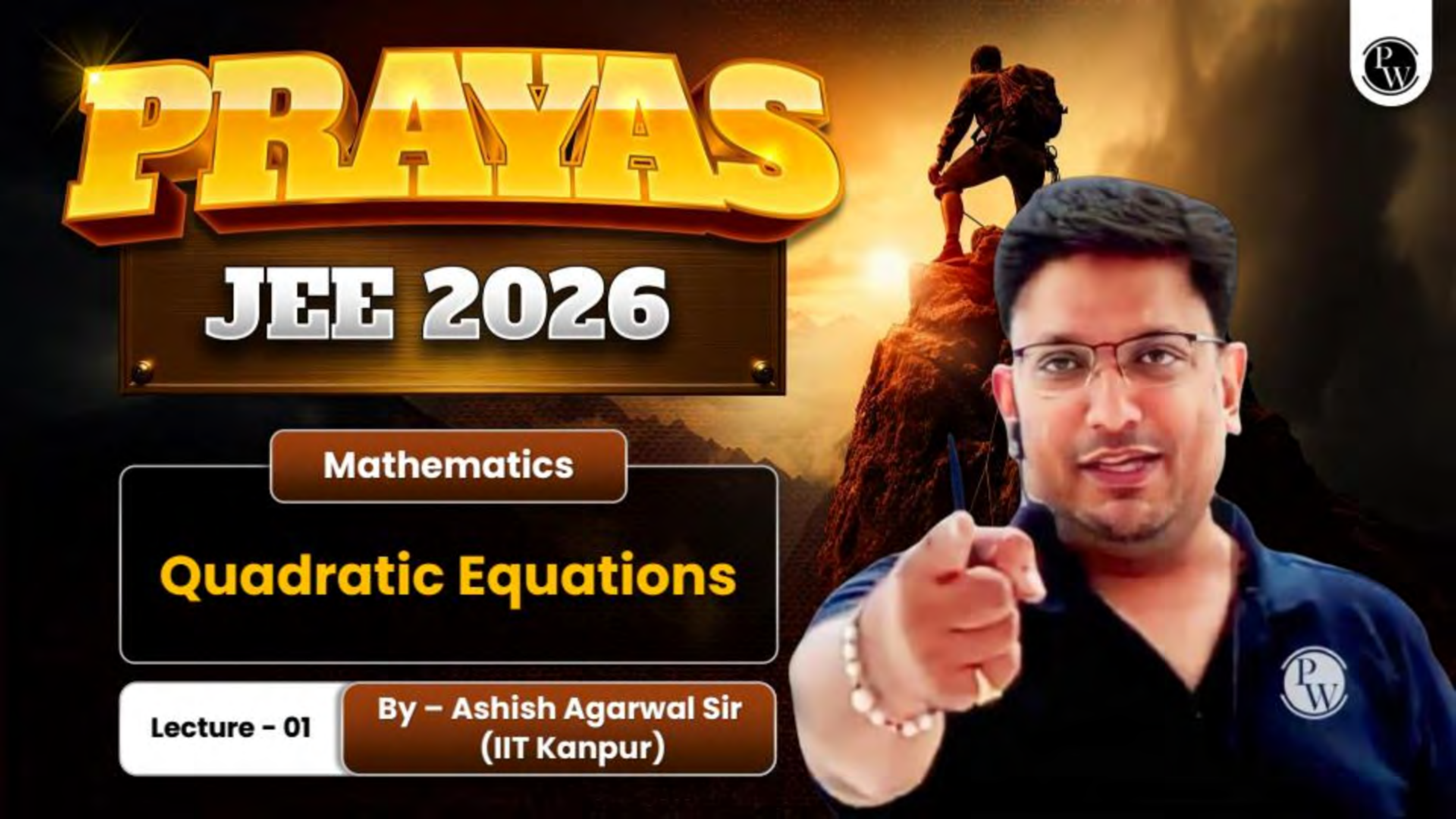
PRAYAS
JEE 2026
Mathematics
Quadratic Equations
Lecture - 01
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Topics *to be covered*

- **A** Quadratic Equation & Its Solution
- **B** Nature of Roots
- **C** Sum of Roots & Product of Roots

Homework Discussion

QUESTION [JEE Mains 2019 (10 April)]

The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is

Case (I) If $2^x - 1 \geq 0 \Rightarrow 2^x \geq 1 = 2^0 \Rightarrow x \geq 0$

$t = 2^x$

$5 + 2^x - 1 = 2^{2x} - 2\cdot 2^x$

$4 + t = t^2 - 2t$

$t^2 - 3t - 4 = 0$

$t = 4, -1$

$2^x = 4, \cancel{-1}$

$x = 2$

Case (II) If $2^x - 1 < 0 \Rightarrow 2^x < 1 = 2^0 \Rightarrow x < 0$

$5 - 2^x + 1 = 2^{2x} - 2\cdot 2^x$

$6 - 2^x = 2^{2x} - 2\cdot 2^x$

$t^2 - t - 6 = 0$

$(t-3)(t+2) = 0$

$2^x = 3, \cancel{-2}$

$x = \phi$ rejected $\because 2^x < 1$

$x = 2$.

(A) 2

(B) 1

(C) 3

(D) 4

Ans. B

QUESTION

9. The equation $4^{\left(\frac{1}{x}-2\right)} = \frac{1}{2}\ln\sqrt{e}$ has the solution-

(A) -1 (B) 1 (C) 2 (D) None [Ans. B]

10. Solution of the equation $2^{x+2} \cdot 27^{\frac{x}{x-1}}$ [redacted] $= 9$ are given by-

(A) $\log_2 (2/3), 1$ (B) $2, 1 - \log_2 3$ (C) $-2, 1 - \log_2 3$ (D) None of these

[Ans. C]

11. If $x^{[\log_3 x^2 + (\log_3 x)^2 - 10]} = \frac{1}{x^2}$ then x is equal to

(A) $9, 1/9$ (B) $9, 1/81$ (C) $1, 1/9$ (D) $2, 2/9$ [Ans. B]

12. Complete set of values of x satisfying the inequality $x - 3 < \sqrt{x^2 + 4x - 5}$ is :

(A) $(-\infty, 5] \cup [1, \infty)$ (B) $(-5, 3]$ (C) $[3, 5)$ (D) $(-5, 3)$

[Ans. A]

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Deh Swaal pe Deh Swaal

Polynomial in One Variable

An algebraic expression of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots + a_2 x^1 + a_0 x^0, \text{ where}$$

(i) $a_n \neq 0$

(ii) Power of x is whole number, is called a polynomial in one variable.

Hence, $a_n, a_{n-1}, a_{n-2}, \ldots..$ are coefficients of x^n, x^{n-1} ......, x^0 respectively and $a_n x^n, a_{n-1} x^{n-1}, a_{n-2} x^{n-2} \ldots\ldots$ are term of the polynomial. [redacted] the term $a_n x^n$ is called is Leading term and its coefficient a_n, the leading coefficient.

NOTE :

If leading coefficient is 'I' then the polynomial is called as monic polynomial.

Degree	Name	General Form	Example
(undefined)	Zero polynomial	0	0
0	(Non – zero) constant polynomial	$a; (a \neq 0)$	1
1	Linear polynomial	$ax + b; (a \neq 0)$	$x + 1$
2	Quadratic polynomial	$ax^2 + bx + c; (a \neq 0)$	$x^2 + 1$
3	Cubic polynomial	$ax^3 + bx^2 + cx + d; (a \neq 0)$	$x^3 + 1$

Quadratic Equation and Its solution

$p(x) = ax^2 + bx + c$, $a \neq 0$ is a quadratic polynomial

$ax^2 + bx + c = 0 \longrightarrow$ (Quadratic Eqn)

Fundamental Theorem of Algebra

Every polynomial Eqn of dergree n shall have exactly n roots (real or imaginary) counted with multiplicity.

Ex: $(x-1)^2(x-2) = 0 \longrightarrow$ cubic Eqn.

$x = 1, 1, 2 \longrightarrow$ roots.

solutions $x = 1, 2$

solutions = no: of distinct roots

Ex: $(x-2)^2(x-1)^3 = 0 \rightsquigarrow$ 5 degree Eqn

$x = 1, 1, 1, 2, 2$ roots

soln $x = 1, 2$

$$ax^2+bx+c=0\ ,\quad a\neq 0$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\underbrace{x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2}-\left(\frac{b}{2a}\right)^2+\frac{c}{a}=0$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}=\frac{b^2-4ac}{4a^2}=\frac{D}{4a^2}$$

$$\underbrace{\text{let } D=b^2-4ac}_{\downarrow}$$

Discriminant

$$\left(x+\frac{b}{2a}\right)^2=\frac{D}{4a^2}$$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{D}}{2a}$$

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$ax^2+bx+c=0 \begin{cases} \alpha=\dfrac{-b+\sqrt{D}}{2a} \\ \beta=\dfrac{-b-\sqrt{D}}{2a} \end{cases}$$

for square completion

$$\underbrace{ax^2+bx}+c$$

$$=a\left(x^2+\frac{b}{a}x\right)+c$$

Add & sub $\left(\frac{\text{coeff of } x}{2}\right)^2$

$$=a\left(\underbrace{x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2}-\left(\frac{b}{2a}\right)^2\right)+c$$

$$=a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}+c.$$

Nature of Roots

D = Discriminant $= b^2 - 4ac$ for $ax^2 + bx + c = 0$

$a, b, c \in R$.

(i) If $D > 0 \Rightarrow$ roots are real and distinct

$\alpha, \beta = \frac{-b \pm \sqrt{D}}{2a}$

equal nahi hongi

(ii) If $D = 0 \Rightarrow$ roots are real and equal

$\alpha, \beta = \frac{-b \pm \sqrt{0}}{2a} = \frac{-b}{2a}$

If $D = 0$ roots are real & equal & each root is equal to $-\frac{b}{2a}$

(iii) If $D < 0 \Rightarrow$ Imaginary roots

Ex: $x^2 + x + 1 = 0$

$D = -3$

$\downarrow$

roots $= \frac{-1 \pm \sqrt{-3}}{2}$

$\alpha, \beta = \frac{-1 + \sqrt{3}i}{2}, \frac{-1 - \sqrt{3}i}{2}$

NOTE:

For real roots $D \geq 0$.

QUESTION [JEE Mains 2025 (3 April)]

Let the equation $x(x+2)(12-k) = 2$ have equal roots. Then the distance of the point $\left(k, \frac{k}{2}\right)$ from the line $3x + 4y + 5 = 0$ is

(A) 15

(B) 12

(C) $5\sqrt{3}$

(D) $15\sqrt{5}$

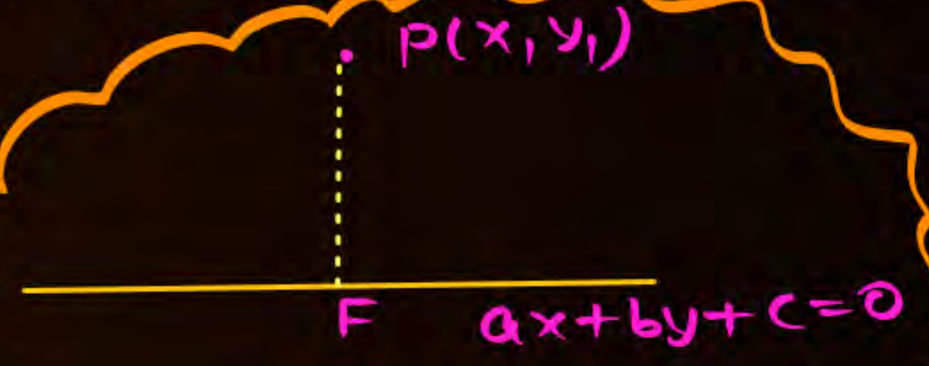
$$x^2 + 2x = \frac{2}{12-k}$$

$$x^2 + 2x - \frac{2}{12-k} = 0 \longrightarrow \text{roots are equal}$$

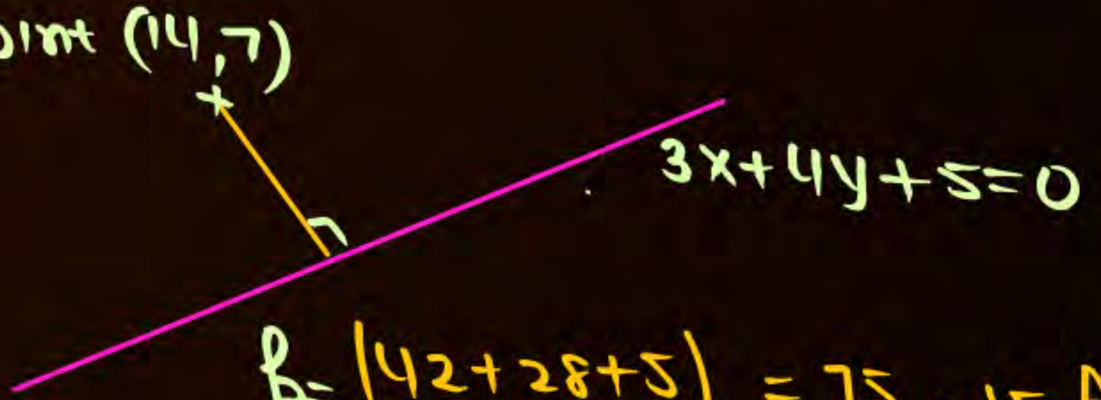
$$D = 0 \Rightarrow \cancel{4} + \cancel{4}\cdot 1 \cdot \frac{2}{12-k} = 0$$

$$12 - k + 2 = 0$$

$$k = 14$$



$$PF = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$



$$p = \frac{|42+28+5|}{\sqrt{3^2+4^2}} = \frac{75}{5} = 15 \ \underline{\text{Ans}}$$

Ans. A

QUESTION [JEE Mains 2025 (29 Jan)]

If the set of all $a \in \mathbf{R}$, for which the equation $2x^2 + (a-5)x + 15 = 3a$ has no real root, is the interval (α, β), and $X = |x \in Z; \alpha < x < \beta|$, then $\sum_{x \in X} x^2$ is equal to:

(A) 2139

(B) 2119

(C) 2109

(D) 2129

$2x^2 + (a-5)x + 15 = 3a \longrightarrow$ No real roots.

$2x^2 + (a-5)x + 15 - 3a = 0$

$D = (a-5)^2 - 8\cdot(15-3a) < 0$

$a^2 - 10a + 25 - 120 + 24a < 0$

$a^2 + 14a - 95 < 0$

$a^2 + 19a - 5a - 95 < 0$

$(a+19)(a-5) < 0$

$a \in (-19, 5)$

$\alpha = -19 \quad \beta = 5$

$-19 < x < 5, \ x \in Z$

$x = -18, -17, \ldots, 2, 3, 4$

Ans. A

$$\sum x^2 = (-18)^2 + (-17)^2 + \cdots + (-1)^2 + 0^2 + \underbrace{1^2 + 2^2 + 3^2 + 4^2}_{30}$$

$$= \frac{18 \cdot 19 \cdot 37}{6} + 30$$

$$= 57 \times 37 + 30$$

$$= 2139$$

* $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$
* $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
* $1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

QUESTION [JEE Mains 2020]

Tanol

The least positive value of 'a' for which the question, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is

QUESTION

$R - \{0\} \Rightarrow a^2, b^2, c^2 > 0$

Let a, b, c $\in R_0$ and 1 be a root of $ax^2 + bx + c = 0$ then comment on nature of roots of $4ax^2 + 3bx + 2c = 0$.

M(1)

$ax^2 + bx + c = 0$ — $x = 1$

Since $x = 1$ is a root $\Rightarrow a + b + c = 0 \rightarrow b = -(a + c)$

$4ax^2 + 3bx + 2c = 0$

$D = 9b^2 - 4 \cdot 4a \cdot 2c = 9b^2 - 32ac = 9(a+c)^2 - 32ac$

$= 9a^2 + 9c^2 + 18ac - 32ac$

$= 9a^2 + 9c^2 - 14ac$

$= 7a^2 + 7c^2 - 14ac + 2(a^2 + c^2)$

$= 7(a^2 + c^2 - 2ac) + 2(a^2 + c^2)$

$= \underbrace{7(a-c)^2}_{\geq 0} + 2\underbrace{(a^2 + c^2)}_{>0 \; >0} > 0$

$\Rightarrow D > 0 \Rightarrow$ Hence roots of $4ax^2 + 3bx + 2c = 0$ are real & distinct.

QUESTION

$R - \{0\} \Rightarrow a^2, b^2, c^2 > 0$

Let a, b, c $\in R_0$ and 1 be a root of $ax^2 + bx + c = 0$ then comment on nature of roots of $4ax^2 + 3bx + 2c = 0$.

M(2)

$ax^2 + bx + c = 0$ — $x = 1$, $x = \beta \in R$ clearly

$D_1 = b^2 - 4ac \geq 0$

$4ax^2 + 3bx + 2c = 0$

$D_2 = 9b^2 - 4 \cdot 4a \cdot 2c = 9b^2 - 32ac$

$D_2 = b^2 + 8b^2 - 32ac$

$= \underbrace{b^2}_{>0} + 8\underbrace{(b^2 - 4ac)}_{\geq 0}$

$D_2 > 0 \Rightarrow$ Roots of $4ax^2 + 3bx + 2c = 0$ are real & distinct.

Relation between Roots & Coefficients

$$ax^2+bx+c=0 \begin{cases} \alpha = \dfrac{-b+\sqrt{D}}{2a} \\ \beta = \dfrac{-b-\sqrt{D}}{2a} \end{cases}$$

(1) Sum of Roots

$$\boxed{S.O.R = -\frac{b}{a} = \alpha+\beta}$$

$$\alpha+\beta = -\frac{2b}{2a} = -\frac{b}{a}$$

$$\alpha\beta = \frac{(-b)^2-(\sqrt{D})^2}{4a^2} = \frac{b^2-(b^2-4ac)}{4a^2} = \frac{c}{a}$$

(2) Product of Roots

$$\boxed{P.O.R = \alpha\beta = \frac{c}{a}}$$

If $D<0 \Rightarrow$ roots are imaginary

$$ax^2+bx+c=0 \begin{cases} p+iq=\alpha \\ p-iq=\beta \end{cases} \quad p,q \in R$$

$$P.O.R = \alpha\beta = p^2-(iq)^2 = p^2+q^2 \geq 0$$

Hence

$$\boxed{P.O.R \text{ is } -ve \Rightarrow \text{Roots are real}}$$

Symmetric Function of Roots

If $f(\alpha, \beta) = f(\beta, \alpha)$ then $f(\alpha, \beta)$ is said to be a symmetric function of roots.

e.g. $f(\alpha, \beta) = \alpha^2 + \beta^2, \alpha^2\beta + \beta^2\alpha + \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$f(\beta, \alpha) = \beta^2 + \alpha^2, \beta^2\alpha + \alpha^2\beta + \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$

$f(\alpha, \beta) = f(\beta, \alpha)$

NOTE :

Every symmetric function in α, β can be expressed in terms of two symmetric functions $\alpha + \beta$ and $\alpha\beta$.

QUESTION

If α, β are the roots of quadratic equation $ax^2 + bx + c = 0$, then which of the following expressions in α, β will denote the symmetric functions of roots. Given proper reason.

(A) $f(\alpha, \beta) = \alpha^2 - \beta$ — $f(\alpha,\beta) = \alpha^2 - \beta$, $f(\beta,\alpha) = \beta^2 - \alpha$ $\Rightarrow f(\alpha,\beta) \neq f(\beta,\alpha)$

(B) $f(\alpha, \beta) = \alpha^2\beta + \alpha\beta^2$ — $f(\beta,\alpha) = \beta^2\alpha + \alpha^2\beta = f(\alpha,\beta)$

(C) $f(\alpha, \beta) = \ln\frac{\alpha}{\beta}$ — $f(\beta,\alpha) = \ln\left(\frac{\beta}{\alpha}\right) = \ln\left(\frac{\alpha}{\beta}\right)^{-1} = -\ln\left(\frac{\alpha}{\beta}\right) \neq f(\alpha,\beta)$

(D) $f(\alpha, \beta) = \cos(\alpha - \beta)$ — $f(\beta,\alpha) = \cos(\beta-\alpha) = \cos(-(\alpha-\beta))$
$= \cos(\alpha-\beta) = f(\alpha,\beta)$

* $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$.

* $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

* $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta) = (\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)$

* $\alpha^3-\beta^3=(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta)=(\alpha-\beta)((\alpha+\beta)^2-\alpha\beta)$

$$(\text{Difference of roots})^2=(\alpha-\beta)^2=\left(-\frac{b}{a}\right)^2-\frac{4c}{a}=\frac{b^2}{a^2}-\frac{4c}{a}$$

$$=\frac{b^2-4ac}{a^2}=\frac{D}{a^2}$$

$$\boxed{(\text{Difference of roots})^2=(\alpha-\beta)^2=\frac{D}{a^2}}$$

QUESTION

$\alpha+\beta=4$

$\alpha\beta=-8$

If α & β are roots of equation $x^2-4x-8=0$ then find value of following :

(i) $\alpha^2+\beta^2$ $\to (\alpha+\beta)^2-2\alpha\beta=\left(+\frac{4}{1}\right)^2-2\cdot\left(-\frac{8}{1}\right)=32$

(ii) $\left(\frac{\alpha^2}{\beta}\right)^{1/3}+\left(\frac{\beta^2}{\alpha}\right)^{1/3}=\frac{\alpha^{2/3}}{\beta^{1/3}}+\frac{\beta^{2/3}}{\alpha^{1/3}}$

$=\frac{\alpha+\beta}{(\alpha\beta)^{1/3}}=\frac{4}{(-8)^{1/3}}$

$=-2$

(iii) $E=\frac{1}{(\alpha-4)^2}+\frac{1}{(\beta-4)^2}$ $\to$ $\alpha^2-4\alpha-8=0$

$\alpha(\alpha-4)=8$

$\alpha-4=\frac{8}{\alpha}$

lly $\beta-4=\frac{8}{\beta}$

$\Downarrow$

$E=\frac{1}{(8/\alpha)^2}+\frac{1}{(8/\beta)^2}$

$E=\frac{\alpha^2+\beta^2}{64}=\frac{32}{64}=1/2.$

(iv) $\alpha^4+\beta^4$ $\to (\alpha^2+\beta^2)^2-2\alpha^2\beta^2$

$=((\alpha+\beta)^2-2\alpha\beta)^2-2(\alpha\beta)^2$

(v) $\alpha^5+\beta^5$ ~ Tah 02 (c)

(vi) $(\alpha^5-\beta^5)^2$ Tah 02 d

(vii) $\frac{\beta}{\alpha^2-8}+\frac{\alpha}{\beta^2-8}$ Tah 02 a

(viii) $\frac{1}{(\alpha+1)(\beta+1)}+\frac{1}{(\alpha-2)(\beta-2)}$ Tah 02 (b)

$$\alpha^5+\beta^5=(\alpha^2+\beta^2)(\alpha^3+\beta^3)-\alpha^2\beta^3-\beta^2\alpha^3$$

$$=(\alpha^2+\beta^2)(\alpha^3+\beta^3)-\alpha^2\beta^2(\alpha+\beta)$$

$$\alpha^5-\beta^5=(\alpha^2+\beta^2)(\alpha^3-\beta^3)+\alpha^2\beta^3-\beta^2\alpha^3.$$

$$=(\alpha^2+\beta^2)(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta)\quad+\alpha^2\beta^2(\beta-\alpha)$$

$$=(\alpha-\beta)\left((\alpha^2+\beta^2)(\alpha^2+\beta^2+\alpha\beta)-\alpha^2\beta^2\right)$$

Some Important Points

P(1): If a, b, c ∈ Q and D is perfect square then roots are rational.

$ax^2+bx+c=0$

Q (rational)

$D = b^2-4ac$ = perfect square $(D \geq 0)$

Ex: $D = 16, 25, \frac{9}{4}, \frac{16}{25}, \ldots$

Q Q

$x = \frac{-b \pm \sqrt{D}}{2a} \in Q$

Q

* Even if D is a perfect square roots may not be rational

Ex: $\sqrt{2}x^2+3x-2\sqrt{2}=0$

$D = 9+4\cdot 2\cdot\sqrt{2}\cdot\sqrt{2} = 25$ (perfect square)

$x = \frac{-3\pm\sqrt{25}}{2\sqrt{2}} = \frac{-3\pm 5}{2\sqrt{2}} \notin Q$

0 = even Integer

P(2): If a = 1, b, c ∈ I and D is square of an integer then roots are integers.

$x^2+bx+c=0$ $\qquad D = b^2 - 4c$ = square of an Integer.

b: odd, even $\qquad 4c$: even

$\sqrt{(ODD)^2} = ODD$

$\sqrt{(Even)^2} = even$

$$x = \frac{-b \pm \sqrt{D}}{2}$$

	b	$\sqrt{D}$	$-b \pm \sqrt{D}$	$x = \frac{-b \pm \sqrt{D}}{2}$
case (I)	even	even	Even	Integer.
case (II)	odd	odd	Even	Integer

Ex: $x^2-5x+6=0$

$a=1,\ b=-5,\ c=6 \in I$

$D = 25-24 = 1 = 1^2$

roots $= \frac{+5 \pm \sqrt{1}}{2} = 3, 2 \in I$

Ex: $x^2-25=0$

$a=1,\ b=0,\ c=-25 \in I$

$D = 0^2 - 4\cdot(-25) = 100 = 10^2$

$x = \frac{0 \pm \sqrt{10^2}}{2} = 5, -5.$

Ex: $x^2+25=0$

$a=1,\ b=0,\ c=25.$

$D = -100 \neq (\text{Integer})^2$

$a+\sqrt[n]{b}$

P(3): If a, b, c $\in$ Q and D is not square but D > 0 then roots are irrational and occurs in conjugate pair of surds i.e.
Roots are of form $p+\sqrt{q}$ & $p-\sqrt{q}$.

Ex: $x^2-3x-6=0$

$a=1, b=-3, c=-6 \in Q$

$D=9+24=33$

$x=\frac{3+\sqrt{33}}{2}, \frac{3-\sqrt{33}}{2}$

$x=\frac{3}{2}+\frac{\sqrt{33}}{2}, \frac{3}{2}-\frac{\sqrt{33}}{2}$

NOTE :

If a, b, c $\in$ Q and if one root of quadratic is $3+\sqrt{7}$ then other root is $3-\sqrt{7}$.

Ex: If a,b,c are rational & one root of quad is $\sqrt{5}+2$ find the other root

Gadho | Gadhiyoo ka Ans: $\sqrt{5}-2$.

Phadne waalay: $2-\sqrt{5}$.

Ex: If $a, b, c \in Q$ in a quad then its roots can be $\frac{2}{3}$ & $8+\sqrt{5}$ (T/F)

False

$\sqrt{2}x^2 - 2\sqrt{2}x - 3\sqrt{2} = 0$

$D = 8 + 4 \cdot 3 \cdot \sqrt{2} \cdot \sqrt{2}$

$= 8 + 24 = 32$ — not a perfect square

$x = \frac{2\sqrt{2} \pm \sqrt{32}}{2\sqrt{2}}$

$x = \frac{2\sqrt{2} \pm 4\sqrt{2}}{2\sqrt{2}}$

$x = 1 \pm 2$

$x = -1, 3.$

If a, b, c are not all rational & D is not a perfect square then root may or may not occur in conjugate pair of surds.

$(x-2)(x-\sqrt{2})=0 < \begin{matrix} 2 \\ \sqrt{2} \end{matrix}$

$x^2-(2+\sqrt{2})x+2\sqrt{2}=0$

★ $a, b, c \in Q$ — D is perfect square $(D \geq 0)$ — roots are rational

★ $a, b, c \in Q$ — D is not a perfect square & $D > 0$ — roots occur in conjugate pair of surds.

P(4): If a, b, c ∈ R and D < 0 then roots are imaginary and occur in conjugate pair i.e. if one roots is p + iq then other root is p – iq.

NOTE :

If a, b, c ∈ R and one root of quad is p + iq then other root is p – iq.

Ex: If a, b, c ∈ R & one root of quad is

(a) $2+3i$ find other root — $2-3i$

(b) $\sqrt{3}i+2$ find other root — $-\sqrt{3}i+2$

(c) $\sqrt{2}i-\sqrt{3}$ find other root — $-\sqrt{2}i-\sqrt{3}$.

Ex: If one root of a quadratic is $1+i$ find other root

Gadho|Gadhiyoo ka answer : $1-i$

Phadne waalay : can not say

$(x-i)(x-2)=0$ root $=2, i$

$x^2-(2+i)x+2i=0$

coeff of are not real

NOTE :

Every odd degree polynomial equation with real coefficient must have at least one real root. Because imaginary roots occur in conjugate pair.

$$x^3 + 3x^2 + 6x + 5 = 0 \begin{cases} p+iq \\ p-iq \\ r \in R \end{cases}$$

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION [JEE Mains 2024 (8 April)]

(KTK 1)

The sum of all the solutions of the equation $(8)^{2x} - 16 \cdot (8)^{x} + 48 = 0$ is :

(A) $1 + \log_8(6)$

(B) $1 + \log_6(8)$

(C) $\log_8(6)$

(D) $\log_8(4)$

Ans. A

QUESTION [JEE Mains 2023 (24 Jan)]

(KTK 2)

The number of real solutions of the equation $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$, is

(A) 3

(B) 4

(C) 0

(D) 2

Ans. C

QUESTION

(KTK 3)

The complete solution set of the inequality $\frac{3^x(2x-5)(x^2+x+2)}{(\cos x-2)(x^2+x)} \le 0$ is

(A) $(-\infty, -1)$

(B) $\left(\frac{5}{2}, \infty\right)$

(C) $\left(-1, \frac{5}{2}\right]$

(D) $(-1,0) \cup \left[\frac{5}{2}, \infty\right)$

Ans. D

QUESTION

(KTK 4)

If S is the set of all real 'x' such that $\frac{x^2(5-x)(1-2x)}{(5x+1)(x+2)}$ is negative and $\frac{3x+1}{6x^3+x^2-x}$ is positive, then S contains

(A) $(1, 4)$

(B) $(5, 11)$

(C) $\left(-\frac{3}{2}, \frac{-1}{2}\right)$

(D) $(-10, -4)$

Ans. A, D

QUESTION [WB JEE 2011]

(KTK 5)

Let a, b, c be three real numbers such that $a + 2b + 4c = 0$. Then the equation $ax^2 + bx + c = 0$

(A) has both the roots complex

(B) has its roots lying within $-1 < x < 0$

(C) has one of the roots equal to $\frac{1}{2}$

(D) has its roots lying within $2 < x < 6$

Ans. C

QUESTION [JEE Mains 2025 (24 Jan)]

(KTK 6)

The product of all the rational roots of the equation $(x^2 - 9x + 11)^2 - (x - 4)(x - 5) = 3$, is equal to

(A) 7

(B) 21

(C) 28

(D) 14

Ans. D

QUESTION [WB JEE 2011]

(KTK 7)

If $\sin\theta$ and $\cos\theta$ are the roots of equation $ax^2 - bx + c = 0$, then a, b and c satisfy the relation

(A) $a^2 + b^2 + 2ac = 0$

(B) $a^2 - b^2 + 2ac = 0$

(C) $a^2 + c^2 + 2ab = 0$

(D) $a^2 - b^2 - 2ac = 0$

Ans. B

QUESTION [WB JEE 2010]

(KTK 8)

The roots of the quadratic equation $x^2 - 2\sqrt{3}x - 22 = 0$ are

(A) imaginary

(B) real, rational and equal

(C) real, irrational and unequal

(D) real, rational and unequal

Ans. C

Homework From Module

Prarambh (Topicwise) : Q1 to Q25

Prabal (JEE Main Level) : Q1 to Q33

Solution to Previous TAH

QUESTION

Find the number of integral values of x satisfying the inequality

$$\left(\frac{3}{4}\right)^{6x+10-x^2} < \frac{27}{64}$$

TAH-1

Q. Find the no. of integral values of x

$$\left(\frac{3}{4}\right)^{6x+10-x^2} < \frac{27}{64}$$

$$\left(\frac{3}{4}\right)^{6x+10-x^2} < \left(\frac{3}{4}\right)^{3}$$

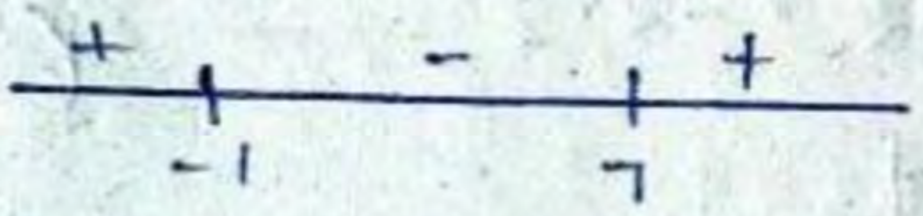
Rajkanya
From Bihar

$$6x+10-x^2 > 3$$

$$-x^2+6x+7>0$$

$$x^2-6x-7<0$$

$$(x-7)(x+1)<0$$



$$\boxed{x \in (-1, 7)}$$

Date
Page

10th May 2025 Lecture - 17

Tan - 01

Find the number of integeral values of x satisfying the inequality.

$$\left(\frac{3}{4}\right)^{6x+10-x^2} < \frac{27}{64}$$

$$\left(\frac{3}{4}\right)^{6x+10-x^2} < \left(\frac{3}{4}\right)^3$$

$$6x+10-x^2 < 3$$

$$x^2-6x-7>0$$

$$x^2-6x-7>0$$

$$D = 36+28 = 64$$

$$\frac{6 \pm \sqrt{64}}{2} = \frac{6+8}{2}, \frac{6-8}{2}$$

$$x = 7, -1$$

$$x \in (-\infty, -1) \cup (7, \infty)$$

So, no. of integral values = (0, 1, 2, 3, 4, 5, 6) = 7 Ans

QUESTION [JEE Mains 2020 (8 Jan)]

Let S be the set of all real roots of the equation,

$3^x(3^x - 1) + 2 = |\underbrace{3^x - 1}_{T_1}| + |\underbrace{3^x - 2}_{T_2}|$. Then S :

(A) contains exactly two elements.

(B) is an empty set.

(C) is a singleton.

(D) contains at least four elements.

let $3^x = t$

$t^2 - t + 2 = |t-1| + |t-2|$

-ve, +ve, +ve

-ve, 1, -ve, 2, +ve

case ① $t \le 1$

$t^2 - t + 2 = 1 - t + 2 - t = 3 - 2t$

$t^2 + t - 1 = 0$

$t = \frac{-1 \pm \sqrt{5}}{2} = \frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}$

$3^x = \underbrace{\frac{-1-\sqrt{5}}{2}}_{-ve}$ (N.P), $3^x = \frac{\sqrt{5}-1}{2}$

$x = \log_3\left(\frac{\sqrt{5}-1}{2}\right)$

case ⑪ if $1 < t < 2$

$t^2 - t + 2 = t - 1 + 2 - t$

$t^2 - t + 1 = 0$

No real roots.

UNION

case ⑪ if $t \geq 2$

$t^2 - t + 2 = 2t - 3$

$t^2 - 3t + 5 = 0$

No real roots

$x = \log_3\left(\frac{\sqrt{5}-1}{2}\right)$

Ans. C

JAH-2

Q. $3^x(3^x-1)+2=|3^x-1|+|3^x-2|$. then S. (set of all real roots)

Put $3^x=t$

$t^2-t+2=|\overset{T_1}{t-1}|+|\overset{T_2}{t-2}|$

T_1	$-$		$+$		$+$
		1		2	
T_2	$-$		$-$		$+$

case (i) if $t\geq 2$

$t^2-t+2=t-1+t-2$

$t^2-3t+4=0$

$\hookrightarrow D<0$

no real roots.

case (ii) if $1<t<2$

$t^2-t+2=t-1-t+2$

$t^2-t+1=0$

$\hookrightarrow D<0$

no real roots.

case (iii) if $t\leq 1$

$t^2-t+2=1-t+2-t$

$t^2+t-1=0$

$t=\frac{-1\pm\sqrt{5}}{2}=\frac{-1-\sqrt{5}}{2},\ \frac{-1+\sqrt{5}}{2}$

$3^x=\frac{-1-\sqrt{5}}{2}\times,\ \frac{-1+\sqrt{5}}{2}$

$x\log_3 3=\log_3\left(\frac{-1+\sqrt{5}}{2}\right)$

$x=\log_3\left(\frac{-1+\sqrt{5}}{2}\right)$

$S=\left\{\log_3\left(\frac{-1+\sqrt{5}}{2}\right)\right\}$ (c) is a singleton.

Tan-(02) (Mains 2020)

Let S be the set of all real roots of the eqn.
$3^x(3^x-1)+2=|3^x-1|+|3^x-2|$. Then S:-

let $3^x=t$

$t(t-1)+2=|t-1|+|t-2|$

$t^2-t+2=\underbrace{|t-1|}_{T_1}+\underbrace{|t-2|}_{T_2}$

T_1: $-$ $+$ $+$

T_2: $-$ 1 $-$ 2 $+$

Case (1) $x\le 1$	Case (2) $1<x<2$	Case (3) $x\ge 2$
$t^2-\cancel{t}+2=-t+1-\cancel{t}+2$	$t^2-t+2=\cancel{t}-1-\cancel{t}+2$	$t^2-t+2=t-1+t-2$
$t^2+t-1=0$	$t^2-t+1=0$	$t^2-t+2=2t-3$
$t=\frac{-1+\sqrt{5}}{2},\frac{-1-\sqrt{5}}{2}$	$D=1-4<0, a>0$	$t^2-3t+5=0$
	always Positive	$D=9-20<0$
	$x\in\phi$	always +ve
		$x\in\phi$

$\cup$

$t=\frac{-1+\sqrt{5}}{2},\frac{-1-\sqrt{5}}{2}$

$3^x=\frac{-1+\sqrt{5}}{2}$ ✓, $\frac{-1-\sqrt{5}}{2}$ ✗ $\rightarrow$ bcz (positive)$^{\text{anything}}$ = always +ve

S is a singleton set.

QUESTION [JEE Mains 2019 (10 April)]

The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is

(A) 2

(B) 1

(C) 3

(D) 4

Ans. B

TAH-3

Q. The no. of real roots of the equation

$$5+|2^x-1| = 2^x(2^x-2)$$

Case (i) $2^x - 1 \geq 0$

$2^x \geq 1 = 2^0$

$x \geq 0$

$5 + 2^x - 1 = (2^x)^2 - 2\cdot 2^x$

Put $2^x = t$

$5 + t - 1 = t^2 - 2t$

$t^2 - 3t - 4 = 0$

$(t-4)(t+1) = 0$

$t = 4, -1$

$2^x = 2^2, -1$ (rejected)

$\boxed{x = 2.}$

$\longrightarrow$ only 1 real sol^n

Case (ii) $2^x - 1 < 0$

$x < 1$

$5 + 1 - 2^x = (2^x)^2 - 2\cdot 2^x$

Put $2^x = t$

$6 - t = t^2 - 2t$

$t^2 - t - 6 = 0$

$(t-3)(t+2) = 0$

$t = 3, -2$

$2^x = \cancel{-2}, \; 2^x = \cancel{3}$

$\because 2^x < 1$

$x \in \phi$

Rajkanya
From Bihar

Tah 03

$$5+|2^x-1| = 2^x(2^x-2)$$

$2^x = t$

$$5+|t-1| = t(t-2)$$

Case 01

$t-1 \geq 0$

$t \geq 1$

$5+t-1 = t^2-2t$

$t^2-3t-4=0$

$t^2-4t+t-4=0$

$t(t-4)+1(t-4)=0$

$(t+1)(t-4)=0$

$t=4$ or $t=-1$

$\cap$

$t=4$

Case 02

$t-1<0$

$t<1$

$5-t+1 = t^2-2t$

$t^2-t-6=0$

$t^2-3t+2t-6=0$

$t(t-3)+2(t-3)=0$

$t=3,\ t=-2$

$\cap$

$t=-2$

$\cup$

$t = 4, -2$

$2^x = 4$

$2^x = 2^2$

$\boxed{x=2}$

$2^x = -2$

$\times$

(B) is correct

QUESTION [JEE Mains 2016]

If x is a solution of the equation, $\sqrt{2x+1}-\sqrt{2x-1}=1$, $\left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2-1}$ is equal to:

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) 2

(D) $2\sqrt{2}$

IAH-04

Q. If x is a solⁿ of $\sqrt{2x+1}-\sqrt{2x-1}=1,\ (x\geq\frac{1}{2})$ then $\sqrt{4x^2-1}$ is.

Rajkanya From Bihar

$$\sqrt{2x+1}-\sqrt{2x-1}=1$$

S.B.S.

$$2x+1+2x-1-2\sqrt{(2x+1)(2x-1)}=1$$

$$4x-2\sqrt{(2x+1)(2x-1)}=1$$

$$4x-2\sqrt{4x^2-1}=1$$

$$2\sqrt{4x^2-1}=4x-1$$

$$\sqrt{4x^2-1}=\frac{4x-1}{2}$$

S.B.S.

$$4x^2-1=\frac{(4x-1)^2}{4}$$

$$4(4x^2-1)=(4x-1)^2$$

$$16x^2-4=16x^2+1-8x$$

$$8x=+5$$

$$x=+\frac{5}{8}=0.62$$

& $4x^2-1\geq 0$

$$(2x-1)(2x+1)\geq 0$$

$+$ $-\frac{1}{2}$ $-$ $\frac{1}{2}$ $+$

$$x\left(-\infty,-\tfrac{1}{2}\right)\cup\left(\tfrac{1}{2},\infty\right)$$

A/Q.

$$\sqrt{4x^2-1}=\sqrt{4\times\frac{25}{64}-1}=\sqrt{\frac{25}{16}-1}$$

$$=\sqrt{\frac{9}{16}}=\boxed{\frac{3}{4}}$$

QUESTION [JEE Mains 2022 (30 June)]

Let $S_1 = \left\{x \in R - \{1,2\}: \frac{(x+2)(x^2+3x+5)}{-2+3x-x^2} \geq 0\right\}$ and $S_2 = \{x \in R: 3^{2x} - 3^{x+1} - 3^{x+2} + 27 \leq 0\}$.

Then, $S_1 \cup S_2$ is equal to :

(A) $(-\infty, -2] \cup (1, 2)$

(B) $(-\infty, -2] \cup [1, 2]$

(C) $(-2, 1] \cup [2, \infty)$

(D) $(-\infty, 2]$

Ans. B

JAH-05

Q. $S_1 = \left\{x \in R - \{1,2\} : \dfrac{(x+2)(x^2+3x+5)}{-2+3x-x^2} \geq 0\right\}$

& $S_2 = \{x \in R : 3^{2x} - 3^{x+1} - 3^{x+2} + 27 \leq 0\}$.

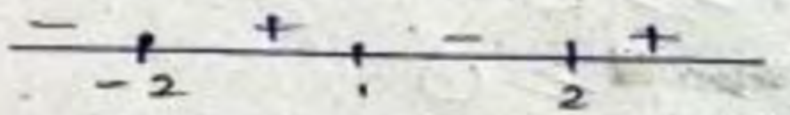
then, $S_1 \cup S_2 = ?$

for S_1:-

$$\frac{(x+2)(x^2+3x+5)}{-2+3x-x^2} \geq 0$$

$$\frac{(x+2)\overbrace{(x^2+3x+5)}^{\text{always +ve}}}{x^2-3x+2} \leq 0$$

$$\frac{(x+2)}{(x-1)(x-2)} \leq 0$$

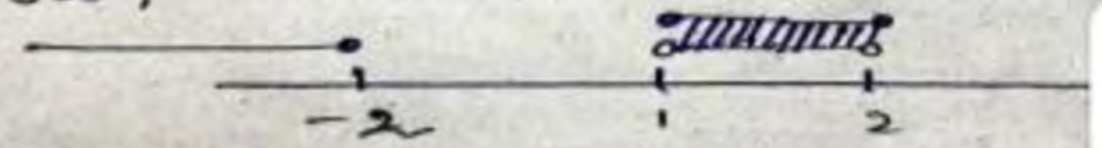


$x \in (-\infty, -2] \cup (1,2)$

& $x \in R - \{1,2\}$

So, $x \in (-\infty, -2] \cup (1,2)$

Now, $S_1 \cup S_2$



$x \in (-\infty, -2] \cup [1,2]$

for S_2:- for $x \in R$

$3^{2x} - 3^{x+1} - 3^{x+2} + 27 \leq 0$

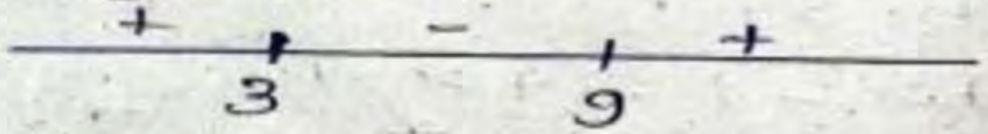
$3^{2x} - 3 \cdot 3^x - 9 \cdot 3^x + 27 \leq 0$

$3^{2x} - 12 \cdot 3^x + 27 \leq 0$

Put $3^x = t$

$t^2 - 12t + 27 \leq 0$

$(t-9)(t-3) \leq 0$



$t \in [3,9]$

$3^x \in [3, 3^2]$

$x \in [1,2]$

Rajkanya From Bihar

IAH → 05 Let $S_1 = \{x \in R - \{1, 2\} : \frac{(x+2)(x^2+3x+5)}{-2+3x-x^2}\}$

and $S_2 = \{x \in R : 3^{2x} - 3^{x+1} - 3^{x+2} + 27 \leq 0\}$.

Then, $S_1 \cup S_2$ is equal to:

As $S_1 = \frac{(x+2)(x^2+3x+5)}{-2+3x-x^2} \geq 0$

$$\frac{(x+2)(x^2+3x+5)}{x^2-3x+2} \leq 0$$

(x^2+3x+5)

$\rightarrow D = -ve$ so, It is always +ve.

$$\frac{(x+2)}{x^2-2x-x+2} \leq 0$$

$$\frac{(x+2)}{(x-2)(x-1)} \leq 0$$

$-\ \underset{-2}{|}\ +\ \underset{1}{|}\ -\ \underset{2}{|}\ +$

$$\boxed{S_1 = x \in (-\infty, -2] \cup (1, 2)}$$

$$S_2 = 3^{2x} - 3^{x+1} - 3^{x+2} + 27 \le 0$$

$$S_2 = 3^x \cdot 3^x - 3^x \cdot 3 - 3^x \cdot 3^2 + 27 \le 0$$

Put $3^x = t$

$\cancel{S_2}$

$$t^2 - 3t - 9t + 27 \le 0$$

$$t(t-3) - 9(t-3) \le 0$$

$$(t\ 9)(t-3) \le 0$$

$+\ \underset{3}{|}\ -\ \underset{9}{|}\ +$

$t = 3^x$

$$\boxed{\cancel{S_2} = t \in [3, 9]}$$

$$\boxed{S_2 = x \in [1, 2]}$$

$$S_1 \cup S_2 \Rightarrow x \in (-\infty, -2] \cup [1, 2]$$

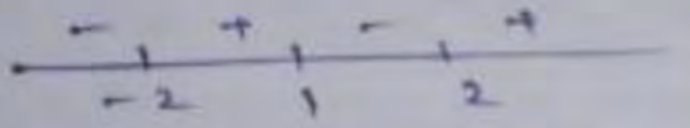
Tah - 05

Q.1. $\longrightarrow \dfrac{(x+2)(x^2+3x+5)}{-2+3x-x^2} \geq 0$

$\rightarrow D<0,\ a>0$ $\rightarrow$ always +ve

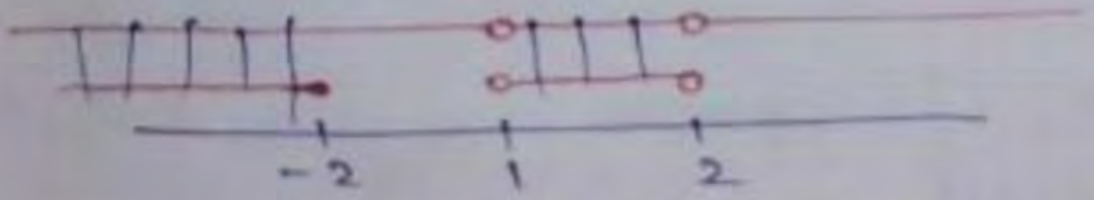
$\dfrac{(x+2)(x^2+3x+5)}{x^2-3x+2} \leq 0$

$\dfrac{(x+2)}{(x-2)(x-1)} \leq 0$



$x \in (-\infty, -2] \cup (1, 2)$

But for S_1: $x \in R - \{1, 2\}$



$\therefore x \in (-\infty, -2] \cup (1, 2)$

$3^{2x} - 3^{x+1} - 3^{x+2} + 27 \leq 0$

$3^{2x} - 3 \cdot 3^x - 9 \cdot 3^x + 27 \leq 0$

let $3^x = t$

$t^2 - 3t - 9t + 27 \leq 0$

$t^2 - 12t + 27 \leq 0$

$(t-9)(t-3) \leq 0$

$t \in [3, 9]$

$3^x \in [3, 9]$

$x \in [1, 2]$

for S_2 : $x \in R$

$\therefore x \in [1, 2]$

$\cup$

$x \in (-\infty, -2] \cup [1, 2]$

(B)

QUESTION [JEE Mains 2022 (25 June)]

Let $A = \{x \in R: |x + 1| < 2\}$ and $B = \{x \in R: |x - 1| \geq 2\}$.
Then which one of the following statements is NOT true?

(A) $A - B = (-1, 1)$

(B) $B - A = R - (-3, 1)$

(C) $A \cap B = (-3, -1]$

(D) $A \cup B = R - [1, 3)$

Ans. B

TAH - 06.

Q. $A = \{x \in R : |x+1| < 2\}$

$B = \{x \in R : |x-1| \geq 2\}$

$A = |x+1| < 2$

$-2 < x+1 < 2$

$-3 < x < 1$

$x \in (-3, 1) \cap x \in R$

$\hookrightarrow x \in (-3, 1)$

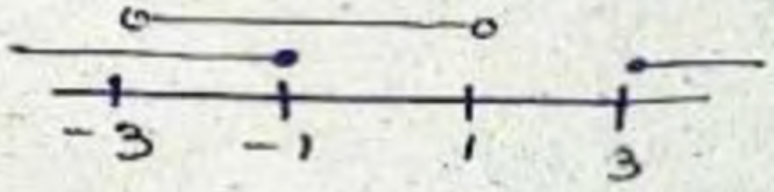
$B = |x-1| \geq 2$

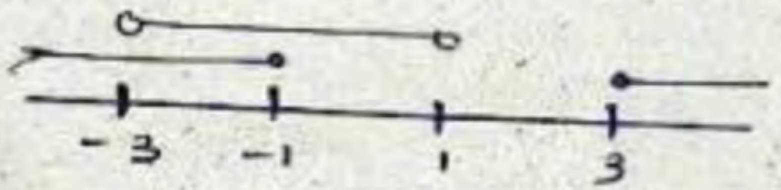
$x-1 \geq 2$ & $x-1 \leq -2$

$x \geq 3$ $\quad$ $x \leq -1$

$x \in R \cap x(-\infty, -1] \cup [3, \infty)$

$x \in (-\infty, -1] \cup [3, \infty)$

(A) $A - B =$  $= (-1, 1)$

(B) $B - A =$  $= (-\infty, -3] \cup [3, \infty)$

(C) $A \cap B = x \in (-3, -1]$

(D) $A \cup B = R - [1, 3)$

(B)

Rajkanya
From Bihar

QUESTION [JEE Mains 2021 (17 March)]

The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\cdots\infty}}}}$ is :

(A) $2+\frac{2}{5}\sqrt{30}$

(B) $2+\frac{4}{\sqrt{5}}\sqrt{30}$

(C) $5+\frac{2}{5}\sqrt{30}$

(D) $4+\frac{4}{\sqrt{5}}\sqrt{30}$

Ans. A

TAH-07

Q. $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\cdots\infty}}}}$

Put $= 4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{\cdots\infty}}} = y$

$y = 4+\cfrac{1}{5+\cfrac{1}{y}}$

$y = 4+\dfrac{y}{5y+1}$

$5y^2+y = 20y+4+y$

$5y^2-20y-4=0$

$y=\dfrac{20\pm4\sqrt{30}}{10}$

$y = 2\pm0.4\sqrt{30},\quad \cancel{2-0.4\sqrt{30}}$

$y = 2+\dfrac{2}{5}\sqrt{30}$

Rajkanya
From Bihar

TAH→07 The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\dots\infty}}}}$ is :

As Put $x = 4+\cfrac{1}{5+\cfrac{1}{4+\dots\infty}}$

$$x = 4+\cfrac{1}{5+\frac{1}{x}} \Rightarrow 4+\frac{x}{5x+1} = \frac{20x+4+x}{5x+1}$$

$$5x^2+x = 21x+4$$

$$5x^2-20x-4=0$$

$$D = 400+4(20) = 480$$

$$x = \frac{20 \pm \sqrt{480}}{10} = \frac{20 \pm 4\sqrt{30}}{10}$$

$$x = \frac{10 \pm 2\sqrt{30}}{5}$$

$$\boxed{x = 2+\frac{2}{5}\sqrt{30}}$$ As

QUESTION [JEE Mains 2020 (5 Sept)]

The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$ is :

(A) $\frac{5}{9}$

(B) $\frac{5}{27}$

(C) $\frac{25}{81}$

(D) $\frac{25}{9}$

Ans. C

TAH-8

Q. Product of the roots of $9x^2-18|x|+5=0$

Rajkanya From Bihar

$$9x^2-18|x|+5=0$$

$$9|x|^2-18|x|+5=0$$

Put $|x|=t$

$$9t^2-18t+5=0$$

$$9t^2-3t-15t+5=0$$

$$3t(3t-1)-5(3t-1)=0$$

$$t=\frac{1}{3}, \frac{5}{3}$$

$|x|=\frac{1}{3}$ & $|x|=\frac{5}{3}$

$x=\frac{1}{3}, -\frac{1}{3}$ $\quad$ $x=\frac{5}{3}, -\frac{5}{3}$

Product $=\frac{1}{3}\times\left(-\frac{1}{3}\right)\times\frac{5}{3}\times\left(-\frac{5}{3}\right)$

$$\boxed{=\frac{25}{81}}$$

TAH → 08 The Product of the Roots of the Equation $9x^2 - 18|x| + 5 = 0$ is:

A) $9x^2 - 18|x| + 5 = 0$

Case 1 If $x \geq 0$

$$9x^2 - 18x + 5 = 0$$
$$9x^2 - 15x - 3x + 5 = 0$$
$$3x(3x-5) - 1(3x-5) = 0$$
$$(3x-1)(3x-5) = 0$$
$$\boxed{x = \frac{1}{3}, \ x = \frac{5}{3}}$$

A

$$\boxed{x = \frac{1}{3}, \frac{5}{3}} \rightarrow A$$

Case 2 If $x < 0$

$$9x^2 + 18x + 5 = 0$$
$$9x^2 + 15x + 3x + 5 = 0$$
$$3x(3x+5) + 1(3x+5) = 0$$
$$\boxed{x = \frac{-1}{3}, \frac{-5}{3}} \rightarrow B$$

$$A \cup B \Rightarrow x = -\frac{1}{3}, \frac{-5}{3}, \frac{1}{3}, \frac{5}{3}$$

Product of Roots $= -\frac{1}{3} \times \left(\frac{-5}{3}\right) \times \frac{1}{3} \times \frac{5}{3}$

$$\boxed{\text{Product of Roots} = \frac{25}{81}}$$

Tah-10

Q $\longrightarrow$ $\log_3 N = \alpha_1 + \beta_1$

$\log_5 N = \alpha_2 + \beta_2$

$\log_7 N = \alpha_3 + \beta_3$

$\alpha_1 = 5$, $\alpha_2 = 3$, $\alpha_3 = 2$

$5 \le \log_3 N = 5 + \beta_1 < 6$

$3^5 \le N < 3^6$

$\boxed{243 \le N < 729}$

$3 \le \log_5 N = 3 + \beta_2 < 4$

$5^3 \le N < 5^4$

$\boxed{125 \le N < 625}$

$2 \le \log_7 N = 2 + \beta_3 < 3$

$7^2 \le N < 7^3$

$\boxed{49 \le N < 343}$

$\Rightarrow 243 \le N < 343$

$\therefore$ largest integral $= 342$

Ans

QUESTION [JEE Mains 2017 (9 April)]

The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is

(A) 16

(B) 14

(C) –4

(D) –5

Ans. C

TAH-9

Q. Sum of real values of x satisfying the eqn.

$$2^{(x-1)(x^2+5x-50)} = 1.$$

$$2^{(x-1)(x^2+5x-50)} = 2^0$$

$$(x-1)(x^2+5x-50) = 0$$

$$(x-1)(x+10)(x-5) = 0$$

$$x = 1, 5, -10$$

Rajkanya
From Bihar

$$\text{Sum} = 1+5-10$$
$$= -4$$

QUESTION

Let $\log_3 N = \alpha_1 + \beta_1$, $\log_5 N = \alpha_2 + \beta_2$ and $\log_7 N = \alpha_3 + \beta_3$ where $\alpha_1, \alpha_2, \alpha_3$ are integers and $\beta_1, \beta_2, \beta_3 \in [0, 1)$.

(i) Find the number of integral values of N if $\alpha_1 = 4$ and $\alpha_2 = 2$

(ii) Find the longest integral values of N if $\alpha_1 = 5$ and $\alpha_2 = 3$ and $\alpha_3 = 2$

(TAH → 10) Let $\log_3 N = a_1 + \beta_1$, $\log_5 N = a_2 + \beta_2$ and $\log_7 N = a_3 + \beta_3$ where a_1, a_2, a_3 are integers and $\beta_1, \beta_2, \beta_3 \in [0, 1)$

Find the Largest integral values of N if $\alpha_1 = 5$ and $\alpha_2 = 3$ and $\alpha_3 = 2$

$\alpha_1 = 5$, $\alpha_2 = 3$, $\alpha_3 = 2$

A) $5 \le \log_3 N < 6$

$243 \le N < 729$

$3 \le \log_5 N < 4$

$125 \le N < 625$

$2 \le \log_7 N < 3 \Rightarrow 49 \le N < 343$

$N \in [243, 343)$

Largest value of $N = 342$

(Tah - 10)

$\log_3 N = \alpha_1 + \beta_1$ $\quad\quad \alpha_1 = 5$

$\log_5 N = \alpha_2 + \beta_2$ $\quad\quad \alpha_2 = 3$

$\log_7 N = \alpha_3 + \beta_3$ $\quad\quad \alpha_3 = 2$

$\therefore \alpha_1 \leq \log_3 N < \alpha_1 + \beta_1$

$5 \leq \log_3 N < 5 + \beta_1$

$3^5 \leq N < 3^6$

$\boxed{243 \leq N < 729}$

$3 \leq \log_5 N < 3 + \beta_2$

$5^3 \leq N < 5^4$

$\boxed{125 \leq N < 625}$

$2 \leq \log_7 N < 2 + \beta_2$

$7^2 \leq N < 7^3$

$\boxed{49 \leq N < 343}$

$\therefore 243 \leq N < 343$

Solution to Previous KTKs

QUESTION

1. The common value of x satisfying $\frac{x-1}{x+2} \geq 0$ and $\frac{2x-5}{x-2} \leq 1$ is

(A) $(2, \infty]$ (B) $(2, 3]$ (C) $(-\infty, 3]$ (D) None of these

[Ans. B]

2. The set of values of x satisfying the inequality $2x - 7 < 4x - 2$ and $-5 \leq 2x + 6 < 4$ is given as

(A) $\left[\frac{-11}{2}, \frac{-5}{2}\right]$ (B) $\left[\frac{-11}{2}, \frac{-5}{2}\right)$ (C) $\left(-\frac{5}{2}, -1\right)$ (D) None of these

[Ans. C]

3. The smallest integer k satisfying the inequality $\frac{x-5}{x^2+5x-14} > 0$ is

(A) –7 (B) –6 (C) 6 (D) None of these

[Ans. B]

4. Number of integer values of x satisfying the inequality $\frac{x^2+6x-7}{|x+4|} < 0$ is

(A) 6 (B) 7 (C) 8 (D) None of these

[Ans. A]

QUESTION

5. Set of real values of x satisfying $|x+4| = 3x - 2$ is given as

(A) $\left\{-\frac{1}{2}, 3\right\}$ (B) $\left\{-\frac{1}{2}\right\}$ (C) $\{3\}$ (D) None of these

[Ans. C]

6. The complete set of solution of $2|x+1| + |x-3| = 4$ is given by

(A) $\left\{\frac{5}{3}\right\}$ (B) $\{-1\}$ (C) $\left\{-1, \frac{5}{3}\right\}$ (D) None of these

[Ans. B]

7. The inequality $\frac{|x-3|}{x^2-5x+6} \geq 2$ is given as

(A) $\left[\frac{3}{2}, 2\right] \cup \left[2, \frac{5}{2}\right]$ (B) $\left[\frac{3}{2}, 2\right]$ (C) $\left[\frac{3}{2}, \frac{5}{2}\right]$ (D) $\left[\frac{3}{2}, 2\right)$ [Ans. D]

8. Values of x satisfying the equality $|x^2 + 8x + 7| = |x^2 + 4x + 4| + |4x + 3|$ for $x \in R$ are

(A) $(-2, \infty)$ (B) $\left(\frac{3}{4}, \infty\right)$ (C) $\{2\} \cup \left[-\frac{3}{4}, \infty\right)$ (D) $\left[-\frac{4}{3}, \infty\right)$

[Ans. C]

QUESTION

9. The equation $4^{\left(\frac{1}{x}-2\right)} = \frac{1}{2}\ln\sqrt{e}$ has the solution-

(A) –1 (B) 1 (C) 2 (D) None [Ans. B]

10. Solution of the equation $2^{x+2} \cdot 27^{x(x-1)} = 9$ are given by-

(A) $\log_2(2/3), 1$ (B) $2, 1-\log_2 3$ (C) $-2, 1-\log_2 3$ (D) None of these

[Ans. C]

11. If $x^{[\log_3 x^2+(\log_3 x)^2-10]} = \frac{1}{x^2}$ then x is equal to

(A) 9, 1/9 (B) 9, 1/81 (C) 1, 1/9 (D) 2, 2/9 [Ans. B]

12. Complete set of values of x satisfying the inequality $x-3<\sqrt{x^2+4x-5}$ is :

(A) $(-\infty, 5] \cup [1, \infty)$ (B) $(-5, 3]$ (C) $[3, 5)$ (D) $(-5, 3)$

[Ans. A]

K.T.K → 01 The common value of x satisfying $\frac{x-1}{x+2} \geq 0$ and $\frac{2x-5}{x-2} \leq 1$ is

A) $\frac{x-1}{x+2} \geq 0$ $\qquad x \in (-\infty, -2) \cup [1, \infty)$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & -2 & & 1 & \end{array}$$

$\frac{2x-5}{x-2} \leq 1$

$\frac{2x-5-x+2}{x-2} \leq 0$ $\qquad \cap$

$\frac{x-3}{x-2} \leq 0$ $\qquad x \in (2, 3]$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & 2 & & 3 & \end{array}$$ $\qquad x \in (2, 3]$

K.T.K → 02 The set of values of x satisfying the inequality $2x-7 < 4x-2$ and $-5 \leq 2x+6 < 4$ is given as

As

$$2x-7 < 4x-2$$
$$2x > -5$$
$$x > -\frac{5}{2}$$

$$-5 \leq 2x+6 < 4$$
$$-11 \leq 2x < -2$$
$$-\frac{11}{2} \leq x < -1$$

$$\cap$$

$$x \in \left(-\frac{5}{2}, -1\right)$$

DATE / /
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(K.T.K→3) The Smallest Integer k satisfying the Inequality $\frac{x-5}{x^2+5x-14} > 0$ is

As $\frac{x-5}{x^2+7x-2x-14} > 0$

$$\frac{(x-5)}{(x+7)(x-2)} > 0$$

$$\underset{-7}{-\quad|}\quad + \quad\underset{2}{|}\quad - \quad\underset{5}{|}\quad +$$

$$x \in (-7, 2) \cup (5, \infty)$$

Smallest Integer $= -6$
Satisfies the ineq

(KTK's)

1) $\dfrac{x-1}{x+2} \geq 0$ & $\dfrac{2x-5}{x-2} \leq 1$

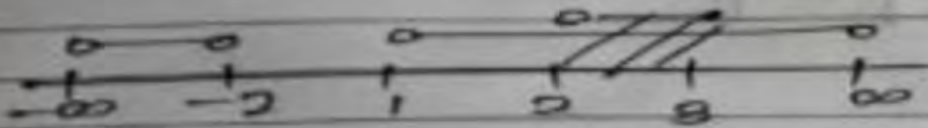
$\therefore x \in (-\infty, -2) \cup [1, \infty)$

$\dfrac{2x-5}{x-2} - 1 \leq 0$

$\dfrac{2x-5-x+2}{(x-2)} \leq 0$

$\dfrac{(x-3)}{(x-2)} \leq 0$

$\therefore x \in (2, 3]$



$\boxed{\therefore x \in (2, 3]}$

2) $2x-7 < 4x-2$

$2x+5 > 0$

$\boxed{\therefore x > -5/2}$

$-5 \leq 2x+6 < 4$

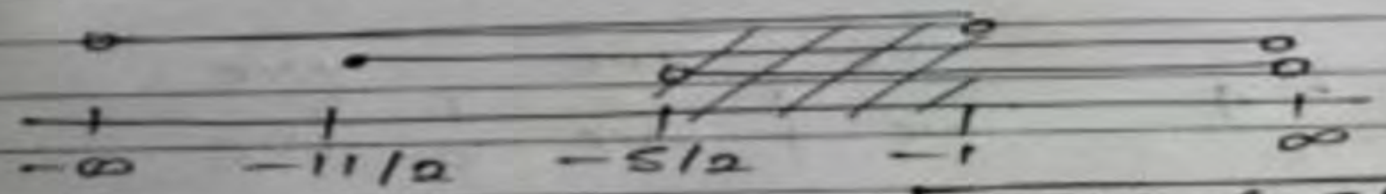
$\therefore 2x+11 \geq 0$

$\boxed{x \geq -11/2}$

$2x+2 < 0$

$2x < -2$

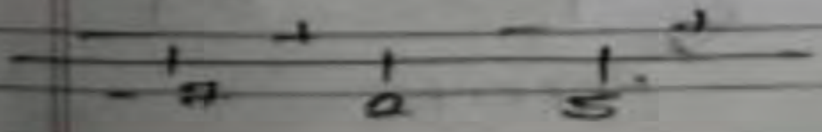
$\boxed{x < -1}$



$\boxed{\therefore x \in (-5/2, -1)}$ Ans

3) $\dfrac{x-5}{x^2+5x-14} > 0$

$\dfrac{(x-5)}{(x+7)(x-2)} > 0$



$\therefore x \in (-7, 2) \cup (5, \infty)$

smallest value of $x = -6$

4. $\dfrac{x^2+6x-7}{|x+4|} < 0$

C-I

$x \geq -4$

$\dfrac{(x+7)(x-1)}{(x+4)} < 0$ $\quad\cap$

$x \in (-\infty,-7) \cup (-4,1)$

$x \in (-4,1)$

C-II

$x < -4$

$\dfrac{(x+7)(x-1)}{-(x+4)} < 0$

$\dfrac{(x+7)(x-1)}{(x+4)} > 0$ $\quad\cap$

$x \in (-7,-4) \cup (1,\infty)$

$x \in (-7,-4)$

$\cap$

$x \in (-7,1) - \{-4\}$

Integral values $= -6,-5,-3,-2,-1,0$

Total $\to$ 6 (A)

K.T.K → 05 Set of Real values of x satisfying $|x+4| = 3x-2$ is given as

A) Case 1 If $x+4 \geq 0$

$x \geq -4$

$x+4 = 3x-2$

$2x = 6$

$\boxed{x = 3}$ $\cap$ $\boxed{x=3}$

Case 2 If $x+4 < 0$

$x < -4$

$-x-4 = 3x-2$

$4x = -2$

$\boxed{x = -\frac{1}{2}}$ $\cap$ ϕ $\cup$

$x \in \{3\}$

6) $2|x+1| + |x-3| = 4$

$-ve \quad +ve \quad +ve$

$-ve \quad -1 \quad -ve \quad 3 \quad +ve$

Case-01

$x \geq 3$

$2x+2+x-3 = 4$

$3x-1 = 4$

$3x-5 = 0$

$\boxed{\therefore x = \frac{5}{3}}$

Reject

Case-02

$-1 \leq x < 3$

$2x+2+3-x = 4$

$x+5 = 4$

$\boxed{x = -1}$ ✓

Final Ans

Case-03

$x < -1$

$-2x-1+3-x = 4$

$-3x+2 = 4$

$-3x = 2$

$\boxed{x = -\frac{2}{3}}$

Reject

Page No. ____
Date

7. $\dfrac{|u-3|}{(u-2)(u-3)} \geq 2$

$u > 3$

$\dfrac{\cancel{(u-3)}}{(u-2)\cancel{(u-3)}} \geq 2$

$u \neq 3$ $\quad \dfrac{1}{(u-2)} - 2 \geq 0$

$\dfrac{1-2u+4}{(u-2)} \geq 0$

$\dfrac{(2u-5)}{(u-2)} \leq 0$

$u \in \left(2, \frac{5}{2}\right]$

$\times$ bcz $u > 3$

$u < 3$

$\dfrac{-\cancel{(u-3)}}{(u-2)\cancel{(u-3)}} \geq 2$

$\dfrac{1}{u-2} \leq -2$

$\dfrac{1+2u-4}{u-2} \leq 0$

$\dfrac{2u-3}{u-2} \leq 0$

$u \in \left[\frac{3}{2}, 2\right)$ ✓

(D)

(7) $\dfrac{(x-3)}{x^2-5x+6} - 2 \geq 0$

Soln: Case 1: $x-3 \geq 0$

$x_1 \geq 3$

$\dfrac{x-3-2(x^2-5x+6)}{x^2-5x+6} \geq 0$

$\dfrac{x-3-2x^2+10x-12}{x^2-5x+6} \geq 0$

$\dfrac{-2x^2+11x-15}{x^2-5x+6} \geq 0$

$\dfrac{2x^2-11x+15}{x^2-5x+6} \leq 0$

$\dfrac{(2x-5)\cancel{(x-3)}}{\cancel{(x-3)}(x-2)} \leq 0$

$\dfrac{2x-5}{x-2} \leq 0 \qquad x \neq 3$

$x_2 \in \left[\frac{5}{2}, 2\right)$

$x_1 \cap x_2 \in \phi$

Case 2: $x-3 < 0$

$x_3 < 3$

$\dfrac{3-x}{x^2-5x+6} - 2 \geq 0$

$\dfrac{3-x-2(x^2-5x+6)}{x^2-5x+6} \geq 0$

$\dfrac{2x^2-9x+9}{x^2-5x+6} \leq 0$

$\dfrac{(2x-3)\cancel{(x-3)}}{\cancel{(x-3)}(x-2)} \leq 0$

$\dfrac{(2x-3)}{x-2} \leq 0 \qquad x \neq 3$

$x_4 \in \left[\frac{3}{2}, 2\right)$

$x_3 \cap x_4 \in \left[\frac{3}{2}, 2\right)$

union of cases $\Rightarrow \left[\frac{3}{2}, 2\right)$ Ans

KTK 08

$$|x^2+8x+7| = |x^2+4x+4| + |4x+3|$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$|a+b| = |a| + |b|$$

$$\rightarrow \quad ab \geq 0$$

$$(x^2+4x+4)(4x+3) \geq 0$$

$$(x+2)^2(4x+3) \geq 0$$

$$4x+3 \geq 0 \qquad x \neq \text{ (x can be } -2 \text{ also)}$$

$$x \geq -\frac{3}{4}$$

$$x \in \left(-\infty, -\frac{3}{4}\right]$$

9) $4^{\left(\frac{1}{x}-2\right)} = \frac{1}{2}\ln\sqrt{e}$

Page No.

Date

$4^{(1/x-2)} = \frac{1}{4}\ln e$

$\therefore 4^{(1/x-2)} = 4^{-1}$

$\frac{1}{x}-2 = -1$

$\frac{1}{x}-1 = 0$

$\therefore \frac{1}{x} = 1 \quad \boxed{\therefore x = 1}$

09
KTK

$$4^{\left(\frac{1}{x}-2\right)} = \frac{1}{2}\ln\sqrt{e}$$

$$4^{\left(\frac{1}{x}-2\right)} = \cancel{\tan}\ \frac{1}{2}\ln e^{1/2}$$

$$4^{\left(\frac{1}{x}-2\right)} = \frac{1}{4}\ln e$$

$$4^{\left(\frac{1}{x}-2\right)} = \frac{1}{4}$$

$$\frac{1}{x}-2 = -1$$

$$\frac{1}{x} = 1$$

$$\boxed{x = 1}$$

(B)

11. $u^{[\log_3 u^2 + (\log_3 u)^2 - 10]} = \frac{1}{u^2}$, $u > 0$

$\Big) \log_3(\,)$

$(2\log_3 u + (\log_3 u)^2 - 10) \cdot \log_3 u = \log_3 \frac{1}{u^2}$

$\hookrightarrow \log_3 u^{-2} \Rightarrow -2\log_3 u$

$(2t + t^2 - 10) \cdot t = -2t \Rightarrow 2t^2 + t^3 - 10t = -2t$

$t^3 + 2t^2 - 8t = 0$

$t^2(t-2) \quad + 4t(t-2) + 0(t-2) = 0$

$(t)(t+4)(t-2) = 0$

$\therefore t = -4, 0, 2$

$\log_3 u = -4, 0, 2$

$u = (3)^{-4}, (3)^0, 3^2$

$u = \frac{1}{81}, 1, 9$ (B)

→ KTK (12)

complete set of values of x satisfying the inequality $x-3 < \sqrt{x^2+4x-5}$

$\Rightarrow \sqrt{x^2+4x-5} > x-3$

$x^2+4x-5 \geq 0 \sim (x+5)(x-1) \geq 0$

$x \in (-\infty, -5] \cup [1, \infty)$ ⋯ (A)

case (I) $x-3 \geq 0$

$x \geq 3$

$\sqrt{x^2+4x-5} > x-3$

(S.B.S)

$\cancel{x^2}+4x-5 > \cancel{x^2}+9-6x$

$10x > 14$

$x > 7/5$

$\cap$

$x \in [3, \infty)$

case (II) $x-3 < 0$

$x < 3$

$\sqrt{x^2+4x-5} > x-3$

$x \in R$

$\cap$

$x \in (-\infty, 3)$

case (I) $\cup$ case (II)

~~$x \in (\ldots)$~~ (B)

$x \in R$

now, (A) $\cap$ (B)

$x \in (-\infty, -5] \cup [1, \infty)$ #

81

Eqn 012
KTK

$x-3 < \sqrt{x^2+4x-5}$

$x^2+4x-5 \geq 0$
$x^2+5x-x-5 \geq 0$
$~~x(x+1)-1(~~$
$x^2-x+5x-5 \geq 0$
$x(x-1)+5(x-1) \geq 0$
$(x+5)(x-1) \geq 0$
$x \in (-\infty,-5] \cup [1,\infty)$

Case 01

$x-3 \geq 0$
$x \geq 3$

$(x-3)^2 < x^2+4x-5$
$x^2+9-6x < x^2+4x-5$
$14 < 10x$
$10x > 14$
$x > \frac{14}{10}$
$x > \frac{7}{5}$

$\cap$
$x \in [3,\infty)$

Case 02

$x-3<0$
$x<3$

$\underset{(-)}{x-3} < \underset{(+)}{\sqrt{x^2+4x-5}}$
always true
$x \in R$
$\cap$
$x \in (-\infty,3)$

$\cup$
$x \in (-\infty,3) \cup (3,\infty)$
$\Rightarrow x \in R$

$\cap$
$x \in (-\infty,-5] \cup (1,\infty)$

(A) option

ay 11, 20

Home Challenge-09

If the least integral value satisfying the equation $\log_3 \sqrt{x^2 - 4x + 4} = 2^{\log_2(\log_3(|x|-2))}$ is α, then find the number of zeroes after decimal and before first significant digit in the number of $(\alpha)^{-4\alpha}$. [Ans. 9]

$\log_3 |x-2| = \log_3(|x|-2)$

$|x-2| = |x|-2$

T_1: $-ve$, $-ve$, $+ve$

T_2: $-ve$, 0, $+ve$, 2, $+ve$

$|x-2| + 2 = |x|$

$|x-2| + |2| = |x|$

$|a| + |b| = |a+b|$

$2(x-2) \geq 0$

$x \geq 2$

$|x-2| \neq 0$, $\log_3(|x|-2) > 0$, $|x|-2 > 0$ (No Need)

$x \neq 2$ $|x|-2 > 1$

$|x| > 3$

$x \in (-\infty, -3) \cup (3, \infty)$

$x > 3 \Rightarrow \alpha = 4$

3 4 5 6

$x = (\alpha)^{-4\alpha} = 4^{-16}$

$\log_{10} x = -16 \log_{10} 4 = -32 \log_{10} 2 = -32(0.3010)$

$= -9.632$

No: of zeros immediately after decimal before sign. digit starts $= 9$

THANK YOU