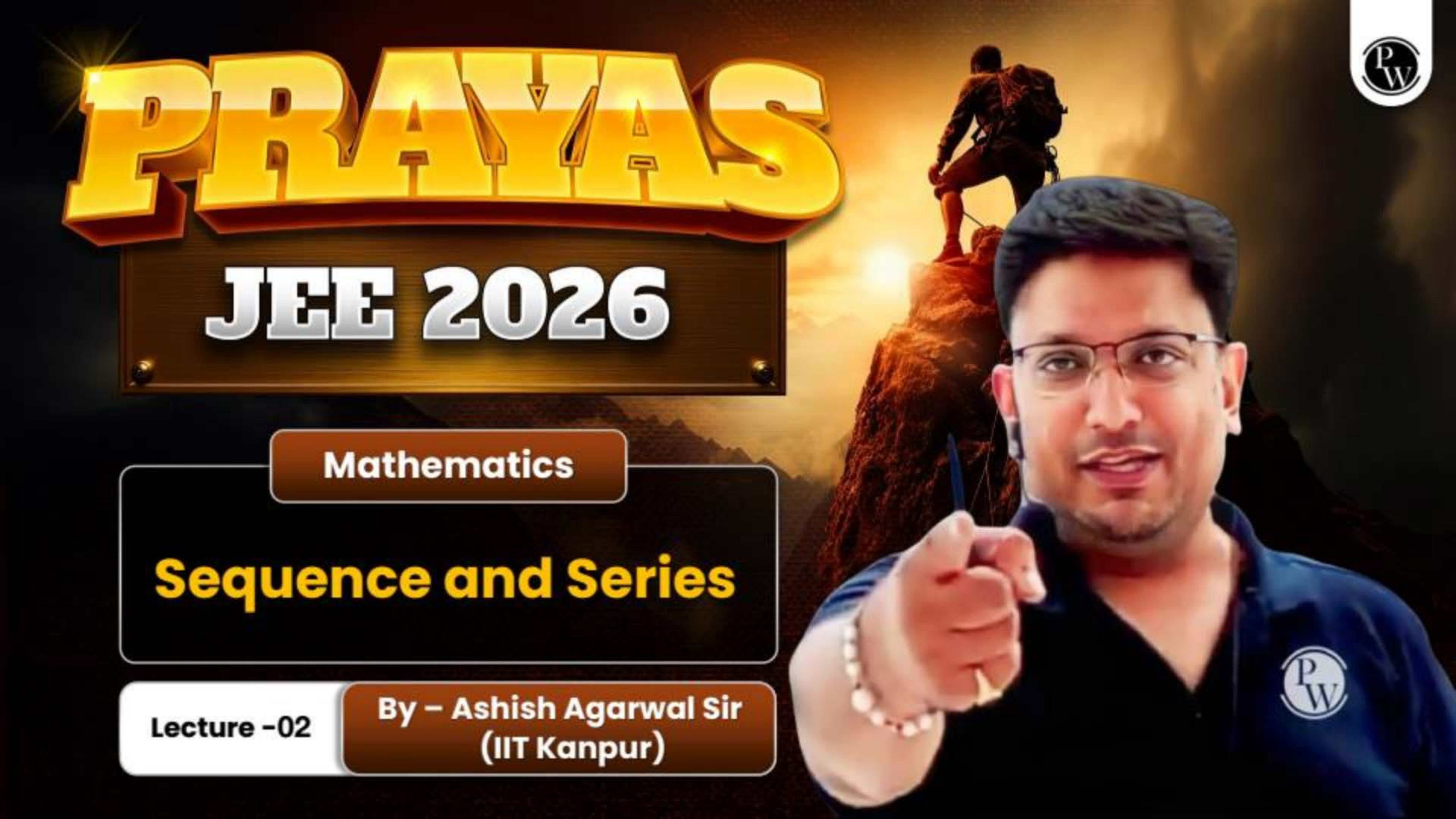
PRAYAS
JEE 2026
Mathematics
Sequence and Series
Lecture -02
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Topics *to be covered*

A Introduction to Sequence & Series

B General term and sum of n terms of an AP

Recap *of previous lecture*

1. For any series if S_n denotes the sum of n terms then $S_n - S_{n-1} =$ T_n

2. For any series with sum of 'n' terms denoted by S_n we have

   $T_1 =$ S_1, $T_2 =$ $S_2 - S_1$, $T_3 =$ $S_3 - S_2$, $T_n =$ $S_n - S_{n-1}$

   $S_1 = T_1$
   $S_2 = T_1 + T_2$
   $S_3 = T_1 + T_2 + T_3$
   ⋮

3. For an A.P. if $d > 0$ then it is Inc A.P, if $d < 0$ then it is Dec A.P & if $d = 0$ then it is Constant A.P

Recap *of previous lecture*

4. For an A.P. $T_n = \underline{a+(n-1)d}$

   $S_n = \underline{\frac{n}{2}(2a+(n-1)d)} = \underline{\frac{n}{2}(a+l)}$

5. For any sequence of n terms k^{th} term from end = $\underline{(n-k+1)}^{th}$ term from beginning.

   $\xrightarrow{d=2}$ $\xleftarrow{d=-2}$

6. 2, 4, 6, ..., 204 then T_{20} from end is $\underline{204+(20-1)(-2)} = 204-38 = 166$ Ans.

Recap *of previous lecture*

7. If $S_n = an^2 + bn$ then it is an A.P with common difference $2a$ & first terms = $a+b$

8. In an A.P. sum of any two terms equidistant from beginning & end is always constant & is equal to sum of first & last term.

Homework Discussion

QUESTION

Find all values of the parameter 'a' for which the inequality $4^x - a \cdot 2^x - a + 3 \le 0$ is satisfied for atleast one real 'x'.

let $2^x = t > 0$

$t^2 - a \cdot t - a + 3 \le 0$ should for some $t > 0$.

Case (I)

$f(0) < 0$

$-a + 3 < 0$

$a > 3$

Case (II)

$f(0) \ge 0 \quad 3 - a \ge 0 \quad a \le 3$

$D \ge 0 \quad a^2 + 4a - 12 \ge 0 \rightarrow (a+6)(a-2) \ge 0$

$-\frac{b}{2a} > 0 \rightarrow \frac{a}{2} > 0 \Rightarrow a > 0 \quad a \in (-\infty, -6] \cup [2, \infty)$

$\Downarrow$

$a \in [2, 3]$

$\cup$

$a \in [2, \infty)$

Ans. $[2, \infty)$

QUESTION

Let $P(x) = (m^2 + 4m + 5)x^2 - 4x + 7, m \in R$. If $3 \leq x \leq 5$ then find the minimum of minimum value of $P(x)$.

$$x_v = \frac{4}{2(m^2+4m+5)} = \frac{2}{m^2+4m+4+1} = \frac{2}{\underbrace{(m+2)^2+1}_{[1,\infty)}}$$

$2(0,1] = (0,2]$

$f(3) = \min$

$\min = 9m^2+36m+45-12+7$

$g(m) = 9m^2+36m+40$

$= 9(m^2+4m+4)+40-36$

$= 9(m+2)^2+4$

$g(m)\Big|_{\min} = 4$

Ans. 4

QUESTION

Let $x^2 - (m - 3)x + m = 0$ $(m \in R)$ be a quadratic equation. Find the value of m for which the roots of the equation are

(a) Positive

(b) Negative

(c) such that at least one is positive

(d) One root is smaller than 2 and other root is greater than 2

(e) Both the roots are greater than 2

(f) Both the roots are smaller than 2

(g) Exactly one root lies in the interval $(1, 2)$

(h) Both the roots lie in the interval $(1, 2)$

(i) Such that at least one root lie in the interval $(1, 2)$

(j) One root is greater than 2 and the other root is smaller than 1

Answers :

(a) $m \in [9, \infty)$

(b) $m \in (0, 1]$

(c) $m \in (-\infty, 0) \cup [9, \infty)$

(d) $m \in (10, \infty)$

(e) $m \in [9, 10)$

(f) $m \in (-\infty, 1]$

(g) $m \in (10, \infty)$

(h) $m \in \phi$

(i) $m \in (10, \infty)$

(j) $m \in \phi$

QUESTION [JEE Advanced 2017]

Paragraph

Let p, q, be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, \ldots$, let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

If $a_4 = 28$, then $p + 2q =$ NF $a_{n+2} - a_{n+1} - a_n = 0$

(A) 12 $a_1 = p\alpha + q\beta$ $n=10$ $a_{12} - a_{11} - a_{10} = 0$

$a_0 = p+q$

$a_{12} = a_{11} + a_{10}$.

(B) 14

$n=2$ $a_4 - a_3 - a_2 = 0 \Rightarrow a_4 = a_3 + a_2 = 2a_1 + a_0 + a_1 + a_0 = 3a_1 + 2a_0$

$n=1$ $a_3 - a_2 - a_1 = 0 \Rightarrow a_3 = a_2 + a_1 = 2a_1 + a_0$

(C) 21

$n=0$ $a_2 - a_1 - a_0 = 0 \Rightarrow a_2 = a_1 + a_0$

(D) 7

$$a_4 = 3a_1 + 2a_0$$

$$28 = 3(p\alpha + q\beta) + 2(p+q)$$

$$x^2 - x - 1 = 0$$

$$\alpha, \beta = \frac{1 \pm \sqrt{5}}{2}$$

$$28 = 3p\left(\frac{1+\sqrt{5}}{2}\right) + 3q\left(\frac{1-\sqrt{5}}{2}\right) + 2p + 2q$$

$$28 = \left(\frac{7p+7q}{2}\right) + \left(\frac{3p - 3q}{2}\right)\sqrt{5}$$

$$\underbrace{\frac{7}{2}(p+q) - 28}_{\text{Rational}} + \underbrace{\frac{3}{2}(p-q)}_{\text{Rational}}\sqrt{5} = 0$$

$$\frac{7}{2}(p+q) = 28 \qquad p = q$$

$$p + q = 8$$

$$p = q = 4$$

QUESTION [JEE Advanced 2017]

Paragraph

Let p, q, be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, \ldots$, let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

$a_{12} =$

(A) $2a_{11} + a_{10}$

(B) $a_{11} - a_{10}$

(C) $a_{11} + 2a_{10}$

(D) $a_{11} + a_{10}$

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

Sequence

आरंभ

Series

QUESTION

If m^{th} term of an A.P. is n & n^{th} term is m, then show that its $(m + n)^{th}$ term is zero.

QUESTION

If $S_1, S_2, S_3, \ldots\ldots, S_p$ are the sums of n terms of 'p' arithmetic series whose first term and 1, 2, 3, 4, ... and whose common difference are 1, 3, 5, 7, ...

Prove that $S_1 + S_2 + S_3 + \ldots\ldots + S_p = \frac{np}{2}(np + 1)$

QUESTION [JEE Mains 2023 (1 Feb)]

Let $a_1 = 8, a_2, a_3, ..., a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is

Ans. 754

QUESTION [JEE Mains 2023 (Jan)]

Number of 4 digit integers less than 2800 which are either divisible by 3 or by 11 is equal to

Ans. 710

QUESTION [JEE Mains 2024 (6 April)]

For $A = \{n \in [100, 700] \cap \mathbb{N} : n \text{ is neither a multiple of 3 nor a multiple of 4}\}$. Then the number of elements in A is

(A) 290

(B) 280

(C) 300

(D) 310

Ans. C

QUESTION

How many terms of the series $20 + 19\frac{1}{3} + 18\frac{2}{3} + ...$ must be taken so that sum is 300. Explain the reason of double answer.

Properties of A.P.

Ex:
2, 4, 6, 8, 10 - - - A.P c.d = 2

$2 \pm \sqrt{3}, 4 \pm \sqrt{3}, 6 \pm \sqrt{3}, \ldots$ A.P c.d = 2

Property 1:

(a) If each term of an A.P. is increased or decreased by the same number then the resulting sequence is also an A.P. having the same common difference.

(b) If each term of an A.P. is multiplied or divided by the same non-zero number (k), then the resulting sequence is also an A.P. whose common difference is kd and d/k respectively, where d is common difference or original A.P.

÷ each term by 2

A.P : 2, 4, 6, 8, 10, 12 - - - - - c.d = 2

A.P: 1, 2, 3, 4, 5, 6 - - - - - c.d = $\frac{2}{2}$ = 1

A.P : 6, 12, 18, 24, 30, 36 - - - c.d = 2×3 = 6

Property 2:

If a_1, a_2, a_3 …. and b_1, b_2, b_3 …. are two A.P.s then $a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3$ …. are also in A.P. but $a_1 b_1, a_2 b_2$, …. and $\frac{a_1}{b_1}, \frac{a_2}{b_2}$, …. may or may not be in A.P.

$A \cdot P_1 \pm A \cdot P_2 = A \cdot P$ — $c.d = d_1 \pm d_2$

$A \cdot P_1 \times A \cdot P_2$, $A \cdot P_1 \div A \cdot P_2$ — may or may not be on A·P

$2, 4, 6, 8, 10, 12 \ldots$ — $d_1 = 2$

$3, 6, 9, 12, 15, \ldots$ — $d_2 = 3$

$5, 10, 15, 20, 25 \ldots$ — $d = d_1 + d_2 = 5$

$-1, -2, -3, -4, -5 \ldots$ — $d = d_1 - d_2 = -1$

An important fact about A.P.

In an A.P. summation of k^{th} term from beginning and k^{th} term from the last is always constant which is equal to summation of first term and last term.

1 4 7 10 13 16

17

17

17

$$T_k + T_{n-k+1} = \text{constant} = a + \ell$$

Property 3:

$T_n = pn + q$ $T_{n-1} = p(n-1) + q$

$T_n - T_{n-1} = p = \text{constant} \Rightarrow A.P$

1. If in any sequence, the n^{th} term is a linear function of n, then the given sequence is arithmetic progression.

2. If in any sequence Sn is a quadratic function of a with constant term zero then the sequence is an AP

Property 4:

Any term of an A.P. (except the first and last) is equal to half the sum of terms which are equidistant from it.

$$T_r = \frac{T_{r-k} + T_{r+k}}{2}, k < r$$

$$2, 4, 6, 8, \overset{-3d}{10}, 12, 14, \boxed{16}, 18, 20, \overset{+3d}{22}, 24 \ldots$$

$$16 = \frac{10+22}{2}$$

$$T_{r+k} = T_r + kd$$

$$T_{r-k} = T_r - kd$$

$$T_{r+k} + T_{r-k} = 2T_r$$

$$T_r = \frac{T_{r+k} + T_{r-k}}{2}.$$

$$10 = 16 - 3d$$

$$22 = 16 + 3d$$

$$10 + 22 = 2(16) \Rightarrow 16 = \frac{10+22}{2}$$

Property 5:

If sum of numbers in A.P. is given then always assume the numbers to be-

3 numbers in A.P. : $a - d, a, a + d$

4 numbers in A.P. : $a - 3d, a - d, a + d, a + 3d$

5 numbers in A.P. : $a - 2d, a - d, a, a + d, a + 2d$

6 numbers in A.P. : $a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$

c.d = d

c.d = 2d

Remember first term is not a

Property 6:

If we pick the term of an A.P. in a particular interval, then picked sequence is also an A.P. with common difference interval times the original common difference.

T_1 $\quad$ T_4 $\quad$ T_7

1 $\quad$ 3 $\quad$ 5 $\quad$ 7 $\quad$ 9 $\quad$ 11 $\quad$ 13 .... $\quad$ c.d = 2

Interval = 4 − 1 = 3

1, 7, 13, 19 - - - A.P = 2 × Interval ka length
= 2 × 3 = 6.

d_f = final common difference = 6

d_i = initial common difference = 2

d_f = interval × d_i

$S = \frac{5}{2}\left(\cancel{2}\cdot 4 + (5-1)\cancel{2}\cdot 2\right) = 5(4+8) = 60.$

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22

M① $S = S_{11} - S_4$

M② $S = \frac{7}{2}\left(2\cdot 10 + (7-1)2\right)$

$= 7(10+6) = 112$

2n terms: n even numbered, n odd numbered

2n+1 terms: n+1 odd numbered, n even numbered

QUESTION [JEE Mains 2025 (2 April)]

The number of terms of an A.P. is even; the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is:

(A) 6

(B) 4

(C) 8

(D) 10

$a_1, a_2, a_3, a_4 \dots a_{2n}$

$a_1 = a$, $a_2 = a+d$, $a_3 = a+2d$

M (1)

$a_2 + a_4 + a_6 + \dots + a_{2n} = 30$

$\frac{n}{2}(2a_2 + (n-1)2d) = 30$

$n(a+d+nd-d) = 30$

$n(a+nd) = 30$

$a_1 + a_3 + a_5 + \dots + a_{2n-1} = 24$

$\frac{n}{2}(2a_1 + (n-1)2d) = 24$

$n(a+(n-1)d) = 24$

$\ominus$

$(n^2 - n - n^2)d = -6.$

$nd = 6$

$a_{2n} - a_1 = \frac{21}{2}$

$a + (2n-1)d - a = \frac{21}{2}$

$(2n-1)d = \frac{21}{2}$

$2nd - d = 21/2$

$12 - d = 21/2$

$d = 3/2$

$n = 4$

Ans. B

$d = \frac{3}{2}$, $n = 4$, $a =$

$n(a + nd) = 30$

$a + 6 = \frac{30}{4} = \frac{15}{2}$

$a = \frac{3}{2}$

$\frac{3}{2}, 3, \frac{9}{2}, 6, \text{-----}$ up to 8 terms.

QUESTION [JEE Mains 2025 (2 April)]

The number of terms of an A.P. is even; the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is:

(A) 6

(B) 4

(C) 8

(D) 10

A.P: $a_1, a_2, a_3, a_4 - - - a_{2n}$

M(2)

$a_2 + a_4 + a_6 + - - + a_{2n} = 30$

$\ominus \quad a_1 + a_3 + a_5 + - - - + a_{2n-1} = 24$

$d + d + d + - - - + d = 6$

$nd = 6$ — (II)

$a_{2n} - a_1 = \frac{21}{2}$ — $\cancel{a} + (2n-1)d - \cancel{a} = \frac{21}{2}$

$(2n-1)d = \frac{21}{2}$ — (I)

$2nd - d = 21/2$

$d = 3/2$

$n = 4$

$a = 3/2$.

Ans. B

QUESTION [JEE Mains 2022 (24 June)]

If $\{a_i\}_{i=1}^{n}$, where n is an even integer, is an arithmetic progression with common difference 1, and $\sum_{i=1}^{n} a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to:

(A) 48

(B) 96

(C) 92

(D) 104

$a_1+a_2+a_3+a_4+a_5+a_6+ \cdots + a_{n-3}+a_{n-2}+a_{n-1}+a_n = 192.$

$\ominus \quad a_2+a_4+a_6+ \cdots + a_{n-2}+a_n = 120 \quad \times 2$

$(a_1-a_2)+(a_3-a_4)+(a_5-a_6)+ \cdots +(a_{n-1}-a_n) = 192-240.$

$-d \cdot \frac{n}{2} = -48$

$nd = 96.$

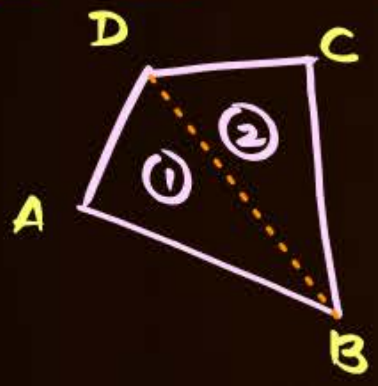
$n(1) = 96. \quad (\because d=1)$

Ans. B

QUESTION [JEE Mains 2025 (28 Jan)]

The interior angles of a polygon with n sides, are in an A.P. with common difference 6°. If the largest interior angle of the polygon is 219°, then n is equal to

Sum of Interior angles of n sided polygon $= (n-2)\times 180$



$(4-2)\times 180^\circ$

$(6-4)\times 180^\circ$

Angles of polygon $a,\ a+6,\ a+12,\ ---,\ a_n = 219^\circ \Rightarrow a_n = 219$

$a+(n-1)6 = 219$

$a = 225-6n$

$\frac{n}{2}\left(2a+(n-1)\cdot 6\right) = (n-2)\cdot 180^\circ$

$n(a+3n-3) = (n-2)\cdot 180^\circ$

$n(225-6n+3n-3) = 180(n-2)$

$n(222-3n) = 180(n-2)$

Ans. 20

$$222n - 3n^2 = 180n - 360$$

$$3n^2 - 42n - 360 = 0$$

$$n^2 - 14n - 120 = 0$$

$$n^2 - 20n + 6n - 120 = 0$$

$$n = 20, \cancel{-6}$$

QUESTION [JEE Mains 2022 (26 July)]

Different A.P.'s are constructed with the first term 100, the last term 199, and integral common differences. The sum of the common differences of all such A.P's having at least 3 terms and at most 33 terms is ______

$a = 100$ $\qquad$ $d \in I$

$l = 199 = 100 + (n-1)d$ $\qquad$ $3 \le n \le 33$.

$(n-1)d = 199 - 100$

$d = \dfrac{99}{n-1} \in I$

$n-1 = 1, 3, 11, 9, 33, 99$

$n = \cancel{2}, 4, 12, 10, \cancel{34}, \cancel{100}$

$d = 33, 9, 11$

Sum of values of $d = 33 + 9 + 11 = 53$

Ans. 53

QUESTION [JEE Advanced 2015]

Tah01

Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

all Terms of A.P are natural given

$$\frac{\frac{7}{2}(2a+6d)}{\frac{11}{2}(2a+10d)}=\frac{6}{11}$$

$$130 < T_7 = a+6d < 140$$

QUESTION [JEE Mains 2022 (27 July)]

TAH 02

Suppose $a_1, a_2, ..., a_n$ ..... be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is 5 : 17 and, $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to

(A) 290

(B) 380

(C) 460

(D) 510

Ans. B

QUESTION [JEE Mains 2025 (29 Jan)]

TAH 03

Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11^{th} term is :

(A) 108

(B) 90

(C) 122

(D) 84

Ans. B

If $m, n, p, q \in N$ such that $m+n = p+q$ then

in an A.P $T_m + T_n = T_p + T_q$

Ex: $a_2 + a_{48} + a_{62} + a_{52} + a_{38} + a_{98}$

$= 3.(a_2 + a_{98})$

$a_2 + a_{98} + a_{48} + a_{52} + a_{62} + a_{38}$

$a_2 + a_{98} + a_2 + a_{98} + a_2 + a_{98}$

$= 3(a_2 + a_{98})$

QUESTION [JEE Mains 2019]

If $a_1, a_2, a_3, \ldots, a_n$ are in A.P. and $a_1 + a_4 + a_7 + \ldots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :

$$a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$$

$$3(a_1 + a_{16}) = 114$$

$$a_1 + a_{16} = 38$$

$$a_1 + a_6 + a_{11} + a_{16}$$

$$= 2(a_1 + a_{16}) = 76.$$

(A) 76

(B) 98

(C) 64

(D) 38

QUESTION [JEE Mains 2016]

Let $a_1, a_2, a_3, \ldots, a_n$ be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$, then the sum of its first 17 terms is equal to:

$$2(a_3 + a_{15}) = 72$$

$$a_3 + a_{15} = 36.$$

$$a_1 + a_2 + \text{---} + a_{17} = \frac{17}{2}(a_1 + a_{17}) = \frac{17}{2} \cdot 36 = 17 \cdot 18 =$$

(A) 306

(B) 204

(C) 153

(D) 612

Ans. A

QUESTION [JEE Mains 2025 (29 Jan)]

Let $a_1, a_2, \ldots, a_{2024}$ be an Arithmetic Progression such that $a_1 + (\underbrace{a_5 + a_{10} + a_{15} + \cdots + a_{2020}}_{(404 \text{ terms})}) + a_{2024} = 2233$. Then $a_1 + a_2 + a_3 + \cdots + a_{2024}$ is equal to

5×404

$a_1 + a_5 + a_{10} + a_{15} + \cdots + a_{2010} + a_{2015} + a_{2020} + a_{2024}$

$203(a_1 + a_{2024}) = 2233.$

$a_1 + a_{2024} = \frac{2233}{203}.$

$S = a_1 + a_2 + \cdots + a_{2024}$

$= \frac{2024}{2}(a_1 + a_{2024})$

$= 1012 \cdot \frac{2233}{203}.$

Ans. 11132

Problems Based on Common Terms

QUESTION [JEE Mains 2024 (1 Feb)]

Let 3, 7, 11, 15, ..., 403 and 2, 5, 8, 11, ..., 404 be two arithmetic progressions. Then the sum, of the common terms in them, is equal to

$3, 7, 11, 15, ---, 403 \quad d_1 = 4$

$2, 5, 8, 11, ---, 404 \quad d_2 = 3$

$\text{LCM}(d_1, d_2) = 12$

$\underbrace{\text{Common Terms}}$. A.P with common diff $= 12$

$T = 0 \quad t = 2$

$t = 3$

$t = 4$

$t = 6$

$t = 12$

$\underbrace{\text{A.P}}$ $11, 23, 35 ---- T_n$

$T_n \le 403$ & $T_n \le 404$

$T_n \le 403$

$11 + (n-1)12 \le 403$

$(n-1) \le 392/12 = 32.-- \Rightarrow n \le 33.---$

Ans. 6699

Common Terms. A.P: 11, 23, 35 --- up to 33 terms

$$S = \frac{33}{2}(2 \cdot 11 + 32 \cdot 12)$$

$$= 33(11 + 192)$$

$$= 6699.$$

QUESTION [JEE Advanced 2019]

$(A\cap B)\cap(B\cap C) = A\cap B\cap C$

Let AP (a, d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference $d > 0$. If $AP(1,3) \cap AP(2,5) \cap AP(3,7) = AP(a,d)$ then a + d equals

$AP(1\ 3):\ 1, 4, 7, 10, 13, ----\quad d_1 = 3$

$AP(2,5):\ 2, 7, 12, \boxed{17}, 22, ----\quad d_2 = 5$

$A\cdot P(3,7):\ 3, 10, \boxed{17}, 24, 31 ----\quad d_3 = 7$

$Lcm(3,5) = 15$

common

$7, 22, 37, 52, ----\quad d = 15$

$Lcm(5,7) = 35$

common

$17, 52, 89, ----\quad d = 35.$

common terms in all 3. $Lcm(15,35) = 105.$

$A\cdot P\ 52, 157 ----$

$a = 52$

$d = 105.$

$a + d = 157$

QUESTION [JEE Advanced 2018]

Tah 04

Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ..., and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set $X \cup Y$ is

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

QUESTION [JEE Mains 2024 (27 Jan)]

Tan 05

The number of common terms in the progressions

4, 9, 14, 19, ..., up to 25^{th} term and 3, 6, 9, 12, ..., up to 37^{th} term is:

(A) 9

(B) 8

(C) 5

(D) 7

Ans. D

QUESTION [JEE Mains 2023 (1 Feb)]

Jan 06

The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15, ..., 399,

2, 5, 8, 11, ..., 359 and

2, 7, 12, 17, ..., 197,

is equal to ______

Ans. 321

QUESTION [JEE Mains 2025 (4 April)]

Jan 07

Let $A = \{1, 6, 11, 16, ...\}$ and $B = \{9, 16, 23, 30, ...\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is

(A) 3814

(B) 4003

(C) 4027

(D) 3761

Ans. D

Problems Based on Ratio of Sum of A.P.

QUESTION

The sum of first n terms of two A.P.s are in ratio $\frac{7n+1}{4n-27}$. Find the ratio of their 11^{th} terms.

$$\frac{\cancel{\frac{n}{2}}(2a+(n-1)d)}{\cancel{\frac{n}{2}}(2A+(n-1)D)} = \frac{7n+1}{4n-27}$$

$$\frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{7n+1}{4n-27}$$

$$\frac{a+10d}{A+10D} = ?$$

$$\frac{n-1}{2} = 10$$

$$n = 21$$

84
27

$$\frac{a+10d}{A+10D} = \frac{148}{57}$$

QUESTION [JEE Advanced 2011]

Let $a_1, a_2, a_3, \ldots\ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is

$$\frac{S_m}{S_n} = \frac{a_1+a_2+\cdots+a_m}{a_1+a_2+\cdots+a_n} = \frac{\frac{m}{2}(2a+(m-1)d)}{\frac{n}{2}(2a+(n-1)d)} = \frac{5n(2a+(5n-1)d)}{n(2a+(n-1)d)}$$

$$\frac{S_m}{S_n} = \frac{5(2a-d+5nd)}{2a-d+nd}$$ does not depend on n.

$\Downarrow$

$2a = d$ or $d = 0$

$a_1 = 3$, $6 = d$ $\Rightarrow a_2 = 9$

$a_1 = 3$, $d=0 \Rightarrow a_2 = 3$.

QUESTION

Let $a_1, a_2, a_3, \ldots$ be terms of an A.P.. If $\frac{a_1+a_2+\cdots+a_p}{a_1+a_2+\cdots+a_q}=\frac{p^2}{q^2}$, where $p \neq q$, then $\frac{a_6}{a_{21}}$ is equal to

(A) 4/11

(B) 7/2

(C) 11/41

(D) 2/7

$$\frac{\frac{p}{2}(2a+(p-1)d)}{\frac{q}{2}(2a+(q-1)d)}=\frac{p^2}{q^2}$$

$$\frac{a+\left(\frac{p-1}{2}\right)d}{a+\left(\frac{q-1}{2}\right)d}=\frac{p}{q}$$

$$\frac{a+5d}{a+20d}$$

$\frac{p-1}{2}=5$, $\frac{q-1}{2}=20$

$p=11$ $q=41$

put $p=11, q=41$

$$\frac{a+5d}{a+20d}=\frac{11}{41}$$

$$\frac{a_6}{a_{21}}=11/41$$

QUESTION [JEE Mains 2021 (31 Aug)]

Tan 08

Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\frac{a_1+a_2+\cdots+a_{10}}{a_1+a_2+\cdots+a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to :

(A) $\frac{19}{21}$

(B) $\frac{100}{121}$

(C) $\frac{21}{19}$

(D) $\frac{121}{100}$

Ans. C

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

KTK 1

For a, b $\in$ R – {0}, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x + \frac{7}{4}\right) = f\left(\frac{7}{4} - x\right)\ \forall\ x \in R$. Also the equation $f(x) = 7x + a$ has only one real and distinct solution.

The value of $(a + b)$ is equal to

(A) 4

(B) 5

(C) 6

(D) 7

Ans. B

QUESTION

KTK 2

For a, b $\in$ R – {0}, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x+\frac{7}{4}\right) = f\left(\frac{7}{4}-x\right)\ \forall\ x \in R$. Also the equation $f(x) = 7x + a$ has only one real and distinct solution.

The minimum value f(x) in $\left[0, \frac{3}{2}\right]$ is equal to

(A) $-\frac{33}{8}$

(B) 0

(C) 4

(D) –2

Ans. D

Homework From Module

Quadratic Equations

Prarambh (Topicwise) : Complete

Prabal (JEE Main Level) : Complete

Parikshit (JEE Advanced Level) : Q1 to Q15

Solution to Previous KTKs

QUESTION

Let $x^2 - (m - 3)x + m = 0$ $(m \in R)$ be a quadratic equation. Find the value of m for which the roots of the equation are

(a) Positive

(b) Negative

(c) such that at least one is positive

(d) One root is smaller than 2 and other root is greater than 2

(e) Both the roots are greater than 2

(f) Both the roots are smaller than 2

(g) Exactly one root lies in the interval (1, 2)

(h) Both the roots lie in the interval (1, 2)

(i) Such that at least one root lie in the interval (1, 2)

(j) One root is greater than 2 and the other root is smaller than 1

Answers :

(a) $m \in [9, \infty)$

(b) $m \in (0, 1]$

(c) $m \in (-\infty, 0) \cup [9, \infty)$

(d) $m \in (10, \infty)$

(e) $m \in [9, 10)$

(f) $m \in (-\infty, 1]$

(g) $m \in (10, \infty)$

(h) $m \in \phi$

(i) $m \in (10, \infty)$

(j) $m \in \phi$

KTK-① let $x^2-(m-3)x+m=0$ $(m\in R)$ be a quadratic eq^n. find the value of m for which the roots of the eq^n are.

(a) Positive

if both roots are (+ve) then

$\alpha+\beta>0$ $\quad$ $\alpha\beta>0$ $\quad$ $D\geq 0$

$m-3>0$ $\quad$ $m>0$ $\quad$ $(m-3)^2-4m\geq 0$

$m>3$ $\quad\quad$ $m^2-10m+9\geq 0$

$(m-9)(m-1)\geq 0$

$m\in(-\infty,1]\cup[9,\infty)$

final Ans. $m\in[9,\infty)$

(b) Negative

if both roots are -ve then

$\alpha+\beta<0$ $\quad$ $\alpha\beta>0$ $\quad$ $D\geq 0$

$m<3$ $\quad$ $m>0$ $\quad$ $m\in(-\infty,1]\cup[9,\infty)$

$m\in(0,1]$

KTK-1 by Nikita from Raj.

(c) such that at least one is +ve.

Case ① $\alpha<0,\beta>0$.

$f(0)<0$.

$\boxed{m<0}$. Ⓐ

$D>0$ (no need)

Case ② $\alpha>0,\beta>0$.

(i) $f(0)>0$

$\boxed{m>0}$

(ii) $-\frac{b}{2a}>0$.

$\frac{m-3}{2}>0$.

$\boxed{m>3}$

(iii) $D\geq 0$.

$m\in(-\infty,1]\cup[9,\infty)$

$m\in[9,\infty)$ Ⓑ

Case ③ $\alpha=0,\beta>0$.

(i) $f(0)=0$.

$m=0$.

(ii) $D>0$.

$(-\infty,1)\cup(9,\infty)$

$m\in(-\infty,1)\cup(9,\infty)$ Ⓒ

final Ans $\Rightarrow$ Ⓐ ∪ Ⓑ ∪ Ⓒ

$m\in(-\infty,0)\cup[9,\infty)$

(d) One root is smaller than 2 and other root is greater than 2.

$f(2)<0$. , $D>0$ (no need)

$4-2m+6+m<0$.

$-m+10<0$.

$m-10>0$.

$\boxed{m>10}$

final Ans $\Rightarrow$ $m\in(10,\infty)$

KTK-1 by Nikita from Raj.

(e) Both roots are greater than 2.

(i) $f(2)>0$. $\Rightarrow$ $-m+10>0$ $\quad m\in(-\infty,10)$

(ii) $\frac{-b}{2a}>2$ $\Rightarrow$ $\frac{m-3}{2}>2$ $\Rightarrow m>7$

(iii) $D\geq 0$. $\Rightarrow$ $m\in(-\infty,1]\cup[9,\infty)$

final Ans. $m\in[9,10)$ Ans

(f) Both roots are smaller than 2.

(i) $f(2)>0$. $\Rightarrow$ $m\in(-\infty,10)$

(ii) $\frac{-b}{2a}<2$ $\Rightarrow$ $m<7$

(iii) $D\geq 0$. $\Rightarrow$ $m\in(-\infty,1]\cup[9,\infty)$

final Ans $\Rightarrow$ $m\in(-\infty,1]$ Ans

(g) exactly one root lies in the interval $(1,2)$

$f(1)f(2)<0$, $D>0$ (no need)

$(1-m+3+m)(-m+10)<0$.

$4(m-10)>0$.

$\boxed{m>10}$

final Ans $\Rightarrow$ $m\in(10,\infty)$ Ans

Two more possibilities are arise.

$f(1)=0$.

$4=0$.

N.P.

$f(2)=0$

$-m+10=0$.

$m=10$.

if $m=10$.

$x^2-7x+10=0$

$(x-5)(x-2)=0$

$x=5,2$

$\notin(1,2)$

so, $m=10$ is rejected.

(h) Both roots lie in the interval $(1,2)$

KTK-1
by Nikita

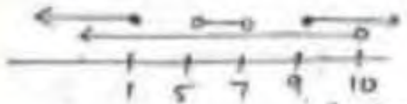
(i) $f(1) > 0 \Rightarrow 4 > 0 \ \forall\, m \in R$

(ii) $f(2) > 0 \Rightarrow -m+10 > 0$

$\boxed{m < 10}$

(iii) $1 < \frac{-b}{2a} < 2 \Rightarrow 1 < \frac{m-3}{2} < 2 \Rightarrow 5 < m < 7$

(iv) $D \geq 0 \Rightarrow m \in (-\infty, 1] \cup [9, \infty)$



final Ans $\Rightarrow$ $\boxed{m \in \phi}$ Ans

No common Area.

(i) Such that at least one root lie in the interval $(1,2)$

C_1:

$\Downarrow$

$\boxed{m \in (10, \infty)}$

C_2:

$\Downarrow$

$\boxed{m \in \phi}$

C_3:

$\boxed{m \in \phi}$ $9=0$ $\Downarrow$ NP

C_4:

$f(2) = 0$
$m = 10$
$\boxed{m = \phi}$

final Ans $\Rightarrow C_1 \cup C_2 \cup C_3 \cup C_4$

$(10, \infty)$ Ans

(j) one root is greater than 2 and the other root is smaller than 1.

$f(1) < 0 \longrightarrow 4 < 0$ (N.P)

$f(2) < 0 \longrightarrow -m+10 < 0$

$m - 10 > 0$

$\boxed{m > 10}$

$\cap$ final Ans. $\boxed{m \in \phi}$ Ans

$D > 0$ (no need)

Q- KTK-1:

KTK 1
by Reed
from WB

→ (a) roots are positive:

$x^2 - (m-3)x + m = 0$

for both roots to be positive.

$S > 0, \quad P > 0, \quad D \geq 0$

$\Rightarrow m - 3 > 0 \Rightarrow m > 3$ | $\Rightarrow m > 0$ | $(m-3)^2 - 4m \geq 0$

$\Rightarrow m^2 - 6m + 9 - 4m \geq 0$

$\Rightarrow m^2 - 10m + 9 \geq 0$

$\Rightarrow (m-1)(m-9) \geq 0$

$\Rightarrow m \in (-\infty, 1] \cup [9, \infty)$

'$\cap$'

$\boxed{m \in [9, \infty)}$

Ans.

→ (b) roots are negative:

$x^2 - (m-3)x + m = 0$

for both roots to be −ve.

$S < 0, \quad P > 0, \quad D \geq 0$

$\Rightarrow m - 3 < 0 \Rightarrow m < 3$ | $m > 0$ | $m \in (-\infty, 1] \cup [9, \infty)$

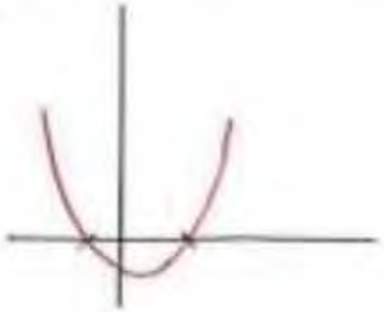
'$\cap$'

$\boxed{m \in (0, 1]}$

→ (c) such that at least one root is +ve!

$x^2-(m-3)x+m=0 \quad m\in R.$

Case-1:



(i) $f(0)<0$

P.O.R = $m<0$

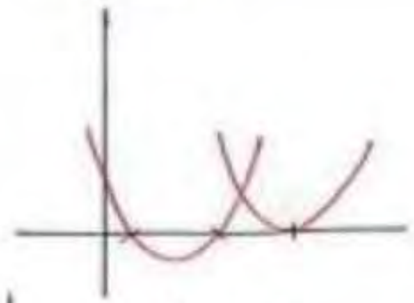
observed

So (i) $f(0)<0$

$\Downarrow$

$\boxed{m<0}$ — (A)

Case-2:



(i) $f(0)>0$

(ii) $\frac{-b}{2a}>0$

(iii) $D\geq 0$

→ (i) $f(0)>0$

$\Downarrow$

$\boxed{m>0}$

→ (ii) $\frac{-b}{2a}>0$

$\Rightarrow \frac{m-3}{2}>0$

$\Rightarrow \boxed{m>3}$

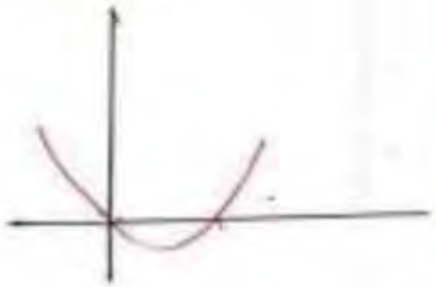
→ (iii) $D\geq 0$

$\Rightarrow (m-9)(m-1)\geq 0$

$\Rightarrow m\in(-\infty,1]\cup[9,\infty)$

"∩" $m\in[9,\infty)$ — (B)

Case-3:



(i) $f(0)=0$

(ii) $\frac{-b}{2a}>0$

(iii) $D>0$ → No need

∴ $f(0)=0$ already.

So, → (i) $f(0)=0$

$\Downarrow$

$m=0.$

→ (ii) $\frac{-b}{2a}>0$

$\Rightarrow m>3.$ "∩"

$m\in\phi$ — (C)

KTK 1 BY REED

Taking union of case-1, case-2, case-3.

$\Downarrow$

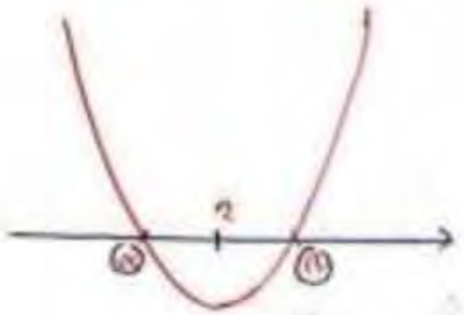
$m\in(-\infty,0)\cup m\in[9,\infty)\cup m\in\phi$

$\Downarrow$

$\boxed{m\in(-\infty,0)\cup[9,\infty)}$ Ans.

→ (d) one root is smaller than 2 and other root is greater than 2:

KTK 1 BY REED



(i) $f(2)<0$

(ii) $D>0$ → no need.

∴ $f(2)<0$

$\Downarrow$

$\Rightarrow 4-2m+6+m<0$

$\Rightarrow m>10.$

∴ $m\in(10,\infty)$ (Ans.)

→ (e) both roots are greater than 2:

(i) $f(2)>0 \Rightarrow 10-m>0 \Rightarrow m<10$ — (A)

(ii) $\frac{-b}{2a}>2 \Rightarrow \frac{m-3}{2}>2 \Rightarrow m>7$ — (B)

(iii) $D\geq 0. \Rightarrow (m-1)(m-9)\geq 0 \Rightarrow m\in(-\infty,1]\cup[9,\infty)$ (C)

→ (A) ∩ (B) ∩ (C):

$\Rightarrow m\in[9,10)$ (Ans)

→ (i) Both roots are smaller than 2:

$f(x) = x^2 - (m-3)x + m$

KTK 1
BY REED

→ (i) $f(2) > 0$

→ (ii) $\frac{-b}{2a} < 2$.

→ (iii) $D \geq 0$

So → (i) $f(2) > 0$

or, $4 - 2m + 6 + m > 0$

or, $10 - m > 0$

or, $m < 10$

→ (ii) $\frac{-b}{2a} < 2$

$\Rightarrow \frac{m-3}{2} < 2$

$\Rightarrow m < 7$

→ (iii) $D \geq 0$

$\Rightarrow (m-1)(m-9) \geq 0$

$\Rightarrow m \in (-\infty, 1] \cup [9, \infty)$

"∩"

$m \in (-\infty, 1]$ (Ans)

→ (9): Exactly one root lies in the interval (1,2):

$f(1) \cdot f(2) < 0$

$\Rightarrow (1 - \cancel{m} + 3 + \cancel{m})(10 - m) < 0$

$\Rightarrow 4(10 - m) < 0$

$\Rightarrow 10 - m < 0$

$\Rightarrow m > 10$ — (i)

KTK 1
by Reed

now, two more possibilities arises:

$f(1) = 0$

⇓

$4 = 0$

Not possible.

$f(2) = 0$

⇓

or, $10 - m = 0$

or, $m = 10$

↓

eqn becomes:

$f(x) = x^2 - 7x + 10 = 0$ → 2, $\frac{10}{2} = 5$

by POR.

∴ other root

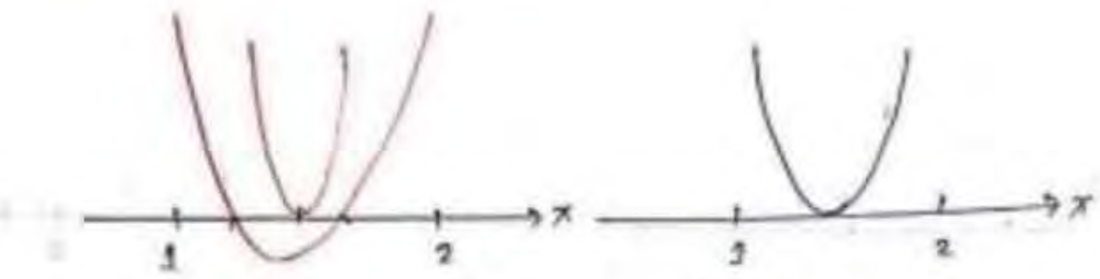
$5 \notin (1,2)$

⇓

∴ $m = 10$ is rejected.

∴ Final Ans: $m > 10$.

→(h) Both the root lies in the interval (1,2):-



(i) $f(1)>0$,
(ii) $f(2)>0$
(iii) $1<\frac{-b}{2a}<2$
(iv) $D\geq 0$.

Sol.

→(i) $f(1)>0$

$\downarrow$

$\Rightarrow \boxed{4>0}$

True for all $m\in R$ — (A)

→(ii) $f(2)>0$

$\Rightarrow 10-m>0$

$\Rightarrow m<10$ — (B)

→(iii) $1<\frac{-b}{2a}<2$.

$\Rightarrow 1<\frac{m-3}{2}<2$

$\Rightarrow 2<m-3<4$

$\Rightarrow 5<m<7$ — (C)

→(iv) $D\geq 0$

$\downarrow$

$\Rightarrow (m-1)(m-9)\geq 0$

$\Rightarrow m\in(-\infty,1]\cup[9,\infty)$ — (D)

(A) ∩ (B) ∩ (C) ∩ (D):

$\Downarrow$

$m\in\phi$. Ans.

→(i) such that at least one root lie in the interval (1,2):

at least one root $\in(1,2)$

$\Downarrow$

means either exactly one root lies in (1,2) (g)

OR Both root lies in (1,2). (h)

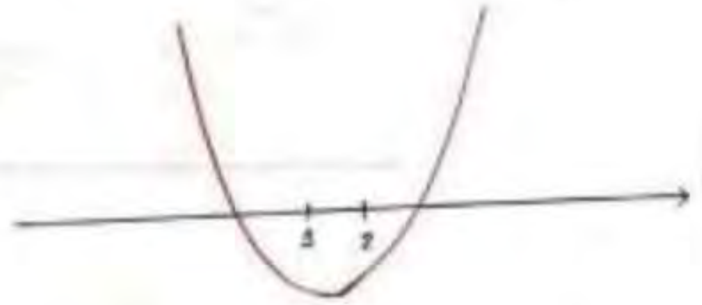
Union

$\Downarrow$

$m\in\phi \cup m\in(10,\infty)$

$\therefore m\in(10,\infty)$

→(j) one root is greater than 2 and other root is smaller than 1:



$f(1)<0$ & $f(2)<0$

$\Downarrow$ / $\downarrow$

$4<0$ / $10-m<0$

$\Downarrow$ / $\Rightarrow m>10$

Not Possible

$\therefore m\in\phi$

$m\in\phi$ (Ans.)

QUESTION [JEE Advanced 2017]

Paragraph

Let p, q, be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, \ldots$, let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

If $a_4 = 28$, then $p + 2q =$

(A) 12

(B) 14

(C) 21

(D) 7

KTK - 2 Paragraph

let p, q be integers and let α, β be the roots of the eqn $x^2 - x - 1 = 0$ where $\alpha \neq \beta$. For $n = 0, 1, 2, \ldots$ let $a_n = p\alpha^n + q\beta^n$

Fact :- If a & b are rational nos. and $a + b\sqrt{5} = 0$ then $a = 0 = b$

If $a_4 = 28$ then $p + 2q = ?$

Part(i) By NF - $x^2 - x - 1 = 0 <^{\alpha}_{\beta}$

$\therefore a_{n+2} - a_{n+1} - a_n = 0.$

$a_1 = p\alpha + q\beta$

$a_0 = p + q$

$a_{n+2} = a_{n+1} + a_n.$

$\alpha, \beta = \frac{1 \pm \sqrt{5}}{2}$

$n = 0, \quad a_2 = a_1 + a_0.$

$\alpha = \frac{1+\sqrt{5}}{2}, \beta = \frac{1-\sqrt{5}}{2}$

$n = 1, \quad a_3 = a_2 + a_1 = 2a_1 + a_0$

$n = 2, \quad a_4 = a_3 + a_2 = 3a_1 + 2a_0$

$\because a_4 = 28.$

$3a_1 + 2a_0 = 28.$

$3p\alpha + 3q\beta + 2p + 2q = 28.$

$(3\alpha + 2)p + (3\beta + 2)q = 28.$

$\left(\frac{3+3\sqrt{5}}{2} + 2\right)p + \left(\frac{3-3\sqrt{5}}{2} + 2\right)q = 28.$

$\frac{7p}{2} + \frac{3\sqrt{5}p}{2} + \frac{7q}{2} - \frac{3\sqrt{5}}{2}q = 28.$

$\left(\frac{7}{2}(p+q) - 28\right) + \left(\frac{3\sqrt{5}}{2}(p-q)\right) = 0.$

$\Downarrow$

$\frac{7}{2}(p+q) = 28. \qquad \frac{3\sqrt{5}}{2}(p-q) = 0 \quad \boxed{p = q}$

$p + q = 8$

$2p = 8$

$\boxed{p = 4}, \boxed{q = 4}$

$p + 2q = 4 + 8 = \boxed{12}$ Ans

KTK-2

[paragraph]

by Nikita

Part (ii) - $a_{12} = ?$

By NF $\Rightarrow$

$a_{n+2} - a_{n+1} - a_n = 0.$

$a_{n+2} = a_{n+1} + a_n.$

$n = 10,$

$\boxed{a_{12} = a_{11} + a_{10}.}$

Ans

QUESTION [JEE Advanced 2017]

Paragraph

Let p, q, be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, \ldots$, let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

$a_{12} =$

(A) $2a_{11} + a_{10}$

(B) $a_{11} - a_{10}$

(C) $a_{11} + 2a_{10}$

(D) $a_{11} + a_{10}$

(2)

$$a_{n+2} = a_{n+1} + a_n$$

$n = 10$

$$a_{12} = a_{11} + a_{10}$$

Ans (D)

THANK YOU