PRAYAS

JEE 2026

Mathematics

Quadratic Equations

Lecture - 09

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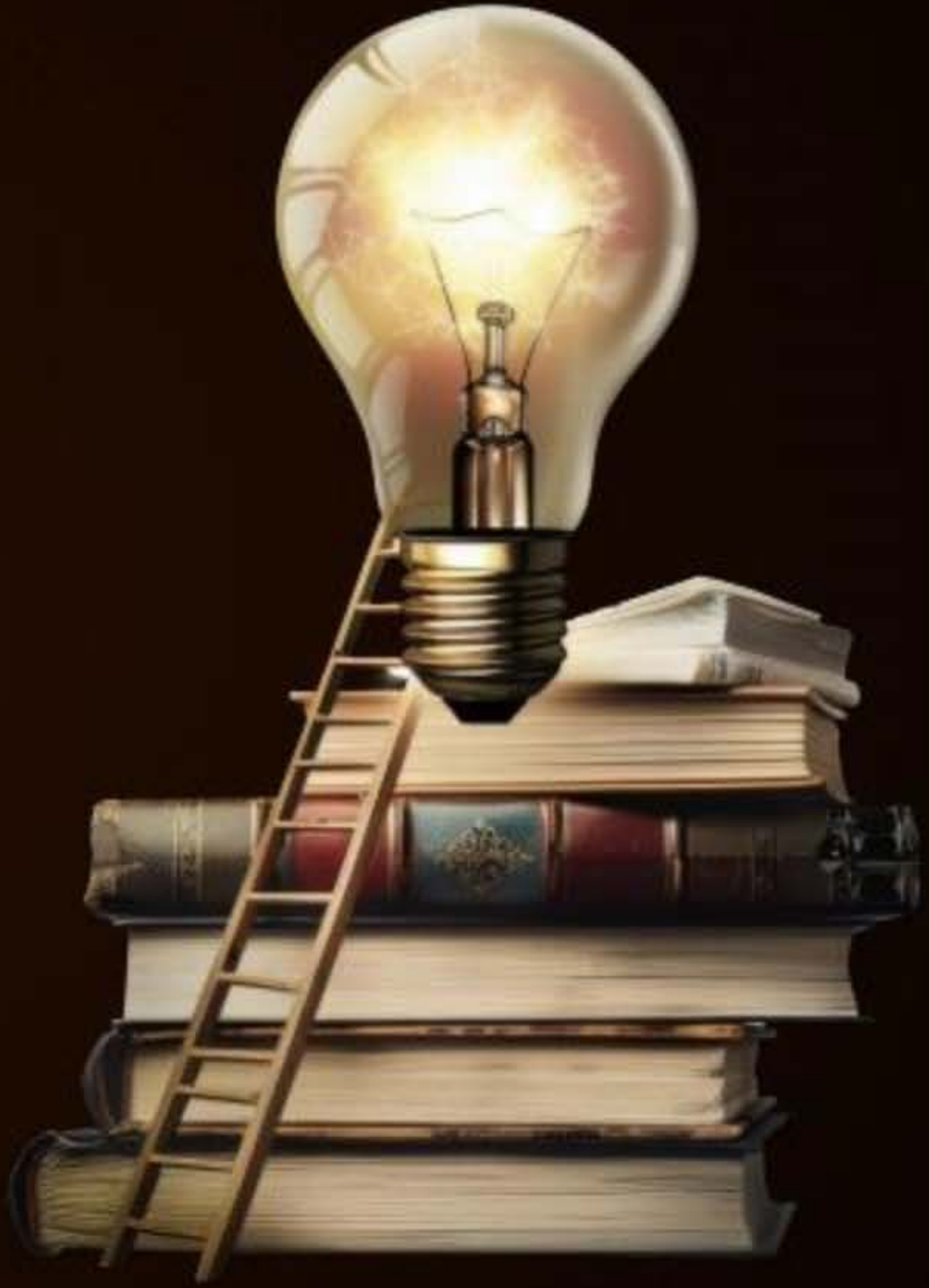
(IIT Kanpur)

Topics *to be covered*

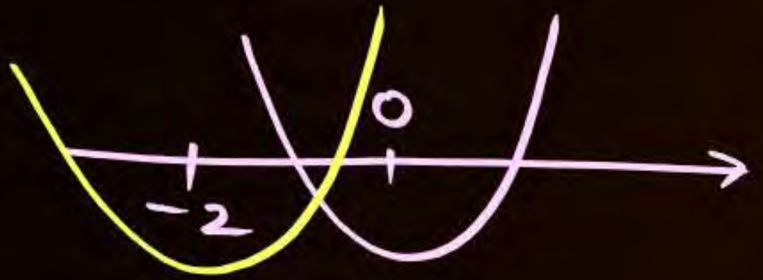
- **A** More Question practice on LOR
- **B** General Second Degree polynomial in x& y



Homework Discussion

QUESTION

Find all possible values of m for which exactly one root of the equation $x^2 + mx + m^2 + 6m = 0$, lies in $(-2, 0)$.



$f(0) \cdot f(-2) < 0$

$(m^2+6m)(4-2m+m^2+6m) < 0$

$m(m+6)(m^2+4m+4) < 0$

$m(m+6)(m+2)^2 < 0$

$m(m+6) < 0, m \neq -2.$

$m \in (-6, 0) - \{-2\}$

P(1)

$f(0) = 0 \rightarrow m = 0, -6$

$m = 0$ Eqn: $x^2 = 0$

(rejected) $x = 0, 0 \notin (-2, 0)$

second root

$m = -6$ $x^2 - 6x = 0$

(rejected) $x = 0, 6 \notin (-2, 0)$

second root

P (2) $f(-2)=0$

$m=-2$ (rejected)

$x^2-2x-8=0$

$(x+2)(x-4)=0$

$x=-2,4 \notin (-2,0)$

Ans: $m \in (-6,0)-\{-2\}$.

QUESTION

(KTK 03)

If the quadratic polynomial $f(x) = (a-3)x^2 - 2ax + 3a - 7$ ranges from $[-1, \infty)$ for every $x \in R$, then the value of a lies in

(A) [0, 2]

(B) [3, 5]

(C) [4, 6)

(D) [5, 7]

clearly $a-3>0$.

$a>3$

$f(x)=(a-3)x^2-2ax+3a-7$ — Range $\left[-\frac{D}{4A}, \infty\right) = [-1, \infty)$

$\frac{-D}{4A} = -1$

$D = 4A$

$4a^2 - 4(a-3)(3a-7) = 4(a-3)$

$a^2 - 3a^2 + 7a + 9a - 21 = a - 3$

$-2a^2 + 15a - 18 = 0$

$2a^2 - 15a + 18 = 0$

$2a^2 - 12a - 3a + 18 = 0$

$(2a-3)(a-6) = 0$

$a = 3/2, 6.$

Ans. D

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

QUESTION

If $f(x) = x^2 + bx + c$ and $f(2 + t) = f(2 - t)$ for all real numbers t, then which of the following is true?

(A) $f(1) < f(2) < f(4)$

(B) $f(2) < f(1) < f(4)$

(C) $f(2) < f(4) < f(1)$

(D) $f(2.1) < f(1.5) < f(3)$

QUESTION [JEE Mains 2020]

Tahol

The set of all real values of λ for which the quadratic equations,

$(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval $(0, 1)$ is:

(A) $(-3, -1)$

(B) $(2, 4]$

(C) $(0, 2)$

(D) $(1, 3]$

QUESTION

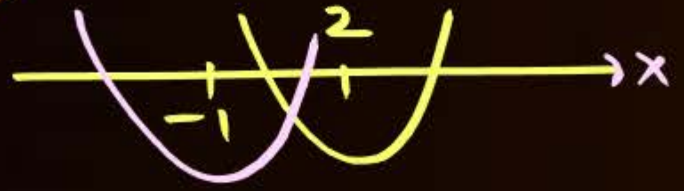
$-1 \notin (-1, 2)$

Find the values of a so that the equation $x^2 + (3 - 2a)x + a = 0$ has exactly one root in

(a) $(-1, 2)$ (b) $[-1, 2]$

(a)

$x^2 + (3-2a)x + a = 0$



$f(-1) \cdot f(2) < 0$

$(1 - 3 + 2a + a)(4 + 6 - 4a + a) < 0$

$(3a - 2)(10 - 3a) < 0$.

$(3a - 2)(3a - 10) > 0$.

$a \in (-\infty, 2/3) \cup (10/3, \infty)$

P① one root lies at $x = -1$

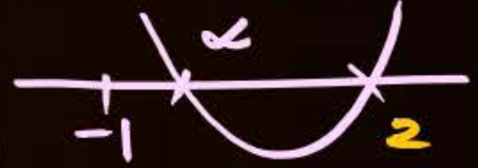
$f(-1) = 0 \Rightarrow a = 2/3$

$P.O.R = a = 2/3$

$-1 \cdot \alpha = 2/3$

$\alpha = -2/3 \in (-1, 2/3)$

P② one root lies at $x = 2$



$f(2) = 0 \Rightarrow a = \frac{10}{3}$

$P.O.R = a = \frac{10}{3}$

$2 \cdot \alpha = \frac{10}{3}$

$\alpha = 5/3 \in (-1, 2)$

Ans: $a \in (-\infty, 2/3] \cup [10/3, \infty)$

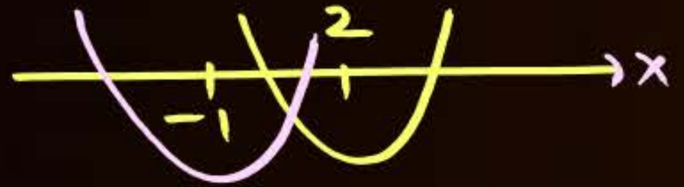
QUESTION

Find the values of a so that the equation $x^2 + (3 - 2a)x + a = 0$ has exactly one root in

(a) $(-1, 2)$ (b) $[-1, 2]$

(b)

$x^2 + (3-2a)x + a = 0$



$f(-1) \cdot f(2) < 0$

$(1 - 3 + 2a + a)(4 + 6 - 4a + a) < 0$

$(3a - 2)(10 - 3a) < 0$.

$(3a - 2)(3a - 10) > 0$.

$a \in (-\infty, 2/3) \cup (10/3, \infty)$

P① one root lies at $x = -1$

$f(-1) = 0 \Rightarrow a = 2/3$ ✗

$P.O.R = a = 2/3$

$-1 \cdot \alpha = 2/3$

$\alpha = -2/3$ is not > 2 or < -1

P② one root lies at $x = 2$

$f(2) = 0 \Rightarrow a = \frac{10}{3}$ (rej)

$P.O.R = a = \frac{10}{3}$

$2 \cdot \alpha = \frac{10}{3}$

$\alpha = 5/3 \not> 2$ or < -1

Ans: $a \in (-\infty, 2/3) \cup (10/3, \infty)$

★ if $D_1 + D_2 \geq 0$ → Both D_1 & D_2 can not be -ve.

↓

atleast one of D_1 or $D_2 \geq 0$

⇓

★ Atleast one of the quadratics has real roots.

$P(x) = a_1x^2 + b_1x + c_1$

↓

$D_1 = b_1^2 - 4a_1c_1$

$f(x) = a_2x^2 + b_2x + c_2$

$D_2 = b_2^2 - 4a_2c_2$.

★ If one of the quadratics has imaginary roots then roots of other quad are real & distinct.

Ex: If $D_2 < 0 \Rightarrow D_1$ should be +ve

If $D_1 < 0 \Rightarrow D_2$ should be +ve

↓

$f(x) = 0$ has real & Distinct roots.

★★★★ If $D_1 + D_2 < 0$ ⟶ Both D_1 & D_2 can not be +ve.

⇓

Atleast one of D_1 or D_2 is −ve.

⇓

★ Atleast one of the quad has imaginary roots.

★ If one of the quad has real roots then roots of the other will be imaginary

A Golden Point

(i) If $D_1 + D_2 \geq 0 \Rightarrow$

(a) Atleast one equation has real roots.

(b) If roots of one equation are imaginary then those of other equation will be real & unequal.

(ii) If $D_1 + D_2 < 0 \Rightarrow$

(a) Atleast one of the equation has imaginary roots.

(b) If roots of one equation are real then those of the other equation will be imaginary.

QUESTION

(IIT JEE)

Let $f(x) = x^2 + ax + b$ and $g(x) = x^2 + cx + d$ be two quadratic polynomial with real coefficients and satisfy $ac = 2(b + d)$. Then which of the following is(are) correct?

(A) Exactly one of either $f(x) = 0$ or $g(x) = 0$ must have real roots.

(B) Atleast one of either $f(x) = 0$ or $g(x) = 0$ must have real roots.

(C) Both $f(x) = 0$ and $g(x) = 0$ must have real roots.

(D) Both $f(x) = 0$ and $g(x) = 0$ must have imaginary roots.

$f(x) = x^2 + ax + b$

$D_1 = a^2 - 4b$

$g(x) = x^2 + cx + d$

$D_2 = c^2 - 4d$

$D_1 + D_2 = a^2 + c^2 - 4(b+d)$

$D_1 + D_2 = a^2 + c^2 - 4 \cdot \frac{ac}{2}$

$= a^2 + c^2 - 2ac$

$D_1 + D_2 = (a-c)^2 \geq 0$

QUESTION

(IIT JEE)

The polynomial $(ax^2 + bx + c)(ax^2 - dx - c)$; $ac \neq 0$ has

(A) Four real zeros

(B) Atmost two real zeros

(C) Atleast two real zeros

(D) Information is insufficient

$h(x) = (ax^2+bx+c)\cdot(ax^2-dx-c)$

let $f(x) = ax^2+bx+c$, $g(x) = ax^2-dx-c$

$D_1 = b^2-4ac$ $\quad$ $D_2 = d^2+4ac$

$D_1 + D_2 = b^2+d^2 \geq 0$.

$\Downarrow$

Atleast one of $f(x)$ or $g(x)$ has real roots.

$\Downarrow$

$h(x)$ has atleast two real roots.

Atmost → Jyaada se Jyaada

Atleast → kum se kum

QUESTION

Let $a, b, c \in R, a > 0$ such that the equation $ax^2 + bcx + b^3 + c^3 - 4abc = 0$ has non real roots let $P(x) = ax^2 + bx + c$ & $Q(x) = ax^2 + cx + b$, then

(A) $P(x) > 0 \forall x \in R$ & $Q(x) = ax^2 + cx + b > 0$

(B) $P(x) < 0 \forall x \in R$ & $Q(x) < 0 \ \forall x \in R$

(C) Neither $P(x)$ nor $Q(x) > 0 \ \forall x \in R$

(D) Exactly one of $P(x)$ or $Q(x)$ is positive for all $x \in R$

$f(x,y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ is general second degree polynomial in x & y.

Condition for $f(x,y)$ to be resolved into two linear factors in x & y

$ax^2 + bx + c = a(x-\alpha)(x-\beta)$

$f(x,y) = ax^2 + x(2hy + 2g) + by^2 + 2fy + c$ — Quad in x.

$$x = \frac{-(2hy+2g) \pm \sqrt{(2hy+2g)^2 - 4 \cdot a(by^2+2fy+c)}}{2a}$$

$$x = \frac{-(hy+g) \pm \sqrt{h^2y^2 + g^2 + 2hyg - aby^2 - 2afy - ac}}{a}$$

$$x = \frac{-(hy+g) \pm \sqrt{(h^2-ab)y^2+(2hg-2af)y+g^2-ac}}{a} = \alpha, \beta$$

$$f(x,y) = a(x-\alpha)(x-\beta)$$

$$= a\left(x - \left(\frac{-(hy+g) + \sqrt{(h^2-ab)y^2+(2hg-2af)y+g^2-ac}}{a}\right)\right)$$

$$\left(\left(x - \left(\frac{-(hy+g) - \sqrt{(h^2-ab)y^2+(2hg-2af)y+g^2-ac}}{a}\right)\right)\right)$$

for linear factors in x & y. $(h^2-ab)y^2+(2hg-2af)y+g^2-ac$ should be a perfect square.

$$\Rightarrow D = (2hg - 2af)^2 - 4(h^2 - ab)(g^2 - ac) = 0$$

$$h^2g^2 + a^2f^2 - 2hgaf - h^2g^2 + h^2ac + abg^2 - a^2bc = 0$$

$$af^2 - 2fgh + h^2c + bg^2 - abc = 0$$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ can be splitted into two linear factors if $\Delta = 0$ i.e $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$a=2,\ 2h=3,\ b=1,\ 2g=3,\ 2f=2,\ c=1$

$$\Delta = \begin{vmatrix} 2 & 3/2 & 3/2 \\ 3/2 & 1 & 1 \\ 3/2 & 1 & 1 \end{vmatrix} = 0 \quad (R_2 = R_3)$$

$\Rightarrow$ above second degree exp can be factorized in to two linear factors.

M① $2x^2+3xy+y^2+2y+3x+1=0$

$$2x^2+x(3y+3)+y^2+2y+1=0$$

$$x = \frac{-3(y+1) \pm \sqrt{9(y+1)^2-8(y+1)^2}}{4} = \frac{-3(y+1) \pm (y+1)}{4}$$

$$x = \frac{-3y-3+y+1}{4}, \frac{-3y-3-y-1}{4}$$

$$x = \frac{-2y-2}{4}, \frac{-4y-4}{4}$$

$$x = \frac{-(y+1)}{2}, -(y+1)$$

$$f(x,y) = 2\left(x + \frac{y+1}{2}\right)(x+y+1)$$

$$f(x,y) = (2x+y+1)(x+y+1)$$

M(2) $2x^2 + 3xy + y^2 + 2y + 3x + 1 = f(x,y)$

Factorize homogenous part

$$2x^2 + 3xy + y^2 = 2x^2 + 2xy + xy + y^2$$

$$= (2x+y)(x+y)$$

$$(2x+y+\lambda)(x+y+\mu) = 2x^2 + 3xy + y^2 + 2y + 3x + 1$$

coeff of x $\quad \lambda + 2\mu = 3$

coeff of y $\quad \lambda + \mu = 2$

$\mu = 1$

$\lambda = 1$

factors $(2x+y+1)(x+y+1)$

QUESTION

Show that in the equation, $x^2 - 3xy + 2y^2 - 2x - 3y - 35 = 0$, for every real value of x there is a real value of y, and for every value of y there is a real value of x.

$x^2 - (3y+2)x + 2y^2 - 3y - 35 = 0$ Let $y \in R$ be any Number

$$D = (3y+2)^2 - 4(2y^2 - 3y - 35)$$
$$= 9y^2 + 4 + 12y - 8y^2 + 12y + 140$$
$$= y^2 + 24y + 144$$
$$= (y+12)^2 \geq 0 \ \forall y \in R$$
$$\Downarrow$$

we get real x for every $y \in R$.

Again $2y^2 - y(3x+3) + x^2 - 2x - 35 = 0$ let $x \in R$ be any Number.

$$D = 9(x+1)^2 - 8(x^2 - 2x - 35)$$
$$= x^2 + 34x + 289 = (x+17)^2 \geq 0 \ \forall x \in R$$

we get real y for every $x \in R$

QUESTION

If the equation $x^2 + 16y^2 - 3x + 2 = 0$ is satisfied by real values of x and y then prove that $1 \leq x \leq 2$ and $-1/8 \leq y \leq 1/8$.

$x^2 + 16y^2 - 3x + 2 = 0$

$x^2 - 3x + 16y^2 + 2 = 0$

for real x

$D = 9 - 4(16y^2 + 2) \geq 0$

$9 - 64y^2 - 8 \geq 0$

$64y^2 - 1 \leq 0$

$(8y-1)(8y+1) \leq 0$

$y \in [-1/8, 1/8]$

$16y^2 + 0 \cdot y + x^2 - 3x + 2 = 0$

for real y

$D = 0 - 4 \cdot 16(x^2 - 3x + 2) \geq 0$

$= -64(x-1)(x-2) \geq 0$

$(x-1)(x-2) \leq 0$

$x \in [1, 2]$

QUESTION

The values of k, for which the equation $x^2 + 2(k-1)x + k + 5 = 0$ possess atleast one positive root, are

(A) $[4, \infty)$

(B) $(-\infty, -1] \cup [4, \infty)$

(C) $[-1, 4]$

(D) $(-\infty, -1]$

Case(i)

$f(0) < 0$

$D > 0 \longrightarrow$ No Need

$k + 5 < 0$

$k < -5$

Case(ii)

$f(0) \geq 0 \Rightarrow k + 5 \geq 0 \Rightarrow k \geq -5$

$-\frac{b}{2a} > 0 \Rightarrow -\frac{2(k-1)}{2} > 0 \Rightarrow k < 1.$

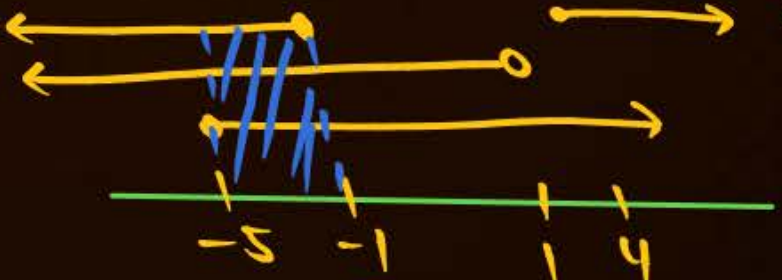
$D \geq 0 \Rightarrow 4(k-1)^2 - 4 \cdot 1(k+5) \geq 0$

$k^2 - 2k + 1 - k - 5 \geq 0$

$k^2 - 3k - 4 \geq 0$

$(k-4)(k+1) \geq 0$

$k \in (-\infty, -1] \cup [4, \infty)$



$k \in [-5, -1]$

$\cup$

$k \in (-\infty, -1]$

QUESTION

Find 'a' for which the inequality $(a^2+3)x^2+\left(\sqrt{5a+3}\right)x-\frac{1}{4}<0$ is satisfied for atleast one real x.

upward opening parabola

should go below x axis

⇓

should have 2 real & distinct roots.

⇓

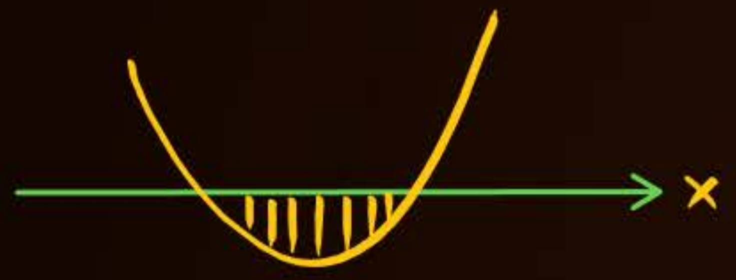
$D>0$

$5a+3-4(a^2+3)(-1/4)>0.$

$a^2+5a+6>0.$

$(a+2)(a+3)>0$

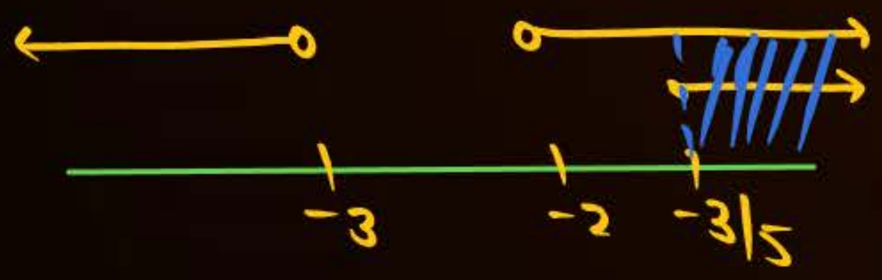
$a\in(-\infty,-3)\cup(-2,\infty)$



Also $5a+3\geq 0.$

$a\geq -3/5$

$\cap$



$a\in[-3/5,\infty)$

Ans. $a\geq -3/5$

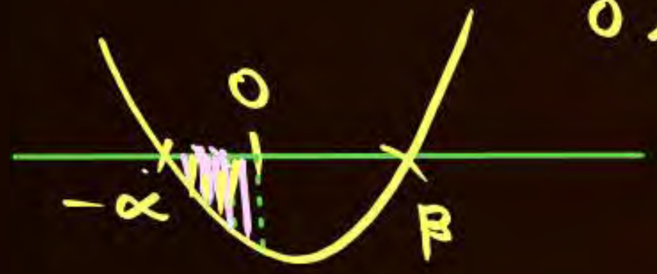
QUESTION

$f(x)$

Prove that for any real value of 'a' the inequality, $(a^2 + 3)x^2 + (a + 2)x - 5 < 0$ is true for at least one negative x.

upward opening parabola should go below x axis for atleast one -ve x.

Observe!! $f(0) = -5$ → 0 lies b/w roots.



⇓

one root is -ve say $-\alpha$ & other +ve say β

$f(x) < 0 \ \forall \ x \in (\alpha, 0)$

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION [JEE Mains 2021 (25 Feb)]

(KTK 01)

The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is:

(A) 4

(B) 2

(C) 3

(D) 0

Ans. C

QUESTION

(KTK 02)

Find the greatest value of $\frac{x+2}{2x^2+3x+6}$ for real values of x.

Ans. 1/3

QUESTION

(KTK 03)

If $\frac{mx^2+3x+4}{x^2+3x+4} < 5$ for all $x \in R$, find possible values of m.

QUESTION

(KTK 04)

Find all values of m for which the equation

$m \in R$, $m \neq -1$, $(1 + m)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ gives roots according to the following conditions:

(i) Exactly one root in the interval (2, 3). Ans. $[4, \infty)$

(ii) One root smaller than 1 and other root greater than 1. Ans. $(-1, 0)$

(iii) Both roots smaller than 2. Ans. $(-1, 0]$

(iv) Atleast one root in the interval (2, 3). Ans. $[3, \infty)$

(v) Atleast one root greater than 2. Ans. $(-\infty, -1) \cup [1, \infty)$

(vi) Roots such that both 1 and 2 lie between them. Ans. ϕ

(vii) One root in (1, 2) and other root in (2, 3). Ans. ϕ

QUESTION

(KTK 05)

If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval $[1, 5]$, then m lies in the interval :

A $(-5, -4)$

B $(4, 5)$

C $(5, 6)$

D $(3, 4)$

Homework From Module

Quadratic Equations

Prarambh (Topicwise) : Q1 to Q27

Prabal (JEE Main Level) : Q1,Q2,Q6 to Q9

Parikshit (JEE Advanced Level) : Abhi Ruko

Solution to Previous TAH

QUESTION

★★KCLS★★

Find number of integral values α for which the quadratic equation $x^2 + \alpha x + \alpha + 1 = 0$ has integral roots.

Q(1) Find num of integral values ~~of~~ α for which the quad eq^n $x^2+\alpha x+\alpha+1=0$ has integral roots.

Sol. integers $\begin{matrix} a \\ b \end{matrix}$ > $x^2+\alpha x+\alpha+1=0$ $\quad \alpha \in I$

↓

D should be perfect square

$\alpha^2-4(\alpha+1)=m^2$

$\alpha^2-4\alpha-4=m^2$

$\alpha^2-4\alpha+4-8=m^2$

$(\alpha-2)^2-m^2=8$

$(\alpha-2-m)(\alpha-2+m)=8$

$\alpha-2-m=+1$	$\alpha-2-m=-8$	$\alpha-2-m=2$	$\alpha-2-m=-4$
$\alpha-2+m=+8$	$\alpha-2+m=-1$	$\alpha-2+m=4$	$\alpha-2+m=-2$
$\alpha=13/2$	$\alpha=-5/2$	$\alpha=5$	$\alpha=-1$
✗	✗	✓	✓

L-8 (quadratic)

classmate Date Page

JA!! (1) $x^2+\alpha x+\alpha+1=0$

$\Rightarrow x^2+\alpha x+\alpha+1=0$ — a, b

here, $a=1$, $b, c = I$

$D = \alpha^2 - 4(\alpha+1)$

$\alpha^2 - 4(\alpha+1) = m^2$ where $m \in I$

$(\alpha-2)^2 - (m^2) = 8 \Rightarrow (\alpha-2-m)(\alpha-2+m) = 8$

rej	1	8	: possible cases
rej	8	1	
✓	2	4	
✓	4	2	
rej	−1	−8	
rej	−8	−1	
✓	−2	−4	
✓	−4	−2	

Now,

$\alpha-2-m = 1$

$\oplus\ \alpha-2+m = 8$

$2\alpha = 13 \Rightarrow \alpha = 13/2$ (rej)

$\alpha-2-m = 2$

$\oplus\ \alpha-2+m = 4$

$2\alpha = 10 \Rightarrow \alpha = 5$

$\alpha-2-m = -2$

$\oplus\ \alpha-2+m = -4$

$2\alpha-4 = -6$

$2\alpha = -2 \Rightarrow \alpha = -1$

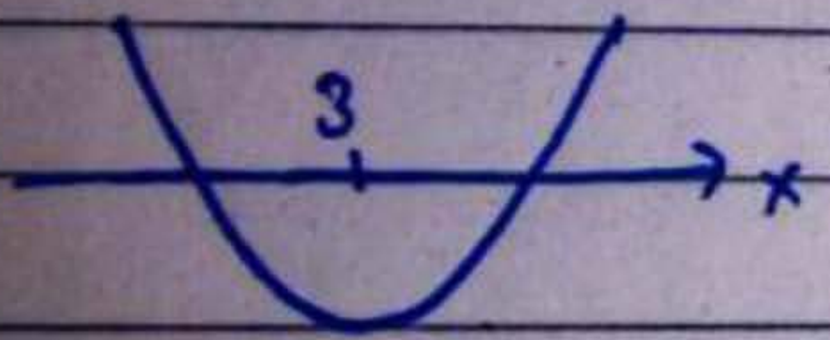
Hence for two integral values of α i.e. 5 & −1 for which quadratic has integral roots//

QUESTION

Find the set of values of 'a' for which zeroes of the quadratic polynomial $(a^2 + a + 1)x^2 + (a - 1)x + a^2$ are located on either side of 3.

TAH-02

$(a^2+a+1)x^2 + (a-1)x + a^2$. are located either side of 3



$a^2+a+1 > 0$. since coefficient of $a^2 > 0$,

& $D < 0$

we dont need to check $D \geq 0$ cause roots are either side

$f(3) < 0$.

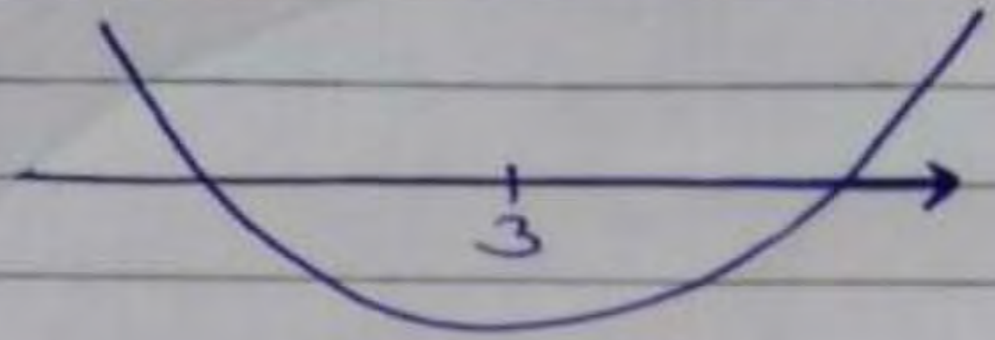
$9a^2+9a+9+3a-3+a^2 < 0$.

$10a^2+12a+6 < 0$.

$5a^2+6a+3 < 0$. No such value

ϕ

TAH-02



$y = \underbrace{(a^2+a+1)}_{D<0} x^2 + (a-1)x + a^2$ — ①

always +ve

$f(3) < 0$

$(a^2+a+1)9 + (a-1)3 + a^2 < 0$

$9a^2 + 9a + 9 + 3a - 3 + a^2 < 0$

$10a^2 + 12a + 6 < 0$

$5a^2 + 6a + 3 < 0$

15, 5, 3

$a > 0$

$D = 36 - 4(5)(3)$

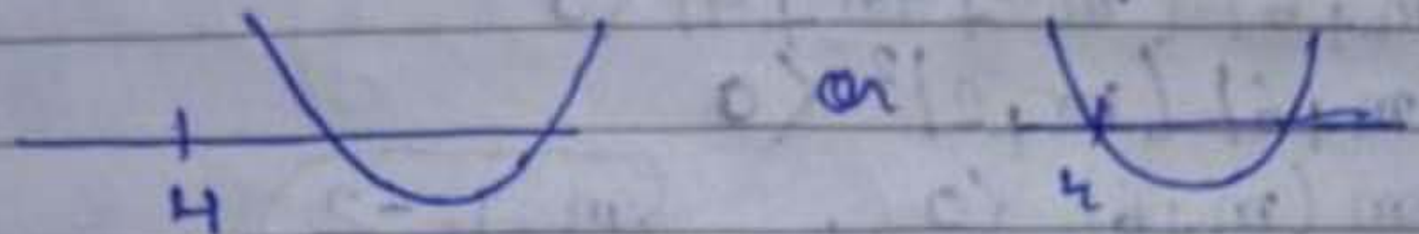
$= -ve$

no such values of a exist.

QUESTION [JEE Advanced 2009]

The smallest value of k, for which both the roots of the equation, $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is

Ans $x^2 - 8kx + 16(k^2 - k + 1) = 0$



(i) $\Delta > 0$

$64k^2 - 64(k^2 - k + 1) > 0$

$k^2 - k^2 + k - 1 > 0$

$k > 1$

(ii) $\frac{-b}{2a} > 4$

$8k > 8$

$k > 1$

(iii) $f(4) \geq 0$

$16 - 32k + 16k^2 - 16k + 16 \geq 0$

$16k^2 - 48k + 32 \geq 0 \rightarrow k^2 - 3k + 2 \geq 0$

$k^2 - 2k - k + 2 \geq 0$

So $(k-2)(k-1) \geq 0$

$k \in (-\infty, 1] \cup [2, \infty)$ Intersection of (i), (ii) & (iii)

$\therefore x \in [2, \infty)$ Smallest Value of $k = 2$ Ans

TAH 03

Soln:

$x^2 - 8kx + 16(k^2 + k + 1) = 0$

① $f(4) \geq 0$

② $-\frac{b}{2a} > 4$

③ $D > 0$

RASIDUL

① $f(4) \geq 0$

$16 - 8k(4) + 16(k^2 - k + 1) \geq 0$

$16 - 32k + 16k^2 - 16k + 16 \geq 0$

$16k^2 - 48k + 32 \geq 0$

$k^2 - 3k + 2 \geq 0$

$k^2 - 2k - k + 2 \geq 0$

$k(k-2) - 1(k-2) \geq 0$

$(k-2)(k-1) \geq 0$

$k \in (-\infty, 1] \cup [2, \infty)$

② $-\frac{b}{2a} > 4$

$\frac{8k}{2} > 4$

$8k > 8$

$k > 1$ $k \in (1, \infty)$

$k \in [2, \infty)$

③ $D > 0$

$64k^2 - 64k^2 + 64k + 64 >$

$64k > -64$

$k > -1$

$k \in (-1, \infty)$

Ans: min. value 2.

TAH-03

$f(x) = x^2 - 8kx + 16(k^2 - k + 1) = 0.$

Conditions

i) $f(4) \geq 0$

$16 - 8(4)k + 16(k^2 - k + 1) \geq 0$

$16 - 32k + 16k^2 - 16k + 16 \geq 0$

$16k^2 - 48k + 32 \geq 0.$

$k^2 - 3k + 2 \geq 0.$

$(k-2)(k-1) \geq 0.$

$k \in (-\infty, 1] \cup [2, \infty).$

(ii) $\frac{-b}{2a} > 4$

$\frac{8k}{2} > 4.$

$k > 1.$

(iii) $D > 0$

$64k^2 - 64(k^2 - k + 1) > 0.$

$k^2 - k^2 + k - 1 > 0.$

$k > 1.$

$\cap$

$k \in [2, \infty)$

$\boxed{k = 2}$ ans

QUESTION

Find all the values of 'a' for which both roots of the equation $x^2 + x + a = 0$ exceed the quantity 'a'.

Ans. $(-\infty, -2)$

TA H 04

Soln:

$$x^2+x+a=0$$

$f(a) > 0$

$a^2+a+a > 0$

$a^2+2a > 0$

$a(a+2) > 0$

~~$x \in (-2,$~~

$x \in (-\infty,-2) \cup (0,\infty)$

$D > 0$

$1-4a > 0$

$4a < 1$

$a < \frac{1}{4}$

$a \in (-\infty, \frac{1}{4})$

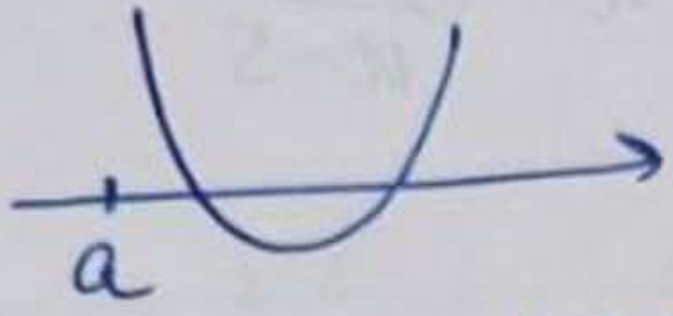
$\cap$

$a \in (-\infty,-2)$

Ans: $a \in (-\infty,-2)$

TAH-04

$f(x) = x^2+x+a=0.$



(i) $\frac{-b}{2a} > a$

$\frac{-1}{2} > a.$

$a < -\frac{1}{2}.$

(ii) $f(a) > 0$

$a^2+a+a > 0.$

$a(a+2) > 0.$

$(-\infty, -2) \cup (0, \infty)$

$D \geq 0.$

$1-4(a) \geq 0.$

$4a-1 \leq 0.$

$a \leq \frac{1}{4}.$

$\cap$

$a \in (-\infty, -2).$

QUESTION

If α, β are roots of the quadratic equation $x^2 + 2(k-3)x + 9 = 0 (\alpha \neq \beta)$.
If $\alpha, \beta \in (-6, 1)$, find k.

TAH (5) $x^2 + 2(K-3)x + 9 = 0$ with roots α, β $(\alpha \neq \beta)$

so, roots are distinct

$a > 0$

$\alpha, \beta \in (-6, 1)$

→ possibility //

$f(-6) > 0$ — (I)

$f(1) > 0$ — (II)

$-6 < \frac{-b}{2a} < 1$ — (III)

$D > 0$ — (IV)

Now,

$f(-6) > 0$ — (I)

$36 + 2(-6)(K-3) + 9 > 0$

$36 - 12K + 36 + 9 > 0$

$-12K + 81 > 0$

$12K < 81$ → $4K < 27$

$\boxed{K < \frac{27}{4}}$

$f(1) > 0$ — (II)

$1 + 2K - 6 + 9 > 0$

$2K > 4$

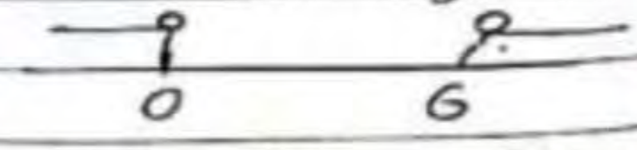
$\boxed{K > 2}$

$D > 0$ — (IV)

$4(K-3)^2 - 4 \cdot 9 > 0$

$K^2 + 9 - 6K - 9 > 0$

$K(K-6) > 0$



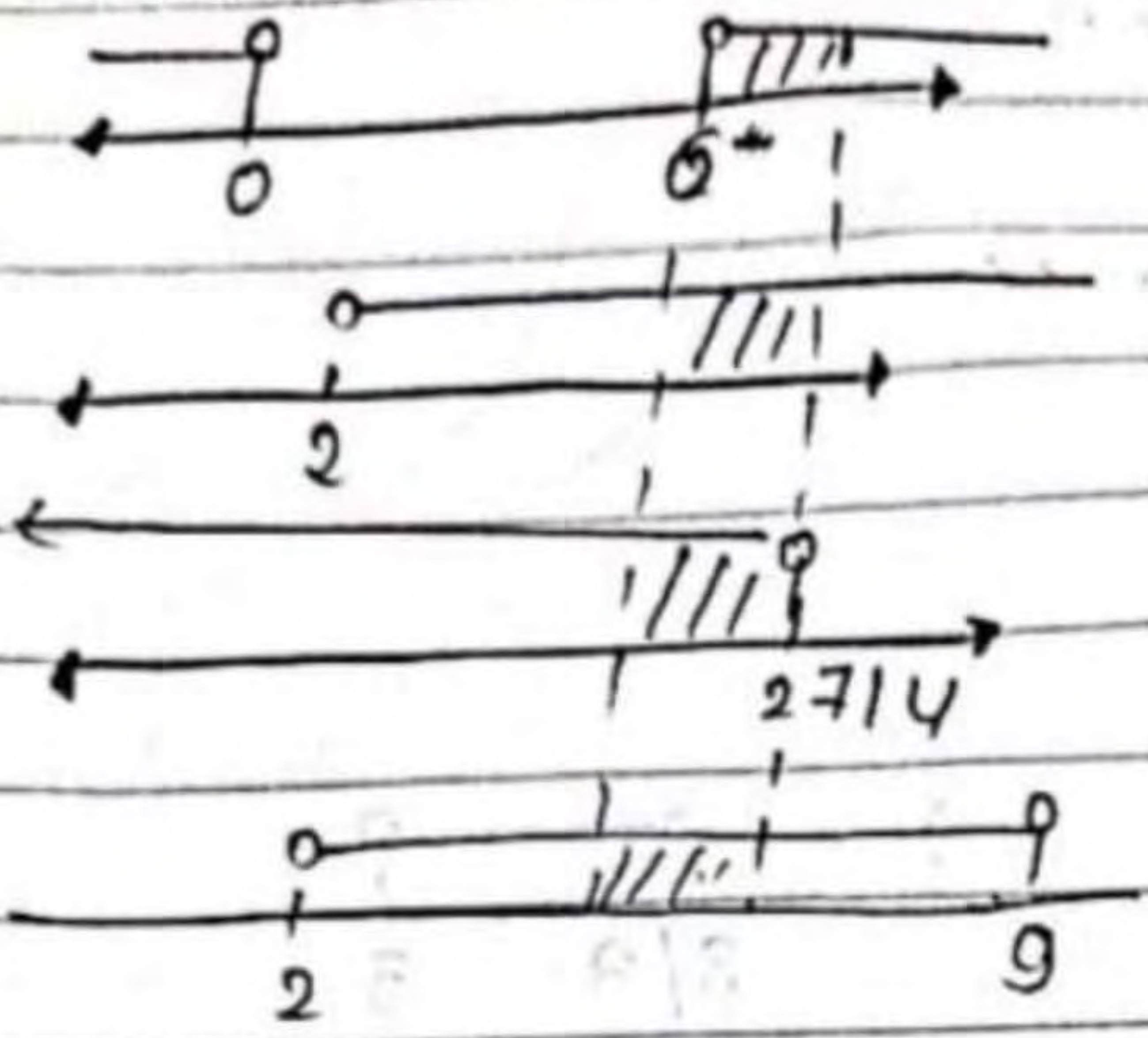
$-6 < \frac{-b}{2a} < 1$ — (III)

$-6 < \frac{-2(K-3)}{2} < 1 \Rightarrow -6 < -(K-3) < 1$

$\Rightarrow -1 < K-3 < 6$

$\Rightarrow \boxed{2 < K < 9}$

Now, (i) ∩ (ii) ∩ (iii) ∩ (iv)



$\Rightarrow K \in (6, 27/4)$

QUESTION

Find the value of k for which one root of the equation of $(k-5)x^2 + 2kx + k - 4 = 0$ is smaller than 1 and the other root exceed 2.

AH ⑥ $(K-5)x^2 + 2Kx + K - 4 = 0$ $\langle^{\alpha}_{\beta}$ $\Rightarrow x^2 + \frac{2K}{K-5}x + \frac{K-4}{K-5} = 0$

where 1 root is smaller than 1 & other exceed 2 $\boxed{a>0}$

$\alpha < 1 < 2 < \beta$

Here $D \neq 0$

$f(1) < 0$ — (i)

$f(2) < 0$ — (ii)

$D > 0 \rightarrow$ NO need (by logic)

So, $f(1) < 0$ — (i)

$K - 5 + 2K + K - 4 < 0$

$4K - 9 < 0$

$\boxed{K < 9/4}$

$f(2) < 0$ — (ii)

$4(K-5) + 4K + K - 4 < 0$

$4K - 20 + 5K - 4 < 0$

$9K < 24$

$\boxed{K < \frac{8}{3}}$

(i) & (ii) $f(1) < 0$ — (i)

$\Rightarrow 1 + \frac{2K}{K-5} + \frac{K-4}{K-5} < 0 \Rightarrow \frac{K - 5 + 3K - 4}{(K-5)} < 0$

$\Rightarrow \frac{(4K-9)}{(K-5)} < 0 \Rightarrow$ (number line: open circles at 9/4 and 5)

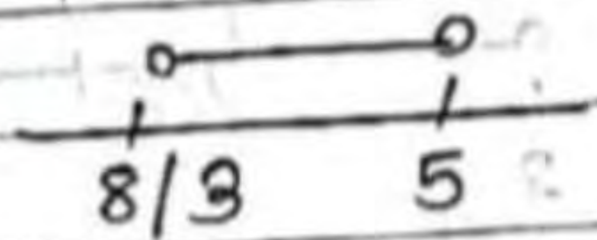
9/4   5

$f(2) < 0$

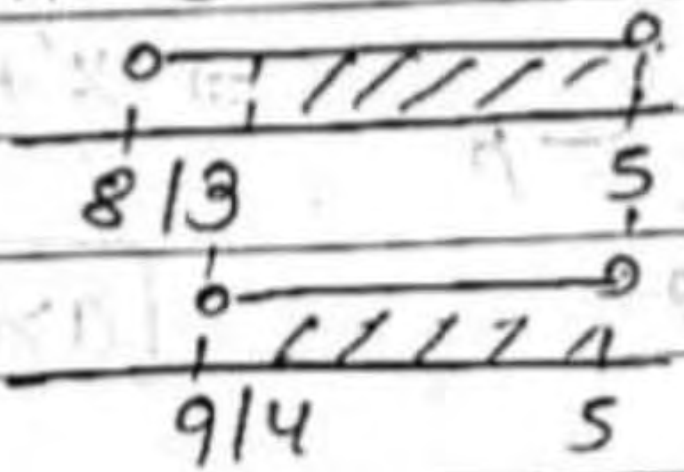
$$4 + \frac{4k}{k-5} + \frac{k-4}{k-5} < 0$$

$$\frac{4k-20+4k+k-4}{k-5} < 0$$

$$\frac{9k-24}{k-5} < 0$$

$$\frac{(k-8/3)}{(k-5)} < 0 \Rightarrow$$ 

(i) ∩ (ii)



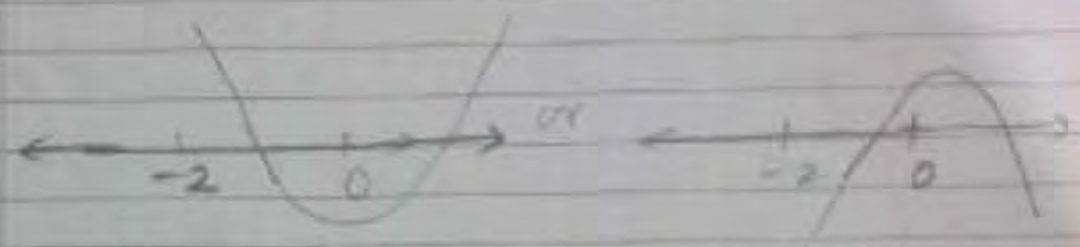
$\Rightarrow k \in (9/4, 5)$ //

QUESTION

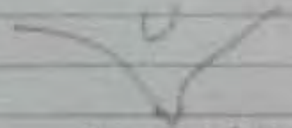
Find all possible values of m for which exactly one root of the equation $x^2 + mx + m^2 + 6m = 0$, lies in $(-2, 0)$.

IANS:- Find all possible values of m for which exactly one root of equation $x^2+mx+m^2+6m=0$, lies in $(-2,0)$.

Ans $x^2+mx+m^2+6m=0$



$f(-2)>0$ | $f(-2)<0$
$f(0)<0$ | $f(0)>0$
$a>0$ | $a<0$
$D>0$ | $D>0$



$f(-2)\,f(0)<0$
$a\neq 0$
$D>0$ {no need

(i) $f(0)f(-2)<0$

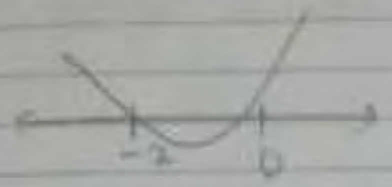
$(m^2+6m)(4-2m+m^2+6m)<0$

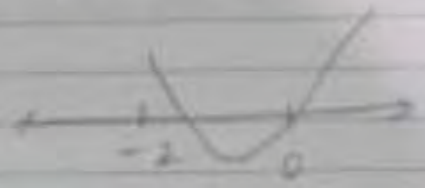
$m(m+6)(m+2)^2<0$

$m(m+6)<0\quad , m\neq -2$

$m\in(-6,0)-\{-2\}$ —(1)

Possibilities —





$f(-2)=0$

$m^2+4m+4=0$
$m+2=0$
$m=-2$

$f(0)=0$

$m^2+6m=0$
$m(m+6)=0$
$m=0,\ m=-6$

So, at $m=-2$,

$x^2-2x+4-12=0$
$x^2-2x-8=0$
$(x-4)(x+2)=0$
$x=4,\ x=-2$ {NP}
✓

at $m=0$,

$x^2=0$
$x=0$ {NP} ∵ Both roots will become equal

at $m=-6$

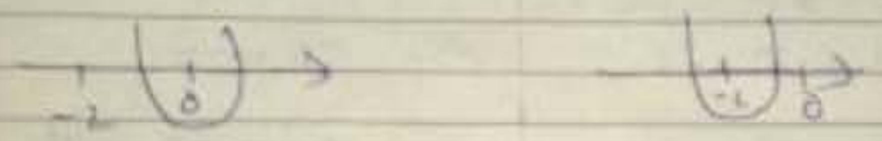
$x^2-6x+36-36=0$
$x(x-6)=0$
$x=0,\ x=6$ {NP}

So, $m\in(0,6)-\{-2\}$

Tah-5
Pg.2

Q Find all possible values for which exactly one root of the eqn $x^2 + mx + m^2 + 6m = 0$ lies in $(-2, 0)$

Sol



$f(0) \cdot f(-2) < 0$

$(m^2+6m)(4-2m+m^2+6m) < 0$

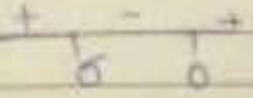
$m(m+6)(m^2+4m+4) < 0$

$m(m+6)(m^2+2m+2m+4) < 0$

$m(m+6)(m+2)(m+2) < 0$

Chirag kardam
From - Delhi

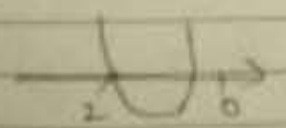
$m(m+6) < 0 \quad m \neq -2$



$m \in (-6, 0) - \{-2\}$ ...(1)

Now two possibilities arise

Case 1



$f(-2) = 0, \ (m+2)^2 = 0$

$(4-2m+m^2+6m) = 0$

$(m+2)^2 = 0$

$m = -2$

$x^2 - 2x + 4 - 12 = 0$

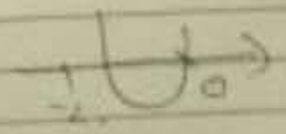
$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, \ x = -2$

4, -2 not lies in $(-2, 0)$

Case 2



$f(0) = 0$

$(m^2+6m) = 0$

$m(m+6) = 0$

$m = 0, \ m = -6.$

$\Rightarrow m = 0$

$x^2 + mx + m^2 + 6m = 0$

$x^2 = 0$

$x = 0.$

$\Rightarrow m = -6$

$x^2 - 6x + 36 - 36 = 0$

$x(x-6) = 0$

$x = 0, \ x = 6$

0, 6 not lies in $(-2, 0)$

Final Ans

$m \in (-6, 0) - \{-2\}$ Ans

Solution to Previous KTKs

QUESTION

(KTK 01)

If $x \in R$ then range of $f(x) = \frac{x^2+2x-3}{2x^2+3x-9}$ is

(A) $(-\infty, \infty)$

(B) $R - \left\{\frac{1}{2}\right\}$

(C) $R - \left\{\frac{4}{9}, \frac{1}{2}\right\}$

(D) $R - \left\{\frac{3}{2}\right\}$

Ans. C

KTK 01

Sol^n:

$$f(x) = \frac{x^2+2x-3}{2x^2+3x-9}$$

$$y = \frac{x^2+3x-x-3}{2x^2+6x-3x-9}$$

$$y = \frac{x(x+3)-1(x+3)}{2x(x+3)-3(x+3)}$$

$$y = \frac{(x+3)(x-1)}{(x+3)(2x-3)}$$

$$y = \frac{x-1}{2x-3}, \quad x \neq -3 \qquad f(-3) = \frac{-3-1}{-6-3} = \frac{4}{9}$$

$$\text{Range} = R - \left\{\tfrac{1}{2}, \tfrac{4}{9}\right\}.$$

$$= R - \left\{\tfrac{4}{9}, \tfrac{1}{2}\right\}.$$

Ans: (C)

RASIDUL

KTK-1.) If $x \in R$ then range of $f(x) = \frac{x^2+2x-3}{2x^2+3x-9}$ is

$$y = \frac{x^2+2x-3}{2x^2+3x-9}$$

Kriti Mathur Raj.

$$y = \frac{(x-1)(x+3)}{(x+3)(2x-3)} \qquad y(-3) = \frac{-4}{-9} = \frac{4}{9}$$

$$y = \frac{x-1}{2x-3}, \quad x \neq -3$$

$$\text{Range}: R - \left\{\frac{1}{2}, \frac{4}{9}\right\}$$ (C) - Ans.

QUESTION

(KTK 02)

If the highest point on the graph of $y = -x^2 - 2kx + 3a$ is $(-1, 2)$ then the value of $(k + 6a)$ is

$x_v \Downarrow -\frac{b}{2a}$ $\quad$ $y_v = -\frac{D}{4a}$

(A) 2

(B) 3

(C) 5

(D) 6

KTK-2.) If the highest point on the graph of $y = -x^2 - 2kx + 3a$ is $(-1, 2)$ then the value of $(k + 6a)$ is

highest point : $\left(\frac{-b}{2a}, \frac{-D}{4a}\right) = (-1, 2)$

$\frac{-b}{2a} = -1 \Rightarrow \frac{2k}{-2} = -1$

$\Rightarrow [k = 1]$

$\frac{-D}{4a} = 2 \Rightarrow \frac{-(4k^2 - 4(-1)(3a))}{-4} = 2$

$= 4k^2 + 12a = 8$

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Raj.

$= 4 + 12a = 8$

$\Rightarrow 12a = 4$

$\Rightarrow a = \frac{1}{3}$

$k + 6a \Rightarrow 1 + 6\left(\frac{1}{3}\right) = 1 + 2$

$= 3$ (B) Ans.

QUESTION

(KTK 03)

If the quadratic polynomial $f(x) = (a-3)x^2 - 2ax + 3a - 7$ ranges from $[-1, \infty)$ for every $x \in R$, then the value of a lies in

(A) $[0, 2]$

(B) $[3, 5]$

(C) $[4, 6)$

(D) $[5, 7]$

Ans. C

QUESTION

(KTK 04)

Find the range of values of a, such that $f(x) = \frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32}$ is always negative.

$(?)\ \frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32}$ — Always (−ve)

$(+)$

$x^2-8x+32$: $a>0,\ D<0$ → always +ve

$ax^2+2(a+1)x+9a+4<0\ \forall x\in R$

$a<0$ & $D<0$.

∩ → Ans.

Ans. $a \in \left(-\infty, -\frac{1}{2}\right)$

KTK 04

Soln.

$$y = \frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32}$$

$\rightarrow D < 0, \rightarrow$ +ve

$a < 0$ (given)

$D < 0$

$4(a^2 + 2a + 1) - 4a(9a+4) < 0$

$4a^2 + 8a + 4 - 36a^2 - 16a < 0$

$-32a^2 - 8a + 4 < 0$

$32a^2 + 8a - 4 > 0$

$8a^2 + 2a - 1 > 0$

$8a^2 + 4a - 2a - 1 > 0$

$4a(2a+1) - 1(2a-1) > 0$

$(2a+1)(4a-1) > 0$

$a \in (-\infty, -\frac{1}{2}) \cup (\frac{1}{4}, \infty)$

$\therefore$ Range : $(-\infty, -\frac{1}{2})$

RASIDUL

Q. [KTK-04]

Find the range of values of a such that:

$f(x) = \frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32}$ is always negative.

Sol: $f(x) = \frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32}$

$\hookrightarrow$ $a > 0$, $D < 0$ always +ve

$\Rightarrow 4(a+1)^2 - 4a(9a+4) < 0$

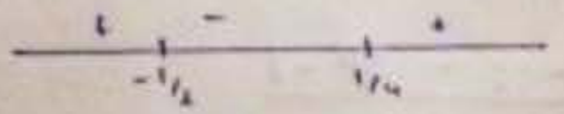
$\Rightarrow 4a^2 + 4 + 8a - 36a^2 - 16a < 0$

$\Rightarrow -32a^2 - 8a + 4 < 0$

$\Rightarrow -8a^2 - 2a + 1 < 0$

$\Rightarrow -(8a^2 + 2a - 1) < 0$

$\Rightarrow (2a+1)(4a-1) > 0$



$a \in (-\infty, -\frac{1}{2}) \cup (\frac{1}{4}, \infty)$

for $a < 0$

$a \in (-\infty, -1/2)$ Ans

Galaxy F15 5G

Aniket raj
From patna

QUESTION [AIEEE 2002]

(KTK 05)

If $\alpha^2 = 5\alpha - 3, \beta^2 = 5\beta - 3$ then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is

(A) $\frac{19}{3}$

(B) $\frac{25}{3}$

(C) $\frac{-19}{3}$

(D) None of these

Ans. A

KTK 05:

$\alpha^2 = 5\alpha - 3$ $\qquad$ $\beta^2 - 5\beta + 3$

$x^2 - 5x + 3 = 0 \rightarrow \alpha, \beta$

$\alpha + \beta = 5$

$\alpha\beta = 3$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$\frac{\alpha^2+\beta^2}{\alpha\beta} \Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$\Rightarrow \frac{(5)^2 - 2\times 3}{3}$$

$$\Rightarrow \frac{25-6}{3}$$

$$\Rightarrow \frac{19}{3}$$

KTK-5.) If $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$ then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is

$\alpha^2 = 5\alpha - 3$

$\beta^2 = 5\beta - 3$ — $x^2 = 5x - 3$

$\Rightarrow x^2 - 5x + 3 = 0$

$\Rightarrow \alpha\beta = \frac{c}{a} = 3$, $\alpha + \beta = 5$

$$\Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} \Rightarrow \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{5\alpha - 3 + 5\beta - 3}{3}$$

$$\Rightarrow \frac{5\alpha + 5\beta - 6}{3}$$

$$\Rightarrow \frac{5(5) - 6}{3} = 19/3 \text{ Ans.}$$

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Raj.

THANK YOU