PRAYAS

JEE 2026

Mathematics

Quadratic Equations

Lecture - 05

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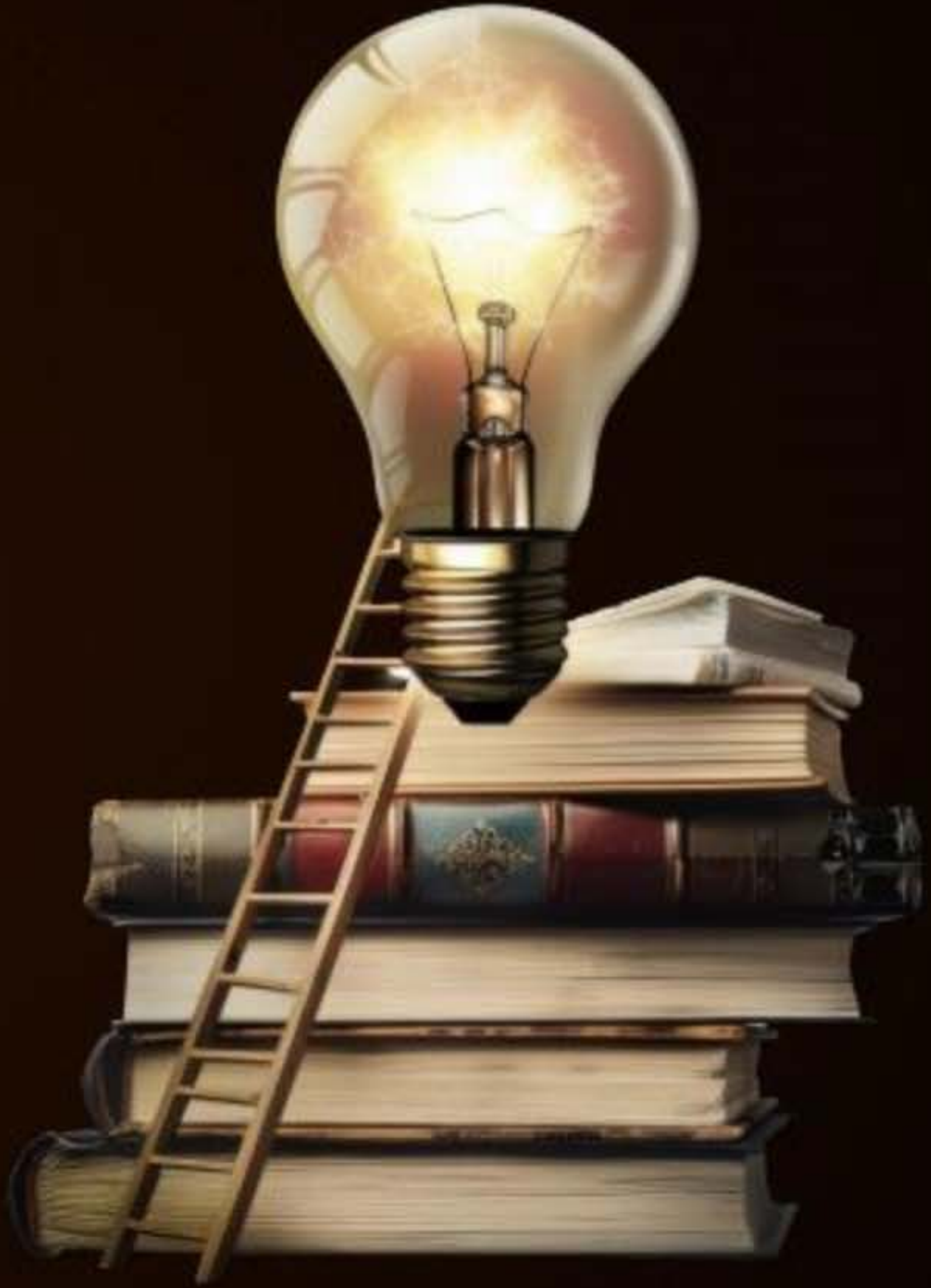
Topics *to be covered*

- **A** Question Practice on Common Root
- **B** Graph of a Quadratic polynomial
- **C** Practice problems



Homework Discussion

QUESTION [JEE Mains 2022 (27 July)]

If α, β are the roots of the equation

0

$$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right)x + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$$

then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is :

$a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$.

$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right)x - 3 = 0$

$x^2 - 5x - 3 = 0$

$3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{1-\frac{1}{3}}}$

$= 3^{(\log_3 5)^{\frac{1}{3}}} - 5^{\log_5 3 \cdot \frac{1}{(\log_5 3)^{\frac{1}{3}}}}$

$= 3^{(\log_3 5)^{\frac{1}{3}}} - \left(5^{\log_5 3}\right)^{(\log_3 5)^{\frac{1}{3}}}$

$= 3^{(\log_3 5)^{1/3}} - 3^{(\log_3 5)^{1/3}}$

$= 0$

(A) $3x^2 - 20x - 12 = 0$

(B) $3x^2 - 10x - 4 = 0$

(C) $3x^2 - 10x + 2 = 0$

(D) $3x^2 - 20x + 16 = 0$

Ans. B

QUESTION [JEE Mains 2021]

Let α, β be two roots of the equation $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$. Then $\alpha^8 + \beta^8$ is equal to

(A) 10

(B) 100

[✓ (C)] 50

(D) 160

M① $x^2+(20)^{1/4}x+5^{1/2}=0$

$\alpha^2+(20^{1/4})\alpha+5^{1/2}=0$

$\alpha^2+\sqrt{5}=-(20^{1/4}\cdot\alpha)$

S.B.S $\alpha^4+5+\cancel{2\sqrt{5}\alpha^2}=20^{1/2}\alpha^2=\sqrt{20}\cdot\alpha^2=\cancel{2\sqrt{5}\alpha^2}$

$\alpha^4=-5.$

$\alpha^8=25.$

Ily $\beta^8=25$ $\Big\rangle$ $\alpha^8+\beta^8=50.$

M② $\alpha+\beta=-(20)^{1/4}$ $\alpha\beta=\sqrt{5}$

$\alpha^2+\beta^2+\cancel{2\sqrt{5}}=\cancel{2\sqrt{5}}.$

$\alpha^4+\beta^4+2\alpha^2\beta^2=0$ ⟶ $\alpha^4+\beta^4=-10.$

$\alpha^8+\beta^8+2\alpha^4\beta^4=100.$

$\alpha^8+\beta^8+50=100 \rightsquigarrow \alpha^8+\beta^8=50.$

Ans. C

M③ $S_n = \alpha^n + \beta^n$

$$S_{n+2} + 20^{1/4} S_{n+1} + \sqrt{5}\, S_n = 0$$

$n = 6$ $$S_8 + 20^{1/4} S_7 + \sqrt{5}\, S_6 = 0$$

lengthy !!

QUESTION [JEE Mains 2023]

Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$. Then $\alpha^{14} + \beta^{14}$ is equal to

- (A) -64
- (B) $-64\sqrt{2}$
- (C) $-128\sqrt{2}$
- [✓ (D)] -128

$\alpha^2 + 2 = \sqrt{2}\alpha$

$\alpha^4 + 4 + 4\alpha^2 = 2\alpha^2$

$\alpha^4 + 4 = -2\alpha^2$

$\alpha^8 + 16 + 8\alpha^4 = 4\alpha^4$

multiply by α^4

$\alpha^8 + 16 = -4\alpha^4 \longrightarrow \alpha^8 + 4\alpha^4 = -16$

$\alpha^{12} + 16\alpha^4 = -4\alpha^8$

$\alpha^{12} = -4(\alpha^8 + 4\alpha^4)$

$\alpha^{12} = 64$

$\alpha^{14} = 64\alpha^2$

lly $\beta^{14} = 64\beta^2$

$\alpha^{14} + \beta^{14} = 64(\alpha^2 + \beta^2)$

$= 64((\alpha+\beta)^2 - 2\alpha\beta)$

$= 64(2 - 2\cdot 2)$

$= -128$

Ans. D

QUESTION

If $x^2 + 3x + 3 = 0$ and $ax^2 + bx + 1 = 0, a, b \in Q$ have a common root, then value of $(3a + b)$ is equal to

A 1/3

B 1

[✓ **C**] 2

D 4

$x^2+3x+3=0$ D<0

$ax^2+bx+1=0$ both roots common.

$\frac{a}{1}=\frac{b}{3}=\frac{1}{3}$

$a=1/3$

$b=1$

$3a+b=2$.

Ans. C

Home Challenge - 07

$\overset{-ve}{x}-3 = -ve$

$18 - \underset{-ve}{x}$

Let n be the number of integers satisfying the inequality $\frac{(x-3)^{\frac{-x}{|x|}}\cdot\sqrt{(x-5)^2}\cdot(18-x)}{\sqrt{-x}(-x^2+x-1)(|x|-37)} < 0$ then value of n is ___________

$-x > 0 \Rightarrow \underline{x<0}$

$$\frac{(x-3)^{\cancel{\frac{-x}{-x}}} \cdot |x-5|\,(18-x)}{\sqrt{-x}\,(-x^2+x-1)\,(-x-37)} < 0$$

$(-x^2+x-1)$: D<0, a<0 $\Rightarrow$ always -ve

$$\frac{\underset{-ve}{\cancel{(x-3)}} \cdot \underset{-ve}{\cancel{(x-5)}} \cdot \underset{+ve}{(18-x)}}{\cancel{-}(x+37)} > 0$$

$\frac{1}{(x+37)} > 0 \quad x \neq 3, 5.$

$x+37 > 0$

$x > -37$

$x \in (-37, 0)$

No: of Integral values of $x = 0-(-37)-1 = 36$ Ans

Home Challenge - 07

$x = -ve$

$|x-5| = -(x-5)$

$-ve$

Let n be the number of integers satisfying the inequality $\frac{(x-3)^{\frac{-x}{|x|}}\cdot\sqrt{(x-5)^2\cdot(18-x)}}{\sqrt{-x}(-x^2+x-1)(|x|-37)} < 0$ then value of n is ________

$-x > 0 \Rightarrow \underline{x < 0}$

$$\frac{(x-3)^{\cancel{\frac{-x}{-x}}}\cdot|x-5|(18-x)}{\sqrt{-x}\,(-x^2+x-1)\,(-x-37)} < 0$$

$(-x^2+x-1)$: $D<0$, $a<0$ $\Rightarrow$ always -ve

$$\frac{(x-3)\cdot\cancel{-}(x-5)\cdot(18-x)}{\cancel{-}(x+37)} > 0$$

$$-\frac{(x-3)(x-5)(x-18)}{x+37} > 0$$

$$\frac{(x-3)(x-5)(x-18)}{x+37} < 0$$

$x \in (-37,3) \cup (5,18)$

But $x<0$

$x \in (-37,0)$

QUESTION

If α, β, γ are roots of the cubic $2011x^3 + 2x^2 + 1 = 0$, then find

(i) $(\alpha\beta)^{-1} + (\beta\gamma)^{-1} + (\gamma\alpha)^{-1}$;

(ii) $\alpha^{-2} + \beta^{-2} + \gamma^{-2}$

Ans. (i) 2 ; (ii) −4

Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal

QUESTION [JEE Mains 2022 (25 June)]

Let a, b ∈ R be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to:

- A 37
- [✓ B] 58
- C 68
- D 92

$ax^2-2bx+15=0 \left\langle\begin{matrix}\alpha\\ \alpha\end{matrix}\right.$

$x^2-2bx+21=0 \left\langle\begin{matrix}\alpha\\ \beta\end{matrix}\right.$

S.O.R = $\alpha+\alpha = +\frac{2b}{a}$

$\alpha = \frac{b}{a}$

$\frac{b^2}{a^2} - 2b\cdot\frac{b}{a} + 21 = 0$

$\frac{b^2}{a^2} - \frac{2b^2}{a} + 21 = 0$

$D = 0 \Rightarrow 4b^2 - 4\cdot 15a = 0$

$b^2 = 15a.$

$\frac{15}{a} - 30 + 21 = 0$

$9a = 15.$

$a = 5/3$

$b^2 = 15a = 15\cdot 5/3$

$b^2 = 25.$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (2b)^2 - 2\cdot 21$

$= 4b^2 - 42 = 4\cdot 25 - 42$

$= \underline{\underline{58}}$

$\Downarrow$

we want b??

Ans. B

QUESTION [JEE Mains 2022 (25 June)]

Let $a, b \in R$ be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to:

Ⓐ 37

[✓ Ⓑ] 58

Ⓒ 68

Ⓓ 92

$$ax^2-2bx+15=0 <^{\alpha}_{\alpha}$$

$$x^2-2bx+21=0 <^{\alpha}_{\beta}$$

$$\begin{vmatrix} a & -2b \\ 1 & -2b \end{vmatrix} \times \begin{vmatrix} -2b & 15 \\ -2b & 21 \end{vmatrix} = \begin{vmatrix} 15 & a \\ 21 & 1 \end{vmatrix}^2$$

$$(-2ab+2b)(-12b) = (15-21a)^2$$

$$b^2(-12)(2-2a) = (15-21a)^2$$

$$\cancelto{5}{15}a \cdot (\cancelto{4}{12})(2)(a-1) = \cancel{9}(5-7a)^2.$$

$$40a(a-1) = 25+49a^2-70a$$

$$9a^2-30a+25=0$$

$$(3a-5)^2=0 \Rightarrow a=5/3$$

$$D=0 \Rightarrow 4b^2-4\cdot15a=0$$

$$b^2=15a.$$

$$b^2=15\cdot5/3=25.$$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$

$$=(2b)^2-2\cdot21$$

$$=4b^2-42$$

$$\Downarrow$$

we want b??

$$\alpha^2+\beta^2=100-42$$

$$=\underline{58}.$$

Ans. B

QUESTION [JEE Mains 2020 (9 Jan)]

Tahol

Let $a, b \in R, a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to:

Ⓐ 28

Ⓑ 24

Ⓒ 26

Ⓓ 25

Ans. D

QUESTION

Tah02

If the equation $x^2 - 4x + 5 = 0$ and $x^2 + ax + b = 0$ have a common root, find a and b.

QUESTION [JEE Mains 2013]

Tah 03

If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$, have a common root, then $a : b : c$ is

A $1 : 2 : 3$

B $3 : 2 : 1$

C $1 : 3 : 2$

D $3 : 1 : 2$

Ans. A

QUESTION [AIEEE 2008]

The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

$x^2 - 6x + a = 0 \;\Big<\; \begin{matrix}\alpha \\ 4\beta\end{matrix}$

$x^2 - cx + 6 = 0 \;\Big<\; \begin{matrix}\alpha \\ 3\beta\end{matrix}$

Integers.

Ⓐ 1

Ⓑ 4

Ⓒ 3

Ⓓ 2

$\alpha + 4\beta = 6$

$\alpha \cdot 3\beta = 6 \Rightarrow \alpha = \frac{2}{\beta}$

$\frac{2}{\beta} + 4\beta = 6$

$\frac{1}{\beta} + 2\beta = 3$

$2\beta^2 + 1 = 3\beta$

$2\beta^2 - 3\beta + 1 = 0$

$2\beta^2 - 2\beta - \beta + 1 = 0$

$(2\beta - 1)(\beta - 1) = 0$

$\cancel{\beta = 1/2}, 1$ ✓

$\alpha = 2$

Ans. D

QUESTION [JEE Mains 2021 (26 Aug)]

Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to

$3 \times \quad x^2 - x + 2\lambda = 0 < \begin{matrix} \alpha \\ \beta \end{matrix}$ ✓ — ①

$3x^2 - 10x + 27\lambda = 0 < \begin{matrix} \alpha \\ \gamma \end{matrix}$ ✓

$7x - 21\lambda = 0$ ✓ α

$x = 3\lambda = \alpha.$

Put in ① $9\lambda^2 - 3\lambda + 2\lambda = 0$

$\lambda(9\lambda - 1) = 0$

$\lambda = \cancel{0}, 1/9 \Rightarrow$

$x^2 - x + 2/9 = 0 < \begin{matrix} 1/3 \\ 2/3 = \beta \end{matrix}$

$3x^2 - 10x + 3 = 0$

$3x^2 - 9x - x + 3 = 0$

$(3x - 1)(x - 3) = 0$ $x = 1/3, 3 = \gamma$

$\frac{\beta\gamma}{\lambda} = \frac{2/3 \cdot 3}{1/9} = 18$ Ans

Ans. 18

QUESTION [IIT-JEE 2006]

★★KCLS★★

Let a and b be the roots of the equation $x^2 - 10cx - 11d = 0$ and those $x^2 - 10ax - 11b = 0$ are c, d then the value of $a + b + c + d$, when $a \neq b \neq c \neq d$, is

$x^2 - 10cx - 11d = 0 <^{a}_{b} \Rightarrow a+b = 10c$

$x^2 - 10ax - 11b = 0 <^{c}_{d} \Rightarrow c+d = 10a$

$b-d = 11(c-a)$

$a+b+c+d = 10(a+c)$

$a^2 - 10ca - 11d = 0$

$c^2 - 10ac - 11b = 0$

$a^2 - c^2 + 11(b-d) = 0$

$(a-c)(a+c) = -11(b-d) = -11 \cdot 11(c-a)$

$\cancel{(a-c)}(a+c) = 121\cancel{(a-c)}$

$a+c = 121$

$\underbrace{a+b+c+d = 1210}$

Ans. 1210

QUESTION

★★ASRQ★★

The equations $kx^2 + x + k = 0$ and $kx^2 + kx + 1 = 0$ have exactly one root in common for

(A) $k = -1/2, 1$

(B) $k = 1$

(C) $k = -1/2$

(D) $k = 1/2$

$$kx^2 + x + k = 0$$

$$kx^2 + kx + 1 = 0$$

$$\begin{vmatrix} k & 1 \\ k & k \end{vmatrix} \times \begin{vmatrix} 1 & k \\ k & 1 \end{vmatrix} = \begin{vmatrix} k & k \\ 1 & k \end{vmatrix}^2$$

$$(k^2 - k)(1 - k^2) = (k^2 - k)^2$$

$$(k^2 - k)(1 - k^2 - k^2 + k) = 0$$

$$k(k-1)(-2k^2 + k + 1) = 0$$

$$k = 0, k = 1, \quad 2k^2 - k - 1 = 0$$

$$2k^2 - 2k + k - 1 = 0$$

$$(2k+1)(k-1) = 0$$

$$k = \cancel{0}, \cancel{1}, -1/2.$$

Both roots are common.

if Both roots are common.

$$\frac{k}{k} = \frac{1}{k} = \frac{k}{1}$$

$$1 = \frac{1}{k} = \frac{k}{1}$$

$$\Downarrow$$

$$k = 1$$

QUESTION

★★ASRQ★★

The equations $kx^2 + x + k = 0$ and $kx^2 + kx + 1 = 0$ have exactly one root in common for

- (A) $k = -1/2, 1$
- (B) $k = 1$
- [✓ (C)] $k = -1/2$
- (D) $k = 1/2$

$$kx^2 + x + k = 0$$
$$kx^2 + kx + 1 = 0$$
$$(1-k)x + k - 1 = 0$$
$$(1-k)x - (1-k) = 0$$
$$(1-k)(x-1) = 0$$

$k = 1$ or $x = 1$

$k = 1$ Both roots are common

if $x = 1$ $\quad k + k + 1 = 0$

$$k = -1/2$$

if Both roots are common.

$$\frac{k}{k} = \frac{1}{k} = \frac{k}{1}$$
$$1 = \frac{1}{k} = \frac{k}{1}$$
$$\Downarrow$$
$$k = 1$$

QUESTION

★★KCLS★★

Tah0 4

If the equations $ax^3 + x + 2 = 0$ and $x^3 + ax + 2 = 0$ have exactly one common root, find the value of $|a|$.

QUESTION

★★KCLS★★

If the quadratic equation $x^2 + bx + c = 0$ & $x^2 + cx + b = 0 (b \neq c)$ have a common root then prove that their uncommon roots are the roots of the equation $x^2 + x + bc = 0$.

① $x^2+bx+c=0 \begin{cases} \alpha \\ \beta \end{cases}$

② $x^2+cx+b=0 \begin{cases} \alpha \\ \gamma \end{cases}$

To show: β, γ are roots of $x^2+x+bc=0$

$\cancel{(b-c)}x + \cancel{c-b}=0 \; \alpha$

$x-1=0 \; \alpha$

$x=1=\alpha$

$\Rightarrow$ from ① $1\cdot\beta=c \Rightarrow \beta=c$

$1\cdot\gamma=b \Rightarrow \gamma=b$

Q·Eqn with β, γ as roots.

$x^2-(b+c)x+bc=0$

Since $\alpha=1$ is a root of ①

$1+b+c=0 \Rightarrow b+c=-1$

$x^2+x+bc=0 \begin{cases} \beta \\ \gamma \end{cases}$

QUESTION

★★ASRQ★★

If the quadratic equations $x^2 + ax + 12 = 0$ and $x^2 + bx + 15 = 0$ and $x^2 + (a + b)\,x + 36 = 0$ have a common positive root find 'a' and 'b' and the root of the equation.

$\alpha > 0$,

$x^2+ax+12=0 \;<\; \alpha,\ \beta$

$x^2+bx+15=0 \;<\; \alpha,\ \gamma$

$\oplus \Rightarrow 2x^2+(a+b)x+27=0 \quad \alpha$

$x^2+(a+b)x+36=0 \;<\; \alpha,\ \delta$

$\ominus \Rightarrow x^2-9=0 \quad \alpha$

$x=-3,3$

Since α is +ve $\boxed{\alpha=3}$

$\alpha\beta=12 \Rightarrow \beta=4 \qquad \alpha+\beta=-a$

$3+4=-a \Rightarrow a=-7$

$\alpha\gamma=15 \Rightarrow \gamma=5 \qquad \alpha+\gamma=-b \Rightarrow b=-8$

[Ans. a = –7, b = –8; roots are (3, 4), (3, 5) and (3, 12)]

QUESTION

★★ASRQ★★

If the equations $x^3 + x^2 - 4x = 4$ and $x^2 + px + 2p = 0 (p \in R)$ have two roots common, then the value of p is

(A) –2

[✓ (B)] –1

(C) 1

(D) 3

$x^3+x^2-4x-4=0$ γ, α, β , $x^2+px+2p=0 < \alpha, \beta$

2 roots common

$x^2(x+1)-4(x+1)=0$

$(x^2-4)(x+1)=0$

$x = 2, -1, -2$

✓ ✓ ✓

× ×

$\alpha+\beta=-p$

$\alpha\beta=2p$

$\alpha=2$

$\beta=-1$

$\alpha+\beta = 1 = -p \Rightarrow p=-1$

$\alpha\beta = 2 \cdot -1 = 2p \Rightarrow p=-1$

$p=-1$

QUESTION

★★KCLS★★

The equations $x^3+4x^2+px+q=0$ and $x^3+6x^2+px+r=0$ have two common roots, where $p, q, r \in R$. If their uncommon roots are the roots of equation $x^2+2ax+8c=0$, then

[✓ (A)] $a+c=8$

(B) $a+c=2$

[✓ (C)] $3q=2r$

(D) $3r=2q$

$x^3+4x^2+px+q=0 < \begin{matrix}\alpha\\ \beta\\ \gamma\end{matrix}$

$x^3+6x^2+px+r=0 < \begin{matrix}\alpha\\ \beta\\ \delta\end{matrix}$

γ, δ are roots of $x^2+2ax+8c=0$

$-2x^2+q-r=0 < \begin{matrix}\alpha\\ \beta\end{matrix}$

$2x^2+r-q=0 < \begin{matrix}\alpha\\ \beta\end{matrix}$

clearly: $\alpha+\beta=0$

Now $\alpha+\beta+\gamma=-4$ $\quad$ $\alpha+\beta+\delta=-6$

$\gamma=-4$ $\quad$ $\delta=-6.$

Eqn with γ & δ as roots.

$x^2+10x+24=0$

$\Rightarrow a=5, c=3$

$\alpha\beta\gamma=-q$

$\alpha\beta\delta=-r$

$\frac{\alpha\beta\gamma}{\alpha\beta\delta}=\frac{q}{r}$

$\frac{2}{3}=\frac{q}{r}$

$3q=2r.$

QUESTION

$(x-2)$ $\quad x^2-3x+2 \frown 2$

$x^2-6x+8 \frown 2$

If $x^2 + 3x + 5$ is the greatest common divisor of $(x^3 + ax^2 + bx + 1)$ and $(2x^3 + 7x^2 + 13x + 5)$ then find the value of $[a + b]$.

[Note: $[k]$ denotes greatest integer less than or equal to k.]

$$\begin{matrix}\alpha \\ \beta\end{matrix} \Big\rangle\, x^3+ax^2+bx+1=0 \times 2$$

Both are divisible by $x^2+3x+5 \begin{matrix}\alpha \\ \beta\end{matrix} = 0$

$$\begin{matrix}\alpha \\ \beta\end{matrix} \Big\rangle\, 2x^3+7x^2+13x+5=0$$

$$(2a-7)x^2+(2b-13)x-3=0 \begin{matrix}\alpha \\ \beta\end{matrix}.$$

$$\frac{2a-7}{1} = \frac{2b-13}{3} = \frac{-3}{5}.$$

$$2a = 7-3/5, \quad 2b = 13-9/5$$

$$a = 16/5 \qquad b = 28/5.$$

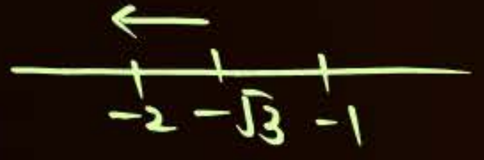
$$[a+b] = \left[\frac{44}{5}\right] = 8 \text{ Ans}$$

$f(x)$ — root α ——— $f(\alpha)=0$

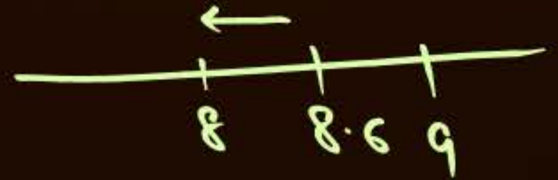
$g(x)$ — root α ——— $g(\alpha)=0$

$h(x) = A f(x) \pm B g(x)$ — root α.

$$h(\alpha) = A f(\alpha) \pm B g(\alpha) = 0$$



$[-\sqrt{3}] = -2.$



$[8\cdot6] = 8$

$[\sqrt{2}] = 1$

QUESTION

★★ASRQ★★

If $Q_1(x) = x^2 + (k - 29)x - k$ and $Q_2(x) = 2x^2 + (2k - 43)x + k$ both are factors of cubic polynomial, then largest value of k is (where $Q_1(x)$ & $Q_2(x)$ are not perfect squares)

- A 0
- B 33
- C 23
- D 30

$Q_1(x) = x^2 + (k-29)x - k <^{\alpha}_{\beta}$.

$Q_2(x) = 2x^2 + (2k-43)x + k <^{r}_{\delta}$

Both are factors of a cubic

$ax^3 + bx^2 + cx + d <^{\alpha,\ \beta}_{r,\ \delta}$

↓

N.P.

$Q_1(x)$ & $Q_2(x)$ should have a common root.

⇓

$x^2 + (k-29)x - k = 0 <^{\alpha}_{\beta}$ $\times 2$ ①

$2x^2 + (2k-43)x + k = 0 <^{\alpha}_{\delta}$.

$-15x - 3k = 0 \to \alpha$

$x = -k/5$. put in ①

$$\left(\frac{-k}{5}\right)^2 - \frac{k}{5}(k-29) - k = 0$$

$$k\left[\frac{k}{25} - \frac{(k-29)}{5} - 1\right] = 0$$

$$k = 0 \quad , \quad k - 5k + 145 - 25 = 0$$

$$4k = 120$$

$$k = 30.$$

QUESTION

Tah05

If two roots of the equation $(x-1)(2x^2-3x+4)=0$ coincide with roots of the equation $x^3+(a+1)x^2+(a+b)x+b=0$ where $a, b \in R$ then $2(a+b)$ equals

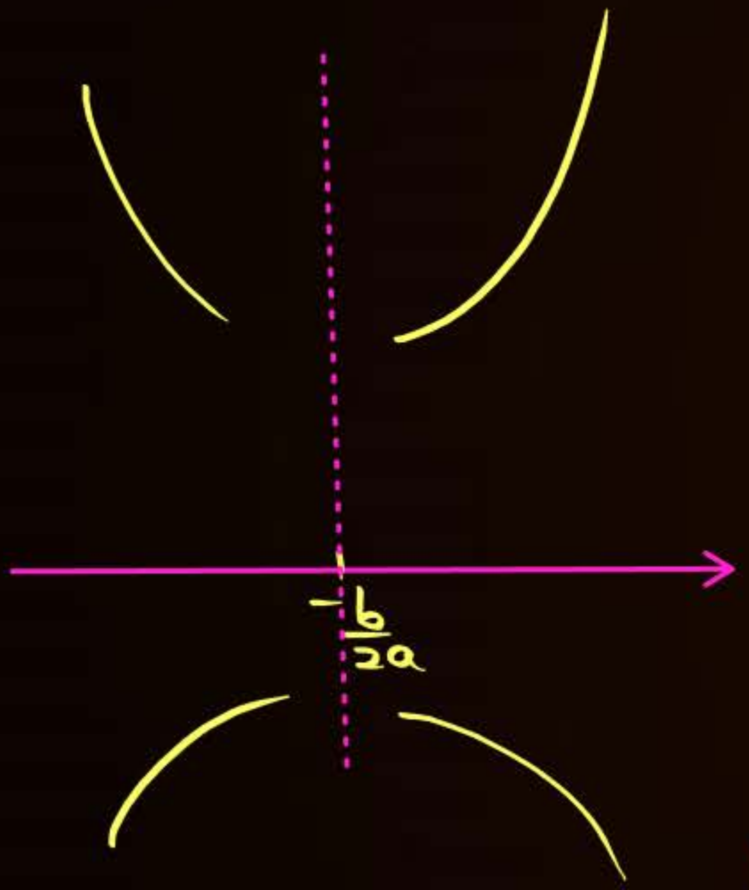
A 4

B 2

C 1

D 0

Analysis of a Quadratic Polynomial



$$p(x) = ax^2+bx+c \quad , a\neq 0$$

$$= a\left(x^2+\frac{b}{a}x\right)+c$$

$$= a\left(x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2\right)+c.$$

$$= a\left(x^2+\left(\frac{b}{a}\right)x+\frac{b^2}{4a^2}\right)-\frac{b^2}{4a}+c.$$

$$= a\left(x+\frac{b}{2a}\right)^2-\left(\frac{b^2}{4a}-c\right)$$

$$= a\left(x+\frac{b}{2a}\right)^2-\left(\frac{b^2-4ac}{4a}\right)$$

$$P(x) = a(x+b/2a)^2-\frac{D}{4a}$$

$$P\left(-\frac{b}{2a}+t\right) = at^2-\frac{D}{4a}$$

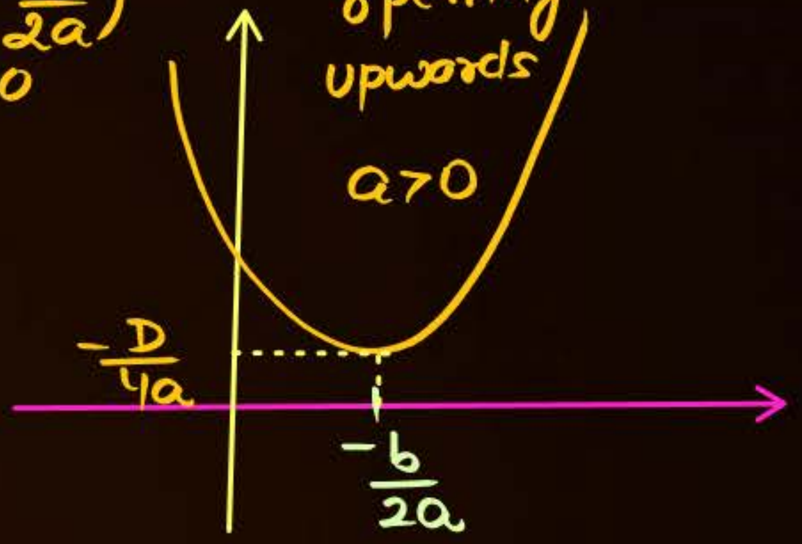
$$P\left(-\frac{b}{2a}-t\right) = at^2-\frac{D}{4a}.$$

$$P(-b/2a+t) = P(-b/2a-t)$$

$$\Downarrow$$

graph of $y=ax^2+bx+c$ is symm about $x=-b/2a$.

$$P(x) = a\underbrace{\left(x+\frac{b}{2a}\right)^2}_{\geq 0} - \frac{D}{4a} = -\frac{D}{4a} + a\underbrace{\left(x+\frac{b}{2a}\right)^2}_{\geq 0}$$



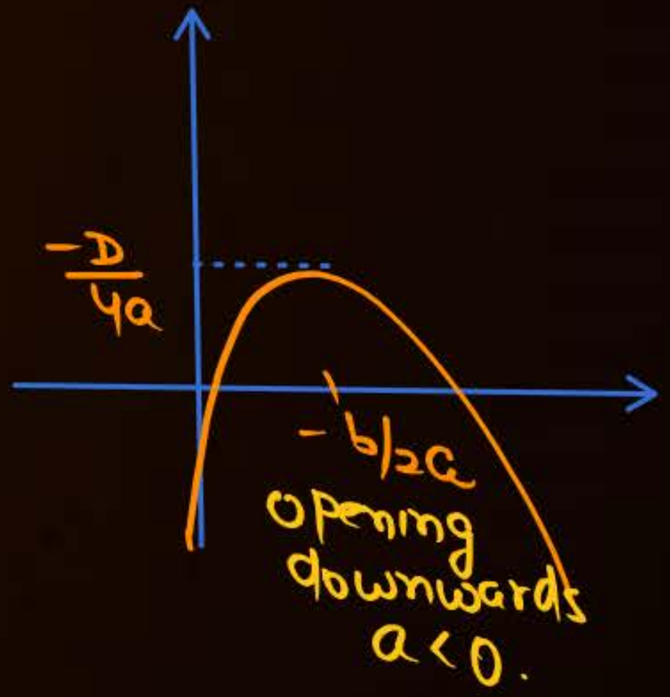
$\underbrace{\text{if } a>0}$ $\quad P(x)\Big|_{min} = -\frac{D}{4a}$ at $x = -\frac{b}{2a}$

$P(x)\Big|_{MAX} \rightsquigarrow \infty$

$\underbrace{\text{if } a<0}$ $\quad P(x) = a\left(x+\frac{b}{2a}\right)^2 - \frac{D}{4a} = -\frac{D}{4a} + \underbrace{a\overset{\geq 0}{\left(x+\frac{b}{2a}\right)^2}}_{\leq 0}$

$P(x)\Big|_{MAX} = -\frac{D}{4a}$ at $x = -\frac{b}{2a}$

$P(x)\Big|_{min} \rightsquigarrow -\infty$



Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Home Challenge - 08

The ordered pair (x, y) satisfying the equation $x^2 = 1 + 6\log_4 y$ and $y^2 = 2^x y + 2^{2x+1}$ are (x_1, y_1) and (x_2, y_2), then find the value of $\log_2|x_1x_2y_1y_2|$.

(Ans:7)

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

(KTK 1)

If α, β are the root of a quadratic equation $x^2 - 3x + 5 = 0$ then the equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$ is

(A) $x^2 + 4x + 1 = 0$

(B) $x^2 - 4x + 4 = 0$

(C) $x^2 - 4x - 1 = 0$

(D) $x^2 + 2x + 3 = 0$

Ans. B

QUESTION

(KTK 2)

The equations $ax^2 + bx + a = 0 (a, b \in R)$ and $x^3 - 2x^2 + 2x - 1 = 0$ have 2 roots common. Then $a + b$ must be equal to

Ⓐ 1

Ⓑ -1

Ⓒ 0

Ⓓ None of these

Ans. C

QUESTION

(KTK 3)

The value of m for which the equation $\frac{a}{x+a+m}+\frac{b}{x+b+m}=1$ has roots equal in magnitude and opposite in signs is

A $\frac{a-b}{a+b}$

B -1

C 0

D $\frac{a+b}{a-b}$

Ans. C

QUESTION

(KTK 4)

Find the values of 'k' so that the equation $x^2 + kx + (k+2) = 0$ and $x^2 + (1-k)x + 3 - k = 0$ have exactly one common root.

Ans. No possible value of k

QUESTION

(KTK 5)

Given a, b are two distinct real number satisfying

$a^2 - 5a + 2 = 0$ and $b^2 - 5b + 2 = 0$ then $(1 - ab + a^2b + b^2a)$

QUESTION

(KTK 6)

Let 'p' is a root of the equation $x^2 - x - 3 = 0$. Then the value of $\frac{p^3+1}{p^5-p^4-p^3+p^2}$ is equal to

(A) $\frac{4}{3}$

(B) $\frac{4}{9}$

(C) $\frac{2}{9}$

(D) $\frac{2}{3}$

QUESTION

(KTK 7)

If α and β are the roots of $ax^2 + bx + c = 0$, then the equation whose roots are $\frac{\alpha+1}{\alpha-2}$ and $\frac{\beta+1}{\beta-2}$ is

A $a(x+1)^2 + b(x+1)(x-2) + c(x-2)^2 = 0$

B $a(x-2)^2 + b(x+1)(x-2) + c(x+1)^2 = 0$

C $a(2x+3)^2 + b(x+1)(x+2) + c(x+2)^2 = 0$

D $a(2x+1)^2 + b(2x+1)(x-1) + c(x-1)^2 = 0$

Ans. D

QUESTION

(KTK 8)

If α, β, γ are roots $x^3 + 2x^2 - 3x + 1 = 0$, then value of $\frac{\alpha\beta}{\alpha+\beta} + \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\beta\gamma}{\beta+\gamma}$ is less than

A 2

B 3

C 4

D 5

Homework From Module

Quadratic Equations

Prarambh (Topicwise) : Q1 to Q27

Prabal (JEE Main Level) : Q1,Q2,Q6 to Q9

Parikshit (JEE Advanced Level) : Abhi Ruko

Solution to Previous TAH

QUESTION

If α, β and γ are roots of $3x^3 - 4x^2 - 3x + 2 = 0$ and

$$(\alpha^5 + \beta^5 + \gamma^5) - (\alpha^3 + \beta^3 + \gamma^3) = \frac{2}{m}\left(2(\alpha^4 + \beta^4 + \gamma^4) - (\alpha^2 + \beta^2 + \gamma^2)\right)$$

Then value of m is ________

Q-1 (TAH-1): If α, β, γ are roots of $3x^3-4x^2-3x+2=0$ and $(\alpha^5+\beta^5+\gamma^5)-(\alpha^3+\beta^3+\gamma^3)=\frac{2}{m}\left(2(\alpha^4+\beta^4+\gamma^4)-(\alpha^2+\beta^2+\gamma^2)\right)$ Then $m=?$

Soln

$$\left.3x^3-4x^2-3x+2=0 \begin{matrix}\nearrow \alpha \\ \rightarrow \beta \\ \searrow \gamma\end{matrix}\right\} \quad S_n=\alpha^n+\beta^n+\gamma^n$$

$$\underbrace{(\alpha^5+\beta^5+\gamma^5)}_{S_5}-\underbrace{(\alpha^3+\beta^3+\gamma^3)}_{S_3}=\frac{2}{m}\left(2\underbrace{(\alpha^4+\beta^4+\gamma^4)}_{S_4}-\underbrace{(\alpha^2+\beta^2+\gamma^2)}_{S_2}\right)$$

$$\Rightarrow S_5-S_3=\frac{2}{m}\left[2S_4-S_2\right]$$

By N.F.: $3S_{n+3}-4S_{n+2}-3S_{n+1}+2S_n=0$.

$n=2 \rightarrow \quad 3S_5-4S_4-3S_3+2S_2=0$

$\Rightarrow 3(S_5-S_3)-2(2S_4-S_2)=0$.

TAH 1
BY REED

$\Rightarrow \quad S_5-S_3=\dfrac{2(2S_4-S_2)}{3}$

$\therefore \quad S_5-S_3=\dfrac{2}{\underset{\searrow m}{\boxed{3}}}(2S_4-S_2)$

$\boxed{\therefore m=3}$ (Ans.)

Homework

Mathematics

Tah : 01

$$3x^3-4x^2-3x+2=0 \begin{array}{l}\to\alpha\\ \to\beta\\ \to\gamma\end{array} \longrightarrow (\alpha^5+\beta^5+\gamma^5)-(\alpha^3+\beta^3+\gamma^3)=\frac{2}{m}\left(2(\alpha^4+\beta^4+\gamma^4)-(\alpha^2+\beta^2+\gamma^2)\right) \longrightarrow \text{find value of } m$$

$$S_n=\alpha^n+\beta^n+\gamma^n$$

$$\Rightarrow 3S_5-4S_4-3S_3+2S_2=0$$

$$\Rightarrow 3(S_5-S_3)=4S_4-2S_2$$

$$\Rightarrow 3(S_5-S_3)=2(2S_4-S_2)$$

$$\Rightarrow S_5-S_3=\frac{2}{3}(2S_4-S_2)$$

$$(\alpha^5+\beta^5+\gamma^5)-(\alpha^3+\beta^3+\gamma^3)=\frac{2}{m}(2(\alpha^4+\beta^4+\gamma^4)-(\alpha^2+\beta^2+\gamma^2)$$

$$S_5-S_3=\frac{2}{m}(2S_4-S_2)$$

On comparing; $m=3$

QUESTION

★★KCLS★★

Let α and β are two real roots of $x^2 + 10x - 7 = 0$. Then

(A) $\dfrac{\alpha^{20} + \beta^{20} - 7(\alpha^{18} + \beta^{18})}{\alpha^{19} + \beta^{19}} = -10$

(B) $\dfrac{\alpha\beta^{18} - 7\alpha\beta^{16} - 10\alpha^{17}\beta}{\alpha^{16} + \beta^{16}} = 70$

(C) $\sqrt{\left(\alpha - \dfrac{7}{\alpha}\right)\left(\beta - \dfrac{7}{\beta}\right)} = 10$

(D) $\dfrac{\alpha^3 + 9\alpha^2 - 17\alpha + 14}{\beta^3 + 11\beta^2 + 3\beta - 8} = 7$

Tah-02 Let α and β are two real roots of $x^2+10x-7=0$ Then

(A) $\frac{\alpha^{20}+\beta^{20}-7(\alpha^{18}+\beta^{18})}{\alpha^{19}+\beta^{19}}=-10$.

Let $S_n=\alpha^n+\beta^n$

$\frac{S_{20}-7(S_{18})}{S_{19}}=E$

$S_{n+2}+10S_{n+1}-7S_n=0$

$n=18 \to S_{20}+10S_{19}-7S_{18}=0$

$S_{20}-7S_{18}=-10S_{19}$

$\therefore E=\frac{S_{20}-7S_{18}}{S_{19}}=-10$ true ✓

Durgesh Up.

(B) $\frac{\alpha\beta^{18}-7\alpha\beta^{16}-10\alpha^{17}\beta}{\alpha^{16}+\beta^{16}}=70$

$\beta^2+10\beta-7=0$

$\beta^2-7=-10\beta$

$\frac{\alpha\beta^{16}(\beta^2-7)-10\alpha^{17}\beta}{\alpha^{16}+\beta^{16}}$

$\Rightarrow \frac{\alpha\beta^{16}(-10\beta)-10\alpha^{17}\beta}{\alpha^{16}+\beta^{16}}$

$\Rightarrow -10(\alpha\beta)\frac{\cancel{(\beta^{16}+\alpha^{16})}}{\cancel{(\alpha^{16}+\beta^{16})}}$ $(\alpha.\beta=-7)$

$\Rightarrow -10(-7)=+70$ True.

(C) $\sqrt{\left(\alpha-\frac{7}{\alpha}\right)\left(\beta-\frac{7}{\beta}\right)}=10$

$\left(\left(\frac{\alpha^2-7}{\alpha}\right)\left(\frac{\beta^2-7}{\beta}\right)\right)^{1/2}$

$\left(\left(\frac{-10\cancel{\alpha}}{\cancel{\alpha}}\right)\times\left(\frac{-10\cancel{\beta}}{\cancel{\beta}}\right)\right)^{1/2}$

$(100)^{1/2}=\boxed{10}$ Ans

True

(D) $\frac{\alpha^3+9\alpha^2-17\alpha+14}{\beta^3+11\beta^2+3\beta-8}=7$

$=\frac{\alpha^3+10\alpha^2-7\alpha-\alpha^2-10\alpha+14}{\beta^3+10\beta^2-7\beta+\beta^2+10\beta-8}$

$=\frac{\alpha(\alpha^2+10\alpha-7)-(\alpha^2+10\alpha-7)+7}{\beta(\beta^2+10\beta-7)+(\beta^2+10\beta-7)-1}$

$\frac{\alpha\cancel{(\alpha^2+10\alpha-7)}-\cancel{(\alpha^2+10\alpha-7)}+7}{\beta\underbrace{\cancel{(\beta^2+10\beta-7)}}_{0}+\underbrace{\cancel{(\beta^2+10\beta-7)}}_{0}-1}$

$=\frac{+7}{-1}=-7$

Q-2: Let α and β are two real roots of $x^2+10x-7=0$ then:

Soln: option(a):

$\frac{\alpha^{20}+\beta^{20}-7(\alpha^{18}+\beta^{18})}{\alpha^{19}+\beta^{19}}$

$=\frac{\alpha^{18}(\alpha^2-7)+\beta^{18}(\beta^2-7)}{\alpha^{19}+\beta^{19}}$

$=\frac{-10\cancel{(\alpha^{19}+\beta^{19})}}{\cancel{(\alpha^{19}+\beta^{19})}}=\boxed{-10}$ ✓

$x^2+10x-7=0 \begin{matrix}\alpha\\ \beta\end{matrix}$

$\alpha^2+10\alpha-7=0$.

$\Rightarrow \alpha^2-7=-10\alpha$

$\beta^2+10\beta-7=0$

$\Rightarrow \beta^2-7=-10\beta$

TAH 2
BY REED

option(b)

$E=\frac{\alpha\beta^{18}-7\alpha\beta^{16}-10\alpha^{17}\beta}{\alpha^{16}+\beta^{16}}$

$\Rightarrow E=\frac{\alpha\beta^{16}(\beta^2-7)-10\alpha^{17}\beta}{\alpha^{16}+\beta^{16}}$

$\beta^2-7=-10\beta$

$\Rightarrow E=\frac{\alpha\beta^{16}(-10\beta)-10\alpha^{17}\beta}{\alpha^{16}+\beta^{16}}$

P.O.R $=\alpha\beta=-7$

$=\frac{-10\alpha\beta\cancel{(\beta^{16}+\alpha^{16})}}{\cancel{(\alpha^{16}+\beta^{16})}}=-10\times(-7)=\boxed{70}$ ✓

option-(c):

$\sqrt{\left(\alpha-\frac{7}{\alpha}\right)\left(\beta-\frac{7}{\beta}\right)}$

$=\sqrt{\left(\frac{\alpha^2-7}{\alpha}\right)\left(\frac{\beta^2-7}{\beta}\right)}$

$=\sqrt{\frac{(-10\alpha)\cdot(-10\beta)}{\alpha\beta}}$

$=\sqrt{100}=\boxed{10}$ ✓

(i) $\alpha^2+10\alpha-7=0$
(ii) $\beta^2+10\beta-7=0$
(iii) $\alpha^2+10\alpha=7$
(iv) $\beta^2+10\beta=7$

option-(d):

$E=\frac{\alpha^3+9\alpha^2-17\alpha+14}{\beta^3+11\beta^2+3\beta-8}$

$\Rightarrow E=\frac{\alpha(\alpha^2+9\alpha-17)+14}{\beta(\beta^2+11\beta+3)-8}$

$\Rightarrow E=\frac{\alpha(\alpha^2+10\alpha-7-\alpha-10)+14}{\beta(\beta^2+10\beta-7+\beta+10)-8}$

$\Rightarrow E=\frac{\alpha(0-\alpha-10)+14}{\beta(0+\beta+10)-8}$

$\Rightarrow E=\frac{-(\alpha^2+10\alpha)+14}{(\beta^2+10\beta)-8}$

$\Rightarrow E=\frac{-7+14}{7-8}=\frac{7}{-1}=-7$

$\therefore$ Ans $\Rightarrow$ (a),(b),(c).

$\therefore$ (d) is incorrect.

QUESTION [JEE Mains 2025 (23 Jan)]

If the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ has equal roots, where $a + c = 15$ and $b = \frac{36}{5}$, then $a^2 + c^2$ is equal to

Ans. 117

Q-3) If the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ has equal roots, where $a+c=15$ and $b=\frac{36}{5}$, then $a^2+c^2=??$

Solⁿ

$$\underbrace{a(b-c)}_{A}x^2+\underbrace{b(c-a)}_{B}x+\underbrace{c(a-b)}_{C}=0 \begin{matrix}\nearrow\alpha\\ \searrow\beta\end{matrix} \quad (\alpha=\beta)$$

$$A+B+C = \cancel{ab}-\cancel{ac}+\cancel{bc}-\cancel{ab}+\cancel{ac}-\cancel{bc}=0.$$

∴ roots are $1, \beta$

but since roots are equal $\Rightarrow \boxed{\beta=1}$

$$S.O.R = 2 = \frac{-b(c-a)}{a(b-c)}$$

$\Rightarrow 2ab-2ac = ab-bc$

$\Rightarrow ab+bc = 2ac.$

$\Rightarrow b(a+c) = 2ac.$

$\Rightarrow \frac{36}{\cancel{5}}\times \overset{3}{\cancel{15}} = 2ac$

$\Rightarrow \boxed{ac = 54}$

∴ a^2+c^2

$=(a+c)^2-2ac$

$= 225-2\times 54$

$= 225-108$

$= 117$ (Ans)

TAH 3
BY REED

Q-4) $\frac{(x-a)(x-b)}{(c-a)(c-b)}+\frac{(x-b)(x-c)}{(a-b)(a-c)}+\frac{(x-c)(x-a)}{(b-c)(b-a)}=1$

TAH 4
BY REED

Solⁿ put $x=a$, $L.H.S. = 0+\frac{(a-b)(a-c)}{(a-b)(a-c)}+0 = 0+1+0$
$= 1 = R.H.S.$

∴ $x=a$ is a root.

a, b, c should be distinct !!

put $x=b$, $L.H.S. = 0+0+\frac{(b-c)(b-a)}{(b-c)(b-a)}$
$= 0+0+1 = R.H.S.$ ∴ $x=b$ is a root.

put $x=c$, $L.H.S. = 1+0+0 = 1 = R.H.S.$ ∴ $x=c$ is a root.

∴ This quadratic has more than 2 roots, hence it is an identity. $\Rightarrow \infty$ly many solⁿs of x.

QUESTION

Given, the cubic equation $x^3 - 5x^2 + 6x - 3 = 0$ has roots α, β, γ. Find the cubic having roots

(i) $\alpha + 1, \beta + 1, \gamma + 1$

(ii) $\alpha - 1, \beta - 1, \gamma - 1$

(iii) $-\alpha, -\beta, -\gamma$

(iv) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

(v) $\frac{3\alpha-2}{\alpha+1}, \frac{3\beta-2}{\beta+1}, \frac{3\gamma-2}{\gamma+1}$

Q-5) Given the cubic equn $x^3-5x^2+6x-3=0$ has roots α, β, γ. Find the cubic having roots:

Soln) **(i)** $\alpha-1, \beta-1, \gamma-1$ are roots of:

$f(x) = x-1$

$\Rightarrow y = x-1 \Rightarrow \boxed{x = y+1}$

Put $x = y+1$ in equn $\Rightarrow (y+1)^3 - 5(y+1)^2 + 6(y+1) - 3 = 0$

$\Rightarrow y^3+1+3y^2+3y-5y^2-10y-5+6y+6-3=0$

$\Rightarrow y^3-2y^2-y-1=0$

replace $y \to x$

$x^3-2x^2-x-1=0$ (Ans)

(ii) $-\alpha, -\beta, -\gamma$ are roots of:

$f(x) = -x \Rightarrow y = -x \Rightarrow x = -y.$

Put $x=-y$, $-y^3-5y^2-6y-3=0$

$\Rightarrow y^3+5y^2+6y+3=0$

replace y by x

$\Rightarrow x^3+5x^2+6x+3=0.$

TAH 5
By Reed
from WB

(iv) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ are roots of:

$\frac{1}{x} = y \Rightarrow x = \frac{1}{y}.$

put $x = \frac{1}{y}$ in equn $\Rightarrow \left(\frac{1}{y}\right)^3 - 5\left(\frac{1}{y^2}\right) + 6\left(\frac{1}{y}\right) - 3 = 0$

$\Rightarrow 1-5y+6y^2-3y^3=0$

replace y by $x \Rightarrow 3x^3-6x^2+5x-1=0$. (Ans.)

(v) $\frac{3\alpha-2}{\alpha+1}, \frac{3\beta-2}{\beta+1}, \frac{3\gamma-2}{\gamma+1}$ are roots of:

put $\frac{3x-2}{x+1} = y$

$\Rightarrow 3x-2 = xy+y$

$\Rightarrow x(3-y) = y+2$

$\Rightarrow x = \frac{y+2}{3-y}$

Put $x = \frac{y+2}{3-y}$ in the main equn,

TAH 5

$$\left(\frac{y+2}{3-y}\right)^3 - 5\left(\frac{y+2}{3-y}\right)^2 + 6\left(\frac{y+2}{3-y}\right) - 3 = 0.$$

or, $(y+2)^3 - 5(y+2)^2(3-y) + 6(y+2)(3-y)^2 - 3(3-y)^3 = 0.$

or, $y^3+8+6y^2+12y-5(y^2+2y+4)(3-y)+6(y+2)(9-y^2+6y)$
$-3(27-y^3-27y+9y^2)=0$

or, $y^3+8+6y^2+12y-5(3y^2-y^3+12-4y+12y-4y^2)$
$+6(y^3+9y-6y^2+2y^2+18-12y)$
$-3(27-y^3-27y+9y^2)=0$

or, $15y^3-40y^2+35y-25=0$

or, $3y^3-8y^2+7y-5=0$

replace $y \to x \Rightarrow 3x^3-8x^2+7x-5=0$

Required Equn.

Homework

$x^3-5x^2+6x-3=0 \begin{cases} \to \alpha \\ \to \beta \\ \to \gamma \end{cases} \longrightarrow$ find cubic

(a) $\alpha+1,\ \beta+1,\ \gamma+1$

$y=x+1$

$\Rightarrow x=y-1$

$\rightarrow (y-1)^3-5(y-1)^2+6(y-1)-3=0$

$\Rightarrow y^3-1-3y^2+3y-5y^2-5+10y+6y-6-3=0$

$\Rightarrow x^3-8x^2+19y-15=0$

(b) $-\alpha,-\beta,-\gamma$

$y=-x$

$\Rightarrow x^3+5x^2+6x+3=0$

(c) $\alpha-1,\beta-1,\gamma-1$

$y=x-1 \Rightarrow x=y+1$

$\Rightarrow y^3+1+3y^2+3y-5y^2-5-10y+6y+6-3=0$

$\Rightarrow x^3-2y^2-y-1=0$

(d) $\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma}$

$y=\frac{1}{x} \Rightarrow x=\frac{1}{y}$

$\Rightarrow \frac{1}{y^3}-\frac{5}{y^2}+\frac{6}{y}-3=0$

$\Rightarrow 3x^3-6x^2+5x-1=0$

(e) $\frac{3\alpha-2}{\alpha+1},\frac{3\beta-2}{\beta+1},\frac{3\gamma-2}{\gamma+1}$

$x^3-8x^2+19y-15=0 \begin{cases} \to \alpha+1 \nearrow \phi \\ \to \beta+1 \nearrow \delta \\ \to \gamma+1 \nearrow \partial \end{cases}$

$\longrightarrow \frac{3\alpha-2}{\alpha+1}=\frac{3\phi-5}{\phi}$

Now we need to find eqⁿ having roots $3-\frac{5}{\phi},3-\frac{5}{\delta},3-\frac{5}{\partial}$

$y=3-\frac{5}{x} \Rightarrow 3-y=\frac{5}{x}$

$\Rightarrow x=\frac{5}{3-y} \to t$

$\Rightarrow \frac{125}{t^3}-\frac{200}{t^2}+\frac{95}{t}-15=0$

$\Rightarrow 15t^3-95t^2+200t-125=0$

$\Rightarrow 3t^3-19t^2+40t-25=0$

$\Rightarrow 3(3-y)^3-19(3-y)^2+40(3-y)-25=0$

$\Rightarrow 3(27-y^3-27y+9y^2)-19(9+y^2-6y)$
$+40(3-y)-25=0$

$\Rightarrow 81-3y^3-81y+27y^2-171-19y^2+114y$
$+120-40y-25$

$\Rightarrow 3x^3-8x^2+7x-5$ Ans

QUESTION

Let α, β and γ are the roots of the cubic $x^3 - 3x^2 + 1 = 0$. Find a cubic whose roots are $\frac{\alpha}{\alpha-2}, \frac{\beta}{\beta-2}$ and $\frac{\gamma}{\gamma-2}$. Hence or otherwise find the value of $(\alpha - 2)(\beta - 2)(\gamma - 2)$.

Ans. $3y^3 - 9y^2 - 3y + 1 = 0; (\alpha - 2)(\beta - 2)(\gamma - 2) = 3$

Q-61 If α, β, γ are roots of $x^3 - 3x^2 + 1 = 0$.
Find a cubic whose roots are $\frac{\alpha}{\alpha-2}, \frac{\beta}{\beta-2}, \frac{\gamma}{\gamma-2}$.
Hence or otherwise find value of $(\alpha-2)\cdot(\beta-2)\cdot(\gamma-2)$:

Soln:
$$\frac{\alpha}{\alpha-2} = f(\alpha), \quad \frac{\beta}{\beta-2} = f(\beta), \quad \frac{\gamma}{\gamma-2} = f(\gamma)$$
$$\Downarrow$$
$$f(x) = \frac{x}{x-2}$$
$$\Rightarrow y = \frac{x}{x-2}$$
$$\Rightarrow x = xy - 2y$$
$$\Rightarrow x(1-y) = -2y$$
$$\Rightarrow \boxed{x = \frac{2y}{y-1}}$$

TAH 6
BY REED
FROM WB

put $x = \frac{2y}{y-1}$
in the main equn
↓
$$\left(\frac{2y}{y-1}\right)^3 - 3\left(\frac{2y}{y-1}\right)^2 + 1 = 0$$
$$\Rightarrow 8y^3 - 3\cdot 4y^2(y-1) + 1(y-1)^3 = 0$$
$$\Rightarrow 8y^3 - 12y^3 + 12y^2 + y^3 - 1 - 3y^2 + 3y = 0$$
$$\Rightarrow -4y^3 + y^3 + 9y^2 + 3y - 1 = 0$$
$$\Rightarrow 3y^3 - 9y^2 - 3y + 1 = 0. \Longrightarrow (y \to x) \quad \boxed{3x^3 - 9x^2 - 3x + 1 = 0}$$

2nd: $x^3 - 3x^2 + 1 = (x-\alpha)(x-\beta)(x-\gamma)$
$$\Rightarrow x^3 - 3x^2 + 1 = -(\alpha - x)(\beta - x)(\gamma - x)$$
↓
put $x = 2 \Rightarrow 8 - 3\cdot 4 + 1 = -(\alpha-2)(\beta-2)(\gamma-2)$
$$\Rightarrow (\alpha-2)(\beta-2)(\gamma-2) = -(-3) = \boxed{3}$$ Ans.

* Tah ob:-

$x^3-3x^2+1=0$ has α, β, γ are roots.

$y=f(x)=\frac{x}{x-2} \Rightarrow x=\frac{2y}{y-1}$ put in eqn.

$$\left(\frac{2y}{y-1}\right)^3-3\left(\frac{2y}{y-1}\right)^2+1=0$$

#Ankush

$$\frac{8y^3}{(y-1)^3}-\frac{12y^2}{(y-1)^2}+1=0$$

$$8y^3-12y^2(y-1)+(y-1)^3=0$$

$$8y^3-12y^3+12y^2+y^3-1-3y^2+3y=0$$

$$-3y^3+9y^2+3y-1=0$$

$$3y^3-9y^2-3y+1=0$$

$\hookrightarrow \boxed{3x^3-9x^2-3x+1=0}$ Ans

$\Rightarrow \quad y = f(x) = x-2 \Rightarrow x = y+2$ put in eqⁿ.

$(y+2)^3 - 3(y+2)^2 + 1 = 0$

$y^3 + 8 + 3y^2 \cdot 2 + 3 \cdot 4 \cdot y - 3(y^2 + 4 + 4y) + 1 = 0$

$y^3 + 6y^2 + 12\cancel{y} + 8 - 3y^2 - 12\cancel{y} - 12 + 1 = 0$

$y^3 + 3y^2 - 3 = 0$

↳ $\left[x^3 + 3x^2 - 3 = 0\right]$ Ans

Ankush!!

QUESTION

If α, β, γ are roots of the cubic $2011x^3 + 2x^2 + 1 = 0$, then find

(i) $(\alpha\beta)^{-1} + (\beta\gamma)^{-1} + (\gamma\alpha)^{-1}$;

(ii) $\alpha^{-2} + \beta^{-2} + \gamma^{-2}$

Ans. (i) 2 ; (ii) -4

• **Q-7:** α, β, γ are roots of $2011x^3+2x^2+1=0$. then find:

(i) $\alpha^{-2}+\beta^{-2}+\gamma^{-2}$

$$2011x^3+2x^2+1=0 \begin{matrix}\rightarrow \alpha \\ \rightarrow \beta \\ \rightarrow \gamma\end{matrix} \quad ①$$

$$\left\{ \begin{matrix} S_1 = \frac{-2}{2011} \\ S_2 = 0. \\ S_3 = \frac{-1}{2011} \end{matrix} \right\}$$

Sol^n: • **M-1:** Normal way!

$$E = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{\alpha^2\beta^2+\beta^2\gamma^2+\alpha^2\gamma^2}{(\alpha\beta\gamma)^2}$$

from ① $\alpha\beta+\beta\gamma+\alpha\gamma = 0.$

S.B.S.

$$\Rightarrow \alpha^2\beta^2+\beta^2\gamma^2+\alpha^2\gamma^2+2\alpha\beta\gamma(\alpha+\beta+\gamma) = 0$$

$$\Rightarrow \alpha^2\beta^2+\beta^2\gamma^2+\alpha^2\gamma^2 = -2\alpha\beta\gamma(\alpha+\beta+\gamma)$$

$$\therefore E = \frac{-2\alpha\beta\gamma(\alpha+\beta+\gamma)}{(\alpha\beta\gamma)^2} = \frac{-2(\alpha+\beta+\gamma)}{\alpha\beta\gamma} = \frac{-2\times\left(\frac{-2}{2011}\right)}{\left(\frac{-1}{2011}\right)}$$

$$\Rightarrow \boxed{E = -4}$$

• **M-2:** Newton's formula!

let, $S_n = \alpha^n+\beta^n+\gamma^n$.

$E = \alpha^{-2}+\beta^{-2}+\gamma^{-2}$

$S_{-2} = \alpha^{-2}+\beta^{-2}+\gamma^{-2}$

from $2011x^3+2x^2+1=0$

By N.F. $2011\,S_{n+3} + 2S_{n+2} + S_n = 0$

put $n=-2$, $2011\,S_1 + 2S_0 + S_{-2} = 0$

$$\Rightarrow \cancel{2011}\left(\frac{-2}{\cancel{2011}}\right) + (2\times3) + S_{-2} = 0.$$

$$\Rightarrow S_{-2} = 2-6 = -4$$

$$\Rightarrow \boxed{S_{-2} = -4 = E}$$

• **M-3:** By manipulation:

$$2011x^3+2x^2+1=0 \begin{matrix}\nearrow\alpha\\ \to\beta\\ \searrow\gamma\end{matrix}$$

$\alpha \hookrightarrow$ $2011\alpha^3+2\alpha^2=-1$

$\Rightarrow \alpha^2(2011\alpha+2)=-1$

$\Rightarrow \frac{1}{\alpha^2}=-(2011\alpha+2)$

$\Big\}$ lly, $\frac{1}{\beta^2}=-(2011\beta+2)$

$\frac{1}{\gamma^2}=-(2011\gamma+2)$

adding $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=-2011(\alpha+\beta+\gamma)-6=E$

$\Rightarrow E=-2011\times\left(\frac{-2}{2011}\right)-6$

$\Rightarrow E=2-6=\boxed{-4}$ (Ans.)

• **M-4:** By transformation:

we have to find a cubic whose roots are $\frac{1}{\alpha^2},\frac{1}{\beta^2},\frac{1}{\gamma^2}$ $\Longleftrightarrow$ $f(x)=\frac{1}{x^2}$

$\Rightarrow y=\frac{1}{x^2}$

$\Rightarrow \boxed{x=\pm\frac{1}{\sqrt{y}}}$

Put $x=\pm\frac{1}{\sqrt{y}}$ in ①,

$$2011\left(\frac{\pm1}{\sqrt{y}}\right)^3+\left(\frac{\pm1}{\sqrt{y}}\right)^2\cdot2+1=0$$

$\Rightarrow \pm2011+2\sqrt{y}+y\sqrt{y}=0.$

$\Rightarrow \pm2011=-\sqrt{y}(2+y)$

S.B.S.

$\Rightarrow (2011)^2=y(4+y^2+4y)$

$$\Rightarrow y^3+4y^2+4y-(2011)^2=0 \begin{matrix}\nearrow\alpha^{-2}\\ \to\beta^{-2}\\ \searrow\gamma^{-2}\end{matrix}$$

$\therefore$ S.O.R $=\alpha^{-2}+\beta^{-2}+\gamma^{-2}=-\frac{4}{1}=\boxed{-4}$

Mathematics

Tah : 06(a)

Homework

$$2011x^3+2x^2+1=0 \begin{matrix}\nearrow \alpha\\ \to \beta\\ \searrow \gamma\end{matrix}$$

$$\to \alpha^{-2}+\beta^{-2}+\gamma^{-2}=?$$

Method - 01

$$2011S_1+2S_0+S_{-2}=0$$

$$S_1=\alpha+\beta+\gamma=\frac{-2}{2011}$$

$$S_0=3$$

$$\Rightarrow -2+6+S_{-2}=0$$

$$S_{-2}=-4$$

$$\Rightarrow \alpha^{-2}+\beta^{-2}+\gamma^{-2}=\boxed{-4}$$

Ans

Method 02

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$$

$$=\frac{\beta^2\gamma^2+\alpha^2\gamma^2+\alpha^2\beta^2}{(\alpha\beta\gamma)^2}\nearrow E$$

$$\alpha\beta\gamma=-1/2011$$

$$\alpha\beta+\beta\gamma+\gamma\alpha=0$$

$$\alpha+\beta+\gamma=-2/2011$$

$$\beta^2\gamma^2+\alpha^2\beta^2+\alpha^2\gamma^2=(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)$$

$$\Rightarrow \beta^2\gamma^2+\alpha^2\beta^2+\alpha^2\gamma^2=\frac{-4}{(2011)}$$

$$\Rightarrow E=\frac{-4\times(2011)^2}{(2011)}=\boxed{-4}$$ Ans

Method 03

$$y=\frac{1}{x^2}\Rightarrow x=\frac{1}{\sqrt{y}}$$

$$\Rightarrow \frac{2011}{y^{3/2}}+\frac{2}{y}+1=0$$

$$\Rightarrow y^{3/2}+2y^{1/2}+2011=0$$

$$\Rightarrow y^{3/2}+2y^{1/2}=-2011$$

$$\Rightarrow y^3+4y+4y^2=(2011)^2$$

$$\Rightarrow x^3+4x^2+4x-(2011)^2=0$$

Sum of roots $=\boxed{-4}$ Ans

Method 04

Method 2.5

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)^2-2\left(\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\alpha\gamma}\right)=-2\left(\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}\right)=\boxed{-4}$$ Ans

$\nearrow 0$

lec01/ QuadraticEquations/ Ashish Sir

Akash

QUESTION

★★ASRQ★★

Let roots of the equation $x^3 + 3x^2 + 4x = 11$ are α, β, γ and the roots of equation $x^3 + lx^2 + mx + n = 0$ (l, m, n $\in$ R) are $\alpha + \beta, \beta + \gamma, \gamma + \alpha$.

Column-I	Column-II
(A) l is equal to	(P) –6
(B) m is equal to	(Q) 6
(C) n is equal to	(R) 13
(D) $(l + m + n)$ is equal to	(S) 23

JAH–06 (b)

$x^3+3x^2+4x-11=0 \begin{cases}\to \alpha \\ \to \beta \\ \to \gamma\end{cases}$ — ①

$\alpha+\beta+\gamma=-3$

$\alpha+\beta \downarrow -3-\gamma$ $\quad$ $\beta+\gamma \downarrow -3-\alpha$ $\quad$ $\gamma+\alpha \downarrow -3-\beta$

$\therefore\ y=-3-x$

$x=-3-y$

$=-(3+y)$ put in ①

$-(3+y)^3+3(3+y)^2-4(3+y)-11=0$

$-(y^3+9y^2+27y+27)+3(9+y^2+6y)$
$-12-4y-11=0$

$\Rightarrow -y^3-9y^2-27y-\cancel{27}+\cancel{27}+3y^2+18y$
$-12-4y-11=0$

$\Rightarrow -y^3-6y^2-13y-23=0$

$\Rightarrow y^3+6y^2+13y+23=0$

Given $x^2+lx^2+mx+n=0$

$\therefore\ l=6 \;/\; m=13 \;/\; n=23$
Ⓠ Ⓡ Ⓢ

$l+m+n = 6+13+23$
$= 42$

Q-8: Let roots of the equation $x^3+3x^2+4x=11$ are α,β,γ and the roots of equation $x^3+lx^2+mx+n=0$ $(l,m,n \in R)$ are $\alpha+\beta, \beta+\gamma, \gamma+\alpha$.

Soln:

$$x^3+3x^2+4x-11=0 \begin{matrix}\nearrow \alpha \\ \rightarrow \beta \\ \searrow \gamma\end{matrix} \text{ -(i)} \quad \left.\right\} \quad \begin{aligned} &\alpha+\beta+\gamma=-3 \\ \Rightarrow\ &\alpha+\beta=-3-\gamma \\ \Rightarrow\ &\beta+\gamma=-3-\gamma \\ \Rightarrow\ &\gamma+\alpha=-3-\beta \end{aligned}$$

to form an equation with roots $\alpha+\beta, \beta+\gamma, \gamma+\alpha$.

$$\underset{-3-\gamma}{\overset{|||}{}}\quad \underset{-3-\gamma}{\overset{|||}{}}\quad \underset{-3-\beta}{\overset{|||}{}}$$

$$y=f(x)=-3-x \Rightarrow x=-3-y$$

So, put $\boxed{x=-3-y}$ in (i):

$$x^3+3x^2+4x-11=0$$
$$\Rightarrow (-3-y)^3+3(-3-y)^2+4(-3-y)-11=0$$
$$\Rightarrow -(3+y)^3+3(3+y)^2-4(3+y)-11=0$$
$$\Rightarrow -27-y^3-27y-9y^2+27+18y+3y^2-4y-23=0$$
$$\Rightarrow y^3+13y+6y^2+23=0.$$ replace $y \rightarrow x$.
$$\Rightarrow x^3+6x^2+13x+23=0$$

Compare with $x^3+lx^2+mx+n=0$ $\rightarrow$ $l=6$, $m=13$, $n=23$.

$\therefore l+m+n=42$.

Ans. ⇒ (A) → (Q) 6, (C) → (S) 23

(B) → (R) 13, (D) → ( ) 42.

QUESTION

If the roots of $p(x) = x^3 + 3x^2 + 4x - 8$ are a, b and c, what is the value of $a^2(1 + a^2) + b^2(1 + b^2) + c^2(1 + c^2)$?

- **Q-14:** If the roots of $p(x) = x^3+3x^2+4x-8$ are a, b, c then what is the value of $a^2(1+a^2)+b^2(1+b^2)+c^2(1+c^2) = ?$

Sol^n: $p(x) = x^3+3x^2+4x-8 = 0 \begin{matrix} \nearrow a \\ \to b \\ \searrow c \end{matrix} \Big\}$ roots —①

$\begin{pmatrix} S_1 = -3 \\ S_2 = ? \end{pmatrix}$

$E = a^2+b^2+c^2+a^4+b^4+c^4 = S_2+S_4$

By N.F. from ① $\Rightarrow$ $S_{n+3}+3S_{n+2}+4S_{n+1}-8S_n = 0$

$\boxed{n=1} \to S_4+3S_3+4S_2-8S_1 = 0.$

$S_2 = (a+b+c)^2 - 2(ab+bc+ca)$ ↗4

$S_2 = (-3)^2 - 2\times4$

$S_2 = 9-8 = 1$

$\Rightarrow S_4+3S_3+4-8(-3) = 0$

$\Rightarrow S_4+3S_3 = -28$ — ⓘ

put $\boxed{n=0} \to S_3+3S_2+4S_1-8S_0 = 0$

$\Rightarrow S_3+3(1)+4(-3)-8(3) = 0$

$\Rightarrow \boxed{S_3 = 33}$ — ⓘⓘ

$S_4 = -28-99$

$\Rightarrow S_4 = -127$

$\therefore E = S_4+S_2 = -127+1 = -126$ (Ans)

method-2: $p(x) = x^2+3x^2+4x-8 = 0 \begin{matrix} \nearrow a \\ \to b \\ \searrow c \end{matrix} \Big\}$ roots.

$a+b+c = -3$

$ab+bc+ac = 4$

$abc = 8.$

now, $(ab+bc+ac)^2 = a^2b^2+b^2c^2+a^2c^2 - 2abc(a+b+c)$

$\Rightarrow (ab+bc+ac)^2 - 2abc(a+b+c) = a^2b^2+b^2c^2+c^2a^2$

$\Rightarrow 16-16(-3) = 64 = \underbrace{a^2b^2+b^2c^2+c^2a^2}_{t}$

$\therefore E = a^2+b^2+c^2+a^4+b^4+c^4$

$\Rightarrow E = \{(a+b+c)^2 - 2(ab+bc+ca)\} + \{(a^2+b^2+c^2)^2 - 2t\}$

$\Rightarrow E = (9-8) + (\underbrace{(a+b+c)^2 - 2(ab+bc+ca)}_{1})^2 - 2(64)$

$\Rightarrow E = 1+1^2-128$

$\Rightarrow E = -126$ (Ans.)

TAH 08
BY REED
FROM WB

TMH-08

$p(x) = x^3+3x^2+4x-8 \begin{cases} \to a \\ \to b \\ \to c \end{cases}$

M₁

$$a^2(1+a^2)+b^2(1+b^2)+c^2(1+c^2)$$
$$= a^2+a^4+b^2+b^4+c^2+c^4$$
$$= (a^2+b^2+c^2)+(a^4+b^4+c^4)$$
$$= S_2+S_4$$

where, $S_n = a^n+b^n+c^n$

By Newton's formula,

$$S_{n+3}+3S_{n+2}+4S_{n+1}-8S_n = 0$$

$n=1$

$$S_4+3S_3+4S_2-8S_1=0$$

$$S_4+3S_3+4S_2+24=0 \quad \{ a+b+c=-3$$

put $n=0$

$$S_3+3S_2+4S_1-8S_0 = 0$$
$$S_3+3S_2+4(-3)-8(3)=0$$
$$S_3+3S_2=36$$
$$S_3 = 36-3S_2$$
$$S_4+108-9S_2+4S_2+24=0$$
$$S_4 = 5S_2-132$$
$$S_4+S_2 = 6S_2-132$$

$$S_2 = a^2+b^2+c^2$$
$$= (a+b+c)^2-2(ab+bc+ca)$$
$$= (-3)^2-2(4)$$
$$= 9-8$$
$$= 1$$

$$\Rightarrow S_4+S_2 = 6\times1-132$$
$$= -126$$

M₂

$$a+b+c=-3$$
$$ab+bc+ca = 4$$
$$abc = 8$$

S.B.S

$$a^2+b^2+c^2+2(ab+bc+ca) = 9$$
$$a^2+b^2+c^2+8=9$$
$$a^2+b^2+c^2 = 1$$

S.B.S

$$a^4+b^4+c^4+2(a^2b^2+b^2c^2+c^2a^2)=1$$

$$ab+bc+ca=4$$

S.B.S

$$a^2b^2+b^2c^2+c^2a^2+2abc(a+b+c)=16$$
$$a^2b^2+b^2c^2+c^2a^2+2(8)(-3)=16$$
$$a^2b^2+b^2c^2+c^2a^2 = 16+48$$
$$= 64$$

$$a^4+b^4+c^4+2\times64=1$$
$$a^4+b^4+c^4+128=1$$
$$a^4+b^4+c^4=-127$$

Ans

$$\therefore (a^2+b^2+c^2)+(a^4+b^4+c^4) = 1-127 = \underline{-126}$$

M_3

$p(1) = 1+3+4-8 = 0$

$p(x) = x^2(x-1)+4x(x-1)+8(x-1)$

$= (x-1)(x^2+4x+8)$

Let, $a=1$ &

then the other roots

b & c is the root of

$x^2+4x+8=0 \begin{matrix}\nearrow b \\ \searrow c\end{matrix}$

$\Rightarrow b+c=-4,\ bc=8$

$\therefore (a^2+b^2+c^2)+(a^4+b^4+c^4)$

Put, $a=1$

$1+b^2+c^2+1+b^4+c^4$

$= 2+(b^2+c^2)+(b^4+c^4)$

$= 2+(b+c)^2-2bc+b^4+c^4$

$= 2+16-16+b^4+c^4$

$= 2+\overset{\rightarrow 0}{(b^2+c^2)^2}-2(bc)^2$

$= 2-2(8)^2$

$= 2-2\times 64$

$= 2-128$

$= -126$

Ans

QUESTION

Let $\alpha_1, \alpha_2, \alpha_3$ and α_4 are the roots of equation $x^4 - 7x + 1 = 0$, then

(A) $\sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{25}{9}$

(B) $\prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = 1$

(C) $\prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{1}{9}$

(D) $\sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{23}{9}$

Q–[illegible]! Let $\alpha_1, \alpha_2, \alpha_3$ and α_4 are the roots of eqn $x^4-7x+1=0$ then:

Soln

$$x^4-7x+1=0 \begin{matrix} \nearrow \alpha_1 \\ \rightarrow \alpha_2 \\ \searrow \alpha_3 \\ \searrow \alpha_4 \end{matrix} \text{———} ① \quad \& \ \alpha_1\alpha_2\alpha_3\alpha_4 = \frac{+1}{1}$$

for ⓑ, ⓒ:
$$\prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{\alpha_1\alpha_2\alpha_3\alpha_4}{(1+\alpha_1)(1+\alpha_2)(1+\alpha_3)(1+\alpha_4)}$$

from ① → $x^4-7x+1 = (x-\alpha_1)(x-\alpha_2)(x-\alpha_3)(x-\alpha_4)$

put $x=-1$, $1+7+1 = (1+\alpha_1)(1+\alpha_2)(1+\alpha_3)(1+\alpha_4)$

$\Rightarrow 9 = (1+\alpha_1)(1+\alpha_2)(1+\alpha_3)(1+\alpha_4)$

$$\therefore \prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{1}{9} \quad (\text{Ans} \Rightarrow ⓒ)$$

for ⓐ, ⓓ:
$$\sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{\alpha_1}{1+\alpha_1} + \frac{\alpha_2}{1+\alpha_2} + \frac{\alpha_3}{1+\alpha_3} + \frac{\alpha_4}{1+\alpha_4}$$

we have to find a 4 degree polynominal with roots, $f(x) = \frac{x}{1+x} = y$

$\Rightarrow x(1-y) = y$

$\Rightarrow \boxed{x = \frac{y}{1-y}}$ put in ①

$$\left(\frac{y}{1-y}\right)^4 - 7\left(\frac{y}{1-y}\right) + 1 = 0$$

$\Rightarrow y^4 - 7y(1-y)^3 + (1-y)^4 = 0.$

$\Rightarrow y^4 - 7y(1-y^3-3y+3y^2) + 1 + y^4 - 4y^3 + 6y^2 - 4y = 0.$

$\Rightarrow y^4 - 7y + 7y^4 + 21y^2 - 21y^3 + y^4 - 4y^3 + 6y^2 - 4y = 0$

$\Rightarrow 9y^4 - 25y^3 + \dots = 0$

$$\therefore \sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{-(-25)}{9} = \frac{25}{9} \quad (\text{Ans} \Rightarrow ⓐ).$$

TAH-07

$$x^4 - 7x + 1 = 0 \quad \alpha_1, \alpha_2, \alpha_3, \alpha_4$$

$$\sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{\alpha_1}{\alpha_1+1} + \frac{\alpha_2}{\alpha_2+1} + \frac{\alpha_3}{1+\alpha_3} + \frac{\alpha_4}{\alpha_4+1}$$

$$y = \frac{x}{x+1} \Rightarrow xy + y = x \Rightarrow x = \frac{y}{1-y}$$

$$\left(\frac{y}{1-y}\right)^4 - 7\left(\frac{y}{1-y}\right) + 1 = 0 \quad (1-y)^4 \quad (1-y)^2 \times (1-y)^2$$

$$\Rightarrow y^4 - 7(y)(1-y)^3 + \cancel{(1-y)^4} = 0$$

$$\Rightarrow y^4 - 7y(1-y^3-3y+3y^2) + (1+y^2-2y)^2 = 0$$

$$\Rightarrow \cancel{y^4 - 7y + 7y^4 + 21y^2 - 21y^3} + \cancel{[1 + y^4 + 4y^2 + 2(y^2 + 2y^3 + 2y)]}$$

$$\Rightarrow \cancel{9y^4 - 17y^3 + 27y^2}$$

$$y^4 - 7y + 7y^4 - 21y^2 - 21y^3 + 1 + y^4 + 4y^2 + 2(y^2 - 2y^3 - 2y) = 0$$

$$\Rightarrow 9y^4 - 25y^3 - 15y^2 - 11y + 1 = 0$$

$$\text{Sum of roots} = -\left(\frac{-25}{9}\right) = \frac{25}{9} \quad (A)$$

$$\prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \left(\frac{\alpha_1}{1+\alpha_1}\right) \times \left(\frac{\alpha_2}{1+\alpha_2}\right)\left(\frac{\alpha_3}{1+\alpha_3}\right)\left(\frac{\alpha_4}{1+\alpha_4}\right) =$$

$$\Rightarrow \frac{(\alpha_1\alpha_2\alpha_3\alpha_4)}{(1+\alpha_1+\alpha_2+\alpha_1\alpha_2)(1+\alpha_3+\alpha_4+\alpha_3\alpha_4)}$$

$$\Rightarrow \frac{1}{1+\alpha_1+\alpha_2+\alpha_1\alpha_2+\alpha_3+\alpha_1\alpha_3+\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_3+\alpha_4+\alpha_1\alpha_4+\alpha_2\alpha_4+\alpha_1\alpha_2\alpha_4 + \alpha_4+\alpha_3\alpha_4+\alpha_3\alpha_4\alpha_1+\alpha_1\alpha_2\alpha_4\alpha_3+\alpha_1\alpha_2\alpha_3\alpha_4}$$

$$\Rightarrow \frac{1}{1+(\alpha_1+\alpha_2+\alpha_3+\alpha_4)+(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_1\alpha_4+\alpha_2\alpha_3+\alpha_2\alpha_4+\alpha_3\alpha_4) + (\alpha_1\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_4+\alpha_1\alpha_3\alpha_4 + \alpha_2\alpha_3\alpha_4)+\alpha_1\alpha_2\alpha_3\alpha_4}$$

$$\Rightarrow \frac{1}{1+(0)+(0)+(7)+(1)}$$

$$= \frac{1}{9} \quad (C)$$

QUESTION [JEE Mains 2023 (30 Jan)]

If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then β is equal to

Ans. 13

Homework

Mathematics

Tah : 09

$x^2-5ax+1=0$ and $x^2-ax-5=0$ → common real root → $\frac{3}{\sqrt{2\beta}}$ then $\beta=?$

$$x^2-5ax+1=0$$
$$x^2-ax-5=0$$

$$\begin{vmatrix}1 & -5a\\ 1 & -a\end{vmatrix} \times \begin{vmatrix}-5a & 1\\ -a & -5\end{vmatrix} = \begin{vmatrix}1 & 1\\ -5 & 1\end{vmatrix}^2$$

$$(4a)(26a) = 36$$

$$a^2 = \frac{36}{4\times 26}$$

$$a^2 = \frac{9}{2\times 13} \nearrow \quad \beta = 13 \ \underline{\text{Ans}}$$

Akash

TAH-09.

$$x^2 - 5ax + 1 = 0$$

$$x^2 - ax - 5 = 0.$$

$$\Rightarrow \begin{vmatrix} 1 & -5a \\ 1 & -a \end{vmatrix} \times \begin{vmatrix} -5a & 1 \\ -a & -5 \end{vmatrix} = \begin{vmatrix} 1 & \ast & 1 \\ -5 & & 1 \end{vmatrix}^2$$

$$\Rightarrow 4a \times 26a = (1+5)^2$$

$$\Rightarrow 4a \times 26a = \cancel{36}\,9.$$

$$\because a > 0$$

$$\Rightarrow a^2 = \frac{9}{26} \Rightarrow a = \pm\frac{3}{\sqrt{26}} = +\frac{3}{\sqrt{2\beta}} \Rightarrow \boxed{\beta = 13}$$

TAH-09

$$x^2 - 5ax + 1 = 0$$
$$x^2 - ax - 5 = 0$$

α is a common root

$$\cancel{\alpha^2} - 5a\alpha + 1 = 0$$
$$\cancel{\alpha^2} - a\alpha - 5 = 0$$
$$\ominus \quad \oplus \quad \oplus$$

$$-4a\alpha + 6 = 0$$
$$a\alpha = 3/2$$

$$\alpha^2 - a\alpha = 5$$
$$\alpha^2 - 3/2 = 5$$
$$\alpha^2 = 13/2$$

$$a^2\alpha^2 = 9/4$$
$$a^2 \times 13/\cancel{2} = 9/\cancel{4}\,2$$
$$a^2 = \frac{9}{2\times 13}$$
$$a = \frac{3}{\sqrt{2\times 13}} \quad (a > 0)$$

Q-11! If the value of real no. $a>0$ for which $x^2-5ax+1=0$ and $x^2-ax-5=0$ have a common root is $\frac{3}{\sqrt{2\beta}}$ then $\beta=?$

Sol^n Method-2!

$$x^2-5ax+1=0 \quad \text{—— (i)}$$

$$\underline{\underset{-}{x^2}\ \underset{+}{-}\ ax\ \underset{+}{-}\ 5=0} \quad \text{—— (ii)}$$

$$-4ax+6=0$$

$$\Rightarrow 4ax=-6$$

$$\Rightarrow \boxed{x=\frac{3}{2a}} \rightarrow x=\frac{3}{2a}=\alpha \text{ (common root)}$$

put $x=\frac{3}{2a}$ in (ii),

$$\left(\frac{3}{2a}\right)^2-\cancel{a}\left(\frac{3}{2\cancel{a}}\right)-5=0$$

$$\Rightarrow \frac{9}{4a^2}=\frac{3}{2}+5$$

$$\Rightarrow \frac{9}{4a^2}=\frac{13}{2}$$

$$\Rightarrow a^2=\frac{9}{26}$$

$$\Rightarrow a=\frac{3}{\sqrt{26}}=\frac{3}{\sqrt{2\beta}} \qquad \therefore 2\beta=26$$

$$\Rightarrow \boxed{\beta=13} \text{ Ans.}$$

Solution to Previous KTKs

QUESTION [JEE Mains 2019 (10 April)]

If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0, \theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})\cdot(\alpha-\beta)^{24}}$ is equal to :

- (A) $\frac{2^{12}}{(\sin\theta-8)^6}$
- (B) $\frac{2^6}{(\sin\theta+4)^{12}}$
- (C) $\frac{2^{12}}{(\sin\theta+8)^{12}}$
- (D) $\frac{2^{12}}{(\sin\theta-4)^{12}}$

Ans. C

Q-131 α, β roots of $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta \in (0, \frac{\pi}{2})$

$$\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})\,(\alpha-\beta)^{24}} \text{ is } = ?$$

KTK 1
BY REED
FROM WB

Soln:

$$x^2 + x\sin\theta - 2\sin\theta = 0 \begin{array}{l}\nearrow \alpha \\ \rightarrow \beta\end{array}$$

$\alpha+\beta = -\sin\theta$

$\alpha\beta = -2\sin\theta.$

$$E = \frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})\,(\alpha-\beta)^{24}}$$

$(\alpha-\beta)^2 = \frac{D}{a^2}$

$(\alpha-\beta)^2 = \frac{\sin^2\theta + 8\sin\theta}{1}$

$$= \frac{\cancel{\alpha^{12}+\beta^{12}}}{\frac{\cancel{(\alpha^{12}+\beta^{12})}}{(\alpha\beta)^{12}} \times (\alpha-\beta)^{24}}$$

$$= \frac{2^{12}\sin^{12}\theta}{(\sin^2\theta + 8\sin\theta)^{12}}$$

$$= \frac{2^{12}\cancel{\sin^{12}}\theta}{\cancel{\sin^{12}}\theta\,(\sin\theta+8)^{12}}$$

$$= \frac{2^{12}}{(\sin\theta+8)^{12}}$$ (Ans = ©)

QUESTION [JEE Mains 2022 (27 July)]

If α, β are the roots of the equation

$$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right)x + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$$

then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is :

A $3x^2 - 20x - 12 = 0$

B $3x^2 - 10x - 4 = 0$

C $3x^2 - 10x + 2 = 0$

D $3x^2 - 20x + 16 = 0$

Ans. B

Q-14: Soln:

$$E = x^2 - \Big(5 + \underbrace{3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}}_{t}\Big)x + 3\Big(\underbrace{3^{(\log_3 5)^{1/3}} - 5^{(\log_5 3)^{\frac{2}{3}}}}_{u} - 1\Big) = 0$$

$$t = 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}$$

we know

$$a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$$

$$\Rightarrow t = 3^{\sqrt{\log_3 5}} - 3^{\sqrt{\log_3 5}}$$

$$\Rightarrow \boxed{t = 0}$$

$$u = 3^{(\log_3 5)^{1/3}} - 5^{(\log_5 3)^{\frac{2}{3}}}$$

$$\Rightarrow u = 3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{1-\frac{1}{3}}}$$

$$\Rightarrow u = 3^{(\log_3 5)^{-\frac{1}{3}}} - 5^{\frac{\log_5 3}{(\log_5 3)^{\frac{1}{3}}}}$$

$$\Rightarrow u = 3^{(\log_3 5)^{\frac{1}{3}}} - \left(5^{\log_5 3}\right)^{\frac{1}{(\log_5 3)^{\frac{1}{3}}}}$$

$$\Rightarrow u = 3^{(\log_3 5)^{-\frac{1}{3}}} - \left(3^{\log_3 5}\right)^{\frac{1}{(1/\log_3 5)^{\frac{1}{3}}}}$$

$$\Rightarrow u = 3^{(\log_3 5)^{-\frac{1}{3}}} - 3^{(\log_3 5)^{-\frac{1}{3}}}$$

$$\Rightarrow \boxed{u = 0}$$

put $t = 0$, $u = 0$ in E,

$$E = x^2 - 5x + 3(0-1) = x^2 - 5x - 3 = 0.$$

∴ quad ~~reqd~~ eqn with roots $\alpha + \frac{1}{\beta}$, $\beta + \frac{1}{\alpha}$ is:

$\alpha + \beta = 5$
$\alpha\beta = -3.$

$$x^2 - \left(\alpha + \frac{1}{\beta} + \beta + \frac{1}{\alpha}\right)x + \left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = 0$$

$$\Rightarrow x^2 - \left(\alpha + \beta + \frac{\alpha+\beta}{\alpha\beta}\right)x + \left(\alpha\beta + \frac{1}{\alpha\beta} + 1 + 1\right) = 0$$

$$\Rightarrow x^2 - \left(5 + \frac{5}{-3}\right)x + \left(-3 - \frac{1}{3} + 2\right) = 0$$

$$\Rightarrow x^2 - \frac{10}{3}x + \left(\frac{-9-1+6}{3}\right) = 0$$

$$\Rightarrow x^2 - \frac{10}{3}x + \left(-\frac{4}{3}\right) = 0 \Rightarrow 3x^2 - 10x - 4 = 0.$$ Ans.

QUESTION [JEE Mains 2021]

Let α, β be two roots of the equation $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$. Then $\alpha^8 + \beta^8$ is equal to

- (A) 10
- (B) 100
- (C) 50
- (D) 160

Ans. C

KTK - 3

$$x^2 + (20)^{\frac{1}{4}} x + (5)^{\frac{1}{2}} = 0$$

$\Rightarrow \alpha + \beta = -(20)^{\frac{1}{4}}, \quad \alpha\beta = (5)^{\frac{1}{2}}$

S.B.S.

$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = \left[-(20)^{\frac{1}{4}}\right]^2$

$\Rightarrow \alpha^2 + \beta^2 = (-)^2 (20)^{\frac{1}{4} \times 2} - 2(\sqrt{5}).$

$= (20)^{\frac{1}{2}} - 2\sqrt{5}$

$= 2\sqrt{5} - 2\sqrt{5} = 0$

S.B.S.

$\Rightarrow \alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 0.$

$\Rightarrow \alpha^4 + \beta^4 = -2(5). = -10.$

$\Rightarrow$ S.B.S.

$\Rightarrow \alpha^8 + \beta^8 + 2\alpha^4 \cdot \beta^4 = 100$

$\Rightarrow \alpha^8 + \beta^8 = 100 - 2((5)^2) = 50$ (C)

Ans

Q-7: Let α, β be two roots of the equation $x^2+(20)^{1/4}x+(5)^{1/2}=0$. Then $\alpha^8+\beta^8$ is equal to:

(a) 10 (b) 100 (c) 50 (d) 160.

KTK 3
BY REED
FROM WB

Soln:
$$x^2+(20)^{1/4}x+(5)^{1/2}=0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix} \text{ —— } ①$$

Since; α, β are the roots of eqn ①,

$$\Rightarrow \alpha^2+(20)^{1/4}\alpha+(5)^{1/2}=0$$

or, $\alpha^2+5^{\frac{1}{2}}=-20^{1/4}\alpha$

S.B.S. → or, $\alpha^4+5+(2)\cdot(5)^{\frac{1}{2}}\cdot\alpha^2=20^{\frac{1}{2}}\alpha^2$

or, $\alpha^4+5+\cancel{(20)^{\frac{1}{2}}\cdot\alpha^2}=\cancel{(20)^{\frac{1}{2}}}\cdot\alpha^2$

or, $\alpha^4+5=0$

or, $\alpha^4=-5$

S.B.S. → or, $\boxed{\alpha^8=25}$ — (A) Similarly $\Rightarrow \boxed{\beta^8=25}$ — (B)

(A) + (B):

$\therefore \alpha^8+\beta^8=25+25=50$. (Ans.)

QUESTION [JEE Mains 2023]

Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$. Then $\alpha^{14} + \beta^{14}$ is equal to

A -64

B $-64\sqrt{2}$

C $-128\sqrt{2}$

D -128

Ans. D

Q-61 Let α, β be the roots of the equation $x^2-\sqrt{2}x+2=0$, then $\alpha^{14}+\beta^{14}$ is equal to!

(a) -64 (b) $-64\sqrt{2}$ (c) $-128\sqrt{2}$ (d) -128

$\boxed{\alpha+\beta=\sqrt{2}}$

KTK 4
DONE BY REED
FROM WB

Soln: $x^2-\sqrt{2}x+2=0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}\Big\}$ roots.

$\Rightarrow \alpha^2-\sqrt{2}\alpha+2=0.$

or, $\alpha^2+2=\sqrt{2}\alpha \Longrightarrow \boxed{\alpha^2=\sqrt{2}\alpha-2}$

S.B.S or, $(\alpha^2+2)^2=(\sqrt{2}\alpha)^2$

or, $\alpha^4+4\alpha^2+4=2\alpha^2$

or, $\alpha^4+4=-2\alpha^2 \longrightarrow \boxed{\alpha^4=(-2\alpha^2-4)}$

S.B.S or, $\alpha^8+16+8\alpha^4=4\alpha^4$

or, $\alpha^8+16=-4\alpha^4 \rightarrow \boxed{\alpha^8=-4\alpha^4-16}$

multiply both sides by α^4.

or, $\alpha^{12}+16\alpha^4=-4\alpha^8$

or, $\alpha^{12}+16\alpha^4=-4(-4\alpha^4-16)$ (put the value of α^8)

or, $\alpha^{12}+16\alpha^4=16\alpha^4+64$

or, $\alpha^{12}=64$

multiply both sides by α^2,

or, $\alpha^{14}=64\alpha^2$

or, $\boxed{\alpha^{14}=64(\sqrt{2}\alpha-2)} \rightarrow$ (A) [put the value of α^2]

Similarly $\Rightarrow \boxed{\beta^{14}=64(\sqrt{2}\beta-2)} \longrightarrow$ (B)

(A) + (B):

$\alpha^{14}+\beta^{14}=64(\sqrt{2}\alpha-2+\sqrt{2}\beta-2)$

or, $\alpha^{14}+\beta^{14}=64[\sqrt{2}(\alpha+\beta)-4]$ $(\because \alpha+\beta=\sqrt{2})$

or, $\alpha^{14}+\beta^{14}=64(2-4)=-128$ (Ans) $\therefore$ Ans $\Rightarrow$ (d) -128.

QUESTION

If $x^2 + 3x + 3 = 0$ and $ax^2 + bx + 1 = 0, a, b \in Q$ have a common root, then value of $(3a + b)$ is equal to

- A $1/3$
- B 1
- C 2
- D 4

Ans. C

- **Q-8:** If $x^2+3x+3=0$ and $ax^2+bx+1=0$; $a, b \in Q$ have a common root, then value of $(3a+b)$ is equal to:

(a) $\frac{1}{3}$ (b) 1 (c) 2 (d) 4.

Soln:

$x^2+3x+3=0 \rightarrow ①$
$ax^2+bx+1=0 \rightarrow (ii)$
have a common root.

now, $x^2+3x+3=0$

2

$D = 9-12 = -3 \ (<0) \rightarrow$ roots are imaginary

⇓

both roots are in pair.

Since, a root is common & roots are imaginary ⇒ both roots are common

Condition for both roots to be common:

$$\frac{a}{1} = \frac{b}{3} = \frac{1}{3}$$

$$\boxed{a = \frac{1}{3}}, \quad \boxed{b = \frac{3}{3} = 1}$$

$$\therefore 3a+b = \left(3a \cdot \frac{1}{3}\right) + 1 = 1+1 = 2. \text{ (Ans)}$$

Solution to Previous Home Challenge

Home Challenge - 07

Let n be the number of integers satisfying the inequality $\frac{(x-3)^{\frac{-x}{|x|}}\cdot\sqrt{(x-5)^2}\cdot(18-x)}{\sqrt{-x}(-x^2+x-1)(|x|-37)} < 0$ then value of n is ________

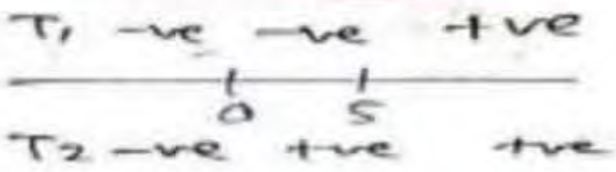
Q- 121 $n \rightarrow$ no. of integers, satisfying the ineq.

$$\frac{(x-3)^{\frac{-x}{|x|}}\sqrt{(x-5)^2}\cdot(18-x)}{\sqrt{-x}\,(-x^2+x-1)(|x|-37)} < 0$$

$x \rightarrow T_2$
$(x-5) \rightarrow T_1$

Sol^n $\Rightarrow$

$$\frac{(x-3)^{\frac{-x}{|x|}}\cdot\overset{T_1}{\overline{|x-5|}}\,(18-x)}{\sqrt{-x}\,(-x^2+x-1)(\underset{T_2}{\underline{|x|}}-37)} < 0$$



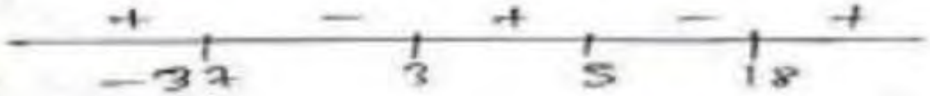
Case- I: $x \le 0$

HC 11
BY REED
FROM WB

$$\frac{(x-3)^1(\cancel{-}(x-5)(x-18))}{\underset{\hookrightarrow +ve}{\sqrt{-x}}\,\underset{\hookrightarrow\, D<0,\ a<0\ \}\,-ve}{(-x^2+x-1)}\,(-x-37)} < 0 \;;\; x \neq 0$$

2 times sign change.

$$\Rightarrow \frac{(x-3)(x-5)(x-18)}{(x+37)} < 0$$



$x \in (-37, 3) \cup (5, 18)$

"∩" $\boxed{x \in (-37, 0)}$ ↓ final interval.

Case-II: $\boxed{x \in (0, 5)}$

but for $\sqrt{-x}$ to be defined $\boxed{x < 0}$

∴ Case II rejected

Case-III: $\boxed{x \ge 5}$ ← Case-III also rejected.

$\boxed{\therefore x \in (-37, 0)}$ → no. of integers $= 0 - (-37) - 1$

$= 36$. (Ans.)

THANK YOU