}{24} = \frac{27 \cdot 3 \cdot 2^3 \cdot 8}{2 \cdot 4} = 216$

(A) 18

(B) 216

(C) 36

(D) 72

x_1, x_2, x_3, x_4 G.P

$x_1 - 2,\ x_2 - 7,\ x_3 - 9,\ x_4 - 5$ A.P

$a - 2,\ ar - 7,\ ar^2 - 9,\ ar^3 - 5$ A.P

$2(ar - 7) = a - 2 + ar^2 - 9$

$2ar - 14 = a - 2 + ar^2 - 9$

$a(r^2 + 1 - 2r) = -3$

$a(1 - r)^2 = -3.$

$2(ar^2 - 9) = ar - 7 + ar^3 - 5.$

$2ar^2 - 18 = ar + ar^3 - 12$

$ar(r^2 + 1 - 2r) = -6.$

$ar(1 - r)^2 = -6.$

$r = 2$

$a = -3$

Ans. B

QUESTION [JEE Mains 2024 (29 Jan)]

If each term of a geometric progression $a_1, a_2, a_3, \ldots$ with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \cdots + a_n$, then $S_{20} - S_{18}$ is equal to

(A) -2^{15}

(B) 2^{15}

(C) -2^{18}

(D) 2^{18}

$$T_m = \frac{T_{m+1} + T_{m+2}}{2}$$

$a_1 = \frac{1}{8}$ $\quad a_2 \neq a_1$

$ar \neq a$

$r \neq 1.$

$$2ar^{m-1} = ar^m + ar^{m+1}$$

$$2 = r + r^2$$

$$r^2 + r - 2 = 0$$

$$(r+2)(r-1) = 0$$

$$r = -2, \cancel{1}$$

$$S_{20} - S_{18} = a_{19} + a_{20}$$

$$= \frac{1}{8} \cdot (-2)^{18} + \frac{1}{8}(-2)^{19}$$

$$= \frac{1}{8}(-2)^{18}(1-2)$$

$$= -2^{15}.$$

Ans. A

$$S_{20} = a_1 + a_2 + a_3 + \cdots + a_{18} + a_{19} + a_{20}$$

$$S_{18} = a_1 + a_2 + a_3 + \cdots + a_{18}$$

$$S_{20} - S_{18} = a_{19} + a_{20}.$$

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Today's SHIKAARS

QUESTION [JEE Mains 2025 (24 Jan)]

S 01

In an arithmetic progression, if $S_{40} = 1030$ and $S_{12} = 57$, then $S_{30} - S_{10}$ is equal to:

(A) 525

(B) 505

(C) 510

(D) 515

Ans. D

QUESTION [JEE Mains 2025 (23 Jan)]

S 02

The roots of the quadratic equation $3x^2 - px + q = 0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88, then $q - 2p$ is equal to

Ans. 474

QUESTION [JEE Mains 2025 (22 Jan)]

S 03

Suppose that the number of terms in an A.P. is $2k, k \in N$. If the sum of all odd terms of the A.P. is 40, the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27, then k is equal to:

(A) 8

(B) 6

(C) 4

(D) 5

Ans. D

QUESTION [JEE Mains 2025 (23 Jan)]

S 04

If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

(A) -120

(B) -1200

(C) -1080

(D) -1020

Ans. C

QUESTION [JEE Mains 2024 (30 Jan)]

S 05

Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{20} = 790$ and $S_{10} = 145$, then $S_{15} - S_5$ is:

(A) 405

(B) 390

(C) 410

(D) 395

Ans. D

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

KTK 01

The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is

Ans. 14

QUESTION [JEE Mains 2024 (6 April)]

KTK 02

For $A = \{n \in [100, 700] \cap \mathbb{N} : n \text{ is neither a multiple of 3 nor a multiple of 4}\}$. Then the number of elements in A is

(A) 290

(B) 280

(C) 300

(D) 310

Ans. C

QUESTION [JEE Mains 2023 (29 Jan)]

KTK 03

Let $\{a_k\}$ and $\{b_k\}, k \in N$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k, k \in N$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then

$\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to

Ans. 9

QUESTION [JEE Mains 2024 (6 April)]

KTK 04

Let α, β be roots of $x^2 + \sqrt{2}x - 8 = 0$. If $U_n = \alpha^n + \beta^n$, then $\frac{U_{10} + \sqrt{2}U_9}{2U_8}$ is equal to

Ans. 4

QUESTION

KTK 05

If m^{th} term of an A.P. is n & n^{th} term is m, then show that its $(m + n)^{th}$ term is zero.

Remmember This!!

Solution to Previous TAH

QUESTION

If $a_1 a_2 a_3$ ..... a_n are n arithmetic means inserted between 7 and 2015 whose sum is 56616 then

(A) n is 56

(B) n is 28

(C) $a_{19} = 2029/3$

(D) $a_{19} = 36543/57$

Q. If $a_1, a_2, a_3 \dots a_n$ are n A.M inserted btw 7 & 2015 whose sum is 56616 then. (TAH-1)

(A) n is 56

(B) n is 28

(C) $a_{19} = 2029/3$

(D) $a_{19} = 36543/57$

$7, a_1, a_2, a_3 \dots a_n, 2015$

$a_1 + a_2 + a_3 \dots + a_n = 56616$

$\frac{n}{2}[2(7+d) + (n-1)d] = 56616$

$\frac{n}{2}[14 + 2d + nd - n] = 56616$

$n[14 + d(n+1)] = 56616 \times 2$

$a + (n+1)d = 2015$

$(n+1)d = 2008$

$= n[14 + 2008] = 56616 \times 2$

$n = \frac{56616 \times \cancel{2}}{\cancel{2022}\ 1011} = 56$

$d = \frac{2008}{57}$

$a_{19} = 7 + \cancel{18} \times \frac{2008}{\cancel{57}\ 3} = 7 + \frac{2008}{3} = \frac{2029}{3}$

QUESTION [JEE Advanced 2018]

The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

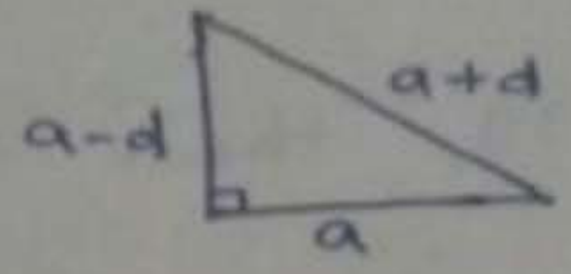
Ans. 6

TAH-02

Q The sides of a right angeled Δ are in A.P. If the Δ has area 24, then what is the length of its smallest side?

$\rightarrow$

Area $= \frac{1}{2}(a-d)\cdot a = 24$



$(a-d)\cdot a = 48$

$a^2 - ad = 48 \Rightarrow 3ad = 48 \Rightarrow ad = 16$

$a^2 + (a-d)^2 = (a+d)^2$ (By pythagores theorem)

$a^2 + \cancel{a^2} + \cancel{d^2} - 2ad = \cancel{a^2} + \cancel{d^2} + 2ad$

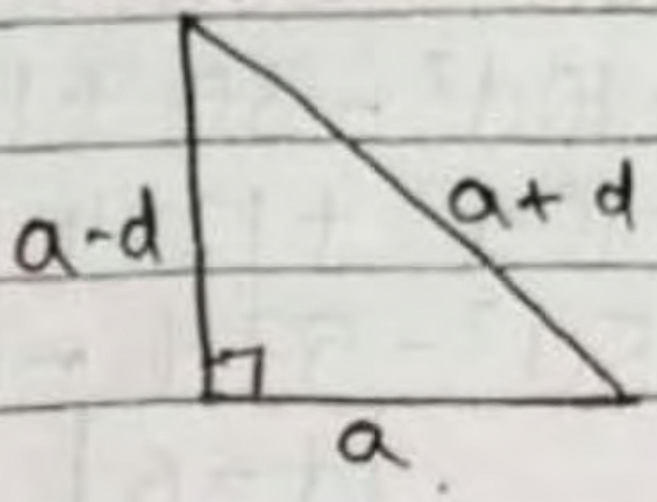
$a^2 = 4ad$ $\qquad ad = 16$

$a^2 = 64$ $\qquad d = 2$

$a = 8$ $\qquad \therefore$ Smallest side $= a-d = 8-2 = 6$

Muskan

TAH2: $\frac{1}{2}\cdot(a-d)\cdot a = 24$



$a^2 - ad = 48$

$= 16d^2 - 4d^2 = 48$

$12d^2 = 48$

$d^2 = 4$

$\boxed{d = \pm 2}$

$\boxed{\begin{matrix} d = 2 \\ a = 8 \end{matrix}}$

By pythageoras theoram

$(a+d)^2 = (a-d)^2 + a^2$

$\cancel{a^2} + \cancel{d^2} + 2ad = \cancel{a^2} + \cancel{d^2} - 2ad + a^2$

$a^2 = 4ad$

$a = 4d$

Length of smallest side $= 8 - 2 = 6$ Ans

QUESTION [JEE Mains 2025 (4 April)]

Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of A.P.'s in A and B respectively such that $D = d + 3, d > 0$. If $\frac{p+q}{p-q} = \frac{19}{5}$, then $p - q$ is equal to

(A) 540

(B) 450

(C) 600

(D) 630

Ans. A

IAH3: Set A : $a-d,\ a,\ a+d$

$\text{Sum} = 3a = 36 \Rightarrow \boxed{a=12}$

$\text{Product} = (a-d)(a+d)(a) = a^3 - ad^2 = p$

$p = (12)^3 - 12d^2$

$\Longrightarrow$ Set B : $A-D,\ A,\ A+D$

$\text{Sum} = 3A = 36 \Rightarrow \boxed{A=12}$

$\text{Product} = A^3 - AD^2 = q$

$= (12)^3 - 12D^2 = q$

$$\Rightarrow \frac{p+q}{p-q} = \frac{19}{5}$$

$$= \frac{(12)^3 - 12d^2 + (12)^3 - 12D^2}{\cancel{(12)^3} - 12d^2 - \cancel{(12)^3} + 12D^2} = \frac{19}{5}$$

$$= \frac{(12)^3 - 12d^2 + (12)^3 - 12(d^2+9+6d)}{12(d^2+9+6d) - 12d^2} = \frac{19}{5}$$

$$= \frac{2\times(12)^3 - 12d^2 - 12d^2 - 108 - 72d}{\cancel{12d^2} + 108 + 72d - \cancel{12d^2}} = \frac{19}{5}$$

$$= \frac{\cancel{12}(288 - d^2 - d^2 - 9 - 6d)}{\cancel{12}(9+6d)} = \frac{19}{5}$$

DATE:

$$\frac{-2d^2 - 6d + 279}{9 + 6d} = \frac{19}{5}$$

$$-10d^2 - 30d + 1395 = 171 + 114d$$

$$= -10d^2 - \cancel{30} + 1395 - 171 = 114d + 30d$$

$$= -10d^2 + 1224 = 144d$$

$$\Rightarrow 5d^2 - 57d - 612 = 0$$

$$\boxed{d = 6}$$

$$p - q = 108 + 72 \times 6$$

$$= 108 + 432 = 540 \Rightarrow \underline{\underline{\text{Ans}}}$$

QUESTION [JEE Mains 2021 (August)]

Let $a_1, a_2, \ldots.. a_{21}$ be an A.P. such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$.

If the sum of this A.P. is 189, then $a_6 \cdot a_{16}$ is equal to:

(A) 57

(B) 72

(C) 48

(D) 36

Q. TAMS

Let $a_1, a_2, a_3 \ldots a_{21}$ be an A.P. such that

$$\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$$ If the sum of this A.P.

is 189, then $a_6 \cdot a_{16}$ is equal to:

Soln:- $S = \frac{1}{a_1 \cdot a_2} + \frac{1}{a_2 \cdot a_3} + \cdots + \frac{1}{a_{20}\, a_{21}} = \frac{4}{9}$

$$\frac{1}{d}\left(\frac{d}{a_1 a_2} + \frac{d}{a_2 a_3} + \cdots + \frac{d}{a_{20} a_{21}}\right) = \frac{4}{9}$$

$$\frac{1}{d}\left(\frac{a_2 - a_1}{a_1 a_2} + \frac{a_3 - a_2}{a_2 a_3} + \cdots + \frac{a_{21} - a_{20}}{a_{20} a_{21}}\right) = \frac{4}{9}$$

$$\frac{1}{d}\left(\frac{1}{a_1} - \cancel{\frac{1}{a_2}} + \cancel{\frac{1}{a_2}} - \cancel{\frac{1}{a_3}} + \cdots \cancel{\frac{1}{a_{20}}} - \frac{1}{a_{21}}\right) = \frac{4}{9}$$

$$\frac{1}{d}\left(\frac{a_{21} - a_1}{a_1 a_{21}}\right) = \frac{4}{9}$$

$$\cancel{\frac{1}{d}}\left(\frac{\cancel{a} + 20\cancel{d} - \cancel{a}}{a(a+20d)}\right) = \frac{4}{9}$$

$$\frac{20\ 5}{a(9+10d)} = \frac{4}{9}$$

$$(9-10d)(9+10d) = 45$$

$$81 - 100d^2 = 45$$

$$100d^2 = 36$$

$$d^2 = \frac{36}{100}, \frac{-36}{100}$$

$$d = \frac{6}{10}, -\frac{6}{10}$$

$$d = \frac{3}{5}, -\frac{3}{5}$$

$S_{21} = 189$

$\frac{21}{2}[2a_1 + 20d] = 189$

$a_1 + 10d = 9$ — (1)

$d = 3/5$	$d = -3/5$
$a + 10 \times \frac{3}{5} = 9$	$a - 10 \times \frac{3}{5} = 9$
$a + 6 = 9$	$a = 9 + 6$
$a = 3$	$a = 15$

$a_6 \cdot a_{16} = (a+5d)(a+15d)$

$= (6)(12)$

$= 72$

$a_6 \cdot a_{16} = (a+5d)(a+15d)$

$= (15-3)(15-9)$

$= 72$

Ans $= 72$

Muskan

TAMS 4

$$\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$$

and $S_{21} = a_1 + a_2 + \cdots + a_{21} = 189$

$= \frac{21}{2}(2a_1 + 20d) = 189$

$\Rightarrow 2a_1 + 20d = 18$

$\Rightarrow a_1 + 10d = 9$ — (i)

$$\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$$

$$\Rightarrow \left[\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \cdots + \frac{1}{a_{20} a_{21}}\right] = \frac{4}{9}$$

$$\Rightarrow \frac{d}{d}\left[\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \cdots + \frac{1}{a_{20} a_{21}}\right] = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\left[\frac{a_2 - a_1}{a_1 a_2} + \frac{a_3 - a_2}{a_2 a_3} + \cdots + \frac{a_{21} - a_{20}}{a_{20} a_{21}}\right] = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\left[\frac{1}{a_1} - \cancel{\frac{1}{a_2}} + \cancel{\frac{1}{a_2}} - \cancel{\frac{1}{a_3}} + \cdots + \cancel{\frac{1}{a_{20}}} - \frac{1}{a_{21}}\right] = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\left[\frac{1}{a_1}-\frac{1}{a_{21}}\right]=\frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\left[\frac{a_{21}-a_1}{a_1 a_{21}}\right]=\frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\left[\frac{\cancel{a_1}+20d-\cancel{a_1}}{a_1 a_{21}}\right]=\frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\left[\frac{20d}{a_1 a_{21}}\right]=\frac{4}{9}$$

$$\Rightarrow 20\times 9 = 4a_1a_{21}$$

$$\Rightarrow a_1a_{21}=45$$

$$\Rightarrow 45 = a_1(a_1+20d)$$

$$\Rightarrow a_1^2+20a_1d = 45$$

$$\Rightarrow a_1(a_1+20d)=45$$

$$\Rightarrow a_1(18-a_1)=45$$

$$\Rightarrow -a_1^2+18a_1=45$$

$$\Rightarrow a_1^2-18a_1+45=0$$

$$\Rightarrow a_1^2-15a_1-3a_1+45=0$$

$$\Rightarrow a_1(a_1-15)-3(a_1-15)=0$$

$$\Rightarrow (a_1-3)(a_1-15)=0$$

$a_1=3 \mid a_1=15$

case ① $a_1=3$

$$a_6\cdot a_{16}$$

$$=(a_1+5d)\times(a_1+15d)$$

$$=\left(3+5\times\frac{6}{10}\right)\times\left(3+15\times\frac{6}{10}\right)$$

$$=\left(3+\frac{6}{2}\right)\times\left(3+\frac{18}{2}\right)$$

$$=6\times 12$$

$$=72$$ [Ans → B]

(Case ii) $a_1 = 15$

$a_6 a_{16}$

$= (a_1 + 5d)(a_1 + 15d)$

$= \left(15 + 5 \times \frac{-6}{10}\right)\left(15 + 15 \times \frac{-6}{10}\right)$

$= (15 - 3)(15 - 9)$

$= 12 \times 6$

$= 72$ [Ans $\rightarrow$ B]

$\boxed{\text{Ans} \rightarrow \text{B}}$

QUESTION [JEE Mains 2021 (27 July)]

If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to

Ans. 3

RHS: $2\log_3(2^x-5) = \log_3 2 + \log_3(2^x - 7/2)$

$\log_3(2^x-5)^2 = \log_3 2(2^x - 7/2)$

$= (2^x-5)^2 = 2(2^x - 7/2)$

$= 2^{2x} + 25 - 10\times 2^x = 2^{x+1} - 7$

$2^x = t$

$t^2 + 25 - 10t = 2t - 7$

$t^2 - 12t + 32 = 0$

$t^2 - 8t - 4t + 32 = 0$

$t(t-8) - 4(t-8) = 0$

$t = 4, 8$

$t = 4$ $\qquad\qquad$ $t = 8$

$2^x = 4$ $\qquad\qquad$ $2^x = 8$

$\boxed{x = 2}$ (NP) $\qquad\qquad$ $\boxed{x = 3}$ ✓

THANK YOU