PRAYAS

JEE 2026

Mathematics

Quadratic Equations

Lecture - 08

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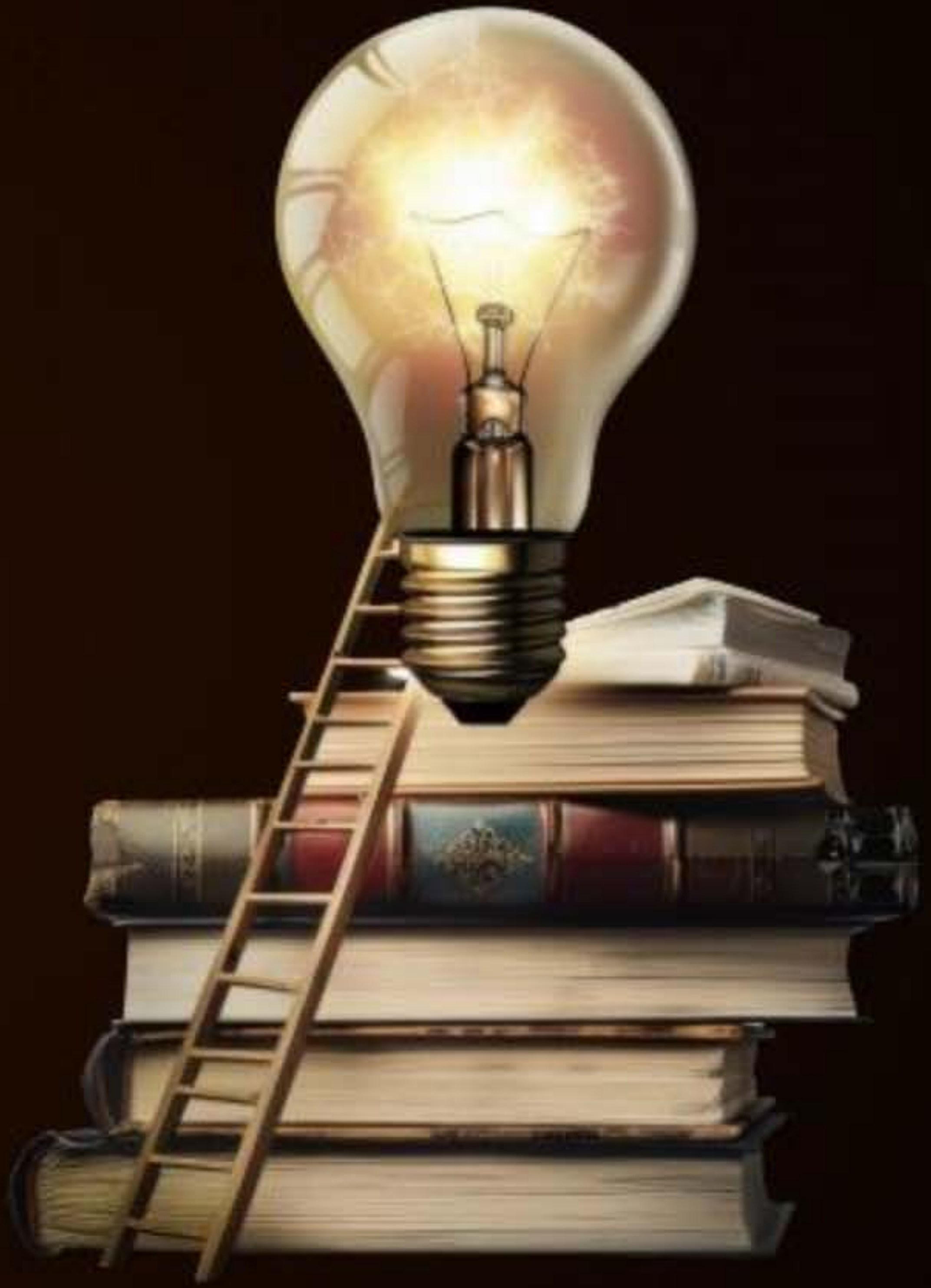
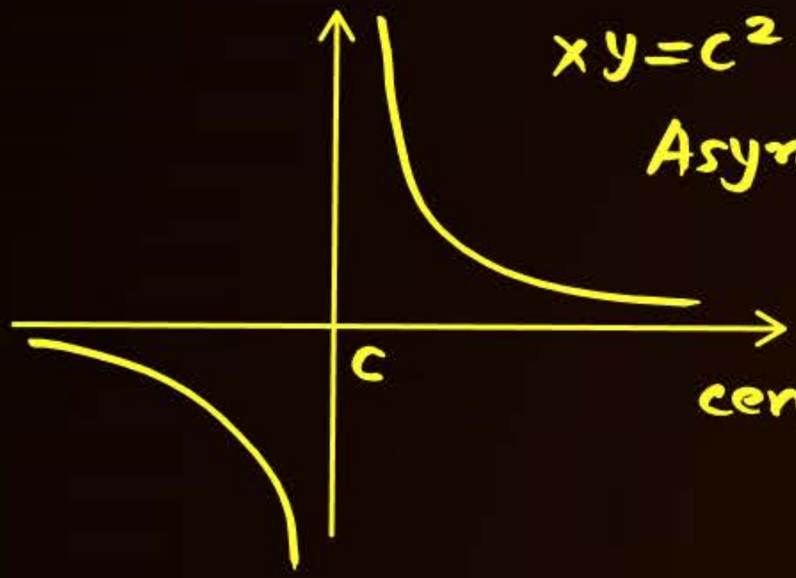
Topics *to be covered*

- **A** Integral Roots
- **B** Location of Roots



Homework Discussion

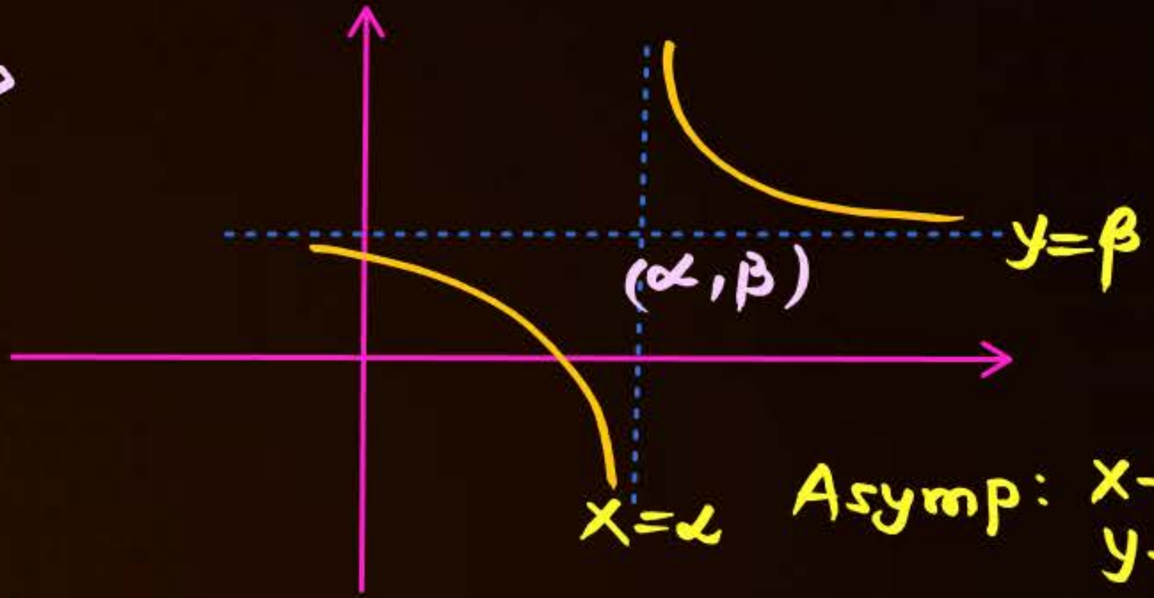
$xy=c^2$



Asympt: $x=0$
$y=0$

centre $(0,0)$

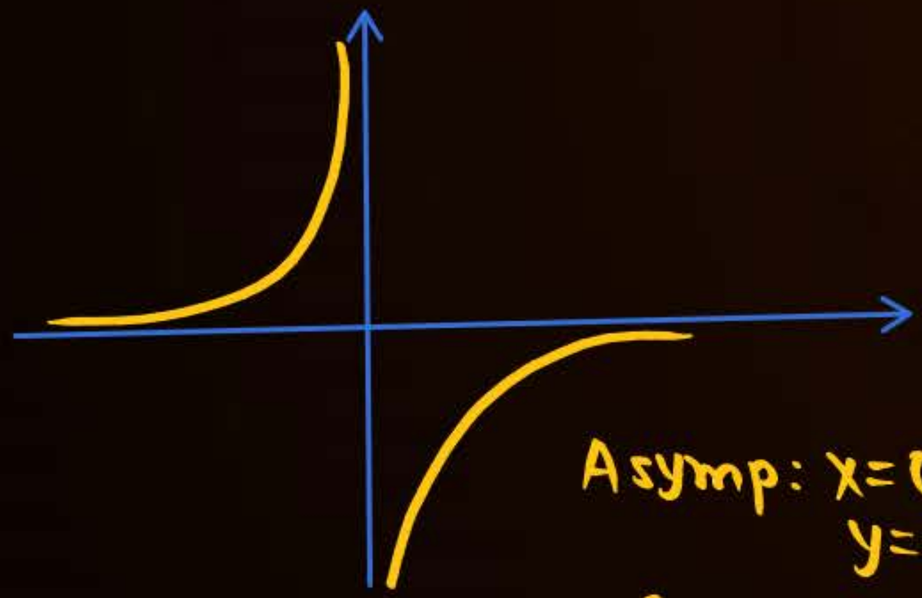
$(x-\alpha)(y-\beta)=c^2$



Asymp: $x-\alpha=0$
$y-\beta=0$
$\Downarrow$
$x=\alpha$
$y=\beta$

centre (α,β)

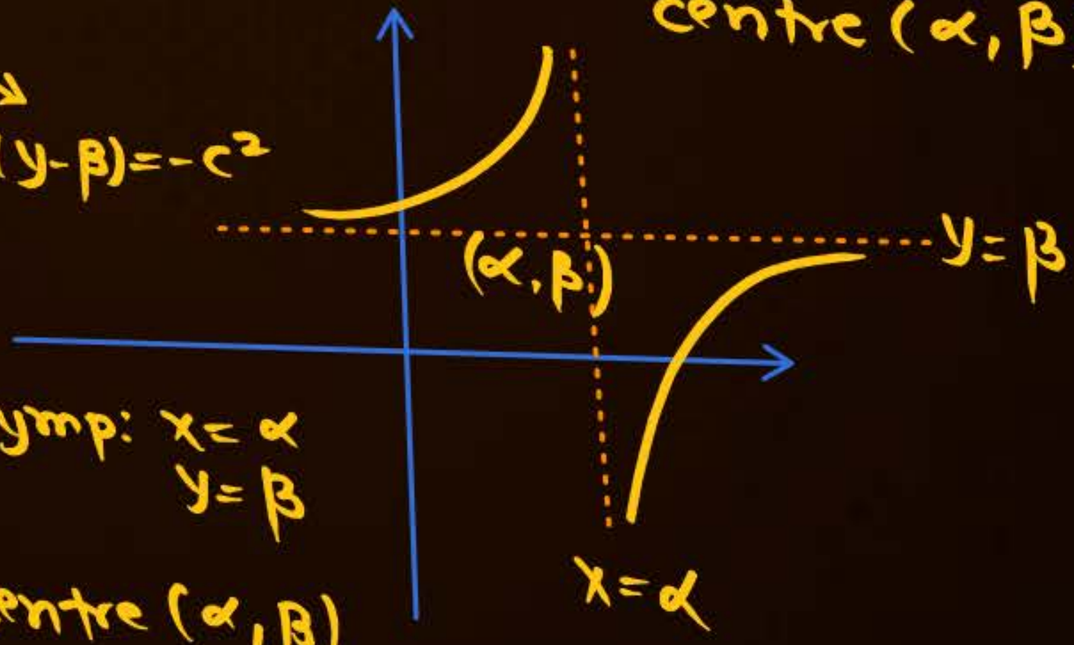
$xy=-c^2$



Asymp: $x=0$
$y=0$

Centre $(0,0)$

$(x-\alpha)(y-\beta)=-c^2$



Asymp: $x=\alpha$
$y=\beta$

centre (α,β)

Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal

QUESTION

★★KCLS★★

If the range of the function $f(x) = \frac{x^2+ax+b}{x^2+2x+3}$ is $[-5, 4]$, $a, b \in N$, then find the value of (a^2+b^2).

$$y = \frac{x^2+ax+b}{x^2+2x+3}$$

$$x^2y + 2xy + 3y = x^2 + ax + b$$

$$x^2(y-1) + (2y-a)x + 3y - b = 0$$

Since $x \in R$

$$D \geq 0$$

$$(2y-a)^2 - 4(y-1)(3y-b) \geq 0$$

$$4y^2 - 4ay + a^2 - 12y^2 + 4by + 12y - 4b \geq 0$$

$$-8y^2 + y(12 - 4a + 4b) + a^2 - 4b \geq 0$$

$$8y^2 - y(12-4a+4b) + 4b - a^2 \le 0$$

$\rightarrow -5$ & 4 should be its roots

$$S.O.R = \frac{12-4a+4b}{8} = -5+4$$

$$12-4a+4b = -8$$

$$4a-4b = 20$$

$$a-b = 5 \quad \text{(1)}$$

$$P.O.R = \frac{4b-a^2}{8} = -5.4 = -20$$

$$a^2-4b = 160 \quad \text{(11)}$$

$$a^2-4(a-5) = 160$$

$$a^2-4a-140 = 0$$

$$(a-14)(a+10) = 0$$

$a = 14, \cancel{-10}$

$b = 9$

$$a^2+b^2 = 196+81 = 277$$

QUESTION

★★KCLS★★

Complete set of values of 'a' such that $y = \frac{x^2 - x}{1 - ax}$ $(x \in R)$ attain all real values is-

(A) $[1, \infty)$

(B) $(0, 4]$

(C) $(0, 1]$

(D) $(1, \infty)$

$a = 1$

$y = \frac{x(x-1)}{(1-x)}$

$y = -x \quad , x \neq 1$

Range: $R - \{-1\}$

$y = \frac{x^2 - x}{1 - ax}$

For range to be R Nr & Den should have no common factor or Root.

$x^2 - x = 0 \Rightarrow x = 0, 1$

$1 - ax = 0 \Rightarrow x = \frac{1}{a} \Rightarrow \frac{1}{a} \neq 1$

$a \neq 1 \longrightarrow (1)$

$y - axy = x^2 - x$.

$x^2 + (ay - 1)x - y = 0$

Since $x \in R$ $\quad D \geq 0$

$(ay-1)^2 + 4y \geq 0$

$a^2y^2 - 2ay + 1 + 4y \geq 0$ should be satisfied $\forall y \in R$

$$a^2y^2 - 2ay + 1 + 4y \geq 0 \quad \forall y \in R$$

$$a^2y^2 + (4-2a)y + 1 \geq 0 \quad \forall y \in R$$

$$\Downarrow$$

$$a^2 > 0, \; D \leq 0$$

$$(4-2a)^2 - 4 \cdot a^2 \leq 0$$

$$16 + \cancel{4a^2} - 16a - \cancel{4a^2} \leq 0$$

$$16a \geq 16$$

$$a \geq 1$$

$$a \in [1, \infty)$$

But $a \neq 1$ $\quad a \in (1, \infty)$

QUESTION

★★KCLS★★

Find the values of 'a' for which $-3 < \left[\frac{(x^2+ax-2)}{(x^2+x+1)}\right] < 2$ is valid for all real x.

Lallu: $y = \frac{x^2+ax-2}{x^2+x+1}$ should have Range $(-3, 2)$

$\downarrow$

$y \in (0, 1)$ ✓

$y \in [-2, 1]$ ✓

Kallu: $-3 < \frac{x^2+ax-2}{x^2+x+1} < 2 \quad \forall x \in R$

$-3(x^2+x+1) < x^2+ax-2 < 2(x^2+x+1) \quad \forall x \in R$

$4x^2+(a+3)x+1 > 0 \ \forall x \in R$

$x^2+(2-a)x+4 > 0 \ \forall x \in R$

$4x^2+(a+3)x+1>0 \ \forall x \in R$ & $x^2+(2-a)x+4>0 \ \forall x \in R$

$\downarrow$

$D=(a+3)^2-16<0$

$(a+7)(a-1)<0$

$a \in (-7,1)$

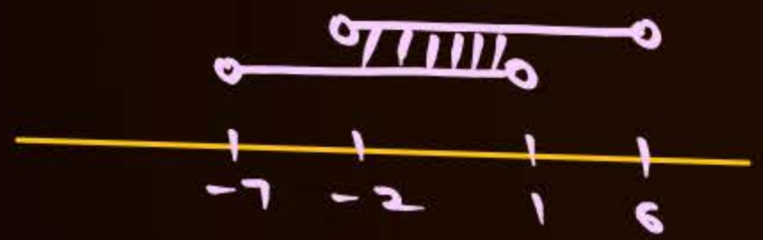
$\downarrow$

$D=(2-a)^2-16<0$

$(a-2)^2-16<0$

$(a-6)(a+2)<0$

$a \in (-2,6)$



$a \in (-2,1)$ Ans

QUESTION

★★KCLS★★

If $f: R \to R$, $f(x) = \dfrac{3x^2+mx+n}{x^2+1}$ has range $[-4, 3)$ then $m^2 + n^2$ is

(A) 10

(B) 25

(C) 16

(D) 2

$y = \dfrac{3x^2+mx+n}{x^2+1}$ has Range $[-4,3)$

$(y-3)x^2 - mx + y - n = 0$

$\underbrace{y \neq 3}$

$\downarrow$

$D = m^2 - 4(y-3)(y-n) \geq 0$

$m^2 - 4y^2 + 4yn + 12y - 12n \geq 0$

$4y^2 - 4yn - 12y + 12n - m^2 \leq 0$

$4y^2 - y(4n+12) + 12n - m^2 \leq 0$

Root $-4, 3$.

$S \cdot O \cdot R = \dfrac{4n+12}{4} = -4+3 \Rightarrow 4n+12 = -4 \Rightarrow n = -4$

$\underbrace{\text{If } y=3}$

$3x^2+3 = 3x^2+mx+n$

$3 = 0 \cdot x + -4$

$3 = -4$ (N·P)

CHECK

$P \cdot O \cdot R = \dfrac{12n-m^2}{4} = -12$

$-48 - m^2 = -48$

$m = 0$

$m^2+n^2 = 16.$

Integral/Rational Roots

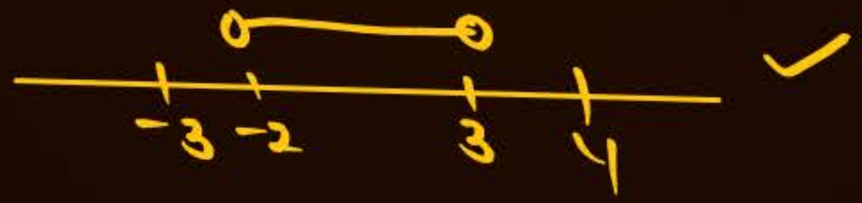
$ax^2+bx+c=0$ If $a=1$ & $b, c \in I$ & D is a perfect square then roots of $ax^2+bx+c=0$ are Integers.

$ax^2+bx+c=0$, $a, b, c \in Q$ then if $D \geq 0$ & D is a perfect square then roots of the quad are rational.

Range of x is $[-2, 4]$

Range of x is $[-2, 2)$

if $-3 < x < 4$



QUESTION

★★KCLS★★

If both the roots of the quadratic equation $x^2-(2n+18)x-n-11=0, n \in I$, are rational, then values of n are n_1 & n_2. Which of the following is CORRECT?

(A) $|n_1-n_2|=9$

(B) $|n_1-n_2|=3$

(C) $n_1^2+n_2^2=185$

(D) $n_1^2+n_2^2=175$

$x^2-(2n+18)x-n-11=0$

Both roots rational
$n \in I$

$1, -(2n+18), -n-11 \in Q$

for rational roots D should be perfect square.

$D=(2n+18)^2+4(n+11)=m^2 \quad m \in I$

$4n^2+324+72n+4n+44=m^2$

$4n^2+76n+368=m^2$

$368=m^2-(4n^2+2\cdot2\cdot19n+361-361)$

$368=m^2-(2n+19)^2+361$

$$(m-2n-19)(m+2n+19)=7.$$

$$(2n+19-m)(2n+19+m)=-7$$

$$2n+19-m=7$$
$$\underline{2n+19+m=-1}$$
$$\underline{n=-8}$$

OR

$$2n+19-m=-1$$
$$\underline{2n+19+m=7}$$
$$4n+38=6.$$
$$4n=-32$$
$$n=-8$$

OR

$$2n+19-m=-7$$
$$\underline{2n+19+m=1}$$
$$4n+38=-6$$
$$n=-11$$

OR

$$2n+19+m=-7$$
$$\underline{2n+19-m=1}$$
$$\underline{n=-11}$$

$$n=-8,-11$$

$$|n_1-n_2|=3$$

$$n_1^2+n_2^2=121+64$$
$$=185.$$

QUESTION

★★KCLS★★

Jah01

Find number of integral values α for which the quadratic equation $x^2 + \alpha x + \alpha + 1 = 0$ has integral roots.

M① Integers $\left\langle \begin{matrix} a \\ b \end{matrix} \right\rangle$ $x^2+\alpha x+\alpha+1=0$ $\quad \alpha \in I$

$\downarrow$

D should be a perfect square

$\alpha^2-4(\alpha+1)=m^2$

$\alpha^2-4\alpha-4=m^2$

$\alpha^2-4\alpha+4-8=m^2$

$(\alpha-2)^2-m^2=8$

$(\alpha-2-m)(\alpha+2+m)=8$

M②

$S.O.R=-\alpha=a+b$

$P.O.R=\alpha+1=ab$

$a+b+ab=1$

$1+a+b(a+1)=1+1$

$1(a+1)+b(a+1)=2$

$(a+1)(b+1)=2$

$a+1=2$, $b+1=1$ $\Rightarrow$ $a=1$, $b=0$ $\Downarrow$ $\alpha=-1$

$a+1=-2$, $b+1=-1$ $\Rightarrow$ $a=-3$, $b=-2$, $\alpha=5.$

QUESTION

★★KCLS★★

The number of integral roots of the equation $x^8 - 24x^7 - 18x^5 + 39x^2 + 1155 = 0$ is

(A) 0

(B) 2

(C) 4

(D) 6

$$x^8 - 24x^7 - 18x^5 + 39x^2 + 1155 = 0$$

$$x^2(x^6 - 24x^5 - 18x^3 + 39) = -1155.$$

If $\alpha \in I$ is a root

$$\underbrace{\alpha^2}_{\text{Integer} \downarrow \text{ perfect square}} \underbrace{(\alpha^6 - 24\alpha^5 - 18\alpha^3 + 39)}_{\text{Integer.}} = -1155. = -5.231. = -5.11.21$$

$$= -5.11.7.3 \;.= 1^2(-1155)$$

↓ No perfect square

Except: $\alpha^2 = 1$

$\alpha = 1, -1$

But $\alpha = 1$ LHS $\neq$ RHS

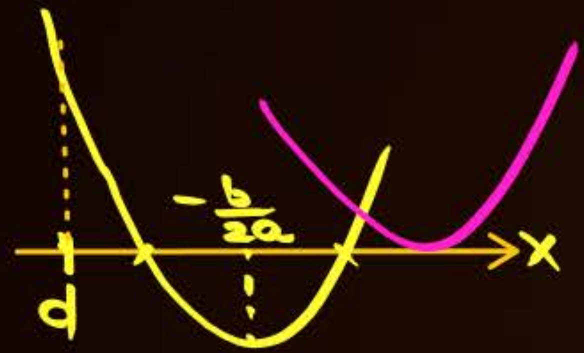
$\alpha \neq -1$ LHS $\neq$ RHS.

Location of Roots

This article deals with an elegant approach of solving problems on quadratic equations when the roots are located/specified on the number line with variety of constraints:

Consider $f(x) = ax^2 + bx + c$

Type ① If both roots of $f(x) = 0$ are greater than a specified no: d.



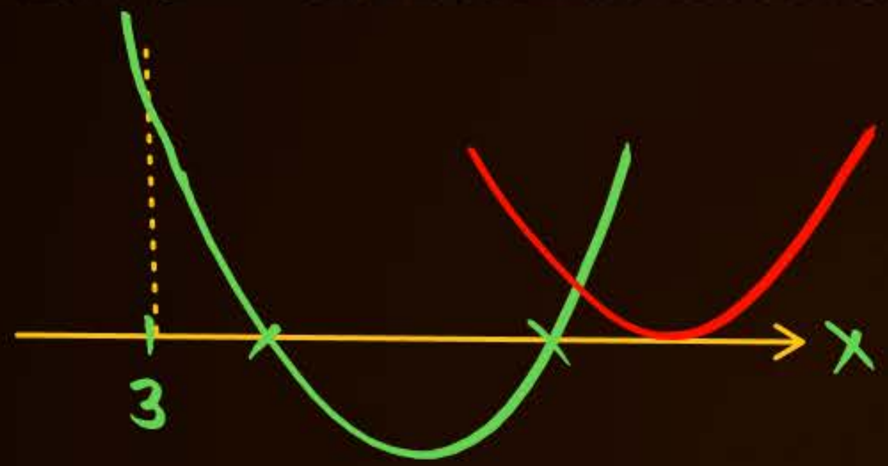
(i) $a > 0$
(ii) $-\frac{b}{2a} > d$
(iii) $D \geq 0$
(iv) $f(d) > 0$

(i) $a < 0$
(ii) $-\frac{b}{2a} > d$
(iii) $D \geq 0$
(iv) $f(d) < 0$

(i) $-\frac{b}{2a} > d$
(ii) $D \geq 0$
(iii) $af(d) > 0$

QUESTION

Find all the values of the parameter 'd' for which both roots of the equation $x^2 - 6dx + (2 - 2d + 9d^2) = 0$ exceed the number 3.



(i) $f(3) > 0$ (ii) $D \geq 0$ (iii) $-\frac{b}{2a} > 3$

$9 - 18d + 2 - 2d + 9d^2 > 0$

$9d^2 - 20d + 11 > 0$

$9d^2 - 11d - 9d + 11 > 0$

$(9d-11)(d-1) > 0$

$d \in (-\infty, 1) \cup (11/9, \infty)$

$\cancel{36}d^2 - \cancel{4}\cdot(2 - 2d + 9d^2) \geq 0$

$9d^2 - 2 + 2d - 9d^2 \geq 0$

$d \geq 1$.

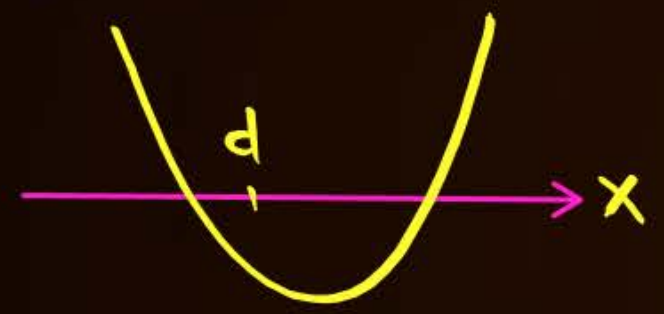
$\frac{6d}{2} > 3$

$d > 1$

$\cap$

$d \in (11/9, \infty)$

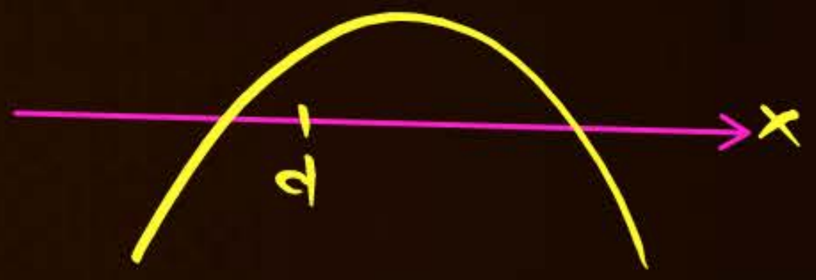
Type (2) one root of $f(x)=0$ is less than d & the other root is greater than d or d lies b/w the roots.



(i) $f(d)<0$

(ii) $a>0$

(iii) $D>0$



(i) $a<0$

(ii) $f(d)>0$

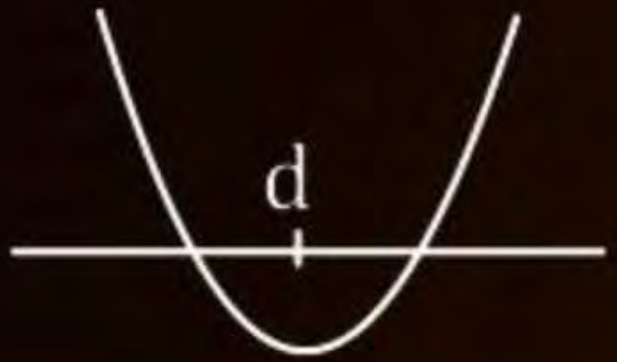
(iii) $D>0$

(i) $af(d)<0$

(ii) $D>0$ ⇝ No need.

Type 2 :

Both roots lie on either side of a fixed number d or alternatively one root is less than d & other root is greater than d or d lies between roots of $f(x) = 0$.



(1) $a > 0$	(1) $a < 0$
(2) $D > 0$	(2) $D > 0$
(3) $f(d) < 0$	(3) $f(d) > 0$

Union

(i) $a\,f(d) < 0$

(ii) $D > 0$ (no need)

Type (3) Both roots of $f(x)=0$ are confined between d & e $(d<e)$

one root is greater than d & other root is less than e $(d<e)$

(i) $a>0$

(ii) $D \geq 0$

(iii) $f(d)>0$

(iv) $f(e)>0$

(v) $d<-\frac{b}{2a}<e$

(i) $a<0$

(ii) $D \geq 0$

(iii) $f(d)<0$

(iv) $f(e)<0$

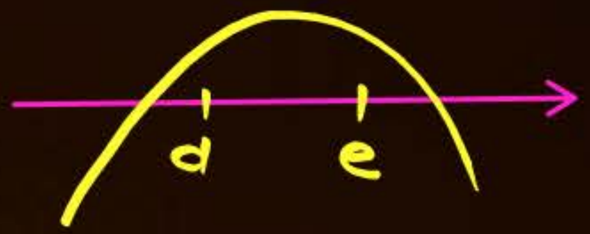
(v) $d<-\frac{b}{2a}<e$

$d<-\frac{b}{2a}<e$ $\quad af(d)>0$

$D \geq 0$ $\quad af(e)>0$

Type (4) one root is less than d & other root is greater than e $(d<e)$

or d, e lie b/w the roots $(d<e)$



(i) $a>0$

(ii) $D>0$

(iii) $f(d)<0$

(iv) $f(e)<0$

(i) $a<0$

(ii) $f(d)>0$

(iii) $f(e)>0$

(iv) $D>0$

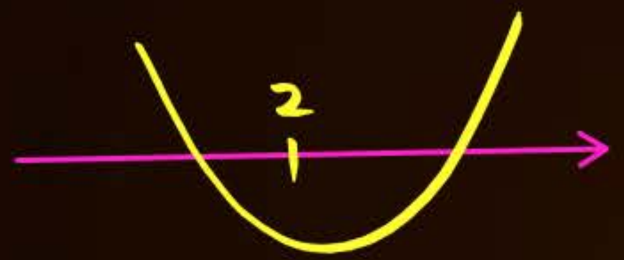
$af(d)<0$

$af(e)<0$

$D>0 \rightsquigarrow$ (No need)

QUESTION

Find the value of k for which one root of the equation of $x^2-(k+1)x+k^2+k-8=0$ exceed 2 and other is smaller than 2.



$f(2)<0 \longrightarrow 4-2k-2+k^2+k-8<0$

$D>0 \longrightarrow$ No Need

$k^2-k-6<0$

$(k-3)(k+2)<0$

$k\in(-2,3)$

QUESTION

Tan 02

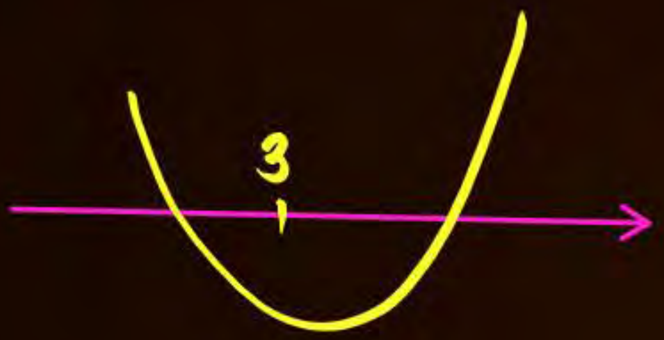
Find the set of values of 'a' for which zeroes of the quadratic polynomial $(a^2 + a + 1)x^2 + (a - 1)x + a^2$ are located on either side of 3.

$\downarrow$

$D < 0,\ A = 1 > 0$

$\downarrow$

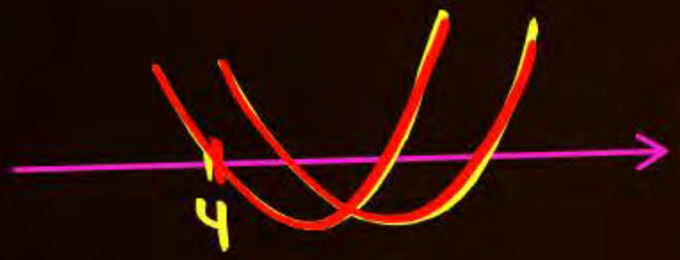
always +ve



QUESTION [JEE Advanced 2009]

Tan03

The smallest value of k, for which both the roots of the equation,
$x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is



(i) $f(4) \geq 0$

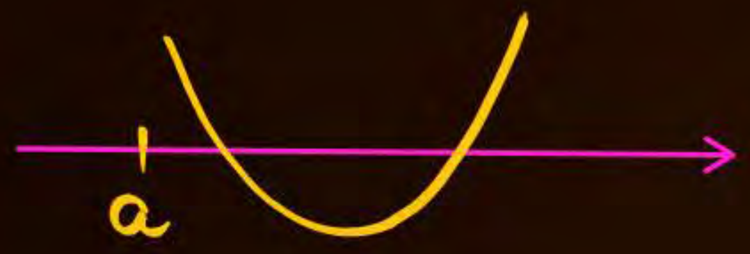
(ii) $-\frac{b}{2a} > 4$

(iii) $D > 0$

QUESTION

Tan 4

Find all the values of 'a' for which both roots of the equation $x^2 + x + a = 0$ exceed the quantity 'a'.

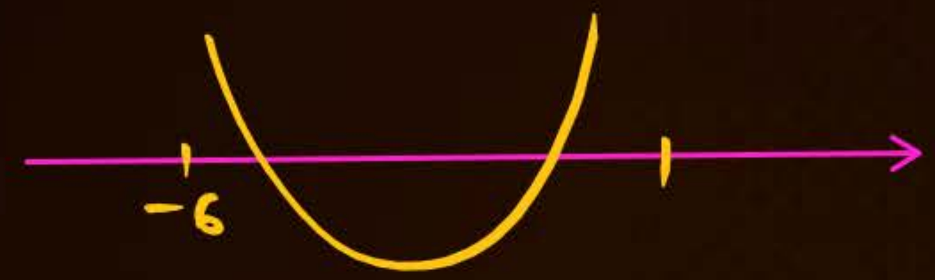


Ans. $(-\infty, -2)$

QUESTION

Tah05

If α, β are roots of the quadratic equation $x^2 + 2(k-3)x + 9 = 0 (\alpha \neq \beta)$.
If $\alpha, \beta \in (-6, 1)$, find k.



(i) $f(-6) > 0$

(ii) $f(1) > 1$

(iii) $-6 < -\frac{b}{2a} < 1$

(iv) $D > 0$

$\cap$

Ans.

QUESTION

Tah06

Find the value of k for which one root of the equation of $(k-5)x^2 + 2kx + k - 4 = 0$ is smaller than 1 and the other root exceed 2.

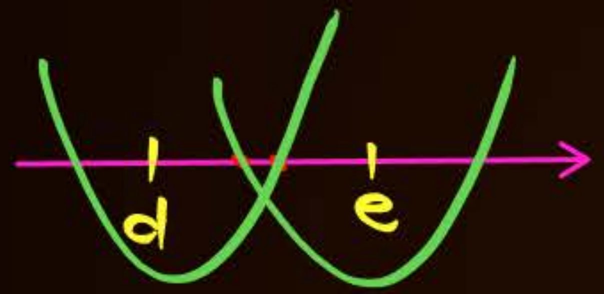
$x^2 + \frac{2k}{k-5}x + \frac{k-4}{k-5} = 0$

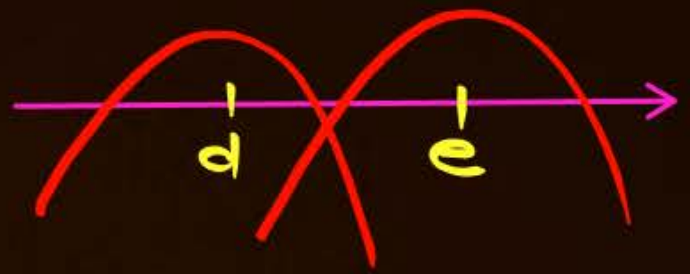
$f(1) < 0$

$f(2) < 0$

$D > 0 \longrightarrow$ No Need

Type ⑤ Exactly one root of $f(x)=0$ lie b/w d & e $(d<e)$





(i) $a>0$

(ii) $f(d)\cdot f(e)<0$

(iii) $D>0$

(i) $a<0$

(ii) $f(d)\cdot f(e)<0$

(iii) $D>0$

(i) $f(d)\cdot f(e)<0$

(ii) $a\neq 0$

Two more possibilities arise.

P 1

$f(d) = 0$

& second root lies b/w d & e

P 2

$f(e) = 0$

& second root lies b/w d & e

QUESTION

Find all possible value 'a' for which exactly one root of equation $x^2 - (a+1)x + 2a = 0$ lies in $(0, 3)$.

Now Two possibilities arise

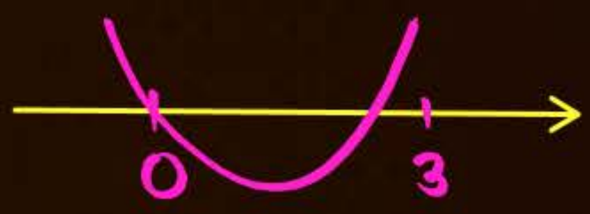
$f(3) \cdot f(0) < 0$

$(9 - 3a - 3 + 2a) \cdot 2a < 0$

$(6 - a)a < 0$

$a(a-6) > 0$

$a \in (-\infty, 0) \cup (6, \infty)$



$f(0) = 0$

$a = 0$

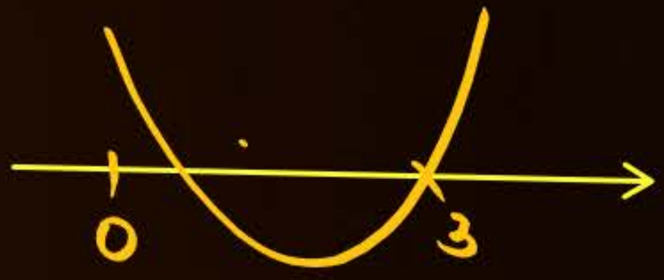
$x^2 - x = 0$

$x = 0, 1$

↳ second root lies b/w 0 & 3

$a = 0$

$a \in (-\infty, 0] \cup (6, \infty)$



$f(3) = 0$

$6 - a = 0$

$a = 6$

$x^2 - 7x + 12 = 0$

$x = 3, 4$

second root in b/w 0 & 3 does not lie

QUESTION

Jan 07

Find all possible values of m for which exactly one root of the equation $x^2 + mx + m^2 + 6m = 0$, lies in $(-2, 0)$.

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

(KTK 01)

If $x \in R$ then range of $f(x) = \frac{x^2+2x-3}{2x^2+3x-9}$ is

(A) $(-\infty, \infty)$

(B) $R - \left\{\frac{1}{2}\right\}$

(C) $R - \left\{\frac{4}{9}, \frac{1}{2}\right\}$

(D) $R - \left\{\frac{3}{2}\right\}$

Ans. C

QUESTION

(KTK 02)

If the highest point on the graph of $y = -x^2 - 2kx + 3a$ is $(-1, 2)$ then the value of $(k + 6a)$ is

A) 2

B) 3

C) 5

D) 6

Ans. C

QUESTION

(KTK 03)

If the quadratic polynomial $f(x) = (a - 3)x^2 - 2ax + 3a - 7$ ranges from $[-1, \infty)$ for every $x \in R$, then the value of a lies in

(A) $[0, 2]$

(B) $[3, 5]$

(C) $[4, 6)$

(D) $[5, 7]$

Ans. C

QUESTION

(KTK 04)

Find the range of values of a, such that $f(x) = \frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32}$ is always negative.

Ans. $a \in \left(-\infty, -\frac{1}{2}\right)$

QUESTION [AIEEE 2002]

(KTK 05)

If $\alpha^2 = 5\alpha - 3, \beta^2 = 5\beta - 3$ then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is

A $\frac{19}{3}$

B $\frac{25}{3}$

C $\frac{-19}{3}$

D None of these

Ans. A

Homework From Module

Quadratic Equations

Prarambh (Topicwise) : Q1 to Q27

Prabal (JEE Main Level) : Q1,Q2,Q6 to Q9

Parikshit (JEE Advanced Level) : Abhi Ruko

Solution to Previous TAH

QUESTION

For $x \in [1,5]$, $y = x^2 - 5x + 3$ has-

A) Least value $= -1.5$

B) Greatest value $= 3$

C) Least value $= -3.25$

D) Greatest value $= \frac{5+\sqrt{13}}{2}$

TAH 01

$$y = x^2 - 5x + 3 + (5/2)^2 - (5/2)^2$$

$$y = (x - 5/2)^2 - 13/4 \quad , \quad x \in [1,5]$$

TAH-1
By Nikita
From Raj.

$$\Rightarrow x \in [1, 5]$$

$$\Rightarrow x - \frac{5}{2} \in \left[-\frac{3}{2}, \frac{5}{2}\right] \Rightarrow \left[-\frac{3}{2}, 0\right] \cup \left[0, \frac{5}{2}\right]$$

$$\Rightarrow \left(x - \frac{5}{2}\right)^2 \in \left[0, \frac{9}{4}\right] \cup \left[0, \frac{25}{4}\right] = \left[0, \frac{25}{4}\right]$$

$$\Rightarrow \left(x - \frac{5}{2}\right)^2 - \frac{13}{4} \in \left[\frac{-13}{4}, 3\right]$$

$$\Rightarrow \boxed{y \in \left[\frac{-13}{4}, 3\right]} \text{ Ans}$$

Max. value = 3

Min. value = $-\frac{13}{4} = -3.25$

Q-11 For $x \in [1,5]$. $y = x^2 - 5x + 3$ has.

(a) least value $= -1.5$ (b) Greatest value $= 3$.

(b) Least value $= -3.25$ (d) Greatest value $= \frac{5+\sqrt{13}}{2}$

Soln:

$$y = x^2 - 5x + 3$$

$$\Rightarrow y = x^2 - 2 \cdot \frac{5}{2} \cdot x + \frac{25}{4} + 3 - \frac{25}{4}$$

TAH 1
BY REED

$$\Rightarrow y = \left(x - \frac{5}{2}\right)^2 - \frac{13}{4}$$

$$x \in [1,5]$$

$$\Rightarrow \left(x - \frac{5}{2}\right) \in \left[-\frac{3}{2}, \frac{5}{2}\right] \equiv \left[-\frac{3}{2}, 0\right] \cup \left[0, \frac{5}{2}\right]$$

$$\Rightarrow \left(x - \frac{5}{2}\right)^2 \in \left[0, \frac{25}{4}\right]$$

$$\Rightarrow \left(x - \frac{5}{2}\right)^2 - \frac{13}{4} \in \left[-\frac{13}{4}, \frac{12}{4}\right] \equiv [-3.25, 3]$$

$\therefore y|_{min} = -3.25$, $y_{max} = 3$. (Ans) $\therefore$ Ans $\Rightarrow$ (b), (c)

QUESTION

Find range of following functions:

(i) $f(x) = 3x^2 - 2x - 7$

(ii) $f(x) = 3x^2 - 2x - 7, x \in (0, 5]$

(iii) $f(x) = 3x^2 - 2x - 7, x \in [-6, -1]$

Ans:[22/3,∞)

Ans: [22/3,58]

Ans: [-2,113]

TAH 2

(i) $y = 3x^2 - 2x - 7$

$a > 0$

$\Rightarrow y \in \left[-\frac{D}{4a}, \infty\right)$

$\Rightarrow y \in \left[-\frac{22}{3}, \infty\right)$ Ans

(ii) $y = 3x^2 - 2x - 7$, $x \in (0, 5]$

$\Rightarrow y = 3\left(x^2 - \frac{2}{3}x\right) - 7$

$\Rightarrow y = 3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3} - 7$

$\Rightarrow y = 3(x - 1/3)^2 - \frac{22}{3}$

$x \in (0, 5]$

$\Rightarrow \left(x - \frac{1}{3}\right) \in \left(-\frac{1}{3}, \frac{14}{3}\right] = \left(-\frac{1}{3}, 0\right] \cup \left[0, \frac{14}{3}\right]$

$\Rightarrow (x - 1/3)^2 \in \left[0, \frac{1}{9}\right) \cup \left[0, \frac{196}{9}\right] =$

$\left[0, \frac{196}{9}\right] - \left\{\frac{1}{9}\right\}$

$\Rightarrow 3\left(x - \frac{1}{3}\right)^2 \in \left[0, \frac{196}{3}\right] - \left\{\frac{1}{3}\right\}$

$\Rightarrow 3\left(x - \frac{1}{3}\right)^2 - \frac{22}{3} \in \left[-\frac{22}{3}, 58\right] - \left\{-\frac{21}{3}\right\}$

$\Rightarrow y \in \left[-\frac{22}{3}, 58\right] - \{7\}$ Ans

(iii) $y = 3x^2 - 2x - 7$, $x \in [-6, -1]$.

$\Rightarrow y = 3(x - 1/3)^2 - \frac{22}{3}$

$\Rightarrow x \in [-6, -1]$

$\Rightarrow x - \frac{1}{3} \in \left[-\frac{19}{3}, -\frac{4}{3}\right]$

$\Rightarrow 3\left(x - \frac{1}{3}\right)^2 \in \left[\frac{16}{3}, \frac{361}{3}\right]$

$\Rightarrow 3\left(x - \frac{1}{3}\right)^2 - \frac{22}{3} \in [-2, 113]$

$\Rightarrow y \in [-2, 113]$ Ans

TAH–2
By Nikita
From Raj.

Q-4! Find range of the following functions!

(i) $f(x) = 3x^2 - 2x - 7$ Ans: $[-\frac{22}{3}, \infty)$

(ii) $f(x) = 3x^2 - 2x - 7$, $x \in (0, 5]$ Ans. $[-\frac{22}{3}, 58]$

(iii) $f(x) = 3x^2 - 2x - 7$, $x \in [-6, -1]$ Ans. $[-2, 113]$

Sol:

$$f(x) = 3x^2 - 2x - 7$$

or, $f(x) = 3\left(x^2 - \frac{2}{3}x - \frac{7}{3}\right)$

or, $f(x) = 3\left[x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2 - \frac{7}{3}\right]$

or, $f(x) = 3\left[\left(x - \frac{1}{3}\right)^2 - \frac{22}{9}\right]$

or, $f(x) = 3\left(x - \frac{1}{3}\right)^2 - \frac{22}{3}$

TAH 2
BY REED

→ **Q** $f(x) = 3x^2 - 2x - 7$, $x \in R$:

method-1:

$f(x) = 3x^2 - 2x - 7$ → $a = 3\ (>0)$, $D = 4 + 84 = 88$

$\therefore$ Range $\in \left[-\frac{D}{4a}, \infty\right)$

or, Range $\in \left[\frac{-88}{4\times 3}, \infty\right) \equiv \left[\frac{-22}{3}, \infty\right)$

or, Range $\in \left[-\frac{22}{3}, \infty\right)$ (Ans)

method-2:

$f(x) = 3\left(x - \frac{1}{3}\right)^2 - \frac{22}{3}$

$\Rightarrow x \in R$ i.e. $x \in (-\infty, \infty)$

$\Rightarrow \left(x - \frac{1}{3}\right) \in (-\infty, \infty)$

$\Rightarrow 3\left(x - \frac{1}{3}\right)^2 \in [0, \infty)$

$\Rightarrow 3\left(x - \frac{1}{3}\right)^2 - \frac{22}{3} \in \left[-\frac{22}{3}, \infty\right)$

→ **Q** $f(x) = 3x^2 - 2x - 7$, $x \in (0, 5]$: Ans $\left[-\frac{22}{3}, 58\right]$

method-1:

$f(x) = 3\left(x - \frac{1}{3}\right)^2 - \frac{22}{3}$

$\because x \in (0, 5]$

$\Rightarrow \left(x - \frac{1}{3}\right) \in \left(-\frac{1}{3}, \frac{14}{3}\right] \equiv \left(-\frac{1}{3}, 0\right] \cup \left[0, \frac{14}{3}\right]$

$\Rightarrow \left(x - \frac{1}{3}\right)^2 \in \left[0, \frac{1}{9}\right) \cup \left[0, \frac{196}{9}\right] \equiv \left[0, \frac{196}{9}\right]$

$\Rightarrow 3\left(x - \frac{1}{3}\right)^2 \in \left[0, \frac{196}{3}\right]$

$\Rightarrow 3\left(x - \frac{1}{3}\right)^2 - \frac{22}{3} \in \left[-\frac{22}{3}, \frac{174}{3}\right] \equiv \left[-\frac{22}{3}, 58\right]$

$\Rightarrow 3\left(x - \frac{1}{3}\right)^2 - \frac{22}{3} \in \left[-\frac{22}{3}, 58\right]$ (Ans)

method-2:

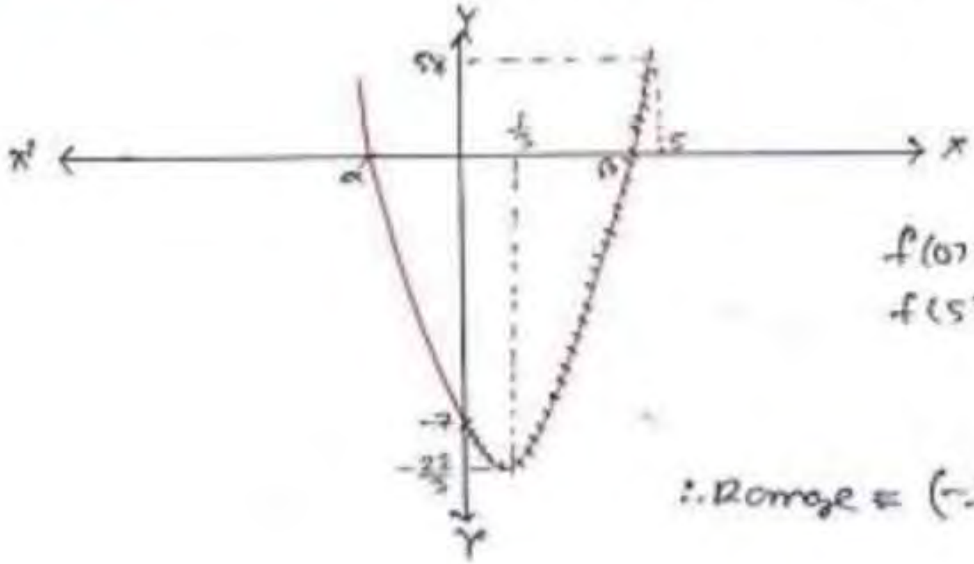
$f(x) = 3x^2 - 2x - 7$.

$a = 3\ (>0)$, $c = -7$. $D = 4 + (21 \times 4)$
or, $D = 88$

vertex $\equiv \left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

or, $V \equiv \left(\frac{1}{3}, -\frac{22}{3}\right)$

TAH 2
BY REED



$f(0) = -7$.

$f(5) = 58$ closed

$\therefore$ Range $\equiv \left(-\frac{22}{3}, 58\right]$ ✗

$\because$ closed at $-\frac{22}{3}$

$\therefore$ Range $\in \left[-\frac{22}{3}, 58\right]$

→ (iii): $f(x) = 3x^2 - 2x - 7,\ x \in [-6, -1]$: Ans. $[-2, 113]$

$$f(x) = 3\left(x - \frac{1}{3}\right)^2 - \frac{22}{3}.$$

$\because\ x \in [-6, -1]$

TAH 2 BY REED

$$\Rightarrow \left(x - \frac{1}{3}\right) \in \left[-\frac{19}{3}, -\frac{4}{3}\right]$$

$$\Rightarrow \left(x - \frac{1}{3}\right)^2 \in \left[\frac{+361}{9}, \frac{16}{9}\right] \text{ i.e } \left[\frac{16}{9}, \frac{361}{9}\right]$$

$$\Rightarrow 3\left(x - \frac{1}{3}\right)^2 \in \left[\frac{16}{3}, \frac{361}{3}\right]$$

$$\Rightarrow 3\left(x - \frac{1}{3}\right)^2 - \frac{22}{3}\ \left[-\frac{6}{3}, \frac{339}{3}\right] \equiv [-2, 113] \text{ (Ans)}$$

QUESTION

Find the range of $f(x)$:

(i) $f(x) = 2x^2 - 3x + 2$ Ans. $\left[\frac{7}{8}, \infty\right)$

(ii) $f(x) = 2x^2 - 3x + 2, x \in [0, 2]$ Ans. $\left[\frac{7}{8}, 4\right]$

(iii) $f(\theta) = 2\cos^2\theta - 6\sin\theta + 1$ Ans. $[-5, 7]$

TAH 03

(i) $f(x) = 2x^2 - 3x + 2$

$y \in [-\frac{D}{4a}, \infty)$

$y \in [+\frac{7}{8}, \infty)$ Ans

(ii) $y = 2x^2 - 3x + 2$, $x \in [0, 2]$

$a > 0$. min. value is $-\frac{D}{4a}$ at $\frac{-b}{2a}$.

max. value is at $x = 2$

$y \in [\frac{7}{8}, 4]$ $\quad y \in [\frac{7}{8}, f(2)]$

(iii) $f(\theta) = 2\cos^2\theta - 6\sin\theta + 1$

$f(\theta) = 2 - 2\sin^2\theta - 6\sin\theta + 1$

$f(\theta) = -2\sin^2\theta - 6\sin\theta + 3$

$f(\theta) = -2(\sin^2\theta + 3\sin\theta) + 3$

$f(\theta) = -2(\sin^2\theta + 3\sin\theta + \frac{9}{4} - \frac{9}{4}) + 3$

$f(\theta) = -2(\sin\theta + \frac{3}{2})^2 + \frac{9}{2} + 3.$

$f(\theta) = -2(\sin\theta + \frac{3}{2})^2 + \frac{15}{2}$

$\sin\theta \in [-1, 1]$

$-2[\frac{1}{2}, \frac{5}{2}]^2 + \frac{15}{2}$

$-2[\frac{1}{4}, \frac{25}{4}]$

$[\frac{-25}{2}, \frac{-1}{2}] + \frac{15}{2}$

$y \in [\frac{-10}{2}, \frac{14}{2}] = [-5, 7]$

TAH-3

By Nikita

From Raj.

Q-5: Find the range of f(x):

(i) $f(x) = 2x^2 - 3x + 2$. Ans: $[\frac{7}{8}, \infty)$

(ii) $f(x) = 2x^2 - 3x + 2,\ x \in [0, 2]$ Ans. $[\frac{7}{8}, 4]$

(iii) $f(\theta) = 2\cos^2\theta - 6\sin\theta + 1$ Ans $[-5, 7]$

Soln:

→ (i) $f(x) = 2x^2 - 3x + 2 ;\ x \in R$.

$a = 2\ (>0)$. $D = 9 - 16 = -7\ (<0)$

$\therefore$ Range $\in [\frac{-D}{4a}, \infty)$

$\Rightarrow$ Range $\in [\frac{7}{8}, \infty)$

TAH 3
BY REED
FROM WB

→ (ii) $f(x) = 2x^2 - 3x + 2 ;\ x \in [0, 2]$

$f(x) = 2(x^2 - \frac{3}{2}x + 1)$

or, $f(x) = 2(x^2 - \frac{3}{2}x + \frac{9}{16} - \frac{9}{16} + 1)$

or, $f(x) = 2[(x - \frac{3}{4})^2 + \frac{7}{16}]$

or, $f(x) = 2(x - \frac{3}{4})^2 + \frac{7}{8}$

$\because x \in [0, 2]$

or, $(x - \frac{3}{4}) \in [-\frac{3}{4}, \frac{5}{4}] \equiv [-\frac{3}{4}, 0] \cup [0, \frac{5}{4}]$

or, $2(x - \frac{3}{4}) \in [-\frac{3}{2}, \frac{5}{2}] \equiv [-\frac{3}{2}, 0] \cup [0, \frac{5}{2}]$

or, $2(x - \frac{3}{4})^2 \in [\frac{9}{16} \times 2, 0] \cup [0, \frac{25}{16} \times 2]$

or, $2(x - \frac{3}{4})^2 \in [0, \frac{25}{8}]$

or, $2(x - \frac{3}{4})^2 + \frac{7}{8} \in [\frac{7}{8}, \frac{32}{8}] \equiv [\frac{7}{8}, 4]$ (Ans.)

→ (iii) $f(\theta) = 2\cos^2\theta - 6\sin\theta + 1$:-

TAH 3
BY REED
FROM WB

$f(\theta) = 2\cos^2\theta - 6\sin\theta + 1$

or, $f(\theta) = 2(1 - \sin^2\theta) - 6\sin\theta + 1$

or, $f(\theta) = 2 - 2\sin^2\theta - 6\sin\theta + 1$

or, $f(\theta) = -2\sin^2\theta - 6\sin\theta + 3$

$\therefore f(\theta) = -2\sin^2\theta - 6\sin\theta + 3$

Let $\sin\theta = x ;\ x \in [-1, 1]$

or, $g(x) = -2x^2 - 6x + 3 ;\ x \in [-1, 1]$

or, $g(x) = -2(x^2 + 3x - \frac{3}{2})$

or, $g(x) = -2(x^2 + 3x + \frac{9}{4} - \frac{9}{4} - \frac{3}{2})$

or, $g(x) = -2[(x + \frac{3}{2})^2 - \frac{15}{4}]$

or, $g(x) = -2(x + \frac{3}{2})^2 + \frac{15}{2}$

$\because x \in [-1, 1]$

or, $(x + \frac{3}{2}) \in [\frac{1}{2}, \frac{5}{2}]$

or, $(x + \frac{3}{2})^2 \in [\frac{1}{4}, \frac{25}{4}]$

or, $-2(x + \frac{3}{2})^2 \in [-\frac{25}{2}, -\frac{1}{2}]$

or, $-2(x + \frac{3}{2})^2 + \frac{15}{2} \in [\frac{-10}{2}, \frac{14}{2}]$

or, $-2(x + \frac{3}{2})^2 + \frac{15}{2} \in [-5, 7]$

QUESTION

Find the maximum and

(i) $f(x) = x^2 + 2x + 4$

(ii) $f(x) = x^2 + 4x + 4$

(iii) $f(x) = x^2 - 5x + 4$

(iv) $f(x) = -x^2 + x - 4$

(v) $f(x) = -x^2 + 6x - 9$

(vi) $f(x) = -x^2 + 6x - 8$

Ans. (i) Min value = 3; (ii) Min value = 0
(iii) Min value = $-9/4$, (iv) Max value = $-15/4$
(v) Max value = 0, (vi) Max value = -1

TAH-04

(i) $f(x) = x^2+2x+4$, $a>0$

$f(x) = (x+1)^2+3$

$f(x)|_{min}$ at $-\frac{b}{2a} = -1$

$f(x)|_{min} = 3$ Ans

(ii) $f(x) = x^2+4x+4$ $a>0$

$f(x) = (x+2)^2$

$f(x)|_{min}$ at $x = -\frac{4}{2} = -2$

$f(x)|_{min} = 0$ Ans

(iii) $f(x) = x^2-5x+4$ $a>0$

$f(x)|_{min}$ at $x = \frac{5}{2}$

$f(x)|_{min} = \frac{25}{4} - \frac{25}{2} + 4$

$= -\frac{25}{4} + 4$

$f(x)|_{min} = -\frac{9}{4}$ Ans

(iv) $f(x) = -x^2+x-4$

$a<0$

It has max. value

$f(x)|_{max}$ at $x = -\frac{b}{2a} = \frac{-1}{-2} = \frac{1}{2}$

$f(x)|_{max} = -\frac{1}{4} + \frac{1}{2} - 4$

$\frac{1}{4} - 4$

$f(x)|_{max} = -\frac{15}{4}$

(v) $f(x) = -x^2+6x-9$

$a<0$

max. value obtained.

$F(x)|_{max}$ at $x = -\frac{b}{2a} = \frac{-6}{2(-1)} = 3$

$f(x)|_{max} = -9+18-9 = 0$ Ans

(vi) $f(x) = -x^2+6x-8$

$a<0$

$f(x)|_{max}$ at $x = \frac{-b}{2a} = \frac{-6}{-2} = 3$

$f(x)|_{max} = -9+18-8$

$= 18-17 = 1$

Ans

TAH-4

BY Nikita,Raj.

Q-2: Find the maximum & minimum values if they exist.

Soln: → (i) $x^2+2x+4 = f(x)$:

⇒ m-1: $y = (x^2+2x+1)+3$

$= \underbrace{(x+1)^2}_{\geq 0}+3.$

$\therefore y_{min} = 3$ at $x=-1$.

$y_{max} \to \infty$ (D.N.E.)

m-2: $y \in \left[\frac{-D}{4a}, \infty\right)$

⇒ $y \in \left[\frac{-(-12)}{4}, \infty\right)$

⇒ $y \in [\underbrace{3}_{y_{min}}, \infty)$

→ (ii) $f(x) = x^2+4x+4$:

⇒ $f(x) = x^2+2.x.2+2^2 = \underbrace{(x+2)^2}_{\geq 0}$

$\therefore y_{min} = 0$ at $x=-2$.

$y_{max} \to \infty$ (D.N.E)

→ (iii) $f(x) = x^2-5x+4$:

⇒ $y = (x-\frac{5}{2})^2+4-\frac{25}{4}$

⇒ $y = \underbrace{(x-\frac{5}{2})^2}_{\geq 0} - \frac{9}{4}$

$\therefore y_{min} = -\frac{9}{4}$. $y_{max} \to \infty$ (D.N.E.)

→ (iv) $f(x) = -x^2+x-4$:

⇒ $f(x) = -(x^2-x+\frac{1}{4})-4+\frac{1}{4}$

$f(x) = -\underbrace{(x-\frac{1}{2})^2}_{\geq 0} - \frac{15}{4}$

(max when $x=\frac{1}{2}$)

$\therefore y_{max} = -\frac{15}{4}$

$y_{min} \to -\infty$ (D.N.E)

→ (v) $f(x) = -x^2+6x-9$:

$f(x) = -\underbrace{(x-3)^2}_{\geq 0}$

y_{max} = '0' at $x=3$

$y_{min} \to -\infty$ (D.N.E.)

TAH 4
BY REED
FROM WB

→ (vi) $f(x) = -x^2+6x-8$:

$f(x) = -(x^2-6x+9)+1$

$= -\underbrace{(x-3)^2}_{\geq 0}+1$

$\therefore y_{max} = 1$ at $x=3$

$y_{min} \to -\infty$ (D.N.E.)

QUESTION

Find the range of $f(x) = \frac{x^2 - 5x + 4}{x^2 + 2x - 3}$.

TAH 05

$$F(x) = \frac{x^2-5x+4}{x^2+2x-3}$$

$$\Rightarrow y = \frac{(x-4)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}}, \quad x \neq 1 \qquad f(1) = -\frac{3}{4}$$

$$\Rightarrow y = \frac{x-4}{x+3}$$

Range $= R - \{1, f(1)\}$

$y \in R - \{1, -\frac{3}{4}\}$ Ans

Graph

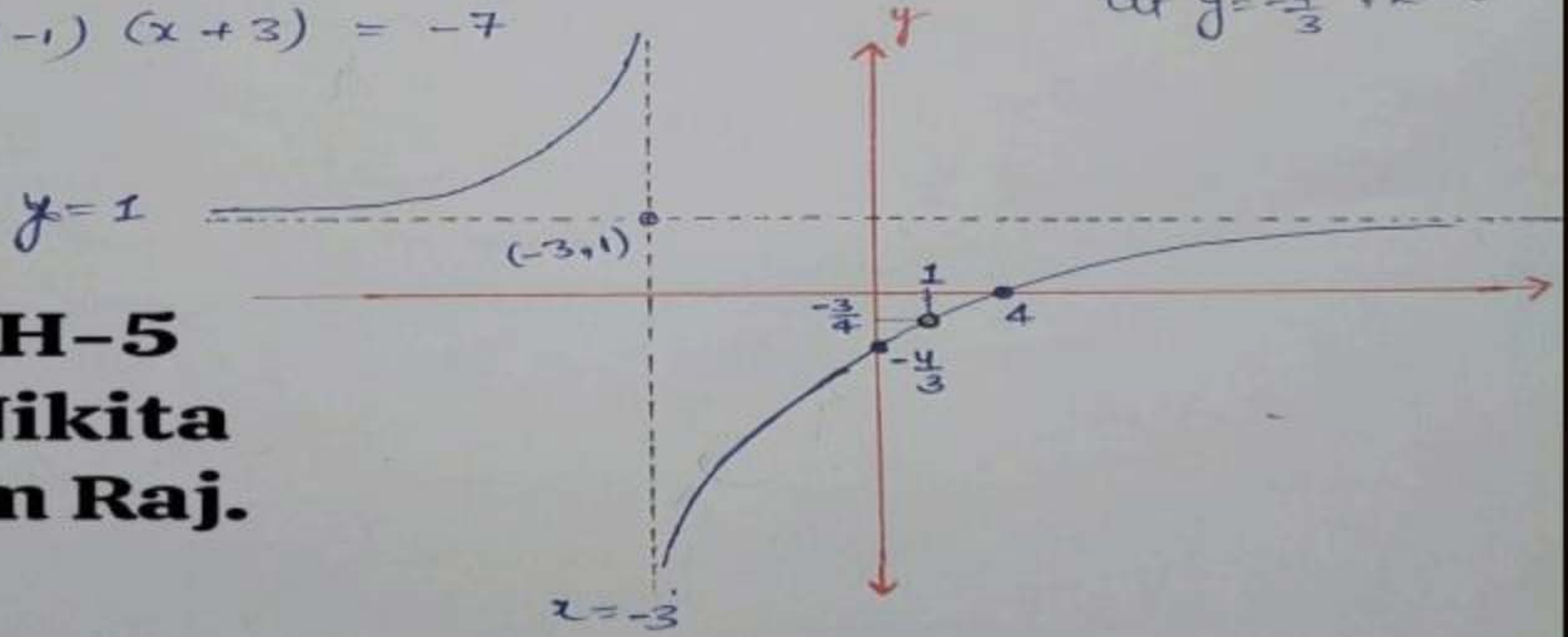
$$xy + 3y = x - 4$$

$$\Rightarrow x(y-1) + 3y - 3 = -7$$

$$\Rightarrow (y-1)(x+3) = -7$$

at $x=4$, $y=0$

at $y=-\frac{4}{3}$, $x=0$



TAH-5
By Nikita
From Raj.

Q-3! Find the range of $f(x) = \dfrac{x^2-5x+4}{x^2+2x-3}$ and also draw its graph.

Sol^n $f(x) = \dfrac{x^2-5x+4}{x^2+2x-3} = \dfrac{(x-4)\cancel{(x-1)}}{\cancel{(x-1)}(x+3)} = \dfrac{x-4}{x+3}$; $x \neq 1$.

$\therefore$ Range $= R - \left\{\dfrac{a}{c}, f(1)\right\}$ $\qquad f(1) = \dfrac{1-4}{1+3} = \dfrac{-3}{4}$.

$= R - \left\{1, \dfrac{-3}{4}\right\}$

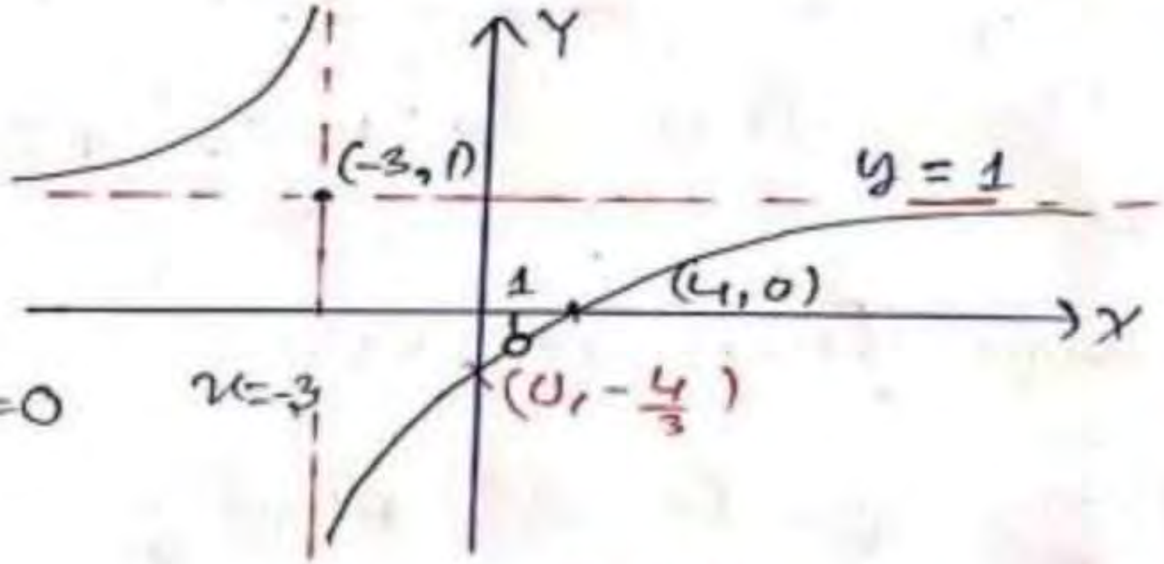
graph: $y = \dfrac{x-4}{x+3}$.

$\Rightarrow xy + 3y = x - 4$

$\Rightarrow x(y-1) + 3y + 4 = 0$

$\Rightarrow x(y-1) + 3(y-1) + 7 = 0$

$\Rightarrow (x+3)(y-1) = -7$.



$x=0 \Rightarrow 3y - 3 = -7$, $y = -\dfrac{4}{3}$ | $y=0$, $-x-3=-7$, $x = 4$

TAH 05
BY REED
FROM WB

QUESTION

Find domain & range of

$$f(x) = \frac{2x^2 + 2x + 3}{x^2 + x + 1}$$

TAH 06

$f(x) = \dfrac{2x^2+2x+3}{x^2+x+1}$ → $D<0, a>0$ always +ve.

x^2+x+1 → $D<0, a>0$ always +ve. So, Domain, $x \in R$

$y = \dfrac{2x^2+2x+3}{x^2+x+1}$

TAH-6
By Nikita
From Raj.

$x^2y + xy + y = 2x^2+2x+3$

$(y-2)x^2 + (y-2)x + y-3 = 0.$

$C_1 \Rightarrow y-2 \neq 0$

$y \neq 2$

since $x \in R$, $D \geq 0$.

$(y-2)^2 - 4(y-2)(y-3) \geq 0$

$(y-2)(y-2-4y+12) \geq 0.$

$(y-2)(-3y+10) \geq 0.$

$(y-2)(3y-10) \leq 0.$

$\cap \quad y \in [2, \frac{10}{3}]$

$y \in (2, \frac{10}{3}]$

$C_2 \Rightarrow y-2=0.$

$y=2$

$y-3=0$

$\boxed{y=3}$

final ans. $C_1 \cup C_2$

$y \in (2, \frac{10}{3}]$ Ans

TAH-6: Find domain & Range of $f(x)=\dfrac{2x^2+2x+3}{x^2+x+1}$

TAH 6
BY REED
FROM WB

Sol: $y=\dfrac{2x^2+2x+3}{x^2+x+1}$

Dom. $x^2+x+1\neq 0$
$\hookrightarrow D<0, a>0$ ∴ always +ve ∴ $\boxed{x\in R}$

Range: $x^2y+xy+y=2x^2+2x+3$

$\Rightarrow (y-2)x^2+(y-2)x+y-3=0$

Case-1: $y-2\neq 0$
$\Rightarrow y\neq 2.$

So, $D\geq 0$

$\Rightarrow (y-2)^2-4(y-2)(y-3)\geq 0$

$\Rightarrow y^2+4-4y-4y^2+20y-24\geq 0$

$\Rightarrow 3y^2-16y+20\leq 0$

$\Rightarrow 3y^2-10y-6y+20\leq 0$

$\Rightarrow (3y-10)(y-2)\leq 0$ $\quad y\in\left(2,\frac{10}{3}\right]$ — (i)

Case-2: If $y-2=0$
$\Rightarrow y=2$

$2-3=0$

$\Rightarrow -1=0$ [N.P]

∴ $y\neq 2$ — (ii)

$\boxed{y\in\left(2,\frac{10}{3}\right]}$

QUESTION

Find domain & range of $f(x) = \frac{2x}{1+x^2}$.

TAH 07

$$y = \frac{2x}{1+x^2}$$ $\to$ $a>0, D<0$, always +ve $x \in R$

$$y + yx^2 = 2x$$

$$yx^2 - 2x + y = 0$$

$C_1 \Rightarrow y \neq 0$,

since $x \in R$, $D \geq 0$

$$4 - 4y^2 \geq 0$$

$$4(y^2 - 1) \leq 0$$

$$(y-1)(y+1) \leq 0$$

$$y \in [-1, 1]$$

$\cap$

$$y \in [-1, 1] - \{0\}$$

$C_2 \Rightarrow$ if $y = 0$

$$-2x = 0$$

$$x = 0$$

at $x = 0$, $y = 0$

final ans. $C_1 \cup C_2$

$$y \in [-1, 1] - \{0\} \cup \{0\}$$

$$y \in [-1, 1]$$ Ans

TAH-7

By Nikita

From Raj.

TAH-7! Dom. & Range of $f(x) = \frac{2x}{1+x^2}$

Soln: $y = \frac{2x}{1+x^2} \Rightarrow x^2y + y = 2x \Rightarrow x^2y - 2x + y = 0$

Case-1! if $y \neq 0 \Rightarrow D \geq 0$.

$4 - 4y^2 \geq 0$

$\Rightarrow y^2 - 1 \leq 0$

$\Rightarrow (y+1)(y-1) \leq 0$

$\therefore y \in [-1, 1] - \{0\}$

Case-2! if $y = 0$.

$2x = 0$

$\Rightarrow x = 0 \rightarrow$ Real.

$\therefore y = 0$ is also possible.

$\boxed{y \in [-1, 1]}$

TAH 7
BY REED

QUESTION

If x be real, then prove that $\frac{x}{x^2-5x+9}$ must lie between $-\frac{1}{11}$ and 1.

TAH-8: If x real, prove $\frac{x}{x^2-5x+9} \in \left[-\frac{1}{11}, 1\right]$

Soln:

$$y = \frac{x}{x^2-5x+9}$$

$$\Rightarrow x^2y - 5xy - x + 9y = 0$$

$$\Rightarrow yx^2 - x(5y+1) + 9y = 0.$$

TAH 7
BY REED

Case-1: $y \neq 0 \Rightarrow D \geq 0.$

$$(5y+1)^2 - 36y^2 \geq 0$$

$$\Rightarrow 25y^2 + 10y + 1 - 36y^2 \geq 0$$

$$\Rightarrow 11y^2 - 10y - 1 \leq 0$$

$$\Rightarrow 11y^2 - 11y + y - 1 \leq 0$$

$$\Rightarrow (11y+1)(y-1) \leq 0$$

$$\Rightarrow y \in \left[-\frac{1}{11}, 1\right] - \{0\}$$

Case-2: If $y = 0$

$$-x = 0$$

$$\Rightarrow x = 0$$

Real.

$\therefore y = 0$ is also possible

$$y \in \left[-\frac{1}{11}, 1\right]$$

proved

TAH 08

$$y = \frac{x}{x^2-5x+9} \rightarrow a>0,\ D<0,\ x \in R$$

$x \in R$

always +ve.

TAH-8
By Nikita
From Raj.

$$x^2y - 5xy + 9y = x$$

$$yx^2 - (5y+1)x + 9y = 0$$

$C_1 \Rightarrow y \neq 0$, $x \in R$, $D \geq 0$

$$(5y+1)^2 - 4 \cdot 9y^2 \geq 0.$$

$$25y^2 + 1 + 10y - 36y^2 \geq 0.$$

$$-11y^2 + 10y + 1 \geq 0.$$

$$11y^2 - 10y - 1 \leq 0.$$

$$(11y+1)(y-1) \leq 0.$$

$\cap$

$y \in [-\frac{1}{11}, 1] - \{0\}$ $\quad y \in [-\frac{1}{11}, 1]$

$C_2 \Rightarrow y = 0$

$-x = 0$

$x = 0$

at $x = 0$, $y = 0$

final ans. $\Rightarrow C_1 \cup C_2$

$y \in [-\frac{1}{11}, 1]$ Ans

QUESTION

If x is real, then maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is

(A) 1

(B) 17/7

(C) 1/4

(D) 4

Ans. 4

TAH-9: max value of $\frac{3x^2+9x+17}{3x^2+9x+7}$; $x \in R$.

Solⁿ:

Dom. $\rightarrow$ $3x^2+9x+7$

$\left.\begin{array}{l} D<0 \\ a>0 \end{array}\right\} >0 \Rightarrow x \in R.$

TAH 8
By Reed

$y = \frac{3x^2+9x+7+10}{3x^2+9x+7} = 1+\frac{10}{\boxed{3x^2+9x+7}} \Rightarrow t$

$y = 1+\frac{10}{t}$

$\left[-\frac{D}{4a}, \infty\right)$

$\therefore t \in \left[-\frac{(81-84)}{12}, \infty\right)$

$\Rightarrow t \in \left[\frac{3}{12}, \infty\right) \equiv \left[\frac{1}{4}, \infty\right)$

$\therefore \frac{1}{t} \in (0, 4]$

$\Rightarrow \frac{10}{t} \in (0, 40]$

$\Rightarrow 1+\frac{10}{t} \in (1, 41]$

$\therefore y \in (1, 41]$

$\therefore y_{max} = 41$. (Ans)

Tah 09 if $x \in R$, then max. value of $\frac{3x^2+9x+17}{3x^2+9x+7}$

$$y = \frac{3x^2+9x+7+10}{3x^2+9x+7}$$

$$y \in 1 + \frac{10}{3x^2+9x+7}$$

$a > 0$, $D < 0$, always +ve

$$\left[\frac{-D}{4a}, \infty\right) = \left[\frac{1}{4}, \infty\right)$$

$$y = 1 + 10(0, 4]$$

$$y = 1 + (0, 40] \Rightarrow y \in (1, 41]$$

max. value of y is $\underline{41}$ Ans

TAH-9
By Nikita
From Raj.

PW

Solution to Previous KTKs

QUESTION

(KTK 01)

The value of 'a' for which the equation $x^7 + ax^2 + 3 = 0$ and $x^8 + ax^3 + 3 = 0$ have a common root, can be

α α

(A) 1

(B) -2

(C) -3

(D) -4

Ans. D

Q-12: The value of 'a' for which the equation $x^7+ax^2+3=0$ and $x^8+ax^3+3=0$ have a common root, can be:

(a) 1 (b) −2 (c) −3 (d) −4.

Soln:

(i) — $x^7+ax^2+3=0 \rightarrow$ multiply by x

(ii) — $x^8+ax^3+3=0$

$$x(x^7+ax^2+3)=0$$

or, $x^8+ax^3+3x=0$ — (iii)

& $x^8+ax^3+3=0$ — (ii)

Subtract: $3x-3=0$

$\Rightarrow (x-1)\cdot 3=0$

$\Rightarrow x-1=0$

$\Rightarrow \boxed{x=1} \longrightarrow$ common root

put $x=1$ in eqn (i):

$$1+a+3=0$$

or, $a+4=0$

or $\boxed{a=-4}$ (Ans)

KTK 1
BY REED
FROM WB

KTK lec → 07

(1) $x^7 + ax^2 + 3 = 0$
$x^8 + ax^3 + 3 = 0$ } a common root

$\cancel{x^8} + \cancel{ax^3} + 3x = 0$
$-\cancel{x^8} \pm \cancel{ax^3} \pm 3 = 0$

$3x - 3 = 0$

$3(x-1) = 0$

$\boxed{x = 1}$

$1 + a \times 1 + 3 = 0$

$\boxed{a = -4}$

ans

QUESTION

(KTK 02)

If α, β are roots of $Ax^2 + Bx + C = 0$ and α^2, β^2 are roots of $x^2 + px + q = 0$ then p is equal to

(A) $\dfrac{B^2 - 4AC}{A^2}$

(B) $\dfrac{2AC - B^2}{A^2}$

(C) $\dfrac{4AC - B^2}{A^2}$

(D) None of these

Ans. B

$Ax^2+Bx+C=0 \to \alpha, \beta$
and
$x^2+px+q=0 \to \alpha^2, \beta^2$ → find P

from eq^n 1

$$\alpha^2+\beta^2 = \frac{B^2}{A^2} - \frac{2C}{A}$$

$$\frac{B^2}{A^2} - \frac{2C}{A} = -P$$

$$\Rightarrow P = \frac{2AC - B^2}{A^2}$$

(B)

Q-13: If α, β are roots of $Ax^2+Bx+C=0$ and α^2, β^2 are roots of $x^2+px+q=0$ then p is equal to:

(a) $\frac{B^2-4AC}{A^2}$ (b) $\frac{2AC-B^2}{A^2}$ (c) $\frac{4AC-B^2}{A^2}$ (d) None of these

Soln:

$$Ax^2+Bx+C=0 \left.\begin{array}{l}\to\alpha\\ \to\beta\end{array}\right\} \text{roots}$$

$$\alpha+\beta = -\frac{B}{A} \qquad \alpha\beta = \frac{C}{A}$$

KTK 2
BY REED

&

$$x^2+px+q=0 \begin{array}{l}\to\alpha^2\\ \to\beta^2\end{array}$$

$$\alpha^2+\beta^2 = -p \qquad \alpha^2\beta^2 = q$$

$\therefore$ $\alpha^2+\beta^2 = -p$

or, $(\alpha+\beta)^2 - 2\alpha\beta = -p$

or, $\frac{B^2}{A^2} - 2\frac{C}{A} = -p$

or $\frac{B^2-2AC}{A^2} = -p$

or, $p = \frac{-(B^2-2AC)}{A^2}$

or, $p = \frac{2AC-B^2}{A^2}$ (Ans.)

$\therefore$ Ans $\Rightarrow$ (b) $\frac{2AC-B^2}{A^2}$

QUESTION

(KTK 03)

Find the values of 'k' so that the equation
$x^2 + kx + (k + 2) = 0$ and $x^2 + (1 - k)x + 3 - k = 0$ have exactly one common root.

Ans. No possible value of k

Homework

Khud Se Try Karo 03

Mathematics

E_1: $x^2+kx+(k+2)=0$

E_2: $x^2+(1-k)x+3-k=0$

exactly one common root.

$$E_1-E_2 = 2kx - x + 2k - 1 = 0$$

$$(x+1)(2k-1)=0$$

$$\Rightarrow x=-1$$

or $k=\frac{1}{2}$ X → both roots in common

@ $x=-1$

$$1-\cancel{k}+\cancel{k}-2=0$$

$$-1 \neq 0 \text{ X}$$

thus no possible values of k

KTK - 03

$$x^2 + kx + k + 2 = 0 \quad \alpha$$

$$x^2 + (1-k)x + 3-k = 0 \quad \alpha$$

$-$

$$(k-(1-k))x + k+2-3+k = 0.$$

$$(2k-1)x + (2k-1) = 0.$$

$$(2k-1)(x+1) = 0.$$

$$\boxed{x=-1} \qquad \boxed{k=1/2} \times$$

KTK-3
By Nikita
From Raj.

If both roots are common a

$$\frac{1}{1} = \frac{k}{1-k} = \frac{k+2}{3-k}$$

$$1-k = k \qquad\qquad 3-k = k+2$$

$$1 = 2k \qquad\qquad \boxed{2k=1}$$

$$\boxed{k=1/2}$$

Here no possible value of k exist.

Ans

(3) $x^2 + Kx + (K+2) = 0$

$$\frac{- \; x^2 + (1-K)x \pm 3-K = 0}{Kx - x + Kx + K + 2 + K - 3 = 0}$$

$2Kx - x + 2K - 1 = 0$

$x(2K-1) + 1(2K-1) = 0$

$(2K-1)(x+1) = 0$

$\boxed{x = -1}$ $\quad \boxed{K = \frac{1}{2}}$ $\times$

If both roots are common

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{1}{1} = \frac{K}{1-K} = \frac{K+2}{3-K}$$

$1 - K = K$

$1 = K + K$

$1 = 2K$ $\quad K \in \phi$

$\boxed{K = \frac{1}{2}}$ No any value it possible

QUESTION

(KTK 04)

In a triangle PQR, $\angle R = \frac{\pi}{2}$, if $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0, a \neq 0$ then

(A) $a = b + c$

(B) $c = a + b$

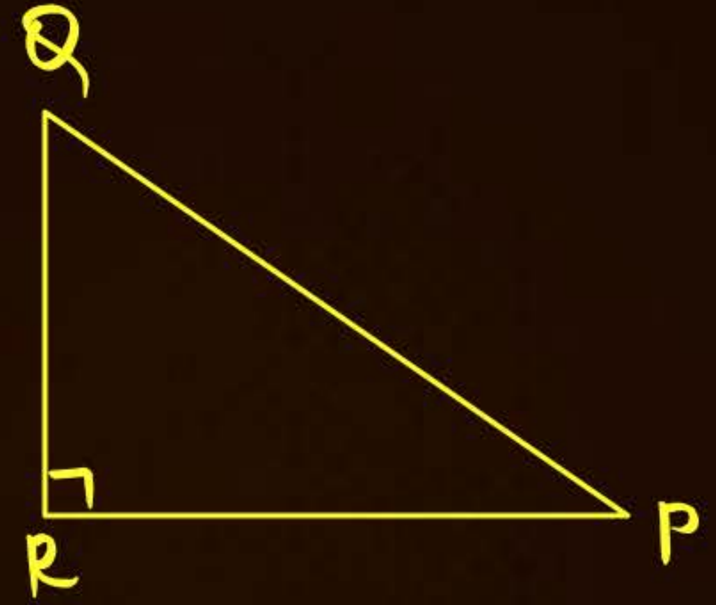
(C) $b = c$

(D) $b = a + c$

$P + Q = \frac{\pi}{2}$

$\frac{P}{2} + \frac{Q}{2} = \frac{\pi}{4}$

$1 = \dfrac{\tan\frac{P}{2} + \tan\frac{Q}{2}}{1 - \tan\frac{P}{2} \cdot \tan\frac{Q}{2}}$



Ans. B

Homework

Khud Se Try Karo 04

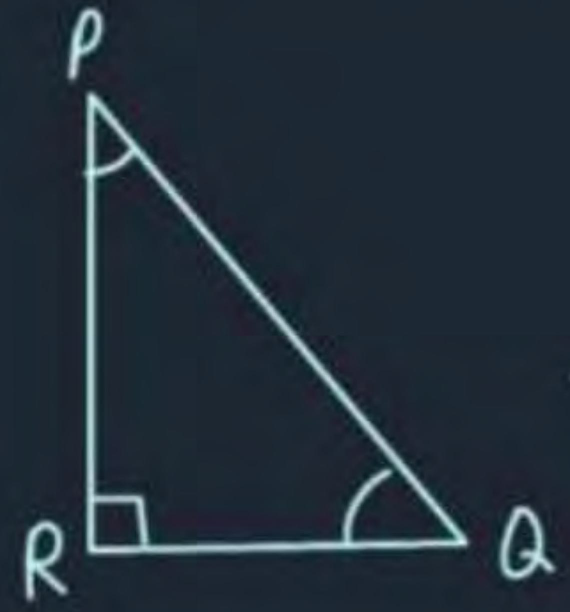
Mathematics

there is a triangle ΔPQR

$\angle R = \pi/2$

$\tan\left(\frac{P}{2}\right)$ and $\tan\frac{Q}{2}$ are roots of E

$E = ax^2 + bx + c = 0$



$P + Q = 90^\circ$

$\frac{P}{2} + \frac{Q}{2} = 45^\circ$

$$\frac{\tan\left(\frac{P}{2}\right) + \tan\frac{Q}{2}}{1 - \tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)} = 1$$

$$\Rightarrow \frac{-b/a}{1 - c/a} = 1$$

$\Rightarrow -b/a = 1 - c/a$

$= c - b = a$

$\Rightarrow c = a + b$ (B) ✓

Akawn

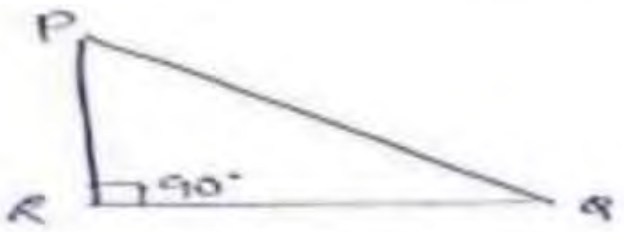
KTK-4! In a triangle PQR, $\angle R = \frac{\pi}{2}$ & if $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are roots of $ax^2+bx+c=0$, $a \neq 0$ then.

(a) $a=b+c$ (b) $c=a+b$

(c) $b=c$ (d) $b=a+c$.

KTK 4
BY REED
FROM WB

Soln:



$$P+Q+R=\pi$$
$$\Rightarrow P+Q=\pi-R=\pi-\frac{\pi}{2}$$
$$\Rightarrow P+Q=\frac{\pi}{2}$$

$$P+Q=\frac{\pi}{2} \Rightarrow \frac{P+Q}{2}=\frac{\pi}{4}$$

$$\Rightarrow \tan\left(\frac{P}{2}+\frac{Q}{2}\right)=\tan\left(\frac{\pi}{4}\right)=1$$

$$\Rightarrow \frac{\tan\frac{P}{2}+\tan\frac{Q}{2}}{1-\tan\frac{P}{2}\cdot\tan\frac{Q}{2}}=1$$

$$\Rightarrow \frac{S.O.R.}{1-P.O.R}=1 \quad \text{—— (i)}$$

now $ax^2+bx+c=0 \nearrow \tan\frac{P}{2}$, $\searrow \tan\frac{Q}{2}$.

$$S.O.R=\tan\frac{P}{2}+\tan\frac{Q}{2}=\frac{-b}{a} \quad \text{—— (ii)}$$
$$P.O.R=\tan\frac{P}{2}\cdot\tan\frac{Q}{2}=\frac{c}{a}. \quad \text{—— (iii)}$$

$\therefore$ from (i):

$$\frac{-b/a}{1-\frac{c}{a}}=1$$
$$\Rightarrow \frac{-b}{a-c}=1$$
$$\Rightarrow -b=a-c$$
$$\Rightarrow \boxed{a+b=c}$$

Ans. $\Rightarrow$ (b)

QUESTION

(KTK 05)

The equations $ax^2 + bx + a = 0 (a, b \in R)$ and $x^3 - 2x^2 + 2x - 1 = 0$ have 2 roots common. Then $a + b$ must be equal to

(A) 1

(B) –1

(C) 0

(D) None of these

Ans. C

Q-8: The equations $ax^2+bx+a=0$ $(a,b \in R)$ and $x^3-2x^2+2x-1=0$ have 2 roots common. Then $a+b$ must be equal to:

(a) 1 (b) -1 (c) 0 (d) None of these

Soln:
$$x^3-2x^2+2x-1=0 = P(x)$$

$P(1) = 1-2+2-1 = 0$

$\therefore (x-1)$ is a factor.

$$\Rightarrow x^2(x-1) - x(x-1) + 1(x-1) = 0$$
$$\Rightarrow (x-1)(x^2-x+1) = 0.$$

$x=1$ (real root)

$x^2-x+1=0$

$D = -3\ (<0) \rightarrow$ imaginary roots.

must be both common with $ax^2+bx+a=0$

KTK 5
BY REED
FROM WB

$$\frac{1}{a} = \frac{-1}{b} = \frac{1}{a}.$$
$$\Rightarrow \frac{a}{1} = \frac{b}{-1}$$
$$\Rightarrow -a = b$$
$$\Rightarrow \boxed{a+b=0}$$

THANK YOU