PRAYAS

JEE 2026

Mathematics

Quadratic Equations

Lecture - 04

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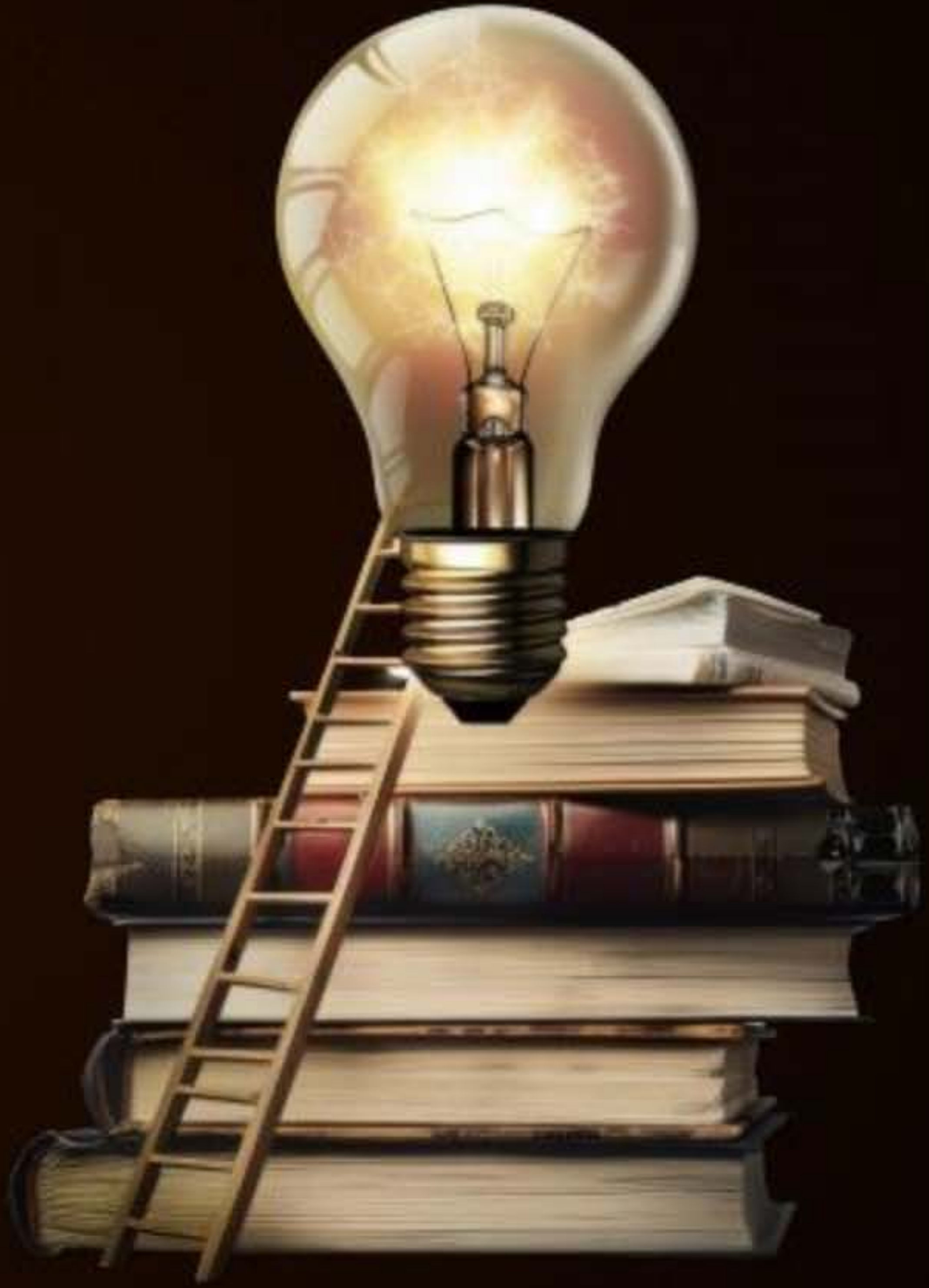
Topics *to be covered*

- **A** Transformation of Equation
- **B** Condition for Common Root
- **C** Practice problems



Homework Discussion

QUESTION

(KTK 5)

Number of integral values of 'a' for which the quadratic equation, $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite sign is,

(A) 1

(B) 2

(C) 3

(D) 4

$$2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$$

Since roots are of opp sign

$$\text{P.O.R} = -\text{ve}$$

$$\frac{a^2 - 4a}{2} < 0$$

$$a(a-4) < 0$$

$$\underbrace{a \in (0,4)}$$

Ans. C

QUESTION

(KTK 6)

If a, b, c are real numbers satisfying the condition $a + b + c = 0$ then the roots of the quadratic equation $3ax^2 + 5bx + 7c = 0$ are

$a \neq 0$

$b = -(a+c)$

$D = 25b^2 - 84ac$

(A) positive

(B) negative

(C) real and distinct

(D) imaginary

$D = 25(a+c)^2 - 84ac$

$= 25a^2 + 25c^2 + 50ac - 84ac$

$= 25a^2 + 25c^2 - 34ac$

$= 8a^2 + 8c^2 + 17a^2 + 17c^2 - 34ac$

$= 8(a^2+c^2) + 17(a^2+c^2-2ac)$

$= \underbrace{8}_{>0}\underbrace{(a^2+c^2)}_{\geq 0} + 17\underbrace{(a-c)^2}_{\geq 0} > 0$

Ans. C

QUESTION

If $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are roots of equation $x^5 - 5x^4 - 1 = 0$, then

(A) $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -\frac{1}{20}$

(B) $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -20$

(C) $\prod_{r=1}^{r=5} \left(\frac{1}{\alpha_r^4} + 5\right)^5 = 1$

(D) $\prod_{r=1}^{r=5} \left(\frac{1}{\alpha_r^4} + 5\right)^3 = \frac{1}{5}$

$\alpha_1^5 - 5\alpha_1^4 - 1 = 0$

$\alpha_1^5 - 5\alpha_1^4 = 1$

$\alpha_1 - 5 = \frac{1}{\alpha_1^4}$

lly $\alpha_2 - 5 = \frac{1}{\alpha_2^4}$

$\vdots$

$\alpha_5 - 5 = \frac{1}{\alpha_5^4}$

~~A~~, ~~B~~ $(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5) - 25 = \sum_{i=1}^{5} \frac{1}{\alpha_i^4}$

$5 - 25 = \sum_{i=1}^{5} \frac{1}{\alpha_i^4}$

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

$ax^3+bx^2+cx+d=0 \begin{cases}\alpha\\ \beta\\ \gamma\end{cases}$ $S_n = p\alpha^n + q\beta^n + t\gamma^n$ p, q, t are constants.

By NF

$$aS_{n+3} + bS_{n+2} + cS_{n+1} + dS_n = 0$$

Newton's Formula can be applied to polynomial eqn of any degree.

Ex: $x^5 - 4x^3 + 6x^2 + 7x - 5 = 0 \begin{cases}\alpha\\ \beta\\ \gamma\\ \delta\\ \Delta\end{cases}$ $S_n = \alpha^n + \beta^n + \gamma^n + \delta^n + \Delta^n$

By NF

$S_{n+5} - 4S_{n+4} + 6S_{n+3} + 7S_{n+2} - 5S_{n+1} = 0 \longrightarrow$ Gadhe / Gadhiyaa aisaay likhegay

Phadne waala : $S_{n+5} - 4S_{n+3} + 6S_{n+2} + 7S_{n+1} - 5S_n = 0$

QUESTION

★★ASRQ★★

If α, β, γ are roots of equation $x^3 - 2x^2 - 1 = 0$ and $T_n = \alpha^n + \beta^n + \gamma^n$, then value of $\frac{T_{11} - T_8}{T_{10}}$ is equal to

$$T_{n+3} - 2T_{n+2} - T_n = 0$$

$\underbrace{n=8}$ $\quad T_{11} - 2T_{10} - T_8 = 0$

$$\frac{T_{11} - T_8}{T_{10}} = 2$$

(A) 1

(B) 2

(C) -1

(D) 3

QUESTION

Tahol

If α, β and γ are roots of $3x^3 - 4x^2 - 3x + 2 = 0$ and

$$(\alpha^5 + \beta^5 + \gamma^5) - (\alpha^3 + \beta^3 + \gamma^3) = \frac{2}{m}\left(2(\alpha^4 + \beta^4 + \gamma^4) - (\alpha^2 + \beta^2 + \gamma^2)\right)$$

Then value of m is ______

$3x^2 - 4x^2 - 3x + 2 = 0 \begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$

$S_n = \alpha^n + \beta^n + \gamma^n$

QUESTION

★★ASRQ★★

If a, b, c are roots of $x^3 - x^2 + 1 = 0$ then find the value of $a^{-2} + b^{-2} + c^{-2}$.

M 1

$x^3 - x^2 + 1 = 0$ — a, b, c

$a+b+c = 1$

$ab+bc+ca = 0$ SBS

$abc = -1$

$S = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{b^2c^2 + c^2a^2 + a^2b^2}{(abc)^2}$

$a^2b^2 + b^2c^2 + c^2a^2 + 2(ab^2c + ac^2b + a^2bc) = 0$

$\underbrace{a^2b^2 + b^2c^2 + c^2a^2} + 2abc(a+b+c) = 0$

$a^2b^2 + b^2c^2 + c^2a^2 + 2(-1)(1) = 0$

$a^2b^2 + b^2c^2 + c^2a^2 = 2$.

$\Rightarrow S = \frac{2}{(-1)^2} = 2$ Ans

QUESTION

★★ASRQ★★

If a, b, c are roots of $x^3 - x^2 + 1 = 0$ then find the value of $a^{-2} + b^{-2} + c^{-2}$.

M② $x^3 - x^2 + 1 = 0$ a, b, c

$S_n = a^n + b^n + c^n$

we want S_{-2}

$abc = -1 \Rightarrow a, b, c \neq 0$

By NF

$S_{n+3} - S_{n+2} + S_n = 0$

put $n = -2$

$S_1 - S_0 + S_{-2} = 0$

$S_{-2} = S_0 - S_1$

$= 3 - (1) = 2$ Ans.

QUESTION

★★ASRQ★★

If a, b, c are roots of $x^3 - x^2 + 1 = 0$ then find the value of $a^{-2} + b^{-2} + c^{-2}$.

M 3 $x^3 - x^2 + 1 = 0 \begin{cases} a \\ b \\ c \end{cases}$

$S_n = a^{-2} + b^{-2} + c^{-2}$

$a^3 - a^2 + 1 = 0$

$a^3 - a^2 = -1$

$a - 1 = -\frac{1}{a^2}$

$\frac{1}{a^2} = 1 - a$

lly $\frac{1}{b^2} = 1 - b$

$\frac{1}{c^2} = 1 - c$

$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = 3 - (a+b+c) = 3 - 1 = 2$ Ans

QUESTION

★★KCLS★★

Tah02

Let α and β are two real roots of $x^2 + 10x - 7 = 0$. Then

(A) $\dfrac{\alpha^{20} + \beta^{20} - 7(\alpha^{18} + \beta^{18})}{\alpha^{19} + \beta^{19}} = -10$

(B) $\dfrac{\alpha\beta^{18} - 7\alpha\beta^{16} - 10\alpha^{17}\beta}{\alpha^{16} + \beta^{16}} = 70$

(C) $\sqrt{\left(\alpha - \dfrac{7}{\alpha}\right)\left(\beta - \dfrac{7}{\beta}\right)} = 10$

(D) $\dfrac{\alpha^3 + 9\alpha^2 - 17\alpha + 14}{\beta^3 + 11\beta^2 + 3\beta - 8} = 7$

QUESTION [JEE Mains 2025 (23 Jan)]

Ja n04

If the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ has equal roots, where $a + c = 15$ and $b = \frac{36}{5}$, then $a^2 + c^2$ is equal to

Ans. 117

QUESTION

★★ASRQ★★

$\pm|x| = \pm x \quad x \geq 0$

$\pm|x| = \pm(-x) \quad x < 0$

$= \mp x$

Let $\frac{-\pi}{6} < \theta < \frac{-\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

(A) $2(\sec\theta - \tan\theta)$

(B) $2\sec\theta$

(C) $-2\tan\theta$

(D) 0

$x^2 - 2x\sec\theta + 1 = 0 \; \langle \; \alpha_1, \beta_1$

$x = \dfrac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2}$

$x = \sec\theta \pm \tan\theta$

$\theta \in (-30^\circ, -15^\circ)$

$\tan\theta = -ve$

$\sec\theta = +ve.$

$\alpha_1 = \sec\theta - \tan\theta$

$\beta_1 = \sec\theta + \tan\theta$

$x^2 + 2x\tan\theta - 1 = 0 \; \langle \; \alpha_2, \beta_2$

$x = \dfrac{-2\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$

$x = -\tan\theta \pm \sec\theta$

$\beta_2 = -\tan\theta - \sec\theta$

$\alpha_2 = -\tan\theta + \sec\theta.$

$\alpha_1 + \beta_2 = -2\tan\theta.$

Quadratic Equation v/s Identity

Identity : a mathematical relation which holds for possible values of variable for which it is defined

If a quadratic equation has more than 2 distinct roots then it becomes an identity.

Ex: $\sin^2\theta + \cos^2\theta = 1$

Ex: $1 + \tan^2\theta = \sec^2\theta$

where $\theta \neq (2n+1)\frac{\pi}{2}$

let if possible have more than 2 say 3 distinct roots.

$ax^2 + bx + c = 0$ with roots α, β, γ

$a\alpha^2 + b\alpha + c = 0$

$a\beta^2 + b\beta + c = 0$

$a\gamma^2 + b\gamma + c = 0$

$\ominus$ $a(\alpha^2 - \beta^2) + b(\alpha - \beta) = 0$

$a(\alpha + \beta) + b = 0$

$\ominus$ $a(\beta + \gamma) + b = 0$

$a(\alpha - \gamma) = 0$

$\Downarrow$

$a = 0$

$b = 0$

$c = 0$

$a = 0, b = 0, c = 0$

If a quad is satisfied by more than 2 distinct values of x then it becomes an identity $0 \cdot x^2 + 0 \cdot x + 0 = 0$

NOTE :

(1) In any polynomial equation, if the number of roots > degree of equation then it becomes an identity.

(2) If any polynomial equation becomes an identity, then its all coefficients are simultaneously zero.

QUESTION

For what values of p, the equation $(p + 2)(p - 1)x^2 + (p - 1)(2p + 1)x + p^2 - 1 = 0$ has more then two roots.

$(p+2)(p-1)=0 \rightarrow p=1,-2$

& $(p-1)(2p+1)=0 \rightarrow p=1,-1/2$

& $p^2-1=0 \rightarrow p=-1,1$

$\cap$

$p=1$

QUESTION

★★ASRQ★★

Let α, β, γ be distinct real number and f(x) is a quadratic polynomial such that

$f(2)\alpha^2 + f(3)\alpha + f(4) = 4\alpha^2 + 4\alpha + 8$ — $(f(2)-4)\alpha^2 + (f(3)-4)\alpha + f(4)-8 = 0$

$f(2)\beta^2 + f(3)\beta + f(4) = 4\beta^2 + 4\beta + 8$ — $(f(2)-4)\beta^2 + (f(3)-4)\beta + f(4)-8 = 0$

$f(2)\gamma^2 + f(3)\gamma + f(4) = 4\gamma^2 + 4\gamma + 8$ — $(f(2)-4)\gamma^2 + (f(3)-4)\gamma + f(4)-8 = 0$

then find the value of f(7).

$\Downarrow$

$(f(2)-4)x^2 + (f(3)-4)x + f(4) - 8 = 0 <$ α, β, γ.

$\Downarrow$

It is an identity

(Aaam Zindagi) $f(2)=4,\ f(3)=4,\ f(4)=8.$

Mazdoori: $f(x) = ax^2+bx+c$

$f(2) = 4a+2b+c = 4$

$f(3) = 9a+3b+c = 4$ ⟩ Solve.

$f(4) = 16a+4b+c = 8$

$f(2)-4=0$

$f(3)-4=0$

$g(x) = f(x)-4$ — quad

roots of $g(x) = 2, 3.$

$g(x) = a(x-2)(x-3)$

$f(x)-4 = a(x-2)(x-3)$

$f(x) = a(x-2)(x-3)+4$

(Mentos Zindagi)

$f(x) = a(x-2)(x-3)+4$

$f(4) = a(2)(1)+4 = 8$

$a=2$

$f(x) = 2(x-2)(x-3)+4$

$f(7) = 2(5)(4)+4 = 44.$

QUESTION

$$\frac{(x-a)(x-b)}{(c-a)(c-b)}+\frac{(x-b)(x-c)}{(a-b)(a-c)}+\frac{(x-c)(x-a)}{(b-c)(b-a)}=1$$

Transformation of Equation

$x^2-5x+6=0 \begin{cases}\alpha=2\\ \beta=3\end{cases}$ form an Eqn whose roots are

(a) $\overset{f(\alpha)}{\frac{1}{\alpha}}, \overset{f(\beta)}{\frac{1}{\beta}}$ $\quad f(x)=\frac{1}{x}$

(b) $\overset{f(\alpha)}{-\alpha}, \overset{f(\beta)}{-\beta}$ $\quad f(x)=-x$

(c) $\overset{f(\alpha)}{\alpha-2}, \overset{f(\beta)}{\beta-2}$ $\quad f(x)=x-2$

Soln (a) $y=\frac{1}{x}=f(x)$

$x=\frac{1}{y}$ put in ①

$\frac{1}{y^2}-\frac{5}{y}+6=0$

$6y^2-5y+1=0 \begin{cases}\frac{1}{\alpha}\\ \frac{1}{\beta}\end{cases}$

$6x^2-5x+1=0 \begin{cases}1/\alpha=1/2\\ 1/\beta=1/3\end{cases}$

(b) $y=-x \Rightarrow x=-y$

put in ①

$(-y)^2-5(-y)+6=0$

$y^2+5y+6=0$

$y^2+5y+6=0$

$x^2+5x+6=0 \begin{cases}-\alpha=2\\ -\beta=3.\end{cases}$

(c) $y=x-2$

$x=y+2$

$(y+2)^2-5(y+2)+6=0$

$y^2+4y+4-5y-10+6=0$

$y^2-y=0$

$x^2-x=0 \begin{cases}\alpha-2=0\\ \beta-2=1\end{cases}$

Transformation of Equation

Suppose we have a polynomial equation of degree n given by

$a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = 0$ with roots $\alpha_1, \alpha_2, \ldots \alpha_n$ and we want to form a polynomial equation whose roots are $f(\alpha_1), f(\alpha_2), \ldots, f(\alpha_n)$ then follow the following steps

Step 1 Put $y = f(x)$

Step 2 Find x in terms of y

Step 3 Put the value obtained above in given Equation

Step 4 Replace y by x to get the desired equation

QUESTION

Tah05

Given, the cubic equation $x^3 - 5x^2 + 6x - 3 = 0$ (1) has roots α, β, γ. Find the cubic having roots

(i) $\alpha + 1, \beta + 1, \gamma + 1$ $\rightarrow y = f(x) = x+1$

$x = y-1$ put in (1)

$(y-1)^3 - 5(y-1)^2 + 6(y-1) - 3 = 0$

$y^3 - 1 - 3y^2 + 3y - 5y^2 - 5 + 10y + 6y - 6 - 3 = 0$

$y^3 - 8y^2 + 19y - 15 = 0$

$x^3 - 8x^2 + 19x - 15 = 0$

(ii) $\alpha - 1, \beta - 1, \gamma - 1$

(iii) $-\alpha, -\beta, -\gamma$

(iv) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

(v) $\frac{3\alpha-2}{\alpha+1}, \frac{3\beta-2}{\beta+1}, \frac{3\gamma-2}{\gamma+1}$

$\rightarrow y = f(x) =$

QUESTION

Jan 06

Let α, β and γ are the roots of the cubic $x^3 - 3x^2 + 1 = 0$. Find a cubic whose roots are $\frac{\alpha}{\alpha-2}, \frac{\beta}{\beta-2}$ and $\frac{\gamma}{\gamma-2}$. Hence or otherwise find the value of $(\alpha - 2)(\beta - 2)(\gamma - 2)$.

Ans. $3y^3 - 9y^2 - 3y + 1 = 0; (\alpha - 2)(\beta - 2)(\gamma - 2) = 3$

QUESTION

Jah06@

$\alpha\beta\gamma=-1$

If α, β, γ are roots of the cubic $2011x^3 + 2x^2 + 1 = 0$, then find

(i) $(\alpha\beta)^{-1} + (\beta\gamma)^{-1} + (\gamma\alpha)^{-1}$; $\rightarrow \frac{1}{\alpha\beta}, \frac{1}{\beta\gamma}, \frac{1}{\gamma\alpha} \frown \frac{\gamma}{\alpha\beta\gamma}, \frac{\alpha}{\alpha\beta\gamma}, \frac{\beta}{\alpha\beta\gamma} \frown -2011\gamma, -2011\alpha, -2011\beta$

~~(ii)~~ $\alpha^{-2} + \beta^{-2} + \gamma^{-2}$

By 4 methods

M(1)

$2011x^3+2x^2+1=0 \leftarrow \alpha, \beta, \gamma$ we want an Eqn with roots $-2011\alpha, -2011\beta, -2011\gamma$.

let $y=-2011x$, $x=\frac{-y}{2011}$

$-\frac{2011y^3}{2011^3}+\frac{2y^2}{2011^2}+1=0$

$-y^3+2y^2+2011^2=0$

$y^3-2y^2-2011^2=0 \leftarrow \frac{1}{\alpha\beta}, \frac{1}{\beta\gamma}, \frac{1}{\gamma\alpha} \Rightarrow S=-\frac{(-2)}{1}=2$

M(2)

$2011x^3+2x^2+1=0 \leftarrow \alpha, \beta, \gamma$.

$\frac{1}{\alpha\beta}, \frac{1}{\beta\gamma}, \frac{1}{\gamma\alpha} = -2011\alpha, -2011\beta, -2011\gamma$

$\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha} = -2011(\alpha+\beta+\gamma)$

$= -2011 \cdot \frac{(-(-2))}{2011}$

$= 2$ Ans

Ans. (i) 2 ; (ii) –4

QUESTION

★★ASRQ★★

Jan 05 (b)

Let roots of the equation $x^3 + 3x^2 + 4x = 11$ are α, β, γ and the roots of equation $x^3 + lx^2 + mx + n = 0$ $(l, m, n \in R)$ are $\alpha + \beta, \beta + \gamma, \gamma + \alpha$.

Column-I	Column-II
(A) l is equal to	(P) -6
(B) m is equal to	(Q) 6
(C) n is equal to	(R) 13
(D) $(l + m + n)$ is equal to	(S) 23

$x^3+3x^2+4x-11=0 < \alpha, \beta, \gamma$ $\quad \alpha+\beta+\gamma=-3$

To form an Eqn with root $\alpha+\beta, \beta+\gamma, \gamma+\alpha$

$\downarrow \quad \downarrow \quad \downarrow$

$-3-\gamma, -3-\alpha, -3-\beta$

$y = f(x) = -3-x$

QUESTION

★★ASRQ★★

The length of sides of a triangle and the 3 distinct roots of the equation $4x^3 - 24x^2 + 47x - 30 = 0$ is area of triangle is Δ, find 100Δ.

M① $4x^3 - 24x^2 + 47x - 30 = 0$ — roots a, b, c

$s = \frac{a+b+c}{2} = \frac{24}{8} = 3$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{3(3-a)(3-b)(3-c)}$

$4x^3 - 24x^2 + 47x - 30 = 4(x-a)(x-b)(x-c)$

put $x=3$ $\quad 108 - 216 + 141 - 30 = 4(3-a)(3-b)(3-c)$

$\frac{3}{4} = (3-a)(3-b)(3-c)$

M② we form an Eqn with roots $3-a, 3-b, 3-c$ $\quad y = f(x) = 3-x$

$x = 3-y$

$4(3-y)^3 - 24(3-y)^2 + 47(3-y) - 30 = 0$

$-4y^3 + y^2(\ ) + y(\ ) + \underbrace{108 - 216 + 141 - 30}_{=0}$

$-4y^3 + y^2(\ ) + y(\ ) + 3 = 0$ — roots $3-a$, $3-b$, $3-c$

$(3-a)(3-b)(3-c) = -\frac{(3)}{-4} = 3/4$

$\Delta = \sqrt{3 \cdot 3/4} = 3/2 \Rightarrow 100\Delta = 150.$

QUESTION

Tan07

Let $\alpha_1, \alpha_2, \alpha_3$ and α_4 are the roots of equation $x^4 - 7x + 1 = 0$, then

(A) $\sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{25}{9}$

(B) $\prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = 1$

(C) $\prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{1}{9}$

(D) $\sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{23}{9}$

QUESTION

Jan 08

If the roots of $p(x) = x^3 + 3x^2 + 4x - 8$ are a, b and c, what is the value of $a^2(1 + a^2) + b^2(1 + b^2) + c^2(1 + c^2)$?

$S_4 + S_2$

M① $S_n = a^n + b^n + c^n$ (NF)

M② $a+b+c=-3$

$ab+bc+ca=4$

$abc=+8$

Using Manipulation

Lengthy Hogaa

M③ $P(1)=0$

$P(x) = x^2(x-1) + 4x(x-1) + 8(x-1)$

$= (x^2+4x+8)(x-1)$

Solution of equations in two variables

$$a_1x + b_1y + c_1 = 0 \times b_2$$
$$a_2x + b_2y + c_2 = 0 \times b_1$$

$$(a_1b_2 - b_1a_2)x + b_2c_1 - b_1c_2 = 0$$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - b_1a_2} = \frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$a_1x + b_1y + c_1 = 0 \times a_2$$
$$a_2x + b_2y + c_2 = 0 \times a_1$$

$$(b_1a_2 - a_1b_2)y + a_2c_1 - a_1c_2 = 0$$

$$y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - b_1a_2} = \frac{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Determinant of 2nd order: $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$

NICHOD

$$a_1x + b_1y + c_1 = 0$$ [redacted]

$$a_2x + b_2y + c_2 = 0$$

1 2

$$\text{Then } x = \frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Condition for Common Root

$$\left.\begin{array}{l} a_1x^2 + b_1x + c_1 = 0 \;\; \alpha \\ \\ a_2x^2 + b_2x + c_2 = 0 \;\; \alpha \end{array}\right\} \text{Have a common root } \alpha$$

$$a_1\alpha^2 + b_1\alpha + c_1 = 0$$

$$a_2\alpha^2 + b_2\alpha + c_2 = 0$$

$$\alpha^2 = \frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} , \quad \alpha = \frac{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\alpha^2 = \frac{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}^2}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}^2} = \frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}^2$$

If $a_1x^2+b_1x+c_1=0$
$a_2x^2+b_2x+c_2=0$ have a common root

then $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}^2$

$$(b_1c_2 - b_2c_1)\cdot(a_1b_2 - a_2b_1) = (a_2c_1 - a_1c_2)^2$$

Condition for common root

In determinant form

$$a_1x^2 + b_1x + c_1 = 0$$

$$a_2x^2 + b_2x + c_2 = 0$$

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}^2$$

Condition for atleast common root

NOTE :

If both roots of the given equations are common then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

$$a_1x^2 + b_1x + c_1 = 0 < \begin{matrix}\alpha \\ \beta\end{matrix}$$

$$a_2x^2 + b_2x + c_2 = 0 < \begin{matrix}\alpha \\ \beta\end{matrix}$$

$$S.O.R = -\frac{b_1}{a_1} = -\frac{b_2}{a_2} \longrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$P.O.R = \frac{c_1}{a_1} = \frac{c_2}{a_2} \longrightarrow \frac{a_1}{a_2} = \frac{c_1}{c_2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Condition for a common root

$$(b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1) = (c_1a_2 - a_1c_2)^2 \quad \text{--- (1)}$$

is satisfied even when both roots are common.

Since

condition for both roots common $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \lambda$

$a_1 = \lambda a_2$

$b_1 = \lambda b_2$ put in (1)

$c_1 = \lambda c_2$

$$\lambda(b_1c_1 - b_1c_1)\,\lambda\cdot(a_1b_1 - a_1b_1) = \lambda^2(c_1a_1 - a_1c_1)^2$$

$$0 = 0$$

NOTE :

$a_1x^2 + b_1x + c_1 = 0 \begin{cases} p + iq \\ p - iq \end{cases}$

$a_2x^2 + b_2x + c_2 = 0 \begin{cases} p + iq \\ p - iq \end{cases}$

$a_1x^2+b_1x+c_1=0 \begin{cases} p+iq \\ p-iq \end{cases}$

$a_2x^2+b_2x+c_2=0 \begin{cases} p+iq \\ p-iq \end{cases}$

Given Both have a common root & roots of one are imaginary

$\Downarrow$

Both roots will be common.

Have a common roots & the roots of one of the equations are imaginary then both roots will be common.

If the equation

$f(x) = 0 \rightarrow \alpha.$ $f(\alpha)=0$

$g(x) = 0 \rightarrow \alpha$ $g(\alpha)=0$

Have a common root say α then $x = \alpha$ is also $Af(x) \pm Bg(x) = 0 \rightarrow \alpha.$

put $x=\alpha$ $Af(\alpha) \pm Bg(\alpha) = A\cdot 0 \pm B\cdot 0 = 0$

$\Downarrow$

$x=\alpha$ is also a root of $Af(x) \pm Bg(x)=0$

QUESTION

If the quadratic equation $3x^2 + ax + 1 = 0 \& 2x^2 + bx + 1 = 0$ have a common root find the value of $5ab - 2a^2 - 3b^2, (a, b \in R)$.

$$\left.\begin{array}{l} 3x^2+ax+1=0 \\ 2x^2+bx+1=0 \end{array}\right\rangle \text{ have a common root}$$

$$\Downarrow$$

$$\begin{vmatrix} 3 & a \\ 2 & b \end{vmatrix} \times \begin{vmatrix} a & 1 \\ b & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}^2$$

$$(3b-2a)(a-b) = (2-3)^2$$

$$3ab - 2a^2 + 2ab - 3b^2 = 1$$

$$5ab - 2a^2 - 3b^2 = 1 \text{ Ans.}$$

QUESTION [JEE Advanced 2011]

A value of b for which the equations $x^2 + bx - 1 = 0, x^2 + x + b = 0$ have one root in common is

(A) $-\sqrt{2}$

(B) $-i\sqrt{3}$

(C) $i\sqrt{5}$

(D) $\sqrt{2}$

$$x^2+bx-1=0$$
$$x^2+x+b=0$$

$$\begin{vmatrix} 1 & b \\ 1 & 1 \end{vmatrix} \times \begin{vmatrix} b & -1 \\ 1 & b \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ b & 1 \end{vmatrix}^2$$

$$(1-b)(b^2+1) = (-1-b)^2$$

$$\cancel{b^2}+\cancel{1}-b^3-b = \cancel{1}+\cancel{b^2}+2b.$$

$$b^3+3b=0$$

$$b(b^2+3)=0$$

$$b=0,\ b=\pm\sqrt{3}i$$

QUESTION [JEE Mains 2023 (30 Jan)]

Jan 09

If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then β is equal to

$x^2 - 5ax - 1 = 0$

$x^2 - ax - 5 = 0$

Ans. 13

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Home Challenge - 07

Let n be the number of integers satisfying the inequality $\dfrac{(x-3)^{\frac{-x}{|x|}}\cdot\sqrt{(x-5)^2}\cdot(18-x)}{\sqrt{-x}(-x^2+x-1)(|x|-37)}<0$ then value of n is ________

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION [JEE Mains 2019 (10 April)]

KTK 01

If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0, \theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})\cdot(\alpha-\beta)^{24}}$ is equal to :

(A) $\frac{2^{12}}{(\sin\theta-8)^6}$

(B) $\frac{2^6}{(\sin\theta+4)^{12}}$

(C) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

(D) $\frac{2^{12}}{(\sin\theta-4)^{12}}$

Ans. C

QUESTION [JEE Mains 2022 (27 July)]

KTK 02

If α, β are the roots of the equation

$$x^2-\left(5+3^{\sqrt{\log_3 5}}-5^{\sqrt{\log_5 3}}\right)x+3\left(3^{(\log_3 5)^{\frac{1}{3}}}-5^{(\log_5 3)^{\frac{2}{3}}}-1\right)=0$$

then the equation, whose roots are $\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$, is :

(A) $3x^2-20x-12=0$

(B) $3x^2-10x-4=0$

(C) $3x^2-10x+2=0$

(D) $3x^2-20x+16=0$

Ans. B

QUESTION [JEE Mains 2021]

KTK 03

Let α, β be two roots of the equation $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$. Then $\alpha^8 + \beta^8$ is equal to

(A) 10

(B) 100

(C) 50

(D) 160

Ans. C

QUESTION [JEE Mains 2023]

KTK 04

Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$. Then $\alpha^{14} + \beta^{14}$ is equal to

(A) -64

(B) $-64\sqrt{2}$

(C) $-128\sqrt{2}$

(D) -128

Ans. D

QUESTION

KTK 5

If $x^2 + 3x + 3 = 0$ and $ax^2 + bx + 1 = 0, a, b \in Q$ have a common root, then value of $(3a + b)$ is equal to

(A) $1/3$

(B) 1

(C) 2

(D) 4

Ans. C

Solution to Previous TAH

QUESTION

Let p & q be the two roots of the equation, $mx^2 + x(2-m) + 3 = 0$. Let m_1, m_2 be the two values of m satisfying $\frac{p}{q} + \frac{q}{p} = \frac{2}{3}$. Determine the numerical value of $\frac{m_1}{m_2^2} + \frac{m_2}{m_1^2}$.

Mathematics

Tah : 01

Homework

$mx^2+x(2-m)+3=0 \begin{matrix} \nearrow p \\ \searrow q \end{matrix}$ and m_1 and $m_2 \to \frac{p}{q}+\frac{q}{p}=\frac{2}{3} \to \frac{m_1}{m_2^2}+\frac{m_2}{m_1^2}$

$$\frac{p}{q}+\frac{q}{p}=\frac{2}{3}$$

$$\Rightarrow \frac{p^2+q^2}{pq}=\frac{2}{3}$$

$$\Rightarrow 3(p^2+q^2)=2pq$$

$pq=\frac{3}{m}$ and $p+q=\frac{m-2}{m}$

$$p^2+q^2=(p+q)^2-2pq$$

$$=\frac{(m-2)^2}{m^2}-\frac{6}{m}$$

$$3(p^2+q^2)=2pq$$

$$3\left(\frac{(m-2)^2}{m^2}-\frac{6}{m}\right)=\frac{6}{m}$$

$$\frac{3(m-2)^2}{m^2}-3\left(\frac{6}{m}\right)=\frac{6}{m}$$

$$\Rightarrow \frac{3(m-2)^2}{m^2}=4\left(\frac{6}{m}\right)$$

$$\Rightarrow 3m^2-12m+12=24m$$

$$\Rightarrow 3m^2-36m+12=0$$

$$\Rightarrow m^2-12m+4=0 \begin{matrix} \nearrow m_1 \\ \searrow m_2 \end{matrix}$$

$$\frac{m_1^3+m_2^3}{(m_1m_2)^2}$$

$m_1+m_2=12 \to m_1^2+m_2^2=136$

$m_1m_2=4$

$$m_1^3+m_2^3=(m_1+m_2)(m_1^2+m_2^2-m_1m_2)$$

$$=12(136-4)$$

$$=12\times 132$$

$$\frac{m_1^3+m_2^3}{m_1^2m_2^2}$$

$$=\frac{12\times 132}{4\times 4}=3\times 33$$

$$=99 \text{ Ans}$$

Akash

Tah-01 Let p & q be the two roots of the equn $mx^2+x(2-m)+3=0$. Let m_1, m_2 be the two values of m satisfying $\frac{p}{q}+\frac{q}{p}=\frac{2}{3}$ — Determine the numerical value of $\frac{m_1}{m_2^2}+\frac{m_2}{m_1^2}$

Sol^n

$$mx^2+x(2-m)+3=0 \begin{matrix} \nearrow p \\ \searrow q \end{matrix}$$

$$\boxed{\begin{aligned} p+q &= -(2-m) \\ p\cdot q &= 3 \end{aligned}}$$

$$\Rightarrow \frac{p}{q}+\frac{q}{p}=\frac{2}{3}$$

$$\Rightarrow \frac{p^2+q^2}{pq}=\frac{2}{3}$$

$$\Rightarrow \frac{(p+q)^2-2pq}{pq}=\frac{2}{3}$$

$$\Rightarrow \frac{(2-m)^2-2(3)}{\cancel{3}}=\frac{2}{\cancel{3}}$$

$$\Rightarrow 4+m^2-4m-8=0$$

$$\Rightarrow m^2-4m-4=0 \begin{matrix} \nearrow m_1 \\ \searrow m_2 \end{matrix}$$

$$\boxed{\begin{aligned} m_1+m_2 &= 4 \\ m_1\cdot m_2 &= -4 \end{aligned}}$$

then

$$\frac{m_1}{m_2^2}+\frac{m_2}{m_1^2}$$

$$=\frac{(m_1)^3+(m_2)^3}{(m_1\cdot m_2)^2}=\frac{(m_1+m_2)(m_1^2+m_2^2-m_1m_2)}{(m_1\cdot m_2)^2}$$

$$=\frac{4((4)^2+2\times4+4)}{(-4)^2}$$

$$=\frac{4(28)}{16}=7$$

Y29

Nitish Adalpur Bihar May 15, 2025, 09:22

QUESTION

Let α, β are the roots of the equation $x^2 + x - 3 = 0$. Then the value of $\alpha^3 - 4\beta^2 + 19$ is equal to

Ans. 0

Homework

Mathematics
Tah : 02

$$x^2+x-3=0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \longrightarrow \alpha^3-4\beta^2+19=?$$

$\alpha^2+\alpha-3=0 \nearrow -\alpha^2=\alpha-3$

$\alpha^3+\alpha^2-3\alpha=0 \longrightarrow \alpha^3=3\alpha-\alpha^2$

$\beta^2+\beta-3=0$

$-4\beta^2-4\beta+12=0 \longrightarrow -4\beta^2=4\beta-12$

$\alpha+\beta=-1$

$\alpha\beta=-3$

Adding both we get

$$\alpha^3-4\beta^2=3\alpha-\alpha^2+4\beta-12$$
$$\alpha^3-4\beta^2+19=3\alpha-\alpha^2+4\beta+7$$
$$\alpha^3-4\beta^2+19=3\alpha+3\beta-\alpha^2+\beta+7$$
$$=-3-\alpha^2+\beta+7$$
$$=\beta-\alpha^2+4$$
$$=\beta+\alpha-3+4$$
$$=-4+4$$
$$\alpha^3-4\beta^2+19=0 \text{ Ans}$$

Akash

lec 03/ Quadratic Equations/ Ashish Sir

Date

Page

* Tah-02 :-

let α, β are the roots of the equation $x^2+x-3=0$. Then the value of $\alpha^3 - 4\beta^2 + 19$ is equal to.

$x^2+x-3=0 - \left[\begin{matrix} \alpha \\ \beta \end{matrix} \right.$

$\{ \alpha+\beta = -1, \alpha\beta = -3 \}$

$\alpha^2 + \alpha - 3 = 0$

$\alpha \times \Rightarrow \alpha^3 + \alpha^2 - 3\alpha = 0$

$\alpha^3 = 3\alpha - \alpha^2$

lly : $\beta^2 + \beta - 3 = 0$

$\beta^2 = 3 - \beta$

Then :-

$\Rightarrow \alpha^3 - 4\beta^2 + 19$

$\Rightarrow 3\alpha - \alpha^2 - 12 + 4\beta + 19$

$\Rightarrow -\alpha^2 + 3\alpha + 4\beta + 7$

$\Rightarrow -\alpha^2 - \alpha + 4\alpha + 4\beta + 7$

$\Rightarrow -\alpha^2 - \alpha + 4(\alpha+\beta) + 7$

$\Rightarrow -\alpha^2 - \alpha + 4(-1) + 7$

$\Rightarrow -\alpha^2 - \alpha - 4 + 7$

$\Rightarrow -\alpha^2 - \alpha + 3 = 0$

$\Rightarrow \alpha^2 + \alpha - \underline{3} = 0$

QUESTION

$P(x) = x^3 + 33x^2 + 327x + 935$

Let $P(x)$ be a polynomial as described above with a, b, c the roots of $P(x)$.

Find $a^2 + b^2 + c^2$ without solving $P(x) = 0$.

Homework

Mathematics

Tah : 03

$$P(x) = x^3 + 33x^2 + 327x + 935 \to a, b, c$$

$$a^2+b^2+c^2 = ?$$

$$a^2+b^2+c^2 = (a+b+c)^2 - 2(ab+bc+ca)$$

$a+b+c = -33$

$ab+bc+ca = 327$

$$\Rightarrow a^2+b^2+c^2 = 33\times 33 - 2\times 327$$

$$= \underset{+990}{99} - 654$$

$$= 1089 - 654$$

$$= 435 \text{ Ans}$$

Akash

$p(x) = x^3 + 33x^2 + 327x + 935$

let $p(x)$ be a polynomial as described above with a, b, c the roots of $p(x)$ find $a^2+b^2+c^2$ without soln $p(x)=0$

Soln $p(x) = x^3 + 33x^2 + 327x + 935 \begin{cases} a \\ b \\ c \end{cases}$

$$\boxed{\begin{aligned} a+b+c &= -33 \\ ab+bc+ca &= 327 \\ a\cdot b\cdot c &= -935 \end{aligned}}$$

Tah-03

$$\Rightarrow a^2+b^2+c^2 = (a+b+c)^2 - 2(ab+bc+ca)$$
$$= (-33)^2 - 2(327.)$$
$$= 1089 - 654$$
$$= \underline{435} \text{ Ans}$$

$$\begin{array}{r} 33 \\ \times 33 \\ \hline 99 \\ 99 \\ \hline 1089 \end{array}$$

Tah - 03

Given, $p(x) = x^3 + 33x^2 + 327x + 935$ — a, b, c

$a+b+c = -33$

$ab+bc+ca = 327$

$abc = 935$

vandana
from Bihar

Now, $a+b+c = -33$

S.B.S

$a^2+b^2+c^2+2(ab+bc+ca) = 1089$

$a^2+b^2+c^2+2\times327 = 1089$

$a^2+b^2+c^2 = 1089-654$

$\boxed{a^2+b^2+c^2 = 435}$ Ans

QUESTION

Let r_1, r_2 and r_3 be the roots of the polynomial $5x^3 - 11x^2 + 7x + 3$.
Evaluate $r_1(1 + r_2 + r_3) + r_2(1 + r_3 + r_1) + r_3(1 + r_1 + r_2)$.

Homework

Mathematics

Tah : 04

$$5x^3 - 11x^2 + 7x + 3 \to \alpha_1, \alpha_2, \alpha_3 \longrightarrow \boxed{\alpha_1(1+\alpha_2+\alpha_3) + \alpha_2(1+\alpha_3+\alpha_1) + \alpha_3(1+\alpha_1+\alpha_2)} = ?$$

E

$$\alpha_1(1+\alpha_2+\alpha_3) + \alpha_2(1+\alpha_3+\alpha_1) + \alpha_3(1+\alpha_1+\alpha_2)$$

$$= \alpha_1 + \alpha_2 + \alpha_3 + \alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3 + \alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3$$

$$= \alpha_1 + \alpha_2 + \alpha_3 + 2(\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1)$$

$$\alpha_1 + \alpha_2 + \alpha_3 = \frac{11}{5} \ ; \ \alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1 = \frac{7}{5}$$

$$\Rightarrow E = \frac{11}{5} + \frac{14}{5} = 5 \text{ Ans}$$

Akash

Let, r_1, r_2 and r_3 be the roots of the Polynomial $5x^3 - 11x^2 + 7x + 3$. Evaluate -
$r_1(1 + r_2 + r_3) + r_2(1 + r_3 + r_1) + r_3(1 + r_1 + r_2)$

Soln $P(x) = 5x^3 - 11x^2 + 7x + 3$ $\langle$ r_1, r_2, r_3

$$\boxed{\begin{aligned} & r_1 + r_2 + r_3 = 11 \\ & r_1 \cdot r_2 + r_2 \cdot r_3 + r_3 \cdot r_1 = 7 \\ & r_1 \cdot r_2 \cdot r_3 = -3 \end{aligned}}$$

Tah-04

$$E = r_1(1 + r_2 + r_3) + r_2(1 + r_3 + r_1) + r_3(1 + r_1 + r_2)$$

$$= r_1 + r_1 r_2 + r_1 r_3 + r_2 + r_2 r_3 + r_1 r_2 + r_3 + r_1 r_3 + r_2 r_3$$

$$= (r_1 + r_2 + r_3) + (r_1 r_2 + r_2 r_3 + r_1 r_3) + (r_1 r_2 + r_2 r_3 + r_3 r_1)$$

$$= 11 + 7 + 7$$

$$= \underline{25} \text{ Ans}$$

QUESTION

If α, β and γ are roots of cubic equation $x^3 + 3x - 1 = 0$ then find value of :

(i) $(2-\alpha)(2-\beta)(2-\gamma)$

(ii) $(3+\alpha)(3+\beta)(3+\gamma)$

(iii) $(4-\alpha^2)(4-\beta^2)(4-\gamma^2)$

(iv) $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)$

if α, β and γ are roots of cubic eqn $x^3+3x-1=0$ then find value of (1) $(2-\alpha)(2-\beta)(2-\gamma)$

Soln

Tah-05

$$x^3+3x-1=0 \quad \alpha, \beta, \gamma$$

$$P(x) = x^3+3x-1 = (x-\alpha)(x-\beta)(x-\gamma)$$

$$P(2) = (2)^3+3(2)-1 = (2-\alpha)(2-\beta)(2-\gamma)$$

$$= 8+6-1 = (2-\alpha)(2-\beta)(2-\gamma)$$

$$= 13 = (2-\alpha)(2-\beta)(2-\gamma)$$

Ans.

(ii) $(3+\alpha)(3+\beta)(3+\gamma)$

$$\boxed{\begin{aligned}\alpha+\beta+\gamma &= 0\\ \alpha\beta+\beta\gamma+\gamma\alpha &= 3\\ \alpha\cdot\beta\cdot\gamma &= 1\end{aligned}}$$

$$= (9+3\beta+3\alpha+\alpha\beta)(3+\gamma)$$

$$= (27+9\gamma+9\beta+3\beta\gamma+9\alpha+3\alpha\gamma+3\alpha\beta+\alpha\beta\gamma)$$

$$= (27+9(\alpha+\beta+\gamma)+3(\alpha\beta+\beta\gamma+\gamma\alpha)+\alpha\beta\gamma)$$

$$\Rightarrow (27+9(0)+3(3)+1)$$

$$= 27+9+1$$

$$= \underline{\underline{37}}$$

Ans

(iii) $(4-\alpha^2)(4-\beta^2)(4-\gamma^2)$

Tah-05

$$\Rightarrow (16-4\beta^2-4\alpha^2+\alpha^2\beta^2)(4-\gamma^2)$$

$$\Rightarrow (64-16\gamma^2-16\beta^2+4\beta^2\gamma^2-16\alpha^2+4\alpha^2\gamma^2+4\alpha^2\beta^2-\alpha^2\beta^2\gamma^2)$$

$$\Rightarrow (64-16(\alpha^2+\beta^2+\gamma^2)+4(\alpha^2\beta^2+\beta^2\gamma^2+\alpha^2\gamma^2)-\alpha^2\beta^2\gamma^2)$$

$$\Rightarrow (64-16\{(0-2(3))\}+4(9)-(1)^2)$$

$$\Rightarrow 64-16(6)+36-1)$$

$$\Rightarrow 64-96+36-1$$

$$\Rightarrow 100-97$$

$$\Rightarrow \underline{3}.$$

~~$64-16(4)$~~

~~$= 64-65$~~

~~$= -59$~~

Ans.

(iv) $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)$

$$\Rightarrow (1+\gamma^2+\beta^2+\beta^2\gamma^2+\alpha^2+1\alpha^2\gamma^2+\alpha^2\beta^2+\alpha^2\beta^2\gamma^2$$

$$\Rightarrow (1+(\alpha^2+\beta^2+\gamma^2)+1(\alpha^2\beta^2+\beta^2\gamma^2+\alpha^2\gamma^2)+(\alpha\beta\gamma)^2)$$

$$\Rightarrow (1+(6)+1(9)+(1)^2)$$

$$\Rightarrow 17 \text{ Ans}$$

Homework

Mathematics

Tah : 05

$x^3+3x-1=0 \to \alpha, \beta, \gamma \longrightarrow$ (a) $(2-\alpha)(2-\beta)(2-\gamma)$ (b) $(3+\alpha)(3+\beta)(3+\gamma)$ (c) $(4-\alpha^2)(4-\beta^2)(4-\gamma^2)$ (d) $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)$

$\alpha+\beta+\gamma=0$; $\alpha\beta+\beta\gamma+\alpha\gamma=3$; $\alpha\beta\gamma=1$

$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)=-6$

$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2=(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)=9$

(a) $(2-\alpha)(2-\beta)(2-\gamma)=(4-2\alpha-2\beta+\alpha\beta)(2-\gamma)$
$=8-4\alpha-4\beta+2\alpha\beta-4\gamma+2\alpha\gamma+2\beta\gamma-\alpha\beta\gamma$
$=8-4(\alpha+\beta+\gamma)+2(\alpha\beta+\beta\gamma+\gamma\alpha)-\alpha\beta\gamma$
$=7-4(0)+2(3)$
$=13$ Ans

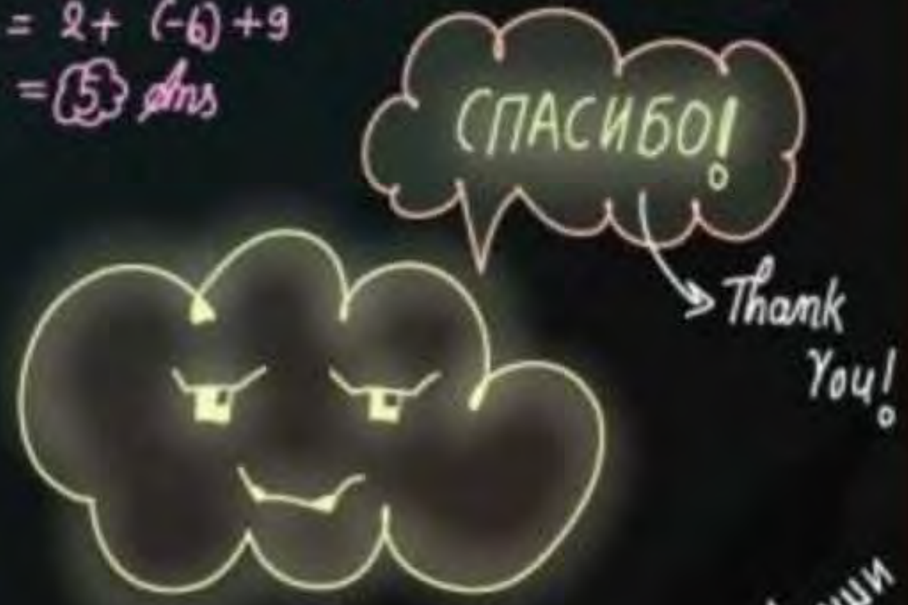
(b) $(3+\alpha)(3+\beta)(3+\gamma)=(9+3\beta+3\alpha+\alpha\beta)(3+\gamma)$
$=27+9\beta+9\alpha+3\alpha\beta+9\gamma+3\beta\gamma+3\alpha\gamma+\alpha\beta\gamma$
$=27+9(\alpha+\beta+\gamma)+3(\alpha\beta+\beta\gamma+\gamma\alpha)+\alpha\beta\gamma$
$=28+9(0)+3(3)$
$=28+9=37$ Ans

(c) $(4-\alpha^2)(4-\beta^2)(4-\gamma)^2=(2+\alpha)(2+\gamma)(2+\beta)(2-\alpha)(2-\beta)(2-\gamma)$
$=13(2+\alpha)(2+\beta)(2+\gamma)$

for this we can replace –ve with +ve in (1)

$=13(8+4(\alpha+\beta+\gamma)+2(\alpha\beta+\beta\gamma+\gamma\alpha)+\alpha\beta\gamma)$
$=13(9+6)=13\times15=195$ Ans

(d) $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)=(1+\beta^2+\alpha^2+\alpha^2\beta^2)(1+\gamma^2)$
$=1+\beta^2+\alpha^2+\alpha^2\beta^2+\gamma^2+\gamma^2\beta^2+\gamma^2\alpha^2+\gamma^2\beta^2\alpha^2$
$=2+(-6)+9$
$=5$ Ans



Akash

QUESTION

If $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are roots of equation $x^5 - 5x^4 - 1 = 0$, then

(A) $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -\frac{1}{20}$

(B) $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -20$

(C) $\prod_{r=1}^{r=5} \left(\frac{1}{\alpha_r^4} + 5\right)^5 = 1$

(D) $\prod_{r=1}^{r=5} \left(\frac{1}{\alpha_r^4} + 5\right)^3 = \frac{1}{5}$

Home challange

if $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are roots of - eqn $x^5 - 5x^4 - 1 = 0$ then.

So $x^5 - 5x^4 - 1 = 0$

$$\alpha_1^5 - 5\alpha_1^4 - 1 = 0$$
$$\alpha_1^4(\alpha_1 - 5) = 1$$
$$\alpha_1^4 = \frac{1}{\alpha_1 - 5}$$

(i) $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -\frac{1}{20}$

$\Rightarrow \frac{1}{\alpha_1^4} + \frac{1}{\alpha_2^4} + \frac{1}{\alpha_3^4} + \frac{1}{\alpha_4^4} + \frac{1}{\alpha_5^4}$

$\Rightarrow \alpha_1 - 5 + \alpha_2 - 5 + \alpha_3 - 5 + \alpha_4 - 5 + \alpha_5 - 5$

$\Rightarrow \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 - 25$

$= 5 - 25$

$= -20$. 1st option $\rightarrow$ wrong.

(ii) $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -20$ 2nd option ✓

(iii) $\prod_{r=1}^{r=5} \left(\frac{1}{\alpha_r^4} + 5\right)^5 = 1$

$\Rightarrow \left(\frac{1}{\alpha_1^4} + 5\right)\left(\frac{1}{\alpha_2^4} + 5\right)\left(\frac{1}{\alpha_3^4} + 5\right)\left(\frac{1}{\alpha_4^4} + 5\right)\left(\frac{1}{\alpha_5^4} + 5\right)$

$\Rightarrow (\alpha_1 - 5 + 5)(\alpha_2 - 5 + 5)(\alpha_3 - 5 + 5)(\alpha_4 - 5 + 5)(\alpha_5 - 5 + 5)$

$\Rightarrow \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_4 \cdot \alpha_5$

Home challange

$\Rightarrow -(-1) = \underline{1}$ Ans

$\boxed{\text{option} \rightarrow \text{B and C is right}}$

Newton's formula :-

$ax^2 + bx + c = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$S_n = p\alpha^n + q\beta^n$

p, q are constant

$a\alpha^2 + b\alpha + c = 0$ $a\beta^2 + b\beta + c = 0$

QUESTION [JEE Mains 2024 (6 April)]

Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and $(4 + x_1^2)(4 + x_2^2)(4 + x_3^2)(4 + x_4^2) = \frac{125}{16}m$. Then the value of m is

M(1)

$P(-1) = 4 - 8 - 17 + 12 + 9 = 0 \rightarrow (x+1)$ is a factor of $P(x)$

$4x^3(x+1) + 4x^2(x+1) - 21x(x+1) + 9(x+1) = 0$

$(x+1)(4x^3 + 4x^2 - 21x + 9) = 0 \rightarrow Q(-3) = -108 + 36 + 63 + 9 = 0$

$(x+1)\left(4x^2(x+3) - 8x(x+3) + 3(x+3)\right) = 0$

$(x+1)(x+3)(4x^2 - 8x + 3) = 0$

$(x+1)(x+3)(4x^2 - 6x - 2x + 3) = 0$

$(x+1)(x+3)(2x-1)(2x-3) = 0$

$x = -1, -3, 1/2, 3/2$.

Ans. 221

QUESTION [JEE Mains 2024 (6 April)]

$i = \sqrt{-1}$

$i^2 = -1$

Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and $(4 + x_1^2)(4 + x_2^2)(4 + x_3^2)(4 + x_4^2) = \frac{125}{16} m$. Then the value of m is

$(2i - x_1)(2i + x_1)$

$= (2i)^2 - x_1^2$

$= 4i^2 - x_1^2$

$= -4 - x_1^2.$

M(2) $P(x) = 4(x - x_1)(x - x_2)(x - x_3)(x - x_4)$

put $x = 2i$

$P(2i) = 4(2i - x_1)(2i - x_2)(2i - x_3)(2i - x_4)$

put $x = -2i$

$P(-2i) = 4(2i + x_1)(2i + x_2)(2i + x_3)(2i + x_4)$

$P(2i) \cdot P(-2i) = 16(-4 - x_1^2)(-4 - x_2^2)(-4 - x_3^2)(-4 - x_4^2)^2$

$P(2i) P(-2i) = 16(4 + x_1^2)(4 + x_2^2)(4 + x_3^2)(4 + x_4^2)$

Ans. 221

$$4+x_1^2 = x_1^2-(2i)^2 = (x-2i)(x+2i)$$

$$x^2+a^2 = x^2-(ia)^2 = (x-ia)(x+ia)$$

$$x^2-a^2 = (x-a)(x+a)$$

Homework

Mathematics

Tah : 06

$4x^4+8x^3-17x^2-12x+9=0 \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} \longrightarrow (4+x_1^2)(4+x_2^2)(4+x_3^2)(4+x_4^2)=\frac{125}{16}m$. find value of m

$(a^2+x^2)=(ai-x)(-ai-x)$

$(4+x_1^2)(4+x_2^2)(4+x_3^2)(4+x_4^2) = \boxed{(2i-x_1)(2i-x_2)(2i-x_3)(2i-x_4)(-2i-x_1)(-2i-x_2)(-2i-x_3)(-2i-x_4)} \to M$

$4(x-x_1)(x-x_2)(x-x_3)(x-x_4)=4x^4+8x^3-17x^2-12x+9$

putting $x=2i$

$E_1 = 4(2i-x_1)(2i-x_2)(2i-x_3)(2i-x_4) = 4(2i)^4+8(2i)^3-17(2i)^2-12(2i)+9$
$= 4(16i^4)+8(8i^3)-17(4i^2)-12(2i)+9$
$= 64-64i+68-24i+9$
$= 141-88i$

putting $x=-2i$

$E_2 = 4(-2-x_1)(-2-x_2)(-2-x_3)(-2-x_4) = 4(-2i)^4+8(-2i)^3-17(-2i)^2-12(-2i)+9$
$= 64+64i+68+24i+9$
$= 141+88i$

multiplying E_1 and E_2

$16(m) = (141-88i)(141+88i)$
$= (141)^2+(88)^2$
$= 141+5640+14100 + (90-2)88$
$= 19881+7920-176$
$= 19881+7744$
$16M = 27625$
$m = \frac{27625}{16}$
$M = \frac{125}{16}\times 221$
$m = 221$ Ans

$$\begin{array}{r} 221 \\ 125\,\overline{)27625} \\ \underline{250} \\ 262 \\ \underline{250} \\ 125 \end{array}$$

lec 03/ Quadratic Equations/ Ashish Sir

Akash

QUESTION [JEE Mains 2024 (9 April)]

★★ASRQ★★

Let $\alpha, \beta; \alpha > \beta$, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$.

Let $P_n = \alpha^n - \beta^n, n \in N$. Then $(11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} - 11P_{12}$ is equal to

(A) $10\sqrt{3}P_9$

(B) $11\sqrt{3}P_9$

(C) $11\sqrt{2}P_9$

(D) $10\sqrt{2}P_9$

Ans. A

Homework

Mathematics

Tah : 07

$$x^2-\sqrt{2}x-\sqrt{3}=0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix} \longrightarrow \alpha > \beta \longrightarrow P_n=\alpha^n-\beta^n \longrightarrow (11\sqrt{3}-10\sqrt{2})P_{10}+(11\sqrt{2}+10)P_{11}-11P_{12} = ?$$

$$P_{12}-\sqrt{2}P_{11}-\sqrt{3}P_{10}=0 \Rightarrow 11P_{12}-11\sqrt{2}P_{11}-11\sqrt{3}P_{10}=0 \quad — (1)$$

$$P_{11}-\sqrt{2}P_{10}-\sqrt{3}P_{9}=0 \Rightarrow 10P_{11}-10\sqrt{2}P_{10}-10\sqrt{3}P_{9}=0 \quad — (2)$$

(2) − (1)

$$= 10P_{11}-10\sqrt{2}P_{10}-10\sqrt{3}P_9-11P_{12}+11\sqrt{2}P_{11}+11\sqrt{3}P_{10}=0$$

$$= (11\sqrt{3}-10\sqrt{2})P_{10}+(11\sqrt{2}+10)P_{11}-11P_1 = 10\sqrt{3}P_9 \quad (A) \checkmark$$

Akash

QUESTION [JEE Mains 2025 (3 April)]

Let α and β be the roots of $x^2 + \sqrt{3}x - 16 = 0$, and γ and δ be the roots of $x^2 + 3x - 1 = 0$. If $P_n = \alpha^n + \beta^n$ and $Q_n = \gamma^n + \delta^n$, then $\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25} - Q_{23}}{Q_{24}}$ is equal to

(A) 4

(B) 3

(C) 5

(D) 7

Ans. C

Homework

Mathematics

Tah : 08

$x^2+\sqrt{3}x-16=0 \begin{smallmatrix}\nearrow \alpha \\ \searrow \beta\end{smallmatrix}$ and $x^2+3x-1=0 \begin{smallmatrix}\nearrow \gamma \\ \searrow \delta\end{smallmatrix}$ and $P_n=\alpha^n+\beta^n$ and $Q_n=\gamma^n+\delta^n \longrightarrow \dfrac{P_{25}+\sqrt{3}P_{24}}{2P_{23}}+\dfrac{Q_{25}-Q_{23}}{Q_{24}}$

$P_{25}+\sqrt{3}P_{24}-16P_{23}=0$

$P_{25}+\sqrt{3}P_{24}=16P_{23}$

$\dfrac{P_{25}+\sqrt{3}P_{24}}{2P_{23}}=8$

$Q_{25}+3Q_{24}-Q_{23}=0$

$\dfrac{Q_{25}-Q_{23}}{Q_{24}}=-3$

adding both we get

$8-3=5$ Ans (C)

Akash

QUESTION [JEE Mains 2025 (2 April)]

Let $P_n = \alpha^n + \beta^n, n \in N$. If $P_{10} = 123, P_9 = 76, P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is :

(A) $x^2 + x - 1 = 0$

(B) $x^2 - x + 1 = 0$

(C) $x^2 + x + 1 = 0$

(D) $x^2 - x - 1 = 0$

let $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$P_1 = \alpha + \beta = 1 = -\frac{b}{a}$

$\frac{b}{a} = -1$

$x^2 - x + \frac{c}{a} = 0$

$\underbrace{NF}$ $P_{n+2} - P_{n+1} + \frac{c}{a} P_n = 0$

$\underbrace{n=8}$ $P_{10} - P_9 + \frac{c}{a} P_8 = 0$

$123 - 76 + 47\frac{c}{a} = 0$

$47\frac{c}{a} = -47$

$\frac{c}{a} = -1$

Eqn: $\underbrace{x^2 - x - 1 = 0}$ $\begin{cases}\alpha \\ \beta\end{cases}$ $\alpha + \beta = 1$, $\alpha\beta = -1$

$S.O.R = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{1}{-1} = -1$

$P.O.R = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = -1$

Eqn: $\underbrace{x^2 + x - 1 = 0}$.

Ans. A

Solution to Previous KTKs

QUESTION

(KTK 1)

If α, β are the roots of the equation $x^2 + px - r = 0$ and $\frac{\alpha}{3}, 3\beta$ are the roots of the equation $x^2 + qx - r = 0$, then r equals

(A) $\frac{3}{8}(p - 3q)(3p + q)$

(B) $\frac{3}{8}(p + 3q)(3p - q)$

(C) $\frac{3}{64}(3p - q)(p - 3q)$

(D) $\frac{3}{64}(3q - p)(p - q)$

Ans. C

QUESTION

(KTK 2)

If one of the root of the equation $4x^2 - 15x + 4p = 0$ is the square of the other then the value of p is

(A) $\frac{125}{64}$

(B) $-\frac{27}{8}$

(C) $-\frac{125}{8}$

(D) $\frac{27}{8}$

Ans. C, D

KTK-02

$4x^2 - 15x + 4P = 0 \rightarrow \alpha, \beta \quad , \alpha^2 = \beta$

$4\alpha^2 - 15\alpha + 4P = 0.$

$\alpha + \beta = \frac{15}{4}$

$\alpha^2 + \alpha = \frac{15}{4}$

$\alpha . \beta = \frac{4P}{4}$

$\alpha^3 = P$

Now,

$4\alpha^2 + 4\alpha = 15$

$4\alpha^2 + 4\alpha - 15 = 0$

$4\alpha^2 + 10\alpha - 6\alpha - 15 = 0$

$2\alpha(2\alpha + 5) - 3(2\alpha + 5) = 0$

$(2\alpha + 5)(2\alpha - 3) = 0$

$\alpha = \frac{3}{2}, -\frac{5}{2}$

$\alpha^3 = \frac{27}{8}$

$\alpha^3 = -\frac{125}{8}$

$P = \frac{27}{8}, -\frac{125}{8}$

QUESTION

(KTK 3)

If $b \in R^{+}$ then roots of the equation $(2+b)x^2+(3+b)x+(4+b)=0$ is

(A) Real and distinct

(B) Real and equal

(C) Imaginary

(D) Cannot be predicted

Ans. C

KTK-03

Q) $(2+b)x^2+(3+b)x+(4+b)=0$ $\in R^+$

$$D = (b+3)^2 - 4(b+2)(b+4)$$
$$= b^2+6b+9-4(b^2+6b+8)$$
$$= b^2+6b+9-4b^2-24b-32$$

$$D = -3b^2 - 19b - 23 < 0$$

$$b \in R^+, \quad b^2 \in R^+$$

So, roots are imaginary

Homework

Mathematics

KTK 03

$b \in R^+ \longrightarrow (2+b)x^2+(3+b)x+(4+b)=0$

$$(3+b)^2-4(4+b)(2+b)$$
$$=9+b^2+6b-4(8+6b+b^2)$$
$$=9+b^2+6b-32-24b-4b^2$$
$$=-3b^2-18b-23$$
$$=-(3b^2+18+23)$$

$b \in R^+$

$-(+ve)$

$=-ve$ imaginary roots

QUESTION

(KTK 4)

If x satisfies $|x-1|+|x-2|+|x-3| \geq 6$, then

(A) $0 \leq x \leq 4$

(B) $x \leq -2$ or $x \geq 4$

(C) $x \leq 0$ or $x \geq 4$

(D) none

Ans. C

KTK-04

$|x-1|+|x-2|+|x-3| \geq 6$

$|x-1|+|x-2|+|x-3|-6 \geq 0$

T_1 T_2 T_3

	$x<1$	$1<x<2$	$2<x<3$	$x>3$
T_1	$-$	$+$	$+$	$+$
T_2	$-$	$-$	$+$	$+$
T_3	$-$	$-$	$-$	$+$

$\boxed{x \leq 1} \rightarrow$ Case (1)

$-x+1-x+2-x+3-6 \geq 0$

$-3x \geq 0$

$3x \leq 0$

$x \leq 0 \rightarrow x \in (-\infty, 0]$

$\boxed{1 < x \leq 2} \rightarrow$ Case (II)

$x-1-x+2-x+3-6 \geq 0$

$-x-2 \geq 0$

$x+2 \leq 0$

$x \leq -2$

$\cap \rightarrow x \in \phi$

Case (III) $\quad 2 < x < 3$

$$x - 1 + x - 2 - x + 3 - 6 \geq 0$$
$$x - 6 \geq 0$$
$$x \geq 6$$

$\cap \rightarrow \phi \in x$

Case (IV) $\quad x \geq 3$

$$x - 1 + x - 2 + x - 3 - 6 \geq 0$$
$$3x \geq 12$$
$$x \geq 4$$

$\cap \rightarrow x \geq 4$

(I) $\cup$ (II) $\cup$ (III) $\cup$ (IV)

$x \in (-\infty, 0] \cup [4, \infty)$

Homework

Mathematics

KTK 07

$|x-1|+|x-2|+|x-3|\geq 6$

Case I $x\geq 3$

$x-1+x-2+x-3\geq 6$

$3x-6\geq 6$

$3x\geq 12$

$\boxed{x\geq 4}$

Case II

$2\leq x\leq 3$

$x-1+x-2-x+3\geq 6$

$x\geq 6$

$x\in\phi$

Case III

$1\leq x\leq 2$

$x-1-x+2-x+3\geq 6$

$-x+4\geq 6$

$-x\geq 2$

$x\leq -2$

$x\in\phi$

Case IV

$x\leq 1$

$-3x+6\geq 6$

$x\leq 0$

union of values of $x=$ $x\in(-\infty,0]\cup[4,\infty)$ (C)

QUESTION

(KTK 5)

Number of integral values of 'a' for which the quadratic equation, $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite sign is,

P.O.R $< 0 \Rightarrow$ Root can not be imaginary

(A) 1

(B) 2

(C) 3

(D) 4

Ans. C

KTK-5

$$2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$$

real & distinct roots $\rightarrow D > 0$

also,

α, β signs are opposite so,

$$\alpha . \beta = -ve = \frac{a^2 - 4a}{2} < 0$$

$$a(a-4) < 0$$

$+ \quad - \quad +$

$0 \quad 4$

$$a \in (0, 4)$$

So, Integral values of a = 1, 2, 3

no of Integral values of a = 3

Homework

Mathematics

KTK 05

$2x^2-(a^2+8a-1)x+a^2-4a=0 \rightarrow$ values of a for which $\rightarrow$ Sign of roots are opposite

$\rightarrow \alpha, \beta \rightarrow \alpha(+ve)\ \beta(-ve)$

$\alpha\beta \rightarrow (-ve)$

$\alpha\beta \leq 0$

$\frac{a^2-4a}{2} < 0$

$a(a-4) < 0$

$\overset{+}{\longleftarrow}\underset{0}{|}\overset{-}{\longrightarrow}\underset{4}{|}\overset{+}{\longrightarrow}$ $a \in (0, 4)$

$\hookrightarrow a = 1, 2, 3 \Rightarrow 3$ (C)

Two roots of opposite sign can have +ve, −ve and 0 as their sum

So a need to satisfy only one condⁿ.

QUESTION

(KTK 6)

If a, b, c are real numbers satisfying the condition $a+b+c=0$ then the roots of the quadratic equation $3ax^2+5bx+7c=0$ are

(A) positive

(B) negative

(C) real and distinct

(D) imaginary

Ans. C

Homework From Module

Basic math

Prarambh (Topicwise) : Q1 to Q32

Prabal (JEE Main Level) : Q1 to Q46

Parikshit (JEE Advanced Level) : Q1 to Q42

THANK YOU