PRAYAS

JEE 2026

Mathematics

Sequence and Series

Lecture - 01

By – Ashish Agarwal Sir
(IIT Kanpur)

Topics *to be covered*

A Introduction to Sequence & Series

B General term and sum of n terms of an AP

Homework Discussion

QUESTION

(KTK 04)

Find all values of m for which the equation

$m \in R$, $m \neq -1$, $(1 + m)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ gives roots according to the following conditions:

(i) Exactly one root in the interval (2, 3). Ans. $[4, \infty)$

(ii) One root smaller than 1 and other root greater than 1. Ans. $(-1, 0)$

(iii) Both roots smaller than 2. Ans. $(-1, 0]$

(iv) Atleast one root in the interval (2, 3). Ans. $[3, \infty)$

(v) Atleast one root greater than 2. Ans. $(-\infty, -1) \cup [1, \infty)$

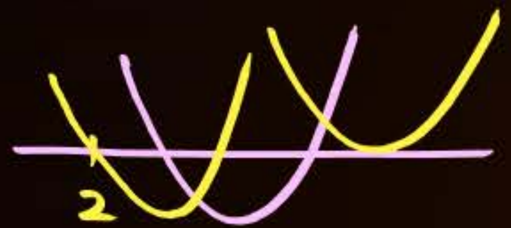
(vi) Roots such that both 1 and 2 lie between them. Ans. ϕ

(vii) One root in (1, 2) and other root in (2, 3). Ans. ϕ

(v) $(1+m)x^2 - 2(1+3m)x + 1+8m = 0$

$$x^2 - \frac{2(1+3m)}{1+m}x + \frac{1+8m}{1+m} = 0$$

Case ①



$f(2) \geq 0 \Rightarrow 4 - \frac{4(1+3m)}{1+m} + \frac{1+8m}{1+m} \geq 0.$

$-\frac{b}{2a} > 2$

$D \geq 0.$

$\frac{\cancel{4}+\cancel{4m}-\cancel{4}-\cancel{12m}+1+\cancel{8m}}{1+m} \geq 0$

$\frac{1+3m}{1+m} > 2.$

$\frac{1}{1+m} \geq 0 \Rightarrow 1+m > 0.$

$\frac{1+3m-2-2m}{1+m} > 0$

$m \in (-1, \infty)$

$\frac{m-1}{1+m} > 0 \Rightarrow m \in (-\infty, -1) \cup (1, \infty)$

$D \geq 0$

$\frac{4(1+3m)^2}{(1+m)^2} - 4 \cdot \frac{1+8m}{1+m} \geq 0$

$1+9m^2+6m-(1+m)(1+8m) \geq 0$

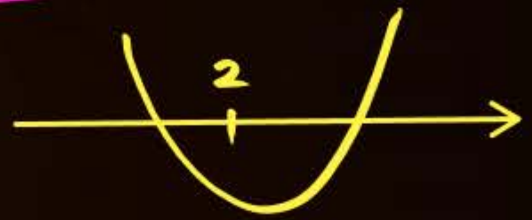
$m^2 - 3m \geq 0$

$\cap$

$m \in (-\infty, 0] \cup [3, \infty)$

$\Rightarrow m \in [3, \infty)$

Case ⑪



$f(2) < 0$

$\frac{1}{1+m} < 0$

$m \in (-\infty, -1)$

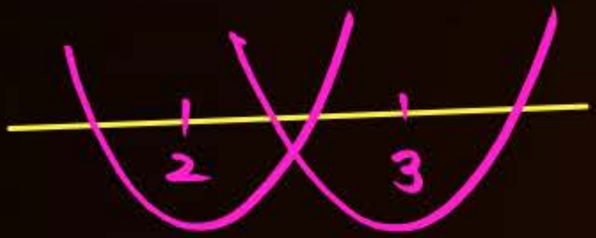
$\cup$

$m \in (-\infty, -1) \cup [3, \infty)$

(iv) $(1+m)x^2 - 2(1+3m)x + 1 + 8m = 0$

$$x^2 - \frac{2(1+3m)}{1+m}x + \frac{1+8m}{1+m} = 0$$

$m \in [3,4)$

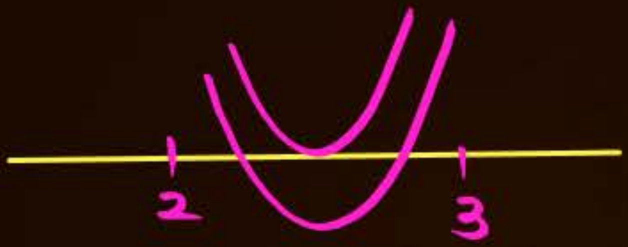


$\Downarrow$

$m \in [4, \infty)$

final Ans

$[3, \infty)$



$f(2) > 0$ — $\frac{1}{1+m} > 0 \Rightarrow \boxed{m \in (-1, \infty)}$

$f(3) > 0 \longrightarrow 9 - \frac{6(1+3m)}{1+m} + \frac{1+8m}{1+m} > 0$

$D \geq 0$ — $\boxed{(-\infty, 0] \cup [3, \infty)}$

$2 < \frac{-b}{2a} < 3 \rightsquigarrow \boxed{m \in (1, \infty)}$

$$\frac{9+9m-6-18m+1+8m}{1+m} > 0$$

$\frac{4-m}{1+m} > 0 \Rightarrow \frac{m-4}{1+m} < 0 \Rightarrow \boxed{m \in (-1, 4)}$

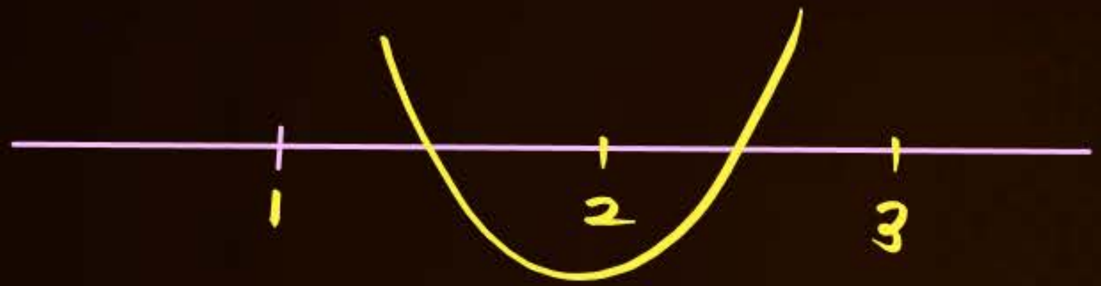
$2 < \frac{(1+3m)}{1+m} < 3$

$m \in (-\infty, -1) \cup (1, \infty)$

$\frac{1+\cancel{3m}-3-\cancel{3m}}{1+m} < 0 \Rightarrow \frac{2}{1+m} > 0$ — $m \in (-1, \infty)$

(vii) $(1+m)x^2 - 2(1+3m)x + 1 + 8m = 0$

$$x^2 - \frac{2(1+3m)}{1+m}x + \frac{1+8m}{1+m} = 0$$



$f(2)<0 \Longrightarrow \frac{1}{1+m}<0 \Rightarrow m\in(-\infty,-1)$

$f(1)>0 \Rightarrow \frac{1+m-2-6m+1+8m}{1+m}>0 \Rightarrow \frac{3m}{1+m}>0 \Rightarrow m\in(-\infty,-1)\cup(0,\infty)$

$f(3)>0 \Rightarrow \frac{4-m}{1+m}>0 \Rightarrow \frac{m-4}{m+1}<0 \Rightarrow m\in(-1,4)$

$m\in\phi$

Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal

QUESTION

★★★ KCLS ★★★

If $f(x) = 4x^2 + ax + (a-3)$ is negative for atleast one negative x, find all possible values of a.

$y = f(x) = 4x^2 + ax + a - 3$ is upward opening parabola & it goes below x axis for atleast one −ve value of x.

case ①

$f(0) \geq 0 \Rightarrow a \geq 3.$

$-\frac{b}{2a} < 0 \Rightarrow -\frac{a}{8} < 0 \Rightarrow a > 0$

$D > 0 \Rightarrow a^2 - 16(a-3) > 0$

$a^2 - 16a + 48 > 0$

$(a-12)(a-4) > 0$

$a \in (-\infty, 4) \cup (12, \infty)$

$\cap$

$a \in [3, 4) \cup (12, \infty)$ — ①

case ⑪

$f(0) < 0$

$a - 3 < 0$

$a < 3$ — ⑪

Ans ① ∪ ⑪

$a \in (-\infty, 4) \cup (12, \infty)$

Ans. $a \in (-\infty, 4) \cup (12, \infty)$

QUESTION

KCLS

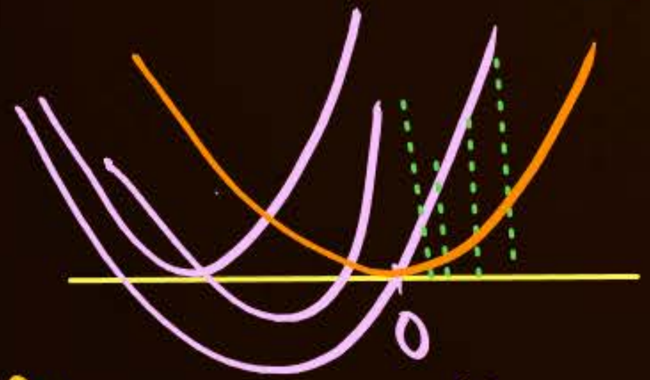
Find all real values of 'm' for which the inequality $mx^2 - 4x + 3m + 1 > 0$ is satisfied for all positive 'x'.

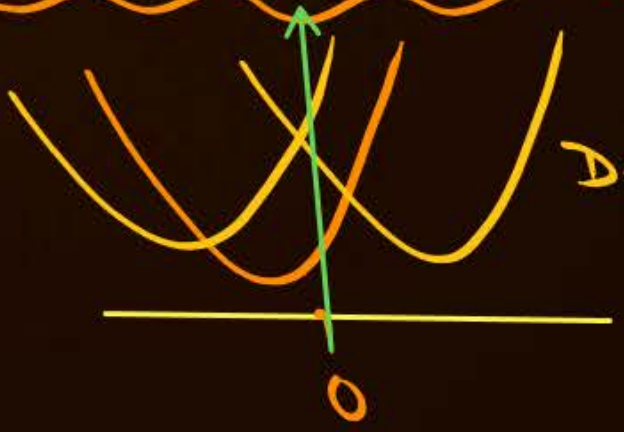
$mx^2 - 4x + 3m + 1 > 0$ should be satisfied for all +ve x.

~~$m > 0,\ D < 0 \longrightarrow$ Gadhe/Gadhiyaa aisay sochege~~

$y = mx^2 - 4x + 3m + 1$ should be above x axis for all +ve x.

clearly $m > 0$





$D < 0 \Rightarrow 16 - 4m(3m+1) < 0$

$4 - 3m^2 - m < 0$

$3m^2 + m - 4 > 0.$

$3m^2 + 4m - 3m - 4 > 0$

$(m-1)(3m+4)$

$m \in (-\infty, -4/3) \cup (1, \infty)$

$m \in \phi$:

- $f(0) \geq 0 \to m \geq -1/3$
- $-\frac{b}{2a} \leq 0 \Rightarrow \frac{4}{2 \cdot m} \leq 0 \to$ N.P ($\because m > 0$)
- $D \geq 0$

Ans. $m \in (1, \infty)$

Ans $m \in (1, \infty)$

QUESTION

Tahol

Let $P(x) = (m^2 + 4m + 5)x^2 - 4x + 7, m \in R$. If $3 \leq x \leq 5$ then find the minimum of minimum value of $P(x)$.

Ans. 4

QUESTION

Jan 02

Find all values of the parameter 'a' for which the inequality $4^x - a \cdot 2^x - a + 3 \leq 0$ is satisfied for atleast one real 'x'.

Ans. $[2, \infty)$

Sequence

आरंभ

Series

Introduction

JEE - MAINS v. imp → scoring

JEE ADVANCE — DIFFICULT PROBLEM.

Sequence

A succession of terms which may be algebraic, real or complex number, written one after other separated by commas is called sequence. It is represented by $\langle a_n \rangle$ or (a_n)

e.g. (i) sequence of prime number 2, 3, 5, 7, 11, ...

(ii) –1, 1, –1, 1, ...

Progression

Special case of sequence in which the terms progress according to a definite rule

e.g. $\langle n^3 - 1 \rangle \equiv 0, 7, 26, \ldots 63, 124 \ldots$

$$\left(\frac{n}{n^2+1}\right) \equiv \frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \ldots\ldots$$

Common Progression

- A.P.
- G.P.
- H.P.
- A.G.P.
- Miscellaneous

Series

If we add all the terms of sequence, then it is called as series :

e.g. $2 + 3 + 5 + 7 + 11 + ...$

$0 + 7 + 26 ......,$

$\frac{1}{2} + \frac{2}{5} + \frac{3}{10} + \frac{7}{17} + \frac{5}{26} .....$

$1 - 3 + 9 - 27 ......$

Golden Point

$T_n \rightarrow$ denotes then n^{th} term of any sequence.

$S_n \rightarrow$ denotes the summation of n terms of any series.

$$\underbrace{\underbrace{T_1 + T_2 + T_3 + \cdots + T_{n-1}}_{S_{n-1}} + T_n}_{S_n}$$

$S_n = T_1 + T_2 + T_3 + \cdots + T_{n-1} + T_n$

$S_{n-1} = T_1 + T_2 + T_3 + \cdots + T_{n-1}$

$S_n - S_{n-1} = T_n$

Note:

For any series $S_n - S_{n-1} = T_n$

Arithmetic Progression

If each and every term except the 1st term bears the constant difference with its immediately preceding term then the sequence is called A.P. i.e., difference between two consecutive terms is constant (common difference)

$$T_{n+1} - T_n = d \qquad n \geq 1$$

A·P ~ Agli Term − Peechli term is a constant

Standard A·P

$$\begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \\ a, & a+d, & a+2d, & a+3d, & a+4d \text{ ----} \end{array}$$

$T_1 = a$, $T_n = a + (n-1)d$ — n^{th} term from Begining

- If $d > 0 \Rightarrow$ A·P is increasing
- If $d < 0 \Rightarrow$ A·P is decreasing
- If $d = 0 \Rightarrow$ A·P is constant.

k^{th} Term from End of an A.P.

M_1 is Good if number of Terms in A.P is known

A.P: $a_1, a_2, a_3, a_4 ---, l$

No: of terms $= n$

M (1) $l =$ last term.

k^{th} Term from End $= (n-k+1)^{th}$ term from start

Ex: 3, 5, 7, 9 ------ 223

find 50^{th} term from End

soln: $223 = T_n = 3 + (n-1)2$

$220 = (n-1)2$

$n = 111$

T_{50} from End $= T_{111-50+1}$ from start $= T_{62} = 3 + (62-1)2 = 125$ Ans.

2, 4, 6, 8, 10, 12, 14, 16, 18

$n = 9$

T_3 from End $= T_7$ from Begining

T_3 from End $= T_{9-3+1}$ from start

$10 = T_5$ from end $= T_5$ from start

$= T_{9-5+1}$ from start.

k^{th} Term from End of an A.P.

M② is good if last term is known.

A.P: $a_1, a_2, a_3, a_4 - - -, l$ — ①

No: of terms $= n$

common diff $= d$

$a_1 = a,\ a+d,\ a+2d,\ a+3d,\ - -\ l = a+(n-1)d$

$\xrightarrow{+d}$ $\xleftarrow{-d}$

Reverse the A.P

$l,\ l-d,\ l-2d, - - - - - -$ A.P — ②

common diff $= -d$.

T_k from last in ① $= T_k$ from start in ②

$= l + (k-1)(-d)$

$= l - (k-1)d$.

$d=2$ $\qquad$ $d=-2$

Ex: $3, 5, 7, 9 - - -, 223$.

50th term from End

T_{50} from End $= 223 + (50-1)(-2)$

$= 223 - 98$

$= 125$ Ans

$a_1, a_2, a_3, a_4, ---, a_n$ is an A.P

T_k from start $= a_1 + (k-1)d$

T_k from End $= a_n + (k-1)(-d)$

T_k from Begining + T_k from End in an A.P $= a_1 + a_n$ = Sum of first & last term.

Ex: 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32.

2+32 2+32 2+32

17+17 = 2+32

Summation of n terms of an A.P.

$T_1 + T_2 + T_3 + \cdots + T_{n-1} + T_n = l = a_n$

$S_n = a + (a+d) + (a+2d) + \cdots + a+(n-2)d + a+(n-1)d$

$S_n = a+(n-1)d + a+(n-2)d + a+(n-3)d + \cdots + a+d + a$

$2S_n = (2a+(n-1)d) + (2a+(n-1)d) + (2a+(n-1)d) + \cdots + (2a+(n-1)d) + 2a+(n-1)d$

$s = 1+3+5+7$

$s = 7+5+3+1$

$(n-2)d + d$

$d(n-2+1)$

$d(n-1)$

$2S_n = n(2a+(n-1)d)$

$S_n = \frac{n}{2}(2a+(n-1)d) \longleftrightarrow S_n = \frac{n}{2}(a+a+(n-1)d)$

$S_n = \frac{n}{2}(a+l)$

NICHOD !!

A.P : $a_1, a_2, a_3, a_4, ---, a_n = l = T_n$

A.P : $a, a+d, a+2d, a+3d, ---, a+(n-1)d$

- $T_n = a+(n-1)d$
- T_k from End $= T_{n-k+1}$ from start $= l+(k-1)(-d)$
- $S_n = \frac{n}{2}(2a+(n-1)d) = \frac{n}{2}(a+l)$

$an + (n^2-n)\frac{d}{2}$

$\frac{d}{2}n^2 + \left(a-\frac{d}{2}\right)n.$

$a = \text{coeff of } n + d/2 = b+a$

$S_n = an^2 + bn$ i.e S_n is a quad in n with constant term 0 then it represents the sum of an A.P with first term $a+b$ & common difference $2a$

$S_n = an^2 + bn$

$S_{n-1} = a(n-1)^2 + b(n-1) = an^2 - 2an + a + bn - b.$

$$\boxed{T_n = S_n - S_{n-1} = 2an - a + b}$$

$T_1 = 2a - a + b = a + b$

$T_n = 2an - a + b$

$T_{n-1} = 2a(n-1) - a + b.$

$T_n - T_{n-1} = 2a = \text{constant}$

Ex: $S_n = 5n^2 + 6n$ find T_5 of the sequence.

Clearly it sum of an A.P

with $T_1 = 5 + 6 = 11$

$d = 10$

$T_5 = 11 + (5-1)10 = 51.$

$\Rightarrow$ Given sum is of an A.P with first term $a+b$ & common diff $2a$.

Remember That

(i) Sum of first n natural number is $\frac{n(n+1)}{2}$. $= 1+2+3+ - - + n = \frac{n}{2}(1+n)$

(ii) Sum of first n odd natural number is n^2. $= 1+3+5+7+ - -$ up to n terms

$= \frac{n}{2}(2\cdot 1+(n-1)2)$

(iii) Sum of first n even natural number is $n(n + 1)$. $= \frac{n}{2}\cdot 2(1+n-1) = n^2$

$\parallel$

$2+4+6+ \text{---}$ upto n terms

$= \frac{n}{2}(2\cdot 2+(n-1)2)$

$= \frac{n}{2}\cdot 2(2+n-1)$

$= n(n+1)$

QUESTION [JEE Mains 2024 (27 Jan)]

The 20^{th} term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, -129\frac{1}{4}$ is :

(A) –115

(B) –100

(C) –110

(D) –118

$\frac{77}{4} - 20 = \frac{-3}{4}$ $\quad \frac{37}{2} - \frac{77}{4}$

$= \frac{74-77}{4} = -3/4$

T_{20} from End $= -129\frac{1}{4} + (20-1)(3/4)$

$= \frac{-517}{4} + \frac{57}{4}$

$= \frac{-460}{4}$

$= -115$ Ans

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

Let $x^2 - (m - 3)x + m = 0$ $(m \in R)$ be a quadratic equation. Find the value of m for which the roots of the equation are

(a) Positive

(b) Negative

(c) such that at least one is positive

(d) One root is smaller than 2 and other root is greater than 2

(e) Both the roots are greater than 2

(f) Both the roots are smaller than 2

(g) Exactly one root lies in the interval (1, 2)

(h) Both the roots lie in the interval (1, 2)

(i) Such that at least one root lie in the interval (1, 2)

(j) One root is greater than 2 and the other root is smaller than 1

Answers :

(a) $m \in [9, \infty)$

(b) $m \in (0, 1]$

(c) $m \in (-\infty, 0) \cup [9, \infty)$

(d) $m \in (10, \infty)$

(e) $m \in [9, 10)$

(f) $m \in (-\infty, 1]$

(g) $m \in (10, \infty)$

(h) $m \in \phi$

(i) $m \in (10, \infty)$

(j) $m \in \phi$

QUESTION [JEE Advanced 2017]

Paragraph

Let p, q, be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, \ldots$, let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

If $a_4 = 28$, then $p + 2q =$

(A) 12

(B) 14

(C) 21

(D) 7

QUESTION [JEE Advanced 2017]

Paragraph

Let p, q, be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, ...,$ let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

$a_{12} =$

(A) $2a_{11} + a_{10}$

(B) $a_{11} - a_{10}$

(C) $a_{11} + 2a_{10}$

(D) $a_{11} + a_{10}$

Solution to Previous TAH

QUESTION [JEE Mains 2020]

Tahol

The set of all real values of λ for which the quadratic equations, $(\lambda^2+1)x^2-4\lambda x+2=0$ always have exactly one root in the interval $(0,1)$ is:

(A) $(-3,-1)$

(B) $(2,4]$

(C) $(0,2)$

(D) $(1,3]$

Case-1:-

$f(0)\,f(1) < 0$

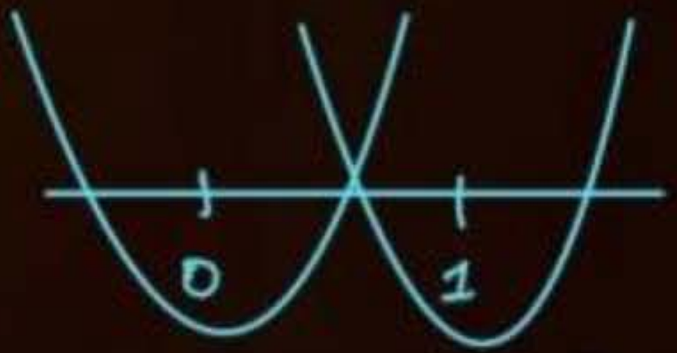
$(\lambda^2+1)x^2-4\lambda x+2=0$

$(2)[\lambda^2+1-4\lambda+2] < 0$

$2[\lambda^2-4\lambda+3] < 0$

$(\lambda-3)(\lambda-1) < 0$

$\lambda \in (1,3)$



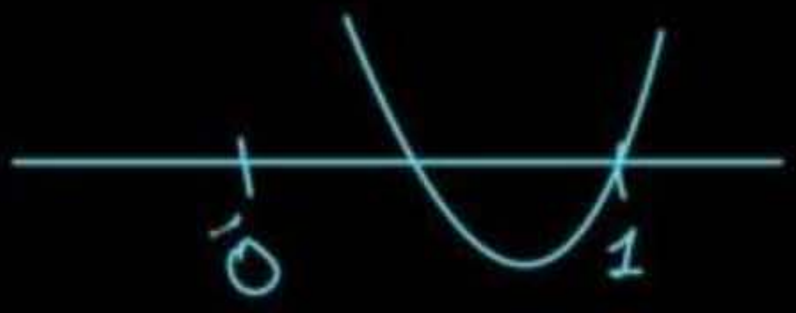
By Pranav Goel
Haryana

Case-2

$f(0)=0$

not possible

Case-3:-



$f(1)=0$

$\lambda^2+1-4\lambda+2=0$

$\lambda^2-4\lambda+3=0$

$(\lambda-3)(\lambda-1)=0$

$\lambda=3$ or $\lambda=1 \implies 2x^2-4x+2=0$

$x^2-2x+1=0$

$(x-1)^2=0$

$x=1 \rightarrow$ rejected

$10x^2-12x+2=0$

$10x^2-10x-2x+2=0$

$(x-1)(10x-2)=0$

$x=1$ or $x=0.2 \rightarrow$ accepted

Ans $\Rightarrow$ $\lambda \in (1,3) \cup \{3\}$

$\in (1,3]$

Solution to Previous KTKs

QUESTION [JEE Mains 2021 (25 Feb)]

(KTK 01)

The integer 'k', for which the inequality $x^2 - 2(3k-1)x + 8k^2 - 7 > 0$ is valid for every x in R, is:

(A) 4

(B) 2

(C) 3

(D) 0

Ans. C

QUESTION [JEE Mains 2021 (25 Feb)]

(KTK 01)

The integer 'k', for which the inequality $x^2 - 2(3k-1)x + 8k^2 - 7 > 0$ is valid for every x in R, is:

$\Rightarrow x^2 - 2(3k-1)x + 8k^2 - 7 > 0$

↳ for quad. to be +ve, $\forall x \in R$, $a > 0$, $D < 0$

(A) 4

(B) 2

(C) 3

(D) 0

$\Rightarrow b^2 - 4ac < 0$

$4(3k-1)^2 - 4(1)(8k^2-7) < 0$

$9k^2 + 1 - 6k - 8k^2 + 7 < 0$

$k^2 - 6k + 8 < 0$

$k^2 - 2k - 4k + 8 < 0$

$(k-2)(k-4) < 0$, $k \in (2, 4)$

By Pranav Goel
Haryana

Ans. C

KTK-01 The integer 'k' for which the inequality $x^2-2(3k-1)x+8k^2-7>0$ is valid for every x in R is.

$$f(x) = x^2-2(3k-1)x+8k^2-7>0$$

since, $x \in R$

$D<0$

$$(-2(3k-1))^2-4\times(8k^2-7)<0$$
$$4(9k^2+1-6k)-32k^2+28<0$$
$$36k^2-24k+4-32k^2+28<0$$
$$4k^2-24k+32<0$$
$$k^2-6k+8<0$$
$$k^2-4k-2k+8<0$$
$$k(k-4)-2(k-4)<0$$
$$(k-2)(k-4)<0$$
$$k\in(2,4)$$

[ony 3 is valid]

Q: [KTH-01] (JEE mains - 2021)

The integer 'k', for which the inequality $x^2 - 2(3k-1)x + 8k^2 - 7 > 0$ is valid for every x in $\mathbb{R}$, is:

(A) 4
(B) 2
(C) 3
(D) 0

$$x^2 - 2(3k-1)x + 8k^2 - 7 > 0$$

$a > 0, \; D < 0$

$\Rightarrow D < 0$

$$(-2(3k-1))^2 - 4 \cdot 1 \cdot (8k^2-7) < 0$$

$\Rightarrow 4(3k-1)^2 - 32k^2 + 28 < 0$

$\Rightarrow 36k^2 + 4 - 24k - 32k^2 + 28 < 0$

$\Rightarrow 4k^2 - 24k + 32 < 0$

$\Rightarrow 4(k^2 - 6k + 8) < 0$

$\Rightarrow k^2 - 6k + 8 < 0$

$\Rightarrow k^2 - 4k - 2k + 8 < 0$

$\Rightarrow k(k-4) - 2(k-4) < 0$

$\Rightarrow (k-4)(k-2) < 0$

$k \in (2, 4)$

$\Rightarrow$ The integer of 'k' lies b/w 2 & 4 is 3.

$\therefore \boxed{k = 3}$ Ans.

Galaxy F15 5G

Aniket raj
From patna

QUESTION

(KTK 02)

Find the greatest value of $\frac{x+2}{2x^2+3x+6}$ for real values of x.

Ans. 1/3

KTK 02

Soln :

$$y = \frac{x+2}{2x^2+3x+6}$$

$\Rightarrow 2yx^2 + 3yx + 6y = x+2$

$\Rightarrow \cancel{x^2(2y-1)+3}$

$\Rightarrow 2yx^2 + x(3y-1) + (6y-2) = 0$

for $x \in R$

$D \geq 0$

$(3y-1)^2 - 4 \cdot 2y(6y-2) \geq 0$

$9y^2 - 6y + 1 - 48y^2 + 16y \geq 0$

$-39y^2 + 10y + 1 \geq 0$

$39y^2 - 10y - 1 \leq 0$

$(13y+1)(3y-1) \leq 0$

$y \in \left[-\frac{1}{13}, \frac{1}{3}\right]$

maximum value is $\frac{1}{3}$

Ans: $\frac{1}{3}$

RASIDUL

KTK-02 Find the greatest value of $\frac{x+2}{2x^2+3x+6}$ for real value of x.

$$y = \frac{x+2}{2x^2+3x+6}$$

$$2x^2y + 3xy + 6y = x+2$$

$$2x^2y + 3xy - x + 6y - 2 = 0$$

$$2y\,x^2 + (3y-1)x + (6y-2) = 0$$

since, $x \in R$

$$D \geq 0$$

$$(3y-1)^2 - 4 \times 2y\,(6y-2) \geq 0$$

$$9y^2+1-6y - 48y^2 + 16y \geq 0$$

$$-39y^2 + 10y + 1 \geq 0$$

$$39y^2 - 10y - 1 \leq 0$$

$$39y^2 - 13y + 3y - 1 \leq 0$$

$$13y(3y-1) + 1(3y-1) \leq 0$$

$$(13y+1)(3y-1) \leq 0$$

$$y \in \left[\frac{-1}{13}, \frac{1}{3}\right]$$

Hence, greatest value $= \frac{1}{3}$

Q. [KTK-02]

Find the greatest value of $\frac{x+2}{2x^2+3x+6}$ for real values of x.

Sol

$$y = \frac{x+2}{2x^2+3x+6}$$

$\Rightarrow 2x^2y + 3xy + 6y = x+2$

$\Rightarrow 2x^2y + 3xy + 6y - x - 2 = 0$

$\Rightarrow 2x^2y + 3xy - x + 6y - 2 = 0$

$\Rightarrow 2x^2y + (3y-1)x + 6y - 2 = 0$

$D \geq 0$

$\Rightarrow (3y-1)^2 - 4 \cdot 2y \cdot (6y-2) \geq 0$

$\Rightarrow (3y)^2 + (1)^2 - 2 \cdot 3y \cdot 1 - 8y(6y-2) \geq 0$

$\Rightarrow 9y^2 + 1 - 6y - 48y^2 + 16y \geq 0$

$\Rightarrow -39y^2 + 10y + 1 \geq 0$

$\Rightarrow 39y^2 - 10y - 1 \leq 0$

$\Rightarrow (13y+1)(3y-1) \leq 0$

$y \in \left[\frac{-1}{13}, \frac{1}{3}\right]$

Rough

$3\,|\,\frac{39}{13}$

$39y^2 - 13y + 3y - 1$

$13y(3y-1) + 1(3y-1)$

$(3y-1)(13y+1)$

$\therefore$ the greatest value of $x = \frac{1}{3}$ Ans

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Aniket raj
From patna

QUESTION

(KTK 03)

If $\frac{mx^2+3x+4}{x^2+3x+4} < 5$ for all $x \in R$, find possible values of m.

KTK 03

Sol^n: $\dfrac{mx^2+3x+4}{x^2+3x+4} < 5$

RASIDUL

$a > 0 \quad D < 0 \rightarrow$ +ve.

$mx^2+3x+4 < 5x^2+15x+20$

$x^2(m-5)+x(-12)-16 < 0$

Ans: $m \in \left(-\infty, \frac{11}{4}\right)$

Here, $a < 0 \quad D < 0$

$m-5 < 0 \quad \Rightarrow 144+64(m-5) < 0$

$m < 5$

$9+4(m-5) < 0 \Rightarrow 4m-11 < 0$

$m < \frac{11}{4}$

max

$\boxed{\text{KTK-03}}$

If $\dfrac{mx^2+3x+4}{x^2+3x+4} < 5$ for all $x \in R$ find possible value of m.

$$\frac{mx^2+3x+4}{\underset{\substack{a>0 \\ D<0}}{x^2+3x+4}} < 5 \quad \left[\begin{array}{c}\text{always} \\ +\text{ve}\end{array}\right]$$

$$mx^2+3x+4 < 5x^2+15x+20$$

$$mx^2-5x^2+3x-15x+4-20 < 0$$

$$x^2(m-5)-12x-16 < 0$$

since, $x \in R$

$D < 0$ & $a < 0$

$(-12)^2 - 4(m-5) \times (-16) < 0$

$144 + 64m + - 320 < 0$

$64m - 176 < 0$

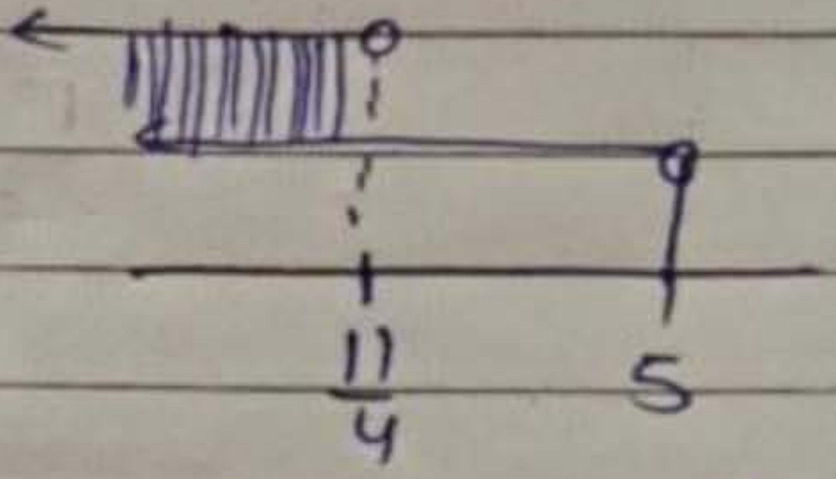
$8m - 22 < 0$

$4m - 11 < 0$

$a < 0$

$m - 5 < 0$

$m < 5$



$m \in \left(-\infty, -\frac{11}{4}\right)$

QUESTION

(KTK 04)

Find all values of m for which the equation $m \in R$, $m \neq -1$, $(1 + m)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ gives roots according to the following conditions:

(i) Exactly one root in the interval $(2, 3)$. Ans. $[4, \infty)$

(ii) One root smaller than 1 and other root greater than 1. Ans. $(-1, 0)$

(iii) Both roots smaller than 2. Ans. $(-1, 0]$

(iv) Atleast one root in the interval $(2, 3)$. Ans. $[3, \infty)$

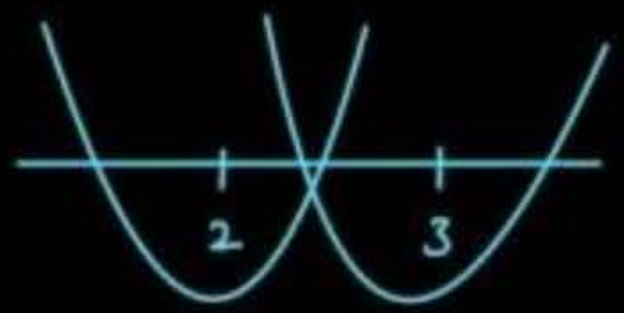
(v) Atleast one root greater than 2. Ans. $(-\infty, -1) \cup [1, \infty)$

(vi) Roots such that both 1 and 2 lie between them. Ans. ϕ

(vii) One root in $(1, 2)$ and other root in $(2, 3)$. Ans. ϕ

$\Rightarrow \quad (1+m)x^2 - 2(1+3m)x + (1+8m) = 0 \,, \quad m \neq -1$

$\hookrightarrow \quad x^2 - \dfrac{2(1+3m)x}{1+m} + \dfrac{1+8m}{1+m} = 0$

(i) 

$f(2)\cdot f(3) < 0$

$\big(4(1+m) - 4(1+3m) + 1+8m\big)\big(9(1+m) - 6(1+3m) + 1+8m\big) < 0$

$[\cancel{4} + \cancel{4m} - \cancel{4} - \cancel{12m} + 1 + \cancel{8m}][9 + 9m - 6 - 18m + 1 + 8m] < 0$

$(1)(4-m) < 0$

$(4-m) < 0$

$(m-4) > 0 \Rightarrow \boxed{m > 4}$

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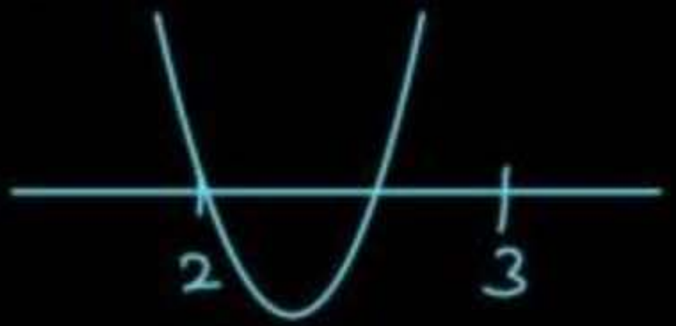
Made with Goodnotes

more possibilities!

I

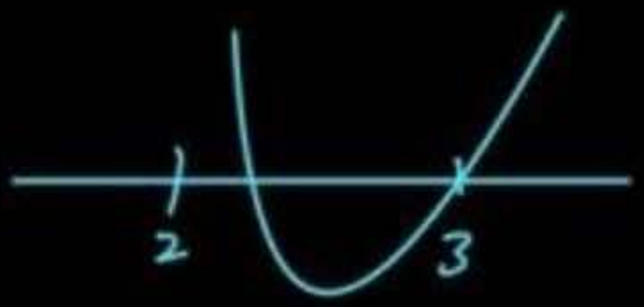


$f(2) = 0$

$1 = 0$

↳ not true

II



$f(3) = 0$

$4 - m = 0$

$m = 4$

$$x^2 - \frac{2}{5}(13)x + \frac{1+32}{5} = 0$$

$$5x^2 - 26x + 33 = 0$$

$$5x^2 - 11x - 15x + 33 = 0$$

$$(5x - 11)(x - 3) = 0$$

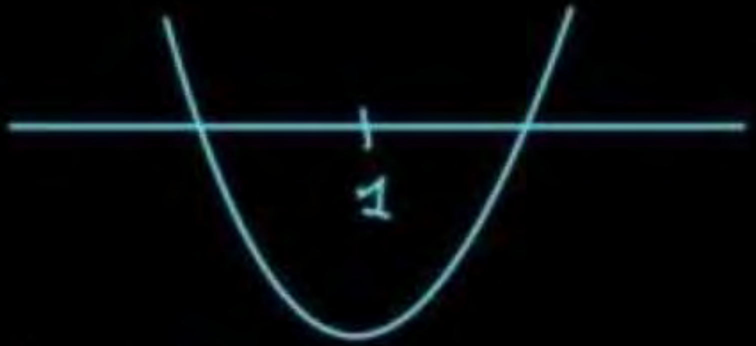
$x = 3$ or $x = 11/5$

↳ accepted

Union of cases

$$\boxed{m \in [4, \infty)}$$

ii.)



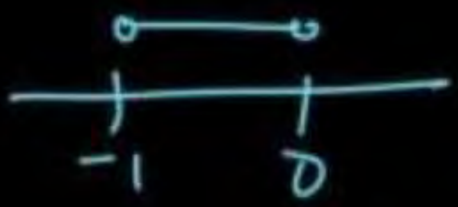
$$x^2 - 2\left[\frac{1+3m}{1+m}\right]x + \left[\frac{1+8m}{1+m}\right] = 0$$

$f(1) < 0$

$D > 0 \to$ no need.

$$\frac{1+m-2-6m+1+8m}{1+m} < 0$$

$$\frac{3m}{1+m} < 0$$



$$\boxed{m \in (-1,0)}$$

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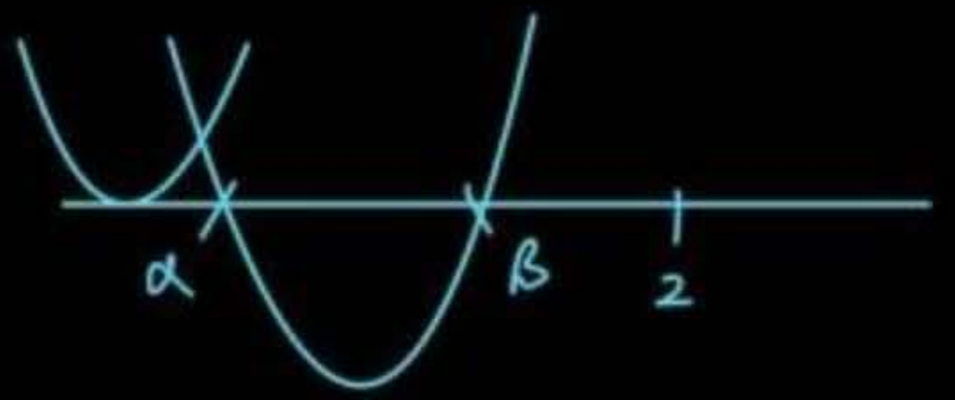
(iii) $x^2 - \dfrac{2(1+3m)}{1+m}x + \dfrac{1+8m}{1+m} = 0$



$\Rightarrow$ $-b/2a < 2$

$D \geq 0$

$f(2) > 0$

$-b/2a < 2$

$\dfrac{1+3m}{1+m} < 2$

$\dfrac{1+3m-2-2m}{1+m} < 0$

$\dfrac{m-1}{m+1} < 0$

$m \in (-1, 1)$

$D \geq 0$

$\cancel{4}\left[\dfrac{1+3m}{1+m}\right]^2 - \cancel{4}\left[\dfrac{1+8m}{1+m}\right] \geq 0$

$\dfrac{1+9m^2+6m-(1+8m)(1+m)}{(1+m)^2} \geq 0$

$\dfrac{9m^2+6m+1-1-m-8m-8m^2}{(1+m)^2} \geq 0$

$\dfrac{m^2-3m}{(1+m)^2} \geq 0$

$\dfrac{m(m-3)}{(1+m)^2} \geq 0$

$+ \quad + \quad - \quad +$

$-1 \quad 0 \quad 3$

$\underline{m \in (-\infty,-1) \cup (-1,0] \cup [3,\infty)}$

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$f(2) > 0$

$$\frac{4(1+m) - 4(1+3m) + 1 + 8m}{1+m} > 0$$

$$\frac{\cancel{4} + \cancel{4m} - \cancel{4} - \cancel{12m} + 1 + \cancel{8m}}{1+m} > 0$$

$$\frac{1}{1+m} > 0$$

$\hookrightarrow m > -1$

intersection of all condⁿ :-

$$\boxed{(-1, 0]}$$

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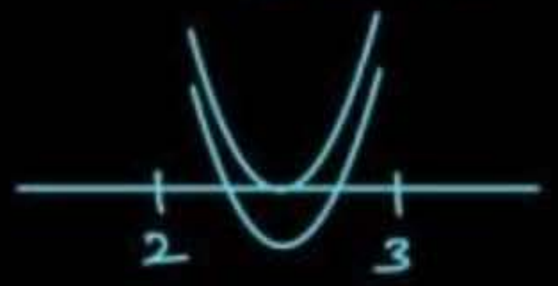
iv.) $x^2 - 2\left[\frac{1+3m}{1+m}\right]x + \left[\frac{1+8m}{1+m}\right] = 0$

Condⁿ of for exactly one root $\cup$ condⁿ for both roots in $(2,3)$

$[4, \infty)$ + condⁿ

Condⁿ :-



$D \geq 0$

$f(2) > 0$

$f(3) > 0$

$2 < \frac{-b}{2a} < 3$

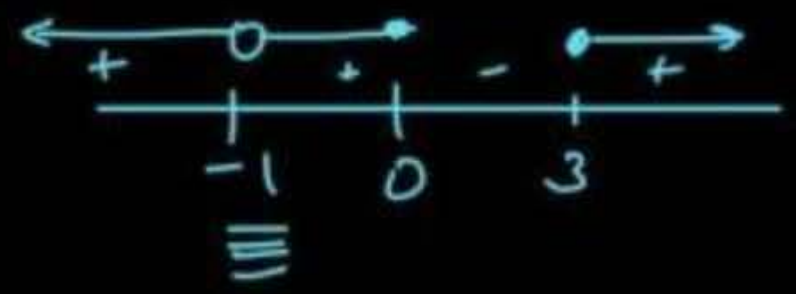
$D \geq 0$

$$4\left[\frac{1+3m}{1+m}\right]^2 - 4\left[\frac{1+8m}{1+m}\right] \geq 0$$

$$\frac{1 + 9m^2 + 6m - (1+8m)(1+m)}{(1+m)^2} \geq 0$$

$$\frac{9m^2 + 6m + 1 - 1 - m - 8m - 8m^2}{(1+m)^2} \geq 0$$

$$\frac{m^2 - 3m}{(1+m)^2} \geq 0 \Rightarrow \frac{m(m-3)}{(1+m)^2} \geq 0$$



$m \in (-\infty, -1) \cup (-1, 0] \cup [3, \infty)$

$$2 < \frac{1+3m}{1+m} < 3$$

$$0 < \frac{-2-2m+1+3m}{1+m} \quad \cap \quad \frac{1+\cancel{3m}-3-\cancel{3m}}{1+m} < 0$$

$$\frac{m-1}{m+1} > 0 \qquad \frac{1}{1+m} > 0$$

$$m \in (-\infty, -1) \cup (1, \infty) \quad \cap \quad m \in (-1, \infty)$$

$$m \in (1, \infty)$$

Made with Goodnotes

$f(2) > 0$

$$\frac{4(1+m) - 4(1+3m) + (1+8m)}{1+m} > 0$$

$$\frac{4 + 4m - 4 - 12m + 1 + 8m}{1+m} > 0$$

$\underline{m > -1}$

$f(3) > 0$

$$\frac{9(1+m) - 6(1+3m) + 1 + 8m}{1+m} > 0$$

$$\frac{9 + 9m - 6 - 18m + 1 + 8m}{1+m} > 0$$

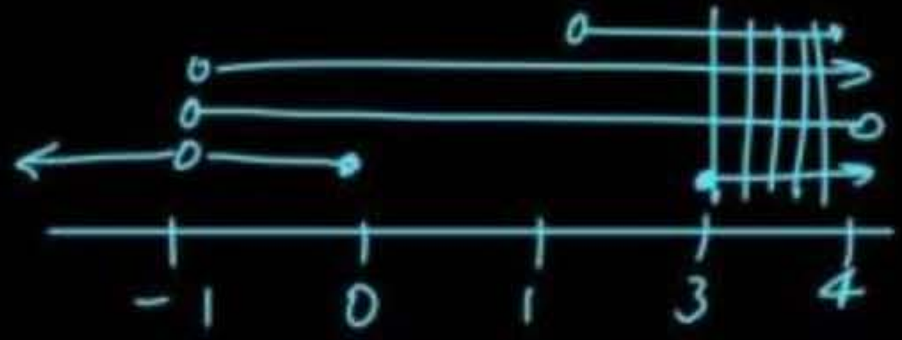
$$\frac{4-m}{1+m} > 0$$

$$\frac{m-4}{m+1} < 0$$

$\underline{m \in (-1, 4)}$

$\boxed{m \in [3, \infty)}$

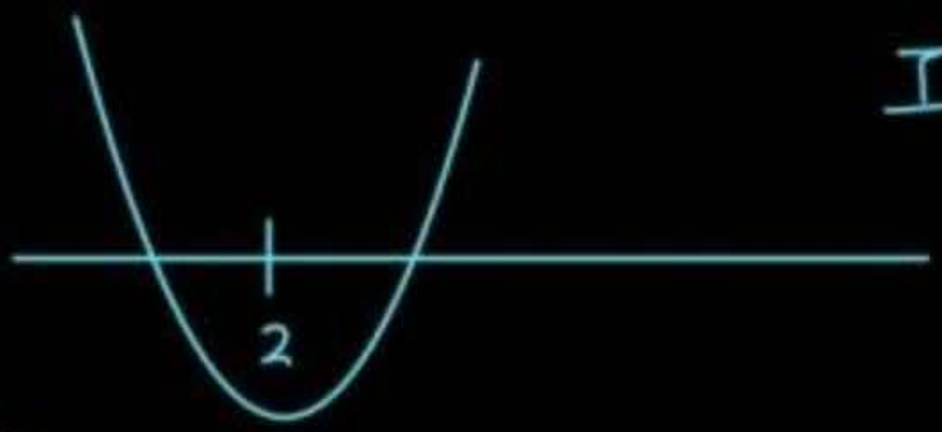
intersec^n of all cases $\Rightarrow$



$m \in [3, 4)$

$\cup\ [4, \infty)$

v) $x^2 - 2\left[\frac{1+3m}{1+m}\right]x + \left[\frac{1+8m}{1+m}\right] = 0$



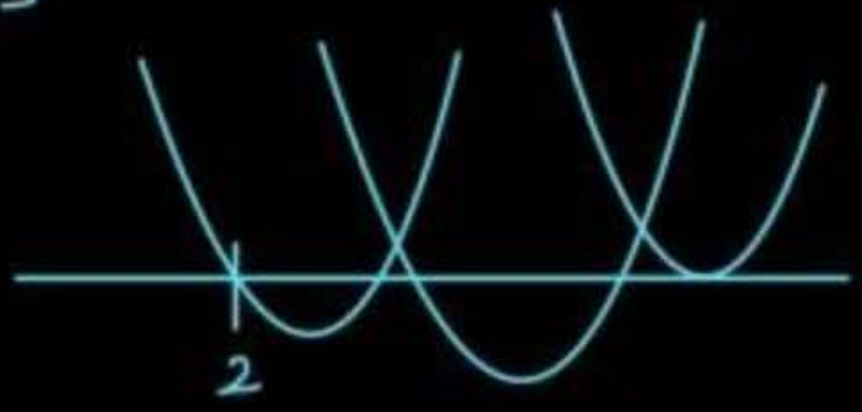
I

$f(2) < 0$

$\Rightarrow \frac{4(1+m) - 4(1+3m) + (1+8m)}{1+m} < 0$

$\frac{4 + 4m - 4 - 12m + 1 + 8m}{1+m} < 0$

$\underline{m < -1}$ $\quad m \in (-\infty, -1)$



II

$f(2) \geq 0$

$D \geq 0$

$\frac{-b}{2a} > 2$

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$$f(2) \geq 0$$

$$\frac{4(1+m) - 4(1+3m) + (1+8m)}{1+m} \geq 0$$

$$\frac{4 + 4m - 4 - 12m + 1 + 8m}{1+m} \geq 0$$

$$m \geq -1$$

$$m \in [-1, \infty)$$

$$D \geq 0$$

$$\cancel{4}\left[\frac{1+3m}{1+m}\right]^2 - \cancel{4}\left[\frac{1+8m}{1+m}\right] \geq 0$$

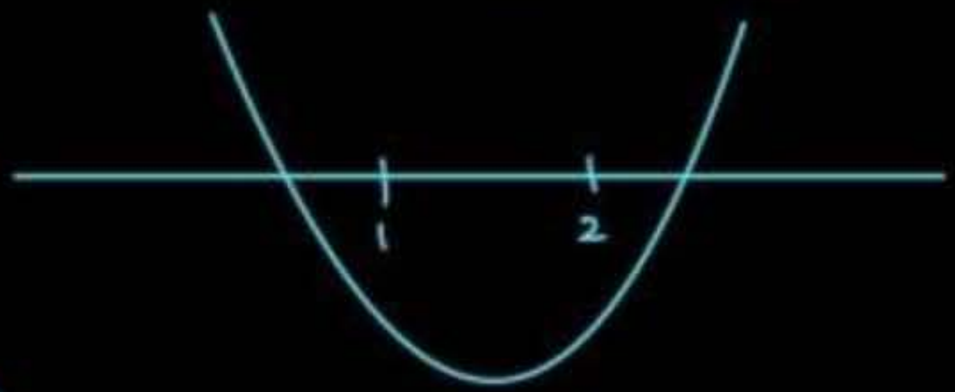
$$\frac{1 + 9m^2 + 6m - (1+8m)(1+m)}{(1+m)^2} \geq 0$$

$$\frac{9m^2 + 6m + 1 - 1 - m - 8m - 8m^2}{(1+m)^2} \geq 0$$

$$\frac{m(m-3)}{(m+1)^2} \geq 0$$

$$m \in (-\infty, 0] \cup [3, \infty) - \{-1\}$$

vi.) $x^2 - 2\left[\frac{1+3m}{1+m}\right]x + \left[\frac{1+8m}{1+m}\right] = 0$



$f(1) < 0$

$f(2) < 0$

$f(1) < 0$

$$\frac{1+m-2-6m+1+8m}{1+m} \leq 0$$

$$\frac{+3m}{1+m} < 0 \qquad m \in (-1, 0)$$

$f(2) < 0$

$$\frac{4+4m-4-12m+1+8m}{1+m} < 0$$

$$\frac{1}{1+m} < 0$$

$m \in (-\infty, -1)$

$\cap$

$\rightarrow \boxed{\phi}$

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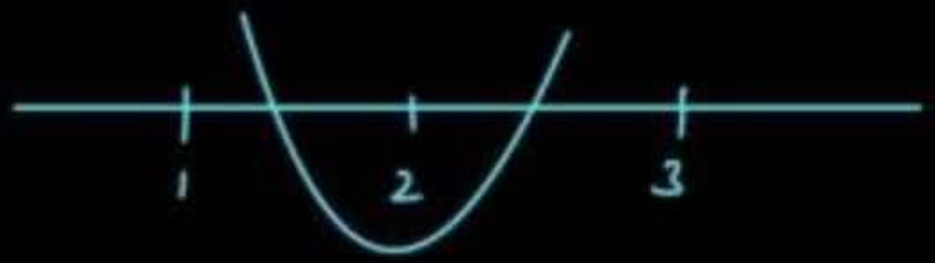
vii) $x^2 - 2\left[\frac{1+3m}{1+m}\right]x + \left[\frac{1+8m}{1+m}\right] = 0$



$f(2) < 0$

$$\frac{4+4m-4-12m+1+8m}{1+m} < 0$$

$$\frac{1}{1+m} < 0$$

$m \in (-\infty, -1)$

$f(2) < 0$

$f(1) > 0$

$f(3) > 0$

$1 < \frac{-b}{2a} < 3 \rightarrow$ no need

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Haryana

Made with Goodnotes

$f(1) > 0, \quad \dfrac{1+m-2-6m+1+8m}{1+m} > 0$

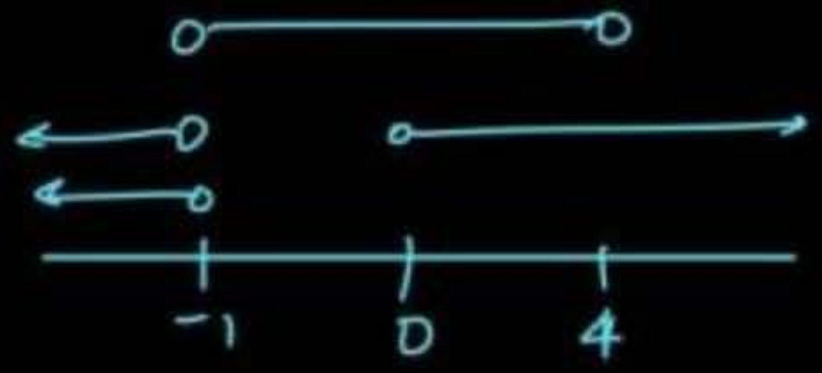
$$\frac{+3m}{1+m} > 0 \qquad m \in (-\infty,-1) \cup (0,\infty)$$

$f(3) > 0, \quad \dfrac{9(1+m)-6(1+3m)+1+8m}{1+m} > 0$

$$\frac{9+9m-6-18m+1+8m}{1+m} > 0$$

$$\frac{4-m}{1+m} > 0$$

$$\frac{m-4}{m+1} < 0, \quad m \in (-1,4)$$



Ans (ϕ)

QUESTION

(KTK 05)

If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval $[1, 5]$, then m lies in the interval :

A $(-5, -4)$

B $(4, 5)$

C $(5, 6)$

D $(3, 4)$

Homework From Module

Quadratic Equations

Prarambh (Topicwise) : Complete

Prabal (JEE Main Level) : Complete

Parikshit (JEE Advanced Level) : Q1 to Q15

THANK YOU