

PRAYAS

JEE 2025

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Lecture - 07

Physics

Capacitor

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Topics *to be covered*

1 RC Circuit

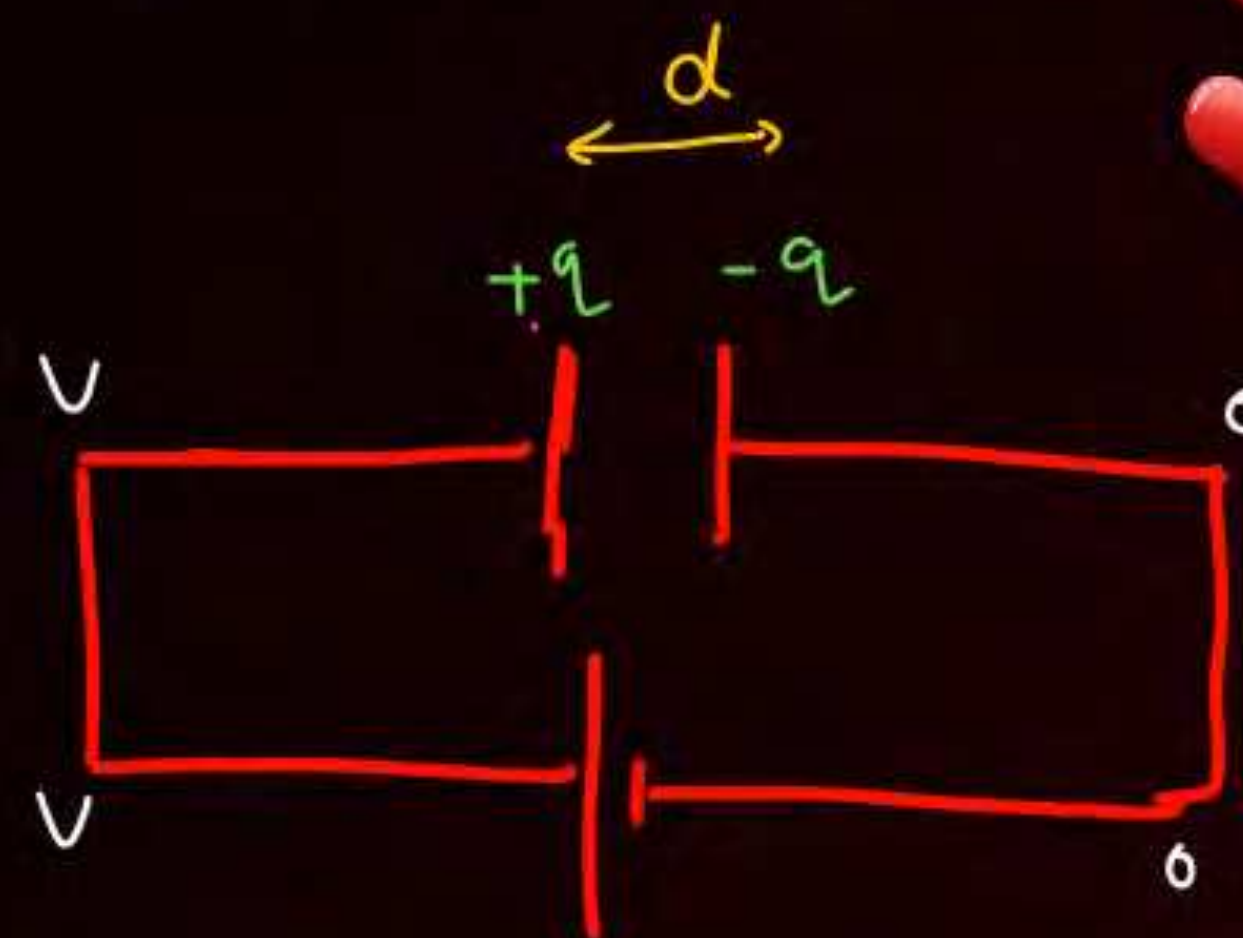
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2

3

4

Battery is Connected



$$C = \frac{A\epsilon_0}{d}$$

$$q = CV$$

$$U = \frac{1}{2} CV^2$$

$$E = \frac{V}{d}$$

$d \longrightarrow \text{half}, \frac{d}{2}$

$C \longrightarrow 2C$

$V \longrightarrow \text{Same}$

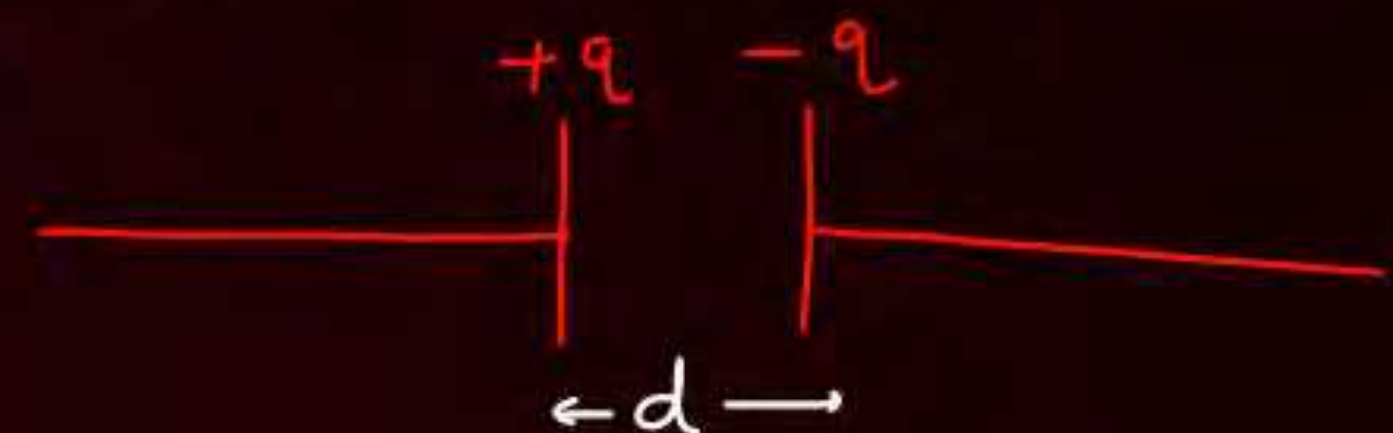
$q \longrightarrow 2q$

$U \longrightarrow 2U$

$E \longrightarrow 2E$

$$(W)_{\text{battery}} = (2q - q)E$$

Battery is Dis-Connected



$$q = CV$$

$$U = \frac{1}{2} \frac{q^2}{C}$$

$q \longrightarrow \text{Same}$

$d \longrightarrow \frac{d}{2}$

$C \longrightarrow 2C$

$V \longrightarrow \text{half}$

$U \longrightarrow \frac{U}{2}$

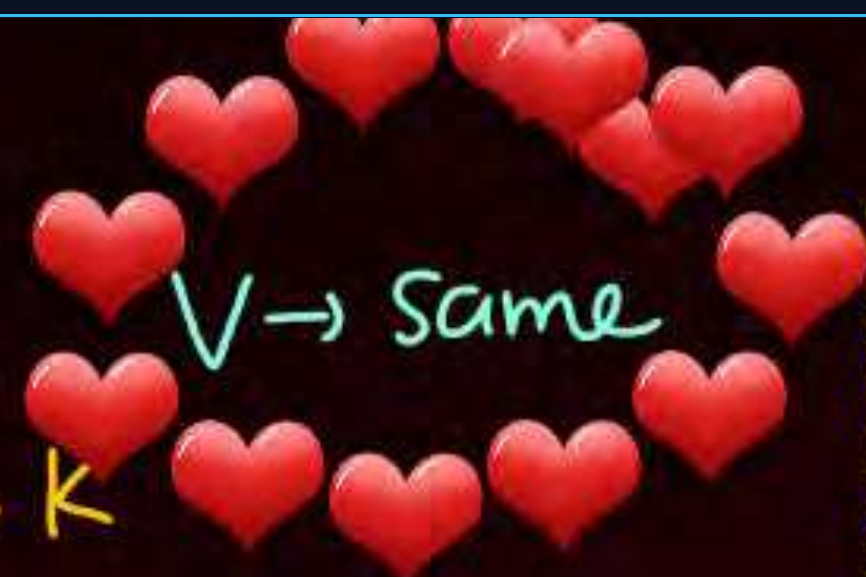
$E \longrightarrow \text{Same}$

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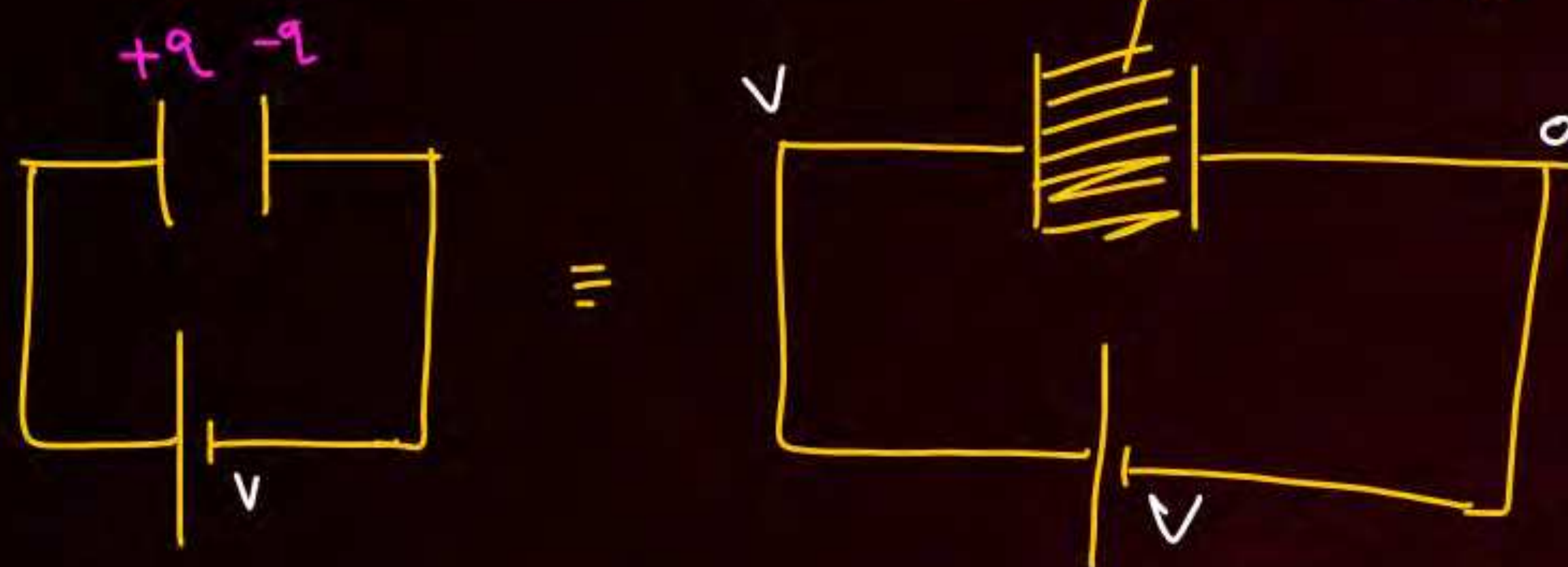




Battery is Connected



$V \rightarrow \text{Same}$



$C \rightarrow KC$

$V \rightarrow \text{Same}$

$q \rightarrow Kq$

$q_i = q \rightarrow q_f = Kq$

$$C = \frac{A\epsilon_0}{d}$$

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \frac{q^2}{C}$$

$$q = CV$$

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(wo) by battery $= (Kq - q)E = qE(K - 1)$

$E \rightarrow \text{Same}$

$V \rightarrow KV$

Force on one plate $\rightarrow K^2(\text{times})$

Battery is disconnected



$C \rightarrow KC$

$q \rightarrow \text{Same}$

$V \rightarrow \frac{V}{K}$

$U \rightarrow \frac{U}{K}$

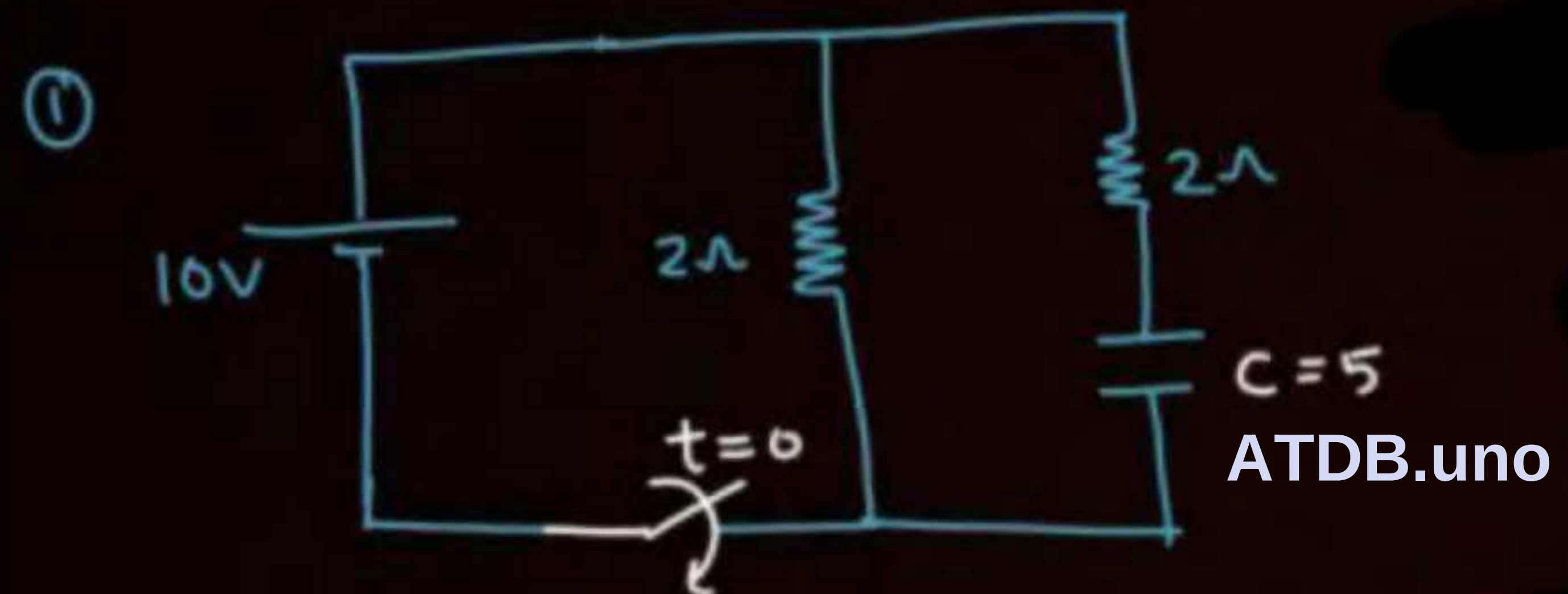
$E \rightarrow \frac{E}{K}$

Force on one plate $\rightarrow \text{Same}$

$$U = \frac{1}{2} \frac{q^2}{C}$$

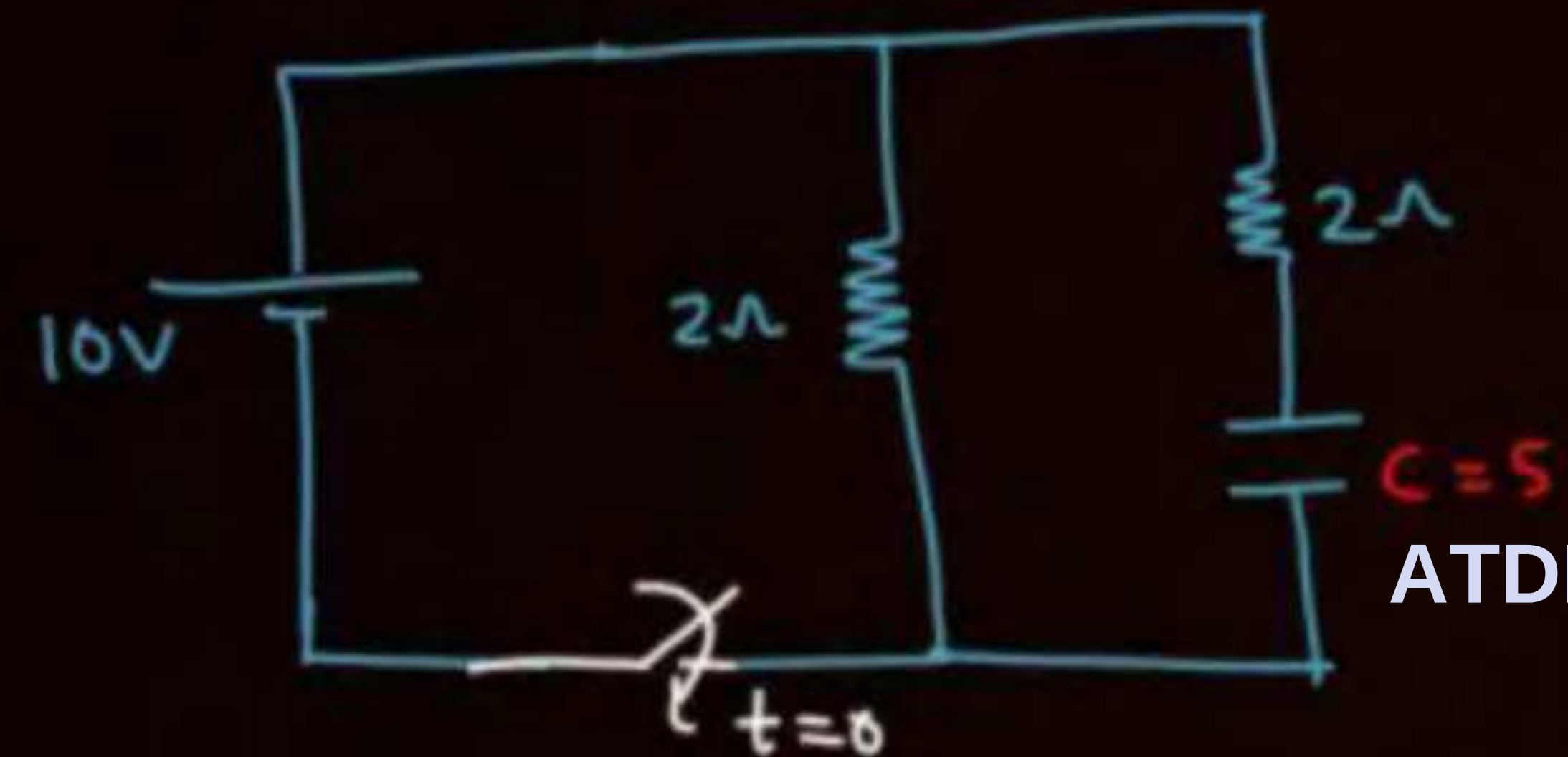


Q Find current in each resistance at $t=0$ & $t=\infty$



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⑥ Find charge on capacitor at $t = \infty$



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$t = \infty$



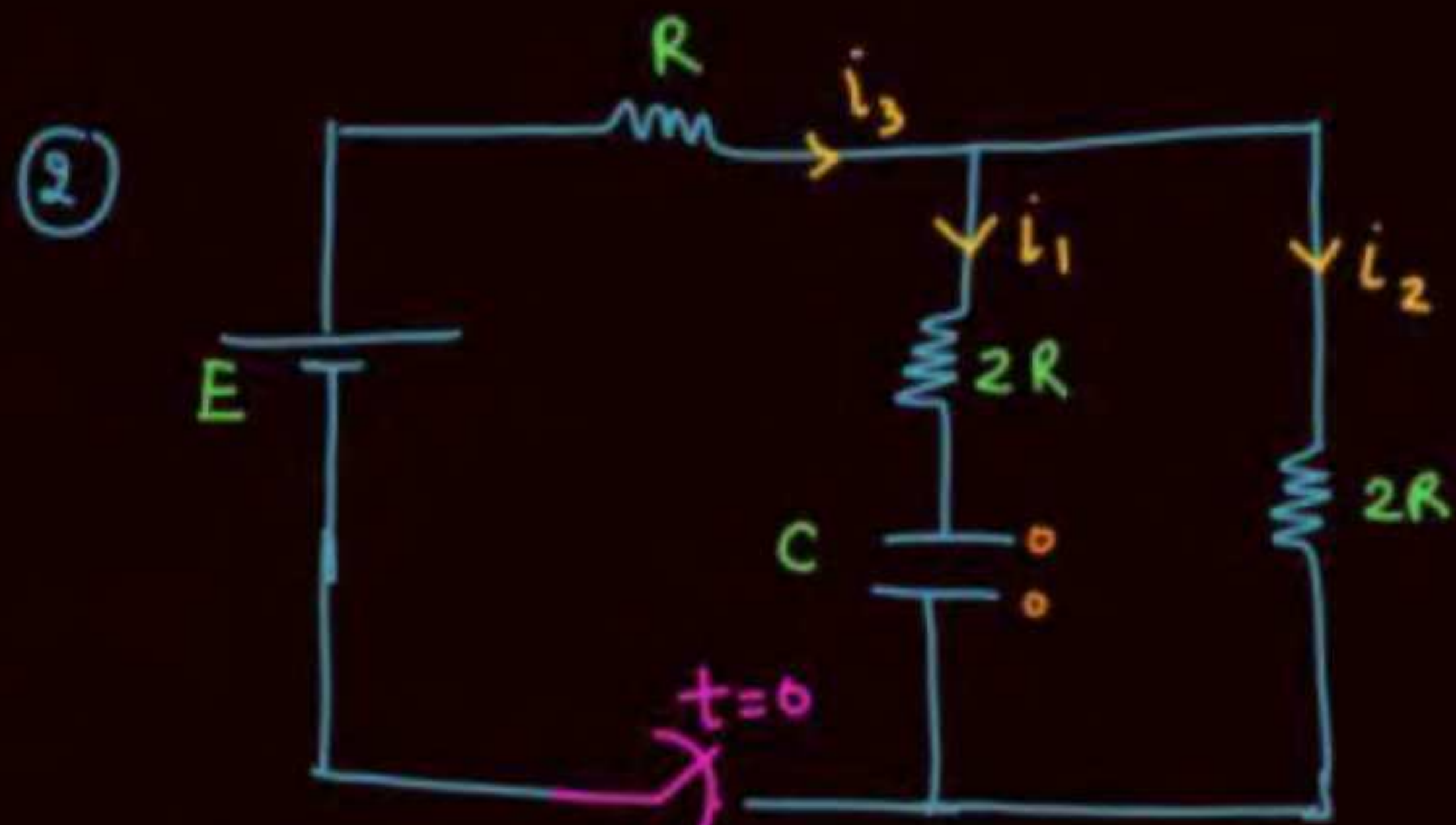
$t = \infty \rightarrow$ After very long time
 $\rightarrow$ At steady state

यहाँ जो Cap है

$$q = CV$$

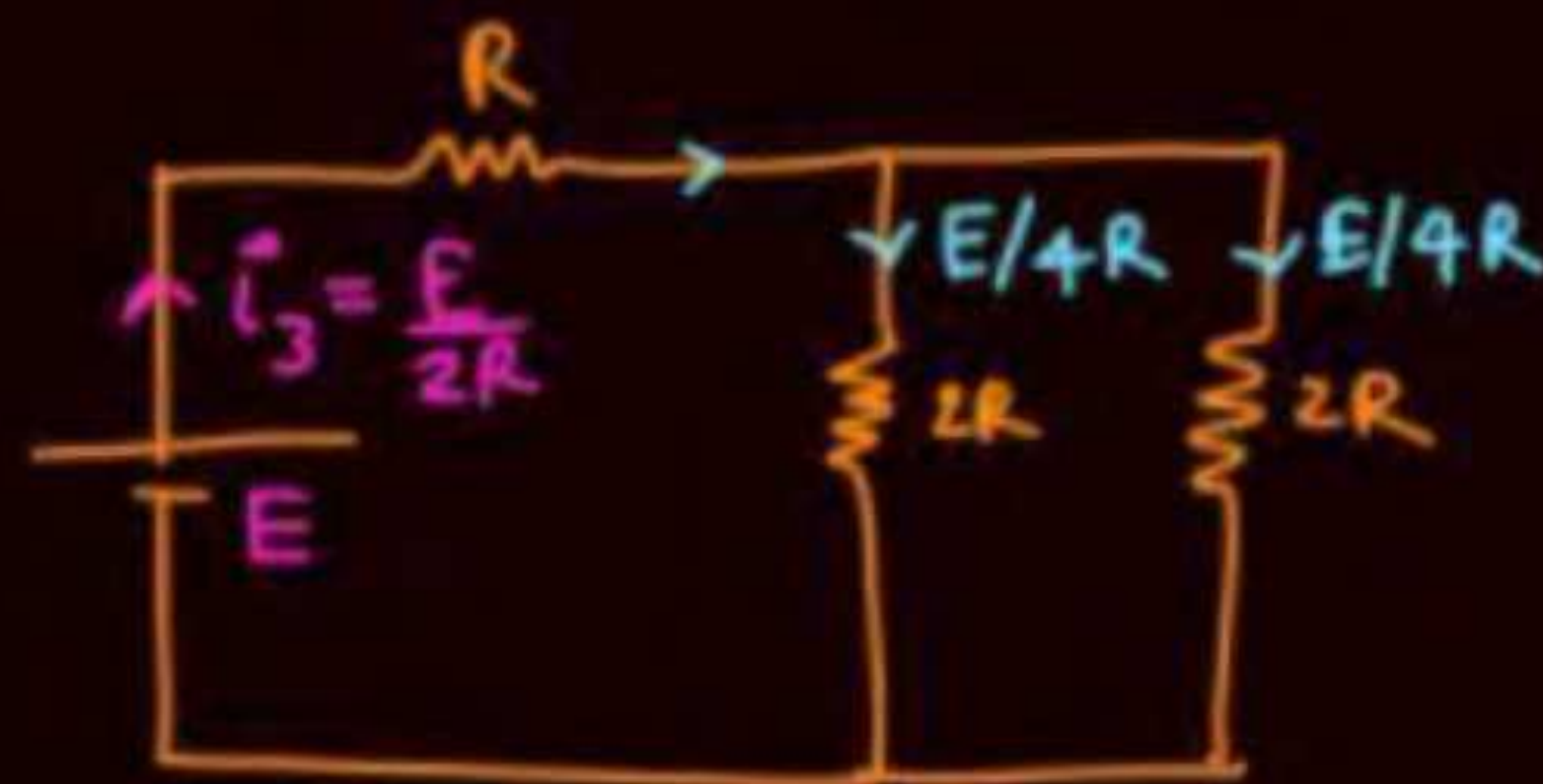
$$= 5 \times 10 = 50$$



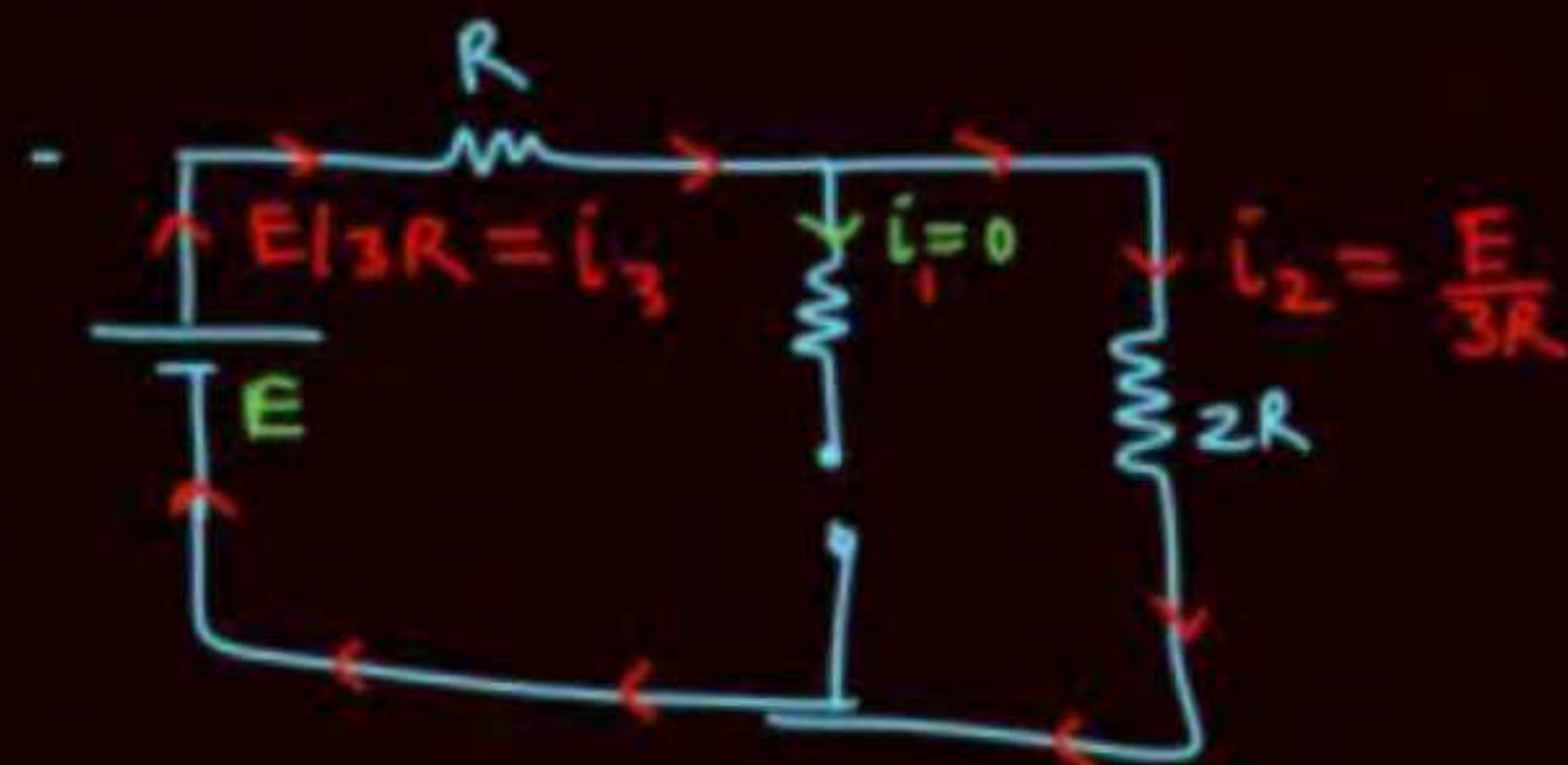


find i_1, i_2, i_3, q on cap at $t=0$ & $t=\infty$

$t=0$



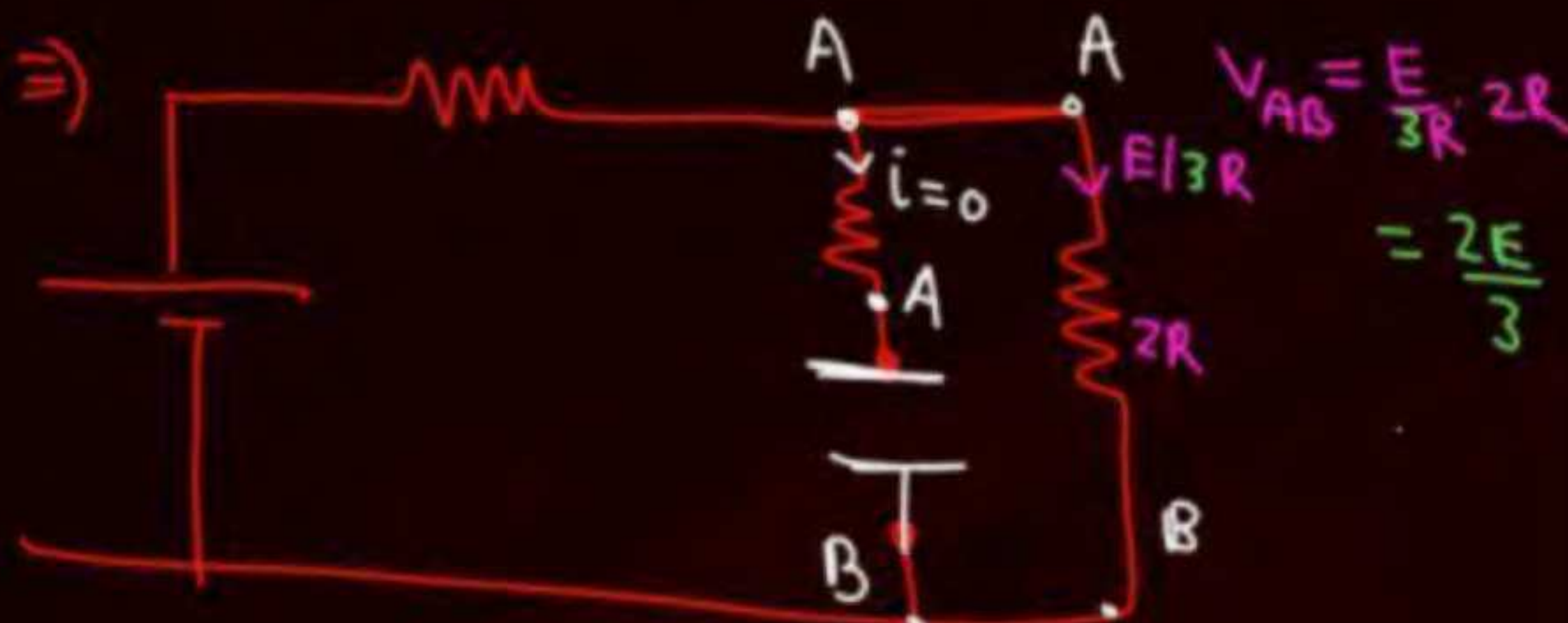
$t=\infty$

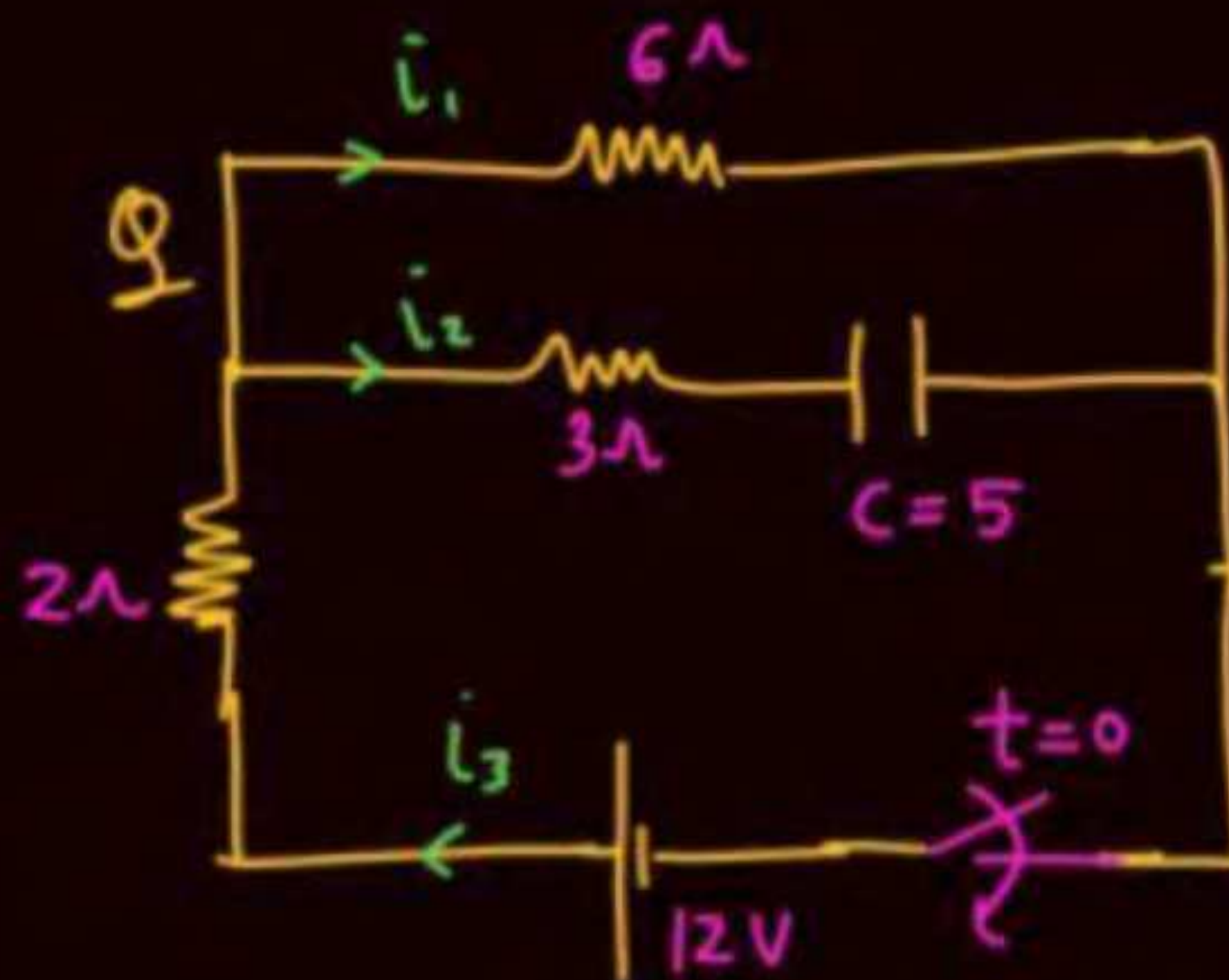


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$t=\infty$ or change on Capacitor $\Rightarrow$

$$q = CV_{AB} = C \cdot \frac{2E}{3}$$

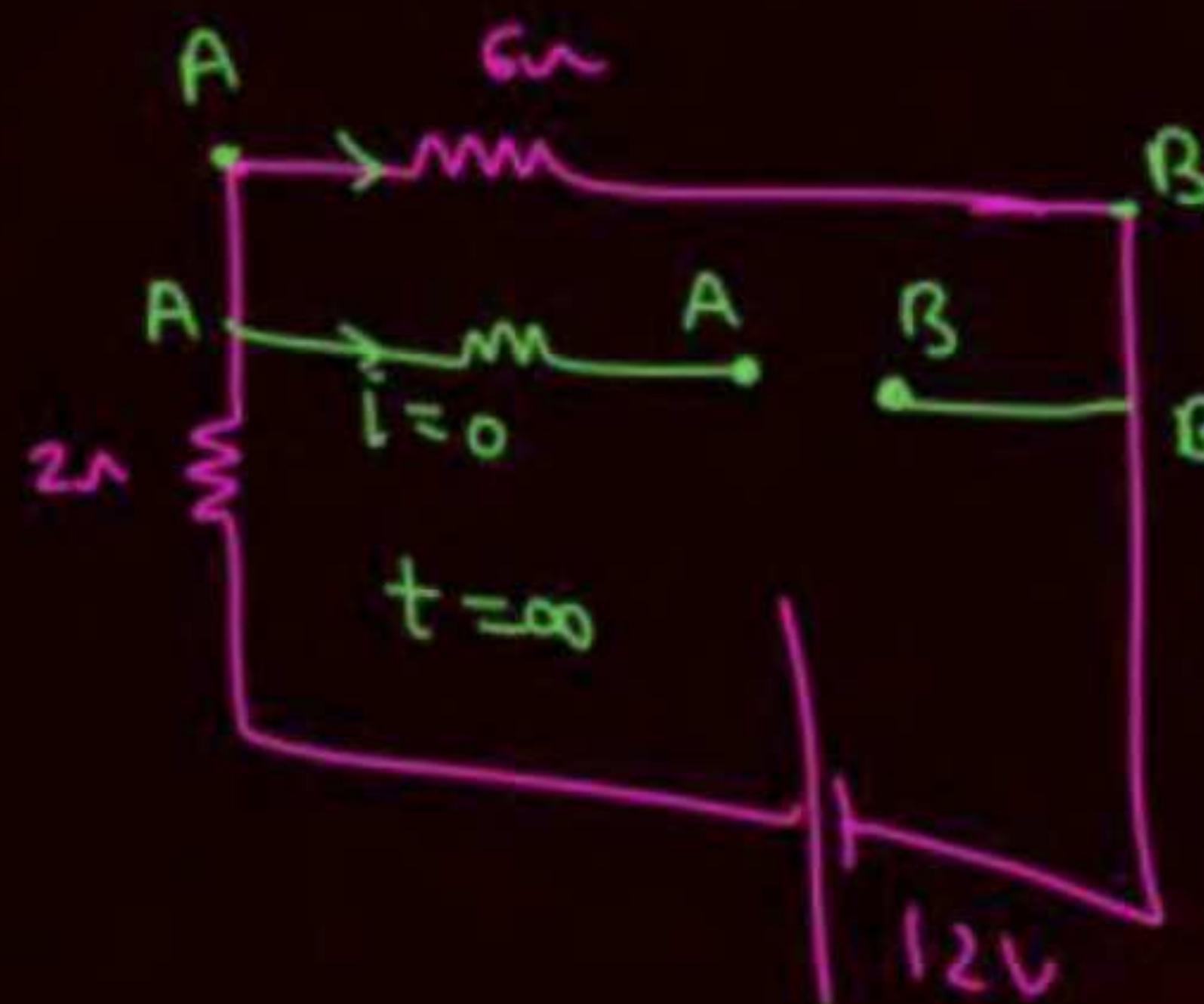




	i_1	i_2	$i_{net} = i_3$
$t=0$	1	2	3
$t=\infty$	$\frac{12}{8}$	0	$\frac{12}{8}$

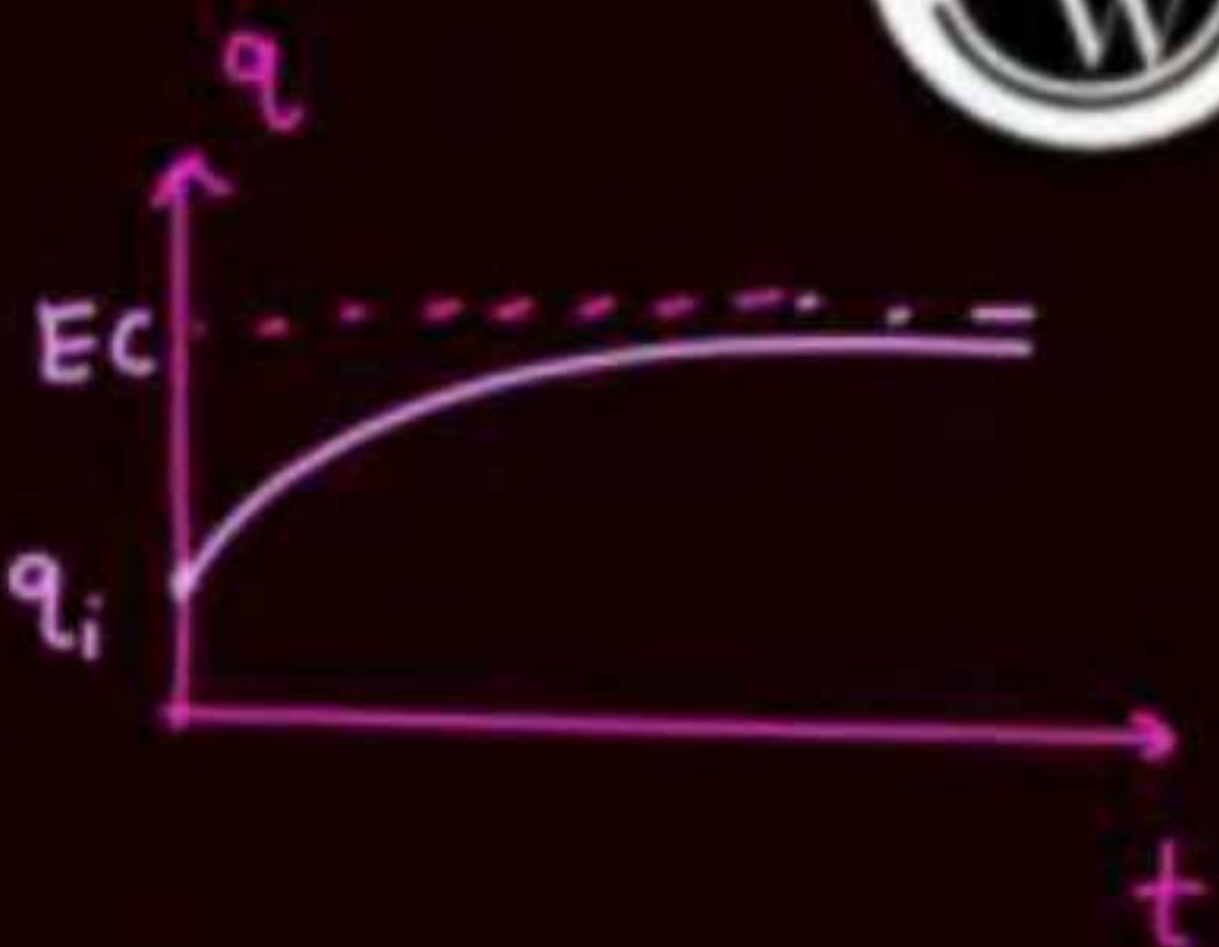
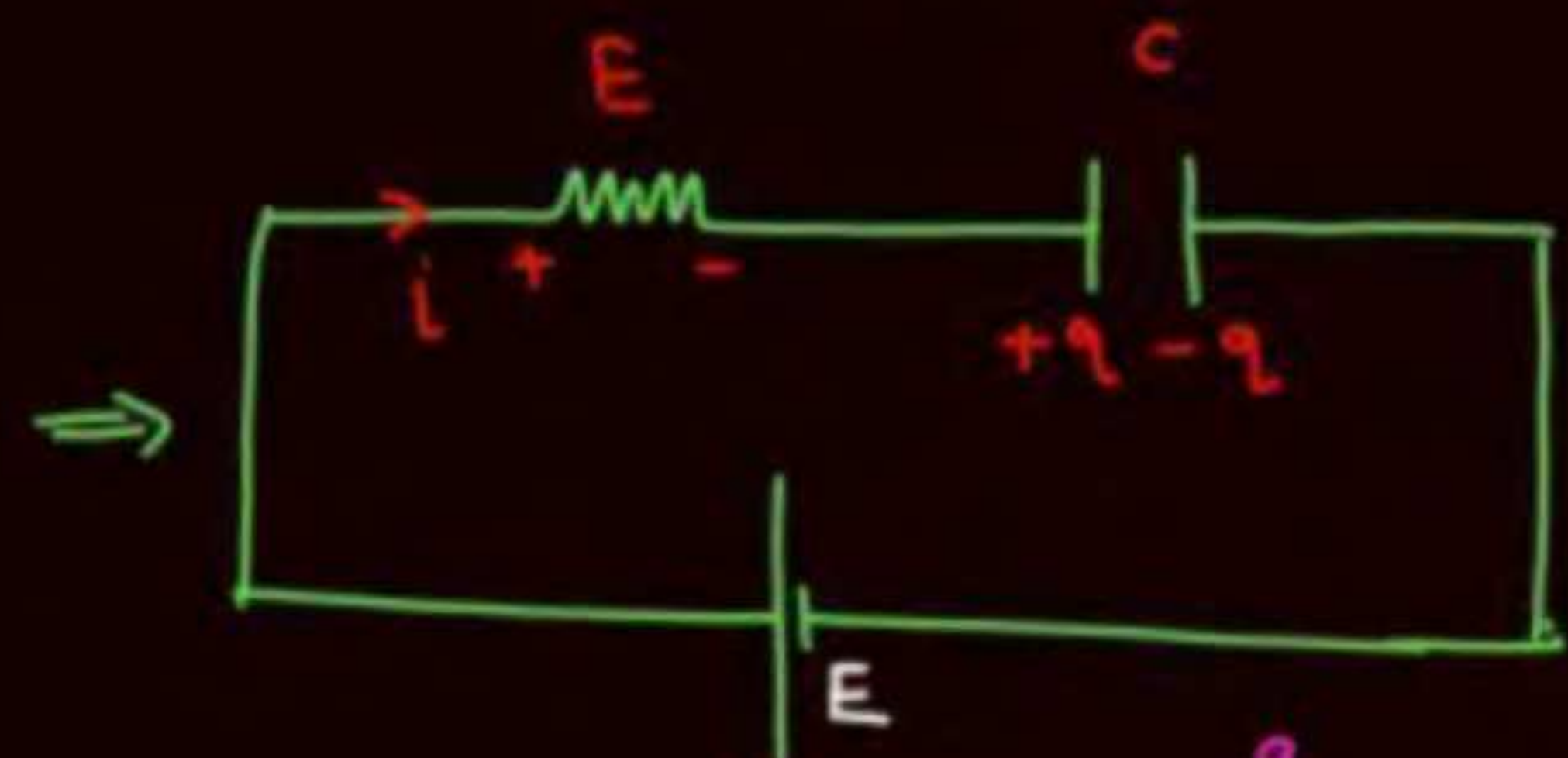
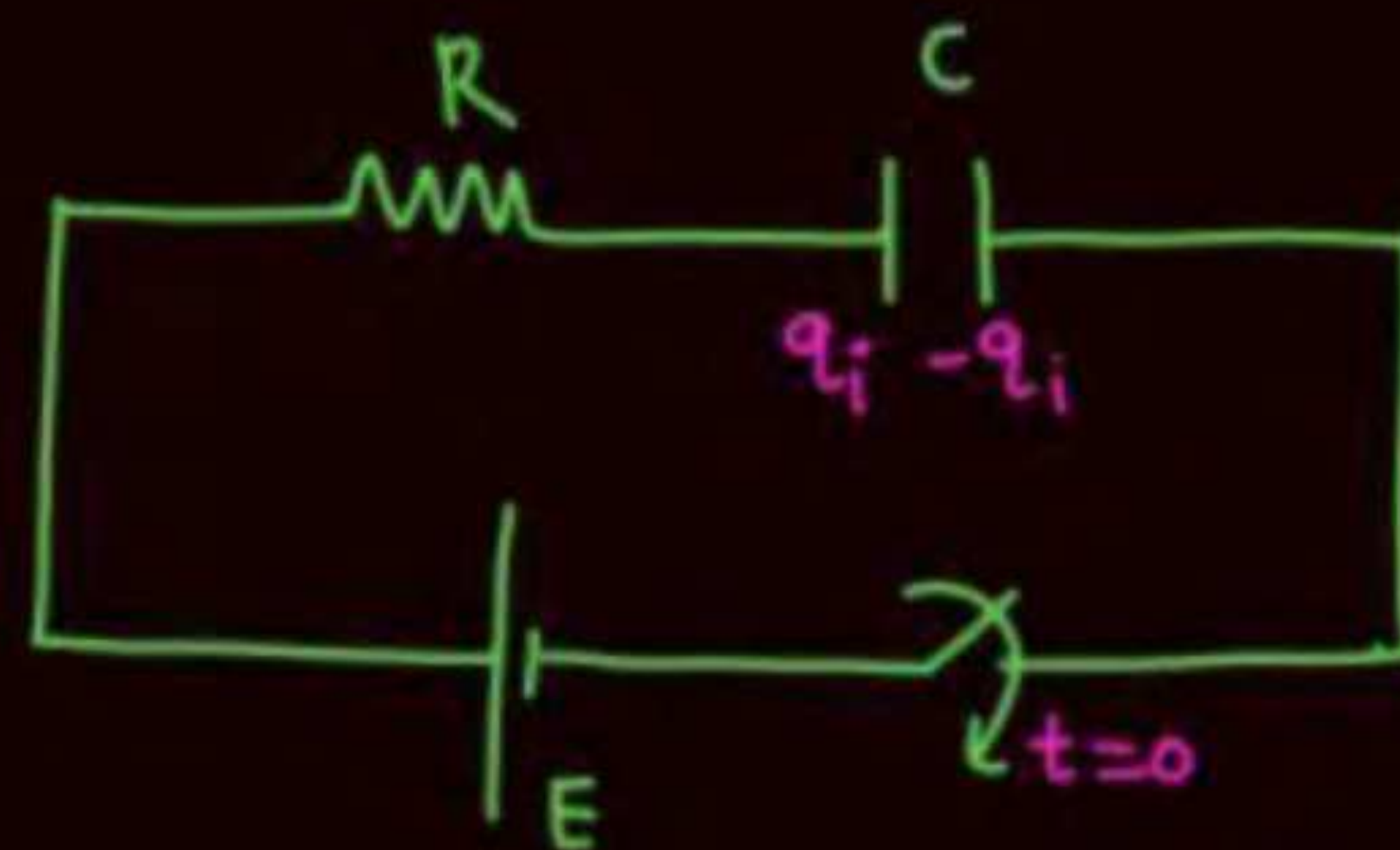
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Charge on cap. at $t=\infty$



$$q = 5 \times 9 = 45$$

$$V_{AB} = \frac{12}{8} \times 6 = 9$$



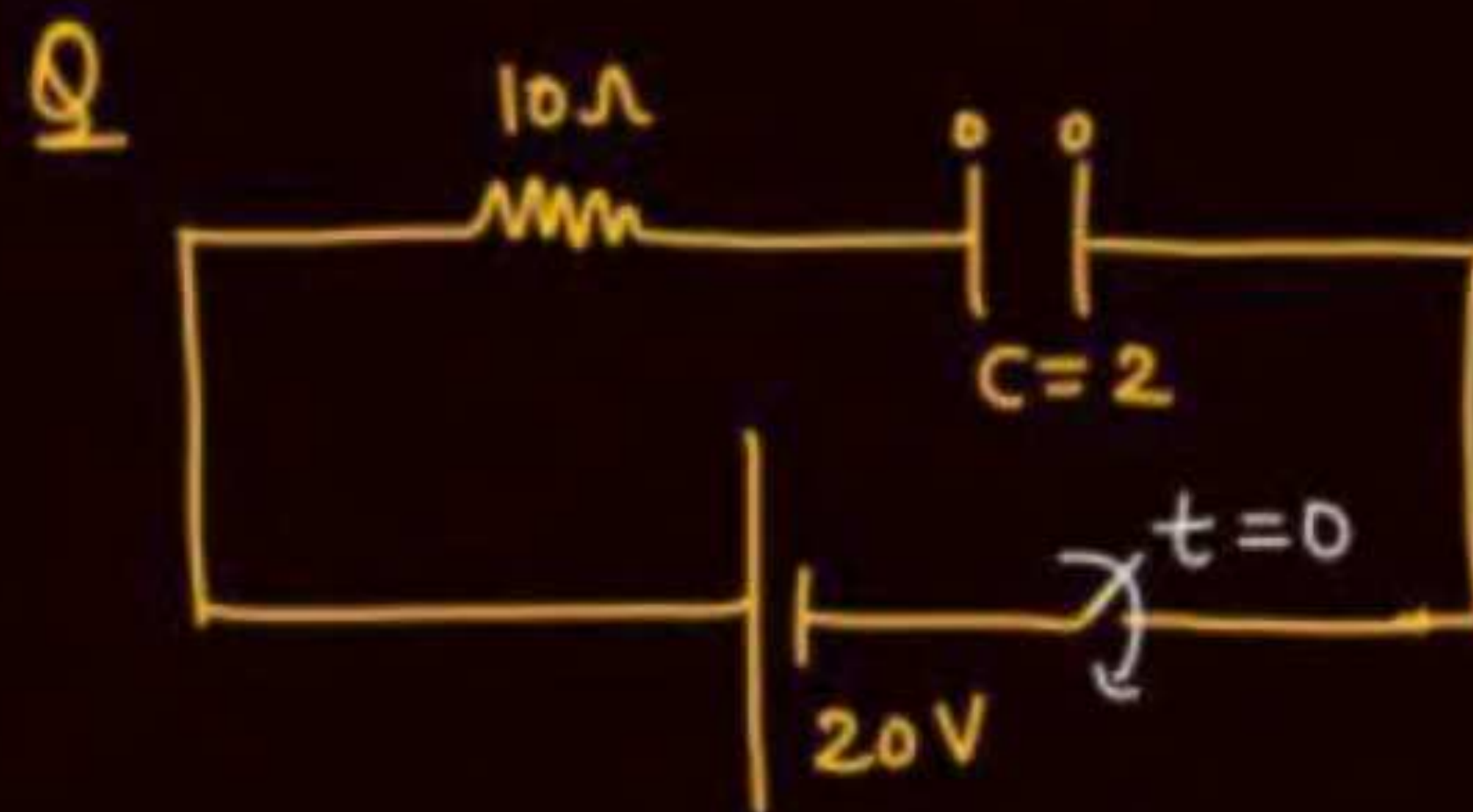
$$E - iR - \frac{1}{C} = 0$$

$$EC - iRC - q = 0$$

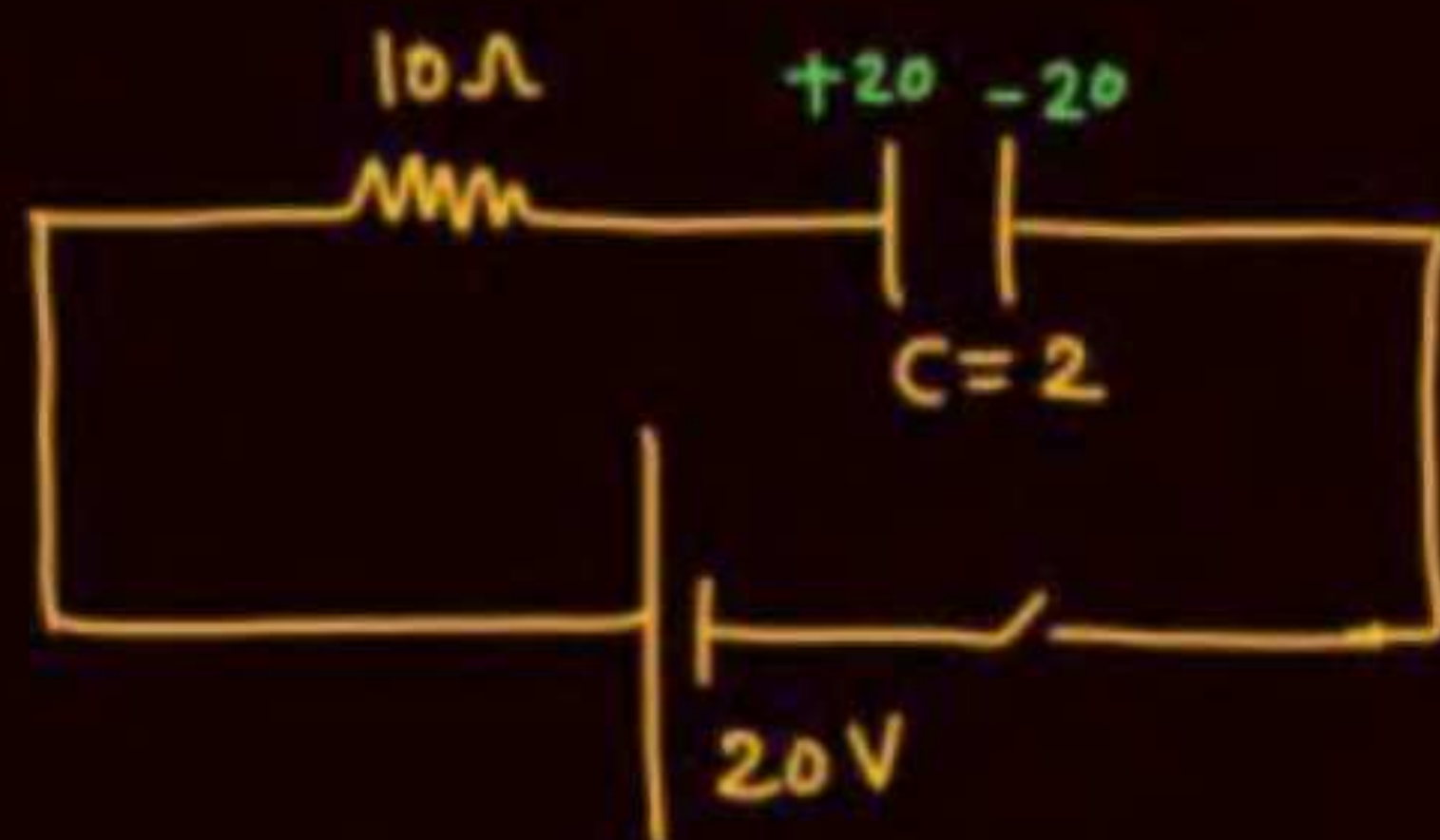
$$EC - q = \frac{dq}{dt} RC$$

$$\int_{q_i}^q \frac{dq}{EC - q} = \int_0^t \frac{dt}{RC}$$

$$q - q_i = (Ec - q_i) (1 - e^{-t/RC})$$



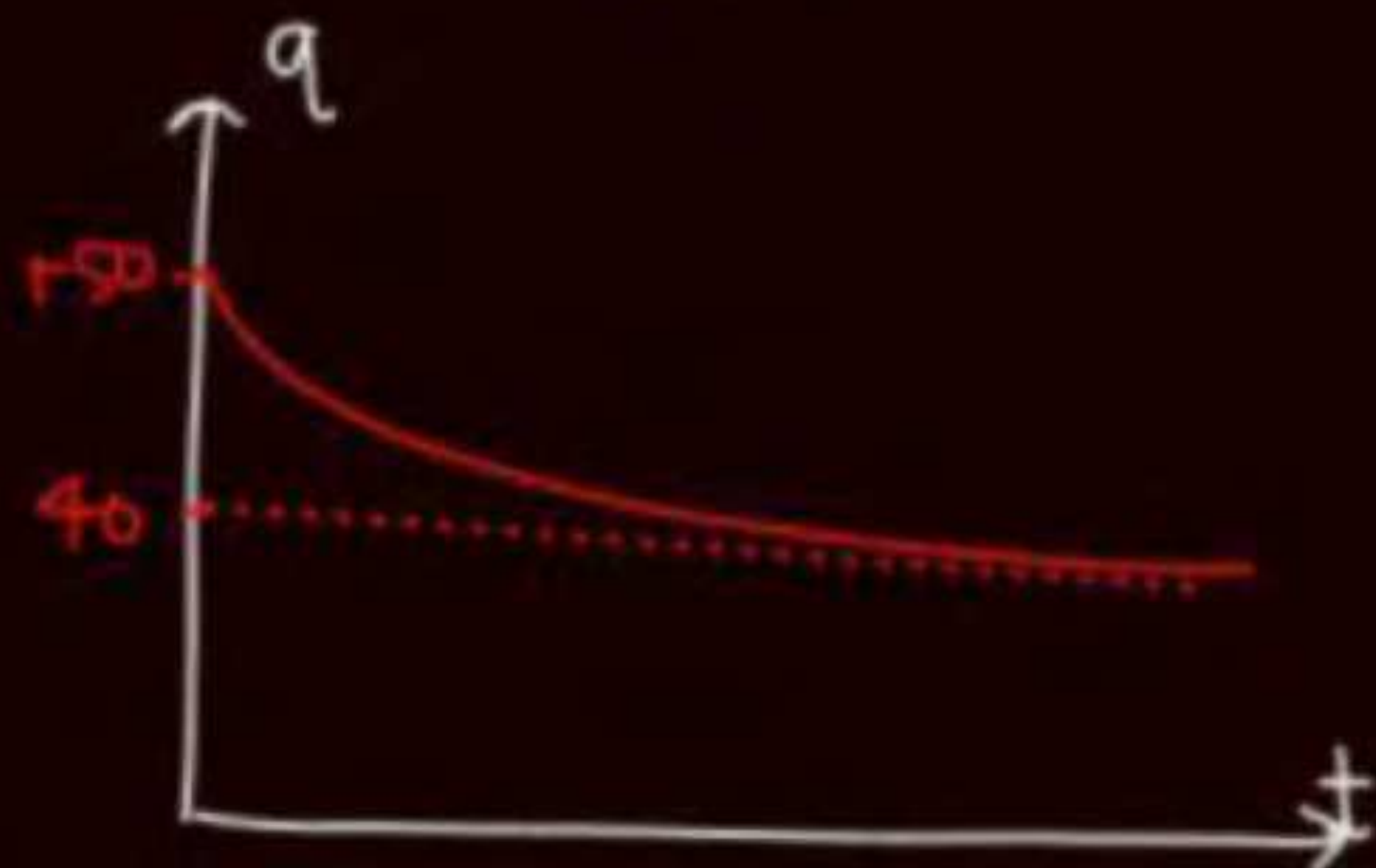
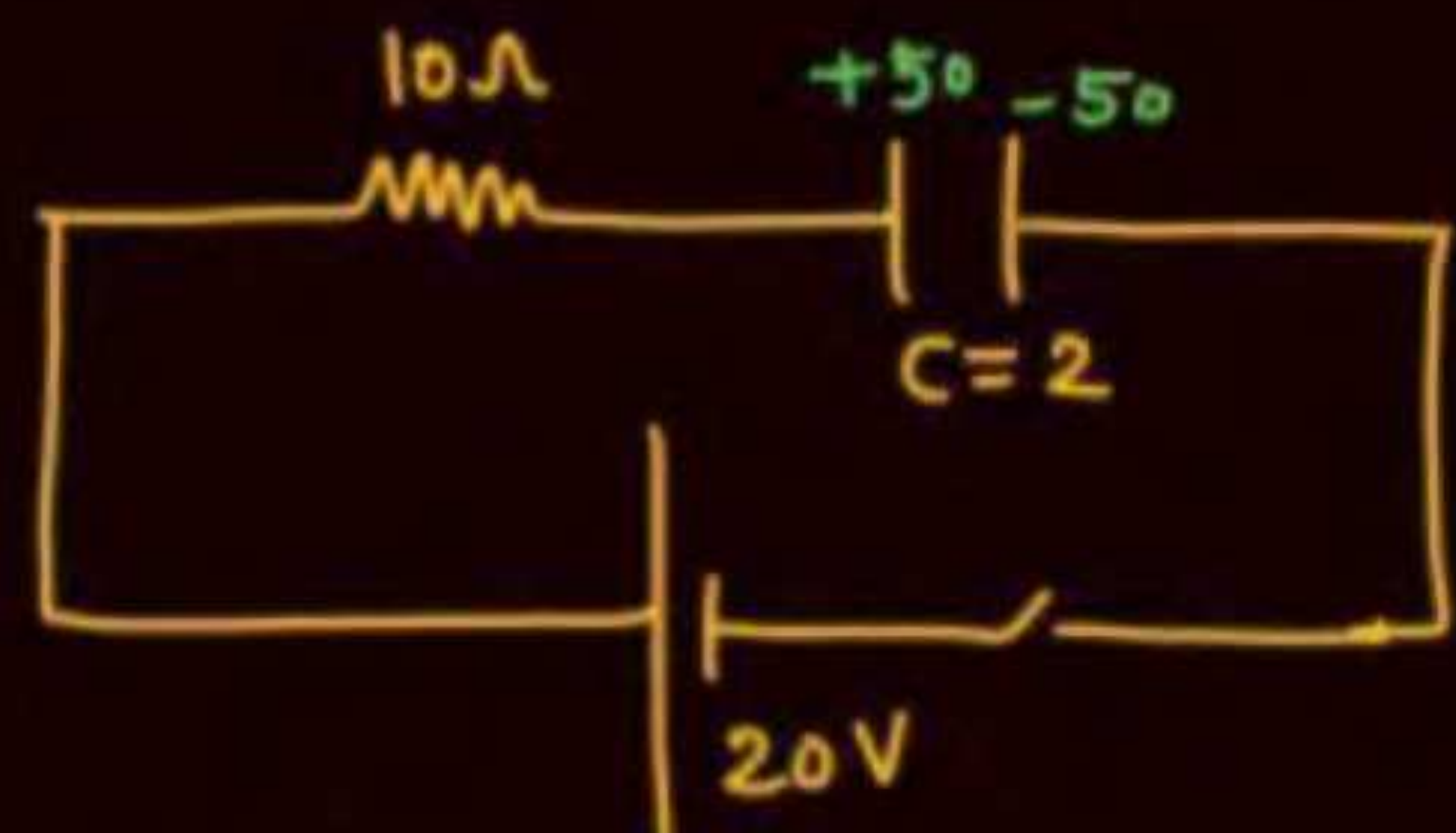
$$Q_{\max} = EC = 40$$



$$Q_{\max} = 40$$

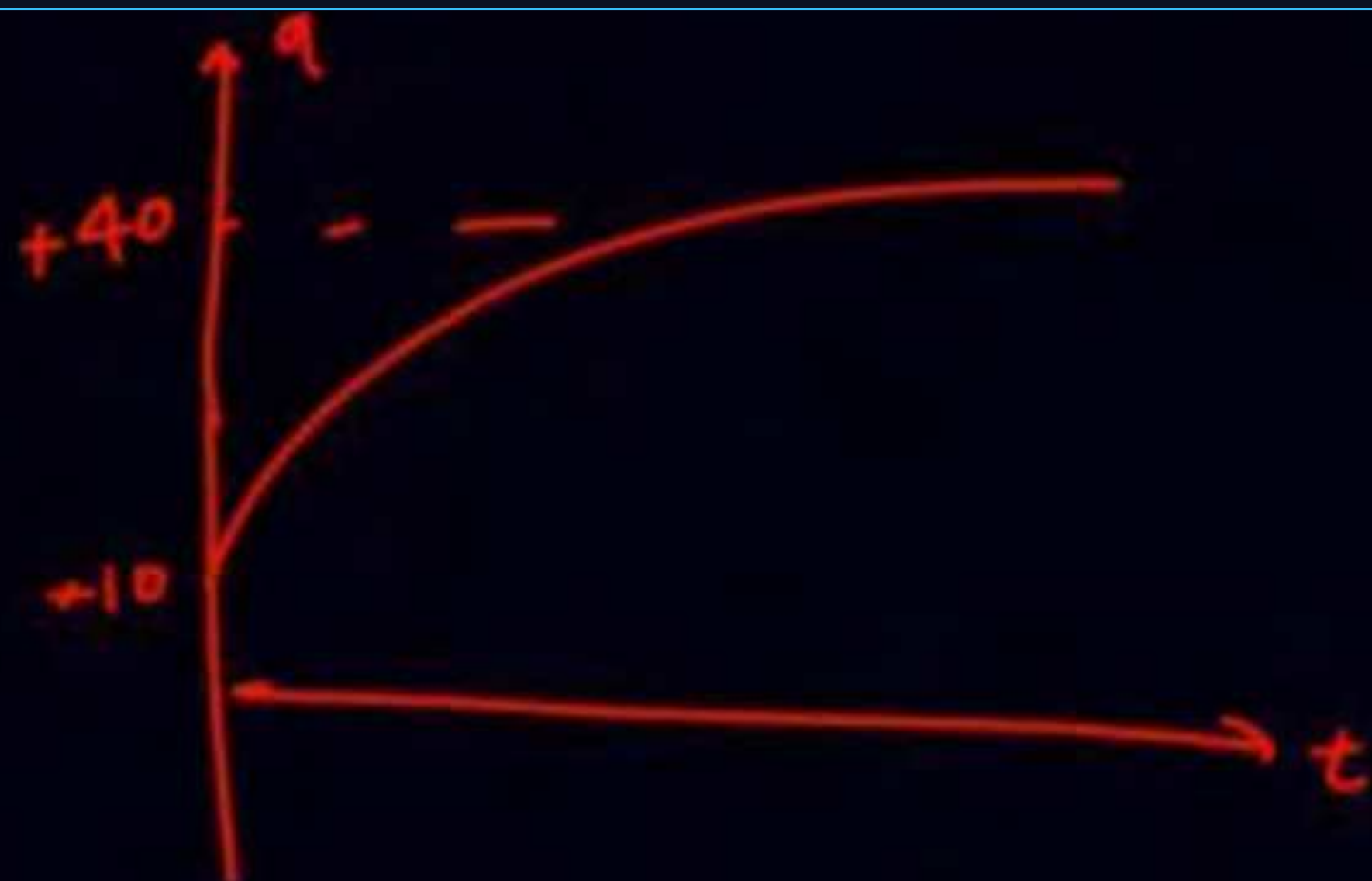


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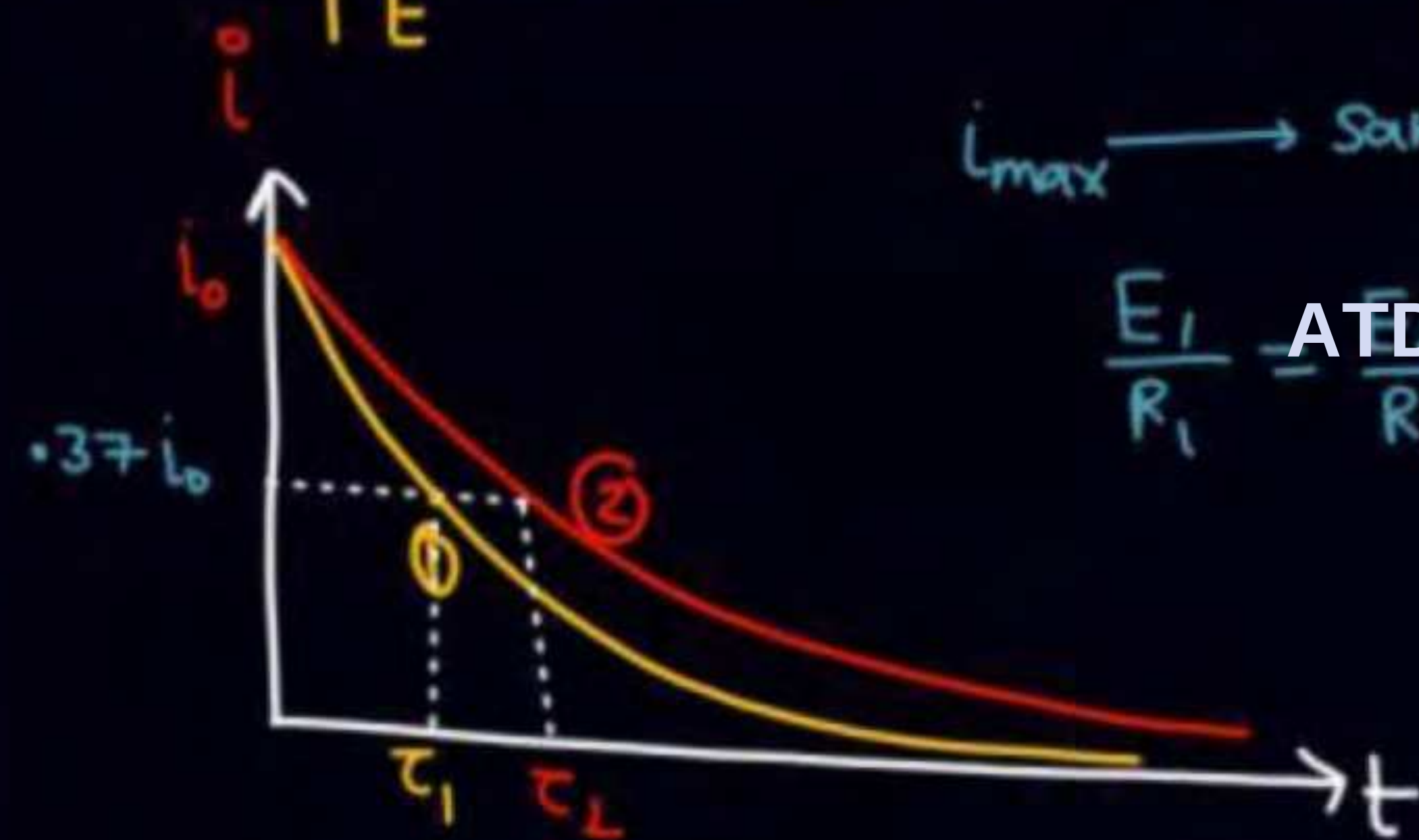
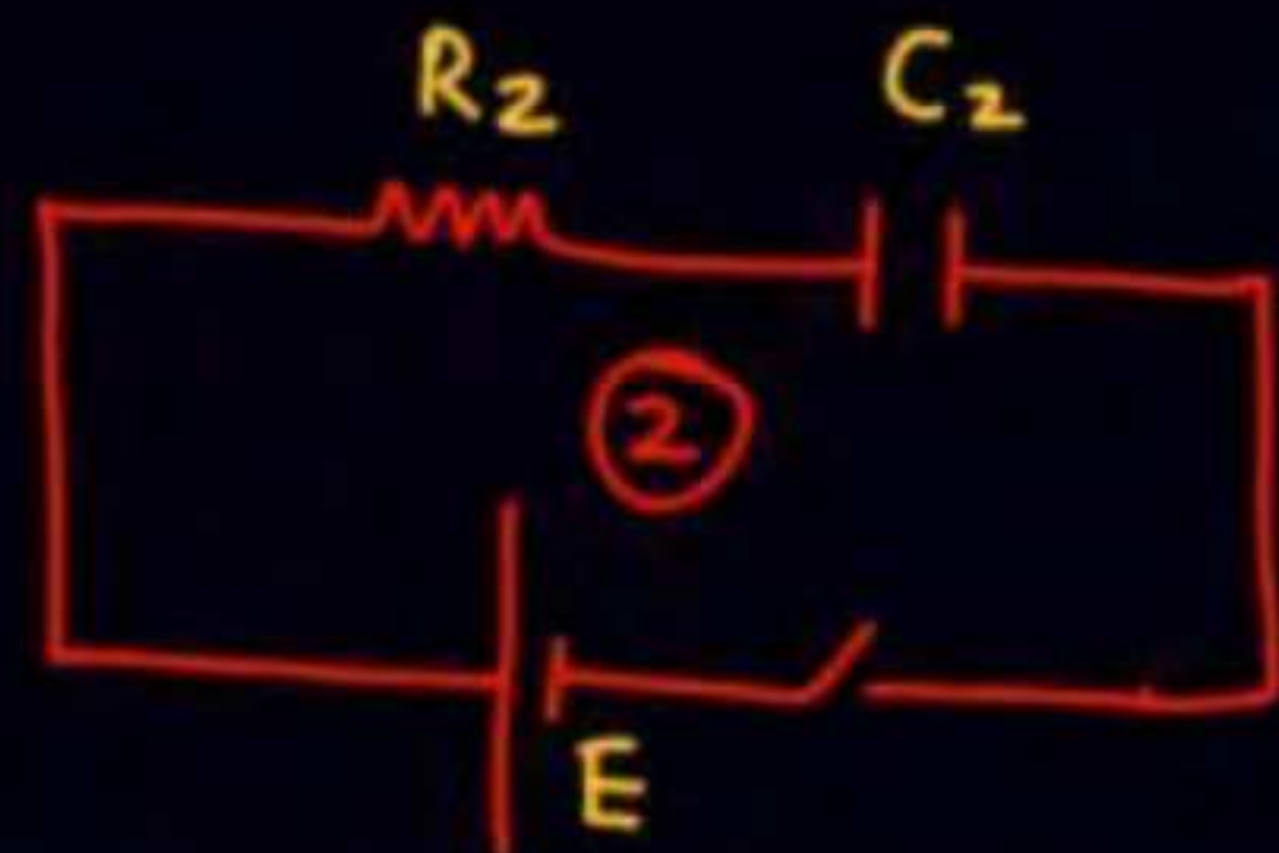




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$$q - 10 = (40 - 10) \left(1 - e^{-t/10}\right)$$



$$i_{\max} \rightarrow \text{Same} = \frac{E}{R}$$

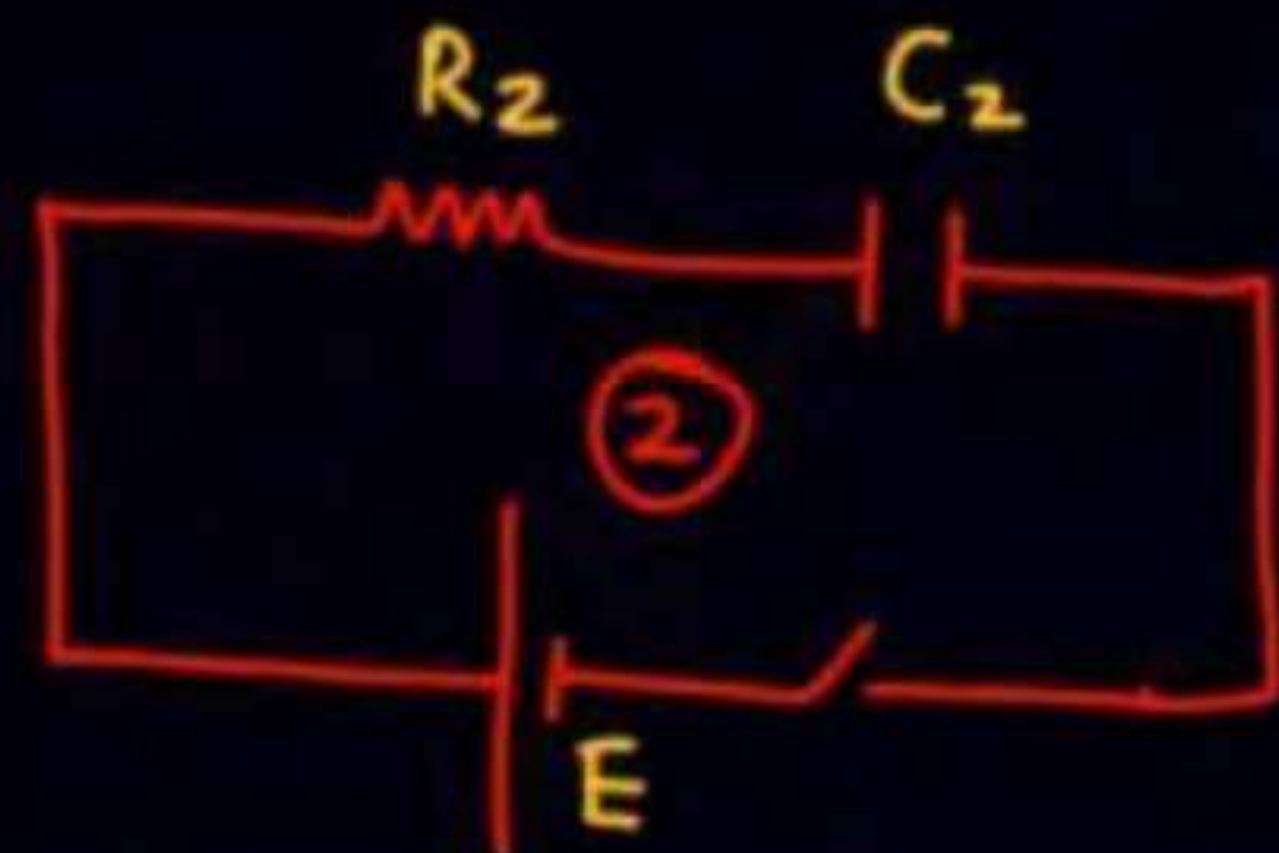
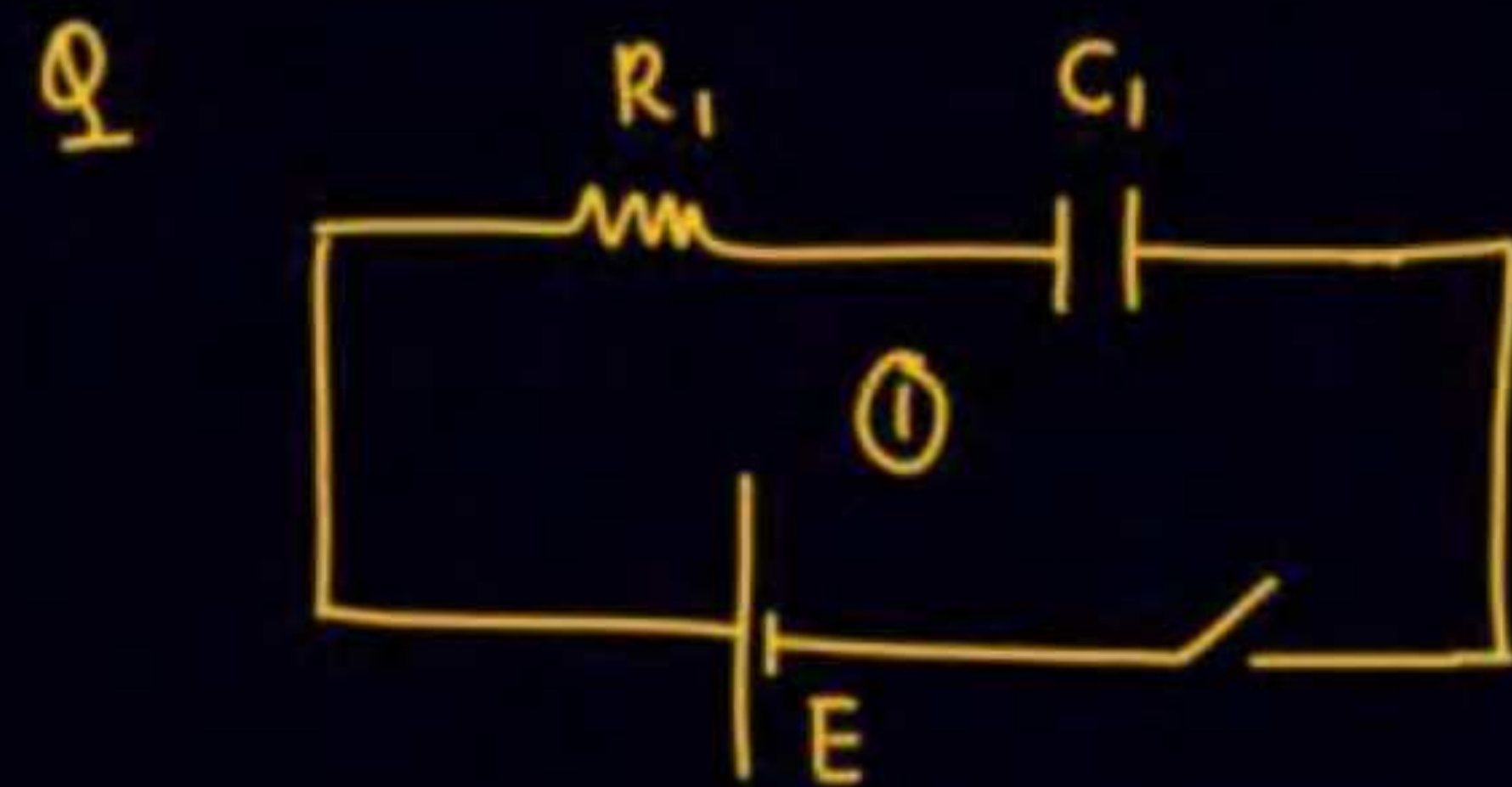
$$\frac{E_1}{R_1} = \frac{E_2}{R_2} \quad (E_1 = E_2)$$

$$R_1 = R_2$$

$$\tau_2 > \tau_1$$

$$R_2 C_2 > R_1 C_1$$

$$C_2 > C_1$$



$$i = \frac{E}{R} e^{-t/\tau}$$

$$\ln i = \ln \frac{E}{R} - \frac{t}{\tau}$$

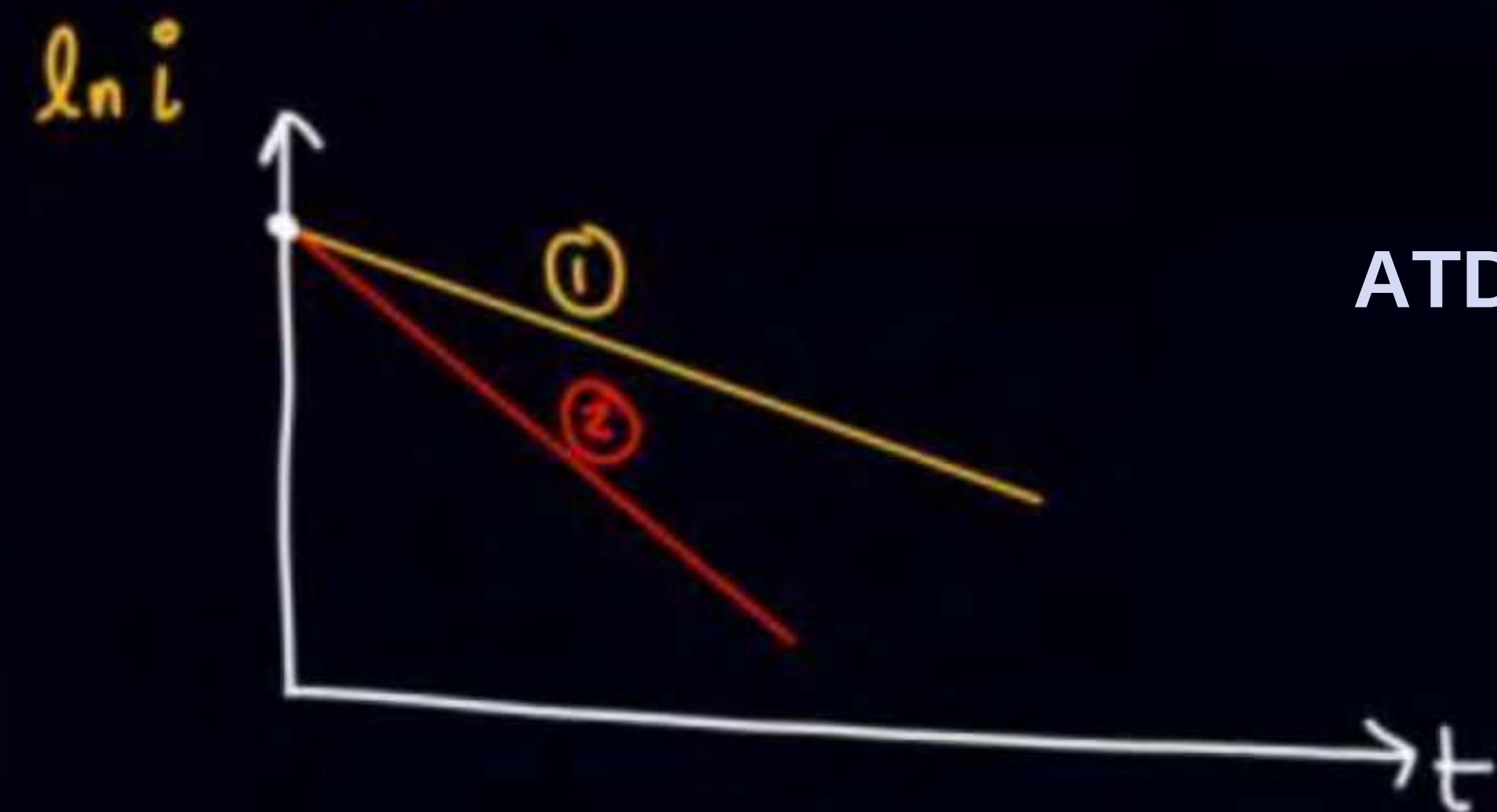
$$\ln i = -\frac{1}{\tau} t + \ln \frac{E}{R}$$

$$y = -mx + c$$

$$t=0, \ln i \rightarrow \text{same}$$

$$\frac{E}{R} \rightarrow \text{same}$$

$$\begin{aligned} (R_1 = R_2) \\ E_1 = E_2 \text{ (same)} \end{aligned}$$



$$\begin{aligned} |\text{slope}|_2 &> |\text{slope}|_1 \\ \left(\frac{1}{\tau_2}\right) &> \left(\frac{1}{\tau_1}\right) \end{aligned}$$

$$\begin{aligned} \tau_2 < \tau_1 \\ \boxed{C_2 < C_1} \end{aligned}$$

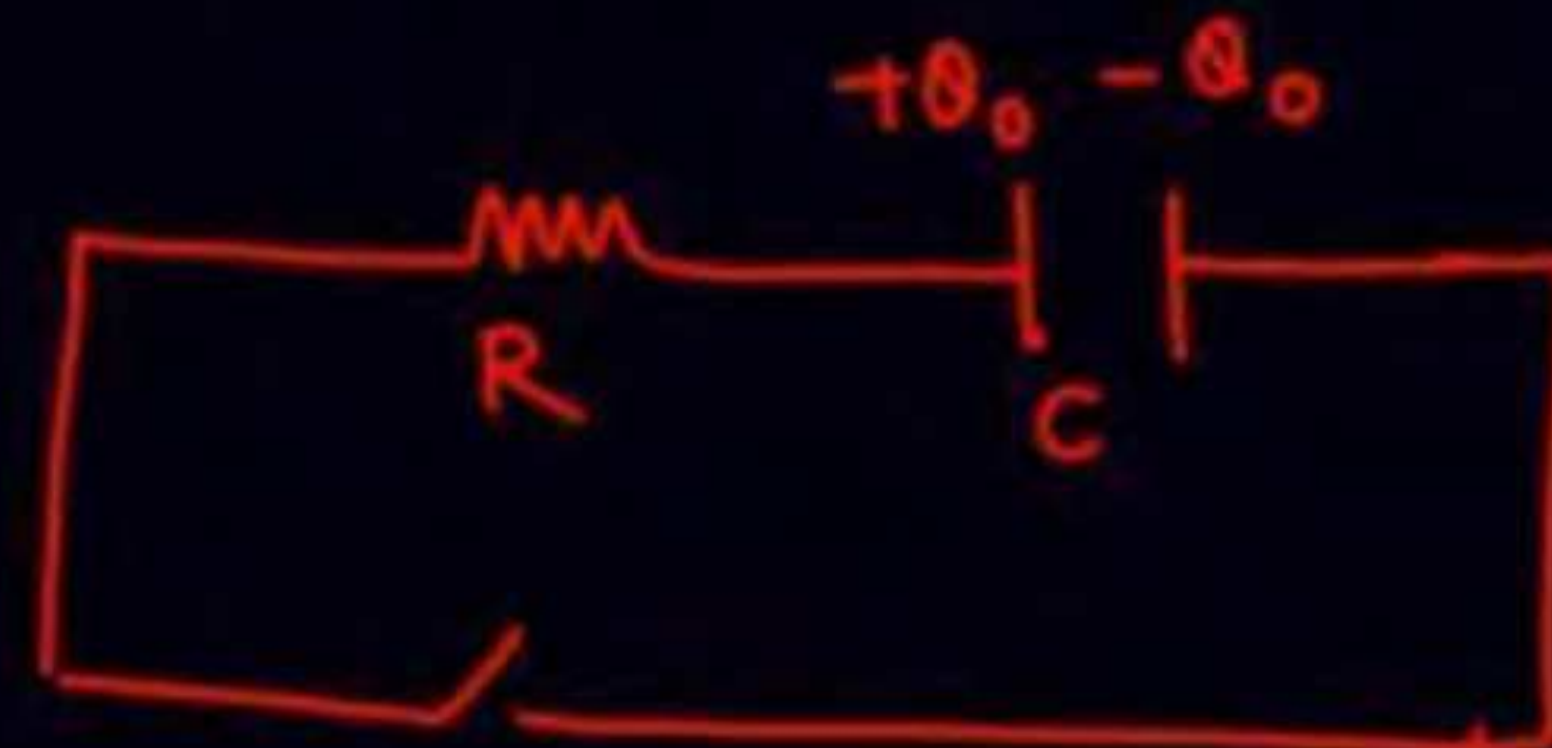


Discharging of Capacitor

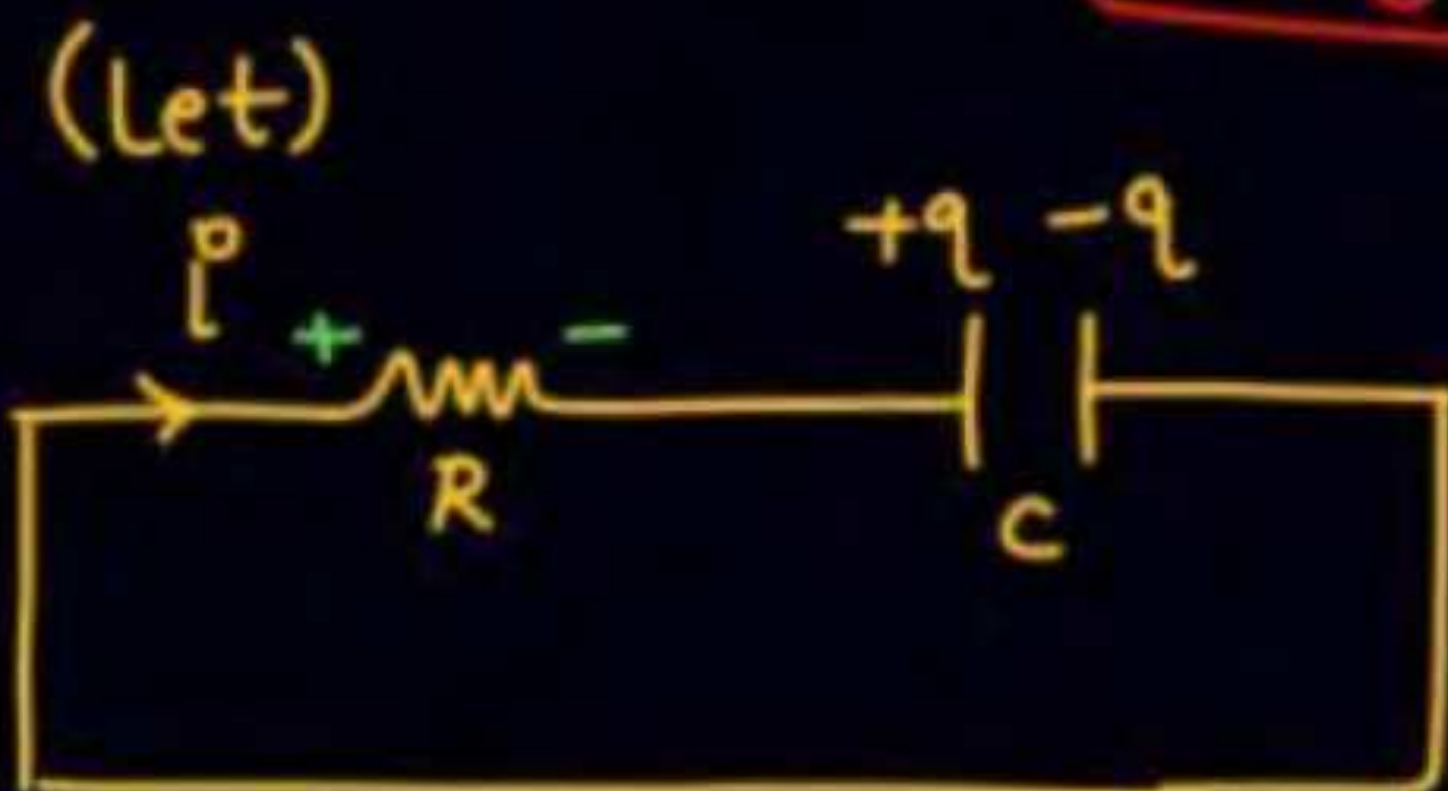
$$q = Q_0 e^{-t/RC}$$

$$V_C = V_R$$

$t=0$



$\Rightarrow$



$$i = \left| \frac{dq}{dt} \right| = \left| \frac{Q_0}{RC} e^{-t/RC} \right|$$

magnitude

find $q = f(t)$ on capacitor

$$q = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$$

$$i = i_0 e^{-t/RC}$$

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$$-iR - \frac{q}{C} = 0$$

$$iR = -\frac{q}{C}$$

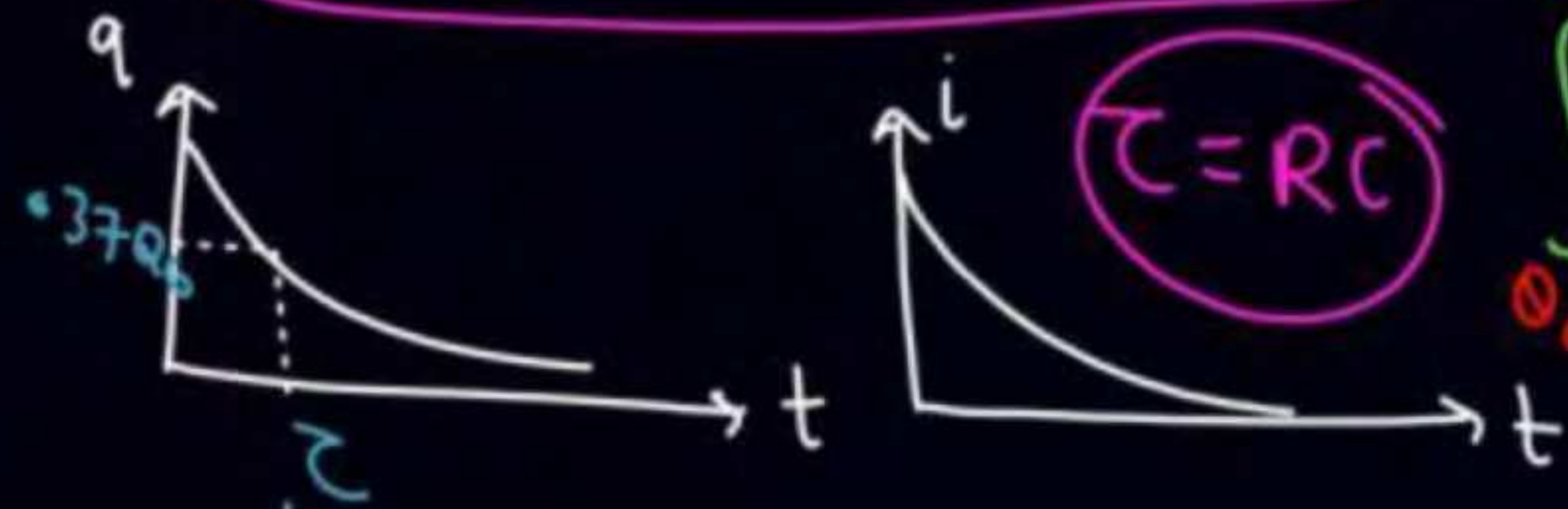
$$\frac{dq}{dt} = -\frac{q}{RC}$$

$$\int_{Q_0}^q \frac{dq}{q} = \int_0^t \frac{-dt}{RC}$$

$$\ln \frac{q}{Q_0} = -\frac{t}{RC}$$

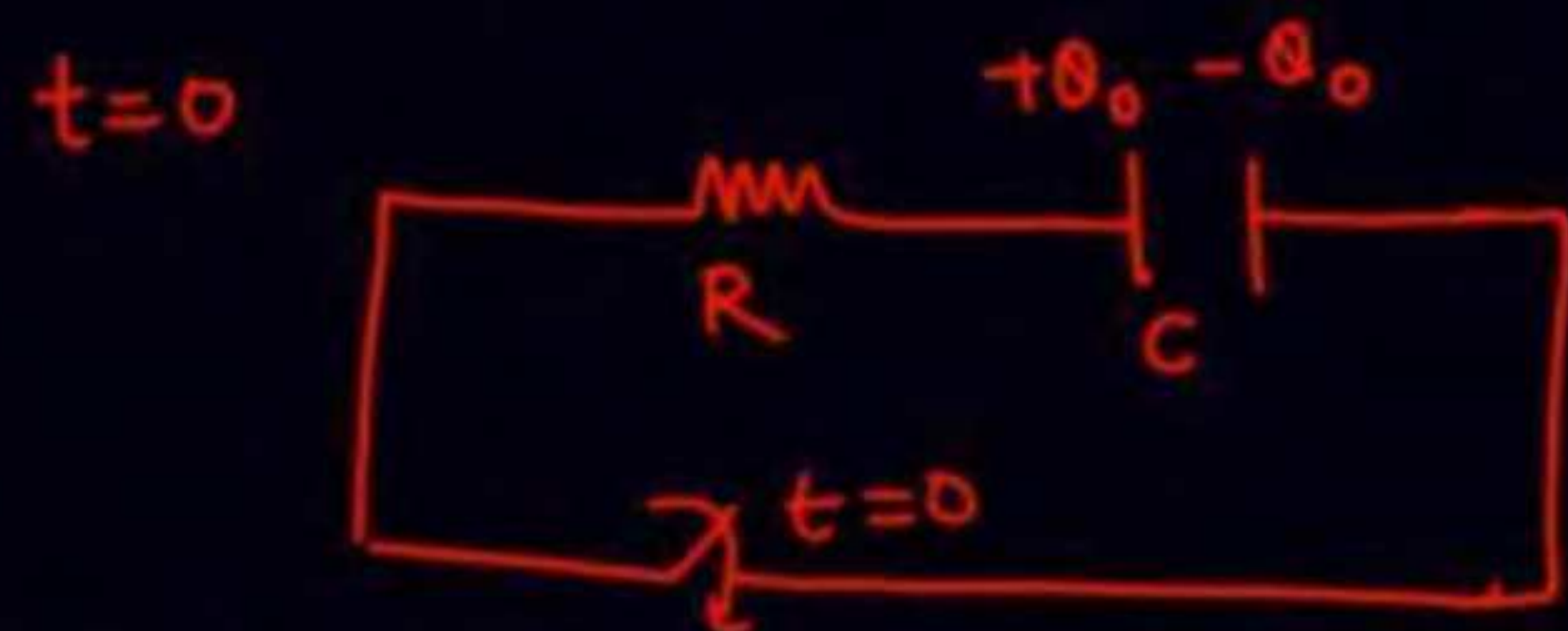
$$q = Q_0 e^{-t/RC}$$

$t \uparrow \Rightarrow q \downarrow, V \downarrow$

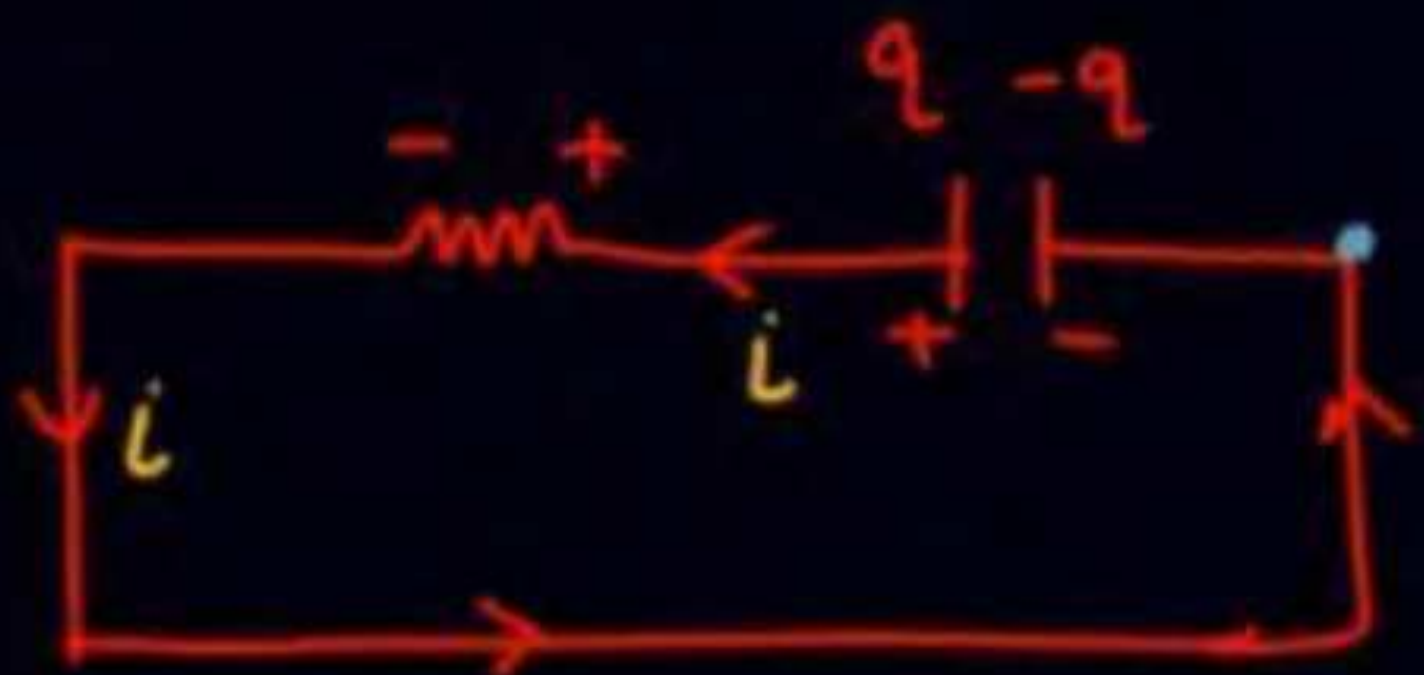




Discharging of Capacitor



find $q = f(t)$ on capacitor



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$$\frac{q}{C} - iR = 0$$

$$\frac{q}{C} = iR \quad i = \ominus \frac{dq}{dt}$$

$$\frac{q}{C} = -\frac{dq}{dt} \cdot R$$

$$\int_{Q_0}^q \frac{dq}{q} = \int_0^t -\frac{dt}{RC}$$

$$\ln \frac{q}{Q_0} = -\frac{t}{RC}$$

$$q = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$$



Discharging of Capacitor



find $q = f(t)$ on capacitor



$$-iR - \frac{q}{C} = 0$$

$$iR = -\frac{q}{C}$$

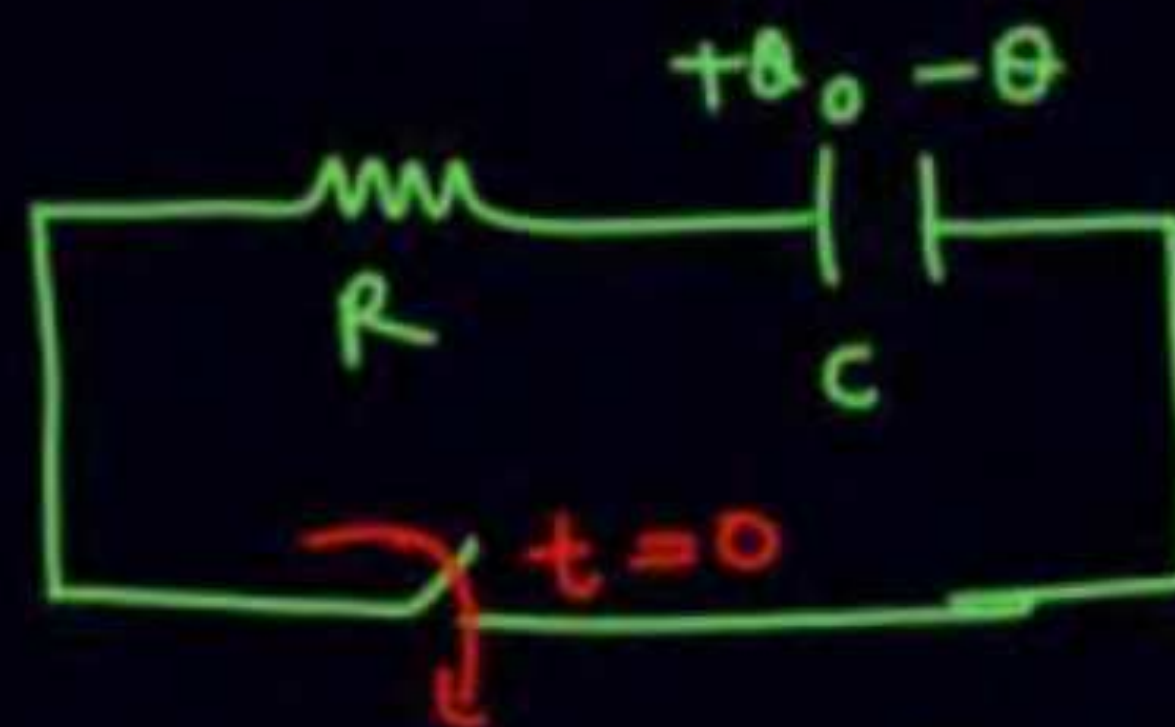
$$\frac{dq}{dt} \cdot R = -\frac{q}{C}$$

$$\int_{Q_0}^q \frac{dq}{q} = \int_0^t -\frac{1}{RC} dt$$

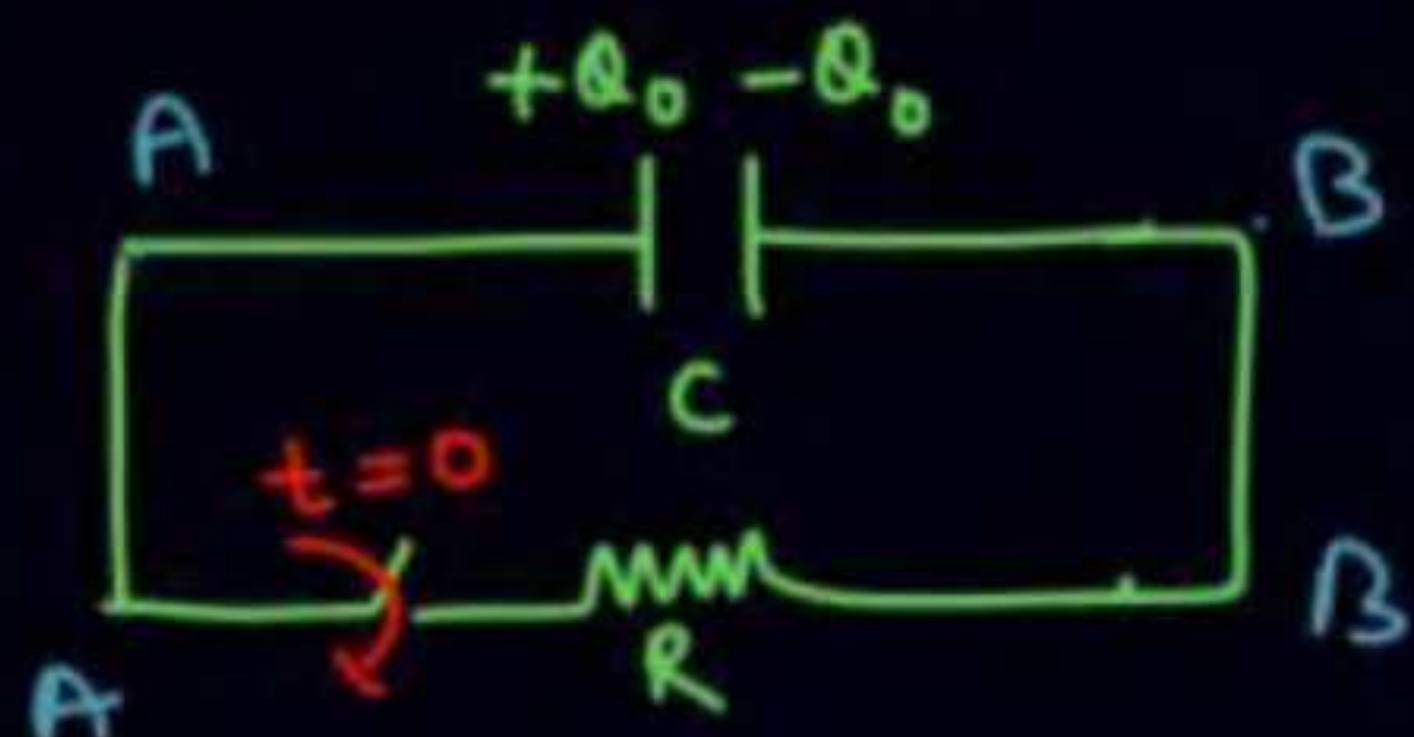
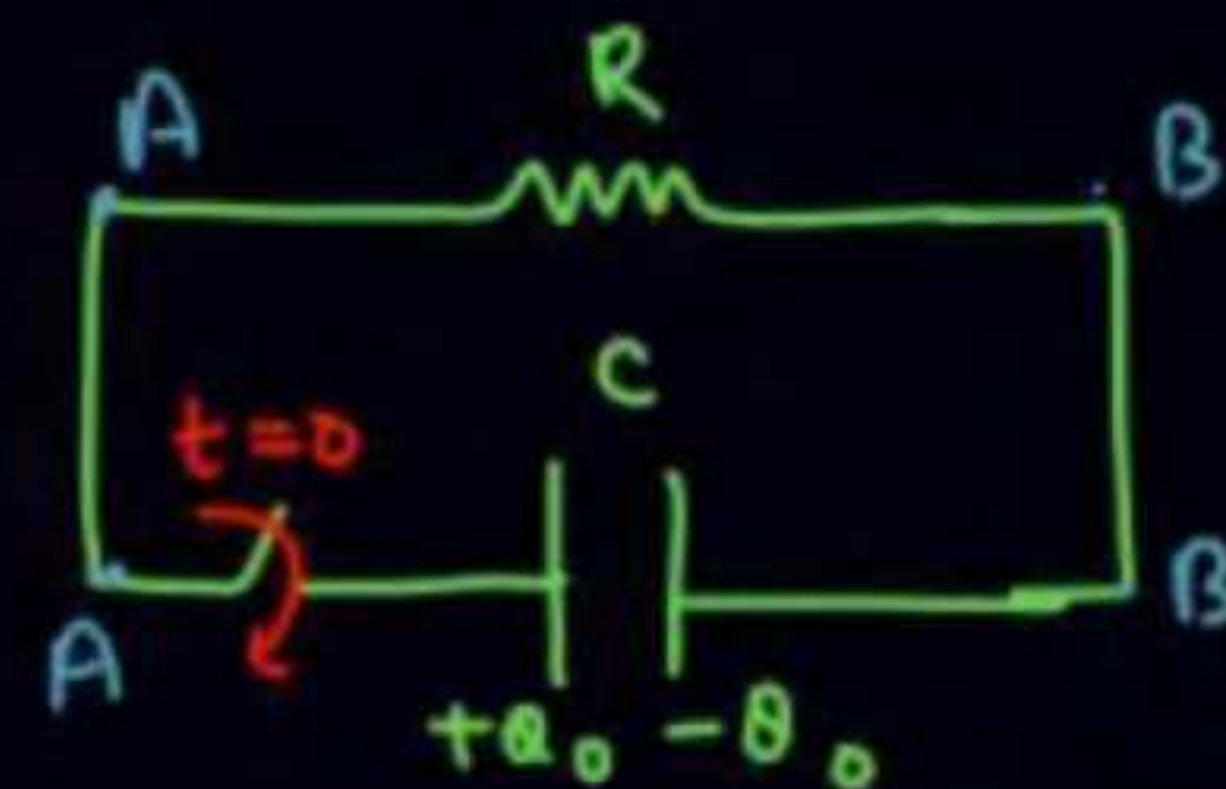
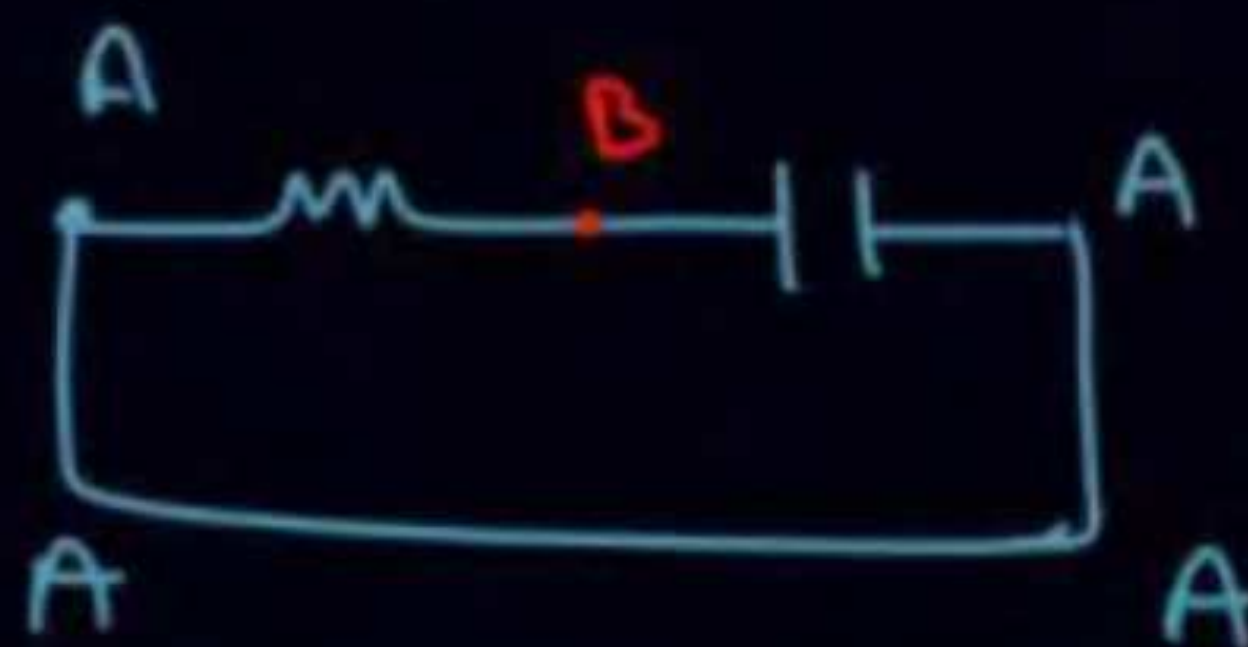
$$\ln \frac{q}{Q_0} = -\frac{t}{RC}$$

$$\frac{q}{Q_0} = e^{-t/RC}$$

$$q = Q_0 e^{-t/RC}$$



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All are same

$t=0$, Switch closed

$$V_C = V_R$$

$$q = Q_0 e^{-t/RC}$$

$$i = i_0 e^{-t/RC}$$



Q



$$\begin{aligned}
 &t = \tau \text{ पर charge} \\
 &q = Q_0 e^{-t/\tau} \\
 &q = Q_0 e^{-1} = \frac{Q_0}{e} \\
 &q = 0.37 Q_0
 \end{aligned}$$

① $\tau = RC = 5 \times 2 = 10$

② change as function of time on capacitor

$$q = Q_0 e^{-t/RC} = 40 e^{-t/10}$$

③ $i, V_R, V_C, (P_{\text{loss}})_{\text{resistance}}$ at time 't'

$$i = \left| \frac{dq}{dt} \right| = \frac{40}{10} e^{-t/10}$$

$$\begin{aligned}
 V_R &= \checkmark \\
 V_C &= \checkmark
 \end{aligned}$$

$$\begin{aligned}
 P &= i^2 R \\
 &= (4 e^{-t/10})^2 \times 5
 \end{aligned}$$

④ heat loss in 10 sec

$$H = \int_0^{10} i^2 R dt$$

⑤ total heat loss.

$$\frac{1}{2} \frac{Q_0^2}{C} \rightarrow \text{heat loss}$$

$$\text{Heat loss} = \int_0^{\infty} i^2 R dt = \frac{Q_0^2}{2C}$$



Q



① Initially S_1 is closed & S_2 is open. at $t=0$
find $q = f(t)$ $t=0$

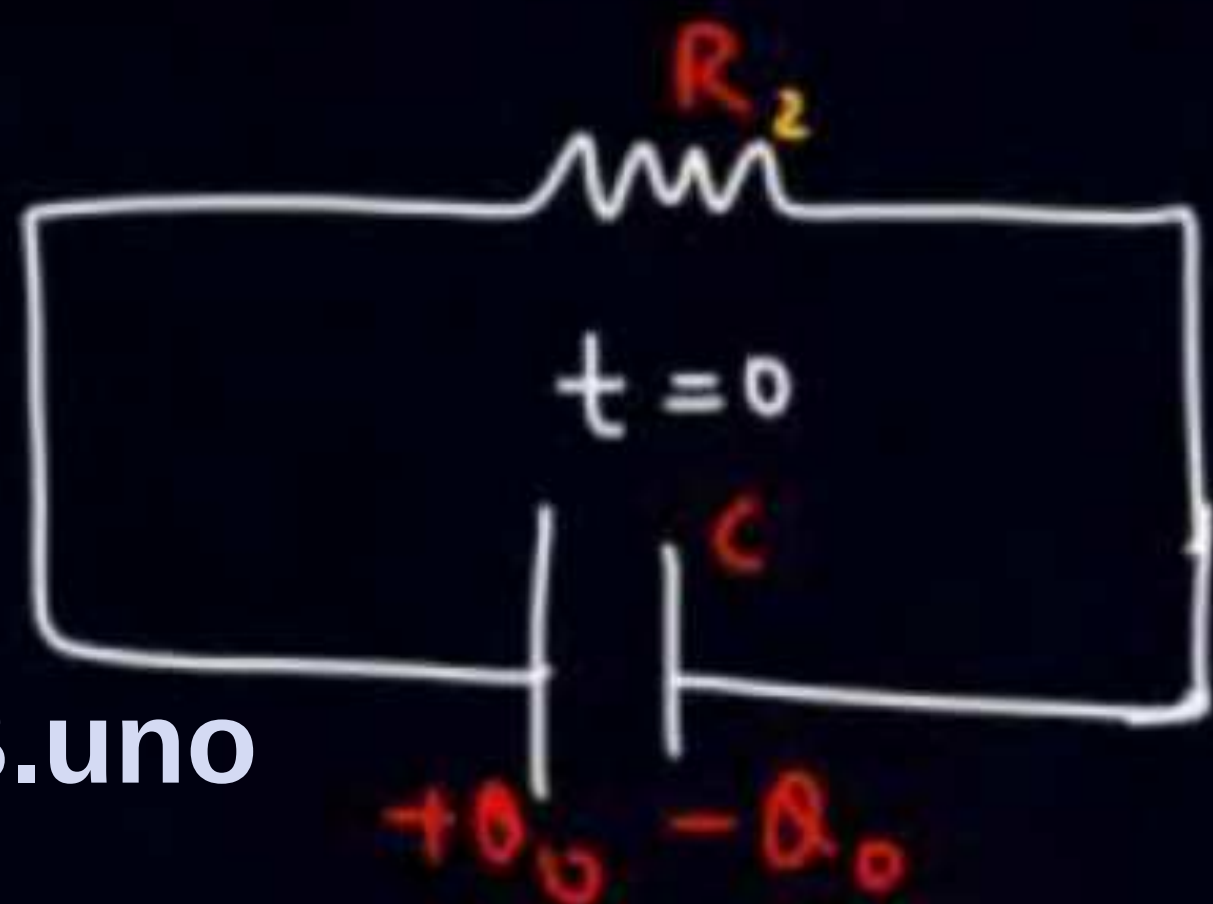
$$q = EC(1 - e^{-t/R_1 C})$$

$$t = \infty \Rightarrow q = Q_0 = EC$$

$$i = 0$$

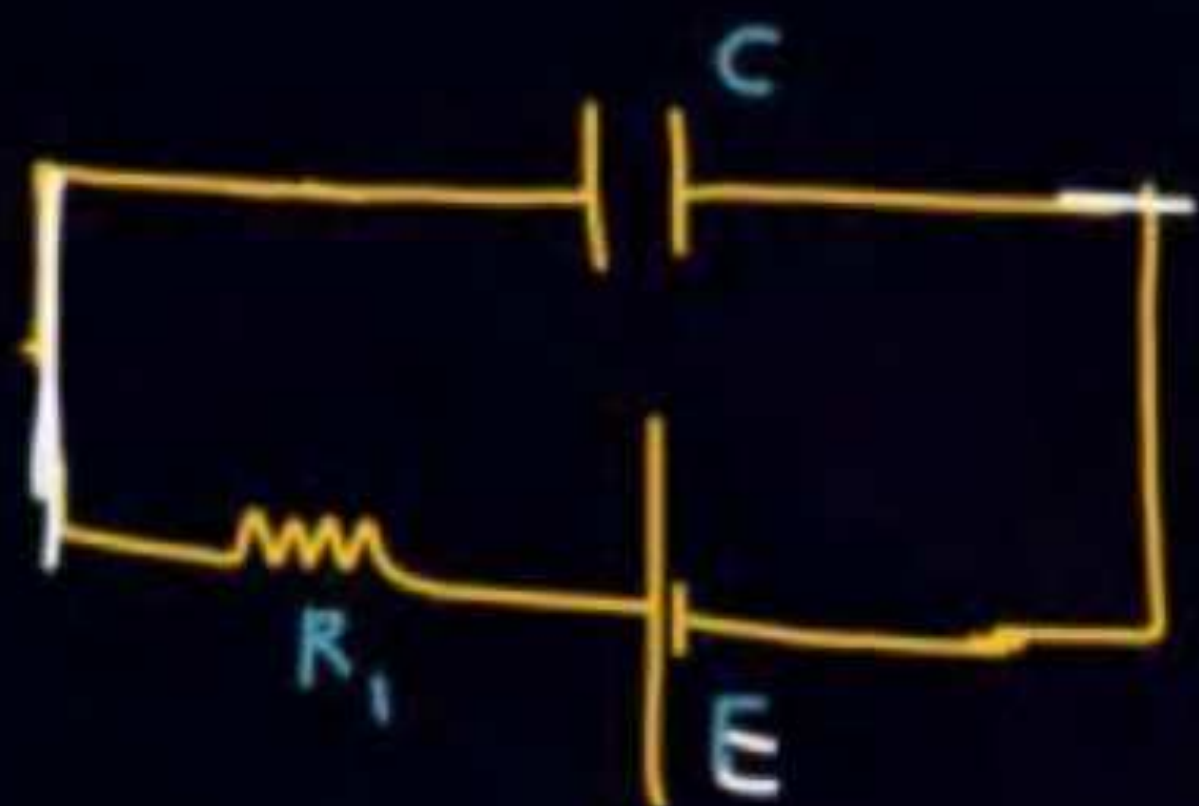
$$i = \frac{E}{R_1} e^{-t/R_1 C}$$

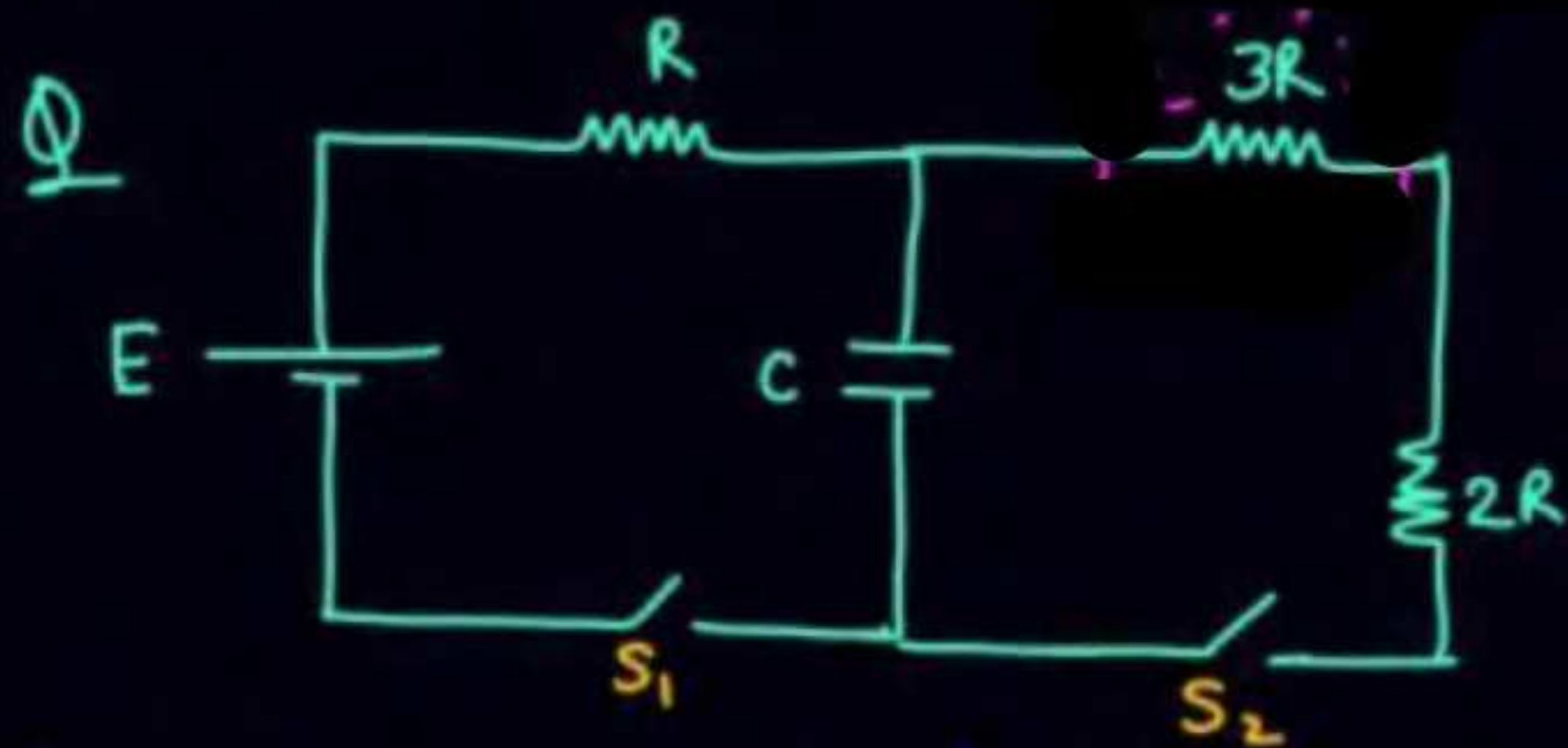
② After very long time S_1 is open and S_2 is closed
and another new stopwatch is start at $t=0$
find $q = f(t)$ according to new observation
total heat loss in resistor.



Discharging $\rightarrow q = Q_0 e^{-t/R_2 C}$

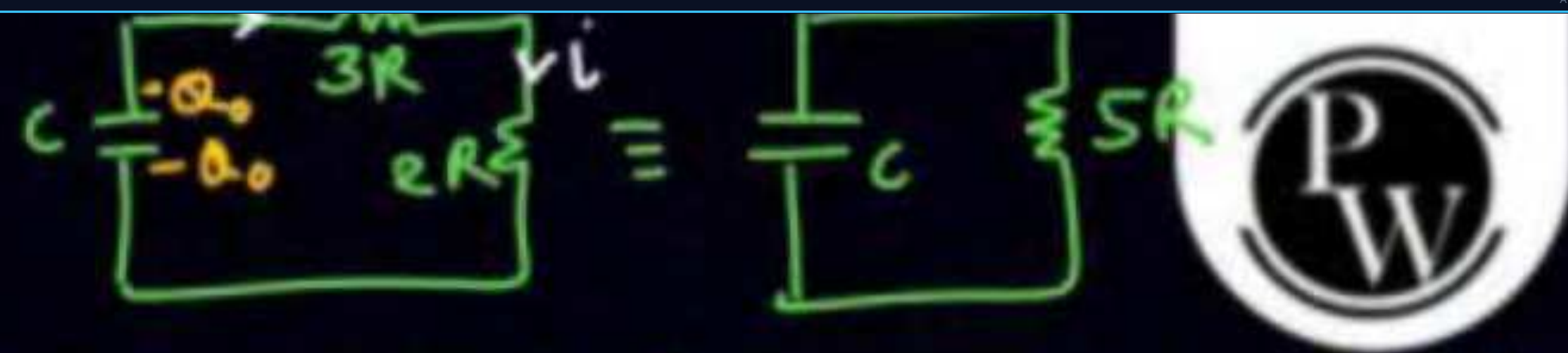
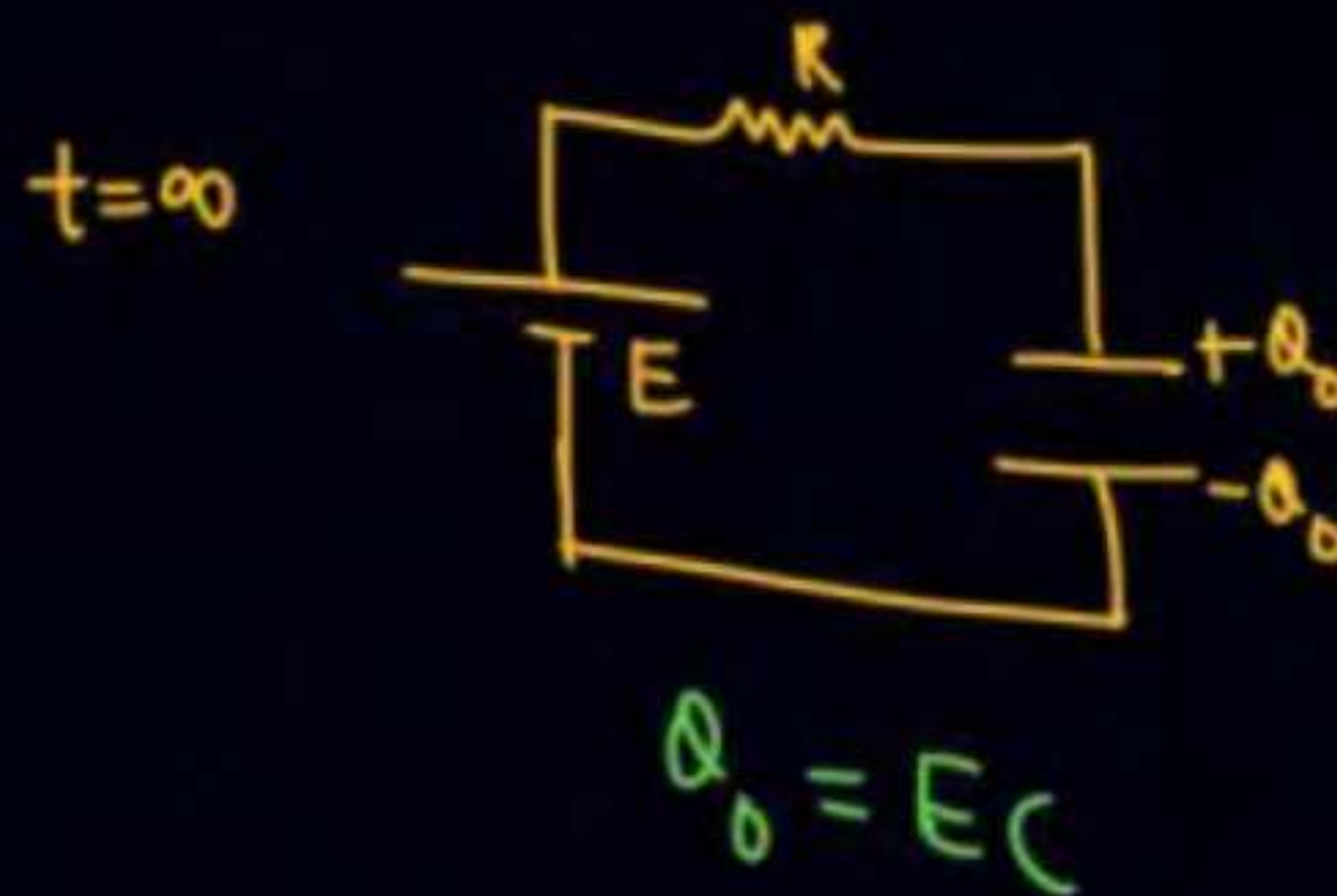
$$q = EC e^{-t/R_2 C}$$





① Initially $S_1 \rightarrow$ closed $S_2 \rightarrow$ open

$$q = EC(1 - e^{-t/RC})$$



② After very long time for new observation S_1 open S_2 closed at $t = 0$

$$q = Q_0 e^{-t/5RC}$$

③ In new observation total heat loss (In above part)

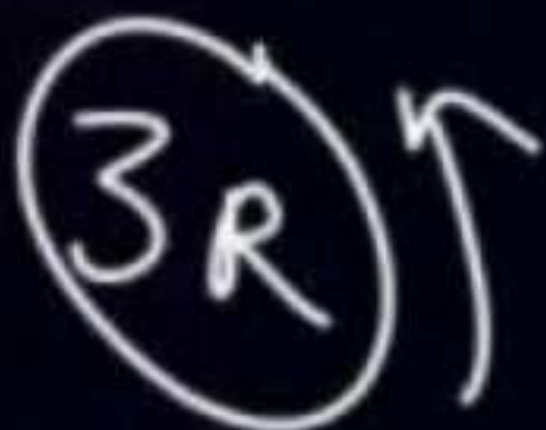
$$H = \frac{1}{2} \frac{Q_0^2}{C} = U_i$$

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④

④ In above part total heat loss in $3R$ resistance.

$$\frac{3H}{5}$$





$$\frac{H_1}{H_2} = \frac{3R}{2R} = \frac{3}{2}$$

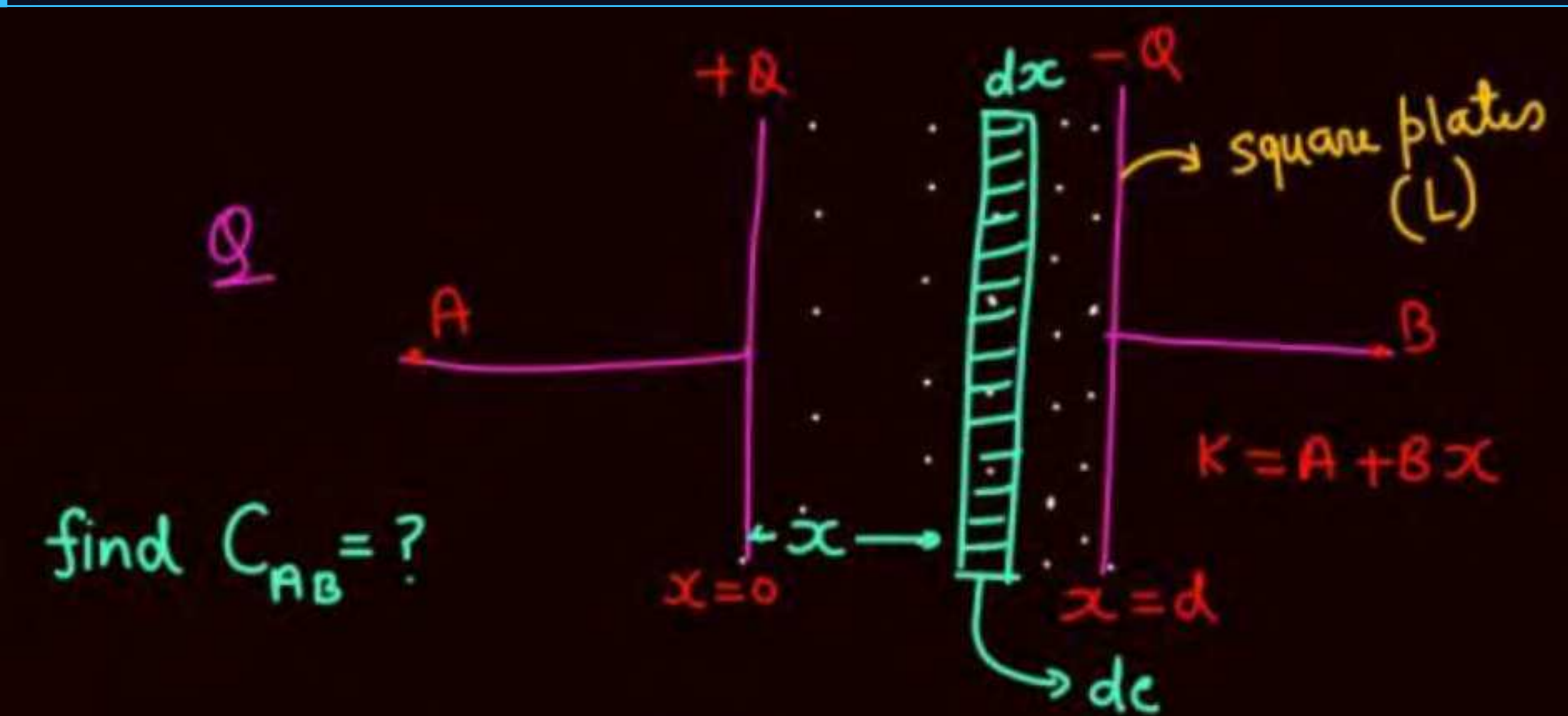
$$H_1 + H_2 = H$$

$$H_1 + \frac{2}{3}H_1 = H$$

$$\frac{5H_1}{3} = H$$

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$$H_1 = \frac{3H}{5}, H_2 = \frac{2H}{5}$$



find $C_{AB} = ?$

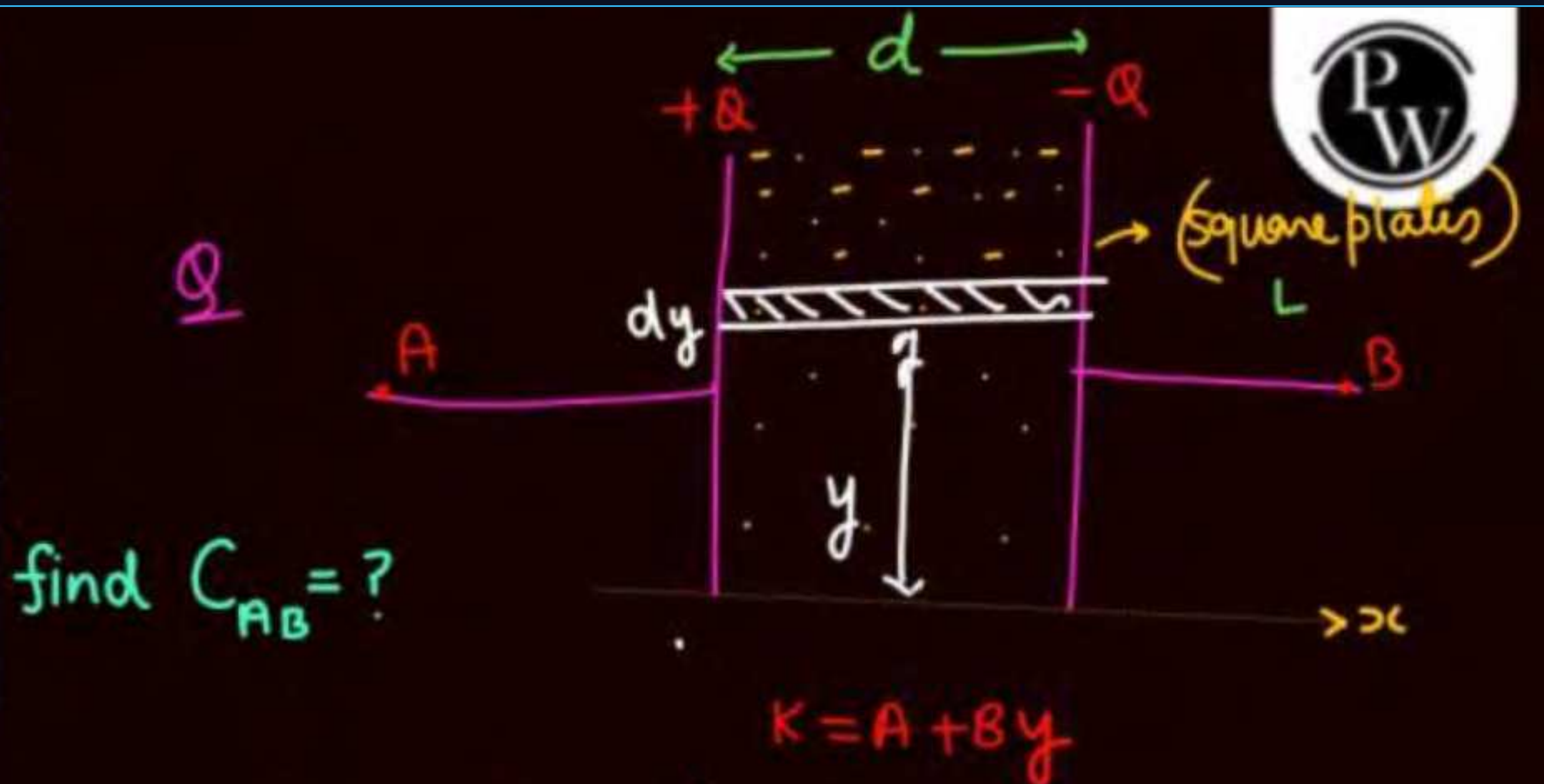
$$dc = \frac{A\epsilon_0(A+Bx)}{dx}$$

$$\frac{1}{C_{eq}} = \int \frac{1}{dc} = \int_0^d \frac{dx}{A\epsilon_0(A+Bx)}$$

$$\frac{1}{C_{eq}} = \frac{1}{A\epsilon_0 B} \ln\left(\frac{A+Bd}{A}\right)$$

$$C_{eq} = \checkmark$$

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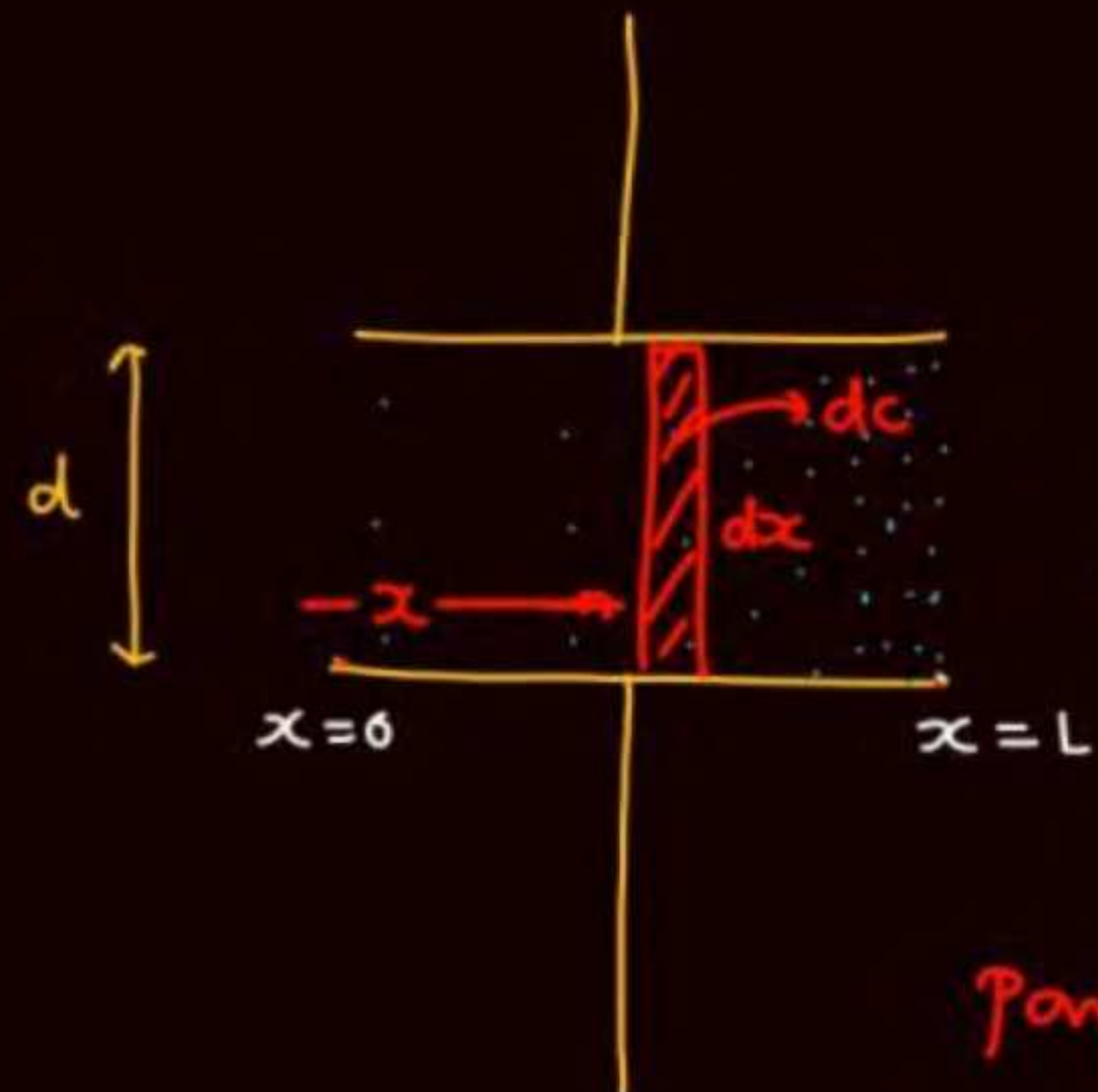
find $C_{AB} = ?$

$$dc = \frac{Ldy \cdot \epsilon_0(A+By)}{d}$$

$$C_{eq} = \int_0^L \frac{L\epsilon_0}{d} (A+By) dy$$



Q



$$K = A + Bx$$

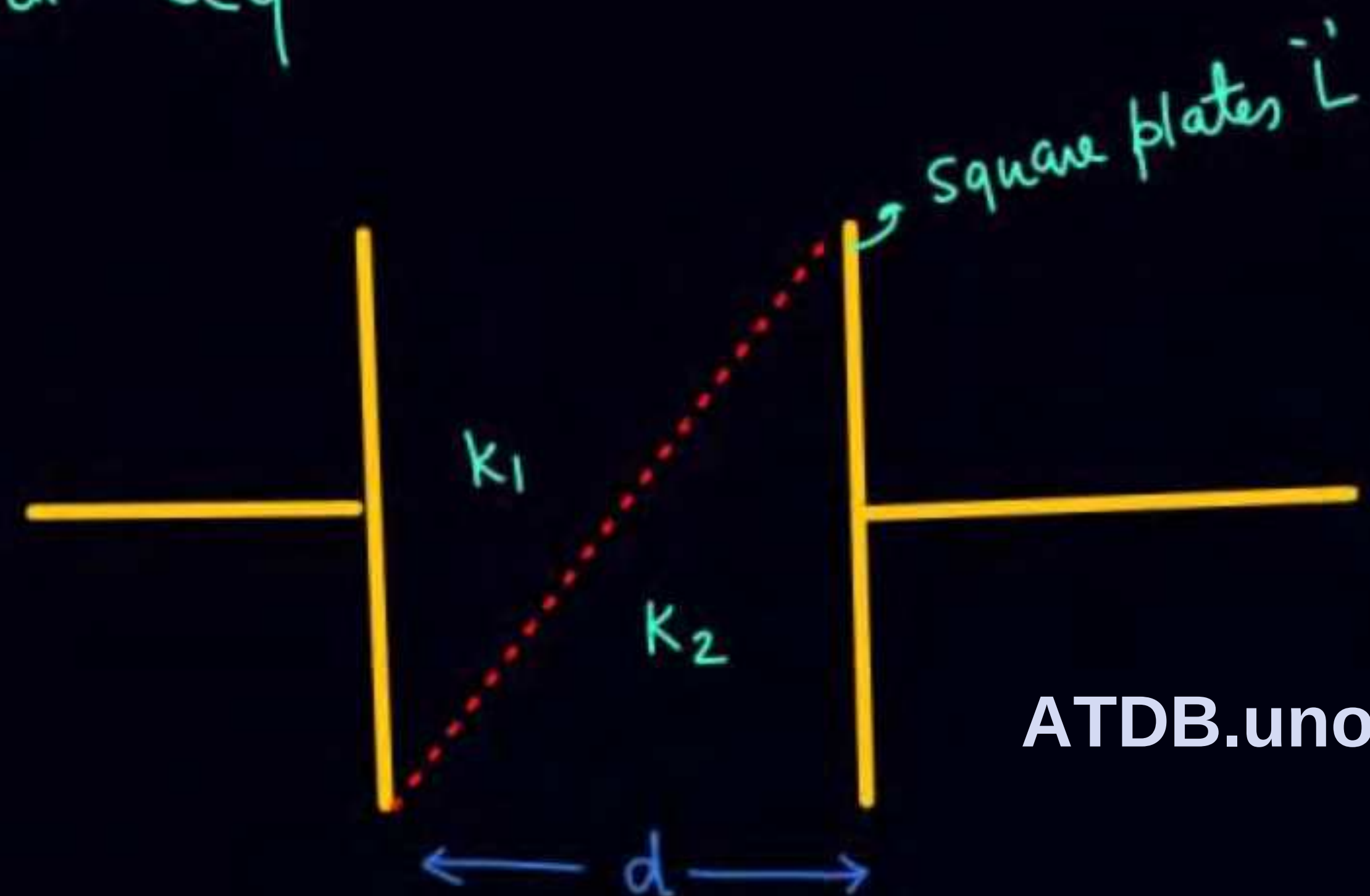
$$\int_0^L dc = \int_0^L \frac{L dx \cdot \epsilon_0 (A + Bx)}{d} = Ceq$$

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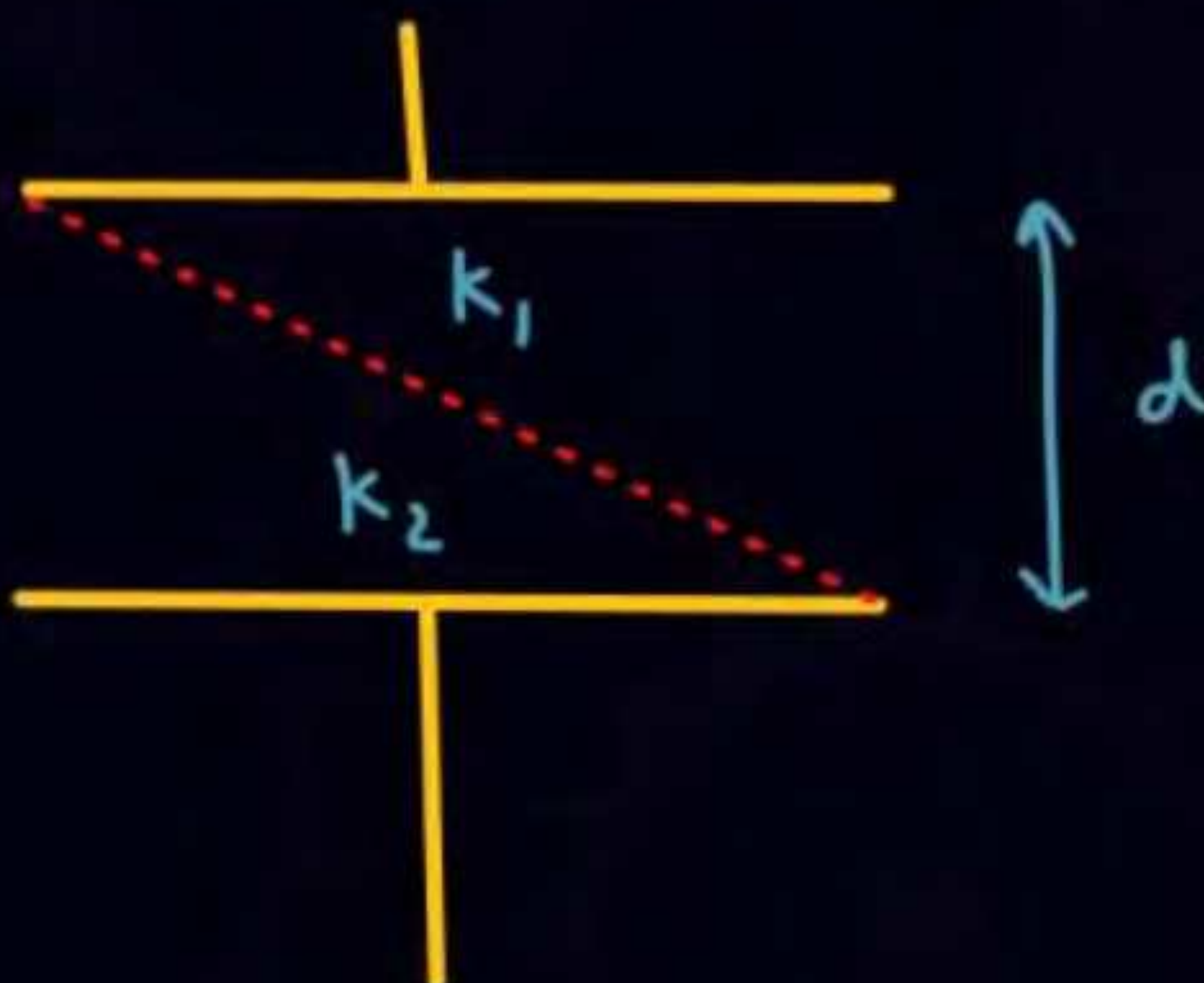
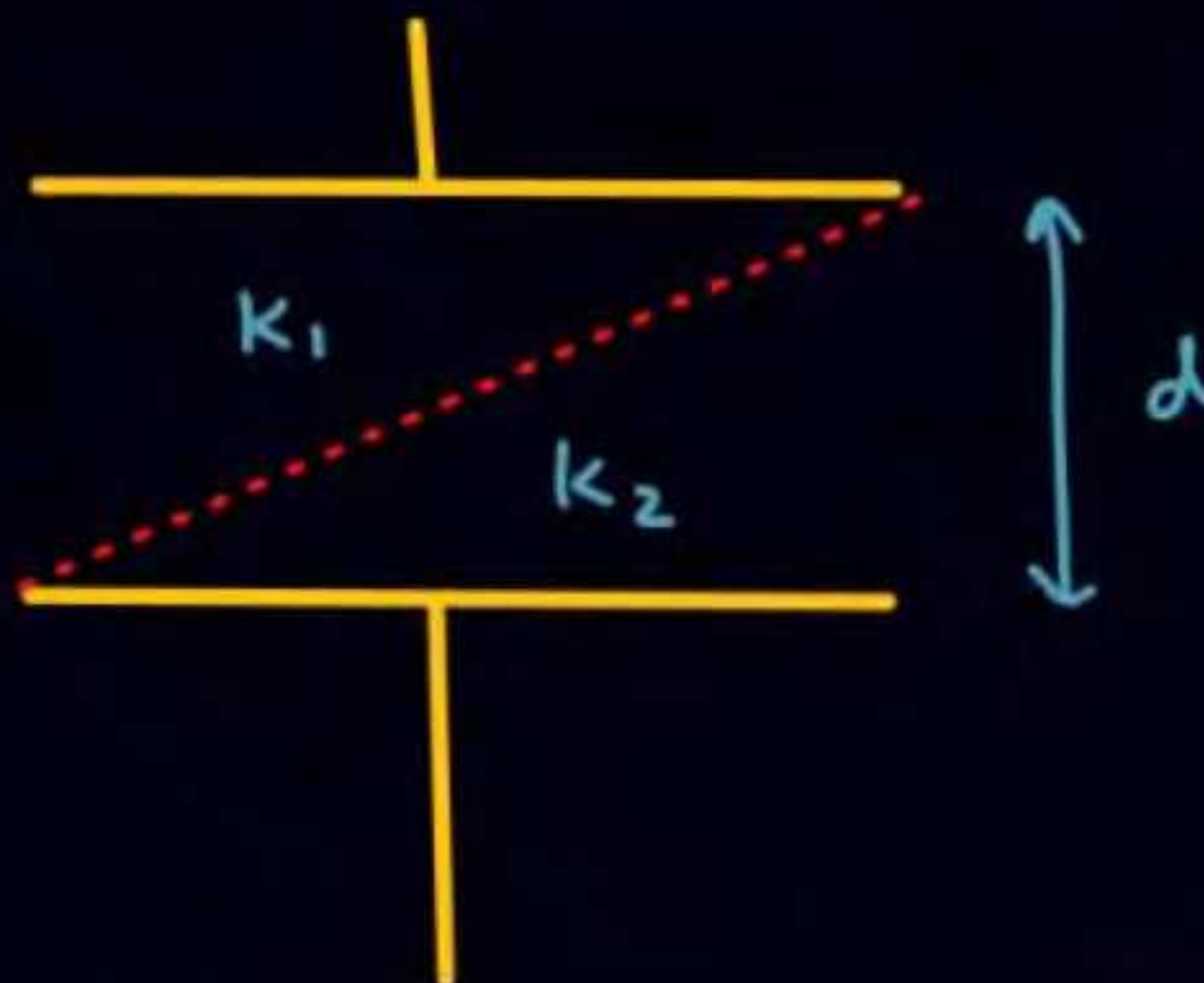
Parallel



Q find C_{eq}

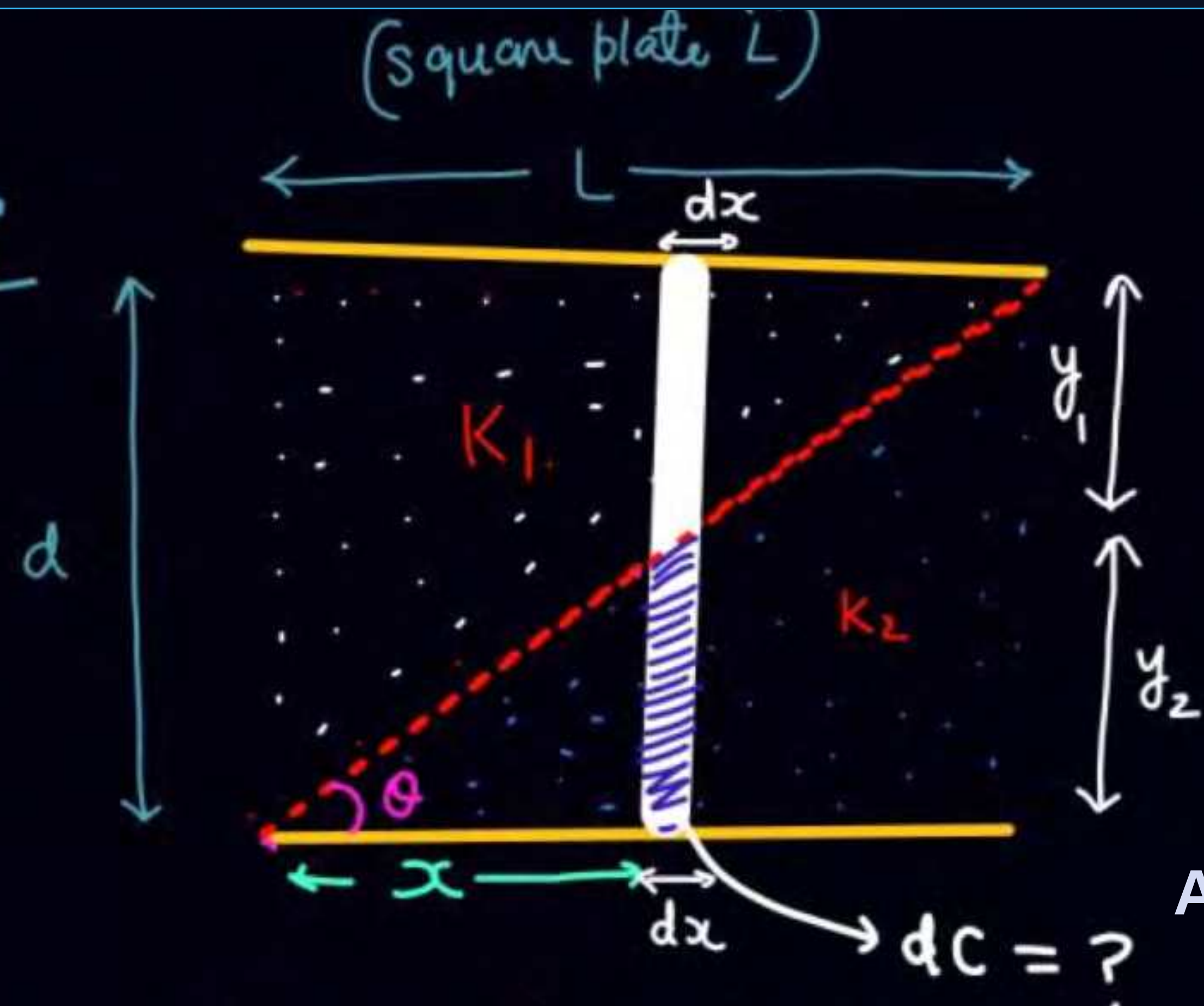


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sol.



$$\tan \theta = \frac{d}{L}$$

$$\tan \theta = \frac{y_1}{x}$$

$$y_1 = x \tan \theta$$

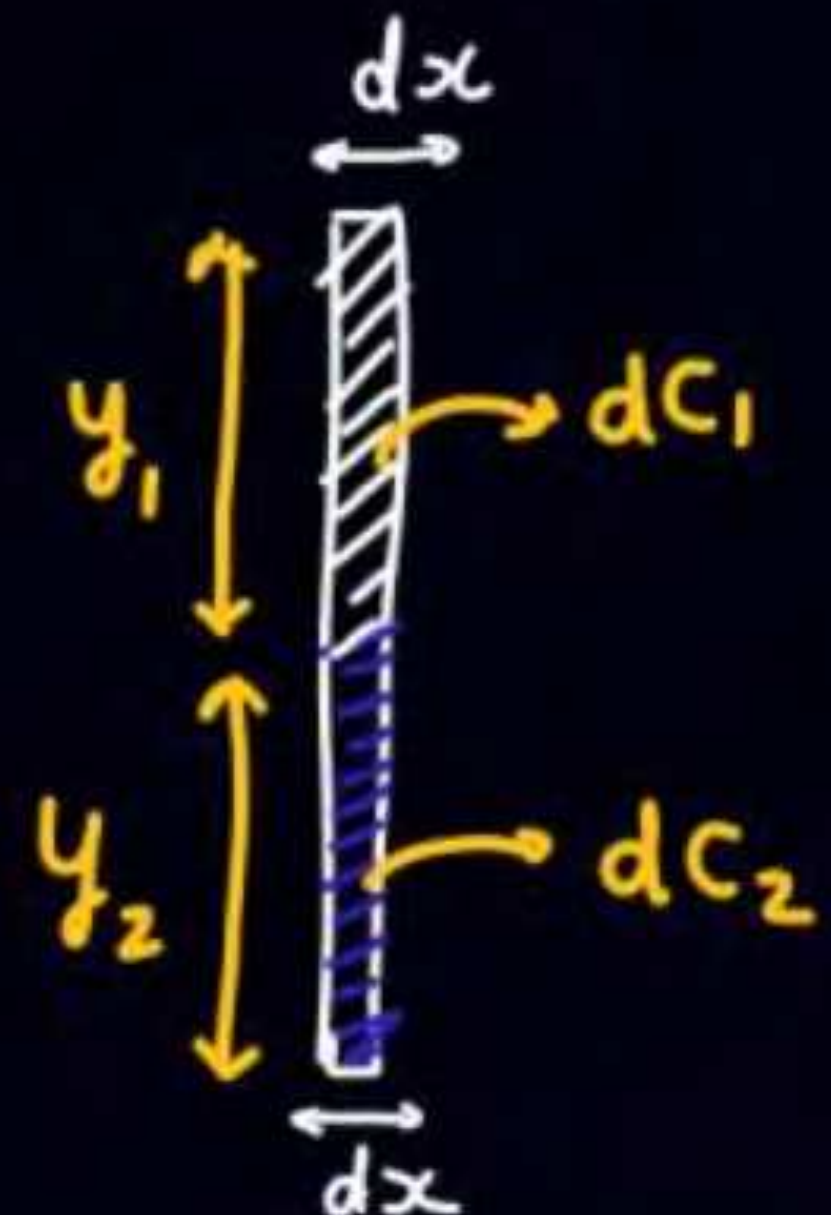
$$y_2 = d - x \tan \theta$$

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$$\frac{1}{dc} = \frac{1}{dc_1} + \frac{1}{dc_2}$$

$$dc = \int$$

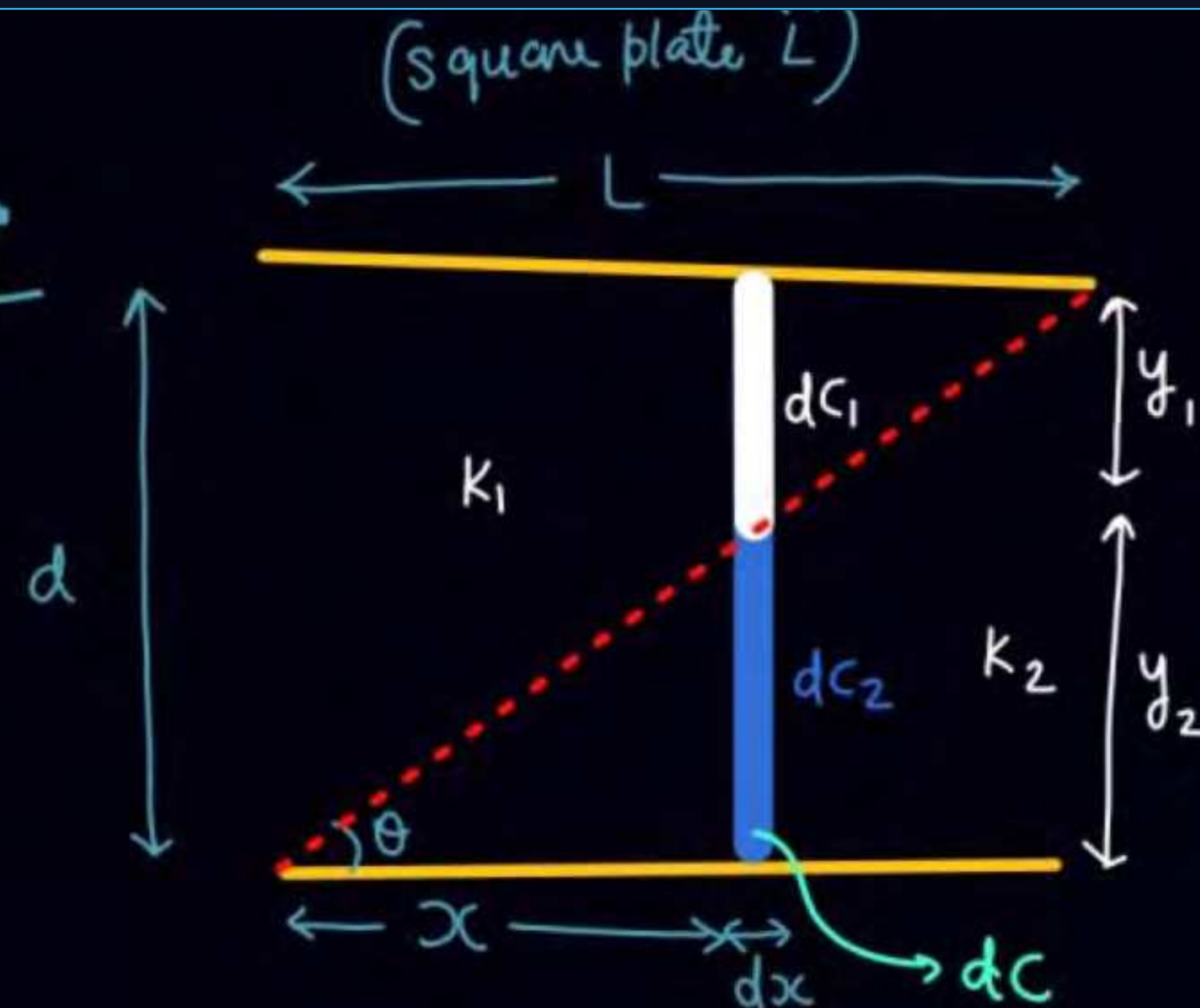
$$\Rightarrow \int dc = C_{eq} = \underline{A_{eq}}$$



$$dc_1 = \frac{L dx \cdot \epsilon_0 K_1}{y_1}$$

$$dc_2 = \frac{L dx \cdot \epsilon_0 K_2}{y_2}$$

dc_1 & $dc_2 \rightarrow$ Series



$$\tan \theta = \frac{d}{L}$$

$$y_2 = x \tan \theta = \frac{x d}{L}$$

$$y_1 = d - x \tan \theta$$

$$y_1 = d - x \frac{d}{L}$$

$$dC_1 = \frac{(L dx) \epsilon_0 K_1}{y_1}$$

$$dC_2 = \frac{L dx \epsilon_0 k_2}{y_2}$$

$$\frac{1}{dc} = \frac{1}{dc_1} + \frac{1}{dc_2} = \frac{y_1}{Ldx \epsilon_0 k_1} + \frac{y_2}{Ldx \epsilon_0 k_2}$$

$$\frac{1}{dc} = \frac{1}{L \cdot dx} \cdot \epsilon_0 \left[\frac{y_1}{K_1} + \frac{y_2}{K_2} \right]$$

$$dc = \frac{L \epsilon_0 dx}{\frac{y_1}{K_1} + \frac{y_2}{K_2}} = \frac{L \epsilon_0 dx}{\frac{d - \frac{x d}{L}}{K_1} + \frac{\frac{x d}{L}}{L \cdot K_2}}$$

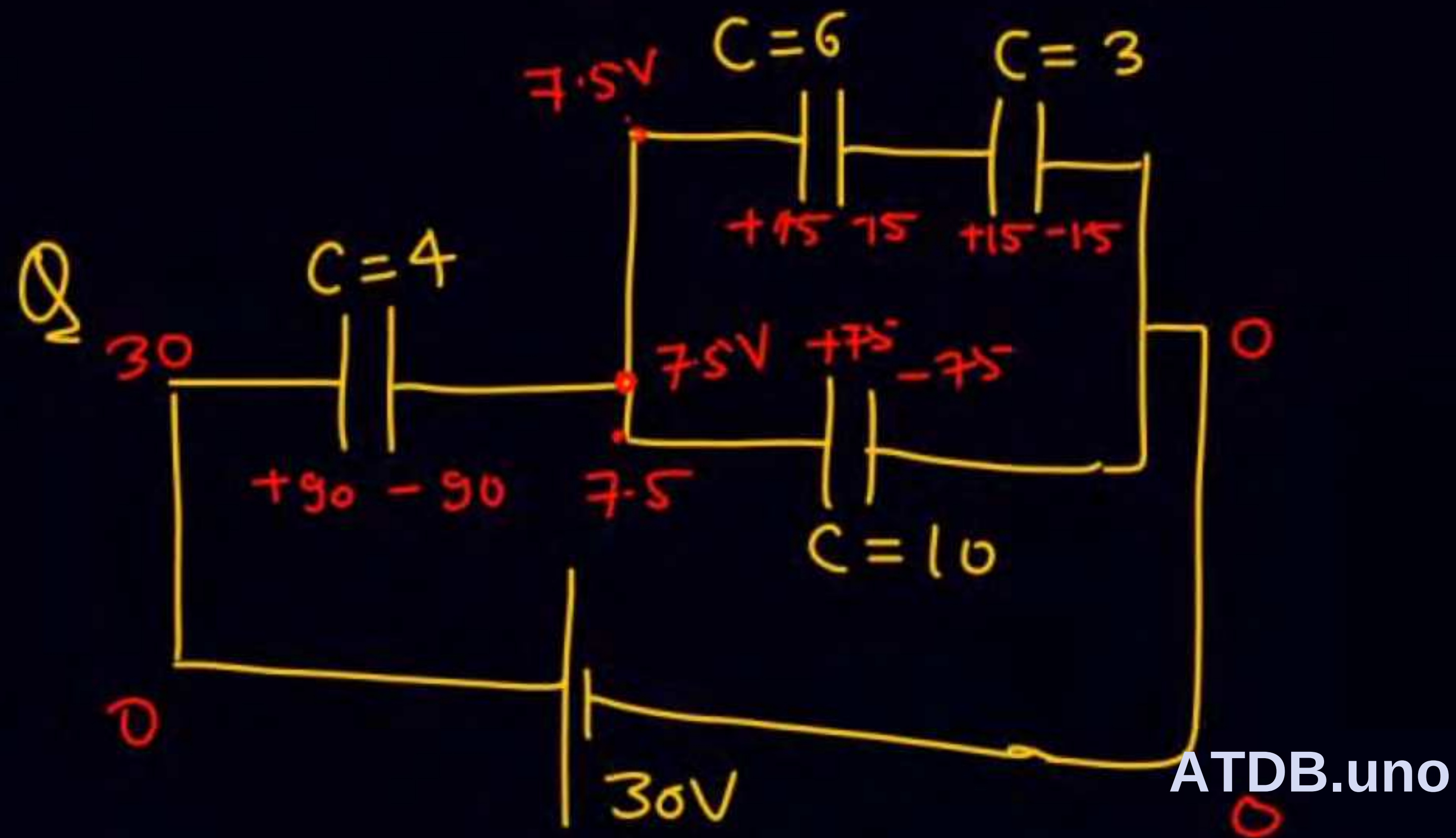
$$dc = \frac{L\epsilon_0 dx}{\frac{d}{k_1} + x \left[\frac{d}{Lk_2} - \frac{d}{Lk_1} \right]}$$



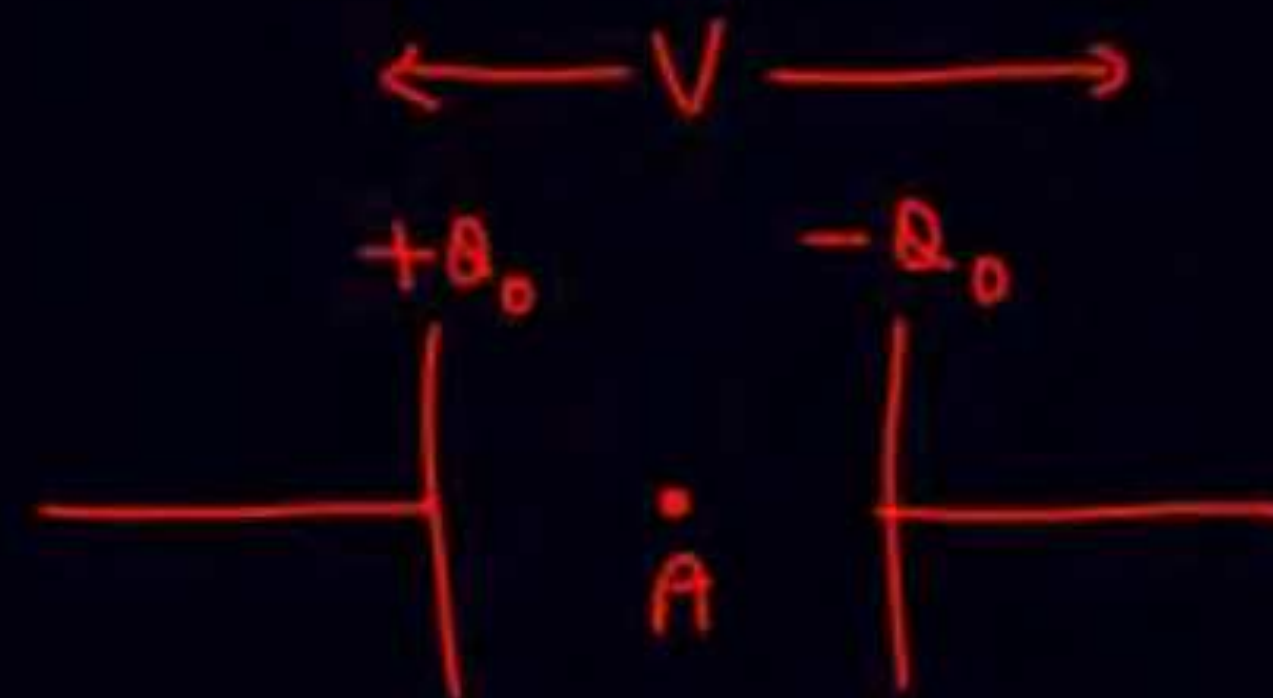
$$dc = \frac{L\epsilon_0 dx}{\frac{d}{k_1} + x \left[\frac{d}{Lk_2} - \frac{d}{Lk_1} \right]}$$

$$\int_0^L dc = \int_0^L \frac{L\epsilon_0 dx}{\frac{d}{k_1} + x \left[\frac{d}{Lk_2} - \frac{d}{Lk_1} \right]} \Rightarrow \boxed{\int \frac{A dx}{B + cx} = \frac{A}{c} \ln(B + cx)}$$

$$C_{eq} = \frac{L\epsilon_0}{\left(\frac{d}{Lk_1} - \frac{d}{Lk_2} \right)} \ln \left\{ \frac{\frac{d}{k_1} + L \left[\frac{d}{Lk_2} - \frac{d}{Lk_1} \right]}{\frac{d}{k_1}} \right\} = \frac{L^2 \epsilon_0 k_1 k_2}{d \cdot (k_2 - k_1)} \ln \left(\frac{k_1}{k_2} \right)$$

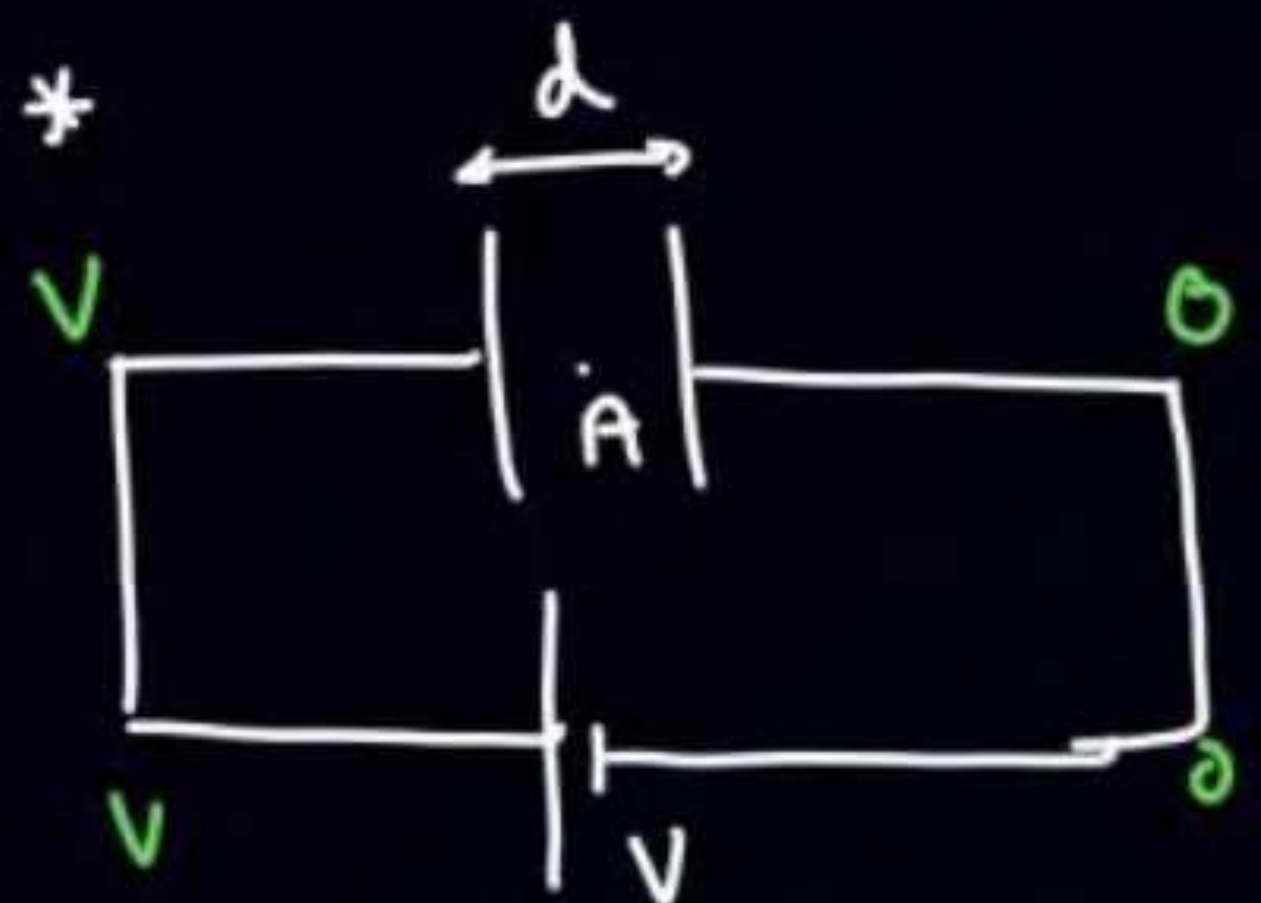


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$$E_A = \frac{V}{d}$$

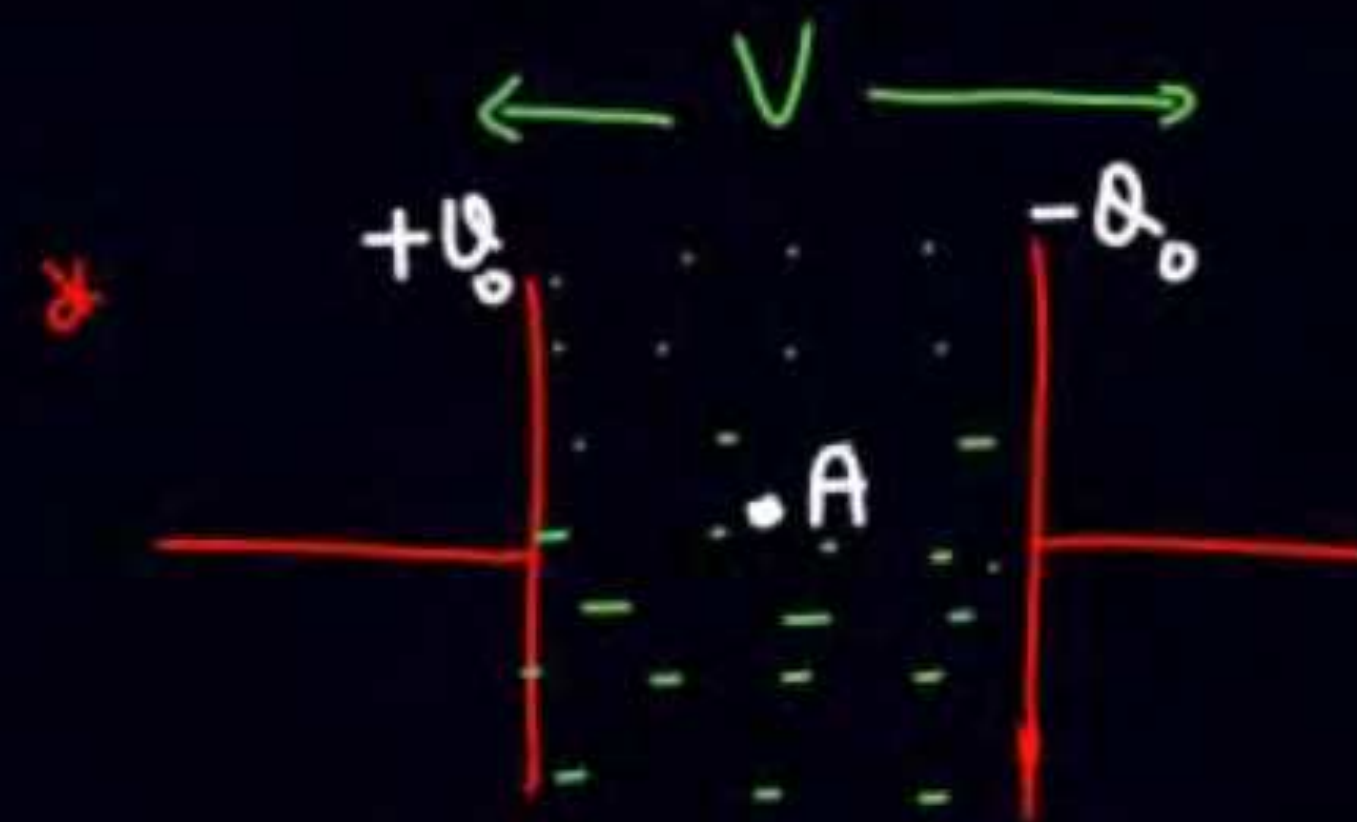
$$E_0 = \frac{Q_0}{A\epsilon_0}$$



$$E_A = \frac{V}{d}$$



$$E_A = \frac{V}{d}$$



$$E_A = \frac{V}{d}$$

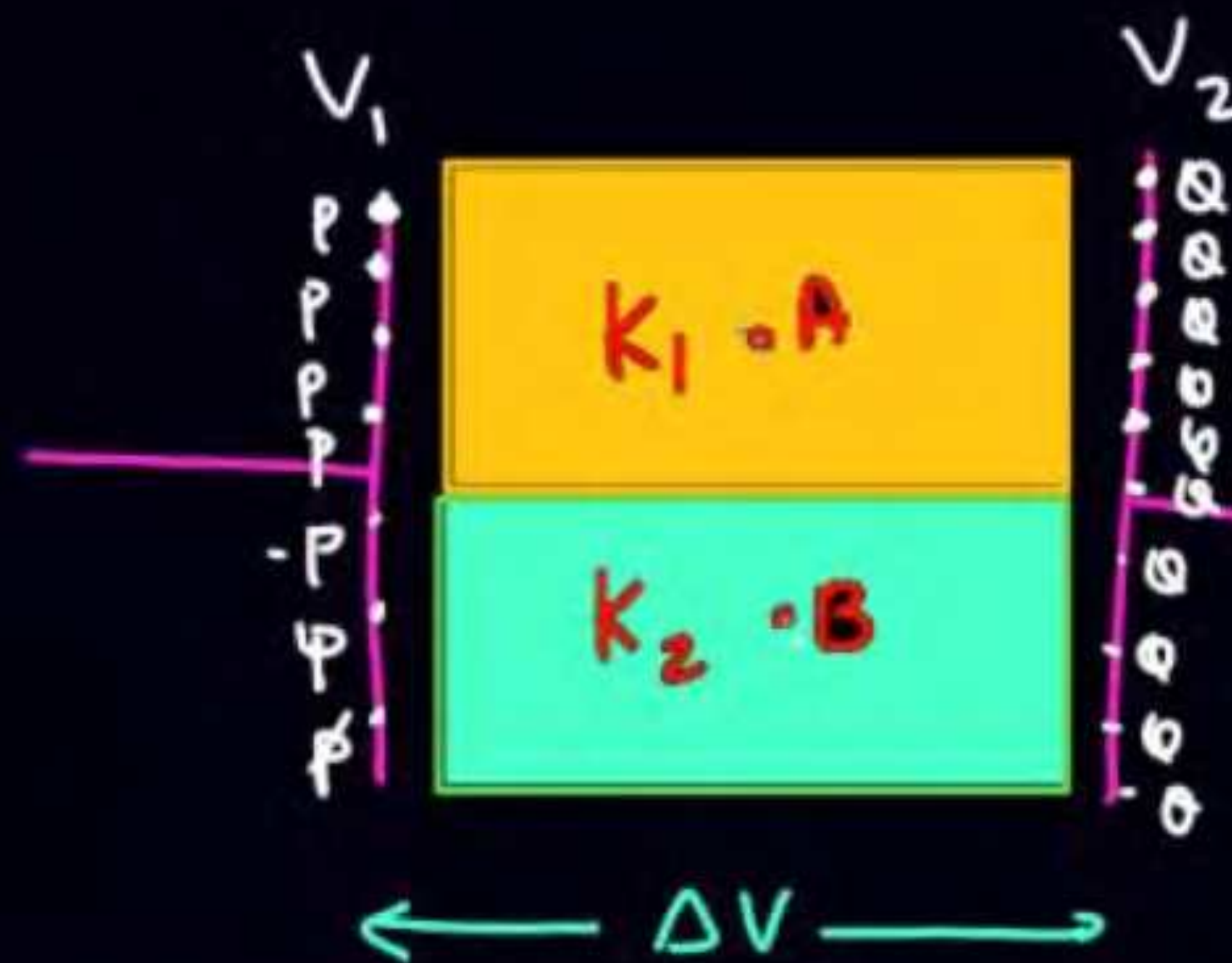
$$E_A = \frac{E_0}{K}$$

$Q = CV$
 $\downarrow$ Same $\downarrow$ k times

$$V \rightarrow \frac{V}{k}$$

$$E_0 \rightarrow \frac{E_0}{k}$$

$$E_A = \frac{V}{d}$$

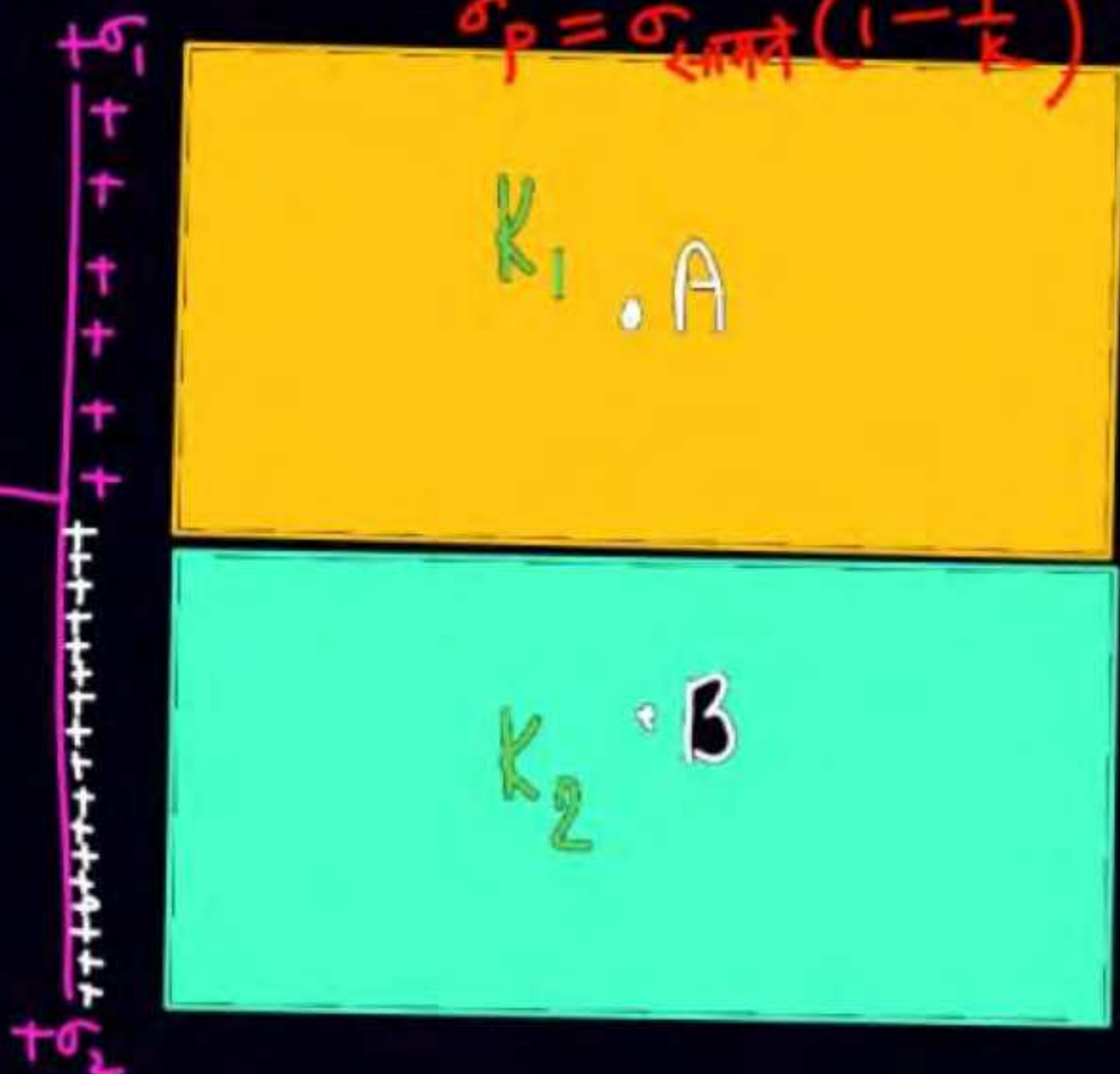


Let $K_1 = K$
 $K_2 = 2K$

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$$E_A = E_B$$

$E = \frac{\Delta V}{d}$ → same
 → same



$\alpha_p = \alpha_0 (1 - K)$
 $\sigma_p = \sigma_{\text{induced}} (1 - \frac{1}{K})$

$(E_{\text{inside}})_A = \frac{\sigma_1}{\epsilon_0 K_1}$

$E_A = E_B$

$(E_{\text{inside}})_B = \frac{\sigma_2}{\epsilon_0 K_2}$

$$\frac{\sigma_1}{\epsilon_0 K_1} = \frac{\sigma_2}{\epsilon_0 K_2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{K_1}{K_2}$$

$\frac{\sigma_1}{\sigma_2} = \frac{K}{2K} = \frac{1}{2}$

QUESTION

$$C_{\text{अपर}} = C_1 = \frac{A \epsilon_0 K}{3d}$$

$$C_{\text{नीचे}} = \frac{2A \epsilon_0}{3d}$$

$$C_{\text{total}} = C_{\text{अपर}} + C_{\text{नीचे}}$$

$$C = \frac{A \epsilon_0}{d} \left(\frac{2}{3} + \frac{K}{3} \right)$$



15. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers $1/3$ of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1 . When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2 . The electric field in the dielectric is E_1 and that in the other portion is E_2 . Choose the correct option/options, ignoring edge effects

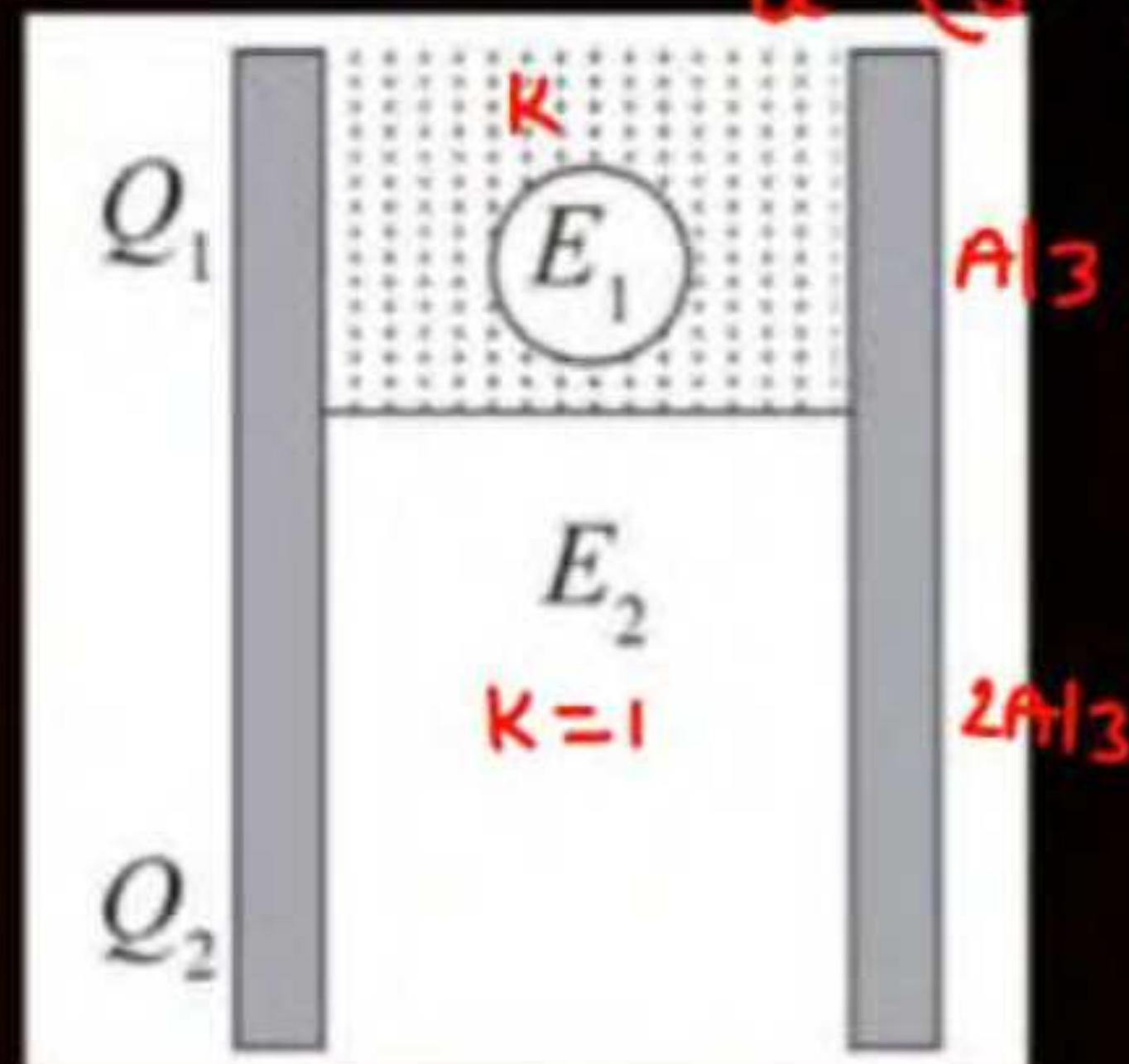
(JEE Adv. 2014)

(1) $\frac{E_1}{E_2} = 1$

~~(2) $\frac{E_1}{E_2} = \frac{1}{K}$~~

~~(3) $\frac{Q_1}{Q_2} = \frac{3}{K}$~~

(4) $\frac{C}{C_1} = \frac{2+K}{K}$

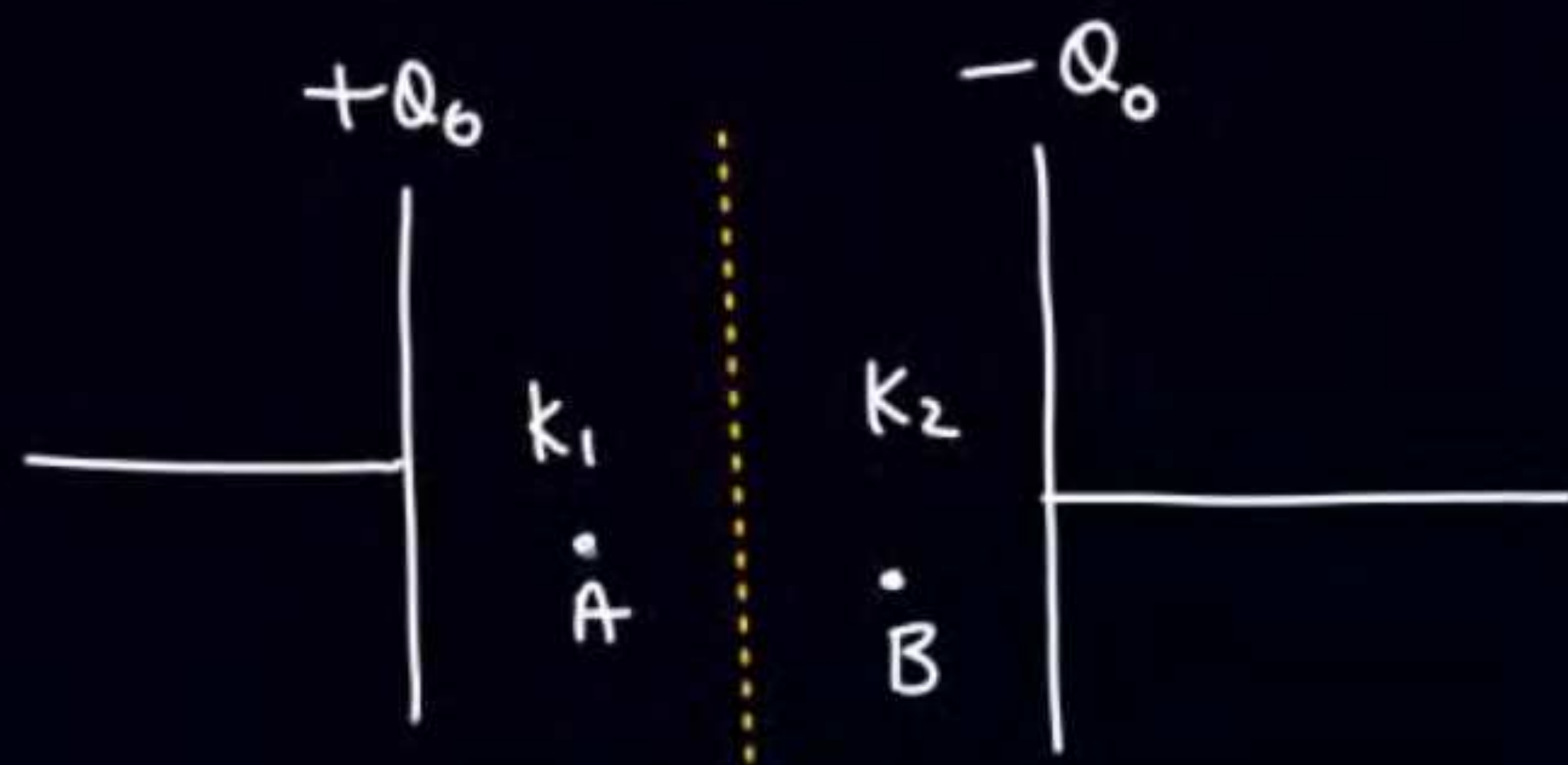
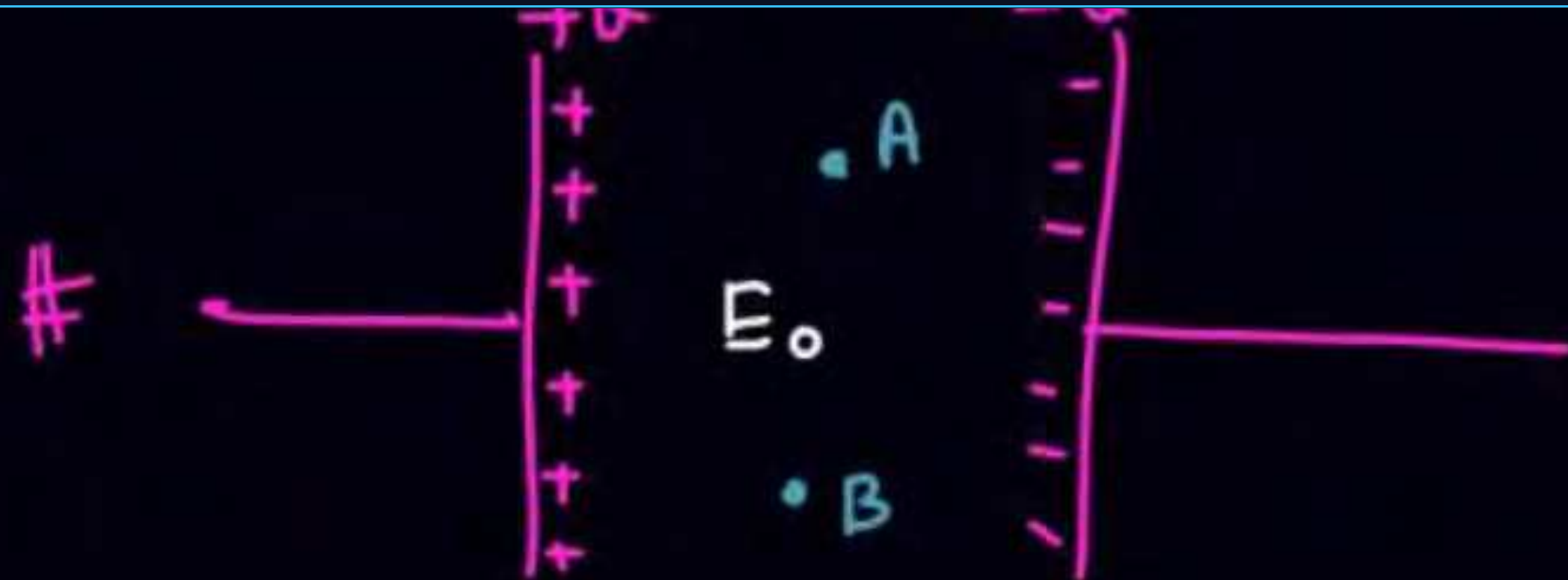


$$\frac{Q_1}{Q_2} = \frac{K}{1}$$

$$\frac{Q_1 \cdot 2A/3}{A/3 \cdot Q_2} \Rightarrow \frac{2Q_1}{Q_2} = K$$

$$\frac{C}{C_1} = \frac{(K+2)}{\frac{2}{3}K} = \frac{K+2}{\frac{2}{3}K}$$

Ans. (1, 4)

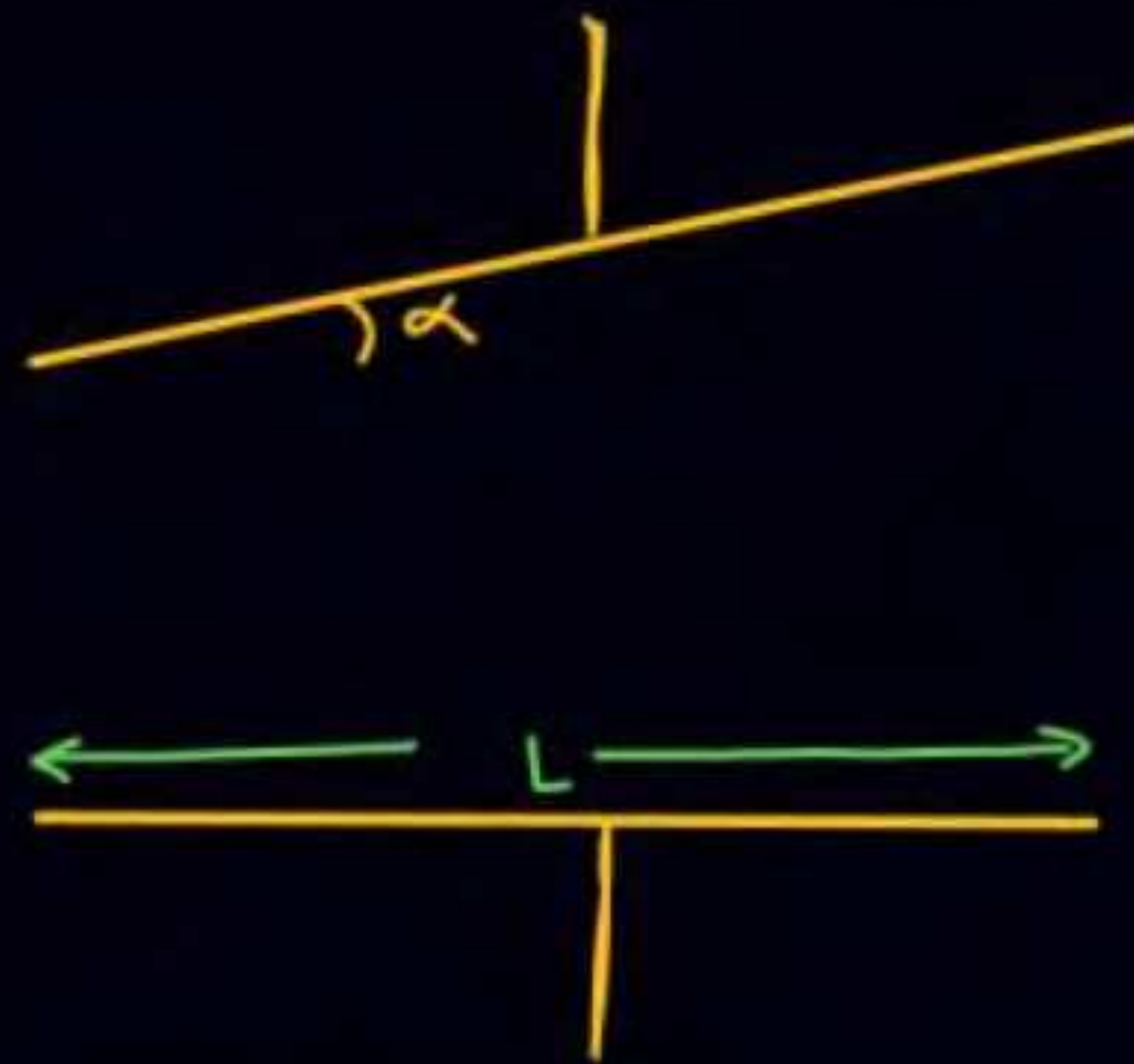
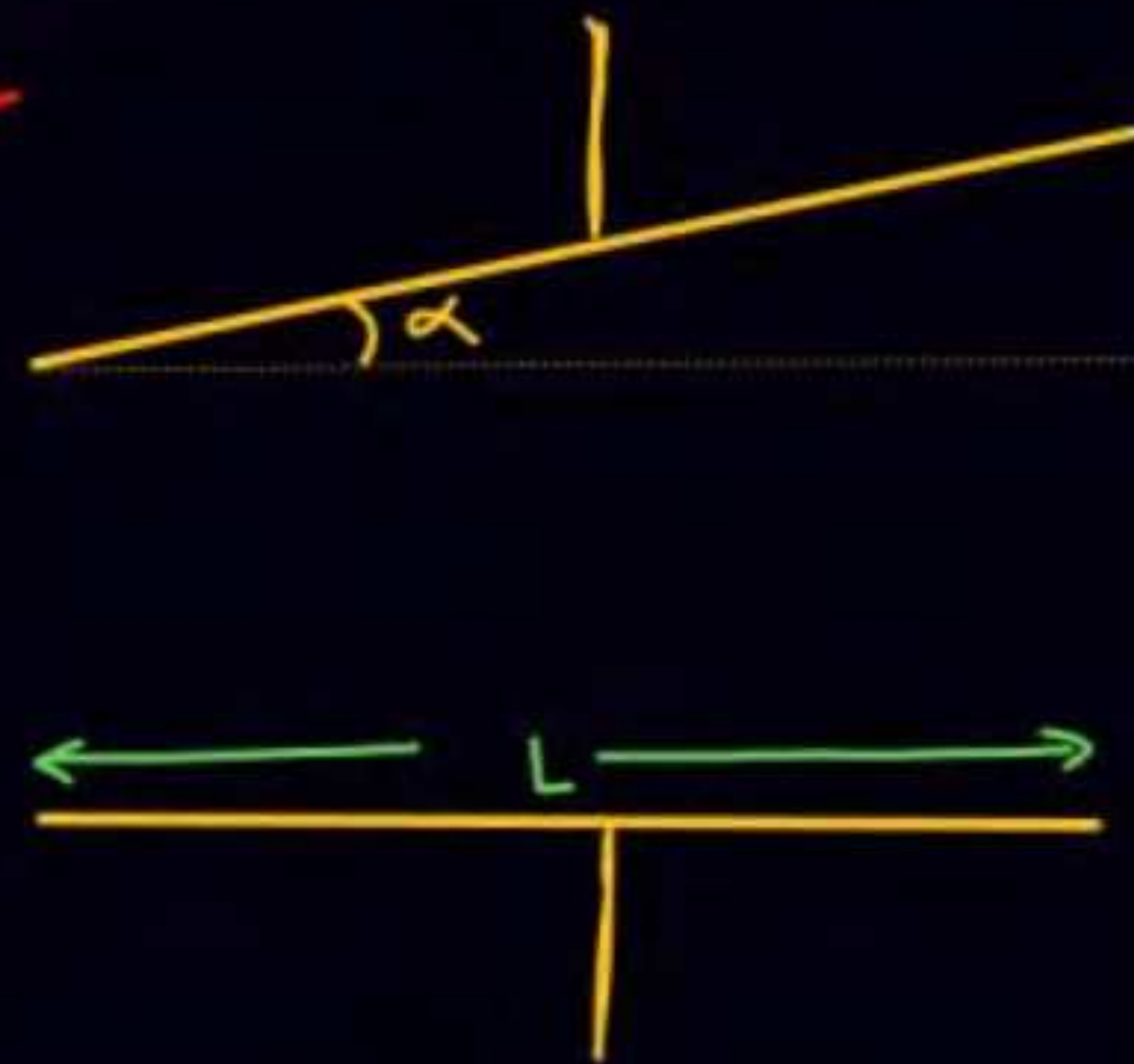


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$$E_A = \frac{E_0}{k_1}$$

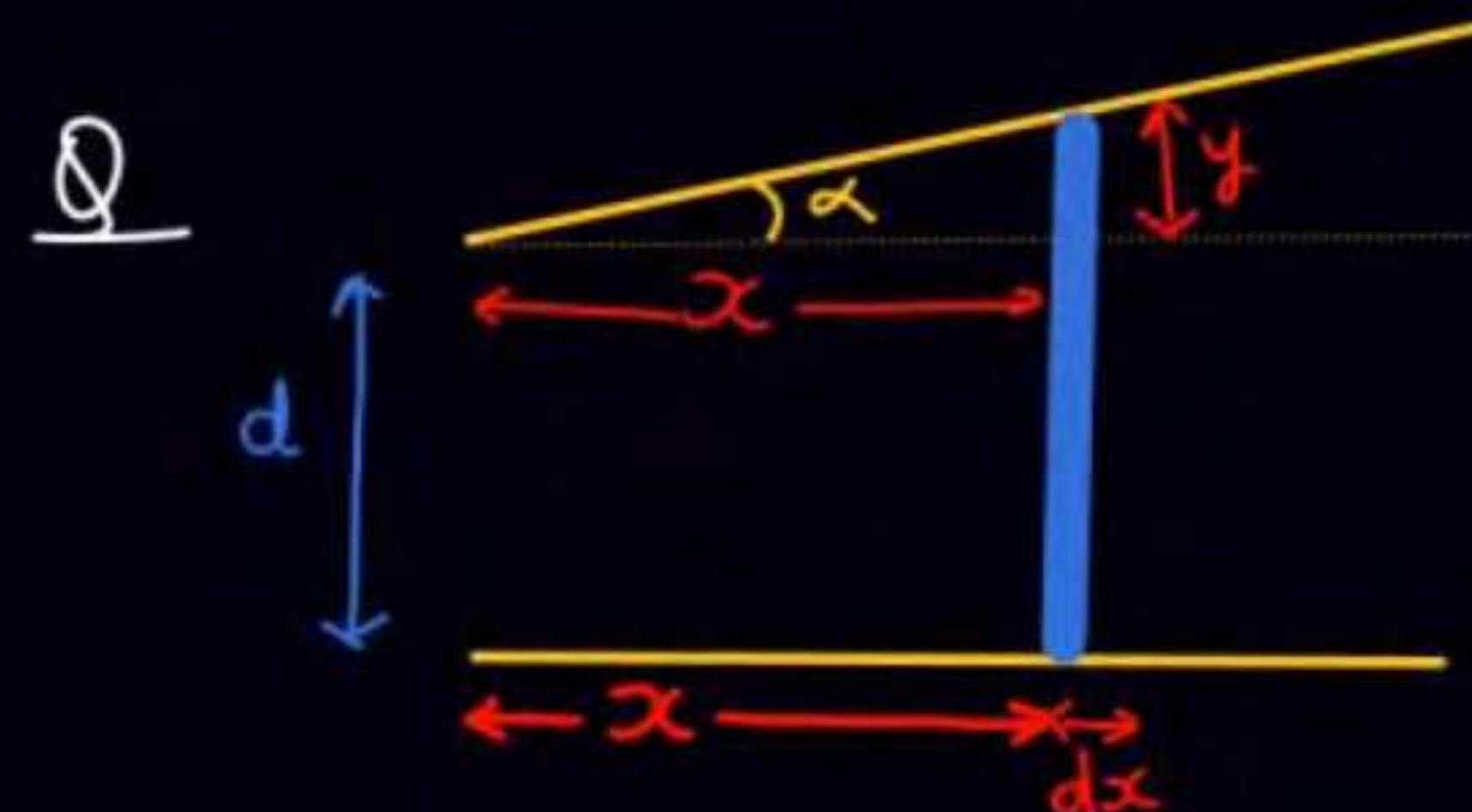
$$E_B = \frac{E_0}{k_2}$$

Q H/W



find C_{eq} . (if α is very small)

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$$\tan \alpha = \frac{y}{x} = \alpha$$

$$y = x\alpha$$

$$dc = \frac{L dx \cdot \epsilon_0}{d + y} = \frac{L dx \epsilon_0}{d + x\alpha}$$

$$C_{eq} = \int dc = \int_0^L \frac{L \epsilon_0 dx}{d + x\alpha} = \frac{L \epsilon_0}{\alpha} \ln \left(\frac{d + L\alpha}{d} \right)$$

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$$C_{eq} = \frac{L \epsilon_0}{\alpha} \ln \left(1 + \frac{L\alpha}{d} \right) \quad \underline{\underline{Ans}}$$

Log Expansion

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$



$$C_{eq} = \frac{L\epsilon_0}{\alpha} \ln\left(1 + \frac{L\alpha}{d}\right)$$

Log Expansion

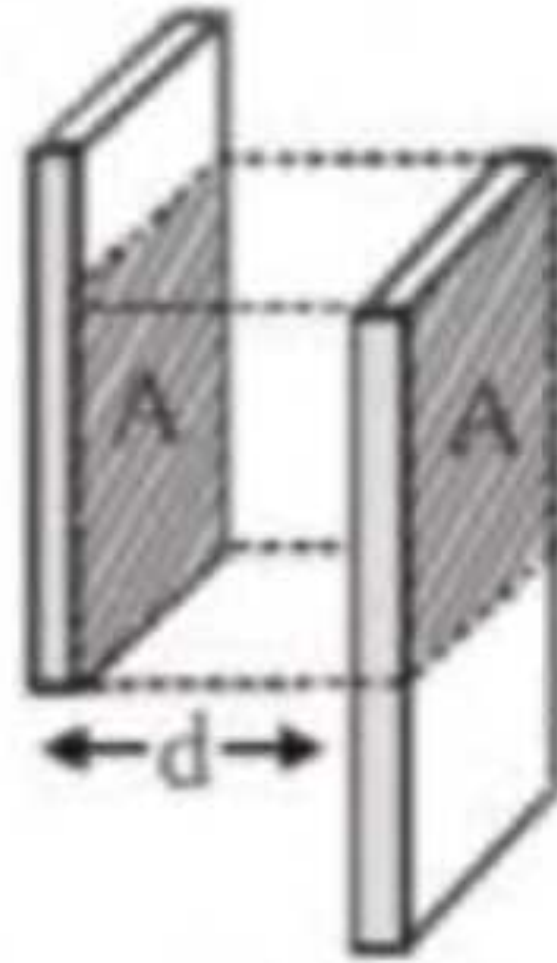
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$C_{eq} = \frac{L\epsilon_0}{\alpha} \left[\frac{L\alpha}{d} - \left(\frac{L\alpha}{d}\right)^2 \frac{1}{2} + \dots \right]$$

$$C_{eq} = \frac{L\epsilon_0}{\alpha} \frac{L\alpha}{d} \left[1 - \frac{L\alpha}{2d} + \dots \right] \quad (\alpha \text{ is very small})$$

$$C_{eq} = \frac{\epsilon_0 L^2}{d} \left[1 - \frac{L\alpha}{2d} \right]$$

- If one of the plates of parallel plate capacitor slides relatively than C decrease (As overlapping area decreases).



$$C = \frac{A_{\text{common}} \epsilon_0}{d}$$

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$$C = \frac{A \epsilon_0}{d}$$



Conducting plates each are placed face to face & equi-spaced at distance d . Area of each plate is half the previous plate. If area of first plate is A . Then the equivalent capacitance of the system shown is :-

पाँच चालक प्लेटों को समान दूरी d पर एक दूसरे के सम्मुख रखा गया है। प्रत्येक प्लेट का क्षेत्रफल पहले वाली प्लेट से आधा है। यदि पहली प्लेट का क्षेत्रफल A हो तो प्रदर्शित निकाय की तुल्यांकी धारिता होगी :-

$$C_1 = \frac{(A/2) \epsilon_0}{d}$$

$$C_2 = \frac{A/4 \epsilon_0}{d}$$

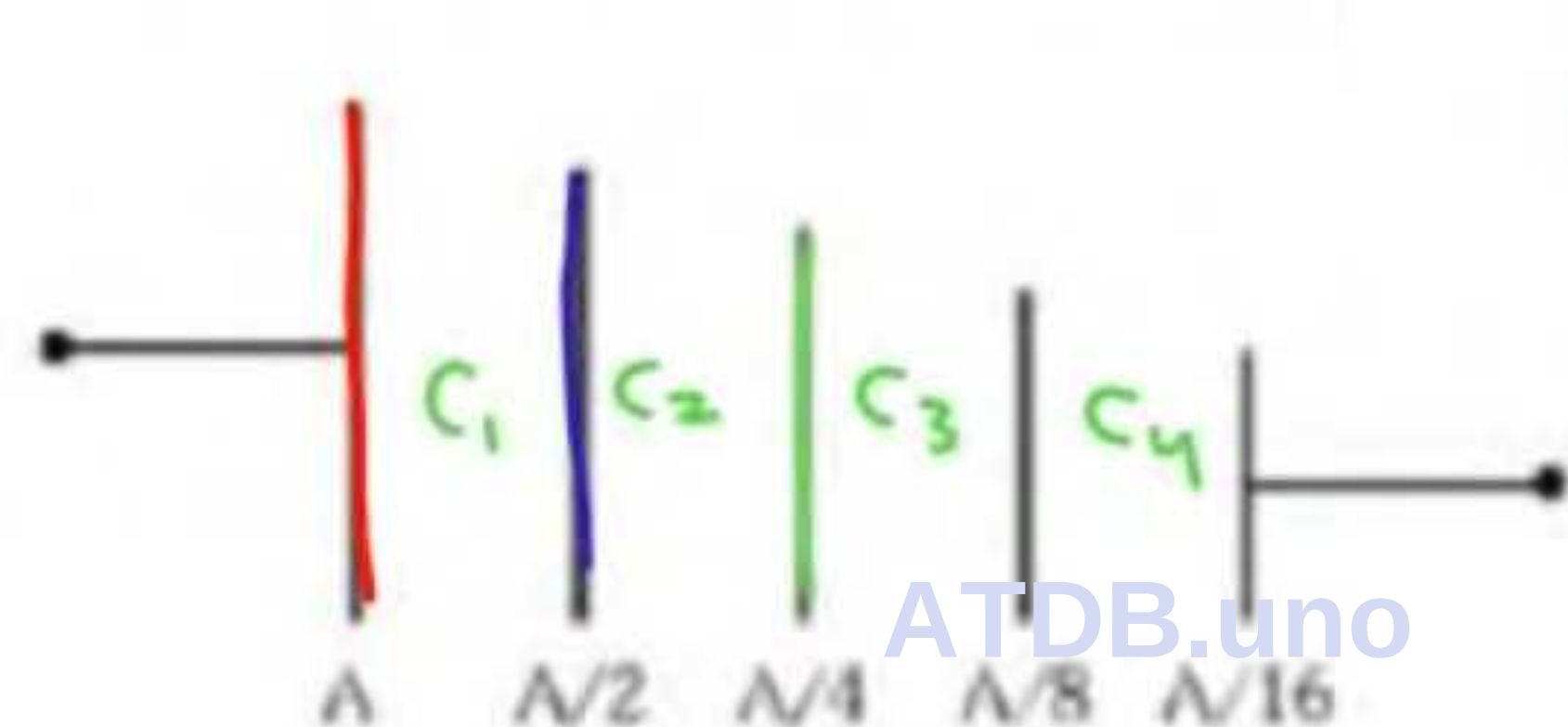
$$C_3 = \frac{(A/8) \epsilon_0}{d}$$

$$(A) \frac{\epsilon_0 A}{d}$$

$$(B) \frac{\epsilon_0 A}{10d}$$

$$(C) \frac{\epsilon_0 A}{20d}$$

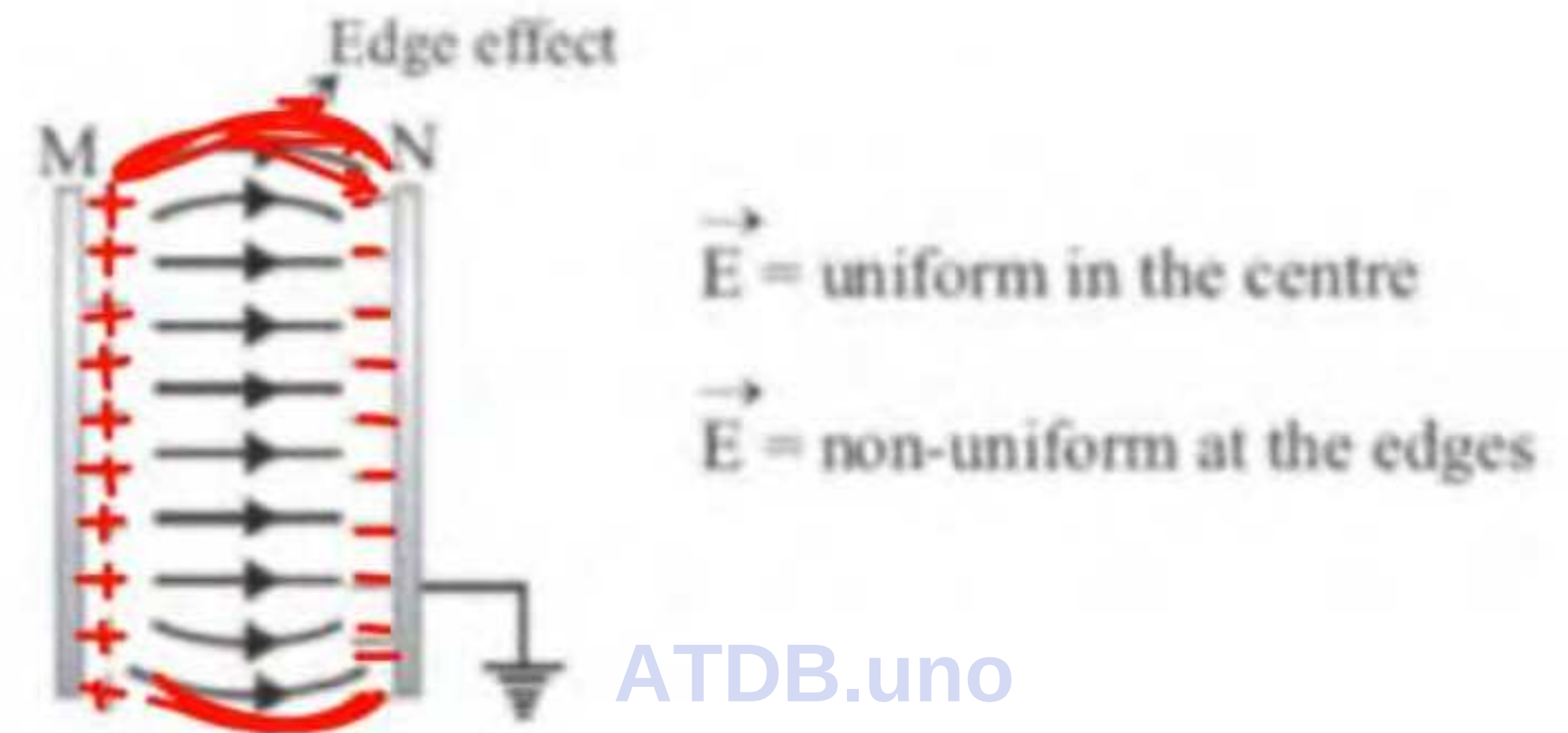
$$(D) \frac{\epsilon_0 A}{30d}$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

$$C_4 = \frac{(A/16) \epsilon_0}{d}$$

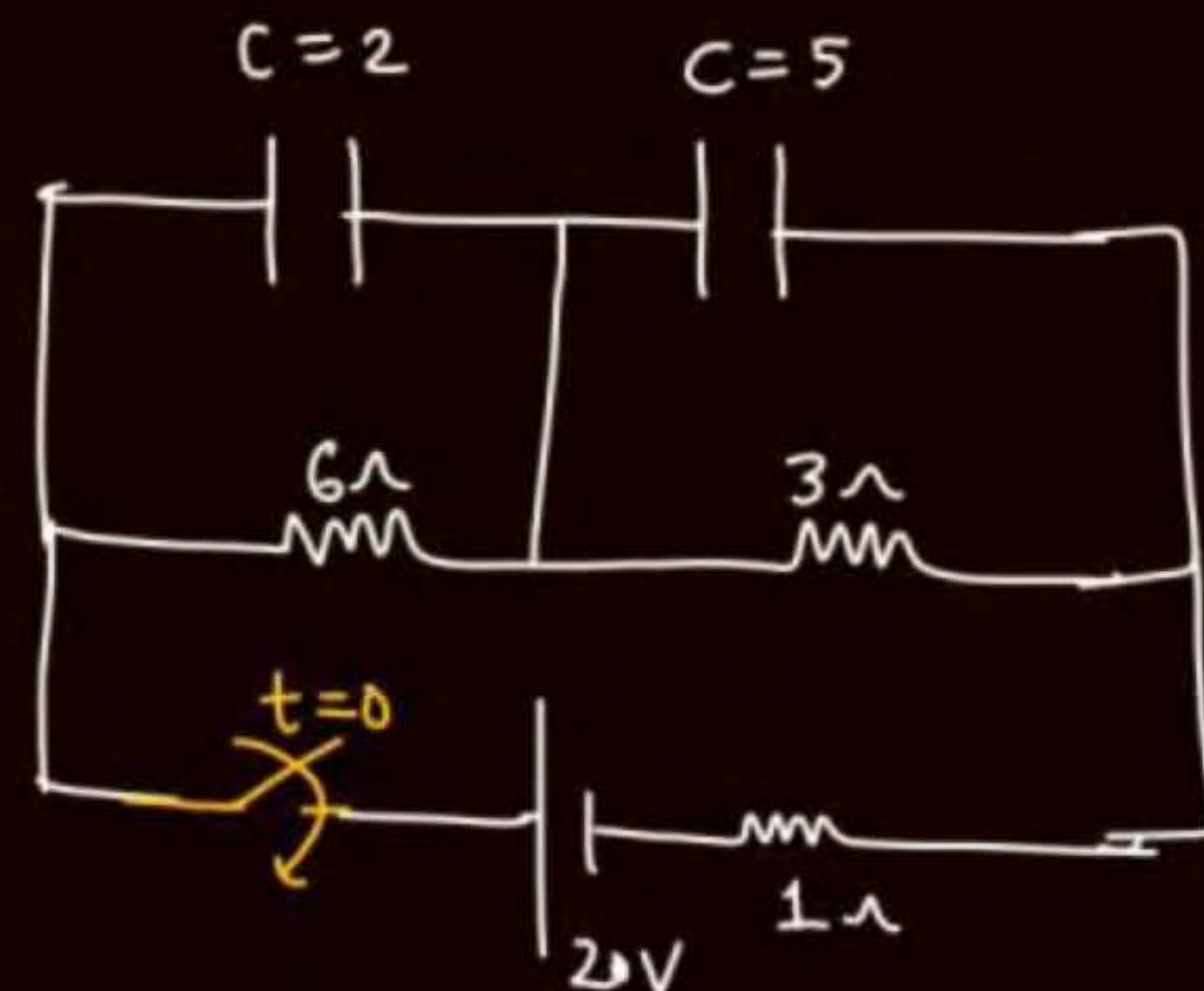
Electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the distance between the plates is very small as compared to the length of the plates.



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Q



① Find current through battery at $t=0^+$

② charge on capacitor at steady state and current passing through battery.

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Ans ① 20A

② 2A, $q_1 = 24$

$q_2 = 6 \times 5 = 30$



$$E_0 = \frac{Q_0}{A\epsilon_0}$$

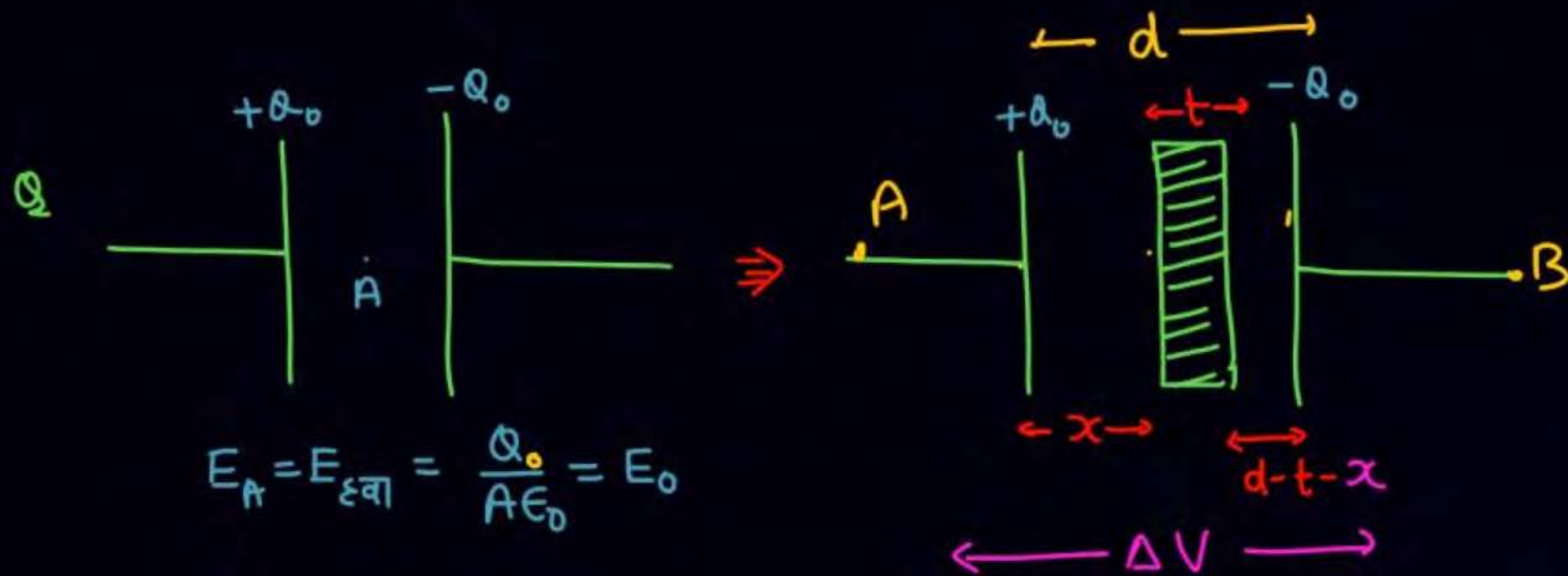
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$$E_A = \frac{E_0}{K}$$

$$C_{\text{नया}} = \frac{A\epsilon_0}{d-t\left(1-\frac{1}{K}\right)}$$

!!!

$$C = \frac{A\epsilon_0}{d}$$



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$$V_A - V_B = \Delta V = E_0 x + \frac{E_0}{K} (t) + E_0 (d - t - x)$$

$$C = \frac{A\epsilon_0}{d - t \left(1 - \frac{1}{K}\right)}$$

$$\Delta V = E_0 \left[\frac{t}{K} + (d - t) \right] = \frac{Q_0}{A\epsilon_0} \left[d - t \left(1 - \frac{1}{K}\right) \right]$$

$$\Delta V = \frac{Q_0}{A\epsilon_0 / d - t \left(1 - \frac{1}{K}\right)} = \frac{Q}{C}$$



THANK YOU

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