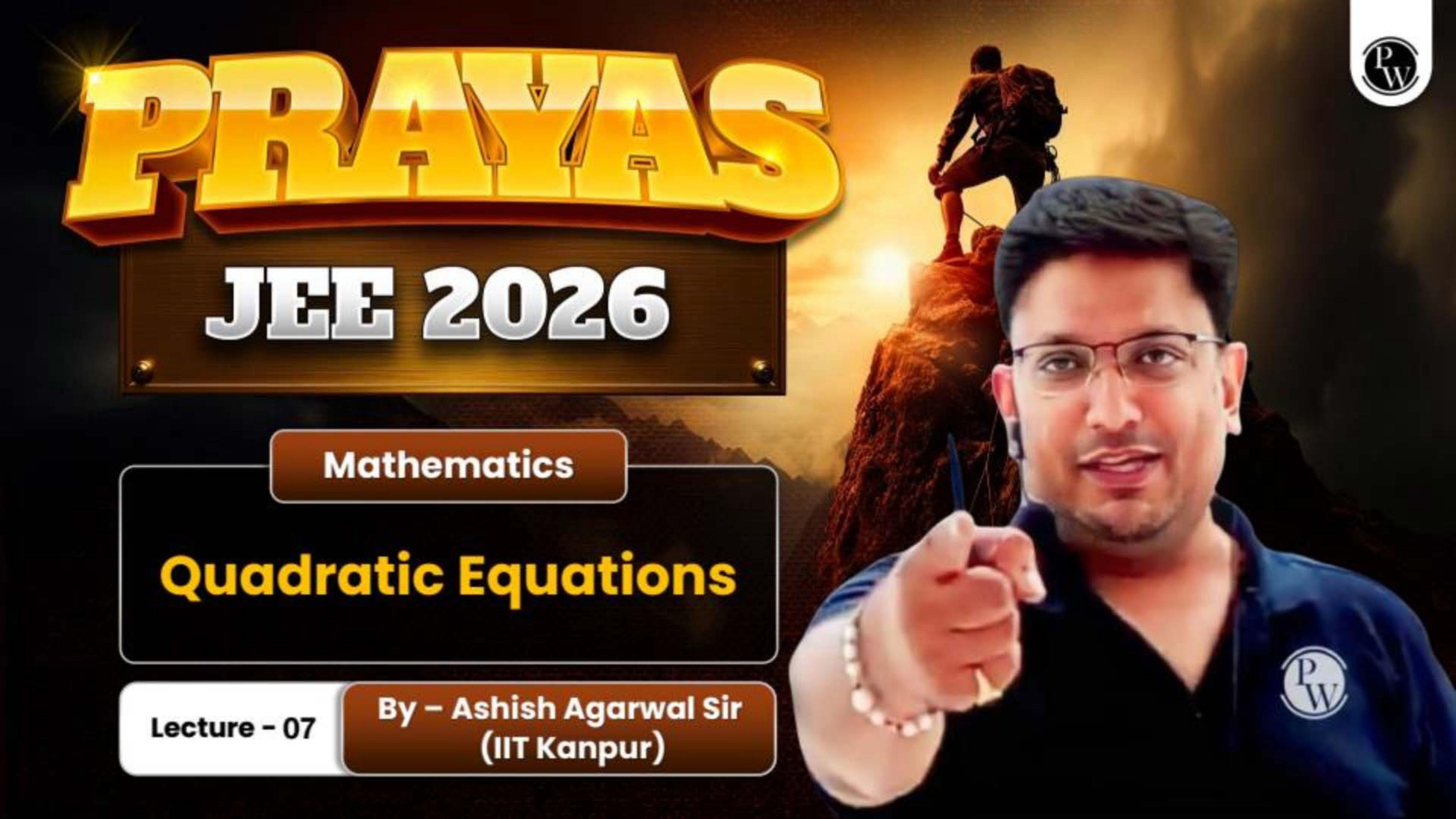
PRAYAS
JEE 2026
Mathematics
Quadratic Equations
Lecture - 07
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Topics *to be covered*

- **A** Range of Rational Functions
- **B** Practice problems

Recap *of previous lecture*

1. If $x \in [-1, 5]$ then

a. $2x + 3 \in$ $[1, 13]$ $2x$: $[-2, 10]$

b. $5 - 6x \in$ $[-25, 11]$

c. $\frac{1}{x} \in$ $(-\infty, -1] \cup [1/5, \infty)$

d. $x^2 \in$ $[0, 25]$

e. $\frac{1}{x^2 - 1} \in$ $(-\infty, -1] \cup [1/24, \infty)$

$5 - 6x$: $6x \in [-6, 30]$, $5 - 6x \in [-25, 11]$

$\frac{1}{x}$: $x \in [-1, 0] \cup [0, 5]$, $\rightarrow 0^+$; $\frac{1}{x} \in (-\infty, -1] \cup [\frac{1}{5}, \infty)$

x^2: $x \in [-1, 0] \cup [0, 5]$; $x^2 \in [0, 1] \cup [0, 25] = [0, 25]$

$\frac{1}{x^2 - 1}$: $x^2 \in [0, 25]$; $x^2 - 1 \in [-1, 24] = [-1, 0] \cup [0, 24]$

Recap *of previous lecture*

2. $y = f(x) = x^2 - 5x + 4, x \in R$. Then find range of
   (i) $y = x^2 - 5x + 4$ where $x \in R$ is $\left[-\frac{D}{4a}, \infty\right) = \left[-\frac{(25-16)}{4}, \infty\right) = \left[-\frac{9}{4}, \infty\right)$
   (ii) $y = x^2 - 5x + 4$ where $x \in [0, 2]$ is $[-2, 4]$
   (iii) $y = x^2 - 5x + 4$ where $x \in [2, 5]$ is $[-9/4, 4]$
   (iv) $y = x^2 - 5x + 4$ where $x \in (-1, 3)$ is $[-9/4, 10)$

$\left(\frac{x \text{ ke coeff}}{2}\right)^2$ add & sub.

3. $y = ax^2 + bx + c \geq 0 \ \forall \ x \in R$ if $a>0$ & $D \leq 0$
4. $y = ax^2 + bx + c \leq 0 \ \forall \ x \in R$ if $a<0, D \leq 0$.
5. $y = ax^2 + bx + c > 0 \ \forall \ x \in R$ if $a>0, D<0$
6. $y = ax^2 + bx + c < 0 \ \forall \ x \in R$ if $a<0, D<0$

$f(x) = x^2 - 5x + 4$
$= x^2 - 5x + \frac{25}{4} - \frac{25}{4} + 4$
$= \left(x - \frac{5}{2}\right)^2 + 4 - 25/4 = \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}$

(ii) $f(x) = \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}$

$[0, 2]$

$\left[-\frac{5}{2}, -\frac{1}{2}\right]$

$\left[\frac{1}{4}, \frac{25}{4}\right]$

$\left[-\frac{8}{4}, \frac{16}{4}\right] = [-2, 4]$

(iii) $f(x) = \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}$

$[2, 5]$

$\left[-\frac{1}{2}, \frac{5}{2}\right] = \left[-\frac{1}{2}, 0\right] \cup \left[0, \frac{5}{2}\right]$

$[0, 1/4] \cup \left[0, \frac{25}{4}\right]$

$[0, 25/4]$

$\left[-9/4, \frac{16}{4}\right] = [-9/4, 4]$

(iv) $f(x) = \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}$ $\quad x \in (-1, 3)$

$\left(-\frac{7}{2}, \frac{1}{2}\right) = \left(-\frac{7}{2}, 0\right] \cup \left[0, \frac{1}{2}\right)$

$\left[0, \frac{49}{4}\right) \cup \left[0, 1/4\right) = \left[0, \frac{49}{4}\right)$

$\left[-\frac{9}{4}, 10\right)$

Recap *of previous lecture*

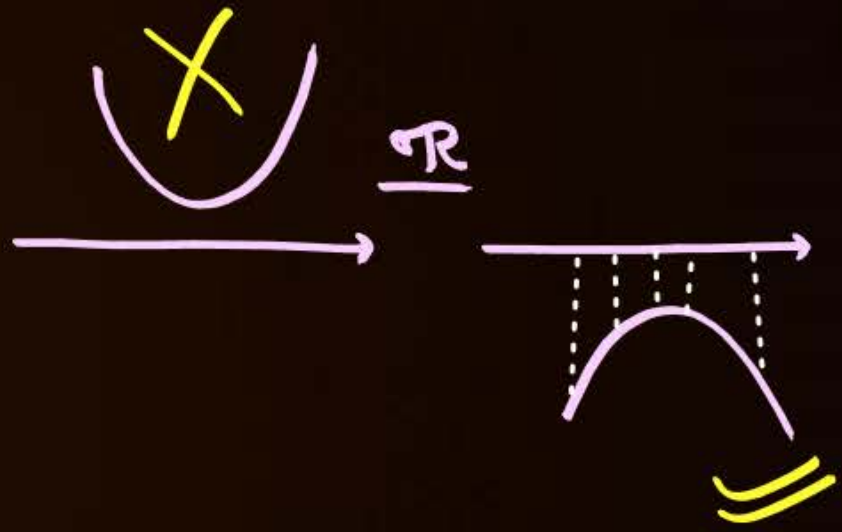
7. If $D < 0$ then graph of $y = ax^2 + bx + c$ lies entirely above x-axis or entirely below x-axis.

8. If $ax^2 + bx + c$ has non real roots & $a + 2b + 4c < 0$ then

$f(x)$ $4f(1/2)$

$a - b + c$ < 0, $4a - 2b + c$ < 0 $f(1/2) < 0$

$f(-1)$ $f(-2)$

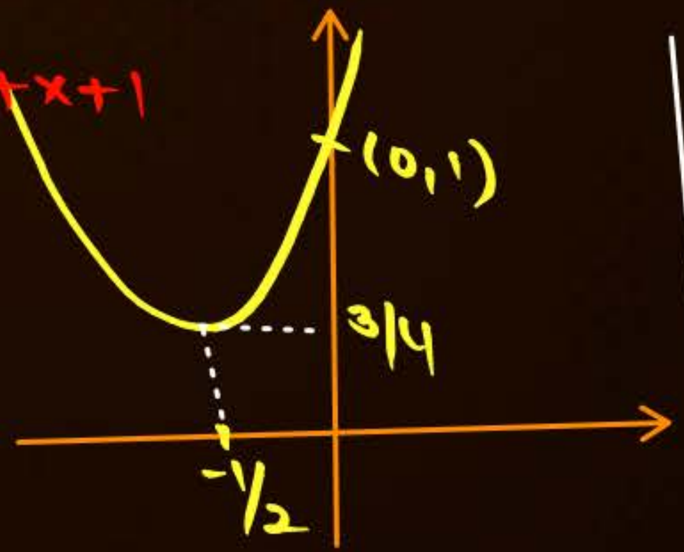


9. Graph of 1. $y = x^2 + x + 1$ 2. $y = x^2 + 2x + 1$

* upward opening parabola
* $D<0, a>0$ ✓ lies above x axis.
* $y_{min} = \frac{-D}{4a} = -\frac{(1-4)}{4} = 3/4$ at $x = -1/2$

$V(-1/2, 3/4)$

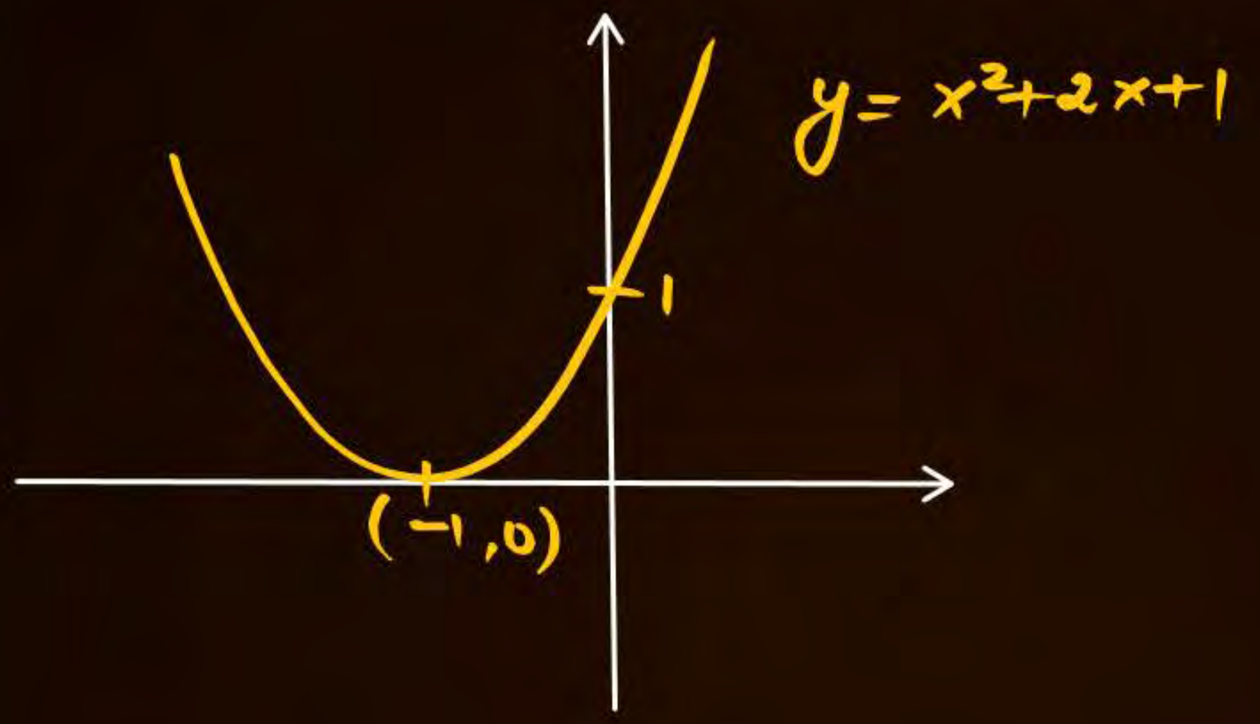
* POI y axis $(0,1)$



* upward opening parabola
* Roots $-1, -1$

$D = 0$ touches x axis.

* $V\left(-\frac{2}{2}, 0\right) = (-1, 0)$
* POI y axis $(0,1)$



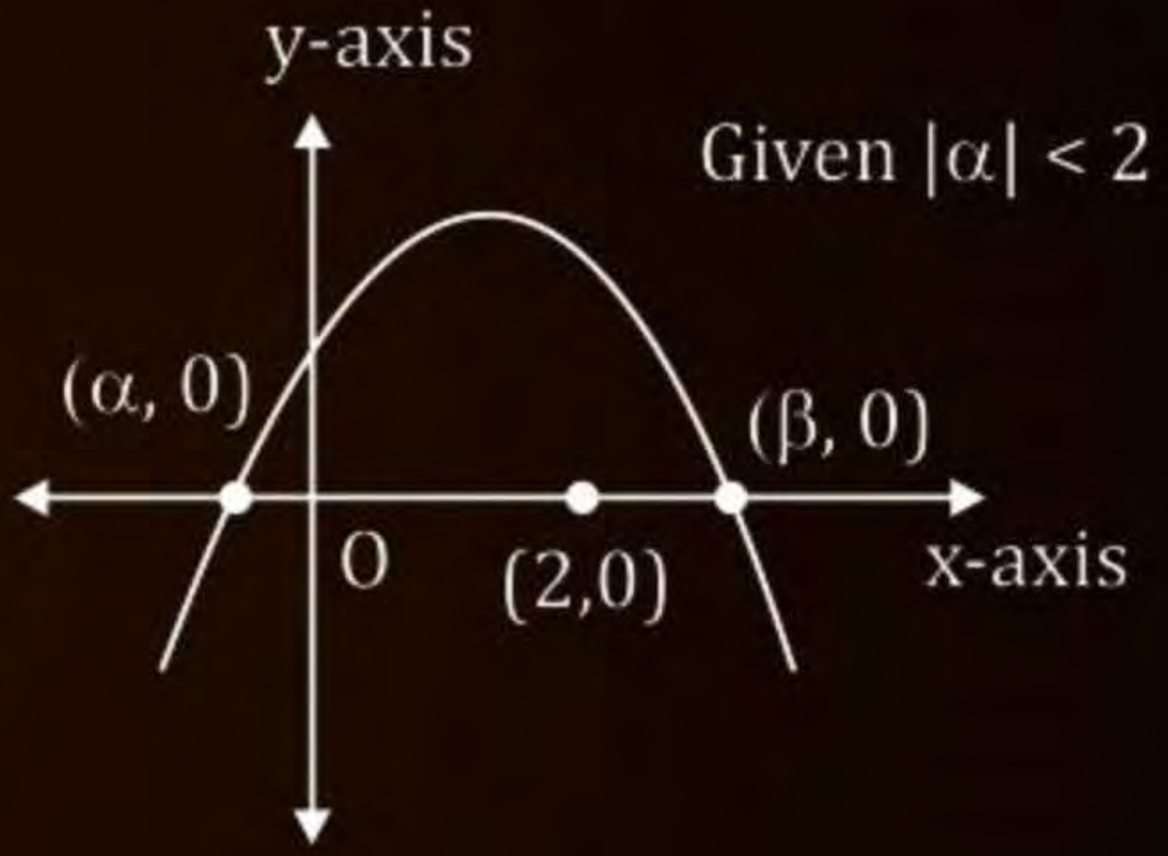
$y = x^2 + 2x + 1$
1
(-1, 0)

Homework Discussion

QUESTION

(ADBST)

The graph of $y = ax^2 + bx + c$ is shown in the figure, then which of the following is(are) correct?



(A) $ab^2c^3 < 0$

(B) $ab < 0$

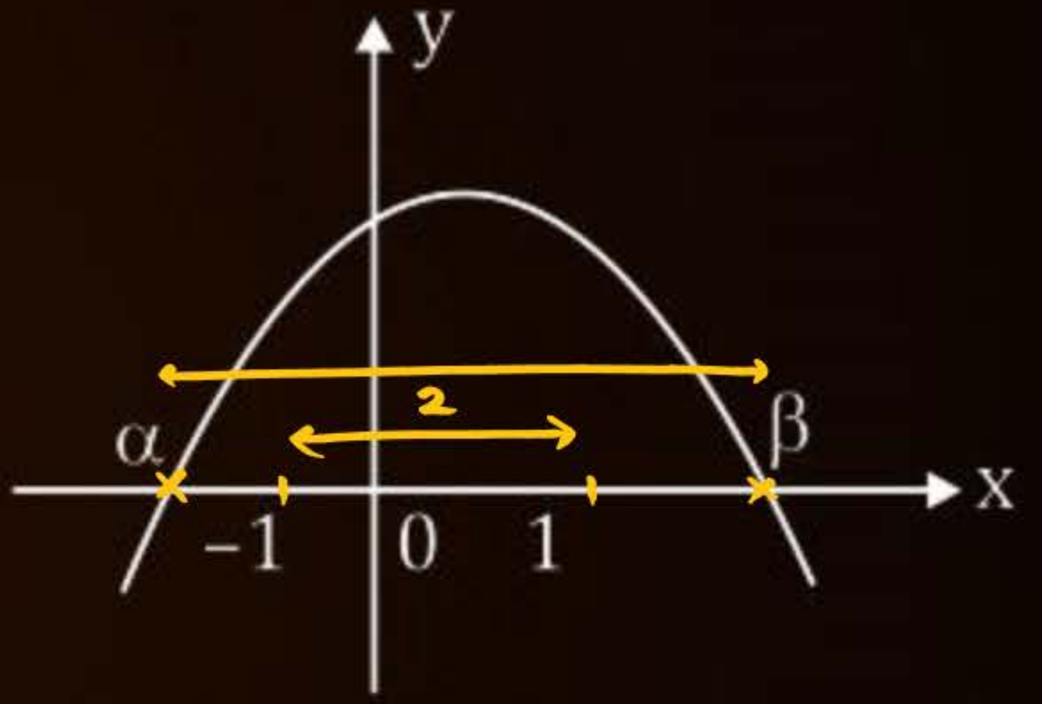
(C) $bc(4a + 2b + c) > 0$

(D) $ab(4a - 2b + c) > 0$

QUESTION

(ADBST)

The graph of quadratic polynomial $f(x) = ax^2 + bx + c$ is shown in below. Which of the following are correct?



(A) $\frac{c}{a} < -1$

(B) $|\beta - \alpha| > 2$

(C) $f(x) > 0 \ \forall \ x \in (0, \beta)$

(D) $abc < 0$

-ve +ve >0

(B) $|\beta-\alpha| > 2$

(A) $\alpha < -1$

$\beta > 1$

$\alpha\beta < -1$

$\frac{c}{a} < -1$

$\alpha = -1.5$

$\beta = 2$ $\alpha\beta = -3 < -1$

$S.O.R = -\frac{b}{a} > 0$

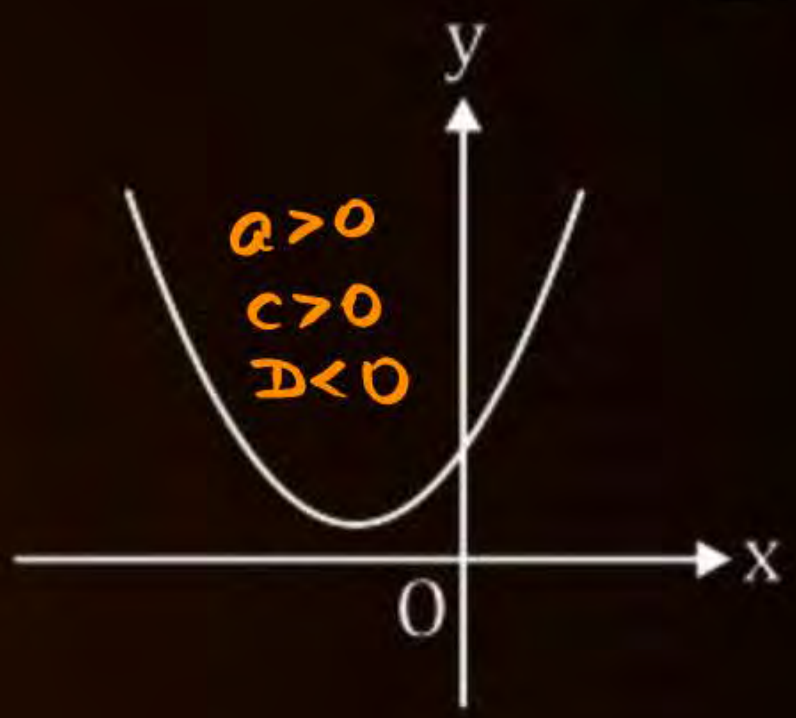
$-b < 0$

$b > 0$

QUESTION

(ADBST)

The curve of the quadratic expression $y = ax^2 + bx + c$ is shown in the figure and α, β be the roots of the equation $ax^2 + bx + c = 0$ then correct option is
[D is the discriminant]



(A) $a > 0,\ b > 0, c > 0, D > 0, \alpha + \beta > 0, \alpha\beta > 0$

(B) $a > 0, b > 0, c > 0, D < 0, \alpha + \beta < 0, \alpha\beta < 0$

(C) $a > 0,\ b > 0, c > 0, D < 0, \alpha + \beta < 0, \alpha\beta > 0$

(D) $a > 0, b < 0, c > 0, D < 0, \alpha + \beta > 0, \alpha\beta > 0$

Ans. C

QUESTION

Find the set of values of a for which $(a - 1)x^2 - (a + 1)x + a + 1 > 0$ for all $x \in R$.

$a-1>0$ & $D<0$

$\cap$

Ans.

QUESTION

Find the set of values of a for which $(a + 4)x^2 - 2ax + 2a - 6 < 0$ for all $x \in R$.

$a+4<0$ & $D<0$

$\cap$

Ans

QUESTION

For what values of p the vertex of $x^2 + px + 13$ lies at a distance 5 unit from origin.

$$V\left(\frac{-p}{2}, -\frac{(p^2-52)}{4}\right)$$

$$O(0, 0)$$

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

QUESTION

Find the range of $f(x) = 2x^2 - 3x + 2$ in $[0, 2]$ Ans. : $\left[\frac{7}{8}, 4\right]$

Find the range of $f(x) = -x^2 + 6x - 1$ in $[0, 4]$ Ans.: $[-1, 8]$

$$y = 2x^2 - 3x + 2$$
$$= 2\left(x^2 - \frac{3}{2}x\right) + 2$$
$$= 2\left(\underbrace{x^2 - \frac{3}{2}x + \frac{9}{16}} - \frac{9}{16}\right) + 2$$
$$= 2\left(\left(x - 3/4\right)^2 - \frac{9}{16}\right) + 2$$
$$= 2(x - 3/4)^2 - \frac{9}{8} + 2 = 2(x - 3/4)^2 + 2 - \frac{9}{8}$$
$$= 2(x - 3/4)^2 + 7/8$$

$[7/8, 4]$

$y = 2\left(x - \frac{3}{4}\right)^2 + \frac{7}{8}$

$[0, 2]$

$\left[-\frac{3}{4}, 5/4\right]$

$[-3/4, 0] \cup [0, 5/4]$

$\left[0, \frac{9}{16}\right] \cup \left[0, \frac{25}{16}\right]$

$[0, 25/16]$

$\left[0, \frac{25}{8}\right]$

QUESTION

Find the range of $f(x) = 2x^2 - 3x + 2$ in $[0, 2]$ Ans. : $\left[\frac{7}{8}, 4\right]$

Find the range of $f(x) = -x^2 + 6x - 1$ in $[0, 4]$ Ans.: $[-1, 8]$

$f(x) = -x^2 + 6x - 1$

$= -1 - (x^2 - 6x + 9 - 9)$

$= -1 - (x-3)^2 + 9$

$= 8 - (x-3)^2$

$[-3, 1] = [-3, 0] \cup [0, 1]$

$[0, 9] \cup [0, 1] = [0, 9]$

$[-1, 8]$

QUESTION

Tan01

For $x \in [1, 5], y = x^2 - 5x + 3$ has-

A) Least value $= -1.5$

B) Greatest value $= 3$

C) Least value $= -3.25$

D) Greatest value $= \frac{5+\sqrt{13}}{2}$

QUESTION

Tah02

Find range of following functions:

(i) $f(x) = 3x^2 - 2x - 7$

(ii) $f(x) = 3x^2 - 2x - 7, x \in (0, 5]$

(iii) $f(x) = 3x^2 - 2x - 7, x \in [-6, -1]$

Ans:[22/3,∞)

Ans: [22/3,58]

Ans: [-2,113]

QUESTION

Tah03

Find the range of $f(x)$:

(i) $f(x) = 2x^2 - 3x + 2$ Ans. $\left[\frac{7}{8}, \infty\right)$

(ii) $f(x) = 2x^2 - 3x + 2, x \in [0, 2]$ Ans. $\left[\frac{7}{8}, 4\right]$

(iii) $f(\theta) = 2\cos^2\theta - 6\sin\theta + 1$ Ans. $[-5, 7]$

QUESTION

Ta^04

Find the maximum and Minimum values If they exist

(i) $f(x) = x^2 + 2x + 4$

(ii) $f(x) = x^2 + 4x + 4$

(iii) $f(x) = x^2 - 5x + 4$

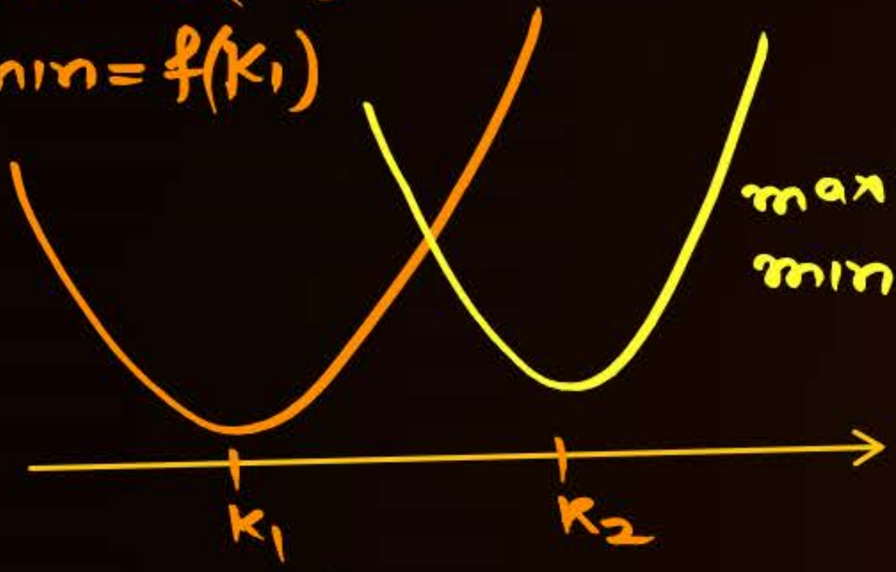
(iv) $f(x) = -x^2 + x - 4$

(v) $f(x) = -x^2 + 6x - 9$

(vi) $f(x) = -x^2 + 6x - 8$

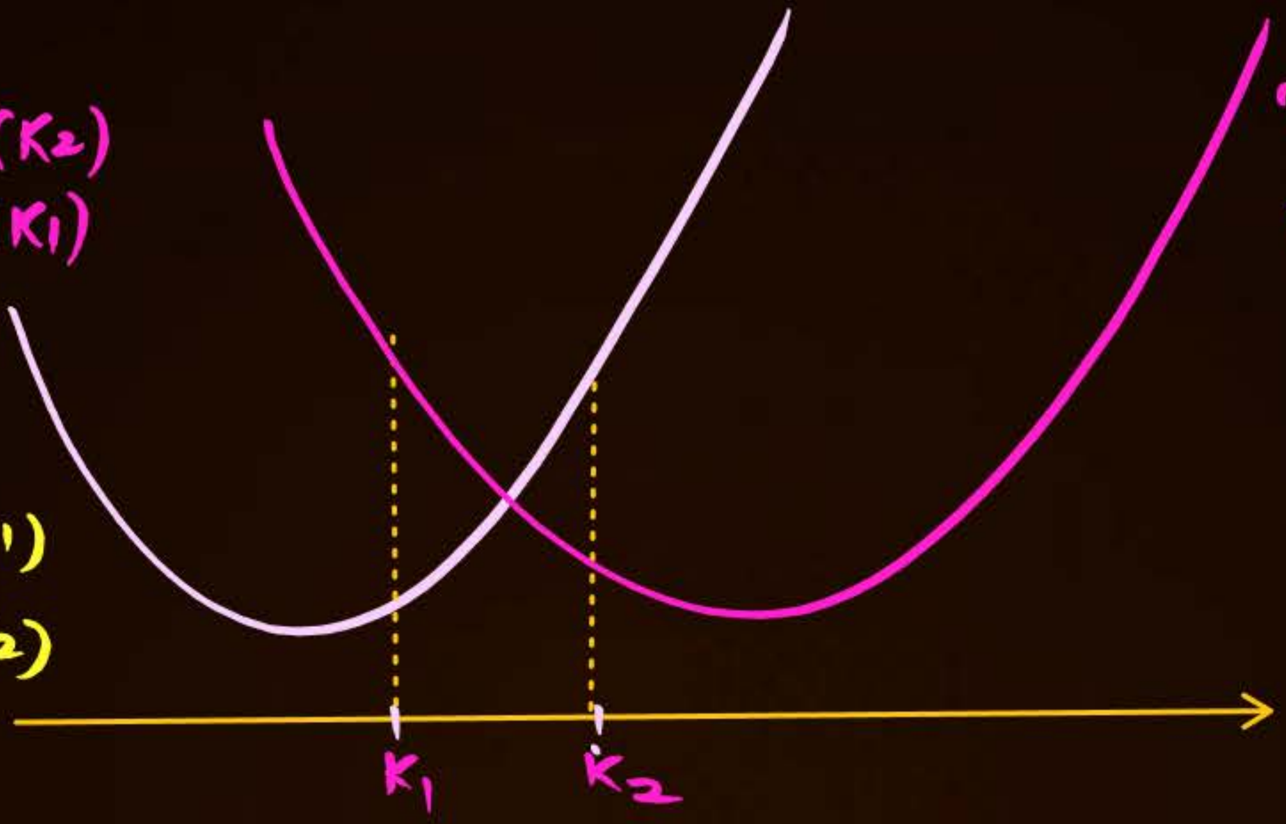
Ans. (i) Min value = 3; (ii) Min value = 0
(iii) Min value = $-9/4$, (iv) Max value = $-15/4$
(v) Max value = 0, (vi) Max value = -1

max = $f(k_2)$
min = $f(k_1)$



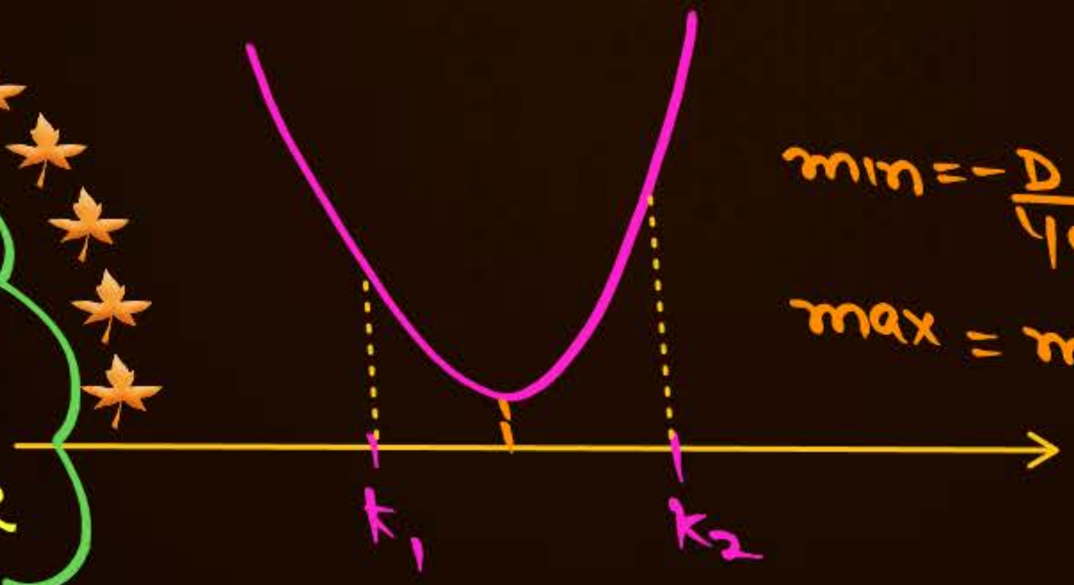
max = $f(k_1)$
min = $f(k_2)$

max = $f(k_2)$
min = $f(k_1)$



max = $f(k_1)$
min = $f(k_2)$

min & max value of
a quad $f(x) = ax^2 + bx + c$
in $[k_1, k_2]$ depends on
position of vertex relative
to $[k_1, k_2]$



min $= -\frac{D}{4a} = f\left(-\frac{b}{2a}\right)$

max $= \max\{f(k_1), f(k_2)\}$

QUESTION

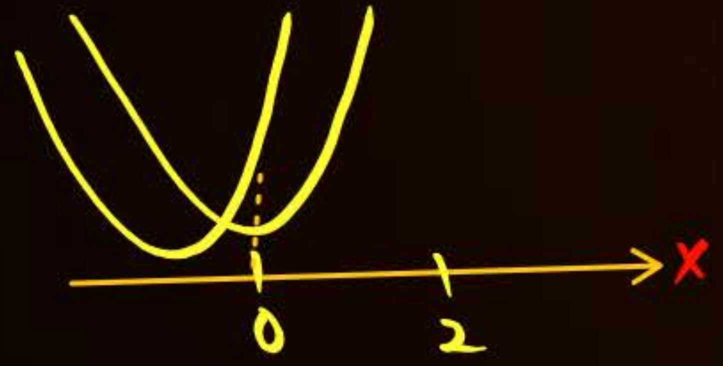
★★★KCLS★★★

Find all numbers p for each of which the least value of the quadratic trinomial $4x^2 - 4px + p^2 - 2p + 2$ on the interval $0 \le x \le 2$ is equal to 3

Given $f(x) = 4x^2 - 4px + p^2 - 2p + 2$ has min value $= 3$ in $x \in [0,2]$

$$x_v = \frac{-(-4p)}{2 \cdot 4} = \frac{p}{2}$$

Case (I) $\frac{p}{2} \le 0 \rightarrow p \le 0$



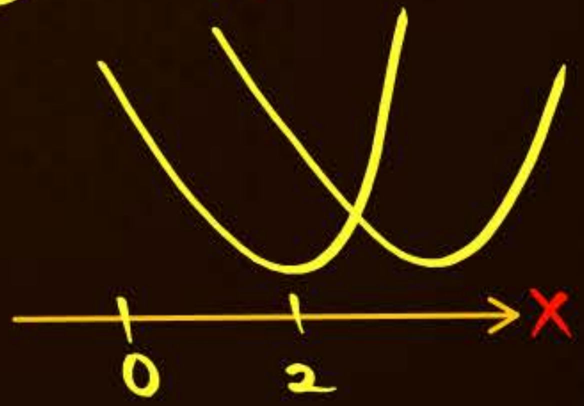
min $= f(0) = 3 \Rightarrow p^2 - 2p + 2 = 3$

$p^2 - 2p - 1 = 0$

$p = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

$p = 1 - \sqrt{2}$

Case (II) if $\frac{p}{2} \ge 2 \rightarrow p \ge 4$



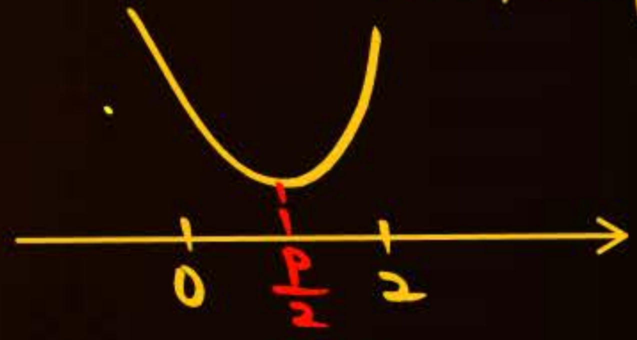
min $= f(2) = 3$

$16 - 8p + p^2 - 2p + 2 = 3$

$p^2 - 10p + 15 = 0$

Case (III) if $0 < \frac{p}{2} < 2$

$0 < p < 4$



min $= f\left(\frac{p}{2}\right) = 3$

$\cancel{p^2} - \cancel{2p^2} + \cancel{p^2} - 2p + 2 = 3$

$p = -\frac{1}{2}$ (rejected)

Ans. $P = 1 - \sqrt{2}$ or $5 + \sqrt{10}$

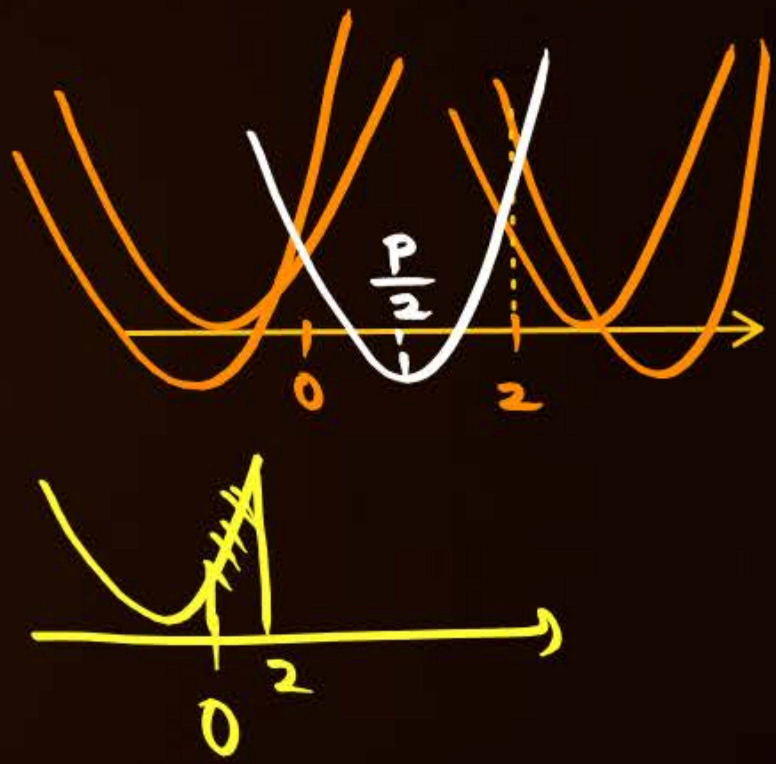
$$p^2-10p+15=0$$

$$p=\frac{10\pm\sqrt{40}}{2}$$

$$p=5\pm\sqrt{10}$$

$$p\geq 4 \rightarrow \cap$$

$$p=5+\sqrt{10}$$



Ans $p=5+\sqrt{10},\ 1-\sqrt{2}$

Range of Rational functions

$f(x) = \frac{(\text{Polynomial})_1}{(\text{Polynomial})_2}$

Type 1:

$$f(x) = \frac{\text{linear}_1}{\text{linear}_2} \text{ i.e } f(x) = \frac{ax+b}{cx+d}\left(\frac{a}{c} \neq \frac{b}{d}\right)$$

Ex: $y = \frac{3x+2}{4x+5}$

Range matlab y kaha se kaha tak hogaa

$4xy + 5y = 3x + 2$

$4xy - 3x = 2 - 5y$

$x(4y-3) = 2 - 5y$

$x = \frac{2-5y}{4y-3}$ $y \in R - \{3/4\}$

* $y = \frac{ax+b}{cx+d}$, $\frac{a}{c} \neq \frac{b}{d}$

$\downarrow$

Range: $R - \{a/c\}$.

$y = \frac{3x+5}{6x+10}$ $\frac{3}{6} = \frac{5}{10}$

$y = \frac{3x+5}{2(3x+5)} = 1/2$, $x \neq -5$.

$y = 1/2$ Range $= \{1/2\}$.

$f(x) = \dfrac{1}{ax+b}$ Range $(-\infty,\infty)-\{0\}$.

$(-\infty,\infty)$

$(-\infty,\infty) = (-\infty,0] \cup [0,\infty)$

$\dfrac{1}{\infty}, \dfrac{-1}{\infty} \to 0$

$(-\infty,0) \cup (0,\infty)$

Domain: $R - \left\{\dfrac{-b}{a}\right\}$.

Range $= (-\infty,\infty)-\{0\}$

QUESTION

Find range of :

(1) $f(x) = \frac{2x-3}{x-1}$ — $R - \{\frac{2}{1}\} = R - \{2\}$

(2) $f(x) = \frac{x+3}{2-5x}$ $\longrightarrow$ $R - \{-\frac{1}{5}\}$

(3) $f(x) = \frac{6}{4x+7} = 6 \cdot \frac{1}{4x+7}$

$(-\infty, \infty) - \{0\}$

$(-\infty, \infty) - \{0\}$

(4) $f(x) = \frac{7x+5}{3}$ — $(-\infty, \infty)$

$(-\infty, \infty)$

Type 2 :

$f(x) = \frac{(\text{quad})_1}{(\text{quad})_2}$ where numerator & denominator contain a common factor

$$f(x) = \frac{x^2 - 7x + 12}{x - 10x + 21} = \frac{(x-3)(x-4)}{(x-3)(x-7)}$$

Steps:

$$f(x) = \frac{x-4}{x-7}, \; x \neq 3$$

$$f(3) = \frac{3-4}{3-7} = \frac{-1}{-4}$$

(i) Factorize numerator & Denominator

(ii) Say $(ax - b)$ is a common factor, cancel $(ax - b)$ from numerator & denominator and write $x \neq b/a$.

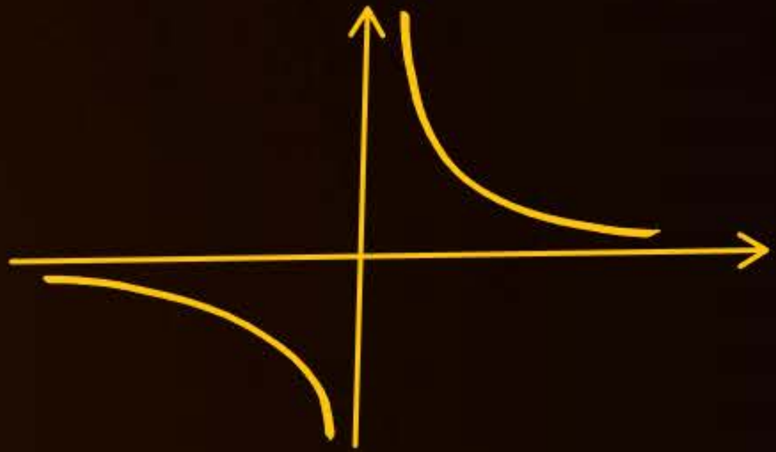
Range $= R - \{1, 1/4\}$

(iii) Now $f(x) = \frac{px+q}{rx+s}, x \neq b/a$ & range $= R - \left\{\frac{p}{r}, f\left(\frac{b}{a}\right)\right\}$

QUESTION

Find range of $f(x) = \dfrac{x^2 - 5x + 6}{x^2 - 6x + 8}$.

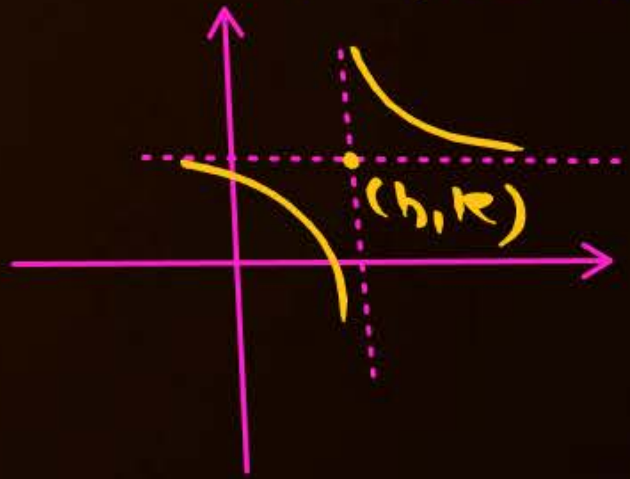
$xy = c^2$



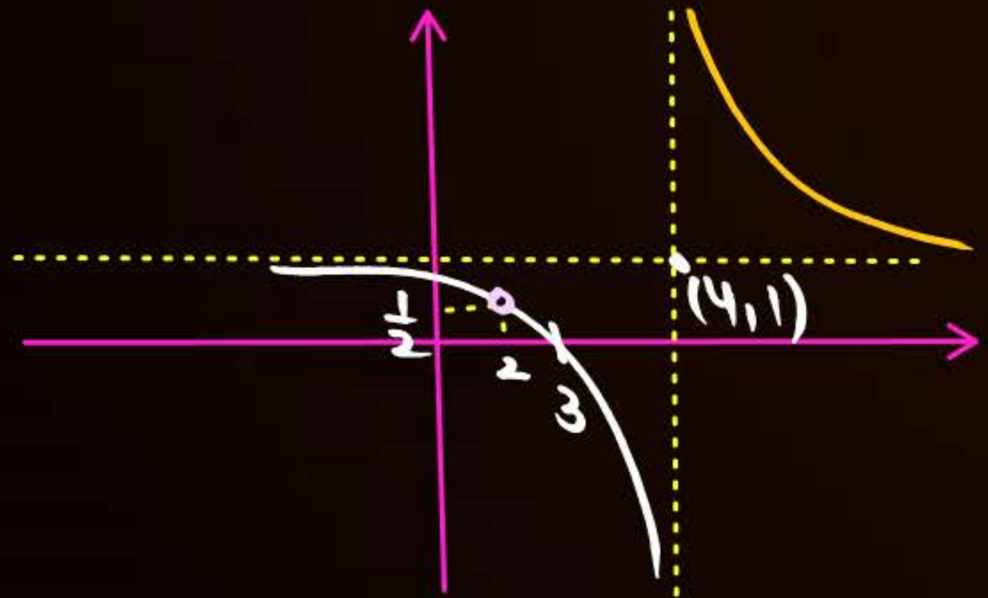
$$y = \frac{(x-2)(x-3)}{(x-2)(x-4)} \quad y(2) = \frac{2-3}{2-4} = \frac{-1}{-2} = \frac{1}{2}$$

$$y = \frac{1\cdot x - 3}{1\cdot x - 4},\ x \neq 2 \quad \text{Range } R - \left\{\frac{a}{c}, f(2)\right\}$$

$(x-h)(y-k) = c^2$



Range $R - \{1, 1/2\}$.



$$y = \frac{x-3}{x-4}$$

$$xy - 4y = x - 3$$

$$xy - x - 4y + 4 = -3 + 4$$

$$x(y-1) - 4(y-1) = 1$$

$$(x-4)(y-1) = 1$$

QUESTION

Find the range of $f(x) = \dfrac{x^2 - 5x + 4}{x^2 + 2x - 3}$.

Jan05

Also Draw its graph

Type 3 :

$f(x) = \frac{\text{quad}}{\text{quad}}, \frac{\text{linear}}{\text{quad}}, \frac{\text{quad}}{\text{linear}}$ where numerator & denominator contain no common factor.

Steps:

(i) Equate given expression to y

(ii) Cross multiply & make quad in X

(iii) Since $x \in R$, put $D \geq 0$ & set range of y.

(vi) Equate coefficient of $x^2 = 0$ to get $y = y_1$ put expression $= y_1$ & solve for x

If we get real x answer of step 3 is final

If we do not get real x exclude y_1 from answer.

QUESTION

Find domain & range of $f(x) = \dfrac{x^2+x+1}{x^2-x+1}$.

$x^2-x+1 \neq 0 \to x \in R$

$D<0,\ a>0$

$\downarrow$

always +ve

$$y = \frac{x^2+x+1}{x^2-x+1}$$

$$x^2y - xy + y = x^2+x+1$$

$$x^2(y-1) - x(y+1) + y-1 = 0$$

case ① $y-1 \neq 0$ i.e $y \neq 1$ since $x \in R$, $D \geq 0$

$$D = (y+1)^2 - 4(y-1)(y-1) \geq 0$$

$$\Rightarrow (y+1)^2 - (2(y-1))^2 \geq 0$$

$$(y+1+2y-2)(y+1-2y+2) \geq 0$$

$$(3y-1)(3-y) \geq 0$$

$$(3y-1)(y-3) \leq 0 \Rightarrow y \in [1/3, 3] - \{1\}$$

Ex: $\dfrac{x^2+x+1}{x^2-x+1} = 3 \in$ Range

$$x^2+x+1 = 3x^2-3x+3$$

$$2x^2-4x+2=0$$

$$x^2-2x+1=0$$

$$x=1$$

Ex: $\dfrac{x^2+x+1}{x^2-x+1} = 4 \notin$ Range

$$x^2+x+1 = 4x^2-4x+4$$

$$3x^2-5x+3=0$$

$D<0$ No real x

Case (ii) if $y=1$

$$-2x = 0$$

$$x = 0$$

$$\Downarrow$$

$y=1$ is also possible

$$y \in \left[\tfrac{1}{3}, 3\right] - \{1\} \cup \{1\}.$$

$$y \in \left[\tfrac{1}{3}, 3\right]$$

QUESTION

M① TaH 06

M②

Find domain & range of

$$f(x) = \frac{2x^2 + 2x + 3}{x^2 + x + 1}$$

Domain : R

$x^2+x+1 \neq 0$

$D<0$

$a>0$

always +ve

$x \in R$

$$y = \frac{2x^2+2x+3}{x^2+x+1} = \frac{2x^2+2x+2-2+3}{x^2+x+1}$$

$$= \frac{2(x^2+x+1)+1}{x^2+x+1} = 2 + \frac{1}{x^2+x+1}$$

$$y = 2 + \frac{1}{x^2+x+1}$$

$$= 2 + \frac{1}{x^2+x+1/4-1/4+1}$$

$$= 2 + \frac{1}{(x+1/2)^2+3/4}$$

$[0,\infty)$

$[3/4,\infty)$

$(0, \frac{4}{3}]$

$(2, \frac{10}{3}]$ = Range.

M③ $y = 2 + \frac{1}{x^2+x+1}$

$[-\frac{D}{4a},\infty) = [\frac{3}{4},\infty)$

$(0, \frac{4}{3}]$

$(2, \frac{10}{3}]$

QUESTION

Find domain & range of $f(x) = \frac{2x}{1+x^2}$.

$1+x^2 \neq 0 \rightarrow x \in R$ = Domain

$a>0,\ D<0$

always +ve

M① $y = \frac{2x}{1+x^2}$ Tanθ

M② $y = \frac{2x}{1+x^2} = \begin{cases} \frac{2}{x+\frac{1}{x}} & x>0 \\ 0 & x=0 \\ \frac{2}{x+1/x} & x<0 \end{cases}$

$x>0$ — $y = \frac{2}{x+\frac{1}{x}}$, $x+\frac{1}{x} \in [2,\infty)$, $2\left(0,\frac{1}{2}\right] = (0,1]$

$x=0 \longrightarrow y=0$

$x<0$ — $y = \frac{2}{x+\frac{1}{x}}$, $x+\frac{1}{x} \in (-\infty,-2]$, $2\left[-\frac{1}{2},0\right) = [-1,0)$

Range: $(0,1] \cup \{0\} \cup [-1,0)$

Range $= [-1,1]$

M③ $y = \frac{2x}{1+x^2}$ $x = \tan\theta$

$y = \frac{2\tan\theta}{1+\tan^2\theta}$

$y = \sin 2\theta$.

$y \in [-1,1]$

QUESTION

If x be real, then prove that $\frac{x}{x^2 - 5x + 9}$ must lie between $-\frac{1}{11}$ and 1.

Tan 08

QUESTION

Tahoq

If x is real, then maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is

Domain

$3x^2+9x+7 \neq 0$

$x \in R$

$y = \frac{3x^2+9x+7+10}{3x^2+9x+7}$

$y = 1 + \frac{10}{3x^2+9x+7}$

$\left[-\frac{D}{4a}, \infty\right)$

(A) 1

(B) 17/7

(C) 1/4

(D) 4

Ans. 4

Ex: $\dfrac{1}{x^2-5x+6}$ $\quad x \neq 2,3$

$\left[-\dfrac{D}{4a}, \infty\right) = \left[-\dfrac{1}{4}, \infty\right) = \left[-\dfrac{1}{4}, 0\right] \cup [0, \infty)$

Range $(-\infty, -4] \cup (0, \infty)$

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

(KTK 01)

The value of 'a' for which the equation $x^7 + ax^2 + 3 = 0$ and $x^8 + ax^3 + 3 = 0$ have a common root, can be

(A) 1

(B) -2

(C) -3

(D) -4

Ans. D

QUESTION

(KTK 02)

If α, β are roots of $Ax^2 + Bx + C = 0$ and α^2, β^2 are roots of $x^2 + px + q = 0$ then p is equal to

A $\frac{B^2 - 4AC}{A^2}$

B $\frac{2AC - B^2}{A^2}$

C $\frac{4AC - B^2}{A^2}$

D None of these

Ans. B

QUESTION

(KTK 03)

Find the values of 'k' so that the equation

$x^2 + kx + (k+2) = 0$ and $x^2 + (1-k)x + 3 - k = 0$ have exactly one common root.

Ans. No possible value of k

QUESTION

(KTK 04)

In a triangle PQR, $\angle R = \frac{\pi}{2}$, if $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then

A) $a = b + c$

B) $c = a + b$

C) $b = c$

D) $b = a + c$

Ans. B

QUESTION

(KTK 05)

The equations $ax^2 + bx + a = 0 (a, b \in R)$ and $x^3 - 2x^2 + 2x - 1 = 0$ have 2 roots common. Then $a + b$ must be equal to

(A) 1

(B) –1

(C) 0

(D) None of these

Ans. C

Homework From Module

Quadratic Equations

Prarambh (Topicwise) : Q1 to Q27

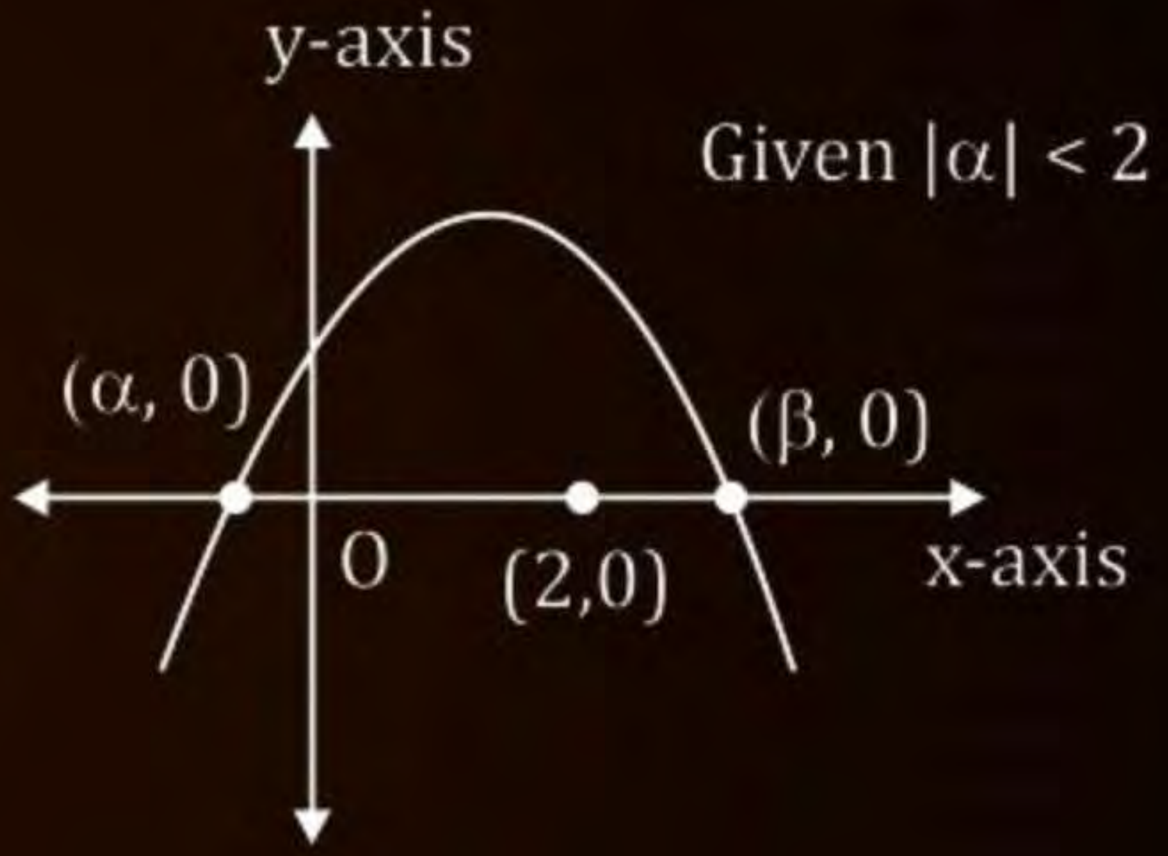
Prabal (JEE Main Level) : Q1,Q2,Q6 to Q9

Parikshit (JEE Advanced Level) : Abhi Ruko

Solution to Previous TAH

QUESTION

The graph of $y = ax^2 + bx + c$ is shown in the figure, then which of the following is(are) correct?



(A) $ab^2c^3 < 0$

(B) $ab < 0$

(C) $bc(4a + 2b + c) > 0$

(D) $ab(4a - 2b + c) > 0$

Lecture-06.

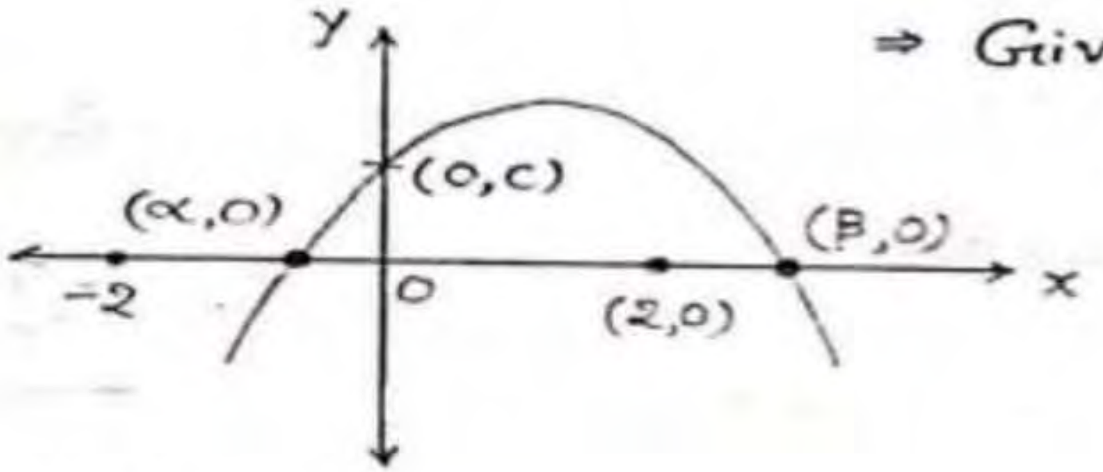
Tah-01. The graph of $y = ax^2 + bx + c$ is shown in the figure, then which of the following is (are) correct?

(A) $ab^2c^3 < 0$

(B) $ab < 0$

(C) $bc(4a+2b+c) > 0$

(D) $ab(4a-2b+c) > 0$.



$\Rightarrow$ Given $|\alpha| < 2$.

$-2 < \alpha < 2$.

from graph

$\alpha < 0$

$-2 < \alpha < 0$.

$a < 0$.

$c > 0$.

$b > 0$.

(A) $\underset{-ve}{a}\,\underset{+ve}{b^2}\,\underset{+ve}{c^3} < 0$. $\Rightarrow -ve$

(B) $\underset{-ve}{a}\,\underset{+ve}{b} < 0$. $\Rightarrow -ve$.

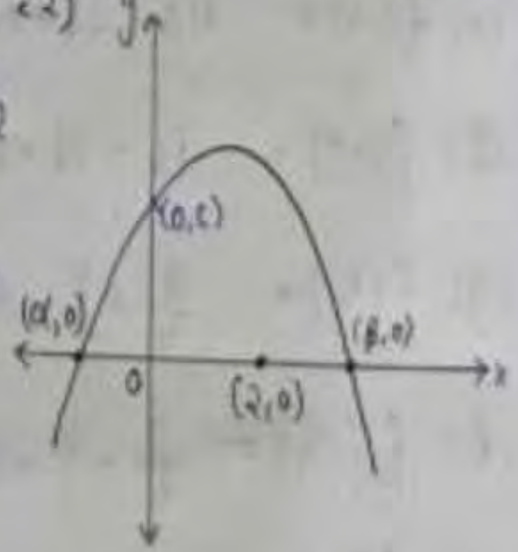
(C) $\underset{+ve}{b}\,\underset{+ve}{c}\,\underbrace{(4a+2b+c)}_{f(2)\,\to\,+ve} > 0$. $\Rightarrow +ve$.

(D) $\underset{-ve}{a}\,\underset{+ve}{b}\,\underbrace{(4a-2b+c)}_{f(-2)\,\to\,-ve} > 0$ $\Rightarrow +ve$

—x—

Qu. The graph of $y = ax^2+bx+c$ is shown in the figure. then which of the following is (are) correct? (given $|\alpha|<2$)

TAH1-A



✓ a) $ab^2c^3 < 0$

$-ve \cdot +ve \cdot +ve < 0$

$-ve < 0$

$a<0$

$c>0$

$\frac{-b}{2a} > 0$

$b>0$

$-2<\alpha<2$

from graph $\alpha<0$

$-2<\alpha<0$

✓ b) $ab<0$

$-ve \cdot +ve < 0$

$-ve<0$

✓ c) $bc(4a+2b+c)>0$

$f(2)$

$bc(f(2))>0$

$+ve \quad +ve \quad +ve$

✗ d) $ab(4a-2b+c)>0$

$ab(f(-2))>0$

$-ve \quad +ve \quad +ve$

Q-1 (TAH-1): The graph of $y=ax^2+bx+c$ is shown in the figure, then which of the following is (are) correct?

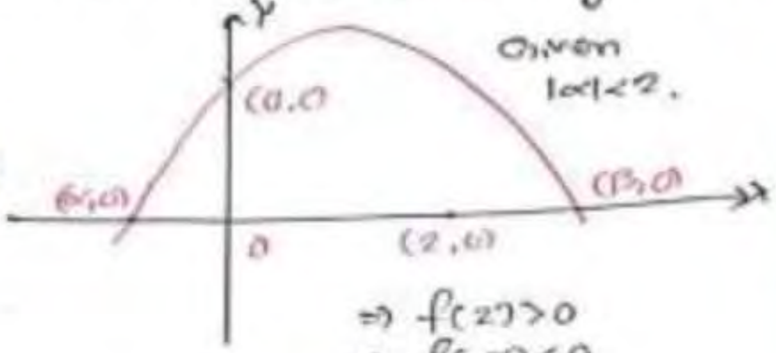
TAH 1
BY REED
FROM WB

(a) $ab^2c^3<0$

(b) $ab<0$

(c) $bc(4a+2b+c)>0$

(d) $ab(4a-2b+c)>0$

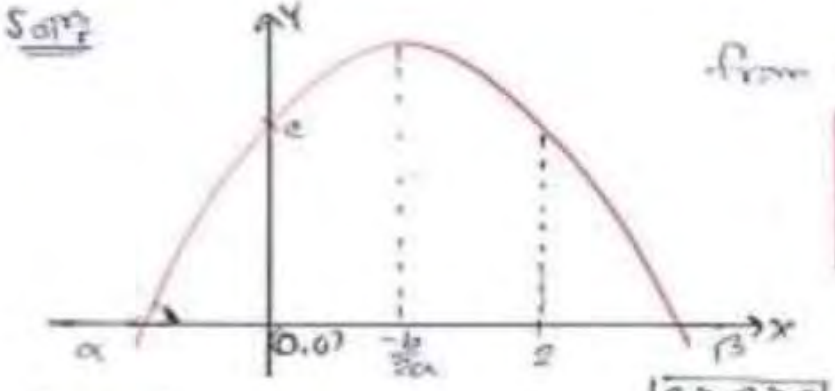


Given $|\alpha|<2$.

$\Rightarrow f(2)>0$

$\Rightarrow f(-2)<0$

Sol^n:



$\boxed{\alpha<0}$ $\boxed{\beta>2>0}$

from figure,

$\boxed{D>0,\ c>0,\ a<0,\ b>0}$

$\because \frac{-b}{2a}>0$

$\Rightarrow -b<0$

$\Rightarrow b>0$

given $|\alpha|<2$

$\Downarrow$

$-2<\alpha<2$

& $\alpha<0$

$\boxed{-2<\alpha<0}$

• **Option-A:** $\left.\begin{matrix} a\ b^2\ c^3 \\ -ve\ +ve\ +ve \end{matrix}\right\} \equiv -ve.$

$\therefore$ $ab^2c^3<0$, is correct.

• **Option-B:** $\left.\begin{matrix} ab \\ -ve\ \ +ve \end{matrix}\right\} \equiv -ve$

$\therefore$ $ab<0$, is correct.

• **Option-C:** $\left.\begin{matrix} b\ c\ \underbrace{(4a+2b+c)}_{f(2)\ \Downarrow\ +ve} \\ +ve\ +ve \end{matrix}\right\} \equiv +ve.$

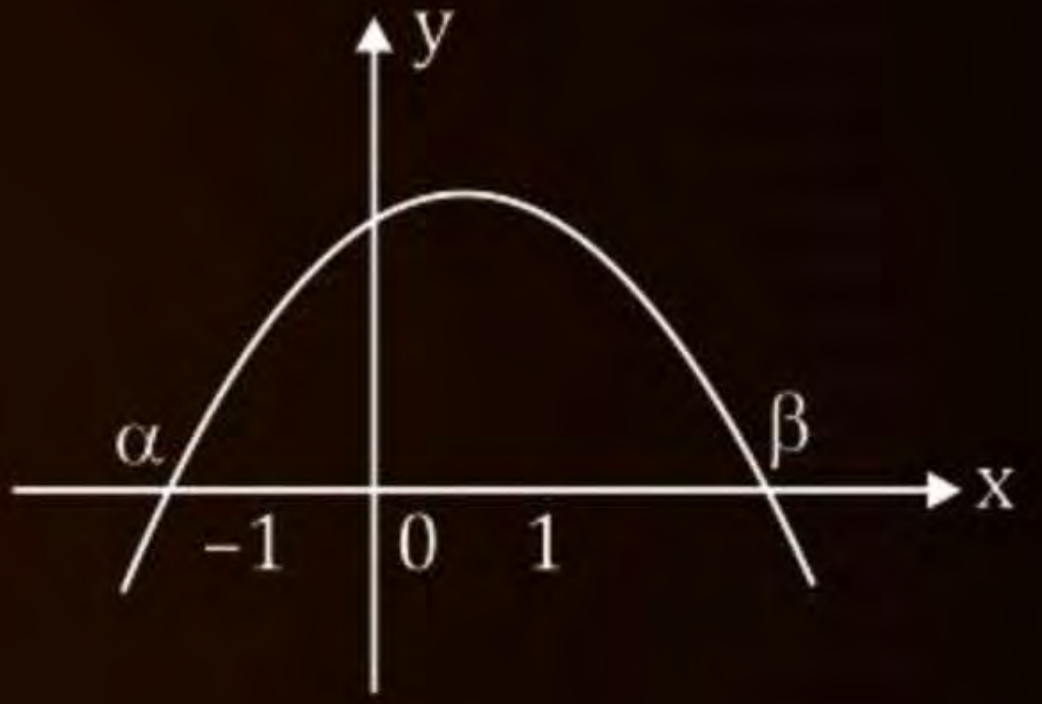
$\therefore$ $bc(4a+2b+c)>0$, is correct.

• **Option-D:** $\left.\begin{matrix} \underbrace{a\ b}_{(-ve \cdot +ve)\ \Downarrow\ -ve}\ \underbrace{(4a-2b+c)}_{f(-2)\ \Downarrow\ -ve} \end{matrix}\right\} \equiv +ve.$

$\therefore$ $ab(4a-2b+c)>0$, is also correct.

QUESTION

The graph of quadratic polynomial $f(x) = ax^2 + bx + c$ is shown in below. Which of the following are correct?



(A) $\frac{c}{a} < -1$

(B) $|\beta - \alpha| > 2$

(C) $f(x) > 0 \ \forall \ x \in (0, \beta)$

(D) $abc < 0$

Tah-02: The graph of Quadratic polynomial $f(x) = ax^2+bx+c$ is shown in below. which of the following are correct ?

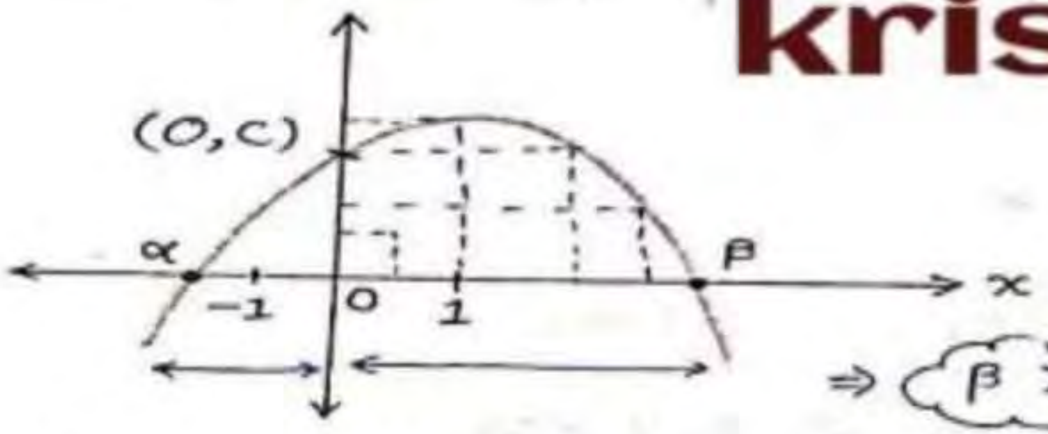
krish

(A) $\frac{c}{a} < -1$.

(B) $|\beta - \alpha| > 2$.

(C) $f(x) > 0 \ \forall \ x \in (0, \beta)$.

(D) $abc < 0$.



$\Rightarrow \beta > \alpha$

$a < 0$.
$b > 0$.
$c > 0$.

$\alpha < -1$
$\beta > 1$.

} from graph.

(A) $\frac{c}{a} < -1$ (c: +ve, a: −ve)

(A) $\frac{c}{a} = \alpha\beta \Rightarrow \frac{c}{a} < -1$ ✓

α: −ve ($\alpha < -1$), β: +ve ($\beta > 1$)

(B) $|\beta - \alpha| > 2$ (β: +ve, α: −ve)

Ex: $\Rightarrow \beta > 1, \alpha < -1$

$\Rightarrow |2-(-2)|$

$\Rightarrow 4 > 2$ ✓

(C) $f(x) > 0 \ \forall \ x \in (0, \beta)$

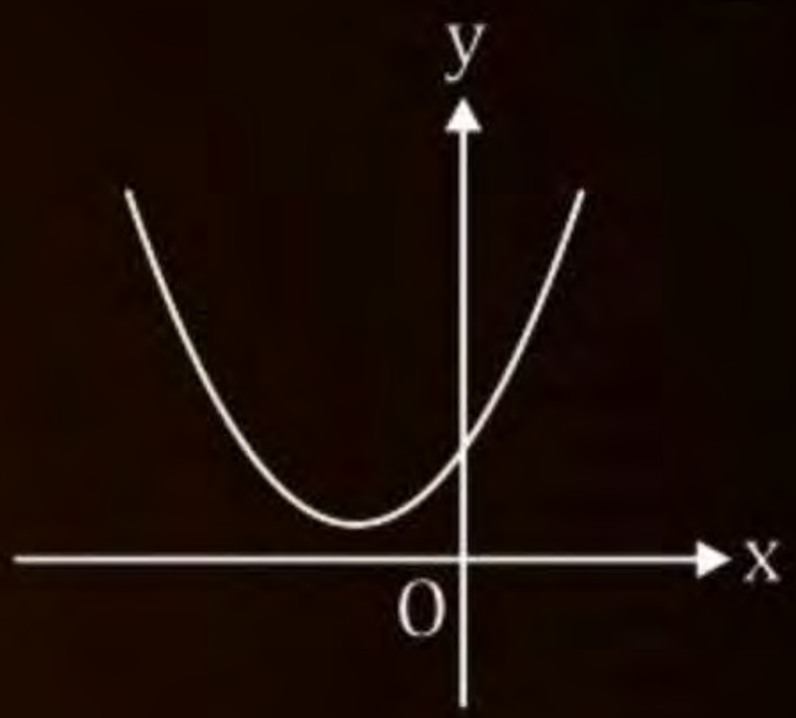
$\Rightarrow$ yes it is true..

(D) $abc < 0$.

a: −ve, b: +ve, c: +ve $\Rightarrow$ −ve

QUESTION

The curve of the quadratic expression $y = ax^2 + bx + c$ is shown in the figure and α, β be the roots of the equation $ax^2 + bx + c = 0$ then correct option is
[D is the discriminant]



(A) $a > 0,\ b > 0, c > 0, D > 0, \alpha + \beta > 0, \alpha\beta > 0$

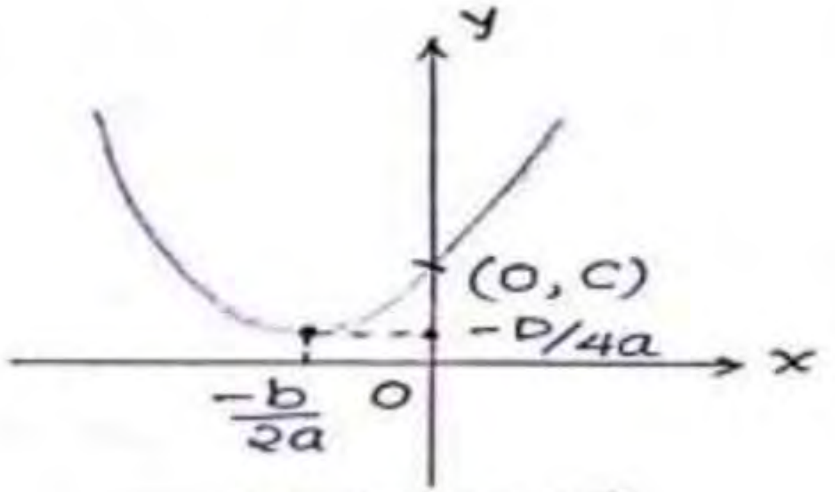
(B) $a > 0, b > 0, c > 0, D < 0, \alpha + \beta < 0, \alpha\beta < 0$

(C) $a > 0,\ b > 0, c > 0, D < 0, \alpha + \beta < 0, \alpha\beta > 0$

(D) $a > 0, b < 0, c > 0, D < 0, \alpha + \beta > 0, \alpha\beta > 0$

Ans. C

Tah-03. The curve of the Quadratic expression $y = ax^2 + bx + c$ is shown in figure and α, β be the roots of the equation $ax^2 + bx + c = 0$ then correct option is :
[D is the dicriminant].



(A) $a>0, b>0, c>0, D>0, \alpha+\beta>0, \alpha\beta>0$.

(B) $a>0, b>0, c>0, D<0, \alpha+\beta<0, \alpha\beta<0$.

(C) $a>0, b>0, c>0, D<0, \alpha+\beta<0, \alpha\beta>0$.

(D) $a>0, b<0, c>0, D<0, \alpha+\beta>0, \alpha\beta>0$.

$\Rightarrow -\underbrace{\frac{b}{2a}}_{+ve} < 0$

$\Rightarrow -b < 0$

$\Rightarrow \boxed{b>0}$.

$\Rightarrow a>0, c>0, b>0$.

$\Rightarrow \frac{-D}{\underbrace{4a}_{+ve}} > 0$

$\Rightarrow -D > 0$

$\Rightarrow \boxed{D<0}$

$\alpha+\beta = -\frac{\overset{-ve}{b}}{\underbrace{a}_{+ve}}$

$\Rightarrow \boxed{\alpha+\beta<0}$

$\alpha\beta = \frac{\overset{+ve}{c}}{\underbrace{a}_{+ve}}$

$\boxed{\alpha\beta>0}$

$\Rightarrow$ option (C)

QUESTION

Find the set of values of a for which $(a - 1)x^2 - (a + 1)x + a + 1 > 0$ for all $x \in R$.

Tah-04: Find the set of values of a for which $(a-1)x^2 - (a+1)x + a + 1 > 0$, for all $x \in R$.

$$\Rightarrow \quad (a-1)x^2 - (a+1)x + a + 1 > 0.$$

$$\Rightarrow \quad (a-1) > 0 \quad \underline{\text{and}} \quad D < 0.$$

$$\Downarrow$$

$$\Rightarrow \quad a > 1 \quad \underline{\text{and}} \quad (-(a+1))^2 - 4(a-1)(a+1) < 0$$

$$\Rightarrow \quad a > 1 \quad \underline{\text{and}} \quad \Rightarrow a^2 + 2a + 1 - 4(a^2 - 1) < 0.$$

$$\Rightarrow a^2 + 2a + 1 - 4a^2 + 4 < 0.$$

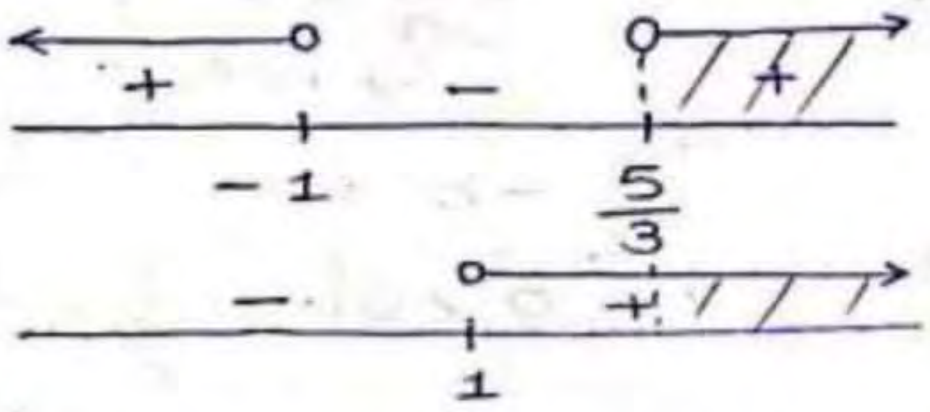
$$\Rightarrow -3a^2 + 2a + 5 < 0$$

$$\Rightarrow 3a^2 - 2a - 5 > 0$$

$$\Rightarrow 3a^2 - 5a + 3a - 5 > 0$$

$$\Rightarrow (a+1)(3a-5) > 0$$

$\Rightarrow$



$$\Rightarrow \quad \boxed{a \in \left(\frac{5}{3}, \infty\right)} \quad \underline{\text{Ans.}}$$

Q-4(TAH-4): Find the set of values of a for which $(a-1)x^2-(a+1)x+a+1>0$ for all ~~x~~ $x\in R$.

Sol^n: $(a-1)x^2-(a+1)x+(a+1)>0 \ \forall x\in R$.

$\Downarrow$

$D<0$ & $(a-1)>0$

$(a+1)^2-4(a-1)(a+1)<0$

$\Rightarrow (a+1)[a+1-4a+4]<0$

$\Rightarrow (a+1)(3a-5)>0$

$\therefore a\in(-\infty,-1)\cup\left(\frac{5}{3},\infty\right)$

$a>1$

$\Downarrow$

$a\in(1,\infty)$

'∩'

$\boxed{a\in \frac{5}{3},\infty}$ (Ans.)

TAH 4
BY REED

QUESTION

Find the set of values of a for which $(a + 4)x^2 - 2ax + 2a - 6 < 0$ for all $x \in R$.

Tah-05: Find the set of value of a for which $(a+4)x^2 - 2ax + 2a - 6 < 0$ for all $x \in R$.

$$\Rightarrow (a+4)x^2 - 2ax + 2a - 6 < 0.$$

$$\Rightarrow (a+4) < 0 \quad \underline{\text{and}} \quad D < 0.$$

$$\Downarrow$$

$$\Rightarrow a < -4 \quad \underline{\text{and}} \quad \Rightarrow (-2a)^2 - 4(a+4)(2a-6) < 0$$

$$\Rightarrow a < -4 \quad \underline{\text{and}} \quad \Rightarrow 4a^2 - 4(2a^2 - 6a + 8a - 24) < 0$$

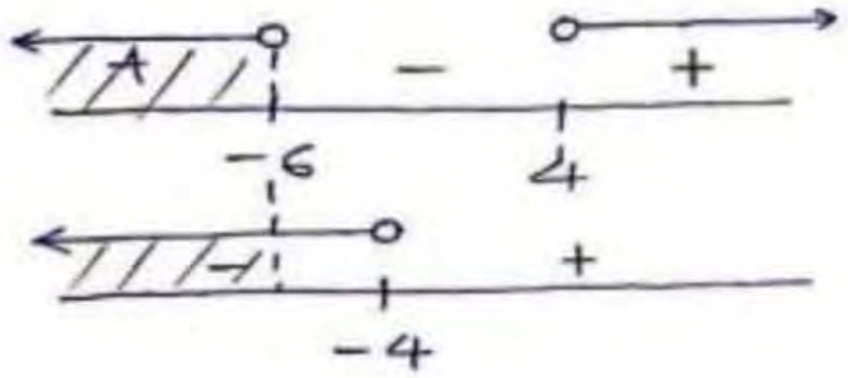
$$\Rightarrow 4a^2 - 8a^2 + 24a - 32a + 96 < 0$$

$$\Rightarrow -4a^2 - 8a + 96 < 0.$$

$$\Rightarrow 4a^2 + 8a - 96 > 0.$$

$$\Rightarrow a^2 + 2a - 24 > 0.$$

$$\Rightarrow (a-4)(a+6) > 0.$$

$\Rightarrow$ 

$\Rightarrow \boxed{a \in (-\infty, -6)}$ Ans.

Qu. Find the set of a for which $(a+4)x^2 - 2ax + 2a - 6 < 0 \ \forall x \in R$

$D < 0$ $\qquad\qquad$ $a+4 < 0 \ \cap \ D < 0$

$\boxed{a < -4}$

$4a^2 - 4(a+4)(2a-6) < 0$

$4a^2 - (4a+16)(2a-6) < 0$

$4a^2 - (8a^2 - 24a + 32a - 96) < 0$

$4a^2 - 8a^2 - 8a + 96 < 0$

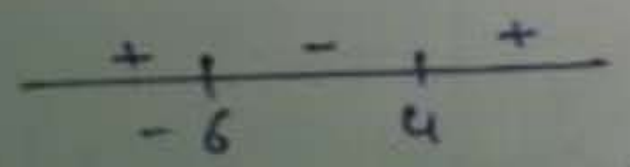
$4a^2 + 8a - 96 > 0$

$a^2 + 2a - 24 > 0$

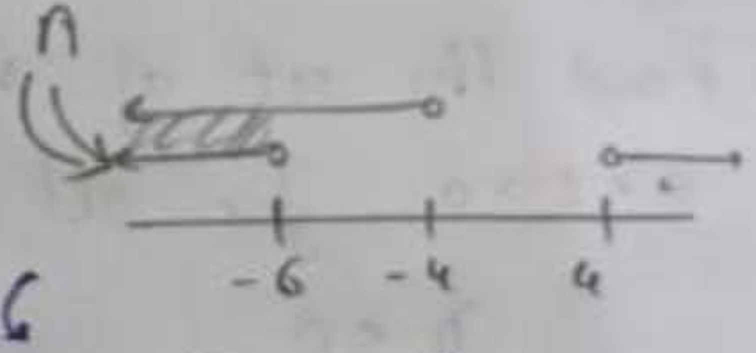
$a^2 + 6a - 4a - 24 > 0$

$a(a+6) - 4(a+6) > 0$

$(a-4)(a+6) > 0$



$a \in (-\infty, -6) \cup (4, \infty)$



$a \in (-\infty, -6)$

QUESTION

For what values of p the vertex of $x^2 + px + 13$ lies at a distance 5 unit from origin.

TDh-06. For what values of P the vertex of $x^2+Px+13$ lies at a distance 5 unit from origin:

$$\Rightarrow \quad x^2 + Px + 13 = 0. \qquad \Rightarrow \text{vertex} = \left(-\frac{b}{2a}, \frac{-D}{4a}\right).$$

$$\Rightarrow \text{vertex at} \Rightarrow \left(-\frac{P}{2}, -\frac{P^2-52}{4}\right).$$

$$\Rightarrow \quad 5 = \sqrt{\left(0+\frac{P}{2}\right)^2 + \left(0+\frac{P^2-52}{4}\right)^2}.$$

$$\Rightarrow \quad 5 = \sqrt{\frac{P^2}{4} + \frac{(P^2-52)^2}{16}}.$$

$$\Rightarrow 25 = \frac{P^2}{4} + \frac{P^4 - 104P^2 + 2704}{16}$$

$$\Rightarrow \quad 25 = \frac{4P^2 + P^4 - 104P^2 + 2704}{16}.$$

$$\Rightarrow \quad 4P^2 + P^4 - 104P^2 + 2704 - 400 = 0.$$

$$\Rightarrow P^4 - 100P^2 + 2304 = 0.$$

$$\Rightarrow \quad P^4 - 64P^2 - 36P^2 + 2304 = 0.$$

$$\Rightarrow \quad P^2(P^2-64) - 36(P^2-64) = 0.$$

$$\Rightarrow \quad (P^2-64)\ (P^2-36).$$

$$\Rightarrow \quad P^2 = 64, \quad P^2 = 36$$

$$\Rightarrow \quad P = \pm 8, \quad P = \pm 6 \qquad \text{Ans.}$$

—x—

A) Tah-06

vertex $= \left(-\frac{b}{2a}, \frac{-D}{4a}\right) = \left(\frac{-p}{2}, \frac{-(p^2-52)}{4}\right)$

origin $= (0,0)$

According to distance formula:

$D = \sqrt{\left(0+\frac{p}{2}\right)^2 + \left(0 + \frac{p^2-52}{4}\right)^2}$

$\Rightarrow 5 = \sqrt{\frac{p^2}{4} + \frac{(p^2-52)^2}{16}} = \sqrt{\frac{4p^2+(p^2-52)^2}{16}}$

$\Rightarrow 16\times 25 = 4p^2 + p^4 + (52)^2 - 104p^2$

$\Rightarrow p^4 - 100p^2 + (52)^2 - 25\times16 = 0$

$\Rightarrow p^4 - 100p^2 + 4^2\times13^2 - 4^2\times25 = 0$

$\Rightarrow p^4 - 100p^2 + 4^2(169-25) = 0$

$\Rightarrow p^4 - 100p^2 + 4^2\times144 = 0$

$\Rightarrow p^4 - 100p^2 + 2304 = 0$

$\Rightarrow p^4 - 64p^2 - 36p^2 + 2304 = 0$

$\Rightarrow (p^2-64)(p^2-36) = 0$

$\Rightarrow p^2 = 64, 36 \Rightarrow p = \pm\sqrt{64}, \pm\sqrt{36}$

$\Rightarrow p = +8, -8, +6, -6$

Ans: values of p : +8, −8, +6, −6

Q-6 (TAH-6): For what values of p the vertex of $x^2+px+13$ lies at a distance 5 unit from origin?

Soln: $5 = \sqrt{\left(-\frac{p}{2}-0\right)^2 + \left(\frac{p^2-52}{4}-0\right)^2}$

$\Rightarrow 25 = \frac{p^2}{4} + \frac{(p^2-52)^2}{16}$

$\Rightarrow 25 = \frac{4p^2+p^4-104p^2+52^2}{16}$

$\Rightarrow 25\times16 = p^4 - 100p^2 + 2704$

$\Rightarrow p^4 - 100p^2 + 2704 - 400 = 0$

$\Rightarrow p^4 - 100p^2 + 2304 = 0$

$\Rightarrow (p^2-64)(p^2-36) = 0$

$\Rightarrow p^2 = 64$ OR, $p^2 = 36$

$\Rightarrow \boxed{p = \pm 8}$ OR, $\boxed{p = \pm 6}$. $\therefore p = -8, -6, 6, 8$. (Ans)

$f(x) = x^2+px+13$

$D = p^2 - 52$.

vertex $= \left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

$\Rightarrow V = \left(\frac{-p}{2}, \frac{-(p^2-52)}{4}\right)$

distance from $(0,0)$ is 5 units.

$$\begin{array}{r|l} 4 & 2304 \\ 4 & 576 \\ 4 & 144 \\ 4 & 36 \\ & 9 \end{array}$$

64, 36

TAH 6

BY REED

THANK YOU