PRAYAS

JEE 2026

Mathematics

Quadratic Equations

Lecture - 03

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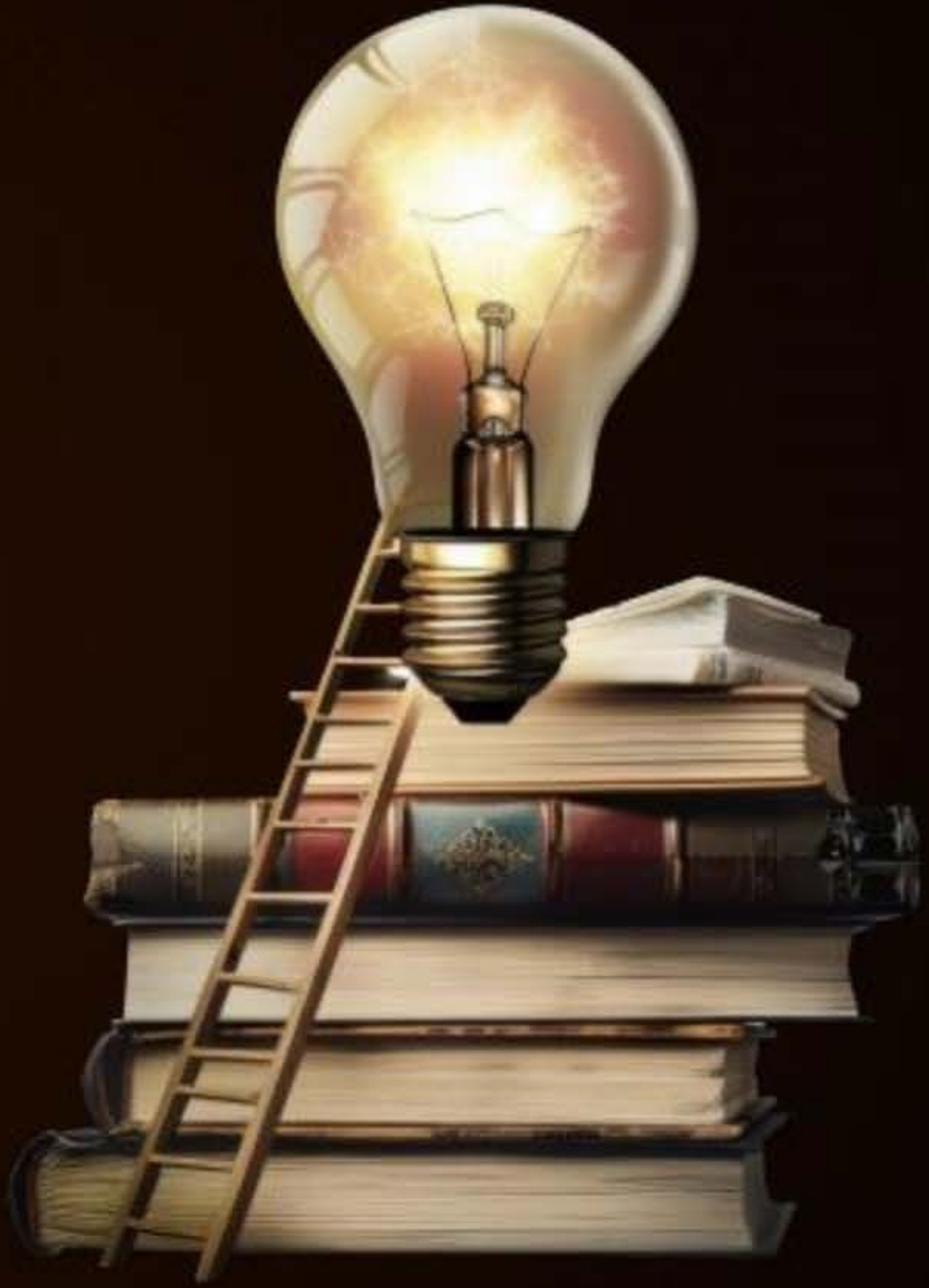
Topics *to be covered*

A General polynomial equation

B Newton's formula

C Practice problems



Recap *of previous lecture*

1. Factors of $ax^2 + bx + c \begin{cases}\alpha \\ \beta\end{cases}$ is $a(x-\alpha)(x-\beta)$

2. $ax^2 + bx + c$ is a perfect square if $D = 0$

3. $ax^2 + bx + c$ is square of a real linear expression if $D = 0$ & $a > 0$

4. $(\alpha - \beta)^2 = \frac{D}{a^2}$, where α, β are roots of $ax^2 + bx + c = 0$.

5. If $a, b, c \in Q$ & $D \geq 0$, also 'D' is a perfect square then roots of $ax^2 + bx + c = 0$ are Rational

Recap *of previous lecture*

6. If a quadratic p(x) takes the value 0 at $\underbrace{x = 2 \text{ only}}$ & P(3) = 4 then the value of p(4) is 16

$P(x) = a(x-2)^2$ also $P(3) = 4 \Rightarrow 4 = a(3-2)^2$

$a = 4$

$P(x) = 4(x-2)^2$

$\Rightarrow P(4) = 4 \cdot 4 = 16$

7. $\alpha^5 - \beta^5 = (\alpha^2 - \beta^2)(\alpha^3 + \beta^3) + \alpha^2\beta^2(\beta - \alpha)$

8. If coefficient of a quadratic are all odd integers, then roots of the quadratic cannot be Rational

9. If product of roots of a quadratic over coefficients is negative, then roots of the quadratic cannot be imaginary

Recap *of previous lecture*

11. If $a, b, c \in R, D < 0$ then roots of $ax^2 + bx + c = 0$ are Imaginary hence if one root is $i - 3$ then other root $-i-3$

12. If $a + b + c = 0$ then roots of $ax^2 + bx + c = 0$ then roots are $1, \frac{c}{a}$

13. If $a - b + c = 0$ then roots of $ax^2 + bx + c = 0$ then roots are $-1, -c/a$

14. If $a = 1, b, c \in I$ & D is a perfect square. $\Rightarrow$ roots are Integers.

15. If α, β are roots of $x^2 - x + 7 = 0$ then equation of with roots $2\alpha - 1$ & $2\beta - 1$ is $x^2 + 27 = 0$

$S = (2\alpha - 1) + 2\beta - 1 = 2(\alpha + \beta) - 2 = 0$

$P = (2\alpha - 1)(2\beta - 1) = 4\alpha\beta - 2(\alpha + \beta) + 1 = 27$

Homework Discussion

QUESTION [JEE Mains 2023 (25 Jan)]

★★KCLS★★

Let $\alpha \in \mathbb{R}$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is

$$(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2 = -30$$

$$((\alpha+\beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2 = -30$$

Ans. 45

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

QUESTION

Let λ_1 and λ_2 be two values of λ for which the expression $x^2 + (2-\lambda)x + \lambda - \frac{3}{4}$ becomes a perfect square. The value of $(\lambda_1^2 + \lambda_2^2)$ equals

(A) 8

(B) 25

(C) 50

(D) 100

$D = 0$

$(2-\lambda)^2 - 4\left(\lambda - \frac{3}{4}\right) = 0$

$4 + \lambda^2 - 4\lambda - 4\lambda + 3 = 0$

$\lambda^2 - 8\lambda + 7 = 0 \quad \lambda_1, \lambda_2$

$\lambda_1^2 + \lambda_2^2 = (\lambda_1 + \lambda_2)^2 - 2\lambda_1\lambda_2$

$= 8^2 - 14 = 50$ Ans

QUESTION [AIEEE 2011]

★★KCLS★★

Let for $a \neq a_1 \neq 0$, $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(2)$ is:

(A) 18

(B) 3

(C) 9

(D) 6

$P(x) = f(x) - g(x) = $ quadratic

zero at only $x = -1$

$P(x) = A(x+1)^2$

$P(-2) = A(-1)^2 = A = 2$

$A = 2$

$P(x) = 2(x+1)^2$

$P(2) = 2(3)^2 = 18$.

QUESTION

Tah 01

Let p & q be the two roots of the equation, $mx^2 + x(2-m) + 3 = 0$. Let m_1, m_2 be the two values of m satisfying $\frac{p}{q} + \frac{q}{p} = \frac{2}{3}$. Determine the numerical value of $\frac{m_1}{m_2^2} + \frac{m_2}{m_1^2}$.

$mx^2 + x(2-m) + 3 = 0 \begin{cases} p \\ q \end{cases}$

$S.O.R = p+q = \frac{m-2}{m}$

$P.O.R = pq = \frac{3}{m}$

$\frac{p}{q} + \frac{q}{p} = \frac{2}{3}$

$\frac{p^2+q^2}{pq} = \frac{2}{3}$

$\frac{(p+q)^2 - 2pq}{pq} = \frac{2}{3}$

QUESTION

Kallu and Lallu solve a quadratic equation. Kallu reads its constant term wrongly and finds its roots as 8 and 2 where as Lallu reads the coefficients of x wrongly and finds its roots as –11, 1. The correct root of the equation are

(A) 11, 1

(B) –11, 1

(C) 11, –1

(D) None of these

$ax^2+bx+c=0 \rightarrow x^2+\frac{b}{a}x+\frac{c}{a}=0$

Kallu : reads wrong value of c.
- P.O.R = Galat aayaygaa
- S.O.R $= -\frac{b}{a}$ = does not depend on c

$\downarrow$ roots = 8, 2.

$\Downarrow$ S.O.R will be correct.

$\Downarrow$ S.O.R$\big|_{correct} = 10$

Lallu reads wrong value of b
- S.O.R = Galat aayaygaa
- P.O.R $= \frac{c}{a}$ = Indp of b

$\downarrow$ roots = –11, 1

$\Downarrow$ P.O.R will be correct

P.O.R$\big|_{correct} = -11$

Eqn: $x^2-10x-11=0$

$(x-11)(x+1)=0 \rightarrow x=-1, 11.$

QUESTION

★★KCLS★★

Given that the quadratic equation $ax^2 + bx + c = 0$ has no real roots, but Mr. X got two roots 2 and 4 since he wrote down a wrong value of 'a'. Mr. Y also got two roots –1 and 4 because he wrote the sign of a term wrongly. Then the value of $\frac{2b+3c}{a}$ is equal to

$ax^2+bx+c=0$ has no real roots $\Rightarrow D = b^2-4ac<0 \Rightarrow 0 \le b^2 < 4ac \Rightarrow 4ac = +ve$

$\Downarrow$

a, c are of same sign

Mr X: Reads wrong value of a

$a'x^2+bx+c=0 \;\; 2, 4$

$S.O.R = 6 = -\frac{b}{a'}$

$P.O.R = 8 = \frac{c}{a'}$

$-\frac{b}{c} = \frac{3}{4}$

$4b = -3c$

b, c are of opp sign

Mr Y: wrote sign of a term wrongly

$\downarrow$

roots $-1, 4$

$b=-3a \quad -\frac{b}{a} = 3 \quad S.O.R = 3$ — a, b sahi sign hai

$P.O.R = -4$ — wrong sign of c.

$\frac{(-c)}{a} = -4$

$c = 4a$

$P.O.R = \frac{c}{a} = +ve$ — a, c have same sign

b, c have opp sign

$S.O.R = -\frac{b}{a} = +ve$ — a, b have opp sign.

$$\frac{2b+3c}{a} = \frac{-6a+12a}{a} = 6. \underline{\text{Ans}}$$

QUESTION [JEE Advanced 2020]

★★ASRQ★★

Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c, d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of $ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$ is

(A) 0

(B) 8000

(C) 8080

(D) 16000

$x^2+20x-2020 \genfrac{}{}{0pt}{}{a}{b}$

$x^2-20x+2020 \genfrac{}{}{0pt}{}{c}{d}$

$a+b=-20,\ ab=-2020$

$c+d=20,\ cd=2020$

$E = a^2c - ac^2 + a^2d - ad^2 + b^2c - bc^2 + b^2d - bd^2$

$= a^2(c+d) + b^2(c+d) - c^2(a+b) - d^2(a+b)$

$= (a^2+b^2)(c+d) - (a+b)(c^2+d^2)$

$= ((a+b)^2 - 2ab)(c+d) - (a+b)((c+d)^2 - 2cd)$

$$ax^2+bx+c=0 \begin{cases}\alpha\\ \beta\end{cases}$$

$$(\alpha-\beta)^2=\frac{D}{a^2}$$

$$\cancel{|\alpha-\beta|=\frac{\sqrt{D}}{|a|}}$$

$$|a+ib|=\sqrt{a^2+b^2}$$

$$|\alpha-\beta|=\frac{\sqrt{D}}{|a|}$$ only for real roots

$$x^2+x+1=0 \begin{cases}\frac{-1-\sqrt{3}i}{2}=\beta\\ \frac{-1+\sqrt{3}i}{2}=\alpha\end{cases}$$

$$D=1-4=-3$$

$$|\alpha-\beta|=|\sqrt{3}i|=\sqrt{3}\neq\frac{\sqrt{-3}}{|1|}$$

QUESTION [JEE Advanced 2016]

★★ASRQ★★

Let S be the set of all non-zero numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S?

(A) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$

(B) $\left(-\frac{1}{\sqrt{5}}, 0\right)$

(C) $\left(0, \frac{1}{\sqrt{5}}\right)$

(D) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

$\alpha x^2 - x + \alpha = 0 \begin{array}{l} x_1 \\ x_2 \end{array}$

$|x_1 - x_2| < 1$

$\frac{\sqrt{D}}{|a|} < 1$

$\frac{\sqrt{1-4\alpha^2}}{|\alpha|} < 1.$

$\sqrt{1-4\alpha^2} < |\alpha|$

$1-4\alpha^2 < \alpha^2$

$5\alpha^2 > 1$

$(\sqrt{5}\alpha - 1)(\sqrt{5}\alpha + 1) > 0$

$\alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(1/\sqrt{5}, \infty\right)$

since $x_1, x_2 \in R$

$D = 1 - 4\alpha^2 > 0$

$4\alpha^2 < 1$

$(2\alpha - 1)(2\alpha + 1) < 0$

$\alpha \in (-1/2, 1/2)$

$\cap$

$\alpha \in \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, 1/2\right)$

Ans. A, D

QUESTION

Tan 02

Let α, β are the roots of the equation $x^2 + x - 3 = 0$. Then the value of $\alpha^3 - 4\beta^2 + 19$ is equal to

Ans. 0

QUESTION

★★ASRQ★★

If α, β are the roots of $ax^2 + bx + c = 0, (a \neq 0)$ and $\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + Bx + C = 0, (A \neq 0)$ for some constant δ, then prove that $\frac{b^2-4ac}{a^2} = \frac{B^2-4AC}{A^2}$

$ax^2+bx+c=0 \begin{matrix}\alpha \\ \beta\end{matrix} > (D.O.R)^2 = (\alpha-\beta)^2 = \frac{D_1}{a^2}$ — (I)

$Ax^2+Bx+C=0 \begin{matrix}\alpha+\delta \\ \beta+\delta\end{matrix} > (D.O.R)^2 = (\alpha+\delta-\beta-\delta)^2 = (\alpha-\beta)^2 = \frac{D_2}{A^2}$ — (II)

from (I) & (II)

$$\frac{D_1}{a^2} = \frac{D_2}{A^2}$$

$$\frac{b^2-4ac}{a^2} = \frac{B^2-4AC}{A^2}$$

General Polynomial Equation

$$a_0x^3 + a_1x^2 + a_2x + a_3 = 0 \begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$$

$$a_0x^3 + a_1x^2 + a_2x + a_3 = a_0(x-\alpha)(x-\beta)(x-\gamma)$$

$$= a_0(x^2 - (\alpha+\beta)x + \alpha\beta)(x-\gamma)$$

$$= a_0(x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma)$$

$$a_0x^3+a_1x^2+a_2x+a_3 = a_0x^3 - a_0(\alpha+\beta+\gamma)x^2 + a_0(\alpha\beta+\beta\gamma+\gamma\alpha) - a_0\alpha\beta\gamma$$

$$-a_0(\alpha+\beta+\gamma) = a_1 \longrightarrow S_1 = \alpha+\beta+\gamma = \frac{-a_1}{a_0}$$

$$a_0(\alpha\beta+\beta\gamma+\gamma\alpha) = a_2 \longrightarrow S_2 = \alpha\beta+\beta\gamma+\gamma\alpha = \frac{a_2}{a_0}$$

$$-a_0\alpha\beta\gamma = a_3 \longrightarrow S_3 = \alpha\beta\gamma = -\frac{a_3}{a_0}$$

$$ax^3 \overset{-}{+} bx^2 \overset{+}{+} cx \overset{-}{+} d = 0 \begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$$

$$S_1 = \alpha+\beta+\gamma = -\frac{b}{a}$$

$$S_2 = \alpha\beta+\beta\gamma+\gamma\alpha = c/a$$

$$S_3 = \alpha\beta\gamma = -\frac{d}{a}$$

$$ax^4 + bx^3 + cx^2 + dx + e = 0 \;\begin{cases}\alpha\\ \beta\\ \gamma\\ \delta\end{cases}$$

$$S_1 = \alpha+\beta+\gamma+\delta = -\frac{b}{a}$$

$$S_2 = \alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta = \frac{c}{a}$$

$$S_3 = \alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta = -\frac{d}{a}$$

$$S_4 = \alpha\beta\gamma\delta = \frac{e}{a}$$

Ex: $x^3 + 6x^2 - 11x + 5 = 0 \;\begin{cases}\alpha\\ \beta\\ \gamma\end{cases}$

$$\alpha+\beta+\gamma = -\frac{6}{1} = -6$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = +\frac{(-11)}{1} = -11$$

$$\alpha\beta\gamma = -\frac{(5)}{1} = -5$$

Ex: $x^4 - 5x^2 + 6x + 7 = 0 \;\begin{cases}\alpha\\ \beta\\ \gamma\\ \delta\end{cases}$

$$x^4 + 0\cdot x^3 - 5x^2 + 6x + 7 = 0$$

$$\alpha+\beta+\gamma+\delta = 0$$

$$\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta = +\frac{(-5)}{1} = -5.$$

QUESTION

Given that the equation $x^3 - px^2 + qx - r = 0$ has roots α, β and γ, find

(i) $\alpha^2 + \beta^2 + \gamma^2$

$$= (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)$$

$$= p^2 - 2q$$

(ii) $\alpha^3 + \beta^3 + \gamma^3$

$$\alpha^3+\beta^3+\gamma^3 - 3\alpha\beta\gamma = (\alpha+\beta+\gamma)(\underbrace{\alpha^2+\beta^2+\gamma^2} - \alpha\beta - \beta\gamma - \gamma\alpha)$$

$$= p \cdot (p^2 - 2q - (\alpha\beta+\beta\gamma+\gamma\alpha))$$

$$= p \cdot (p^2 - 2q - q)$$

$$\alpha^3+\beta^3+\gamma^3 - 3(r) = p \cdot (p^2 - 3q)$$

$$\alpha^3+\beta^3+\gamma^3 = p^3 - 3pq + 3r.$$

QUESTION

Tah03

$$P(x) = x^3 + 33x^2 + 327x + 935$$

Let $P(x)$ be a polynomial as described above with a, b, c the roots of $P(x)$.

Find $a^2 + b^2 + c^2$ without solving $P(x) = 0$.

QUESTION

Tan04

Let r_1, r_2 and r_3 be the roots of the polynomial $5x^3 - 11x^2 + 7x + 3$.
Evaluate $r_1(1 + r_2 + r_3) + r_2(1 + r_3 + r_1) + r_3(1 + r_1 + r_2)$.

QUESTION

Let α, β, γ be roots of $x^3 + 2x^2 - 4x + 5 = 0$ find value of $\frac{(\alpha^3+5)(\beta^3+5)(\gamma^3+5)}{13\alpha\beta\gamma}$.

$\alpha^3 + 2\alpha^2 - 4\alpha + 5 = 0$

$\alpha^3 + 5 = 4\alpha - 2\alpha^2$

$\alpha^3 + 5 = 2\alpha(2-\alpha)$

lly $\beta^3 + 5 = 2\beta(2-\beta)$

$\gamma^3 + 5 = 2\gamma(2-\gamma)$

$E = \frac{(\alpha^3+5)(\beta^3+5)(\gamma^3+5)}{13\alpha\beta\gamma}$

$E = \frac{2\alpha(2-\alpha)\,2\beta(2-\beta)\,2\gamma(2-\gamma)}{13\alpha\beta\gamma}$

$E = \frac{8}{13}(2-\alpha)(2-\beta)(2-\gamma)$

Now $x^3 + 2x^2 - 4x + 5 = (x-\alpha)(x-\beta)(x-\gamma)$

put $x=2$ $8 + 8 - 8 + 5 = (2-\alpha)(2-\beta)(2-\gamma)$

$(2-\alpha)(2-\beta)(2-\gamma) = 13$

$E = \frac{8}{13} \cdot 13 = 8$ Ans

QUESTION

If α, β, γ are roots of $5x^3 - qx - 1 = 0$, $(q \in R)$ find the value of $\frac{\alpha^2-3}{\beta\gamma} + \frac{\beta^2-3}{\alpha\gamma} + \frac{\gamma^2-3}{\alpha\beta}$.

$5x^3 - qx - 1 = 0$ (roots α, β, γ)

$5x^3 + 0 \cdot x^2 - qx - 1 = 0$

$\alpha + \beta + \gamma = 0$

$\Downarrow$

$\alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$

$$E = \frac{\alpha^3 - 3\alpha + \beta^3 - 3\beta + \gamma^3 - 3\gamma}{\alpha\beta\gamma}$$

$$E = \frac{\alpha^3 + \beta^3 + \gamma^3 - 3(\alpha+\beta+\gamma)}{\alpha\beta\gamma}$$

$$E = \frac{3\alpha\beta\gamma - 3(\overset{0}{\overbrace{\alpha+\beta+\gamma}})}{\alpha\beta\gamma}$$

$E = 3$ Ans

QUESTION

Jan05

If α, β and γ are roots of cubic equation $x^3 + 3x - 1 = 0$ then find value of :

(i) $(2-\alpha)(2-\beta)(2-\gamma)$

(ii) $(3+\alpha)(3+\beta)(3+\gamma)$

(iii) $(4-\alpha^2)(4-\beta^2)(4-\gamma^2)$

(iv) $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)$

QUESTION [JEE Mains 2024 (6 April)]

Tan06

Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and $(4 + x_1^2)(4 + x_2^2)(4 + x_3^2)(4 + x_4^2) = \frac{125}{16} m$. Then the value of m is

Ans. 221

QUESTION

If $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are roots of equation $x^5 - 5x^4 - 1 = 0$, then

Home Challenge

(A) $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -\frac{1}{20}$

(B) $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -20$

(C) $\prod_{r=1}^{r=5} \left(\frac{1}{\alpha_r^4} + 5\right)^5 = 1$

$\left(\frac{1}{\alpha_1^4}+5\right)^5 \cdot \left(\frac{1}{\alpha_2^4}+5\right)^5 \cdot \left(\frac{1}{\alpha_3^4}+5\right)^5 \left(\frac{1}{\alpha_4^4}+5\right)^5 \left(\frac{1}{\alpha_5^4}+5\right)^5$

(D) $\prod_{r=1}^{r=5} \left(\frac{1}{\alpha_r^4} + 5\right)^3 = \frac{1}{5}$

Newton's Formula

$S_n = p\alpha^n + q\beta^n$ p, q are constants.

$ax^2+bx+c=0 \begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}$

$a\alpha^2+b\alpha+c=0$, $a\beta^2+b\beta+c=0$

Multiply both sides by $p\alpha^n$

multiply both sides by $q\beta^n$

$ap\alpha^{n+2}+bp\alpha^{n+1}+cp\alpha^n=0$, $aq\beta^{n+2}+bq\beta^{n+1}+q\beta^n \cdot c=0$

$+$

$a(p\alpha^{n+2}+q\beta^{n+2})+b(p\alpha^{n+1}+q\beta^{n+1})+c(p\alpha^n+q\beta^n)=0$

$aS_{n+2}+bS_{n+1}+cS_n=0 \rightarrow$ where $S_n = p\alpha^n+q\beta^n$

if $P.O.R \neq 0$, $n \in I$

if $P.O.R = 0$, $n \in N$.

$S_n = p\alpha^n + q\beta^n$

$\begin{matrix}\alpha\\ \beta\end{matrix}\Big\rangle\, x^2 - 5x + 7 = 0$ M(1) $V = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{5}{7}$, $V' = \alpha^5 + \beta^5 \to S_5$

$S_1 = \alpha + \beta = 5$

$S_0 = \alpha^0 + \beta^0 = 1 + 1 = 2$

M(2) $S_n = \alpha^n + \beta^n$ — we want S_{-1}

By NF

$S_{n+2} - 5S_{n+1} + 7S_n = 0$

$n=3$: $S_5 - 5S_4 + 7S_3 = 0$

$n=2$: $S_4 - 5S_3 + 7S_2 = 0$

$n=1$: $S_3 - 5S_2 + 7S_1 = 0$

$n=0$: $S_2 - 5S_1 + 7S_0 = 0$

$\Downarrow$

$S_2 - 5(5) + 7\cdot 2 = 0$

$S_2 = 11$

$S_3 - 5\cdot 11 + 7(5) = 0$

$S_3 = 55 - 35 = 20$

$S_4 - 5(20) + 7\cdot 11 = 0$

$S_4 = 100 - 77 = 23$

$S_5 - 5(23) + 7\cdot 20 = 0$

$S_5 = 115 - 140 = -25$

put $n = -1$

$S_1 - 5S_0 + 7S_{-1} = 0$

$5 - 5(2) + 7S_{-1} = 0$

$S_{-1} = 5/7$

QUESTION [JEE Mains 2024 (27 Jan)]

If α, β are the roots of the equation, $x^2 - x - 1 = 0$ and $S_n = 2023\alpha^n + 2024\beta^n$, then

(A) $2S_{12} = S_{11} + S_{10}$

(B) $S_{12} = S_{11} + S_{10}$

(C) $S_{11} = S_{10} + S_{12}$

(D) $2S_{11} = S_{12} + S_{10}$

$x^2 - x - 1 = 0$ $\qquad$ $S_n = 2023\alpha^n + 2024\beta^n$

By NF

$S_{n+2} - S_{n+1} - S_n = 0$

put $n = 10$

$S_{12} - S_{11} - S_{10} = 0$

$S_{12} = S_{11} + S_{10}$

Ans. B

QUESTION [JEE Mains 2020]

Let α and β be the roots of the equations, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, $n = 1, 2, 3, \ldots$ then

By NF $\quad 5S_{n+2} + 6S_{n+1} - 2S_n = 0$

$n=4 \quad 5S_6 + 6S_5 - 2S_4 = 0$

$5S_6 + 6S_5 = 2S_4$

(A) $6S_6 + 5S_5 = 2S_4$

(B) $5S_6 + 6S_5 = 2S_4$

(C) $6S_6 + 5S_5 + 2S_4 = 0$

(D) $5S_6 + 6S_5 + 2S_4 = 0$

QUESTION [IIT-JEE 2011]

Let α, β be the roots of $x^2 - 6x - 2 = 0$ with $\alpha > \beta$ if $a_n = \alpha^n - \beta^n, n \geq 1$. Then find the value of $\frac{a_{10} - 2a_8}{2a_9}$.

M 1 N.F $a_{n+2} - 6a_{n+1} - 2a_n = 0$

put $n = 8$

$a_{10} - 6a_9 - 2a_8 = 0$

$a_{10} - 2a_8 = 6a_9$

$\frac{a_{10} - 2a_8}{2a_9} = 3.$

M 2 $\frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$

$\frac{\alpha^{10} - 2\alpha^8 - \beta^{10} + 2\beta^8}{2(\alpha^9 - \beta^9)}$

$\frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$

$\alpha^2 - 6\alpha - 2 = 0$

$\alpha^2 - 2 = 6\alpha$

lly $\beta^2 - 2 = 6\beta$

$\frac{\alpha^8 \cdot 6\alpha - \beta^8 \cdot 6\beta}{2(\alpha^9 - \beta^9)}$

$\frac{6\cancel{(\alpha^9 - \beta^9)}}{2\cancel{(\alpha^9 - \beta^9)}} = 3.$

QUESTION [JEE Mains 2021]

★★ASRQ★★

If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0, \alpha > \beta$ and $P_n = \alpha^n - \beta^n$ each positive integer n, then the value of $\left(\frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}\right)$ is equal to

$x^2 + 5\sqrt{2}x + 10 = 0, \quad \alpha > \beta$

$P_n = \alpha^n - \beta^n$

$E = \frac{P_{17}(P_{20} + 5\sqrt{2}P_{19})}{P_{18}(P_{19} + 5\sqrt{2}P_{18})}$

By NF $\quad P_{n+2} + 5\sqrt{2}P_{n+1} + 10P_n = 0$

$n=18 \quad P_{20} + 5\sqrt{2}P_{19} + 10P_{18} = 0$

$P_{20} + 5\sqrt{2}P_{19} = -10P_{18}$

$E = \frac{P_{17}(-10 \cdot P_{18})}{P_{18}(-10P_{17})} = 1$

$n=17 \quad P_{19} + 5\sqrt{2}P_{18} + 10P_{17} = 0$

$P_{19} + 5\sqrt{2}P_{18} = -10P_{17}$

QUESTION [JEE Mains 2024 (9 April)]

ASRQ

Tah 07

Let $\alpha, \beta; \alpha > \beta$, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$.

Let $P_n = \alpha^n - \beta^n, n \in N$. Then $(11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} - 11P_{12}$ is equal to

(A) $10\sqrt{3}P_9$

(B) $11\sqrt{3}P_9$

(C) $11\sqrt{2}P_9$

(D) $10\sqrt{2}P_9$

Ans. A

QUESTION [JEE Mains 2020]

★★ASRQ★★

Let α and β be the roots of the equations $x^2 - x - 1 = 0$. If $P_k = (\alpha)^k + (\beta)^k, k \geq 1$, then which one of the following statements is not true,

(A) $P_3 = P_5 - P_4$ ✓

(B) $(P_1 + P_2 + P_3 + P_4 + P_5) = 26$ ✓

(C) $P_5 = 11$ ✓

(D) $P_5 = P_2 \cdot P_3$ ✗

$x^2 - x - 1 = 0$

$P_k = \alpha^k + \beta^k \quad k \geq 1$

$P_{k+2} - P_{k+1} - P_k = 0$

$\underbrace{k=3}$ $P_5 - P_4 - P_3 = 0$

$P_3 = P_5 - P_4$

$P_5 = P_4 + P_3 = 7 + 4 = 11$

$\underbrace{k=2}$ $P_4 = P_3 + P_2 = 4 + 3 = 7$

$\underbrace{k=1}$ $P_3 = P_2 + P_1 = 3 + 1 = 4$

$P_2 = \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 1 + 2 = 3$

$P_1 = \alpha + \beta = 1$

$\Downarrow$

$P_1 + P_2 + P_3 + P_4 + P_5 = 26$

QUESTION [JEE Mains 2025 (3 April)]

Tah 08

Let α and β be the roots of $x^2 + \sqrt{3}x - 16 = 0$, and γ and δ be the roots of $x^2 + 3x - 1 = 0$. If $P_n = \alpha^n + \beta^n$ and $Q_n = \gamma^n + \delta^n$, then $\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25} - Q_{23}}{Q_{24}}$ is equal to

(A) 4

(B) 3

(C) 5

(D) 7

Ans. C

QUESTION [JEE Mains 2025 (2 April)]

Tan09

Let $P_n = \alpha^n + \beta^n, n \in N$. If $P_{10} = 123, P_9 = 76, P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is :

$\alpha + \beta = 1 = -\frac{b}{a}$

(A) $x^2 + x - 1 = 0$

(B) $x^2 - x + 1 = 0$

(C) $x^2 + x + 1 = 0$

(D) $x^2 - x - 1 = 0$

Ans. A

QUESTION [JEE Mains 2023 (11 April)]

ASRQ

If a and b are the roots of equation $x^2 - 7x - 1 = 0$, then the value of $\dfrac{\overbrace{a^{21}+b^{21}}^{S_{21}}+\overbrace{a^{17}+b^{17}}^{S_{17}}}{\underbrace{a^{19}+b^{19}}_{S_{19}}}$ is equal to

$S_n = a^n + b^n$ $\qquad x^2 - 7x - 1 = 0 \begin{cases} a \\ b \end{cases}$

By NF $\quad S_{n+2} - 7S_{n+1} - S_n = 0$

put $n = 19$ $\quad S_{21} - 7\cancel{S_{20}} - S_{19} = 0$ $\quad\longrightarrow\quad S_{21} - 50S_{19} - 7\cancel{S_{18}} = 0$

$\oplus$

put $n = 18$ $\quad \cancel{S_{20}} - 7S_{19} - S_{18} = 0 \quad \times 7$

$\ominus$

put $n = 17$ $\quad S_{19} - 7\cancel{S_{18}} - S_{17} = 0$

$S_{21} - 51S_{19} + S_{17} = 0$

$S_{21} + S_{17} = 51 \cdot S_{19}$

$\dfrac{S_{21} + S_{17}}{S_{19}} = 51$

Ans. 51

QUESTION [JEE Mains 2023 (11 April)]

ASRQ

If a and b are the roots of equation $x^2 - 7x - 1 = 0$, then the value of $\frac{\overbrace{a^{21}+b^{21}}^{S_{21}}+\overbrace{a^{17}+b^{17}}^{S_{17}}}{\underbrace{a^{19}+b^{19}}_{S_{19}}}$ is equal to

M2

$x^2 - 7x - 1 = 0 \begin{cases} a \\ b \end{cases}$

$a^2 - 7a - 1 = 0 \longrightarrow a^2 - 1 = 7a \xrightarrow{\text{SBS}}$

$a^4 + 1 - 2a^2 = 49a^2$

$a^4 + 1 = 51a^2$

lly $b^4 + 1 = 51b^2$

$$\frac{a^{21}+a^{17}+b^{21}+b^{17}}{a^{19}+b^{19}} = \frac{a^{17}(a^4+1)+b^{17}(b^4+1)}{a^{19}+b^{19}}$$

$$= \frac{a^{17}\cdot 51a^2 + b^{17}\cdot 51b^2}{a^{19}+b^{19}}$$

$$= \frac{51\cancel{(a^{19}+b^{19})}}{\cancel{a^{19}+b^{19}}} = 51$$

Ans. 51

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

(KTK 1)

If α, β are the roots of the equation $x^2 + px - r = 0$ and $\frac{\alpha}{3}, 3\beta$ are the roots of the equation $x^2 + qx - r = 0$, then r equals

(A) $\frac{3}{8}(p - 3q)(3p + q)$

(B) $\frac{3}{8}(p + 3q)(3p - q)$

(C) $\frac{3}{64}(3p - q)(p - 3q)$

(D) $\frac{3}{64}(3q - p)(p - q)$

Ans. C

QUESTION

(KTK 2)

If one of the root of the equation $4x^2 - 15x + 4p = 0$ is the square of the other then the value of p is

(A) $\frac{125}{64}$

(B) $-\frac{27}{8}$

(C) $-\frac{125}{8}$

(D) $\frac{27}{8}$

Ans. C, D

QUESTION

(KTK 3)

If $b \in R^+$ then roots of the equation $(2 + b)x^2 + (3 + b)x + (4 + b) = 0$ is

(A) Real and distinct

(B) Real and equal

(C) Imaginary

(D) Cannot be predicted

Ans. C

QUESTION

(KTK 4)

If x satisfies $|x-1|+|x-2|+|x-3| \geq 6$, then

(A) $0 \leq x \leq 4$

(B) $x \leq -2$ or $x \geq 4$

(C) $x \leq 0$ or $x \geq 4$

(D) none

Ans. C

QUESTION

(KTK 5)

Number of integral values of 'a' for which the quadratic equation, $2x^2-(a^3+8a-1)x+a^2-4a=0$ possesses roots of opposite sign is,

(A) 1

(B) 2

(C) 3

(D) 4

Ans. C

QUESTION

(KTK 6)

If a, b, c are real numbers satisfying the condition $a+b+c=0$ then the roots of the quadratic equation $3ax^2+5bx+7c=0$ are

(A) positive

(B) negative

(C) real and distinct

(D) imaginary

Ans. C

Homework From Module

Prarambh (Topicwise) : Q1 to Q32

Prabal (JEE Main Level) : Q1 to Q46

Parikshit (JEE Advanced Level) : Q1 to Q42

Solution to Previous BPPs

Bumper Practice Problems

1. For what values of a does the equation $9x^2 - 2x + a = 6 - ax$ posses equal roots ?

2. Find the values of a for which the roots of the equation $(2a - 5)x^2 - 2(a - 1)x + 3 = 0$ are equal.

3. For what values of m does the equation $x^2 - x + m = 0$ possess no real roots ?

4. For what values of m does the equation $mx^2 - (m + 1)x + 2m - 1 = 0$ possess no real roots?

5. Find integral values of k for which the equation $(k - 12)x^2 + 2(k - 12)x + 2 = 0$ possess no real roots?

6. For what values of 'a' does the equation $x^2 + 2a\sqrt{a^2 - 3}x + 4 = 0$ possess equal roots?

Bumper Practice Problems

7. Form a quadratic equation whose roots are the numbers $\frac{1}{10-\sqrt{72}}$ and $\frac{1}{10+6\sqrt{2}}$.

8. Find the least integral value of k for which the equation $x^2 - 2(k+2)x + 12 + k^2 = 0$ has two different real roots.

9. For what values of a is the sum of the roots of the equation $x^2 + (2 - a - a^2)x - a^2 = 0$ equal to zero?

10. For what values of a do the graphs of the functions $y = 2ax + 1$ and $y = (a-6)x^2 - 2$ not intersect?

$2ax+1=(a-6)x^2-2$ should have no real roots.

$(a-6)x^2-2ax-3=0$

if $a \neq 6$ $D<0$

$\cancel{4}a^2-\cancel{4}(-3)(a-6)<0$.

$a^2+3a-18<0$

$(a+6)(a-3)<0$

$a \in (-6,3)$

if $a=6$ $-12x-3=0$

$x=-1/4$, $a=6$ (rejected)

11. For what values of a is the ratio of the roots of the equation $x^2 + ax + a + 2 = 0$ equal to 2?

Bumper Practice Problems

12. For what values of a do the roots x_1 and x_2 of the equation $x^2-(3a+2)x+a^2=0$ satisfy the relation $x_1=9x_2$?

13. Find a such that one of the roots of the equation $x^2-\frac{15}{4}x+a=0$ is the square of the other.

14. The roots x_1 and x_2 of the equation $x^2+px+12=0$ are such that $x_2-x_1=1$. Find p.

15. Find k in the equation $5x^2-kx+1=0$ such that the difference between the roots of the equation is unity.

Answers

1. $a = 20 \pm 6\sqrt{5}$
2. $a = 4$
3. $m \in \left(\frac{1}{4}, \infty\right)$
4. $m \in \left(-\infty, -\frac{1}{7}\right) \cup (1, \infty)$
5. $k = 13$
6. $a = \pm 2$
7. $28x^2 - 20x + 1 = 0$
8. $k = 3$
9. $a_1 = -2, a_2 = 1$
10. $a \in (-6,3)$
11. $a_1 = -\frac{3}{2}, a_2 = 6$
12. $a = 6, -\frac{6}{19}$
13. $a_1 = -\frac{125}{8}, a_2 = \frac{27}{8}$
14. $p = \pm 7$
15. $k = \pm 3\sqrt{5}$

Bumper Practice problems

1. $9x^2 - 2x + a = 6 - ax$, what values of a has roots equal.

$D = 0$

$9x^2 - x(2-a) + a - 6 = 0$

$D = b^2 - 4ac = 0 \Rightarrow (2-a)^2 - 36(a-6) = 0$

$\Rightarrow 4 + a^2 - 4a - 36a + 216 = 0$

$\Rightarrow a^2 - 40a + 220 = 0$

$a = \dfrac{40 \pm \sqrt{720}}{2}$

Aadya

Jharkhand

$= \dfrac{40 \pm 12\sqrt{5}}{2} = \boxed{20 \pm 6\sqrt{5}}$ Ans.

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Jharkhand

Date ___/___/___

2. $(2a-5)x^2 - 2(a-1)x + 3 = 0$, for what values of a roots are equal.

Sol: $D = 0$

$b^2 - 4ac \Rightarrow (2a-2)^2 - 12(2a-5) = 0$

$\Rightarrow 4a^2 + 4 - 8a - 24a + 60 = 0$

$\Rightarrow 4a^2 - 32a + 64 = 0$

$\Rightarrow a^2 - 8a + 16 = 0$

$\Rightarrow (a-4)^2 = 0$

$\Rightarrow \boxed{a = 4}$

3. Value of m for which $x^2 - x + m = 0$, no real roots

Sol: $D < 0 \Rightarrow b^2 - 4ac \Rightarrow 1 - 4m < 0$

$\Rightarrow \cancel{m = \frac{1}{4}}$

$\Rightarrow 4m - 1 > 0$

$\Rightarrow \boxed{m \in \left(\frac{1}{4}, \infty\right)}$

(4) $mx^2-(m+1)x+2m-1=0$

$D<0$

$(m+1)^2-4(2m-1)m<0$

$m^2+1+2m-8m^2+4m<0$

$-7m^2+6m+1<0$

$7m^2-6m-1>0$

$7m^2-7m+m-1>0$

$7m(m-1)+(m-1)>0$

$(7m+1)(m-1)>0$

$\overset{+}{}\ \underset{-\frac{1}{7}}{|}\ \overset{-}{}\ \underset{1}{|}\ \overset{+}{}$

$m\in\left(-\infty,-\frac{1}{7}\right)\cup(1,\infty)$

Sakshi

(5) $(k-12)x^2+2(k-12)x+2=0$

$D<0$

$4(k-12)^2-8(k-12)<0$

$(k-12)(4k-48-8)<0$

$(k-12)(4k-56)<0$

$(k-12)(k-14)<0$

$\overset{+}{}\ \underset{12}{|}\ \overset{-}{}\ \underset{14}{|}\ \overset{+}{}$

$k\in(12,14)$

integral value of $k=13$

7.

Sol^n $\alpha = \frac{1}{10-\sqrt{72}}$, $\beta = \frac{1}{10+6\sqrt{2}}$

$$S.O.R = \frac{1}{10-6\sqrt{2}} + \frac{1}{10+6\sqrt{2}}$$

$$= \frac{10+6\sqrt{2}+10-6\sqrt{2}}{100-72}$$

$$= \frac{20}{28} = \frac{5}{7}$$

$$P.O.R = \frac{1}{10-6\sqrt{2}}\left(\frac{1}{10+6\sqrt{2}}\right)$$

$$= \frac{1}{28}$$

$\therefore$ Equation $= x^2 - \frac{5}{7}x + \frac{1}{28} = 0$

$$\frac{28x^2-20x+1}{28} = 0$$

$$28x^2-20x+1=0$$

(8)

Sol^n $x^2 - 2(k+2)x + 12 + k^2 = 0$

$D = (2k+4)^2 - 4 \cdot 1 \cdot (12+k^2) > 0$

$\cancel{4k^2} + 16k + 16 - 48 - \cancel{4k^2} > 0$

$16k - 32 > 0$

$k - 2 > 0$

$k > 2$

$k \in (2, \infty)$

$\therefore$ least integral value of $k = 3$

Sakshi

(9) $x^2+(2-a-a^2)x-a^2=0$ $\qquad a=?$

$S.O.R=0$

$$\alpha+\beta=\frac{-b}{a}=\frac{-(2-a-a^2)}{1}=0$$

$$\Rightarrow a^2+a-2=0$$

$$(a+2)(a-1)=0$$

$$a=1,-2$$

$\alpha \qquad \beta$ Ans

(10) $y = 2ax + 1$ & $y = (a-6)x^2 - 2$

$\Rightarrow 2ax + 1 = (a-6)x^2 - 2$ not intersect means

$\Rightarrow (a-6)x^2 - 2ax - 3 = 0$ no real roots.

$\Rightarrow 4a^2 + 12(a-6) < 0$ $(D<0)$.

$\Rightarrow 4a^2 + 12a - 72 < 0$

$\Rightarrow a^2 + 3a - 18 < 0$

$\Rightarrow (a+6)(a-3) < 0$

krish

$\Rightarrow a \in (-6, 3)$ Ans.

(11) $x^2 + ax + (a+2) = 0 \begin{cases} \to \alpha \\ \to \beta \end{cases} \Rightarrow$ given: $\boxed{\dfrac{\alpha}{\beta} = 2}$

S.O.R $= -a$

P.O.R $= a+2$

$\alpha = 2\beta$

Then: $\alpha + \beta = -a$
$\Rightarrow 2\beta + \beta = -a$
$\Rightarrow 3\beta = -a$
$\Rightarrow \beta = -a/3$

$\alpha\beta = a+2$
$\Rightarrow 2\beta^2 = a+2$ $[\beta = -a/3]$
$\Rightarrow 2\left(\dfrac{-a}{3}\right)^2 = a+2$
$\Rightarrow 2a^2 = 9a + 18$
$\Rightarrow 2a^2 - 9a - 18 = 0$
$\Rightarrow (2a+3)(a-6) = 0$
$\Rightarrow a = -\dfrac{3}{2}, 6$. Ans.

(12) $x^2-(3a+2)x+a^2=0$

$x_1+x_2=3a+2 \rightarrow x_2=\dfrac{3a+2}{10}$

$x_1x_2=a^2$

$x_1=9x_2$ put here

$9x_2^2=a^2$

$9\left(\dfrac{3a+2}{10}\right)^2=a^2$

$\dfrac{9(9a^2+4+12a)}{100}=a^2$

$81a^2+36+108a=100a^2$

$19a^2-108a-36=0$

$19a^2-114a+6a-36=0$

$19a(a-6)+6(a-6)$

$(a-6)(19a+6)=0$

$a=6,\ -\dfrac{6}{19}$ Ans

Sakshi

(12) $x^2-(3a+2)x+a^2=0 \rightarrow$ Satisfy the relation

$\Rightarrow x_1 = 9x_2$.

S.O.R $\Rightarrow$ $x_1+x_2=(3a+2)$.

P.O.R $\Rightarrow$ $x_1 \cdot x_2 = a^2$.

$9x_2+x_2 = 3a+2$

$\Rightarrow 10x_2 = 3a+2$

$9x_2^2 = a^2$

$\Rightarrow \frac{10a}{3} = 3a+2$

$x_2 = \sqrt{\frac{a^2}{9}} = \pm\frac{a}{3}$

$\Rightarrow 10a = 9a+6$

$\Rightarrow a = 6$ Ans.

$\Rightarrow -\frac{10a}{3} = 3a+2$

$\Rightarrow -10a = 9a+6$

$\Rightarrow -19a = 6$

$\Rightarrow a = -\frac{6}{19}$ Ans.

krish

(13) $x^2 - \frac{15}{4}x + a = 0$ $\to \alpha$, $\to \alpha^2$.

krish

$\underline{S \cdot O \cdot R} = \alpha + \alpha^2 = \frac{15}{4}$

$\Rightarrow \alpha^2 + \alpha - \frac{15}{4} = 0$ $\Rightarrow 4\alpha^2 + 4\alpha - 15 = 0$

$\Rightarrow 4\alpha^2 + 10\alpha - 6\alpha - 15 = 0$

$\underline{P \cdot O \cdot R} \Rightarrow \alpha^3 = a$ $\Rightarrow 2\alpha(2\alpha + 5) - 3(2\alpha + 5) = 0$

$\Rightarrow (2\alpha - 3)(2\alpha + 5) = 0$

$\Rightarrow \alpha = \frac{3}{2}, \frac{-5}{2}$.

$a = \left(\frac{3}{2}\right)^3$ & $\left(\frac{-5}{2}\right)^3$

$= \frac{27}{8}, \frac{-125}{8}$ Ans.

(14) $x^2 + px + 12 = 0$ with roots x_1, x_2 ; $\boxed{x_2 - x_1 = 1}$

S.O.R : $x_1 + x_2 = -p$

$$x_1 + 1 + x_1 = -p$$

$$2x_1 + 1 = -p \quad \text{—} (1)$$

P.O.R : $x_1 \cdot x_2 = 12$

$$x_1(1 + x_1) = 12$$

$$\Rightarrow x_1^2 + x_1 - 12 = 0$$

$$\Rightarrow (x_1 + 4)(x_1 - 3) = 0$$

$$\Rightarrow \boxed{x_1 = -4, 3}$$

$$\Rightarrow 2(-4) + 1 = -p$$

$$\Rightarrow \boxed{p = 7} \text{ Ans.}$$

$$\Rightarrow 2(3) + 1 = -p$$

$$= \boxed{p = -7} \text{ Ans.}$$

(15) $5x^2 - kx + 1 = 0$, $\alpha - \beta = 1$

$\alpha + \beta = k/5$, # $\alpha\beta = 1/5$

sq.

$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = \frac{k^2}{25}$

$\Rightarrow (\alpha - \beta)^2 + 2\alpha\beta + 2\alpha\beta = \frac{k^2}{25}$

यहाँ $\alpha^2 + \beta^2$ को $(\alpha+\beta)^2 - 2\alpha\beta$ BCZ फिर k के term में आ जाता !

$\Rightarrow 1 + \frac{4}{5} = \frac{k^2}{25}$

$\Rightarrow \frac{9}{5} = \frac{k^2}{25} \Rightarrow k^2 = 45$

$k = \pm 3\sqrt{5}$ Ans.

krish

Solution to Previous TAH

QUESTION [JEE Mains 2019]

The number of integral value of m for which the equation $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real root is

(A) 1

(B) infinitely many

(C) 3

(D) 2

Ans. B

Q-1: $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real roots
$m = ?$ $(m \in \mathbb{Z})$

TAH-1
By Reed
From WB

Soln: for no real roots

$D < 0$

$\Rightarrow \cancel{4}(1+3m)^2 - \cancel{4}(1+m^2)(1+8m) < 0$

$\Rightarrow \cancel{1} + 6m + 9m^2 - (\cancel{1} + 8m + m^2 + 8m^3) < 0$

$\Rightarrow -8m^3 + 8m^2 - 2m < 0$

$\Rightarrow 4m^3 - 4m^2 + m > 0$

$\Rightarrow m(4m^2 - 4m + 1) > 0$

$\Rightarrow m(2m-1)^2 > 0$

$\Rightarrow m > 0 ;\ m \neq \frac{1}{2}$

$\therefore m \in (0, \infty) - \left\{\frac{1}{2}\right\}$ $\Rightarrow$ infinite integral solutions of m. (Ans: (b))

Q (Tah-1) The number of integral value of m for which the eqn $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real root is

Soln:- for no real root, $D<0$

$$(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$$

$$D \Rightarrow (2(1+3m))^2 - 4(1+m^2)(1+8m) < 0$$

$$\Rightarrow 4(1+9m^2+6m) - 4(1+8m+m^2+8m^3) < 0$$

$$\Rightarrow \cancel{4} + 36m^2 + 24m - \cancel{4} - 32m - 4m^2 - 32m^3 < 0$$

$$\Rightarrow 32m^2 - 8m - 32m^3 < 0$$

$$\Rightarrow m(32m - 8 - 32m^2) < 0$$

$$\Rightarrow -8m(4m^2 - 4m + 1) < 0 \quad ; \quad m > 0$$

$\Rightarrow 4m^2 - 4m + 1 > 0$

$m > \dfrac{4 \pm \sqrt{16-16}}{2 \times 4} \Rightarrow m > \dfrac{4}{8} \Rightarrow \dfrac{1}{2}.$

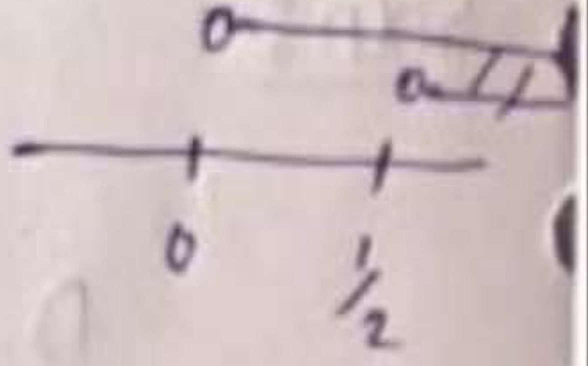
the no. of integral solution is infinitly many solution.

if $m > 0$, $m > \frac{1}{2}.$

$\cup$

$\cap$

$m > \frac{1}{2}.$



Tah-01

The no. of integral value of m for which of the Eqn $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$; has no real root is :

$\Rightarrow$ $D<0$ $\Rightarrow (-(2+6m))^2 - 4(1+m^2)(1+8m) < 0.$

$\Rightarrow (-(2+6m))^2 - (4+4m^2)(1+8m) < 0.$

$\Rightarrow 4 + 36m^2 + 24m - 4 - 32m - 4m^2 - 32m^3 < 0$

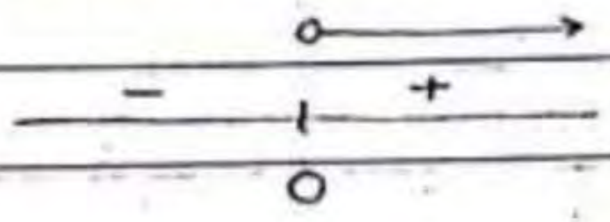
$\Rightarrow -32m^3 + 32m^2 - 8m < 0.$

$\Rightarrow -8m(4m^2 - 4m + 1) < 0.$

$\Rightarrow 8m(4m^2 - 4m + 1) > 0.$

$\Rightarrow 8m(2m-1)^2 > 0.$

$\Rightarrow$



($m = \frac{1}{2}$ is not possible).

$\Rightarrow m \in (0, \infty) - \{1/2\}$

$\Rightarrow$ Option (B) infinite soln. Ans.

QUESTION

If α and β be the roots of the equation $x^2 + 3x + 1 = 0$ then the value of $\left(\frac{\alpha}{1+\beta}\right)^2 + \left(\frac{\beta}{\alpha+1}\right)^2$ is equal to

(A) 15

(B) 18

(C) 21

(D) none

Ans. B

Q-21. If α and β be the roots of the equation $x^2+3x+1=0$ then the value of $\left(\frac{\alpha}{1+\beta}\right)^2+\left(\frac{\beta}{1+\alpha}\right)^2$ is equal to:

TAH-2 By Reed

(a) 15 (b) 18 (c) 21 (d) none.

Soln: $x^2+3x+1=0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$ $\alpha+\beta=-3$ $\alpha\beta=1.$

$\alpha^2+3\alpha+1=0$

or, $\alpha^2+2\alpha+1=-\alpha$ | lly $\beta^2+2\beta+1=-\beta.$

$$E=\frac{\alpha^2}{(1+\beta)^2}+\frac{\beta^2}{(1+\alpha)^2}=\frac{\alpha^2}{1+\beta^2+2\beta}+\frac{\beta^2}{1+\alpha^2+2\alpha}$$

$\alpha^3+\beta^3$

$=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$=-27-3(-3)$

$=-27+9=-18.$

$\Rightarrow E=\frac{\alpha^2}{-\beta}+\frac{\beta^2}{-\alpha}$

$\Rightarrow E=\frac{-(\alpha^3+\beta^3)}{\alpha\beta}$

$\Rightarrow E=-\frac{(-18)}{1}=18$ Ans.

Tah : 2 - if α and β be the roots of the equation $x^2+3x+1=0$ then the value of $\left(\frac{\alpha}{1+\beta}\right)^2+\left(\frac{\beta}{\alpha+1}\right)^2$ is equal to

Soln :- $x^2+3x+1=0 < \begin{matrix}\alpha \\ \beta\end{matrix}$ $\boxed{\begin{matrix}\alpha+\beta=-3 \\ \alpha\beta=1\end{matrix}}$

$\alpha^2+3\alpha+1=0$, $\beta^2+3\beta+1=0$

$\Rightarrow \boxed{\begin{matrix}\alpha^2+1=-3\alpha \\ \Rightarrow \alpha^2+2\alpha+1=-\alpha.\end{matrix}}$ $\boxed{\begin{matrix}\Rightarrow \beta^2+1=-3\beta. \\ \Rightarrow \beta^2+2\beta+1=-\beta\end{matrix}}$

Now, $\left(\frac{\alpha}{1+\beta}\right)^2+\left(\frac{\beta}{\alpha+1}\right)^2$

$\Rightarrow \frac{\alpha^2}{1+\beta^2+2\beta}+\frac{\beta^2}{\alpha^2+1+2\alpha} \Rightarrow \frac{\alpha^2}{-\beta}+\frac{\beta^2}{-\alpha}.$

$\Rightarrow \frac{-\alpha^3-\beta^3}{\alpha\beta}.$

$\Rightarrow -\frac{(\alpha^3+\beta^3)}{\alpha\beta}=-\frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{(\alpha\beta)}$

$\Rightarrow \frac{-(-3)\left((\alpha+\beta)^2-3\alpha\beta\right)}{\alpha\beta}$

$\Rightarrow \frac{3\left((3)^2-3\right)}{1} \Rightarrow 3(6)$

$\Rightarrow 18.$

Tah-02. If α, β be the roots of the equation $x^2+3x+1=0$ then the value of $\left(\frac{\alpha}{1+\beta}\right)^2+\left(\frac{\beta}{1+\alpha}\right)^2$ is equal to.

$\Rightarrow x^2+3x+1=0$ $\begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$

$\Rightarrow \alpha+\beta=-3.$
$\Rightarrow \alpha\beta = 1.$

$\Rightarrow \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$
$= 9-2$
$= 7$

$\Rightarrow \left(\frac{\alpha^2+\alpha+\beta^2+\beta}{\underbrace{(\alpha+1)(\beta+1)}_{\alpha\beta+\alpha+\beta+1}}\right)^2 - \frac{2\alpha\beta}{\underbrace{(\alpha+1)(\beta+1)}_{\alpha\beta+\alpha+\beta+1}}.$

$\Rightarrow \left(\frac{7-3}{1-3+1}\right)^2 - \frac{2}{(1-3+1)}.$

$\Rightarrow \left(\frac{4}{-1}\right)^2 - \frac{2}{-1} \Rightarrow 16+2$

$\Rightarrow 18$ Ans.

krish

QUESTION

If α, β be the roots of $x^2 - a(x-1) + b = 0$, then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$ is

(A) $\frac{4}{(a+b)}$

(B) $\frac{1}{(a+b)}$

(C) 0

(D) $\frac{2}{(a+b)}$

Q-3! α, β be roots of $x^2 - a(x-1) + b = 0$ then value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$ is:

Soln. $x^2 - a(x-1) + b = 0 \begin{smallmatrix} \nearrow \alpha \\ \searrow \beta \end{smallmatrix}$ $\alpha + \beta = a$, $\alpha\beta = a + b$

$\alpha^2 - a\alpha + a + b = 0$
$\Rightarrow \alpha^2 - a\alpha = -(a+b)$
$\Big\}$ lly, $\beta^2 - a\beta = -(a+b)$

TAH 3
BY REED

$\therefore E = \frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$

$\Rightarrow E = \cancel{\frac{1}{-(a+b)}} + \cancel{\frac{1}{-(a+b)}} + \cancel{\frac{2}{a+b}} = \boxed{0}$ (Ans)

Tah - 03 :- if α, β are roots of $x^2 - a(x-1) + b = 0$ then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$

Soln:- $x^2 - a(x-1) + b = 0 < \begin{matrix} \alpha \\ \beta \end{matrix}$

$\alpha^2 - a(\alpha - 1) + b = 0$

$\alpha^2 - a\alpha + a + b = 0$

$\Rightarrow \alpha^2 - \alpha a = -(a+b)$

$\beta^2 - a(\beta - 1) + b = 0$

$\beta^2 - a\beta = -(a+b)$

Now, $\frac{1}{-(a+b)} - \frac{1}{(a+b)} + \frac{2}{(a+b)}$

$\Rightarrow -\frac{2}{a+b} + \frac{2}{a+b} = 0$

Tah-03. If α, β be the roots of $x^2 - a(x-1) + b = 0$, then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$ is :

$\Rightarrow \quad x^2 - a(x-1) + b = 0 \quad \begin{matrix} \alpha \\ \beta \end{matrix}$

$\Rightarrow \quad x^2 - ax + (a+b) = 0$

Put $x = \alpha \Rightarrow \alpha^2 - a\alpha = -(a+b)$.

similarly

$\Rightarrow \beta^2 - a\beta = -(a+b)$.

$\Rightarrow -\frac{1}{a+b} - \frac{1}{a+b} + \frac{2}{a+b}$

$\Rightarrow \frac{-2}{a+b} + \frac{2}{a+b}$

$\Rightarrow 0$ Ans.

krish

QUESTION [JEE Mains 2020]

If α and β be two roots of the equation $x^2 - 64x + 256 = 0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is

(A) 1

(B) 3

(C) 4

(D) 2

Q-4! α, β roots of $x^2-64x+256=0$. Then find value

of ! $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}}+\left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}=?$

Soln

$$x^2-64x+256=0 \begin{array}{l}\nearrow \alpha \\ \searrow \beta\end{array} \qquad \begin{array}{l}\alpha+\beta=64 \\ \alpha\beta=256\end{array}$$

$$E=\left(\frac{\alpha^3}{\beta^5}\right)^{1/8}+\left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}=\frac{\alpha^{\frac{3}{8}}}{\beta^{5/8}}+\frac{\beta^{3/8}}{\alpha^{5/8}}=\frac{\alpha+\beta}{(\alpha\cdot\beta)^{5/8}}$$

TAH 4
BY REED
FROM WB

$$\Rightarrow E=\frac{64}{(256)^{5/8}}=\frac{64}{(2^8)^{5/8}}=\frac{64}{32}=2$$

Ans.

Soln:- $x^2-64x+256=0 \; \begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}$

$$\alpha+\beta=64, \quad \alpha\beta=256$$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8}+\left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$$

$$\Rightarrow \frac{\alpha^{3/8}}{\beta^{5/8}}+\frac{\beta^{3/8}}{\alpha^{5/8}} \Rightarrow \frac{\alpha^{3/8}\cdot\alpha^{5/8}+\beta^{3/8}\cdot\beta^{5/8}}{(\alpha\beta)^{5/8}}$$

$$\Rightarrow \frac{\alpha+\beta}{(\alpha\beta)^{5/8}} \Rightarrow \frac{64}{(2^8)^{5/8}} \Rightarrow \frac{64}{32}=2.$$

Tah-04.

If α and β be two roots of the Eqn :
$x^2 - 64x + 256 = 0$; Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is:

$\Rightarrow \alpha + \beta = 64.$

$\Rightarrow \alpha\beta = 256.$

$$\Rightarrow \frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}}$$

$$\Rightarrow \frac{\alpha + \beta}{(\alpha\beta)^{5/8}}$$

$$\Rightarrow \frac{64}{(2)^5} = \frac{64}{32}$$

$$= 2 \text{ Ans.}$$

$\alpha^{3/8} \cdot \alpha^{5/8}$

$\alpha^{8/8} = \alpha$

$\beta^{3/8} \cdot \beta^{5/8}$

$\beta^{8/8} = \beta$

2	256
2	128
2	64
	32

$= 2^3 \times 2^5 = 2^8$

krish

QUESTION [JEE Mains 2023 (25 Jan)]

★★KCLS★★

Let $\alpha \in \mathbb{R}$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is

Ans. 45

Tah-05. Let $\alpha \in R$ and let α, β be the roots of the eq^n $x^2 + 60^{1/4}x + a = 0$, If $\alpha^4 + \beta^4 = -30$ then the product of all possible value of a is :

$\Rightarrow \alpha + \beta = -60^{1/4}$

$\Rightarrow \alpha\beta = a$

$\Rightarrow$ given: $\alpha^4 + \beta^4 = -30$

$\Rightarrow (\alpha^2 + \beta^2)^2 - 2\cdot\alpha^2\beta^2 = -30$

SBS : $\alpha^2 + \beta^2 = (-60^{1/4})^2 - 2a$

$= 60^{1/2} - 2a$

$= \sqrt{60} - 2a$

$\Rightarrow (\sqrt{60} - 2a)^2 - 2a^2 = -30$

$\Rightarrow 60 + 4a^2 - 4\sqrt{60}a - 2a^2 = -30$

$\Rightarrow 2a^2 - 4\sqrt{60}a + 60 + 30 = 0$

$\Rightarrow 2a^2 - 4\sqrt{60}a + 90 = 0$

$D = (4\sqrt{60})^2 - 4\cdot 2\cdot 90$

$= 960 - 720$

$= 240$

* $a = \dfrac{4\sqrt{60} \pm \sqrt{240}}{4}$

$= \dfrac{\cancel{4}\sqrt{60} \pm \cancel{4}\sqrt{15}}{\cancel{4}}$

$= 2\sqrt{15} \pm \sqrt{15}$

$= \{3\sqrt{15}, \sqrt{15}\}$

Product of value $= (3\sqrt{15}) \times (\sqrt{15})$

$= 45$ Ans.

krish

QUESTION

If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to :

(A) 18

(B) 24

(C) 36

(D) 96

Q-6! If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2-\lambda x-1=0$ is 15, then $6(\alpha^3+\beta^3)^2$ is equal to:

Sol^n:

$$\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2=15.$$

$$3x^2-\lambda x-1=0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$$

$$\alpha+\beta=\lambda/3$$
$$\alpha\beta=-1/3$$

$$\Rightarrow \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}=15$$

$$\Rightarrow \frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}=15$$

$$\Rightarrow \frac{\lambda^2}{9}+\frac{2}{3}=\cancel{15}^{5}\times\frac{1}{\cancel{9}_{3}}.$$

$$\Rightarrow \frac{\lambda^2}{9}=\frac{5}{3}-\frac{2}{3}=1$$

$$\Rightarrow \boxed{\lambda=\pm 3}$$

$$\therefore \alpha+\beta=\frac{\lambda}{3}=\frac{\pm 3}{3}=\pm 1$$

$$\alpha\beta=-\frac{1}{3}$$

$$E=6(\alpha^3+\beta^3)^2$$

$$E=6\left[(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)\right]^2$$

$$E=6\left[(\pm 1)^3-\cancel{3}\left(-\frac{1}{\cancel{3}}\right)(\pm 1)\right]^2$$

$$E=6[\pm 1+(\pm 1)]^2$$

$$6[1+1]^2 = 24 \quad\Big|\quad 6[-1-1]^2 = 24$$

$$E=24$$

Ans.

TAH 6
BY REED
FROM WB

Tah-06.

If the sum of the sq. of the reciprocals of the roots α and β of the eq^n: $3x^2+\lambda x-1=0$ is 15, then $6(\alpha^3+\beta^3)^2$..

Ques उल्टा पढ़ो Hint मिल जाएगा !.

$\Rightarrow \left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2 = 15.$

$\Rightarrow \alpha^2+\beta^2 = 15\alpha^2\beta^2$

$\Rightarrow \left(-\frac{\lambda}{3}\right)^2+\frac{2}{3} = \frac{15}{9}$

$\Rightarrow \frac{\lambda^2}{9}+\frac{2}{3} = \frac{15}{9}$

$\Rightarrow \frac{\lambda^2}{\cancel{9}}+\frac{6}{\cancel{9}} = \frac{15}{\cancel{9}}$

$\Rightarrow \lambda^2 = 9$

$\Rightarrow \boxed{\lambda = \pm 3}$

$\alpha+\beta = \frac{-\lambda}{3}$.

$\alpha\beta = -1/3$.

$\Rightarrow \alpha+\beta = -1, 1$

find : $6(\alpha^3+\beta^3)^2$

$\Rightarrow 6\left[(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)\right]^2$

$\Rightarrow 6\left[(1)^3+\frac{3}{3}(1)\right]^2$

$\Rightarrow 6\times 4 = 24$ Ans.

+1. या -1 कुछ भी put करो Aus same ही आएगा !

krish

QUESTION

Let $f(x)$ be a quadratic polynomial such that $f(-2) + f(3) = 0$. If one of the roots of $f(x) = 0$ is -1, then the sum of the roots of $f(x) = 0$ is equal to:

(A) $\frac{11}{3}$

(B) $\frac{7}{3}$

(C) $\frac{13}{3}$

(D) $\frac{14}{3}$

Ans. A

Tah-06.

(b) Let $f(x)$ be a quad. poly. such that $f(-2)+f(3)=0$. If one of the roots of $f(x)=0$ is -1, then the sum of the roots of $f(x)=0$ is equal to:

$\Rightarrow$ let; $f(x) = ax^2+bx+c=0$ — roots -1, $-c/a$ $\Rightarrow$ $S.O.R = -1-\frac{c}{a} = -\frac{b}{a}$

$-1\cdot\beta = \frac{c}{a} \Rightarrow \beta = -\frac{c}{a}$

$\frac{b}{a} = 1+\frac{c}{a}$

such that:

$\Rightarrow f(-2)+f(3)=0$

$\Rightarrow 4a-2b+c+9a+3b+c=0$

$\Rightarrow 13a+b+2c=0$

$\Rightarrow 13+\frac{b}{a}+\frac{2c}{a}=0$

$\Rightarrow 13+1+\frac{c}{a}+\frac{2c}{a}=0$

$\Rightarrow 3\frac{c}{a}=-14$

$\Rightarrow \frac{c}{a}=-\frac{14}{3}$

$\Rightarrow \alpha+\beta = -\frac{b}{a}$

$\Rightarrow -1-\frac{c}{a} = -\frac{b}{a}$

$\Rightarrow -\frac{b}{a} = -\left(1+\frac{c}{a}\right)$.

Sum of root $= -1+\frac{14}{3}$

$= \frac{11}{3}$ Ans.

krish

THANK YOU