

PRAYAS

JEE 2025

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Lecture -08

Physics

Magnetism

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Today's Goal

- magnetic moment
- Bar magnet

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$$\vec{p} = q \vec{l} \longrightarrow \vec{m} = i \vec{A} \cdot N$$

$$\vec{p} \longrightarrow \vec{m}$$

$$\vec{\tau} = \vec{p} \times \vec{E} \longrightarrow \vec{\tau} = \vec{m} \times \vec{B}$$

$$U = -\vec{p} \cdot \vec{E} \longrightarrow U = -\vec{m} \cdot \vec{B}$$

$$T = 2\pi \sqrt{\frac{I}{pE}}$$

$$T = 2\pi \sqrt{\frac{I}{mB}}$$

$$T = 2\pi \sqrt{\frac{I}{mB}}$$

I → mOI
 m → magnetic moment

→ p
 → E θ = 0
 τ = 0
 Stable

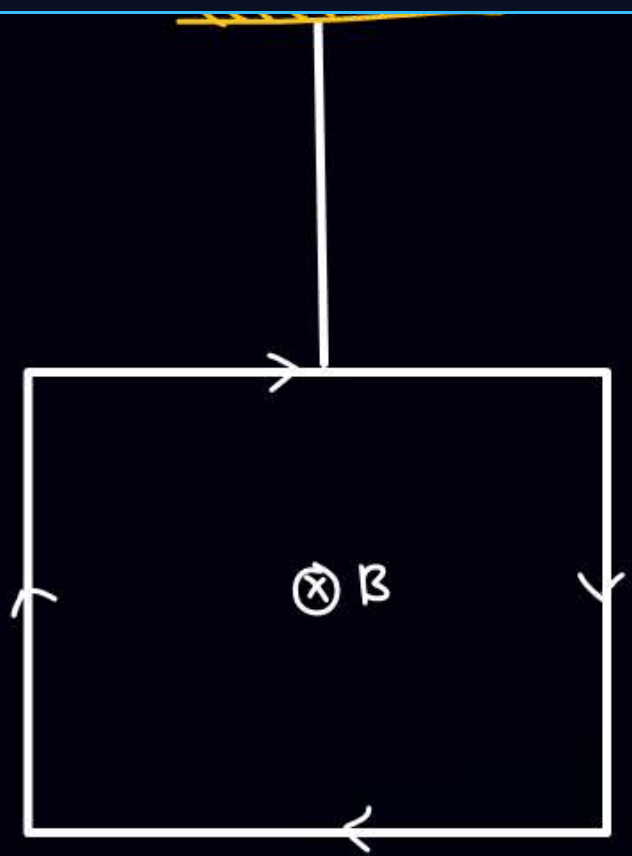
→ m
 → B θ = 0
 τ = 0
 Stable

θ = 0 (stable)
 θ = 180 (Unstable)

θ = 0 (stable)
 θ = 180 (Unstable)

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Q



(4 rod each of mass m & L)
 $\vec{B}_{ext} = -B \hat{k}$

$\tau = mBS \sin \theta$
 $\vec{\tau} = -mB \vec{\theta}$
 $T = 2\pi \sqrt{\frac{I}{mB}}$ (Angular SHM)

If loop is rotated slightly by angle ' θ ', find time period of oscillation.

Solⁿ
 $T = 2\pi \sqrt{\frac{I}{mB}}$

$I = \left[\frac{mL^2}{12} + m \left(\frac{L}{2} \right)^2 \right] \times 2 = \frac{2mL^2}{3}$

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magnetic moment = iL^2

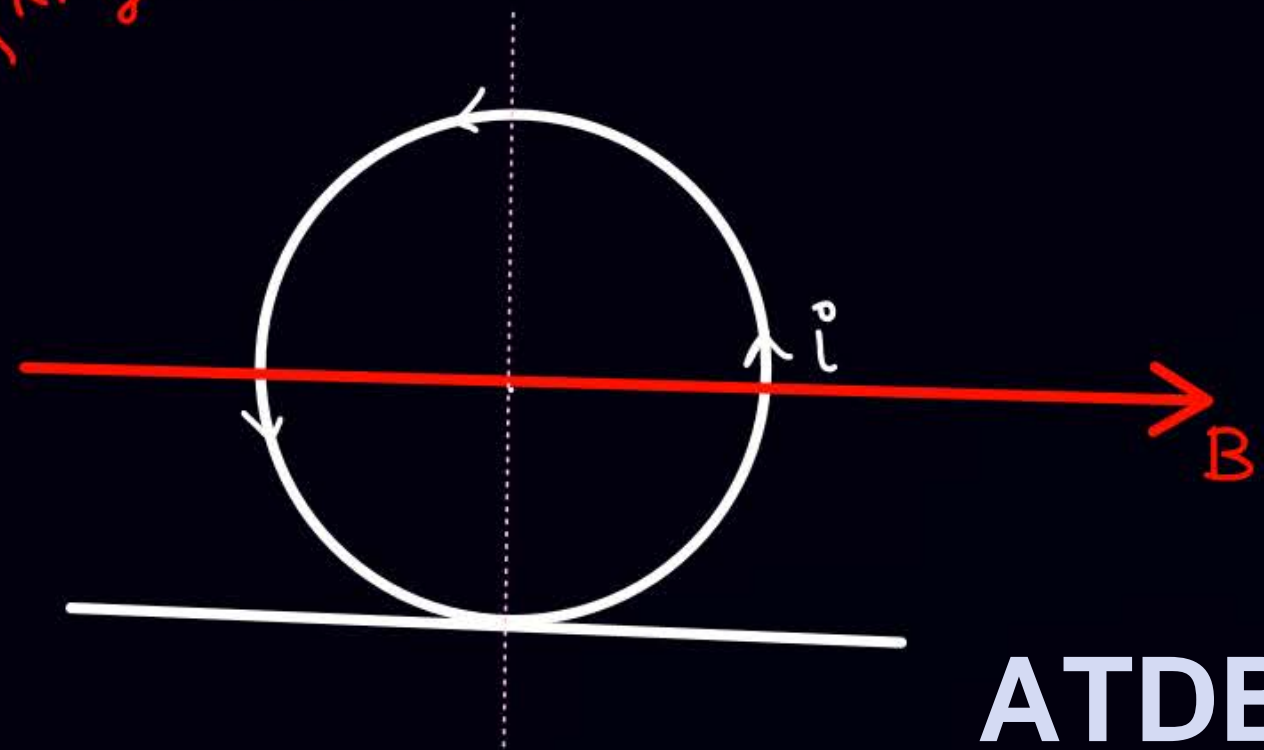
$\theta = 0$, stable

$T = 2\pi \sqrt{\frac{\frac{2mL^2}{3}}{iL^2 B}}$





Q (Ring m, R)



find $\dot{\alpha}$ of ring at given instant.

Solⁿ $\vec{\tau} = \vec{M} \times \vec{B}$

$\vec{M} = i \pi R^2 \hat{k}$

$\vec{B} = B \hat{i}$

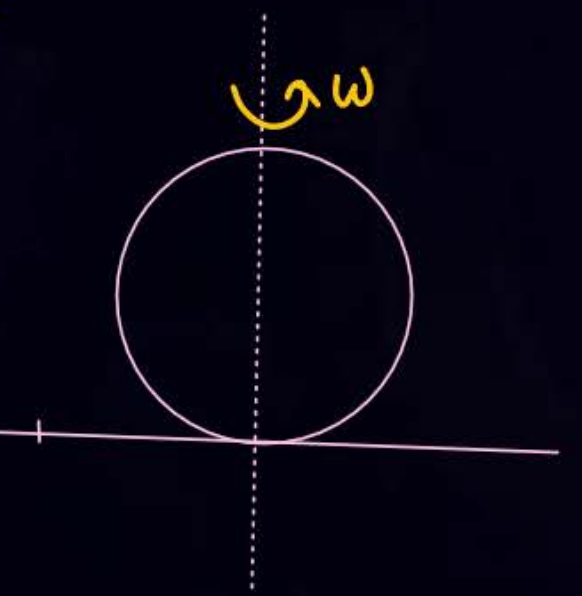
$\tau = i \pi R^2 B = I \alpha$

$\hat{\tau} \equiv \hat{k} \alpha \hat{i} \equiv \hat{j}$

$\tau = i \pi R^2 B = \frac{m R^2}{2} \alpha$

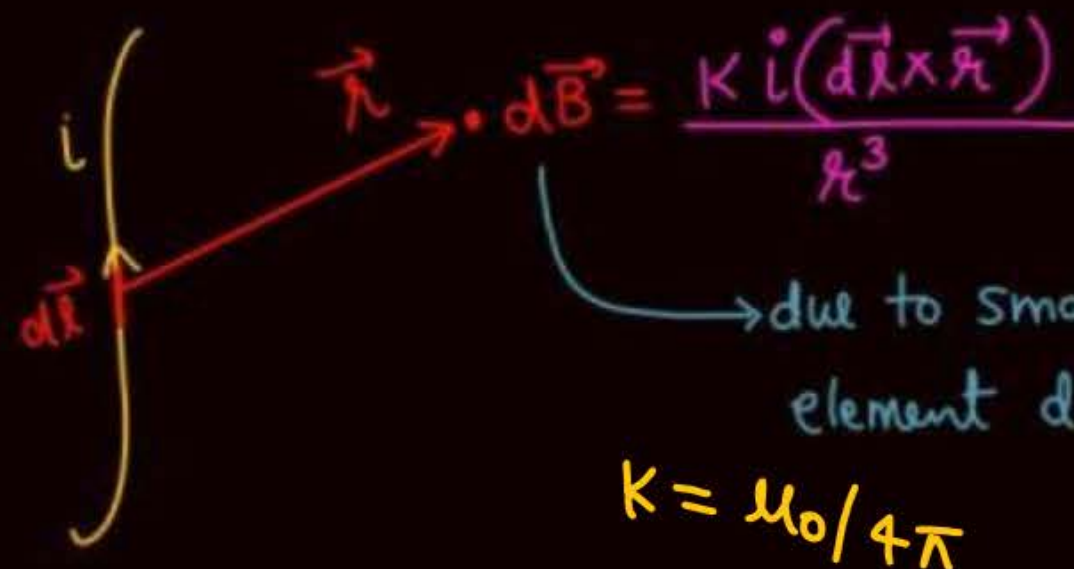
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$\alpha = \frac{2 i \pi B}{m}$



Biot-Savart Law

① for wire

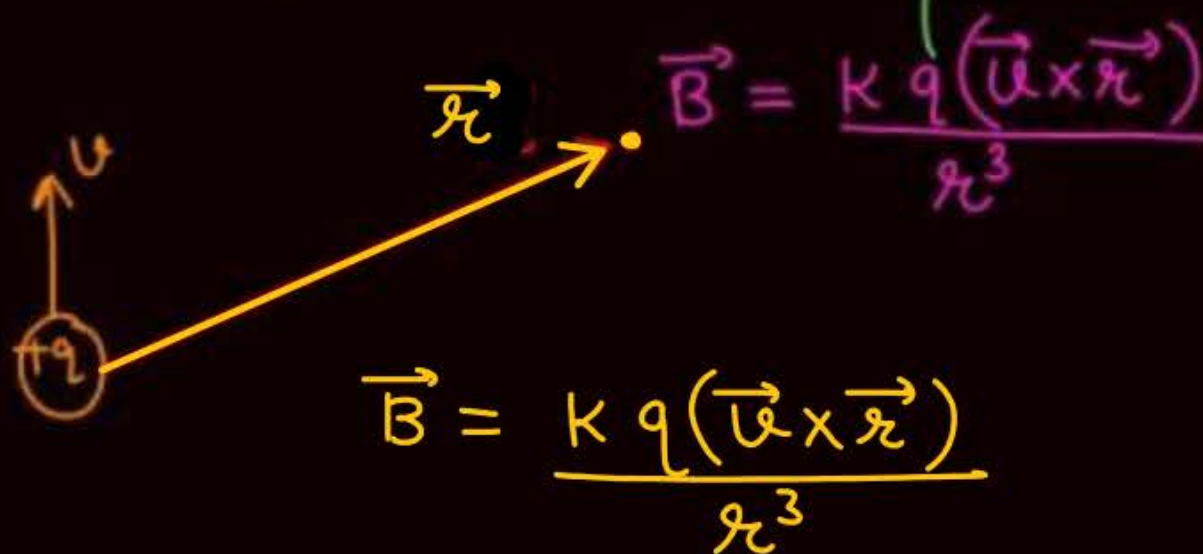


$$d\vec{B} = \frac{\mu_0 i (d\vec{l} \times \vec{r})}{4\pi r^3}$$

due to small element dl .

$$k = \mu_0 / 4\pi$$

② For moving point charge



$$\vec{B} = \frac{\mu_0 q (\vec{u} \times \vec{r})}{4\pi r^3}$$

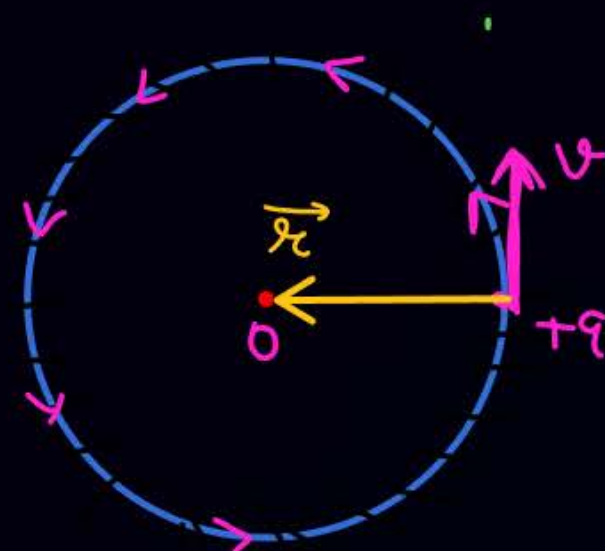
with sign

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Q If a charge particle $(+q, m)$ is moving in a circular path with const speed u, ω . find magnetic field at the center of circle.

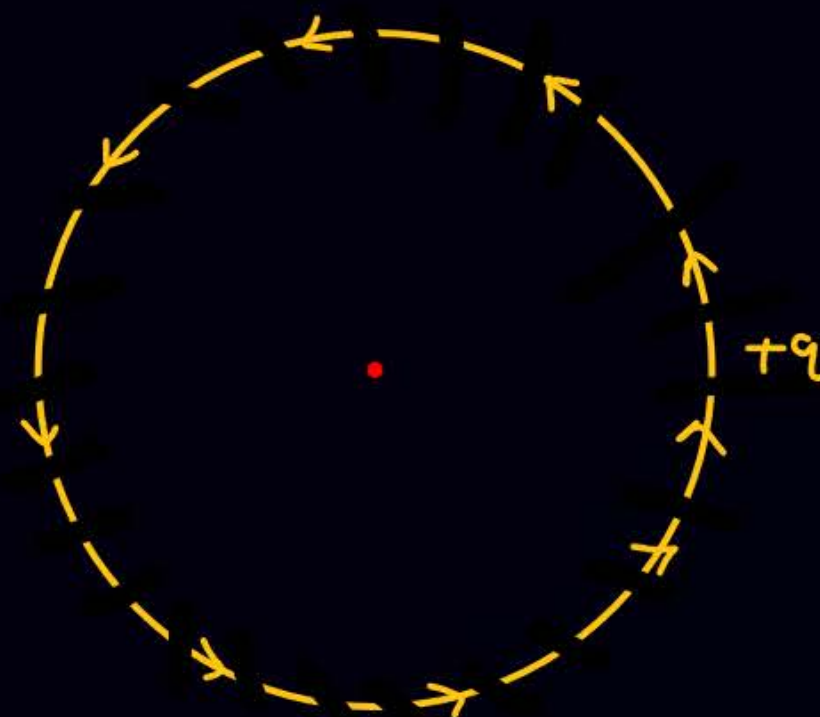
Solⁿ



$$\vec{B} = \frac{kq(\vec{u} \times \vec{r})}{r^3}$$

$$\vec{B} = \frac{kq u r \sin 90^\circ}{r^3} \odot$$

$$\vec{B} = \frac{k \cdot q \cdot r \omega r}{r^3} = \frac{kq\omega}{r}$$



$$i = \frac{q}{T}$$

$T \rightarrow$ time period

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$$K = \frac{Kq\omega}{R} = \frac{\mu_0}{4\pi} \frac{q \cdot 2\pi}{R T}$$

$$= \frac{\mu_0 q}{2RT}$$

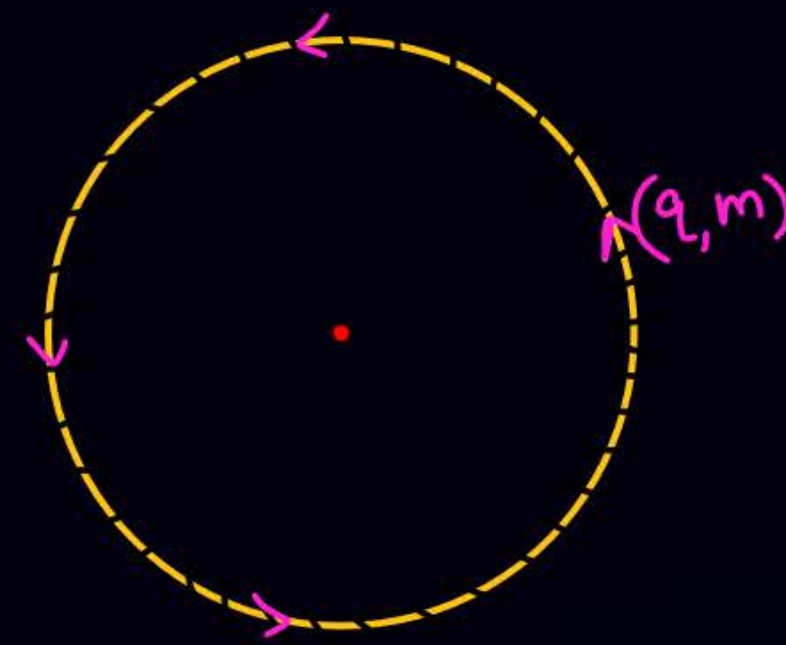
$$\vec{M} = \text{magnetic moment} = ?$$

$$B_{\text{center}} = \frac{\mu_0 i}{2R} = \frac{\mu_0 q}{2RT}$$

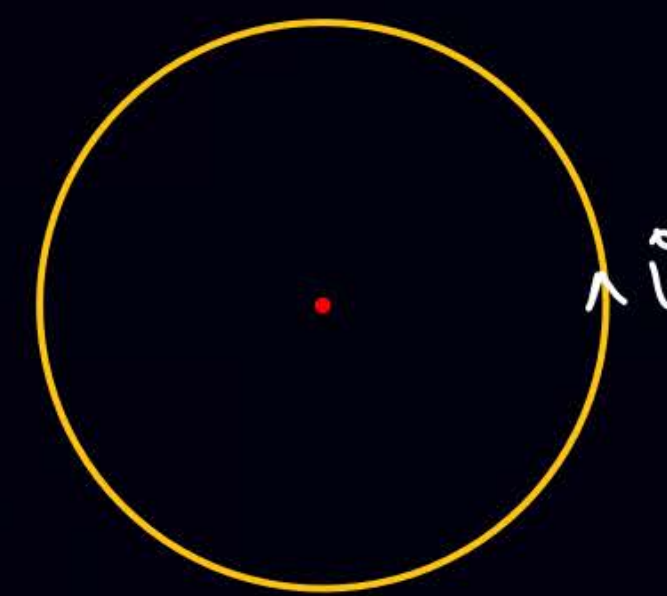
$$\vec{M} = \text{magnetic moment} = iA = \frac{q}{T} \times \pi R^2$$



Q



|||



- ① $B_{center} = \frac{\mu_0 i}{2R} = \frac{\mu_0 (q/T)}{2R} (\hat{k})$
- ② magnetic moment = $iA = \frac{q}{T} \cdot \pi R^2 (\hat{k})$
- ③ Angular momentum = $m v R = m R \omega R = m R^2 \omega (-\hat{k})$

④ $\frac{|\vec{m}|}{|\vec{L}|} = \frac{\frac{q}{T} \cdot \pi R^2}{m R^2 \omega} = \frac{q \pi}{T m \frac{2\pi}{T}} = \frac{q}{2m}$

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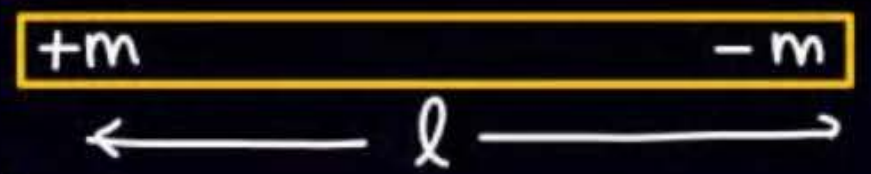
$\vec{m} = \frac{q}{2m} \vec{L}$

$m = \frac{q}{2m} L$

magnetic moment mass Angular momentum

Bar magnet

$m \rightarrow$ pole strength

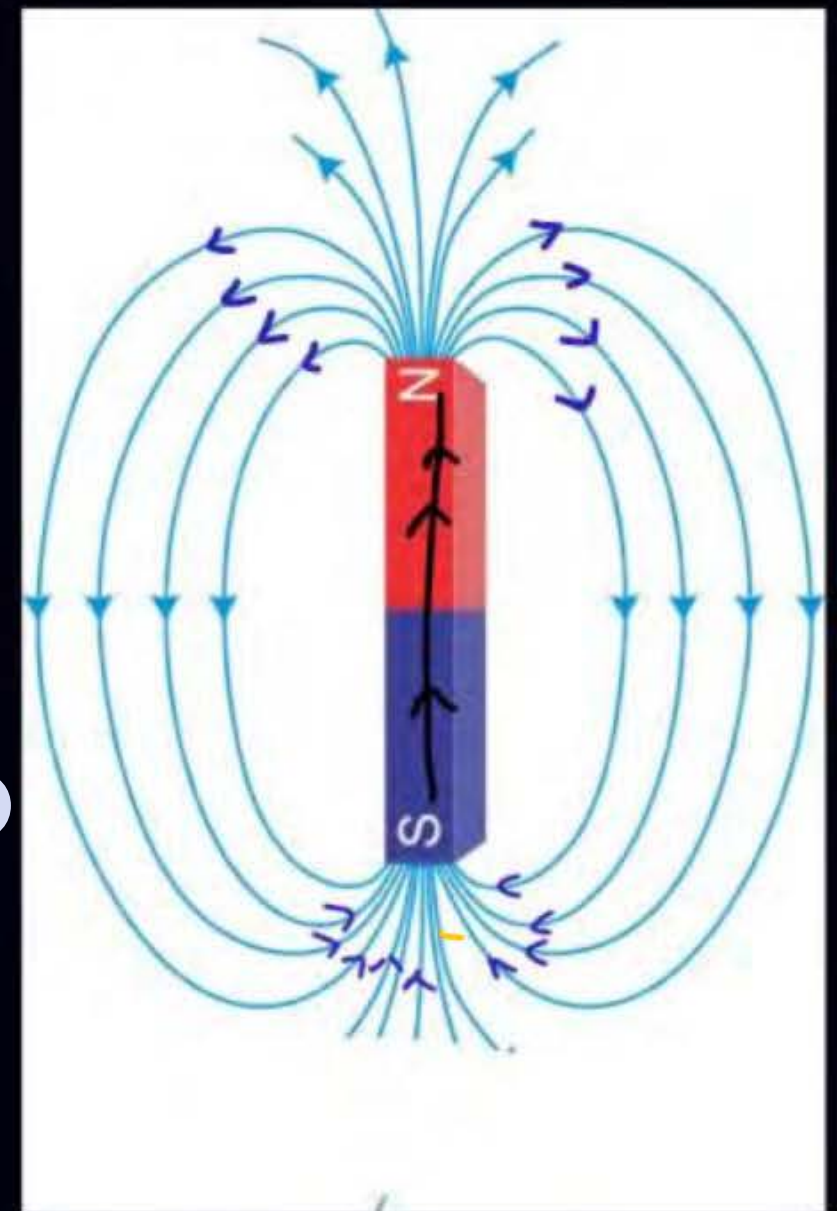


magnetic dipole moment

$$\vec{M} = m \vec{l} \quad (\text{south to north})$$

* Inside magnet magnetic field is from South to North.

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(North pole
|||
+ve charge)

(South pole
|||
-ve charge)



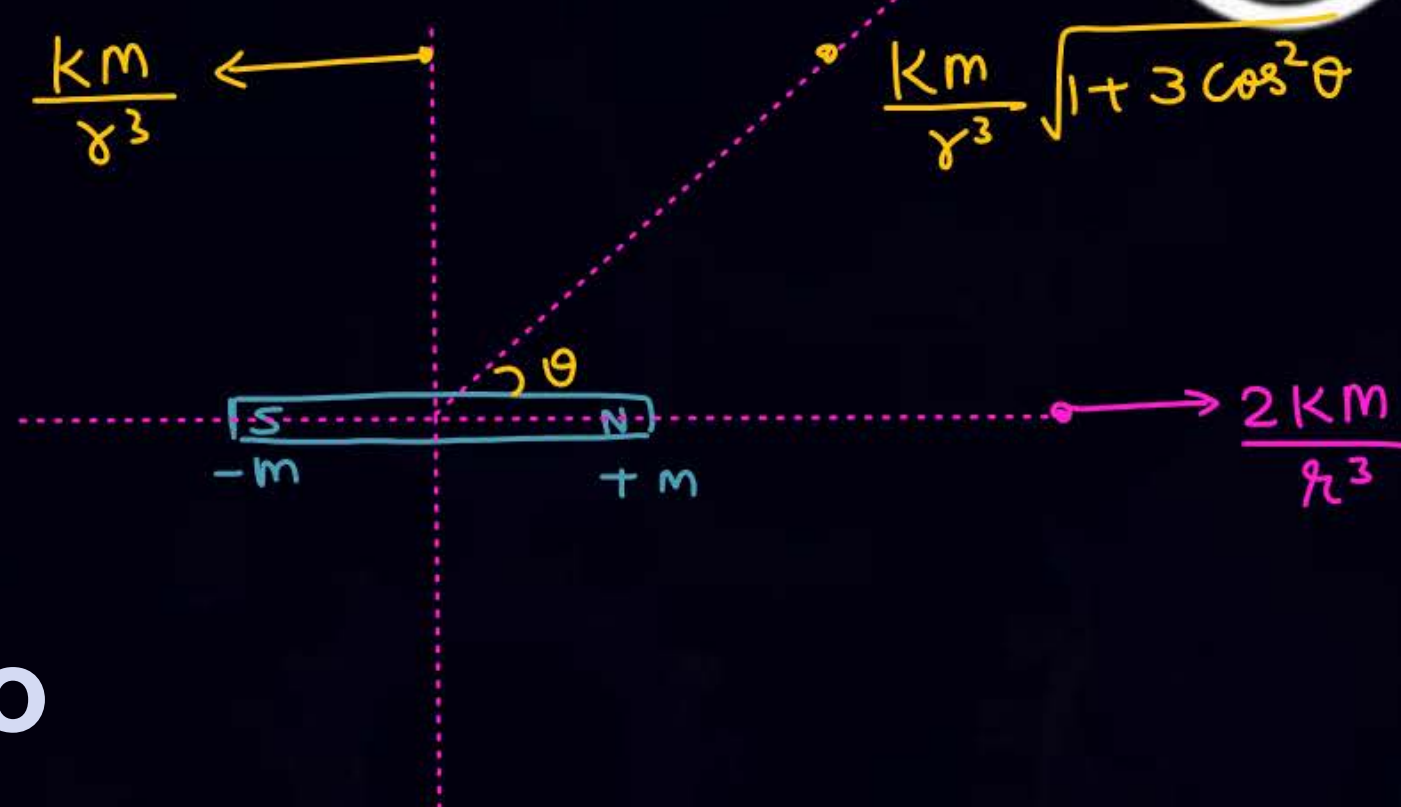
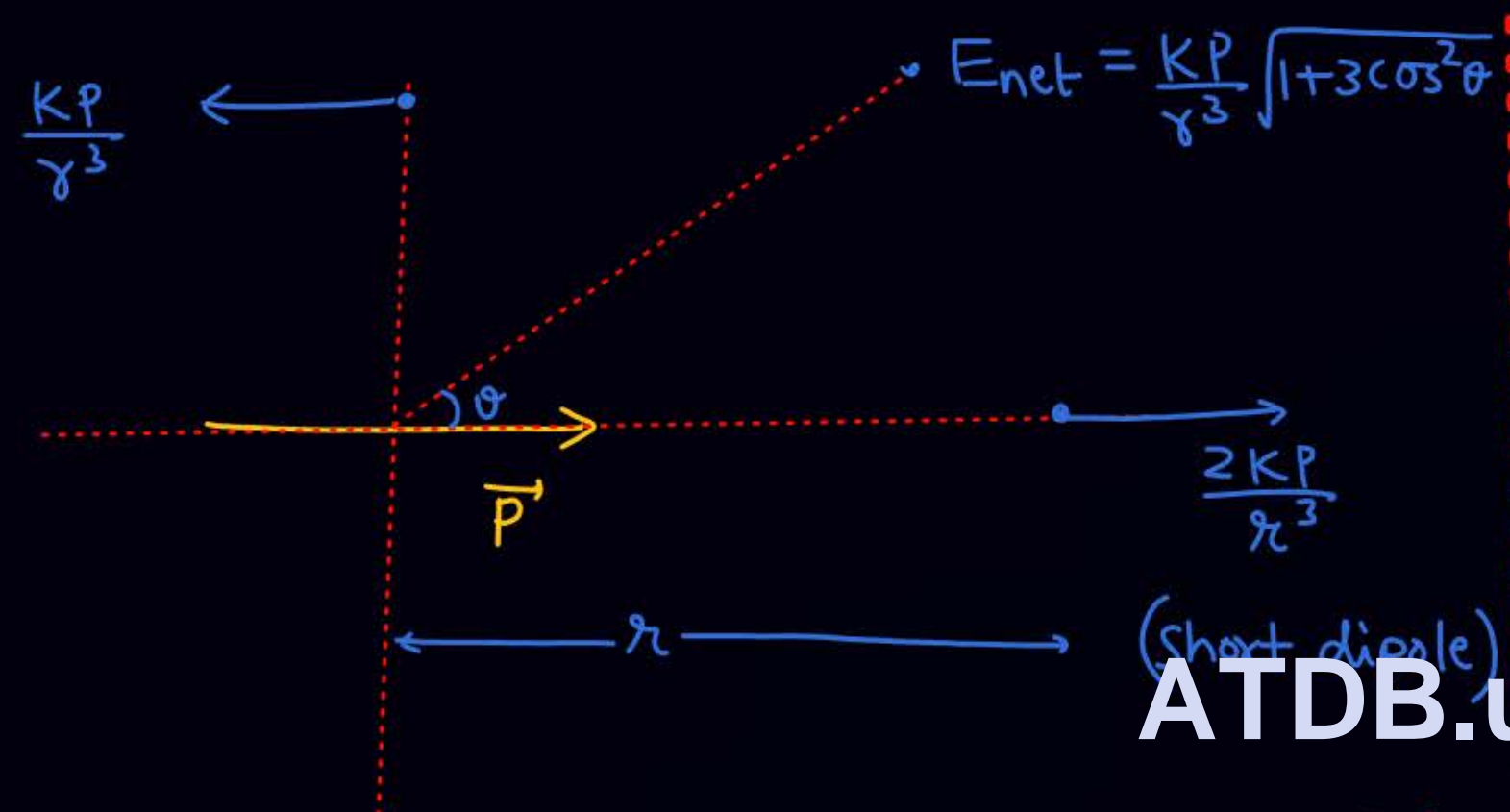


$\vec{m}_{net} = m \vec{l}$
 magnetic dipole moment

$\vec{p} = q \vec{l}$
 Electric dipole moment

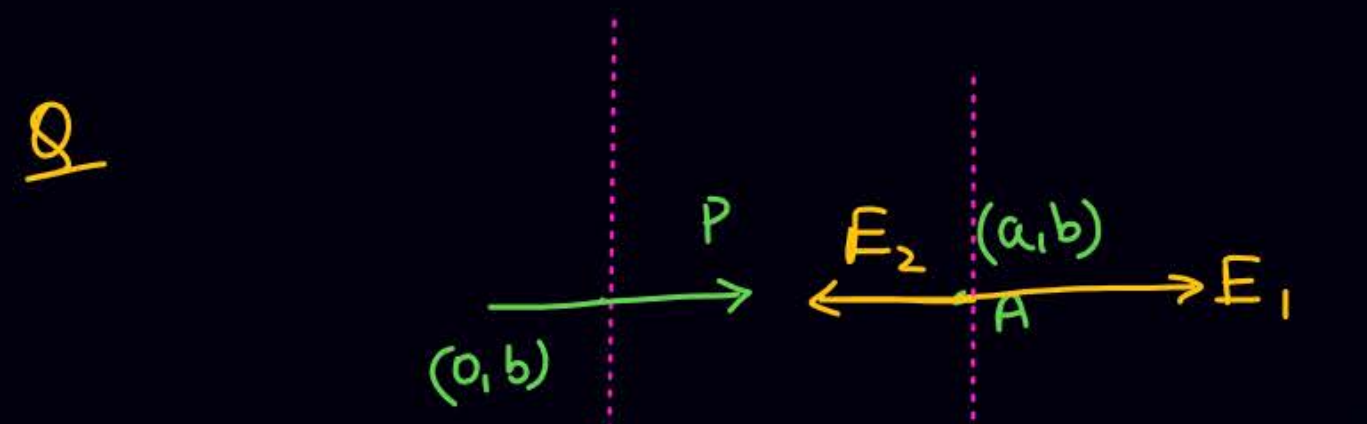
$\vec{c} = \vec{m} \times \vec{b} \longrightarrow \vec{c} = \vec{p} \times \vec{E}$
 $U = -\vec{m} \cdot \vec{B} \longrightarrow U = -\vec{p} \cdot \vec{E}$
 $T = 2\pi \sqrt{\frac{I}{mB}} \longrightarrow T = 2\pi \sqrt{\frac{I}{pE}}$

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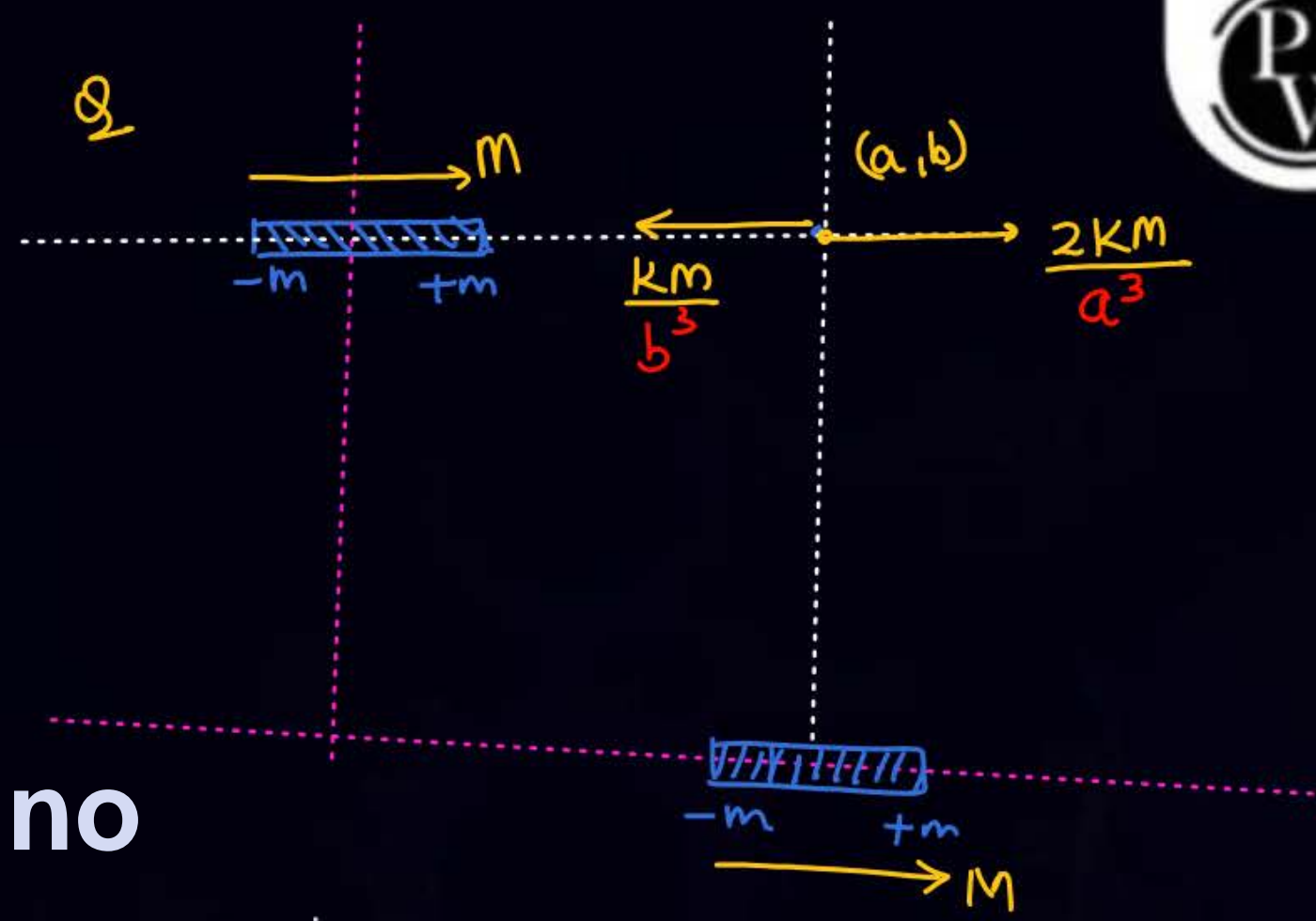


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$\vec{p} \rightarrow \vec{m}$
 $q \rightarrow m$
 $k = \frac{1}{4\pi\epsilon_0} \rightarrow k = \frac{\mu_0}{4\pi}$



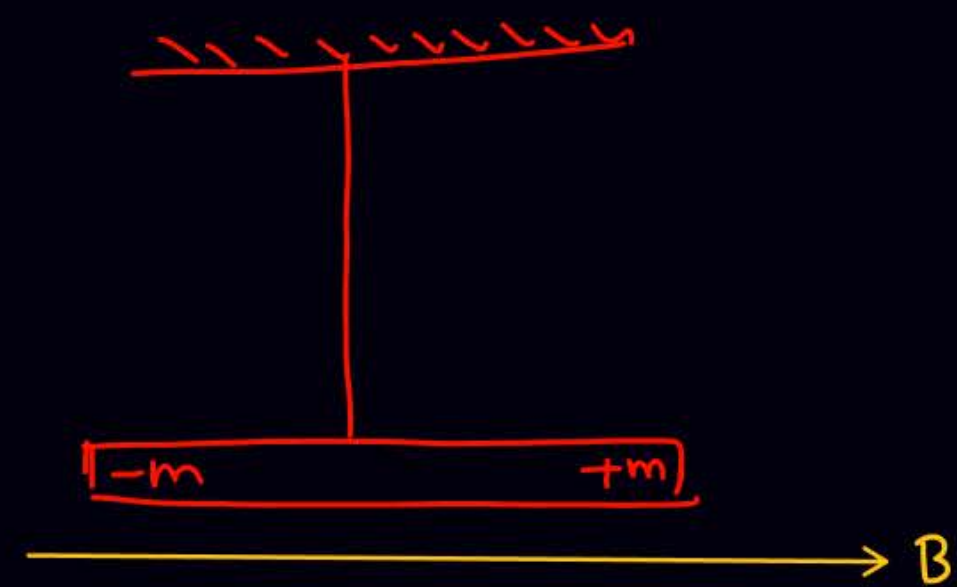
Both are short dipole



$(a, 0)$ P **ATDB.uno**

$$\vec{E}_A = \frac{2Kq}{a^3} - \frac{Kq}{b^3}$$

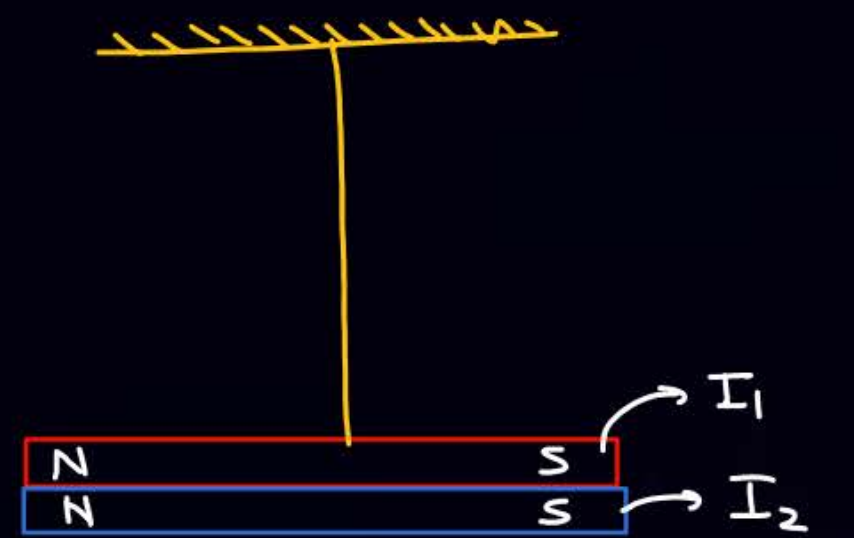
Jm Fvxt
Q



$$T = 2\pi \sqrt{\frac{I}{mB}}$$

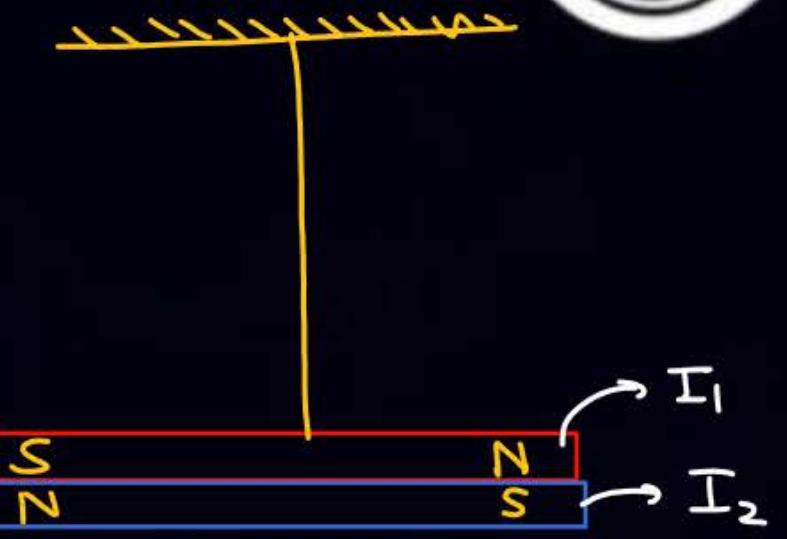
↪ magnetic moment

#



$$T = 2\pi \sqrt{\frac{I_1 + I_2}{(m_1 + m_2) B}}$$

#



$$T = 2\pi \sqrt{\frac{I_1 + I_2}{|(m_1 - m_2)| B}}$$





$$F = \frac{6 K P_1 P_2}{\gamma^4}$$



$$F = \frac{6 K m_1 m_2}{\gamma^4}$$

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$$= \frac{4\pi (r^2 + \ell^2) \sqrt{(r^2 + \ell^2)}}{4\pi (r^2 + \ell^2)^{3/2}}$$

$$B_{eq} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + \ell^2)^{3/2}}, \quad \text{where } M = m(2\ell)$$

If magnet is short $r \gg \ell$, then $B_{eq} = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$

Magnetic shielding

If a soft iron ring is placed in magnetic field, most of the lines are found to pass through the ring and no lines pass through the space inside the ring. The inside of the ring is thus protected against any external magnetic effect. This phenomenon is called magnetic screening or shielding and is used to protect costly wrist-watches and other instruments from



Iron ring in a field

external magnetic fields by enclosing them in a soft-iron case or box.

- (i) Super conductors also provides perfect magnetic shielding due to exclusion of lines of force. This effect is called '**Meissner effect**'
- (ii) Relative magnetic permeability of super conductor is zero. So we can say that super conductors behaves like perfect diamagnetic.



Super conductor in a field

Dipole - Dipole Interactions :

S.No.	Relative position of dipoles	Magnetic force (F_m)
(a)		$\frac{\mu_0}{4\pi} \cdot \frac{6M_1M_2}{r^4}$ (along r)
(b)		$\frac{\mu_0}{4\pi} \cdot \frac{3M_1M_2}{r^4}$ (along r)
		$\mu_0 \cdot 3M_1M_2$



$$= \frac{4\pi (r^2 + l^2) \sqrt{(r^2 + l^2)}}{4\pi (r^2 + l^2)^{3/2}}$$

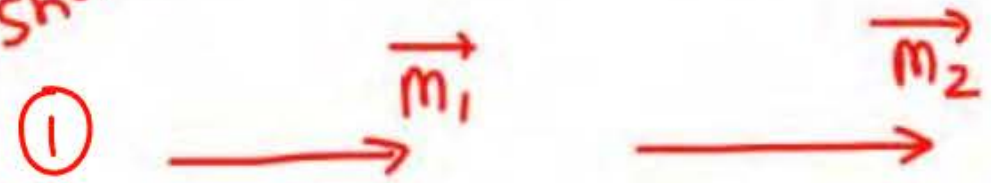
$$B_{eq} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + l^2)^{3/2}}$$

where $M = m(2l)$

$$K = \frac{\mu_0}{4\pi}$$

If magnet is short $r \gg l$, then $B_{eq} = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$

Short



$$F = \frac{6K m_1 m_2}{r^4} \quad \left(\text{Same as electrostatics} \right)$$



$$F = \frac{3K m_1 m_2}{r^4} \quad \left(\text{along } \vec{r} \right)$$



$$F = \frac{3K m_1 m_2}{r^4} \quad \left(\perp \text{ to } r \right)$$

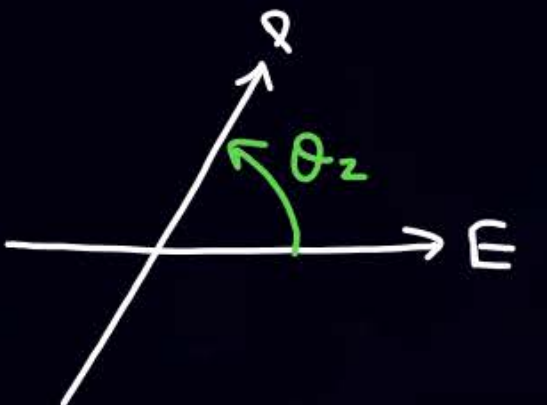
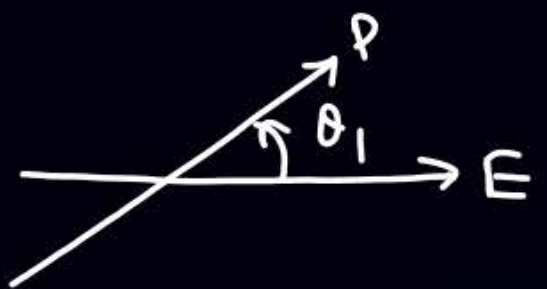
Dipole - Dipole Interactions :

S.No.	Relative position of dipoles	Magnetic force (F_m)
(a)		$\frac{\mu_0}{4\pi} \cdot \frac{6M_1 M_2}{r^4}$ (along r)
(b)		$\frac{\mu_0}{4\pi} \cdot \frac{3M_1 M_2}{r^4}$ (along r)
(c)		$\frac{\mu_0}{4\pi} \cdot \frac{3M_1 M_2}{r^4}$ (along r)



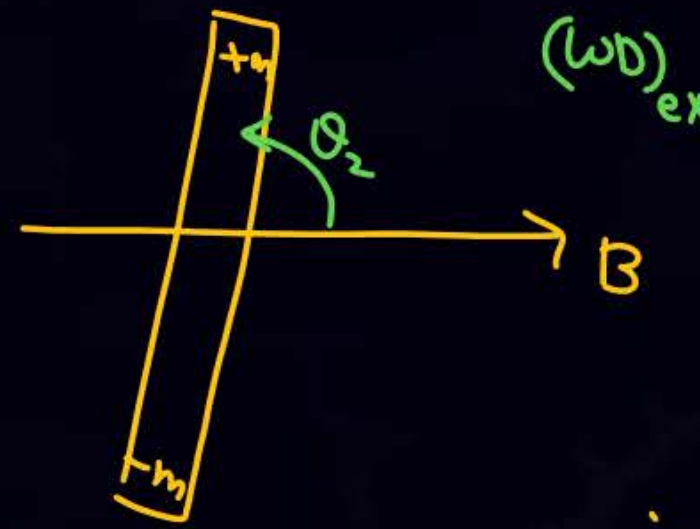
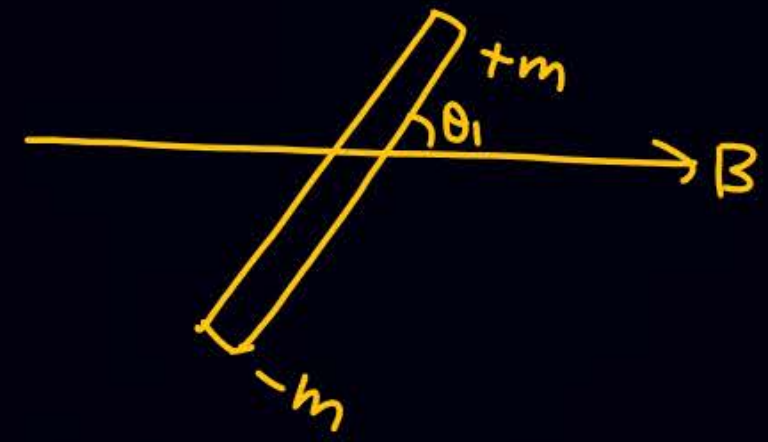
Jm

* $(W)_{\text{by ext-agent required to rotate dipole from } \theta_1 \text{ to } \theta_2 = -PE(\cos \theta_2 - \cos \theta_1)$
 $= PE \cos \theta_1 - PE \cos \theta_2$



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$(W)_{\text{ext}} = -mB(\cos \theta_2 - \cos \theta_1)$
 $= mB(\cos \theta_1 - \cos \theta_2)$

Recalling that the potential energy for an electric dipole is $U = -\vec{p} \cdot \vec{E}$ [see Eq.], a similar form is expected for the magnetic case. The work done by an external agent to rotate the magnetic dipole from an angle θ_0 to θ is given by

$$W_{\text{ext}} = \int_{\theta_0}^{\theta} (\mu B \sin \theta') d\theta' = \mu B (\cos \theta_0 - \cos \theta)$$

$$= \Delta U = U - U_0$$

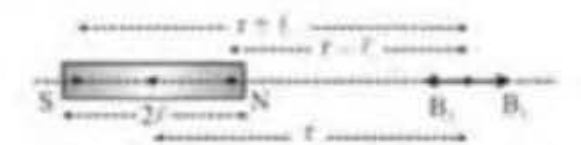
Once again, $W_{\text{ext}} = -W$, where W is the work done by the magnetic field. Choosing $U_0 = 0$ at $\theta_0 = \pi/2$, the dipole in the presence of an external field then has a potential energy as

$$U = -\mu B \cos \theta = -\vec{\mu} \cdot \vec{B}$$

The configuration is at a stable equilibrium when $\vec{\mu}$ is aligned parallel to \vec{B} , making U a minimum with $U_{\text{min}} = -\mu B$. On the other hand, when $\vec{\mu}$ and \vec{B} are anti-parallel, $U_{\text{max}} = +\mu B$ is a maximum and the system is unstable.



Derivation



(i) **At Axial position :-**

Magnetic field at point 'P' due to north pole $B_1 = \frac{\mu_0}{4\pi} \frac{m}{(r-l)^2}$ (away from north pole)

Magnetic field at point 'P' due to south pole $B_2 = \frac{\mu_0}{4\pi} \frac{m}{(r+l)^2}$ (towards north pole)

Net magnetic field at point 'P'

$$B_{axis} = B_1 - B_2, (\because B_1 > B_2) = \frac{\mu_0 m}{4\pi} \left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2} \right] = \frac{\mu_0 m}{4\pi} \left[\frac{4rl}{(r^2 - l^2)^2} \right]$$

$$B_{axis} = \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - l^2)^2}, \text{ where } M = m(2l)$$

If magnet is short $r \gg l$, then $B_{axis} = \frac{\mu_0}{4\pi} \frac{2M}{r^3}$

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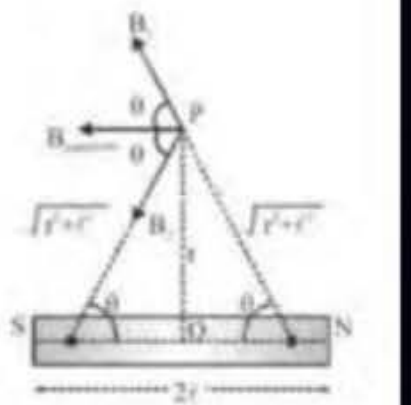
(ii) **At equatorial position :-**

Magnetic field at point 'P' due to north pole :-

$$B_1 = \frac{\mu_0}{4\pi} \frac{m}{(\sqrt{r^2 + l^2})^2} \dots\dots (1) \quad \text{(along NP line)}$$

Magnetic field at point 'P' due to south pole :-

$$B_2 = \frac{\mu_0}{4\pi} \frac{m}{(\sqrt{r^2 + l^2})^2} \dots\dots (2) \quad \text{(along PS line)}$$



From equation (1) & (2) $B_1 = B_2 = \frac{\mu_0}{4\pi} \frac{m}{r^2 + l^2} = B$ (Let)

Net magnetic field at point 'P'

$$B_{eq} = 2 B \cos\theta = 2 \cdot \frac{\mu_0}{4\pi} \frac{m}{r^2 + l^2} \cos\theta, \quad \text{[where } \cos\theta = \frac{l}{\sqrt{r^2 + l^2}} \text{]}$$



Home work

- PYQ attached

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THANK
YOU
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