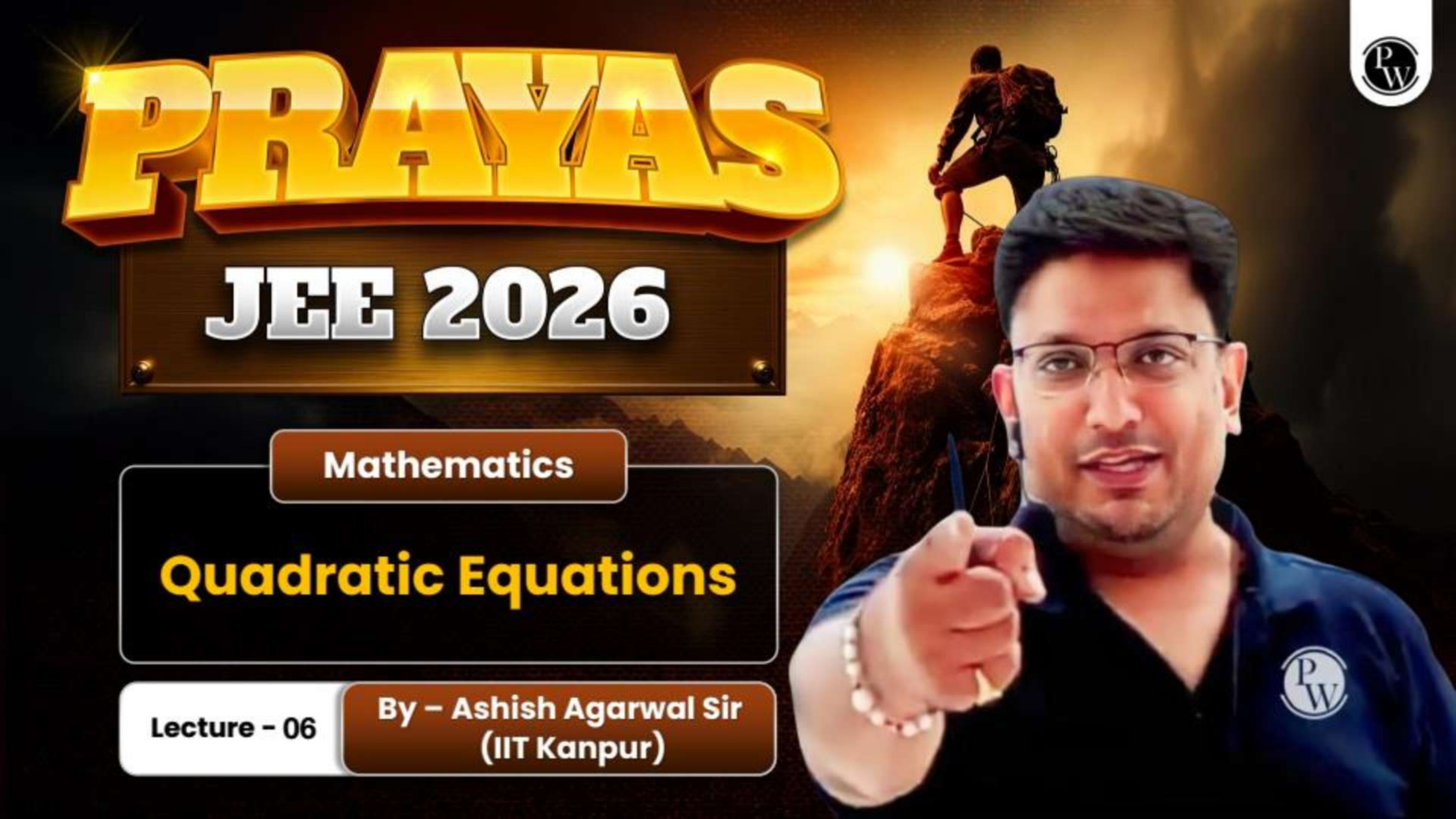

PRAYAS
JEE 2026
Mathematics
Quadratic Equations
Lecture - 06
By – Ashish Agarwal Sir
(IIT Kanpur)

Topics *to be covered*

- **A** Graph of a Quadratic polynomial
- **B** Range of Rational Functions
- **C** Practice problems

Homework Discussion

QUESTION

If two roots of the equation $(x-1)(2x^2-3x+4)=0$ coincide with roots of the equation $x^3+(a+1)x^2+(a+b)x+b=0$ where $a, b \in R$ then $2(a+b)$ equals

(A) 4

(B) 2

(C) 1

(D) 0

$(x-1)(\underbrace{2x^2-3x+4}_{\alpha,\ \beta})=0 < 1, \alpha, \beta$

$x^3+x^2+ax^2+ax+bx+b=0$

$x^2(x+1)+ax(x+1)+b(x+1)=0$

$(\underbrace{x^2+ax+b}_{\alpha,\ \beta})(x+1)=0 < -1, \alpha, \beta$

$\Downarrow$

$2x^2-3x+4=0 < \alpha, \beta$

$x^2+ax+b=0 < \alpha, \beta$

$\frac{1}{2}=-\frac{a}{3}=\frac{b}{4}$

$a=-3/2,\ b=2$

$2(a+b)=-3+4=1$

QUESTION

(KTK 6)

Let 'p' is a root of the equation $x^2-x-3=0$. Then the value of $\frac{p^3+1}{p^5-p^4-p^3+p^2}$ is equal to

(A) $\frac{4}{3}$

(B) $\frac{4}{9}$

(C) $\frac{2}{9}$

(D) $\frac{2}{3}$

$p^2-p-3=0$

$p(p-1)=3.$

$$\frac{(p+1)(p^2-p+1)}{(p^4-p^2)(p-1)}$$

$$\frac{\cancel{(p+1)}(p^2-p+1)}{p\cancel{(p^2-1)}\,p(p-1)}=\frac{p^2-p+1}{p(p-1)\cdot p(p-1)}$$

$$=\frac{p(p-1)+1}{3\cdot 3.}=\frac{3+1}{9}=\frac{4}{9}$$

QUESTION

(KTK 8)

If α, β, γ are roots $x^3 + 2x^2 - 3x + 1 = 0$, then value of $\frac{\alpha\beta}{\alpha+\beta} + \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\beta\gamma}{\beta+\gamma}$ is less than

(A) 2

(B) 3

(C) 4

(D) 5

M(1)

$x^3 + 2x^2 - 3x + 1 = 0 < \alpha, \beta, \gamma$

$\alpha\beta\gamma = -1$

$\alpha + \beta + \gamma = -2$

let $y = \frac{1}{x(x+2)}$

$x(x+2) = \frac{1}{y}$

$x^2 + 2x = 1/y$

$x^2 + 2x + 1 = 1/y + 1$

$(x+1)^2 = 1/y + 1$

$x \cdot x(x+2) - 3x + 1 = 0$

$\frac{x}{y} - 3x + 1 = 0$

$x(1/y - 3) + 1 = 0$

$x(1-3y) + y = 0$

$x(3y-1) = y$

$x = \frac{y}{3y-1} \Rightarrow x + 1 = \frac{4y-1}{3y-1}$ SBS $(x+1)^2 = \left(\frac{4y-1}{3y-1}\right)^2$

$E = \frac{\alpha\beta}{\alpha+\beta} + \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\beta\gamma}{\beta+\gamma}$

$E = \frac{-\frac{1}{\gamma}}{-2-\gamma} + \frac{-\frac{1}{\beta}}{-2-\beta} + \frac{-\frac{1}{\alpha}}{-2-\alpha}$

$E = \underbrace{\frac{1}{\alpha(\alpha+2)}}_{f(\alpha)} + \underbrace{\frac{1}{\beta(\beta+2)}}_{f(\beta)} + \underbrace{\frac{1}{\gamma(\gamma+2)}}_{f(\gamma)}$

$f(x) = \frac{1}{x(x+2)}$

$$\frac{1}{y}+1=\frac{16y^2-8y+1}{9y^2-6y+1}$$

$$(y+1)(9y^2-6y+1)=y(16y^2-8y+1)$$

$$9y^3-6y^2+y+9y^2-6y+1=16y^3-8y^2+y$$

$$7y^3-11y^2+6y-1=0$$

$$\frac{\alpha\beta}{\alpha+\beta}$$

$$\frac{\beta\gamma}{\beta+\gamma}$$

$$\frac{\alpha\gamma}{\alpha+\gamma}$$

$$S_1=E=\frac{11}{7}=1.\ldots$$

QUESTION

(KTK 8)

If α, β, γ are roots $x^3 + 2x^2 - 3x + 1 = 0$, then value of $\frac{\alpha\beta}{\alpha+\beta} + \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\beta\gamma}{\beta+\gamma}$ is less than

(A) 2

(B) 3

(C) 4

(D) 5

M ②

$x^3 + 2x^2 - 3x + 1 = 0 < \alpha, \beta, \gamma$

$\alpha\beta\gamma = -1$

$\alpha + \beta + \gamma = -2$

$E = \frac{1}{2}\left(\frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} - \left(\frac{1}{\alpha+2} + \frac{1}{\beta+2} + \frac{1}{\gamma+2}\right)\right)$

$y = \frac{1}{x+2} \Rightarrow x + 2 = \frac{1}{y}$

$x = 1/y - 2.$

$E = \frac{\alpha\beta}{\alpha+\beta} + \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\beta\gamma}{\beta+\gamma}$

$E = \frac{-\frac{1}{\gamma}}{-2-\gamma} + \frac{-\frac{1}{\beta}}{-2-\beta} + \frac{-\frac{1}{\alpha}}{-2-\alpha}$

$E = \frac{1}{\alpha(\alpha+2)} + \frac{1}{\beta(\beta+2)} + \frac{1}{\gamma(\gamma+2)}$

$E = \frac{\alpha+2-\alpha}{2\alpha(\alpha+2)} + \frac{\beta+2-\beta}{2\beta(\beta+2)} + \frac{\gamma+2-\gamma}{2\gamma(\gamma+2)}$

$E = \frac{1}{2}\left(\frac{1}{\alpha} - \frac{1}{\alpha+2} + \frac{1}{\beta} - \frac{1}{\beta+2} + \frac{1}{\gamma} - \frac{1}{\gamma+2}\right)$

$E = 1/2\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} - \left(\frac{1}{\alpha+2} + \frac{1}{\beta+2} + \frac{1}{\gamma+2}\right)\right)$

Aao Machaay Dhamaal

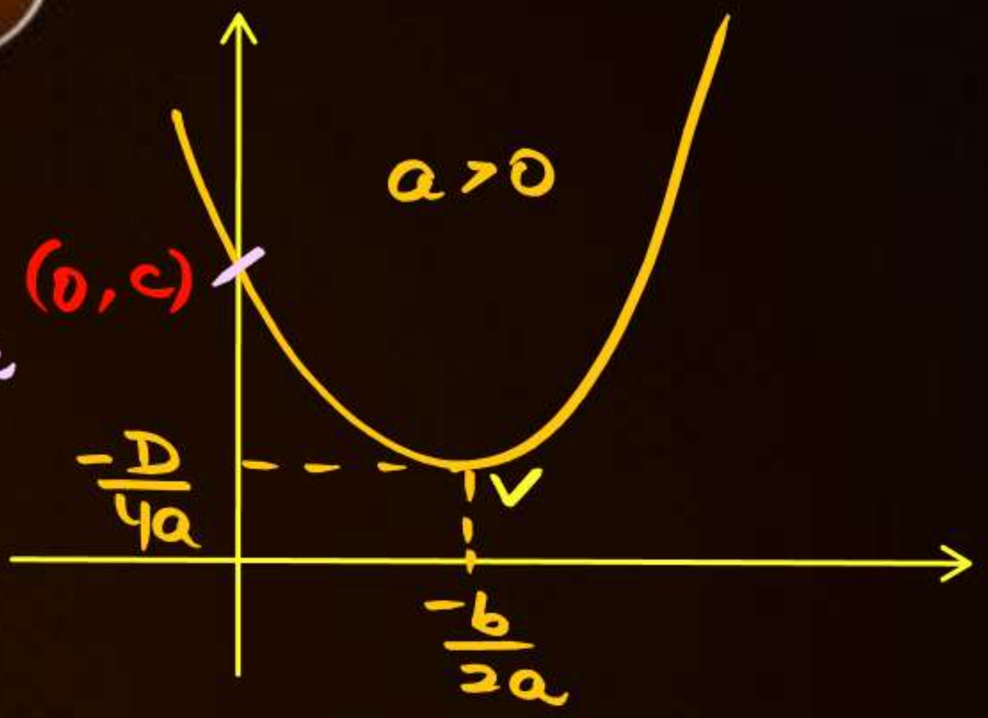
Deh Swaal pe Deh Swaal

Analysis of a Quadratic Polynomial

$y = ax^2 + bx + c \quad , (a>0)$

$\downarrow$

* upward opening parabola
* symmt about $x = -\frac{b}{2a}$
* $y_{min} = -\frac{D}{4a}$ at $x = -\frac{b}{2a}$
* y_{max} D.N.E but $\longrightarrow \infty$
* Parabola Intersects y axis at $(0,c)$
* vertex $V\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$

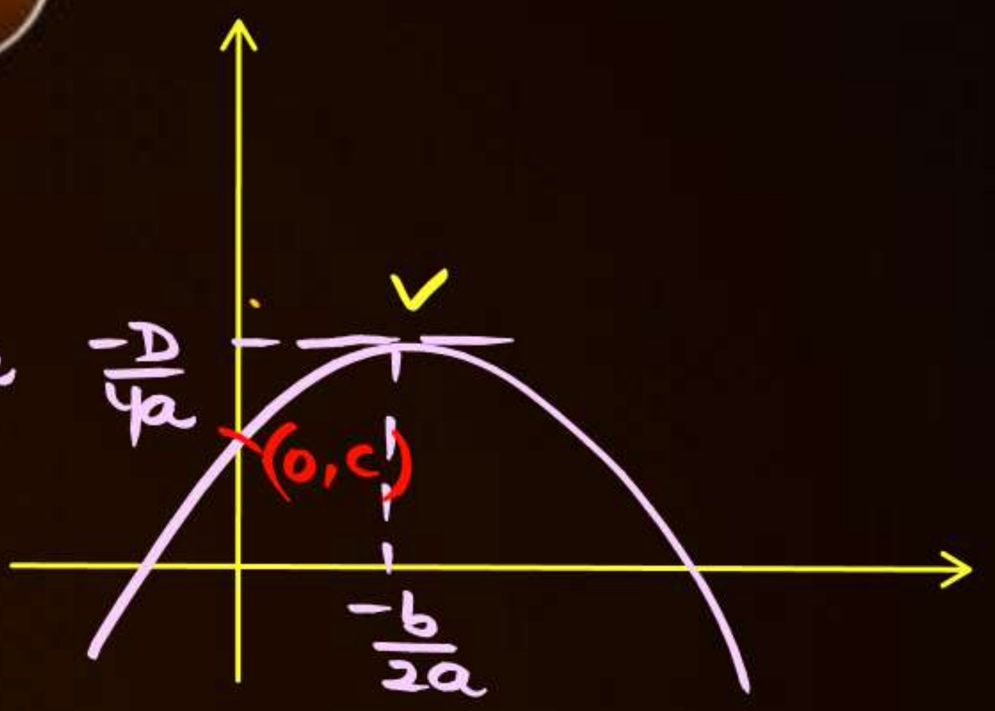


Analysis of a Quadratic Polynomial

$y = ax^2 + bx + c \ , (a<0)$

↓

* Downward opening parabola
* symmt about $x = -\frac{b}{2a}$
* $y_{max} = -\frac{D}{4a}$ at $x = -\frac{b}{2a}$
* y_{min} D.N.E but $\longrightarrow -\infty$
* Parabola Intersects y axis at $(0,c)$
* vertex $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$



Ex: $y = x^2 - 3x + 4$

* Parabola Type ~ upward opening

* Symmt about $x = \frac{-(-3)}{2\cdot 1} = 3/2$

* $y_{min} = -\frac{D}{4a} = \frac{-((-3)^2 - 4\cdot1\cdot4)}{4\cdot 1} = \frac{7}{4}$

M② $y = (3/2)^2 - 3\cdot\frac{3}{2} + 4$

$= \frac{9}{4} - \frac{9}{2} + 4$

$= -9/4 + 4 = \underline{7/4}$

* Intersects y axis at $(0,4)$

Ex: $y = -x^2 + x - 5$.

* Parabola Type: Downward. opening Parabola.

* Symmt about: $x = \frac{-(1)}{2\cdot(-1)} = \frac{1}{2}$

* $y_{max} = -\frac{D}{4a} = \frac{-(1^2 - 4\cdot(-1)(-5))}{4\cdot(-1)} = -\frac{19}{4}$

* Intersects y axis at $(0,-5)$

$y = f(x)$ Intersects x axis at points where $y = 0$

$f(x) = 0 \longrightarrow$ roots $= \alpha, \beta, \gamma$

Real Roots of $f(x)=0$ are nothing but x-coord of POI of curve $y = f(x)$ & X Axis.

$ax^2+bx+c=0$ ke roots are nothing but x-coord of POI of parabola $y = ax^2+bx+c$ with X Axis.

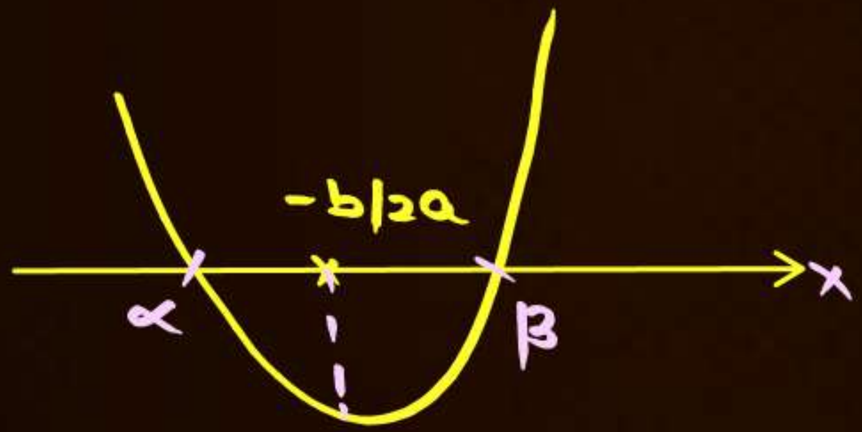
Graph of a Quadratic Polynomial vs D

$y = ax^2 + bx + c$

$ax^2 + bx + c = 0$

Case ① If $D > 0$

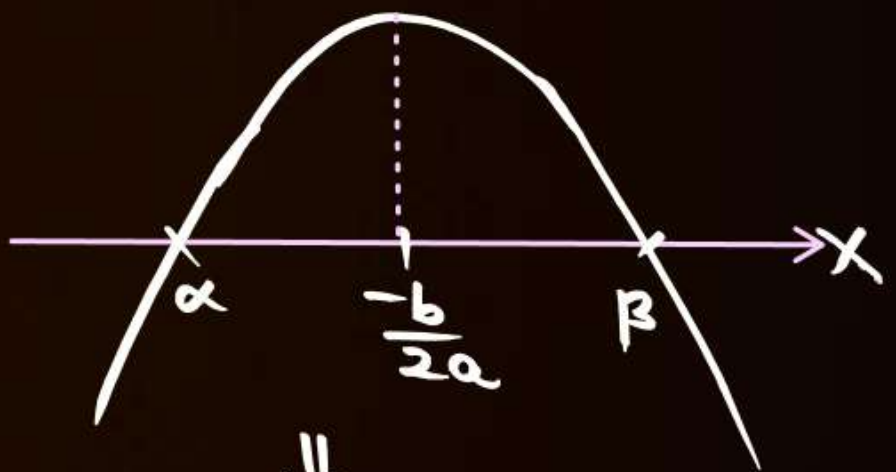
(i) If $a > 0$ $\quad y_{min} = \dfrac{-D}{4a} = -ve.$ (D: +ve, $4a$: +ve)



⇓

2 real & distinct Roots

(ii) If $a < 0$ $\quad y_{max} = -\dfrac{D}{4a} = +ve$ (D: +ve, $4a$: -ve)



⇓

2 real & distinct roots.

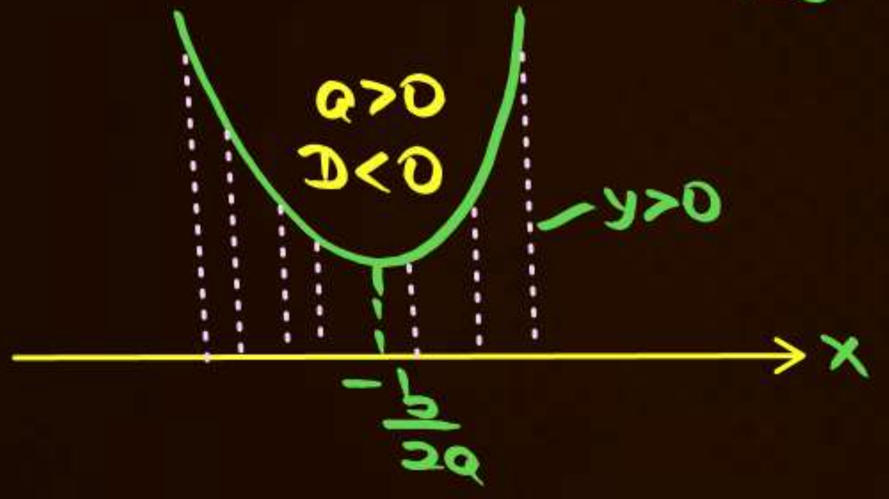
If $D > 0 \Rightarrow$ 2 real & distinct roots.

$y = ax^2 + bx + c$

$ax^2 + bx + c = 0$

Case (II) If $D < 0$

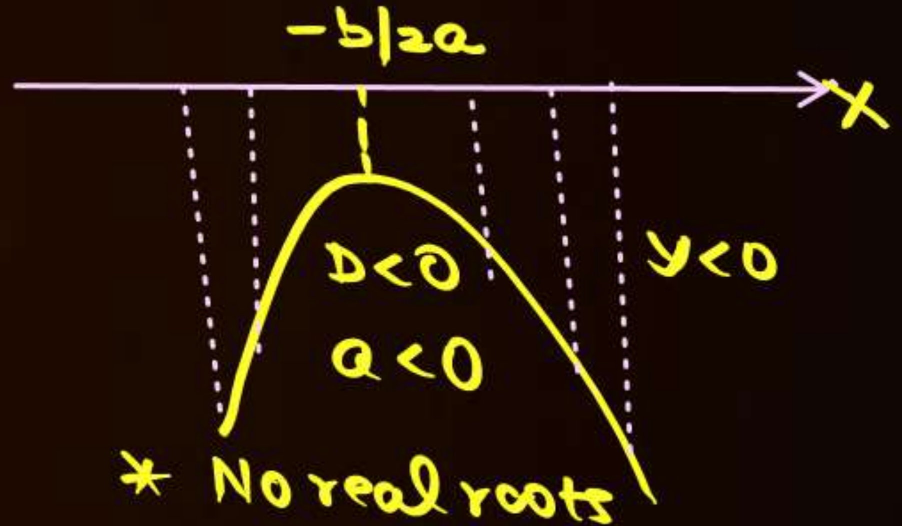
(i) If $a > 0$ $\quad y_{min} = -\dfrac{\overset{-ve}{D}}{\underset{+ve}{4a}} = +ve$



⇓

* No real roots.
* $y = ax^2 + bx + c > 0 \ \forall \ x \in R$

(ii) If $a < 0$ $\quad y_{max} = -\dfrac{\overset{-ve}{D}}{\underset{-ve}{4a}} = -ve$



* No real roots
* $y = ax^2 + bx + c < 0 \ \forall \ x \in R$

NICHOD

$a>0$ & $D<0 \Rightarrow y=ax^2+bx+c>0 \ \forall x \in R$

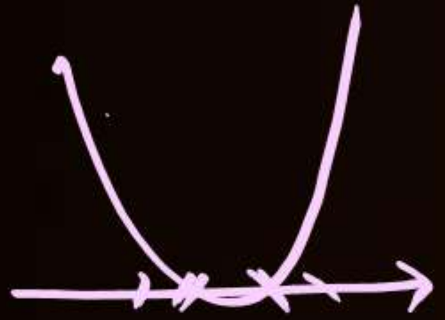
$a<0$ & $D<0 \Rightarrow y=ax^2+bx+c<0 \ \forall x \in R$

Graph of a Quadratic Polynomial vs D

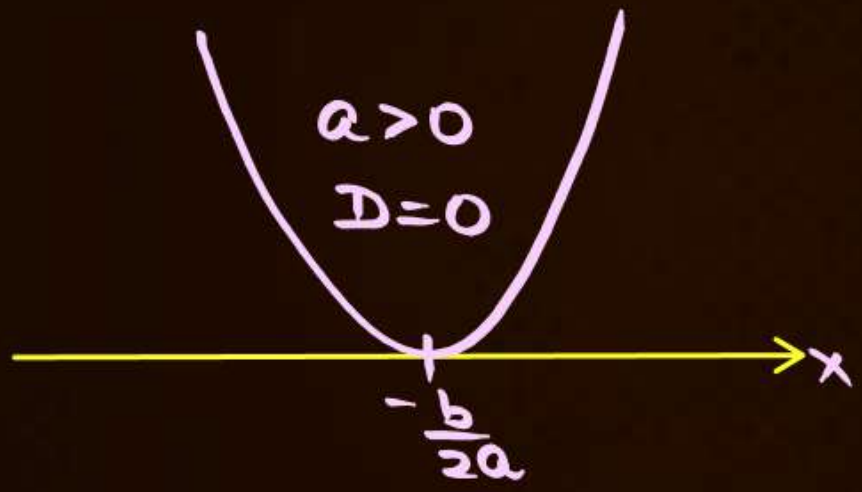
$y = ax^2 + bx + c$

$ax^2 + bx + c = 0$

Case (III) If $D = 0$



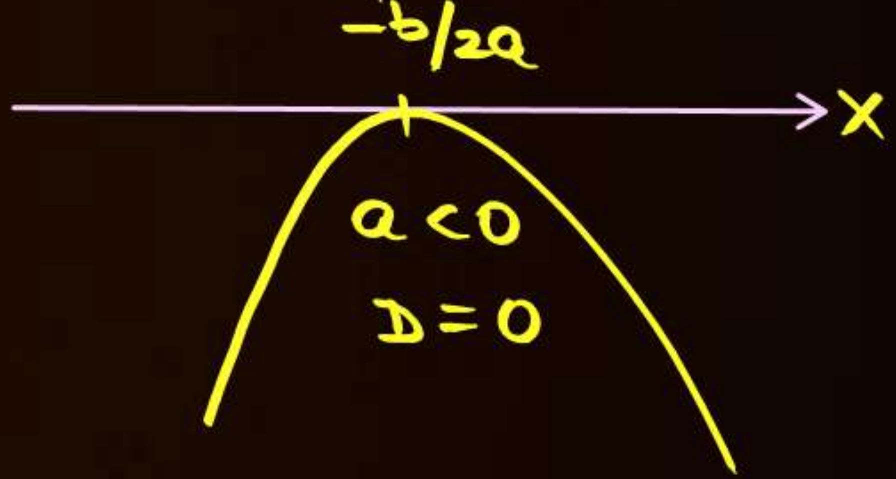
(i) If $a > 0$ $\quad y_{min} = \dfrac{-D}{4a} = 0$



* Two real & Equal roots

(ii) If $a < 0$

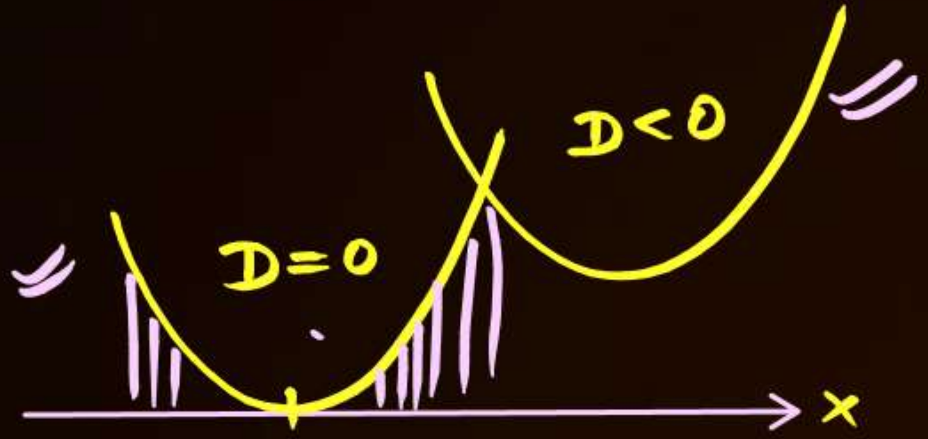
$y_{max} = \dfrac{-D}{4a} = 0$



* Two real & equal roots.

$D = 0 \Rightarrow$ 2 real & Equal roots.

P(1)



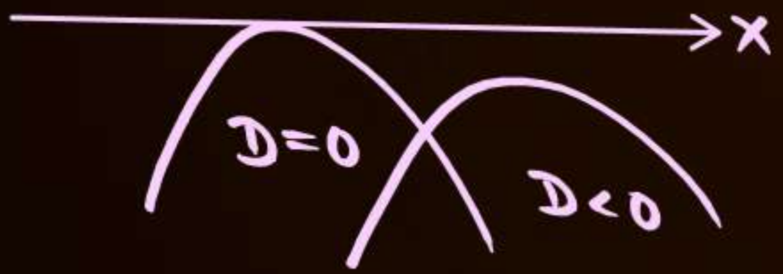
$y = ax^2+bx+c \geq 0 \ \forall x \in R$ if $a>0$ & $D \leq 0$

$a \leq b$

$\downarrow$

$a<b$ या फिर $a=b$

P(2)



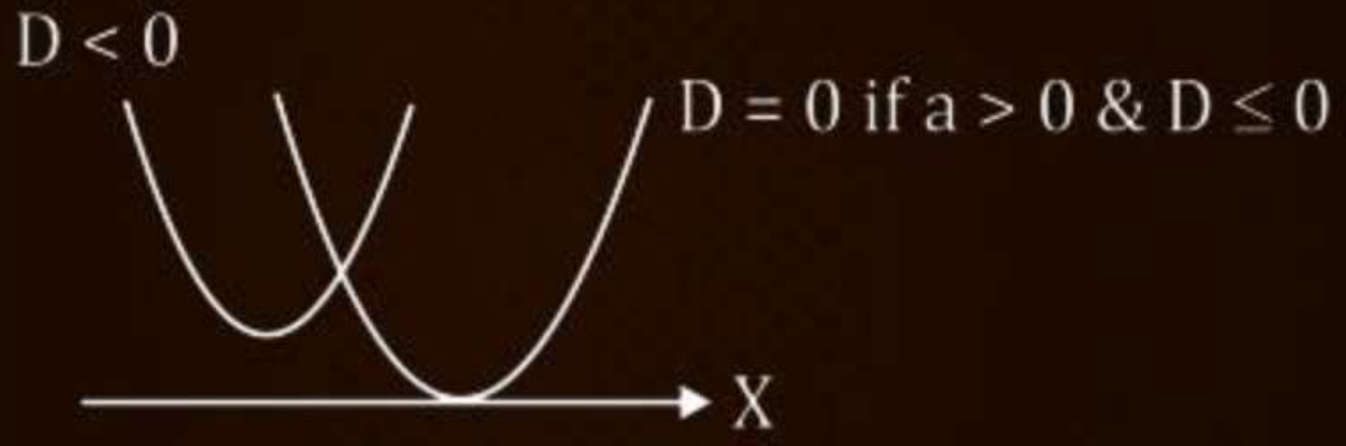
$y = ax^2+bx+c \leq 0$ if $a<0$ & $D \leq 0$.

Important Points

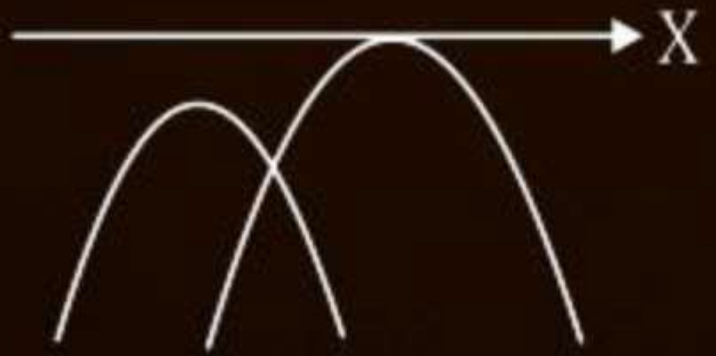
(1) If $a > 0$ and $D < 0$ then $y = ax^2 + bx + c > 0$ for all $x \in R$

(2) If $a < 0$ and $D < 0$ then $y = ax^2 + bx + c < 0 \; \forall \; x \in R$

(3) $ax^2 + bx + c \geq \; \forall \; x \in R$



(4) $ax^2 + bx + c \leq 0 \; \forall \; x \in R$



QUESTION

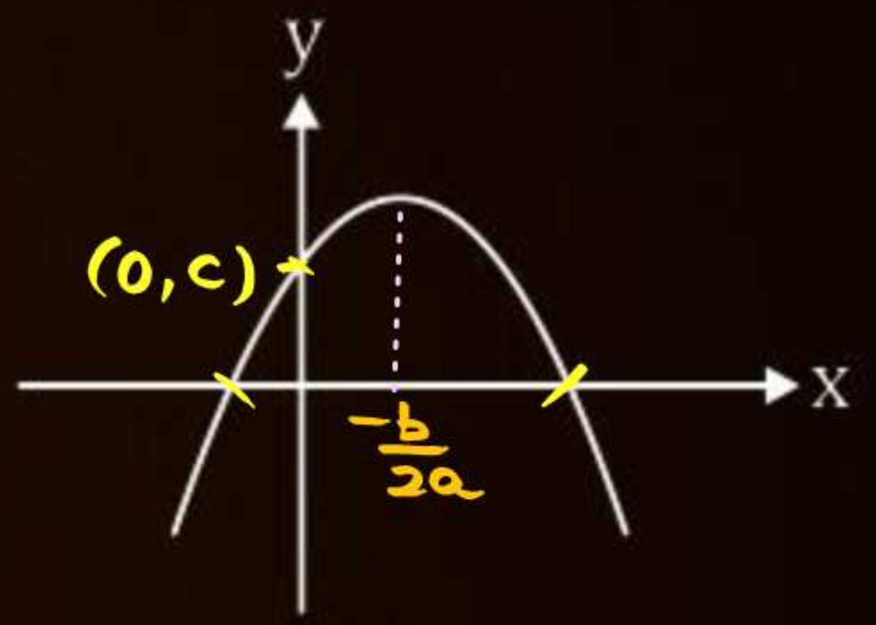
Consider graph of $y = ax^2 + bx + c$ as shown above, comment on signs of

(A) a < 0

(B) b > 0

(C) c > 0

(D) $D > 0$



M① $-\frac{b}{2a} > 0$) a is -ve

$-b < 0$

$b > 0$

M② S.O.R > 0

$-\frac{b}{a} > 0$) a is-ve

$-b < 0$

$b > 0$.

Approach Banao

If $f(x) = ax^2 + bx + c$ then remember

(a) $f(1) = a+b+c$

(b) $f(2) = 4a+2b+c$

(c) $f(3) = 9a+3b+c$

(d) $f(-1) = a-b+c$

(e) $f(-2) = 4a-2b+c$

(f) $f(-3) = 9a-3b+c$

(g) $f(1/2) = \frac{a}{4}+\frac{b}{2}+c = \frac{a+2b+4c}{4}$

(h) $f(-1/2) = \frac{a-2b+4c}{4}$

(i) $f(1/3) = \frac{a+3b+9c}{9}$

(j) $f(-1/3) = \frac{a-3b+9c}{9}$

QUESTION

Consider the graph of quadratic polynomial $y = ax^2 + bx + c$ as shown below. Which of the following is(are) correct?

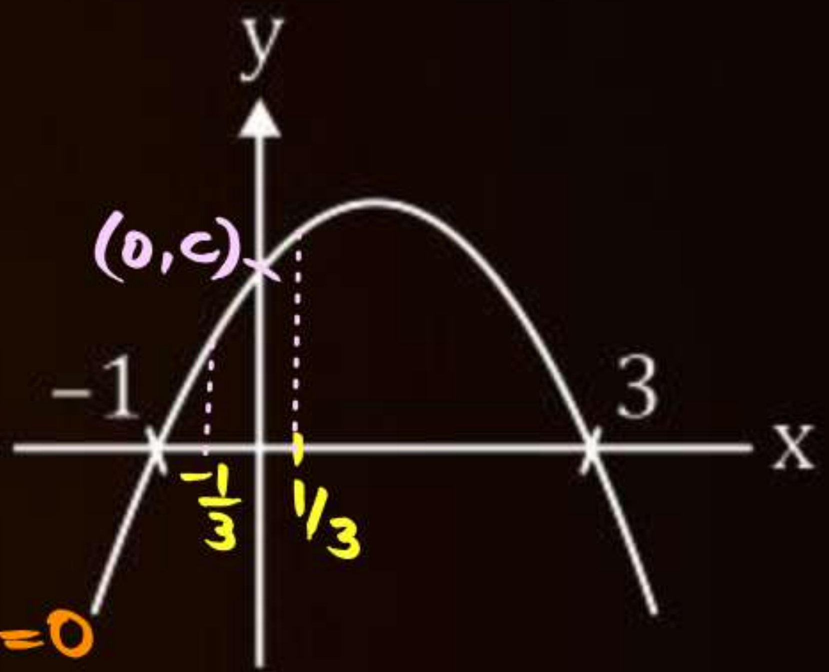
$y = f(x) = ax^2 + bx + c$

$a < 0$

$c > 0$

$-\frac{b}{a} > 0 \rightarrow -b < 0$

$b > 0$



$f(-1) = a - b + c = 0$

$f(3) = 9a + 3b + c = 0$

(A) $\dfrac{a-b+c}{abc} = 0$ $\dfrac{f(-1)}{abc} = \dfrac{0}{abc} = 0$

(B) $abc(9a + \underset{0}{3b + c}) < 0$

(C) $\dfrac{a+3b+9c}{abc} < 0$ $\dfrac{9f(1/3)}{abc} < 0.$ ($f(1/3)$: +ve; a: –ve, b: +ve, c: +ve)

(D) $ab(\underbrace{a - 3b + 9c}) > 0$

$a \cdot b \cdot (9f(-1/3)) < 0.$ (a: –ve, b: +ve, $f(-1/3)$: +ve)

QUESTION

Tan 01

The graph of $y = ax^2 + bx + c$ is shown in the figure, then which of the following is(are) correct?

(1) $a<0$

(2) $c>0$

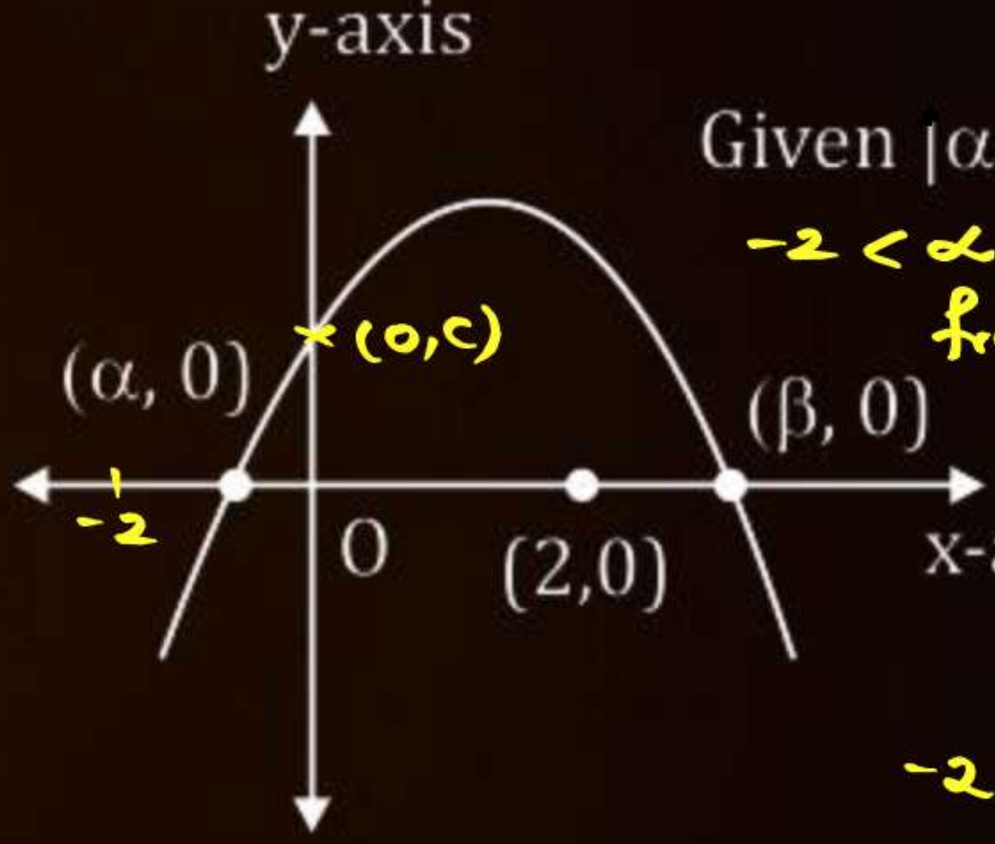
(3) b

(A) $ab^2c^3 < 0$

(B) $ab < 0$

(C) $bc(4a + 2b + c) > 0$

(D) $ab(4a - 2b + c) > 0$



Given $|\alpha| < 2$

$-2<\alpha<2$

from graph

$\alpha<0$

$-2<\alpha<0$

QUESTION

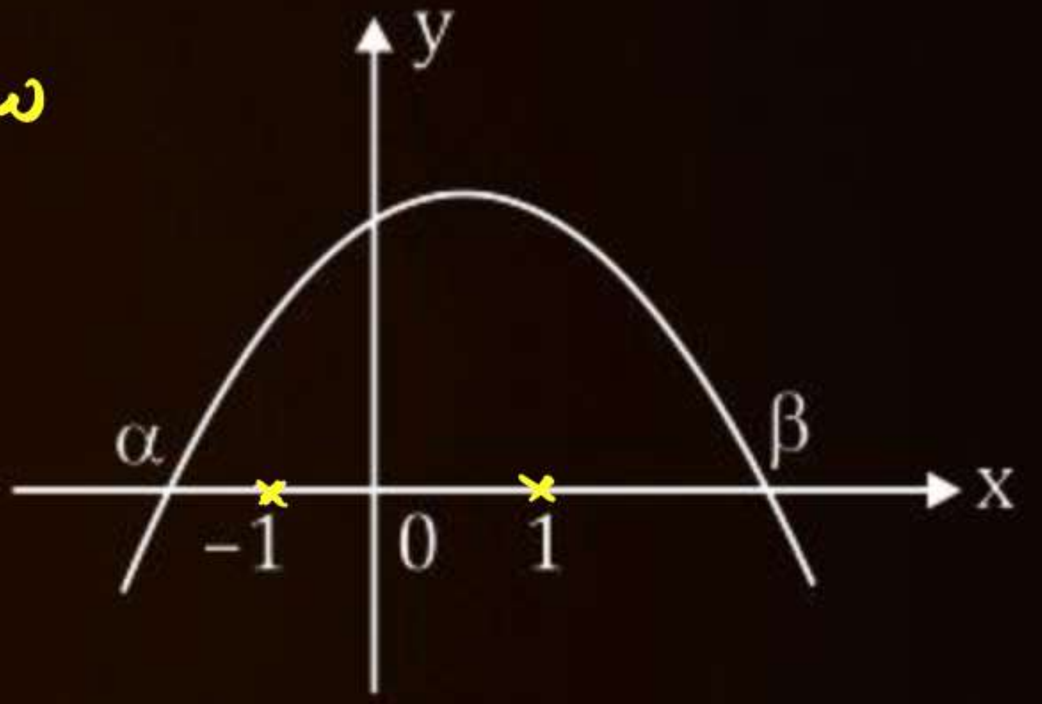
Tah 02

The graph of quadratic polynomial $f(x) = ax^2 + bx + c$ is shown in below. Which of the following are correct?

$|a-b| = $ distance b/w a & b.

Ex: $|-2-3| = 5$

$-2 \qquad 3$



(A) $\dfrac{c}{a} < -1$

(B) $|\beta - \alpha| > 2$

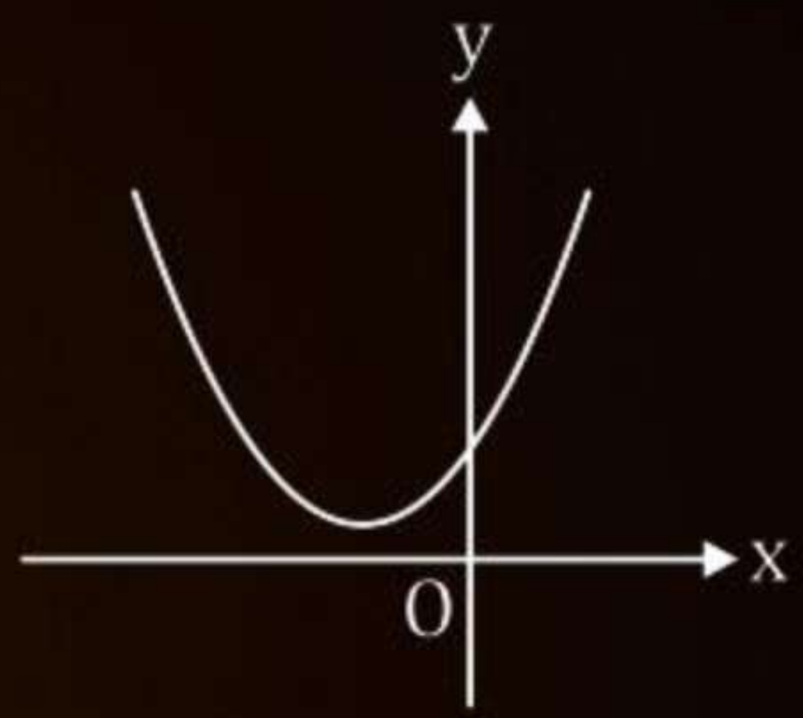
(C) $f(x) > 0 \ \forall \ x \in (0, \beta)$

(D) $abc < 0$

QUESTION

Tah 03

The curve of the quadratic expression $y = ax^2 + bx + c$ is shown in the figure and α, β be the roots of the equation $ax^2 + bx + c = 0$ then correct option is
[D is the discriminant]



(A) $a > 0,\ b > 0, c > 0, D > 0, \alpha + \beta > 0, \alpha\beta > 0$

(B) $a > 0, b > 0, c > 0, D < 0, \alpha + \beta < 0, \alpha\beta < 0$

(C) $a > 0,\ b > 0, c > 0, D < 0, \alpha + \beta < 0, \alpha\beta > 0$

(D) $a > 0, b < 0, c > 0, D < 0, \alpha + \beta > 0, \alpha\beta > 0$

Ans. C

QUESTION

KCLS

If $ax^2 + bx + c = 0$ has no real root and $a + b + c < 0$ then

(A) $4a - 2b + c > 0$

(B) $4a - 2b + c < 0$

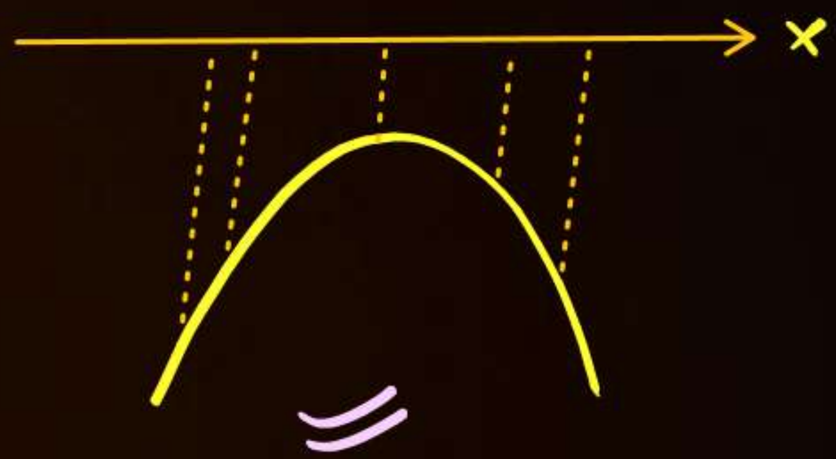
(C) $13a + 5b + 2c < 0$

(D) $5b - 25a - c > 0$

$y = ax^2 + bx + c = f(x)$

or

$y = ax^2 + bx + c = f(x)$



$a + b + c = f(1) < 0$

$f(-2) = 4a - 2b + c$

$13a + 5b + 2c = 9a + 3b + c + 4a + 2b + c$

$= \underbrace{f(3)}_{<0} + \underbrace{f(2)}_{<0} < 0.$

$f(5) = 25a + 5b + c$

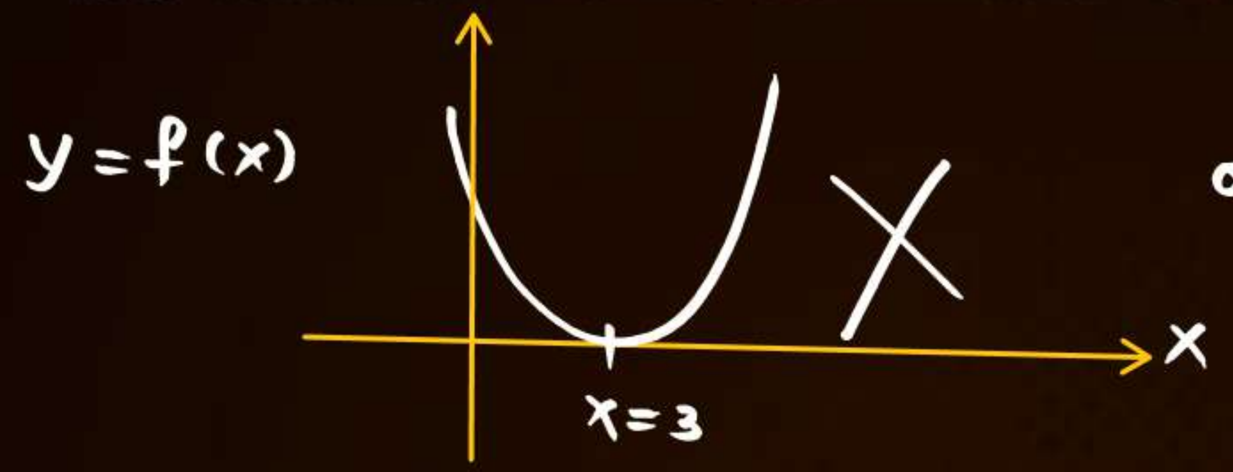
$f(-5) = 25a - 5b + c < 0 \Rightarrow -25a + 5b - c > 0$

Ans. B, C, D

QUESTION

★★★KCLS★★★

If quadratic function f(x) touches x-axis at x = 3 and crosses y-axis at (0, −2), then find f(7).



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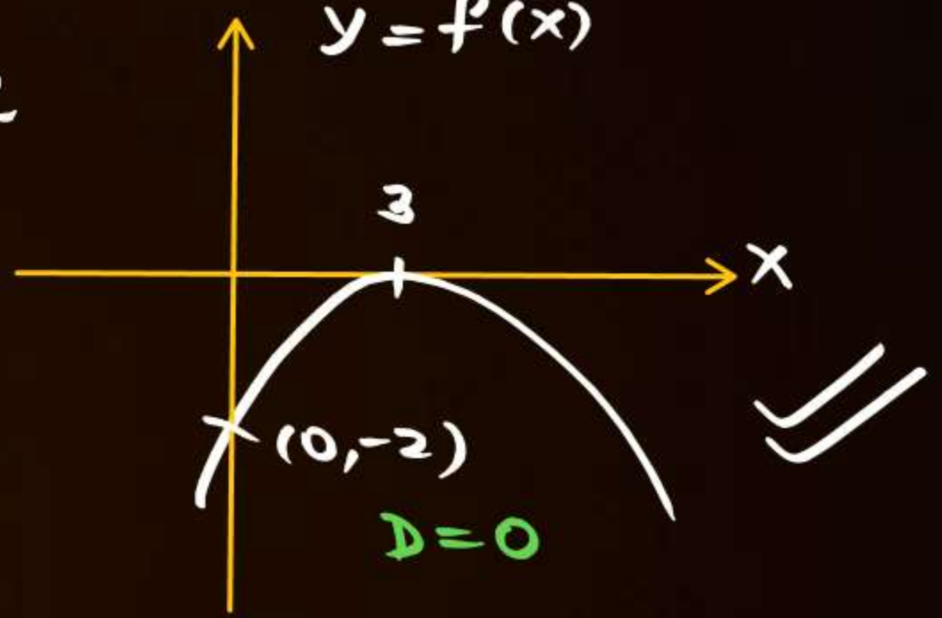


$$y = f(x) = a(x-3)^2$$

M1 passes (0,−2)

$$-2 = a(0-3)^2$$

$$a = -2/9$$

$$f(x) = -2/9\,(x-3)^2$$

$$f(7) = -\frac{2}{9}\cdot 16$$

$$f(7) = -32/9$$

M2 $f(x) = a(x^2-6x+9)$

$$f(x) = ax^2 - 6x + 9a$$

$$c = 9a = -2$$

$$a = -2/9$$

QUESTION

Draw graph of following quadratic :

(i) $f(x) = x^2 + x - 12 = y$

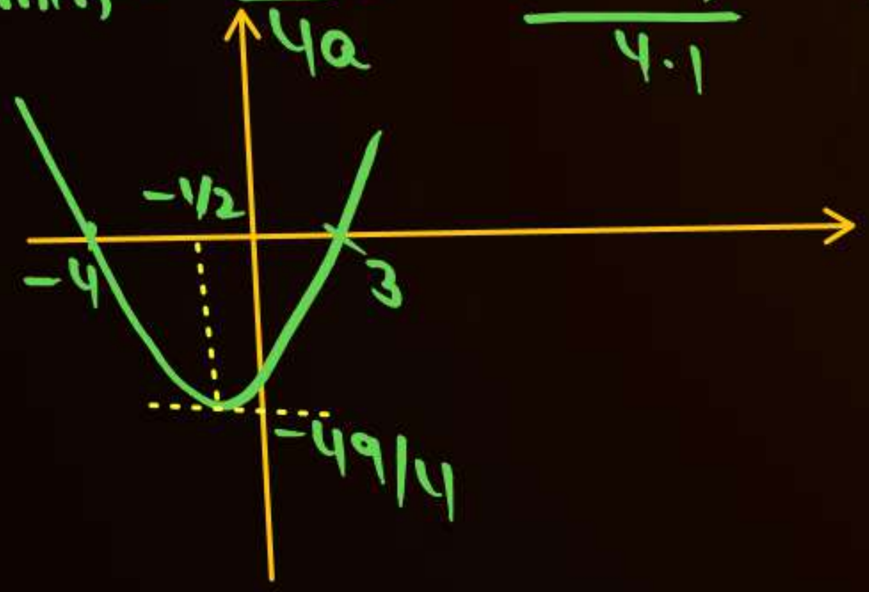
$a > 0 \longrightarrow$ upward opening

$(0,-12)$ — Intersects y axis

Roots $x = \frac{-1 \pm \sqrt{1+48}}{2} = \frac{-1 \pm 7}{2}$

$x = -4, 3$

$y_{min} = -\frac{D}{4a} = -\frac{(1+48)}{4 \cdot 1} = -\frac{49}{4}$ at $x = -\frac{1}{2}$



Range: $\left[-\frac{49}{4}, \infty\right)$

(ii) $f(x) = x - (1 + x^2)$ $y = -x^2 + x - 1$

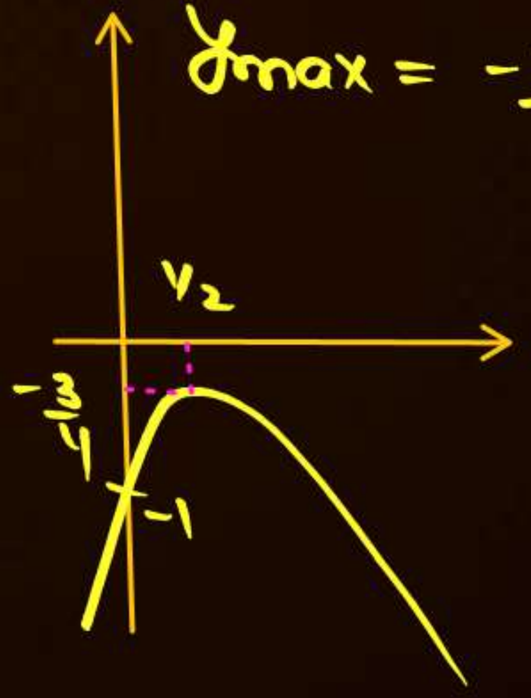
1) $a < 0 \longrightarrow$ downward opening

2) $(0,-1)$ — POI y axis.

3) No real roots.

4) $y_{max} = -\frac{D}{4a} = -\frac{(1-4(-1)(-1))}{4(-1)}$

$y_{max} = -\frac{3}{4}$ at $x = -\frac{1}{2(-1)} = \frac{1}{2}$



QUESTION

★★★KCLS★★★

If $a + b + c > \frac{9c}{4}$ and quadratic equation $ax^2 + 2bx - 5c = 0$ has non-real roots, then

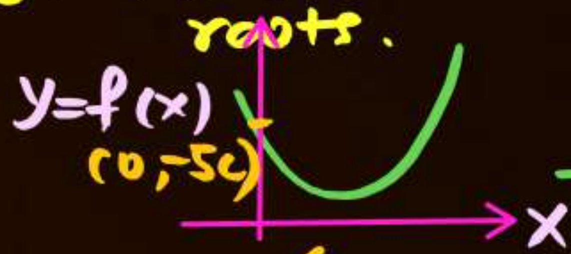
(A) $a > 0, c > 0$

(B) $a > 0, c < 0$

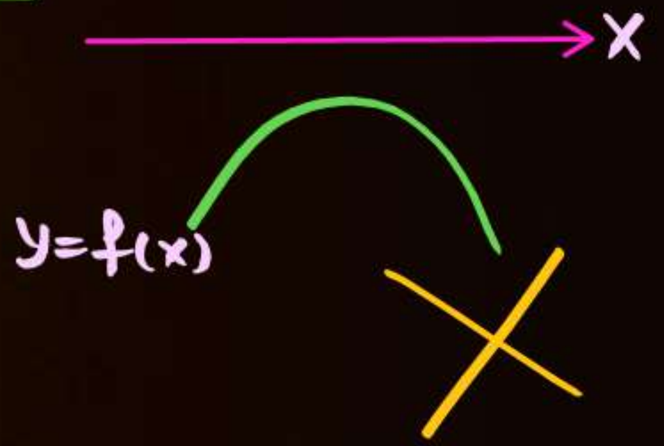
(C) $a < 0, c < 0$

(D) $a < 0, c > 0$

$f(x) = ax^2 + 2bx - 5c = 0$ — No real roots.



OR



$a+b+c > \frac{9c}{4}$

$4a+4b+4c-9c > 0$

$4a+4b-5c > 0$

$f(2) > 0$

$y = ax^2+2bx-5c$ parabola is upward opening

$\Downarrow$

$a>0, -5c>0$

$c<0.$

QUESTION

Tan 04

Find the set of values of a for which $(a - 1)x^2 - (a + 1)x + a + 1 > 0$ for all $x \in R$.

$a-1>0$ & $D<0$

$\cap$

QUESTION

Tah 05

Find the set of values of a for which $(a + 4)x^2 - 2ax + 2a - 6 < 0$ for all $x \in R$.

QUESTION

Tah06

For what values of p the vertex of $x^2 + px + 13$ lies at a distance 5 unit from origin.

$y = x^2 + px + 13$

Vertex $\left(-\frac{p}{2a}, -\frac{(p^2 - 52)}{4}\right)$

origin $(0,0)$ distance $= 5$

$p = ?$

QUESTION

If $y = x^2 - 3x - 4$ then find the range of y when

(i) $x \in R$ (ii) $x \in [0, 3]$ (iii) $x \in [-2, 0]$

$y = x^2 - 3x - 4$

* Range: $\left[-\frac{D}{4a}, \infty\right)$

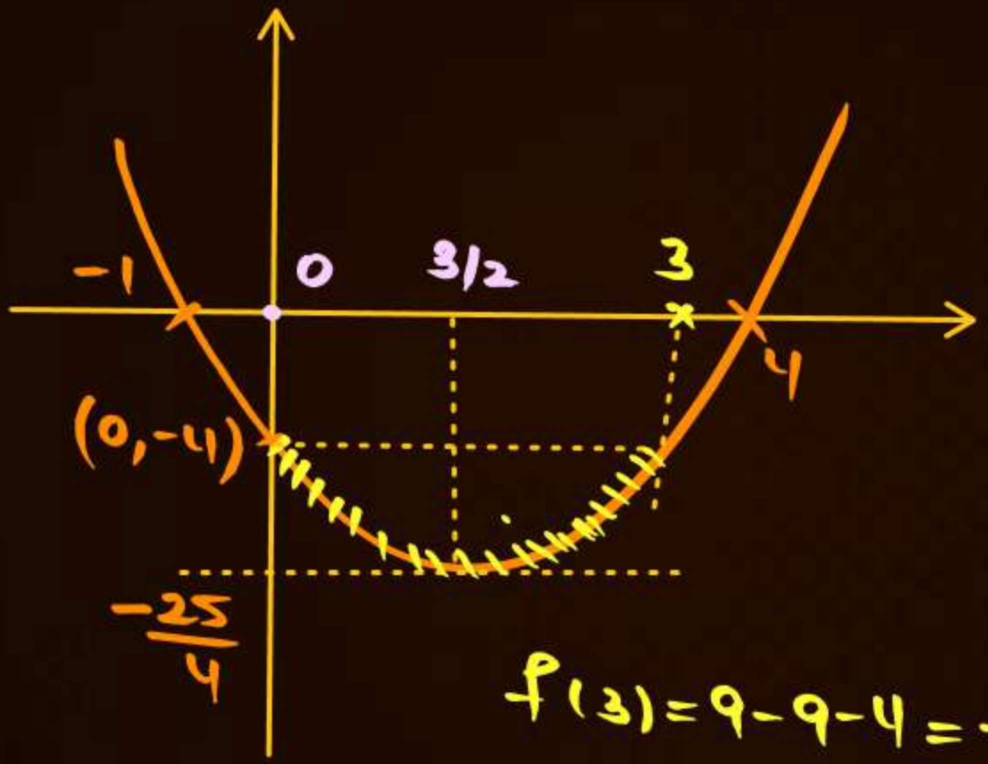
$= \left[-\frac{(9+16)}{4}, \infty\right)$

$= \left[-\frac{25}{4}, \infty\right)$

* Roots $x = \frac{3 \pm \sqrt{25}}{2} = 4, -1$

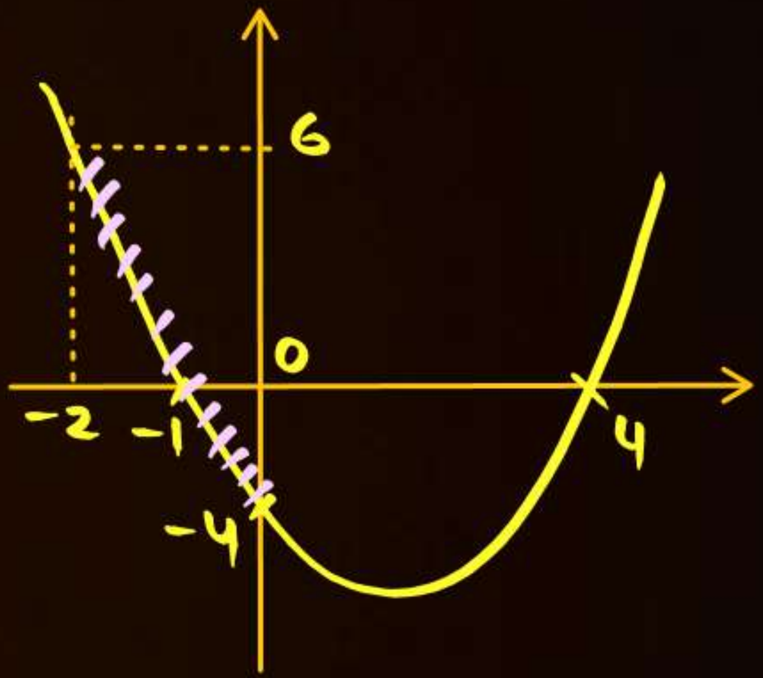
* $y_{min} = -25/4$ at $x = -\frac{(-3)}{2\cdot 1} = \frac{3}{2}$

* POI with y axis $(0, -4)$



$f(3) = 9 - 9 - 4 = -4$

Range: $[-25/4, -4]$

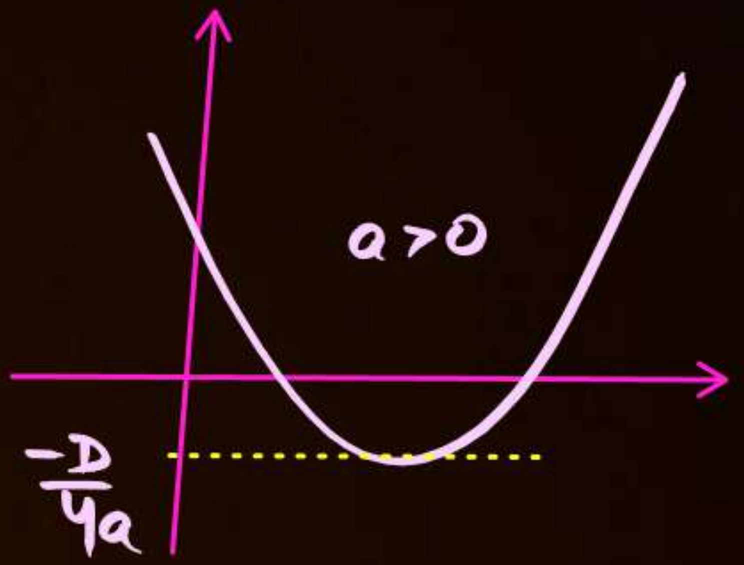


$f(-2) = 4 + 6 - 4 = 6$

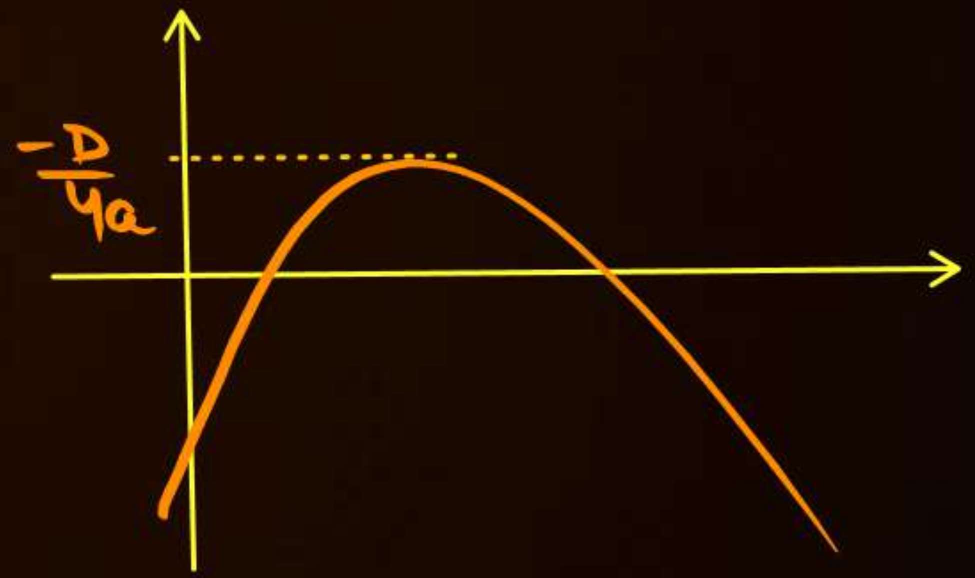
Range $= [-4, 6]$

$y = ax^2 + bx + c$

if $a > 0$ Range : $\left[-\frac{D}{4a}, \infty\right)$



If $a < 0$ Range : $\left(-\infty, -\frac{D}{4a}\right]$



if $x \in [2,3]$

$x^2 \in [4,9]$

$x+2 \in [4,5]$

$x-5 \in [-3,-2]$

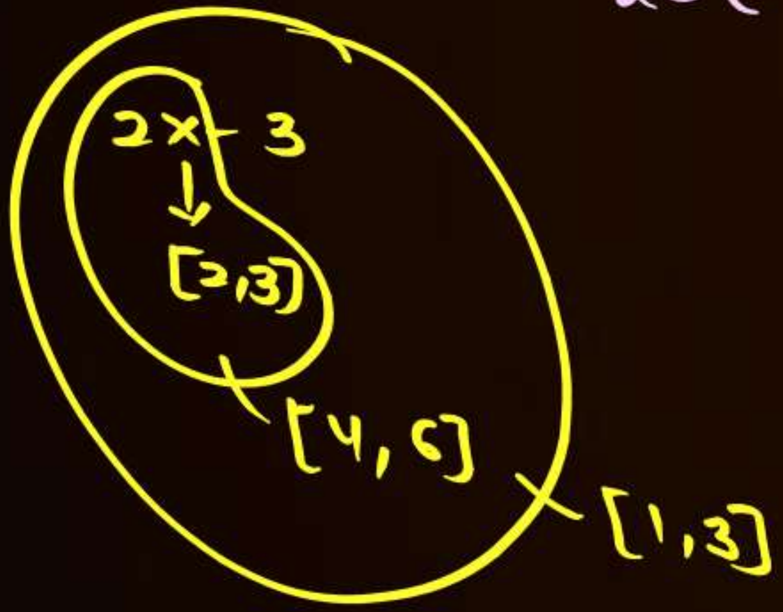
$2x-3 \in [1,3]$

$\frac{1}{x} \in \left[\frac{1}{3}, \frac{1}{2}\right]$

$x^3 \in [8,27]$

$\frac{1}{0^+} \to \infty$

$\frac{1}{0^-} \to -\infty$



Ex: $x \in [-2, 1]$ $\cancel{x^2 \in [0,1],[1,4]}$

$\downarrow$

$x \in [-2,0] \cup [0,1]$

$x^2 \in [0,4] \cup [0,1]$

$x^2 \in [0,4]$

Ex: $x \in [-2,1] = \overleftarrow{[-2,0]} \cup \overrightarrow{[0,1]}$

$\frac{1}{x} \in \left(-\infty, -\frac{1}{2}\right] \cup [1, \infty)$

Ex: $x \in [2,4]$ — find Range $(x-3)^2$

$[-1,1] = [-1,0] \cup [0,1]$

$[0,1] \cup [0,1]$

$(x-3)^2 \in [0,1]$

Ex: $x \in (-\infty,\infty)$ $\quad x^2 \in [0,\infty)$

$\downarrow$

$x \in (-\infty,0] \cup [0,\infty)$

$x^2 \in [0,\infty) \cup [0,\infty)$

QUESTION

If $y = x^2 - 3x - 4$ then find the range of y when

(i) $x \in R$ (ii) $x \in [0, 3]$ (iii) $x \in [-2, 0]$

$y = x^2 - 3x - 4$

$= x^2 - 3x + \frac{9}{4} - \frac{9}{4} - 4$

$= \left(x - \frac{3}{2}\right)^2 - \frac{25}{4}$

$(-\infty, \infty)$

$(-\infty, \infty)$

$[0, \infty)$

$[-25/4, \infty) =$ Range

$y = \left(x - \frac{3}{2}\right)^2 - \frac{25}{4}$

$[0, 3]$

$[-3/2, 3/2] = [-3/2, 0] \cup [0, 3/2]$

$[0, 9/4] \cup [0, 9/4]$

$= [0, 9/4]$

$\left[-25/4, \frac{9-25}{4}\right] = [-25/4, -4]$

$y = \left(x - \frac{3}{2}\right)^2 - \frac{25}{4}$

$[-2, 0]$

$[-7/2, -3/2]$

$\left[\frac{9}{4}, \frac{49}{4}\right]$

$\left[\frac{-16}{4}, \frac{24}{4}\right]$

$[-4, 6]$

$x \in [0, 3]$

find range: $y = x^2 - 2x + 7.$

$y = ((x-1))^2 + 6$

$[-1, 2] = [-1, 0] \cup [0, 2]$

$[0, 1] \cup [0, 4]$

$= [0, 4]$

Range: $[6, 10]$

Gadhe/Gadhi ne yeh Kiyaa

$y = (x-1)^2 + 6$

$[-1, 2]$

$[1, 4]$

$[7, 11]$

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Solution to Previous TAH

QUESTION [JEE Mains 2020 (9 Jan)]

Let $a, b \in R, a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to:

(A) 28

(B) 24

(C) 26

(D) 25

Ans. D

TAH 01

$ax^2 - 2bx + 5 = 0 <^{\alpha}_{\alpha}$

$2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$

$\alpha^2 = \frac{5}{a} \Rightarrow b = a\alpha$

$x^2 - 2bx - 10 = 0. <^{\alpha}_{\beta}$

$\alpha + \beta = 2b$ — (1)

$\alpha\beta = -10$

$\beta = -\frac{10}{\alpha}$

Put $b = a\alpha$, $\beta = -\frac{10}{\alpha}$ in (1)

$\alpha - \frac{10}{\alpha} = 2a\alpha$

$\alpha^2 - 10 = 2a\alpha^2$

$(1-2a)\alpha^2 = 10$

$\alpha^2 = \frac{10}{1-2a}$ and $\alpha^2 = \frac{5}{a}$

$\frac{\cancel{10}\,2}{1-2a} = \frac{\cancel{5}}{a} \Rightarrow 2a = 1-2a \Rightarrow a = 1/4$

$\boxed{\alpha^2 = 20}$

$\beta^2 = \frac{100}{\alpha^2} = \frac{100}{20} = 5$

$\boxed{\beta^2 = 5}$

$\boxed{\alpha^2 + \beta^2 = 25}$ Ans

TAH-1
By Nikita

QUESTION

If the equation $x^2 - 4x + 5 = 0$ and $x^2 + ax + b = 0$ have a common root, find a and b.

TAH 02

$x^2-4x+5=0$ $\qquad$ $x^2+ax+b=0$

$\Downarrow$

$D<0$

$\Downarrow$

imaginary roots

Both roots are common.

TAH-2
By Nikita

So, using condition,

$$\frac{1}{1}=\frac{-4}{a}=\frac{5}{b} \Rightarrow \boxed{\begin{array}{c} a=-4 \\ b=5 \end{array}}$$ Ans

QUESTION [JEE Mains 2013]

If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0, a, b, c \in R$, have a common root, then $a : b : c$ is

(A) $1 : 2 : 3$

(B) $3 : 2 : 1$

(C) $1 : 3 : 2$

(D) $3 : 1 : 2$

Ans. A

TAH 03

TAH-3
By Nikita

$x^2+2x+3=0$

$\Downarrow$

$D<0$

$\Downarrow$

imaginery roots

$ax^2+bx+c=0.$

Both roots are common.

$\frac{1}{a}=\frac{2}{b}=\frac{3}{c} \Rightarrow a:b:c :: 1:2:3.$ Ans

QUESTION

★★KCLS★★

If the equations $ax^3 + x + 2 = 0$ and $x^3 + ax + 2 = 0$ have exactly one common root, find the value of $|a|$.

$ax^3 + x + 2 = 0$ α

$x^3 + ax + 2 = 0$ α $\times a$

$(1-a^2)x + 2(1-a) = 0$ α

$(1-a)\left((1+a)x + 2\right) = 0$

$a = 1$ or $x = -\dfrac{2}{1+a} = \alpha$

(N.P)

Eqn ① $x^3 + x + 2 = 0$

Eqn ② $x^3 + x + 2 = 0$

All 3 roots common.

TAH 04

$$ax^3 + x + 2 = 0 \quad \alpha, \beta, \gamma \qquad x^3 + ax + 2 = 0 \quad \alpha, \delta, \Delta$$

As, α is common root in both eq^n.

$$a\alpha^3 + \alpha + 2 = 0. \quad -(1)$$

$$-\ \alpha^3 + a\alpha + 2 = 0 \quad \times a$$

$$(1-a^2)\alpha + (2-2a) = 0.$$

$$(1-a)\left[(1+a)\alpha + 2\right] = 0.$$

$\boxed{a=1}$ → N.P

$$\alpha = \frac{-2}{1+a}.$$

Put in eq^n (1)

$$\Rightarrow a\left(-\frac{2}{1+a}\right)^3 + \left(-\frac{2}{1+a}\right) + 2 = 0.$$

$$\Rightarrow -8a + (-2)(1+a)^2 + 2(1+a)^3 = 0.$$

$$\Rightarrow -8a - 2(1+a)^2 + 2(1+a)^3 = 0$$

$$\Rightarrow -8a - 2a^2 - 2 - 4a + 2 + 2a^3 + 6a + 6a^2 = 0.$$

$$\Rightarrow 2a^3 + 4a^2 - 6a = 0.$$

$$\Rightarrow 2a(a^2 + 2a - 3) = 0.$$

$\boxed{a=0}$. → N.P., $a^2 + 2a - 3 = 0 \Rightarrow (a+3)(a-1) = 0.$

$\boxed{a=-3}$ $\boxed{a=1}$ → rejected.

So, $|a| = |-3| = 3$ Ans

TAH-4 By Nikita Raj.

* Tech 04 :-

$\frac{64}{16} = 4$

$ax^3 + x + 2 = 0$

$x^3 + ax + 2 = 0$

$\begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix} \begin{vmatrix} 1 & 2 \\ a & 2 \end{vmatrix} = \begin{vmatrix} 2 & a \\ 2 & 1 \end{vmatrix}^2$

$(a^2-1)(2-2a) = (2-2a)^2$

$2a^2 - 2a^3 - 2 + 2a = 4 + 4a^2 - 8a$

$-2a^3 - 2a^2 + 10a - 6 = 0$

$2a^3 + 2a^2 - 10a + 6 = 0 \Rightarrow a^3 + a^2 - 5a + 3 = 0$

$a^2(a-1) + 2a(a-1) - 3(a-1) = 0$

$(a-1)(a^2 + 2a - 3) = 0$

$a^2 + 3a + a - 3 = 0$

$(a-1) = 0 \qquad a(a+3) - (a+3) = 0$

$a = 1 \qquad (a-1)(a+3) = 0$

$a = 1, -3$

$a = -3$ or $(a = 1) \Rightarrow$ Not possible

$a = -3 \Rightarrow |a| = |-3| \Rightarrow \boxed{|a| = 3}$ Ans

Sangam

Ankush!!

$x \downarrow \qquad y \uparrow$

$a\alpha^3 + \alpha + 2 = 0$

$\alpha^3 + a\alpha + 2 = 0$

$\alpha^3 = \dfrac{\begin{vmatrix} 1 & 2 \\ a & 2 \end{vmatrix}}{\begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix}}$

$\alpha = \dfrac{\begin{vmatrix} 2 & a \\ 2 & 1 \end{vmatrix}}{\begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix}}$

$\dfrac{\begin{vmatrix} 1 & 2 \\ a & 2 \end{vmatrix}}{\begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix}} = \dfrac{\begin{vmatrix} 2 & a \\ 2 & 1 \end{vmatrix}^3}{\begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix}^3}$

QUESTION

If two roots of the equation $(x-1)(2x^2-3x+4)=0$ coincide with roots of the equation $x^3+(a+1)x^2+(a+b)x+b=0$ where $a, b \in R$ then $2(a+b)$ equals

(A) 4

(B) 2

(C) 1

(D) 0

TAH05

$(x-1)(2x^2-3x+4)=0 \begin{cases}\alpha\\ \alpha\\ \beta\end{cases}$

$\Downarrow$

$x-1=0.$

& $2x^2-3x+4=0. \begin{cases}\alpha\\ \alpha\end{cases}$

$\Downarrow$

$D<0$.

$\Downarrow$

imagenery roots.

$x^3+(a+1)x^2+(a+b)x+b=0 \begin{cases}\alpha\\ \alpha\\ \gamma\end{cases}$

$\Downarrow$

$x^2(x+1)+ax(x+1)+b(x+1)=0.$

$(x+1)(x^2+ax+b)=0.$

$x+1=0.$

$x^2+ax+b=0 \begin{cases}\alpha\\ \alpha\end{cases}$

Born roots are common.

So, using condition $\Rightarrow$

$$\frac{2}{1}=\frac{-3}{a}=\frac{4}{b} \Rightarrow a=-\frac{3}{2},\ b=2$$

TAH-5
By Nikita,Raj.

$$2(a+b)=2\left(-\frac{3}{2}+2\right)=\underline{\underline{1}}\ \underline{\underline{\text{Ans}}}$$

Solution to Previous KTKs

QUESTION

(KTK 1)

If α, β are the root of a quadratic equation $x^2 - 3x + 5 = 0$ then the equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$ is

(A) $x^2 + 4x + 1 = 0$

(B) $x^2 - 4x + 4 = 0$

(C) $x^2 - 4x - 1 = 0$

(D) $x^2 + 2x + 3 = 0$

Ans. B

Q-7! If α, β are the root of a quadratic equation $x^2-3x+5=0$ then the equation whose roots are $(\alpha^2-3\alpha+7)(\beta^2-3\beta+7)$ is:

Soln!

$$x^2-3x+5=0 \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array}$$

$$\Downarrow$$

$$\alpha^2-3\alpha=-5$$

$$\beta^2-3\beta=-5.$$

KTK 1
BY Reed
From WB

for new eqn, roots $= \alpha^2-3\alpha+7,\ \beta^2-3\beta+7$

$= -5+7, -5+7$

$= 2, 2.$

$\therefore$ The eqn is: $\boxed{(x-2)^2=0.}$

Shoyo
KTK 1

KTK-01 If α, β are the roots of a quadratic equation $x^2 - 3x + 5$, then equation whose roots are $\alpha^2 - 3\alpha + 7$ and $\beta^2 - 3\beta + 7$, is?

Sol.

$$x^2 - 3x + 5 \begin{cases} \to \alpha \\ \to \beta \end{cases}$$

$$\alpha^2 - 3\alpha + 5 = 0 \Rightarrow \alpha^2 - 3\alpha = -5$$

$$\text{lly } \beta^2 - 3\beta = -5$$

New roots $\alpha' = \alpha^2 - 3\alpha + 7$
$= -5 + 7$
$= 2$

lly $\beta' = 2$

req. equation
$\Rightarrow x^2 - (\alpha' + \beta')x + \alpha'\beta' = 0$
$$\boxed{\Rightarrow x^2 - 4x + 4 = 0}$$

QUESTION

(KTK 2)

The equations $ax^2 + bx + a = 0 (a, b \in R)$ and $x^3 - 2x^2 + 2x - 1 = 0$ have 2 roots common. Then $a + b$ must be equal to

(A) 1

(B) -1

(C) 0

(D) None of these

Ans. C

KTK : 2

Q. The Eqn $ax^2+bx+a=0$, $(a,b \in R)$ and $x^3-2x^2+2x-1=0$ have 2 roots common. Then $a+b$ must be.

$$x^3-2x^2+2x-1=0$$

$$\cancel{x^2(x-1)}$$

$$x^3-x^2-x^2+2x-1=0$$

$$x^2(x-1)-x(x-1)+1(x-1)=0$$

$$(x-1)\underbrace{(x^2-x+1)}_{D<0}=0$$

Both have Two common Roots.

$\therefore$ Compare $a=1$ & $b=-1$.

$$a+b = 1-1=0 \text{ Ans}$$

KTK-02 The equations $ax^2+bx+a=0$ and $x^3-2x^2+2x-1=0$ have 2 common roots, then $a+b=?$

$ax^2+bx+a=0 \to \alpha, \beta$

$\alpha+\beta = -\frac{b}{a}$

$\alpha\beta = \frac{a}{a} = 1$

Now, $x^3-2x^2+2x-1=0 \to \alpha, \beta, \gamma$

$\alpha+\beta+\gamma = -(-2)$

$-\frac{b}{a}+\gamma = 2$

$\gamma = 2+\frac{b}{a}$ —— (1)

and $\alpha\beta\gamma = 1$

$\gamma = 1 \to$ put in (1)

$1 = 2+\frac{b}{a}$

$a = 2a+b$

$\boxed{a+b=0}$

Shoyo
KTK 2

QUESTION

(KTK 3)

The value of m for which the equation $\frac{a}{x+a+m}+\frac{b}{x+b+m}=1$ has roots equal in magnitude and opposite in signs is

(A) $\frac{a-b}{a+b}$

(B) -1

(C) 0

(D) $\frac{a+b}{a-b}$

Ans. C

KTK : 3 find value of m for which eqn $\frac{a}{x+a+m} + \frac{b}{x+b+m} = 1$ has roots Equal in magn & opp. in signs.

$$\frac{a}{x+a+m} + \frac{b}{x+b+m} = 1$$

$$\cancel{ax} + \cancel{ab} + \cancel{am} + \cancel{bx} + ab + \cancel{bm} = x^2 + \cancel{bx} + mx + \cancel{ax} + \cancel{ab} + \cancel{am} + mx + \cancel{bm} + m^2$$

$$ab = x^2 + mx + mx + m^2$$

$$ab = x^2 + 2mx - ab + m^2$$

$$\Rightarrow SOR = -2m = \alpha - \alpha$$

$$\Rightarrow -2m = 0$$

$$m = 0$$

[KTK 3] The value of m for which the equation $\frac{a}{x+a+m} + \frac{b}{x+b+m} = 1$ has roots equal in magnitude but opposite in signs is?

Sol.

$$\frac{a}{x+a+m} + \frac{b}{x+b+m} = 1$$

$$\frac{ax+ab+am+bx+ab+bm}{x^2+bx+mx+ax+ab+am+mx+mb+m^2} = 1$$

$$\cancel{ax}+\cancel{ab}+\cancel{am}+\cancel{bx}+ab+\cancel{bm} = x^2+\cancel{bx}+mx+\cancel{ax}+\cancel{ab}+\cancel{am}+mx+\cancel{mb}+m^2$$

$$ab = x^2+2mx+m^2$$

$$x^2+2mx+(m^2-ab) = 0$$

SOR ⇒ $\alpha - \alpha = -2m$

$\boxed{m=0}$

Shoyo

KTK 3

Homework

$$\frac{a}{x+a+m}+\frac{b}{x+b+m}=1$$ m roots equal in magnitude but opposite in sign.

$$ax+ab+am+bx+ab+bm=(x+a+m)(x+b+m)$$

$$\cancel{ax}+ab+\cancel{am}+\cancel{bx}+\cancel{ab}+\cancel{bm}=x^2+\cancel{bx}+mx+\cancel{ax}+\cancel{ab}+\cancel{am}$$
$$+mx+\cancel{mb}+m^2$$

$$x^2+2mx+m^2+ab=0$$

if roots have same magnitude + opp sign

then $-2m=0$

$m=0$ (C)

QUESTION

(KTK 4)

Find the values of 'k' so that the equation

$x^2 + kx + (k + 2) = 0$ and $x^2 + (1 - k)x + 3 - k = 0$ have exactly one common root.

Ans. No possible value of k

Homework

$x^2+kx+(k+2)=0$ $\nearrow \alpha$, $\nearrow \beta$
$x^2+(1-k)x+3-k=0$ $\to \alpha$, $\to \gamma$
have exactly one common root.

Both have one root in common, thus

$\alpha^2+k\alpha+k+2=\alpha^2+(1-k)\alpha+3-k$

$\Rightarrow k\alpha-(1-k)\alpha+2k-1=0$

$\Rightarrow \alpha(k-1+k)+2k-1=0$

$\Rightarrow (2k-1)(\alpha+1)=0$

$\Rightarrow k=1/2$ or $\alpha=-1$

$\to$ X $\to$ Both have two roots in common

putting $\alpha=-1$ in E_1

$1-k+k+2=0$

$1=-2$ (Not possible)

$\Rightarrow$ Thus no value of k is possible.

Akash

QUESTION

(KTK 5)

Given a, b are two distinct real number satisfying

$a^2 - 5a + 2 = 0$ and $b^2 - 5b + 2 = 0$ then $(1 - ab + a^2b + b^2a)$

KTK-05 Given a, b are two distinct real ~~so~~ number satisfying

$a^2-5a+2=0$ and $b^2-5b+2=0$ then $(1-ab+a^2b+b^2a)$

$$x^2-5x+2=0 \begin{cases} a \\ b \end{cases}$$

$a+b=5$

$a \cdot b=2$

$1-ab+ab(a+b)$

$1-2+2(5)$

$1-2+10$

$\boxed{9 \text{ Ans}}$

KTK-5
Ayush Patel
Prayagraj up

KTK = 5

$a^2 - 5a + 2 = 0$ $\quad$ $b^2 - 5b + 2 = 0$

$x^2 - 5x + 2 = 0 \begin{matrix} a \\ b \end{matrix}$

$a + b = 5$

$ab = 2$

$\Rightarrow 1 - ab + ab(a+b)$

$= 1 - 2 + 2 \times 5 \Rightarrow 10 - 1 = \underline{\underline{9}}$ Ans

KTK-5
By Nikita

QUESTION

(KTK 6)

Let 'p' is a root of the equation $x^2 - x - 3 = 0$. Then the value of $\frac{p^3+1}{p^5-p^4-p^3+p^2}$ is equal to

(A) $\frac{4}{3}$

(B) $\frac{4}{9}$

(C) $\frac{2}{9}$

(D) $\frac{2}{3}$

$\underline{\underline{KTK = 6}}$

$x^2 - x - 3 = 0 \swarrow p$

$\underbrace{p(p-1) = 3}$

KTK-6

$$\frac{p^3+1}{p^5-p^4-p^3+p^2} = ?$$

$$\frac{p^3+1}{p^5-p^4-p^3+p^2} = \frac{(p+1)(p^2-p+1)}{p^4(p-1)-p^2(p-1)} = \frac{(p+1)(p^2-p+1+3-3)}{(p-1)p^2(p^2-1)}$$

$$\Rightarrow \frac{\cancel{(p+1)}(\cancel{p^2-p-3}^{\,0}+4)}{p^2(p-1)^2\cancel{(p+1)}} = \frac{4}{p^2(p-1)^2} = \frac{4}{(p(p-1))^2} = \frac{4}{9}$$

$\underline{\underline{\text{Ans}}}$

QUESTION

(KTK 7)

If α and β are the roots of $ax^2 + bx + c = 0$, then the equation whose roots are $\frac{\alpha+1}{\alpha-2}$ and $\frac{\beta+1}{\beta-2}$ is

(A) $a(x+1)^2 + b(x+1)(x-2) + c(x-2)^2 = 0$

(B) $a(x-2)^2 + b(x+1)(x-2) + c(x+1)^2 = 0$

(C) $a(2x+3)^2 + b(x+1)(x+2) + c(x+2)^2 = 0$

(D) $a(2x+1)^2 + b(2x+1)(x-1) + c(x-1)^2 = 0$

Ans. D

KTK 07

$$ax^2+bx+c=0 < {}^{\alpha}_{\beta} \quad -(1)$$

KTK-7
By Nikita,Raj.

let $y=f(x)=\dfrac{x+1}{x-2} \Rightarrow$

$$yx-2y=x+1$$
$$(y-1)x=1+2y$$
$$x=\frac{2y+1}{y-1} \quad \text{Put in (1)}$$

$$a\left(\frac{2y+1}{y-1}\right)^2+b\left(\frac{2y+1}{y-1}\right)+c=0.$$

$$\Rightarrow \quad a(2y+1)^2+b(2y+1)(y-1)+c(y-1)^2=0.$$

$$\Rightarrow \quad \underline{a(2x+1)^2+b(2x+1)(x-1)+c(x-1)^2=0} \quad \text{Ans}$$

QUESTION

(KTK 8)

If α, β, γ are roots $x^3 + 2x^2 - 3x + 1 = 0$, then value of $\frac{\alpha\beta}{\alpha+\beta} + \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\beta\gamma}{\beta+\gamma}$ is less than

(A) 2

(B) 3

(C) 4

(D) 5

KTK - 8

$x^3+2x^2-3x+1=0 \quad -(1)$

$\alpha+\beta+\gamma=-2$

$\alpha\beta\gamma=-1$

$\alpha+\beta=-2-\gamma$ & $\alpha\beta=-1/\gamma$

$\beta+\gamma=-2-\alpha \qquad \beta\gamma=-1/\alpha$

$\text{||ly}, \alpha+\gamma=-2-\beta \qquad \alpha\gamma=-1/\beta$

$x^2(x+2)=3x-1$

$x(x+2)=\frac{3x-1}{x}$

$\Rightarrow \frac{1}{x(x+2)}=\frac{x}{3x-1}$

$$\frac{-1/\gamma}{-\gamma-2}+\frac{-1/\beta}{-\beta-2}+\frac{-1/\alpha}{-\alpha-2} \Rightarrow \overset{a}{\frac{1}{\alpha(\alpha+2)}}+\overset{b}{\frac{1}{\beta(\beta+2)}}+\overset{c}{\frac{1}{\gamma(\gamma+2)}}$$

let $y=\frac{x}{3x-1} \Rightarrow 3xy-y=x$

$(3y-1)x=y \Rightarrow x=\frac{y}{3y-1}$ Put in (1)

$$\Rightarrow \frac{y^3}{(3y-1)^3}+\frac{2y^2}{(3y-1)^2}-\frac{3y}{3y-1}+1=0.$$

$$\Rightarrow y^3+2y^2(3y-1)-3y(3y-1)^2+(3y-1)^3=0.$$

$$\Rightarrow y^3+6y^3-2y^2-\cancel{27y^3}-3y+18y^2+\cancel{27y^3}-1-27y^2+9y=0$$

$$\Rightarrow 7y^3-11y^2+6y-1=0.$$

replaced y by $x \rightarrow$

$7x^3-11x^2+6x-1=0 \quad a, b, c$

$a+b+c=\frac{11}{7}=1.5$

which is less than 2, 3, 4, 5

Ans

KTK-8

By Nikita,Raj

Solution to Previous Home Challenge

Home Challenge - 08

The ordered pair (x, y) satisfying the equation $x^2 = 1 + 6\log_4 y$ and $y^2 = 2^x y + 2^{2x+1}$ are (x_1, y_1) and (x_2, y_2), then find the value of $\log_2|x_1x_2y_1y_2|$.

(Ans:7)

Home challenge - 08

$x^2 = 1 + 6\log_4 y \quad , y > 0$

$\Rightarrow x^2 = 1 + 3\log_2 y$

$\Rightarrow x^2 = 1 + 3\log_2 2^{x+1}$

$\Rightarrow x^2 = 1 + 3(x+1)$

$\Rightarrow x^2 - 3x - 4 = 0$

$\Rightarrow (x-4)(x+1) = 0$

$x = 4, -1$

$y = 2^5, 2^0$

$y = 32, 1$

$x_1 = 4 \quad y_1 = 2^5$

$x_2 = -1 \quad y_2 = 1$

$y^2 = 2^x y + 2^{2x+1}$

$y^2 = 2^x y + 2^{2x} \cdot 2$ let $2^x = t$

$y^2 - ty - 2t^2 = 0$

$y = \dfrac{t \pm \sqrt{t^2 + 8t^2}}{2}$

$y = \dfrac{t \pm 3t}{2}$

$y = 2t, -t$ since, $y > 0$

$y = 2 \cdot 2^x, \; -2^x$ × N.P

$y = 2^{x+1}$

Home challenge-8
By Nikita, Raj.

$\log_2 |x_1 x_2 y_1 y_2|$

$= \log_2 |4(-1)2^5 \cdot 1|$

$\Rightarrow \log_2 (2^7) = \underline{\underline{7}}$ Ans

THANK YOU