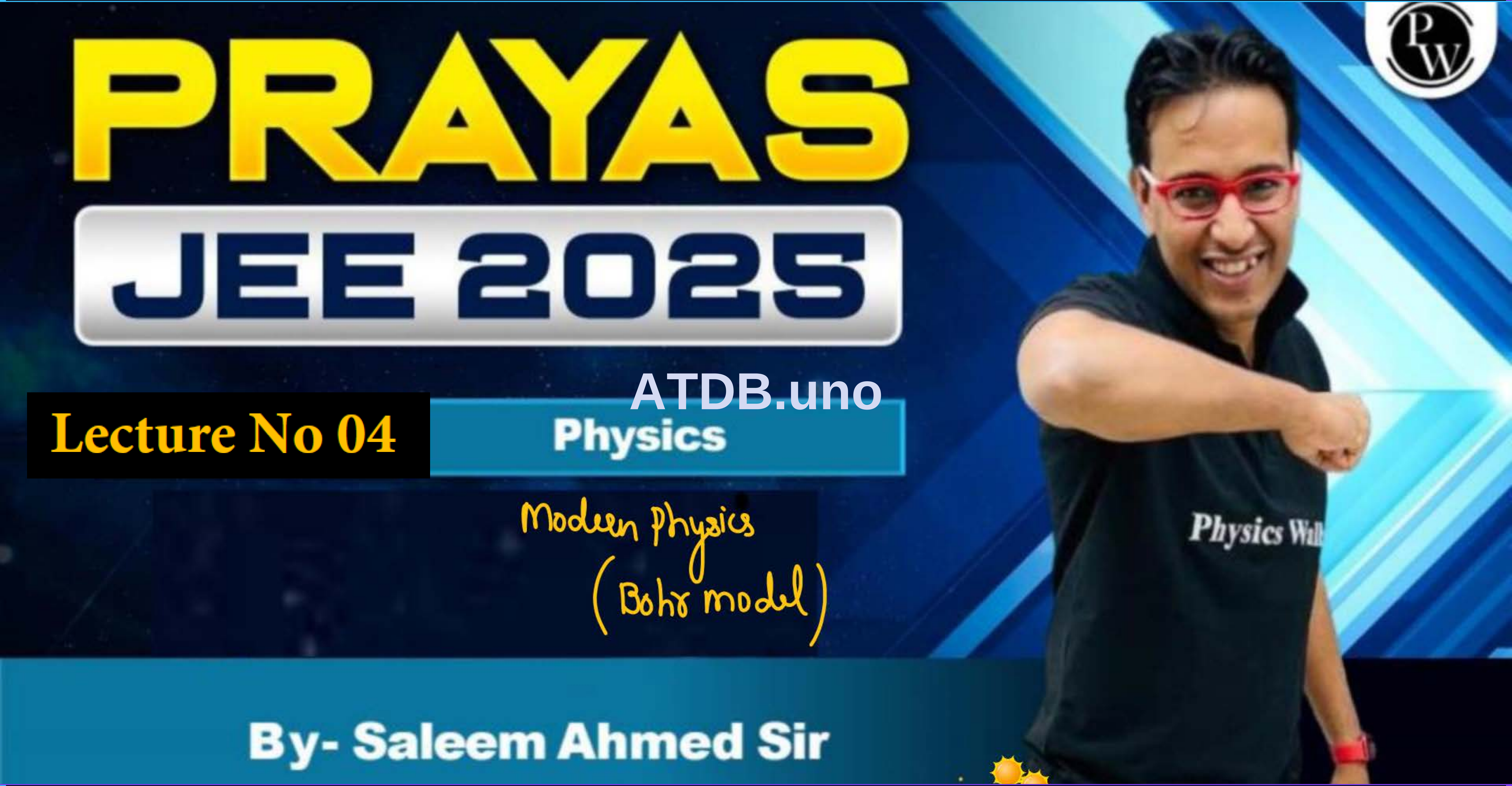

PRAYAS
JEE 2025
ATDB.uno
Lecture No 04
Physics
Modern Physics
(Bohr model)
By- Saleem Ahmed Sir
Physics Wall

# Topics *to be covered*

1. Bohr Model and Energy Level Diagram
2. 
3. 
4. 

Bohr incorporated the following new ideas now regarded as postulates of Bohr's theory:

1. The centripetal force required for an encircling electron is provided by the electrostatic attraction between the nucleus and the electron i.e. $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{mv^2}{r}$ ...(i)

$\varepsilon_0$ = Absolute permittivity of free space = $8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

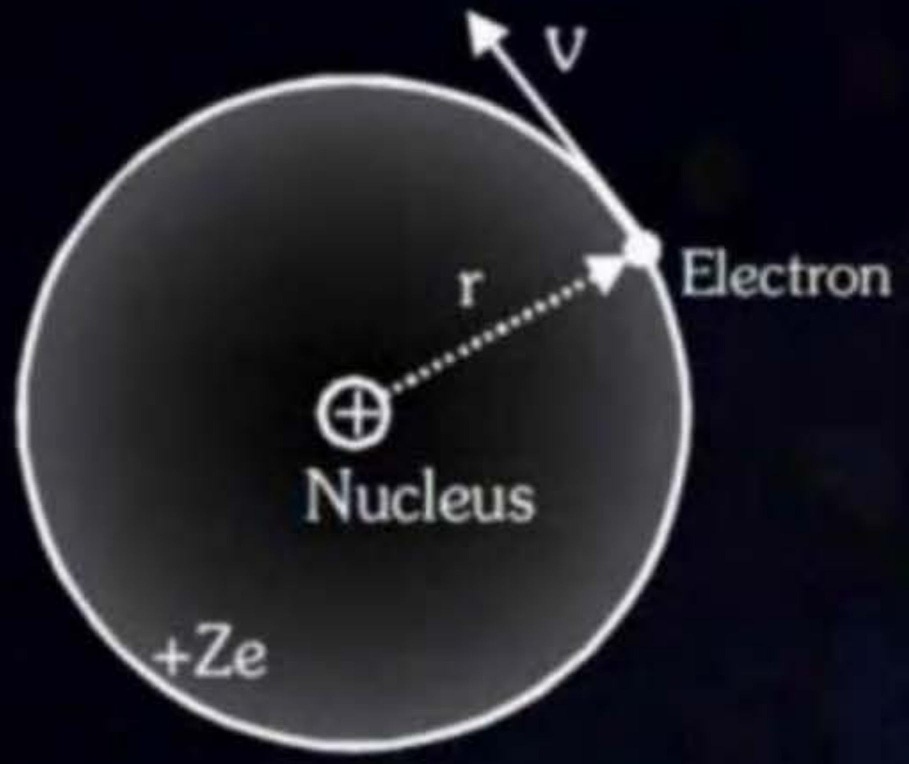


$m$ = Mass of electron

$v$ = Velocity (linear) of electron

$r$ = Radius of the orbit in which electron is revolving.

$Z$ = Atomic number of hydrogen like atom.



Bohr incorporated the following new ideas now regarded as postulates of Bohr's theory.

2. Electrons can revolve only in those orbits in which angular momentum of electron about nucleus is an integral multiple of $\frac{h}{2\pi}$. $i.e., mvr = \frac{nh}{2\pi} \ldots (ii)$

$$mvr = \frac{nh}{2\pi}$$

$n$ = Principal quantum number of the orbit in which electron is revolving.



3. In stable orbit (specific orbit), total energy of $e^-$ remains const its energy changes when it makes a transition from one orbit to another

Bohr incorporated the following new ideas now regarded as postulates of Bohr's theory.

3. Electrons in an atom can revolve only in discrete circular orbits called stationary energy levels (shells). An electron in a shell is characterised by a definite energy, angular momentum and orbit number. While in any of these orbits, an electron does not radiate energy although it is accelerated.

$$\frac{kzee}{r^2} = \frac{mv^2}{r}$$

$$mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$$

$$\frac{kze^2}{r^2} = \frac{m}{r}\frac{n^2h^2}{4\pi^2 m^2 r^2}$$

$$r = \frac{1}{m}\frac{n^2h^2}{4\pi^2 \cdot Ze^2k}$$

$$r = \frac{h^2}{m4\pi^2 e^2k}\left(\frac{n^2}{Z}\right) = .529\frac{n^2}{Z} \text{ Å}$$

mass of $e^-$

$r = ?$

$n \rightarrow$ no. of bohr orbit

$$\boxed{r \propto \frac{n^2}{Z}}$$

$$v = \frac{nh}{2\pi mr} = \frac{nh \, m4\pi^2e^2kZ}{2\pi m \, h^2 n^2}$$

$$v = \frac{h}{2\pi}\frac{4\pi^2e^2k}{h^2}\left(\frac{Z}{n}\right)$$

$$\boxed{v = 2.18 \times 10^6 \frac{Z}{n} \text{ m/s}}$$

① $r \propto \frac{n^2}{Z}$

② $v \propto \frac{Z}{n}$

③ Time period $T = \frac{2\pi r}{v}$

$$T \propto \frac{r}{v} \propto \frac{n^2/Z}{Z/n} \propto \frac{n^3}{Z^2}$$

$$\boxed{T \propto \frac{n^3}{Z^2}}$$

⑥ $a_{centripital} = \frac{v^2}{r} \propto \frac{(Z/n)^2}{n^2/Z}$

$$a_c \propto \frac{Z^3}{n^4}$$

④ frequency $f = \frac{1}{T} \Rightarrow f \propto \frac{Z^2}{n^3}$



⑤ Angular velocity $\omega = \frac{V}{r} = \frac{2\pi}{T}$

$$\omega \propto \frac{Z/n}{n^2/Z} \propto \frac{Z^2}{n^3}$$

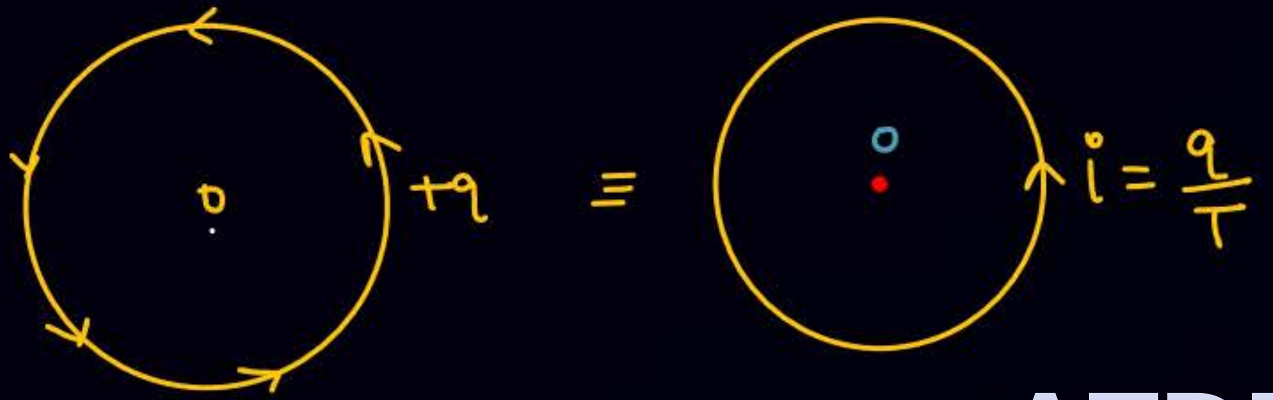
⑦ momentum $= mv \propto \frac{Z}{n}$

⑧ $KE = \frac{1}{2}mv^2 \propto \frac{Z^2}{n^2}$

⑨ $PE = \frac{-K \cdot Ze \cdot e}{r}$

$$PE \propto \frac{Z}{r} \propto \frac{Z}{n^2/Z} \propto \frac{Z^2}{n^2}$$

(10) orbital current $= \dfrac{q}{T} \propto \dfrac{1}{T} \propto \dfrac{Z^2}{n^3}$




(11) Magnetic field at center due to revolving $e^-$



$$B_{center} = \frac{\mu_0 i}{2r}$$

$$B \propto \frac{i}{r} \propto \frac{Z^2/n^3}{n^2/Z}$$

$$\boxed{B \propto \frac{Z^3}{n^5}}$$

(12) magnetic moment

$$\vec{m} = i\vec{A}$$

$$m \propto i\pi r^2 \propto \frac{Z^2}{n^3}\left(\frac{n^2}{Z}\right)^2$$

$$\boxed{m \propto n}$$

(13) Angular momentum $= mvr \propto n$

$$L \propto vr \propto \frac{Z}{n}\cdot\frac{n^2}{Z}$$

$$\boxed{L \propto n}$$

$$\boxed{L = mvr = \frac{nh}{2\pi}}$$

| Physical quantity | Formula | Ratio Formulae of hydrogen atom |
|---|---|---|
| Radius of Bohr orbit ($r_n$) | $r_n = \frac{n^2h^2}{4\pi^2 mkZe^2}$ ; $r_n = 0.53\frac{n^2}{Z}$ Å | $r_1 : r_2 : r_3 ... r_n = 1:4:9 ... n^2$ |
| Velocity of electron in $n^{th}$ Bohr orbit ($v_n$) | $v_n = \frac{2\pi kZe^2}{nh}$ ; $v_n = 2.2 \times 10^6 \frac{Z}{n}$ | $v_1 : v_2 : v_3 ... v_n = 1 : \frac{1}{2} : \frac{1}{3} ... \frac{1}{n}$ |
| Momentum of electron ($p_n$) | $p_n = \frac{2\pi mke^2 z}{nh}$ ; $p_n \propto \frac{Z}{n}$ | $p_1 : p_2 : p_3 ... p_n = 1 : \frac{1}{2} : \frac{1}{3} ... \frac{1}{n}$ |
| Angular velocity of electron($\omega_n$) | $\omega_n = \frac{8\pi^3 k^2 Z^2 mc^4}{n^3h^3}$ ; $\omega_n \propto \frac{Z^2}{n^3}$ | $\omega_1 : \omega_2 : \omega_3 ... \omega_n = 1 : \frac{1}{8} : \frac{1}{27} ... \frac{1}{n^3}$ |

| Physical quantity | Formula | Ratio Formulae of hydrogen atom |
|---|---|---|
| Time Period of electron ($T_n$) | $T_n = \frac{n^3h^3}{4\pi^2k^2Z^2me^4}$ ; $T_n \propto \frac{n^3}{Z^2}$ | $T_1:T_2:T_3...T_n = 1:8:27: ... :n^3$ |
| Frequency ($f_n$) | $f_n = \frac{4\pi^2k^2Z^2e^4m}{n^3h^3}$ ; $f_n \propto \frac{Z^2}{n^3}$ | $f_1 : f_2 : f_3 ... f_n = 1 : \frac{1}{8} : \frac{1}{27} ... \frac{1}{n^3}$ |
| Orbital current ($I_n$) | $I_n = \frac{4\pi^2k^2Z^2me^5}{n^3h^3}$ ; $I_n \propto \frac{Z^2}{n^3}$ | $I_1 : I_2 : I_3 ... I_n = 1 : \frac{1}{8} : \frac{1}{27} ... \frac{1}{n^3}$ |
| Angular momentum ($J_n$) | $J_n = \frac{nh}{2\pi}$ ; $J_n \propto n$ | $J_1:J_2:J_3...J_n = 1 : 2 : 3...n$ |

| Physical quantity | Formula | Ratio Formulae of hydrogen atom |
|---|---|---|
| Centripetal acceleration ($a_n$) | $a_n = \frac{16\pi^4 k^3 Z^3 m e^6}{n^4 h^4}$ ; $a_n \propto \frac{Z^3}{n^4}$ | $a_1 : a_2 : a_3 \ldots a_n = 1 : \frac{1}{16} : \frac{1}{81} \ldots \frac{1}{n^4}$ |
| Kinetic energy ($E_{k_n}$) | $E_{K_n} = \frac{RchZ^2}{n^2}$ ; $E_{K_n} \propto \frac{Z^2}{n^2}$ | $E_{K_1} : E_{K_2} \ldots E_{K_n} = 1 : \frac{1}{4} : \frac{1}{9} \ldots \frac{1}{n^2}$ |
| Potential energy ($U_n$) | $U_n = \frac{-2RchZ^2}{n^2}$ ; $U_n \propto \frac{Z^2}{n^2}$ | $U_1 : U_2 : U_3 \ldots U_n = 1 : \frac{1}{4} : \frac{1}{9} \ldots \frac{1}{n^2}$ |
| Total energy ($E_n$) | $E_n = \frac{-RchZ^2}{n^2}$ ; $E_n \propto \frac{Z^2}{n^2}$ | $E_1 : E_2 : E_3 \ldots E_n = 1 : \frac{1}{4} : \frac{1}{9} \ldots \frac{1}{n^2}$ |

Q Plot the graph between

$\ln A$ Vs $\ln n$ for H atom

$A \longrightarrow$ Area $\quad n \longrightarrow$ no. of orbit

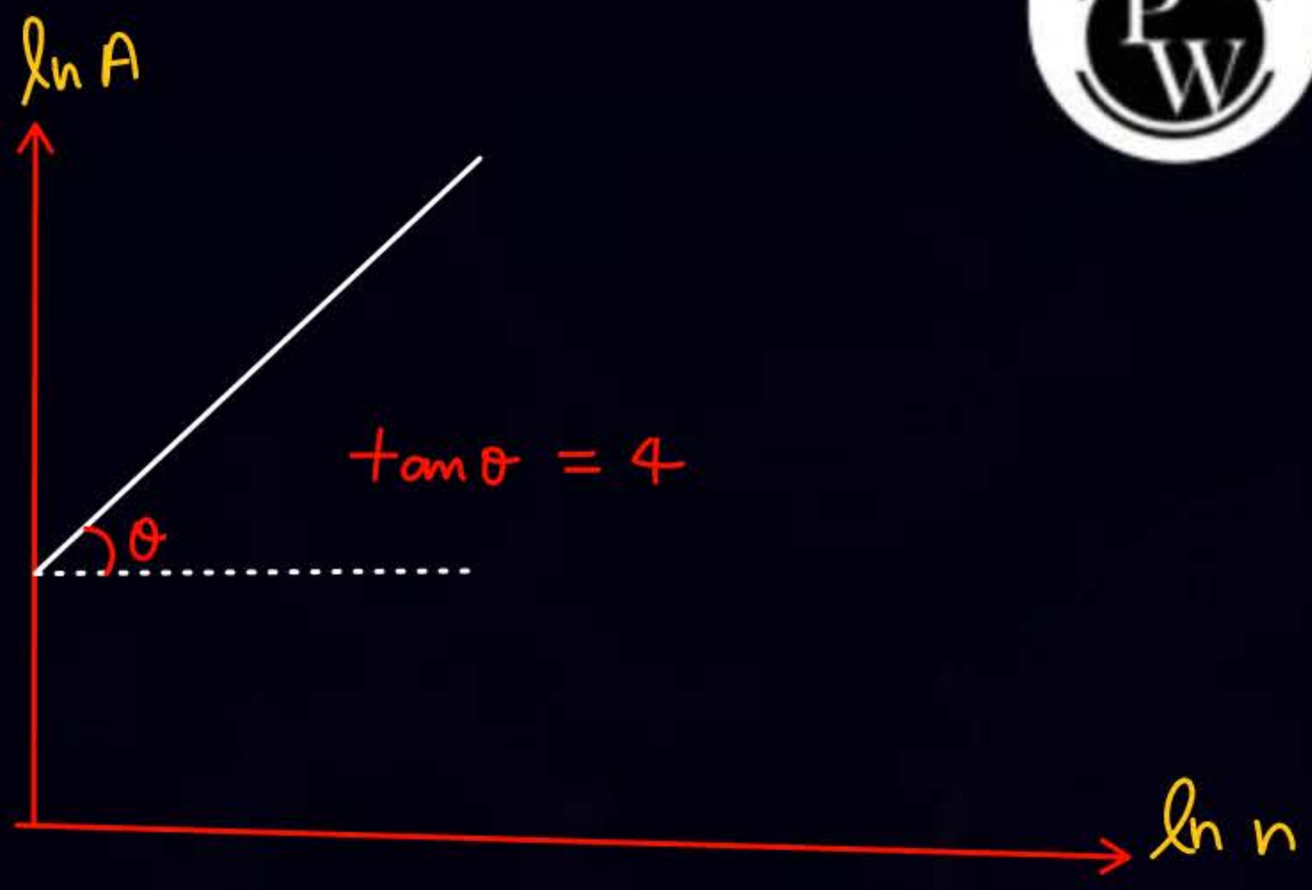
$$A = \pi r^2 \qquad r = K\frac{n^2}{Z}$$

$$A = \pi\left(\frac{Kn^2}{Z}\right)^2$$

$$A = K'n^4$$

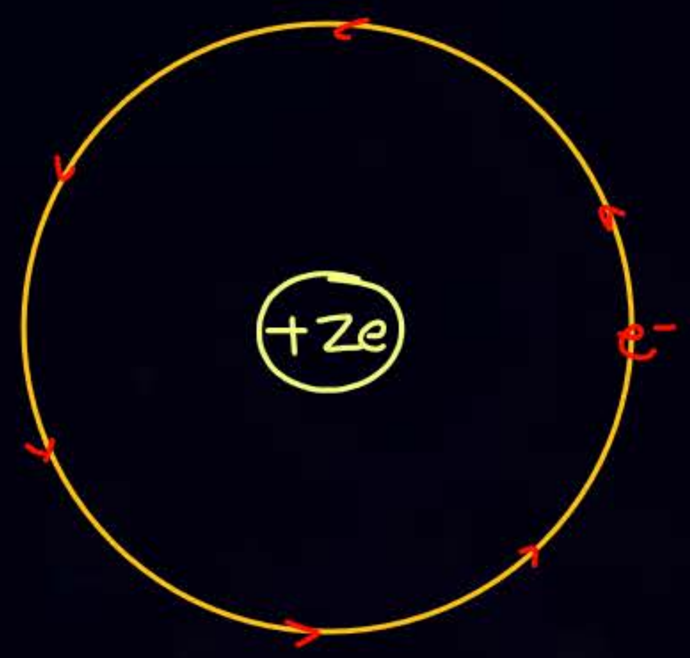
$$\ln A = \ln K' + 4\ln n$$

$$y = mx + c$$



# Energy

$$* \quad \frac{KZee}{r^2} = \frac{mv^2}{r}$$




$$K.E. = \frac{1}{2}mv^2 = \frac{KZe^2}{2r}$$

$$P.E = \frac{KZe(-e)}{r} = -\frac{KZe^2}{r}$$



$$\text{Total Energy} = KE + PE = -\frac{KZe^2}{2r}$$

$$T.E = -\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{2r}$$

$$r = .529\frac{n^2}{Z}\ A^0$$

$$\boxed{KE = |T.E| = \frac{|PE|}{2}}$$

$$\boxed{T.E = -13.6\ \frac{Z^2}{n^2}\ eV}$$

$\ast\; KE = |T.E| = \dfrac{|P.E|}{2}$

for $Z = 1$

(Energy level daigram)

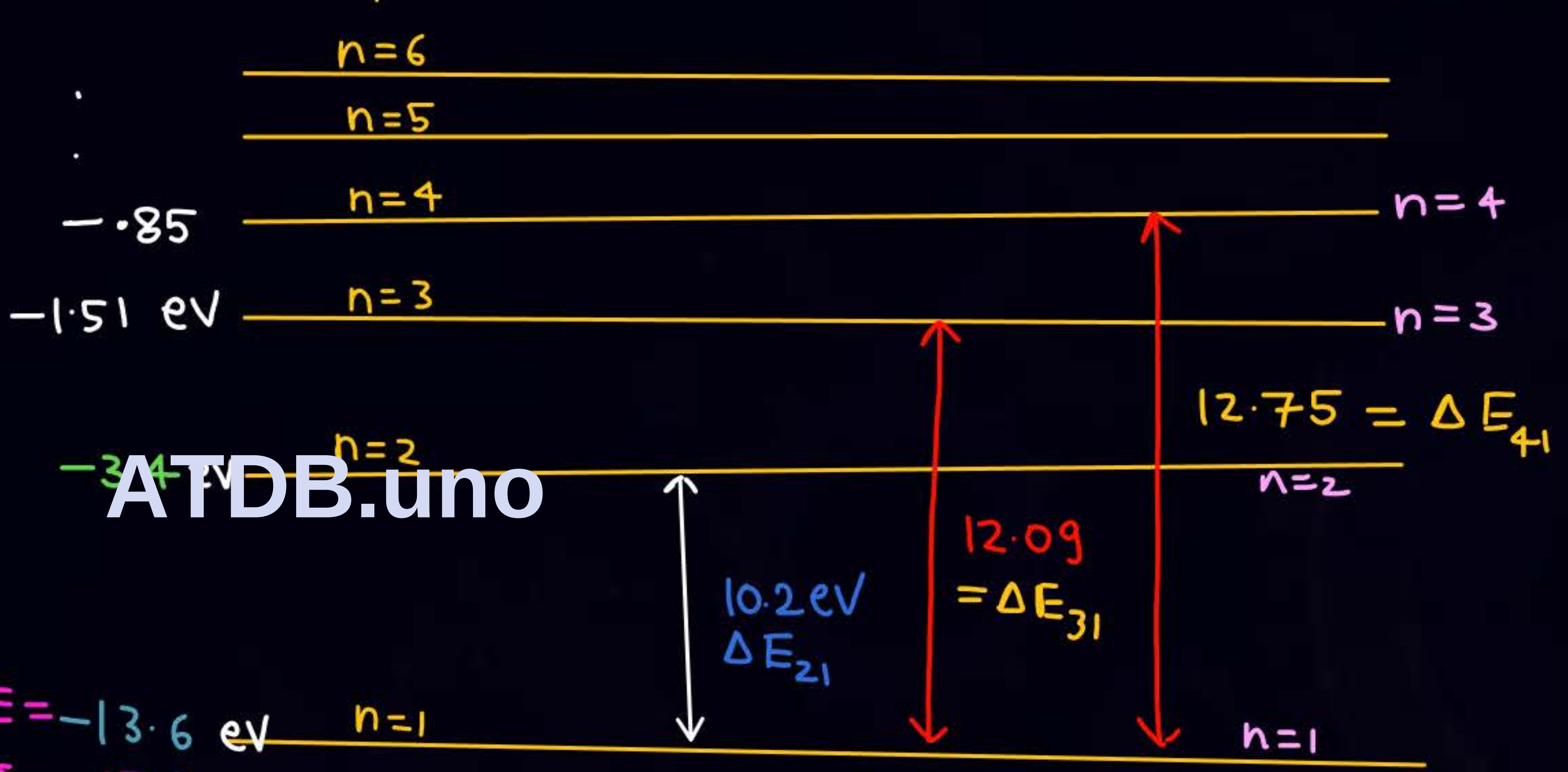
$$\boxed{T.E = -13.6\,\frac{Z^2}{n^2}}$$

$\ast\; n = 2,\quad T.E = \dfrac{-13.6 \times 1^2}{2^2}$

$= -3.4$

$\ast\; n = 3,\quad T.E = \dfrac{-13.6 \times 1^2}{3^2}$

$= -1.51$




$T.E = -13.6$ eV

$KE = 13.6$

$PE = -27.2$

- Electron in lower energy state absorb a photon of energy Equal to energy difference between two state to get excited to higher energy state.

- when an electron make transition from a higher energy level to lower energy level it emit a photon with energy equal to the energy difference between initial & final levels.

# 1 - In ground state $e^-$ can absorb only those photon which have energies equal to the difference in energies of the stable energy level with ground state.

# Energy required to ionise an atom is called ionisation energy of atom for that particular energy level



$$\Delta E_{n \to \infty} = E_{\infty} - E_n = 0 - \left(-\frac{13.6 Z^2}{n^2}\right)$$

$$= \frac{13.6 Z^2}{n^2}$$

Energy of photon required to excite $e^-$ from $n_i = n_1$ to $n_f = n_2$ will be equal to

$$\Delta E = 13.6\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{hc}{\lambda} = \frac{1240}{\lambda\,(\text{nm})}$$

If an electron de-excite from $n_2$ to $n_1$ (heigher $\longrightarrow$ Lower) then energy of photon emitted $= \Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{1240}{\lambda}$

Q. Find energy of photon required to excite electron of $z=1$ atom from $n=1$ to $n=5$

$$\Delta E = 13.6 \times 1^2\left(\frac{1}{1^2}-\frac{1}{5^2}\right) = \frac{1240}{\lambda} \rightarrow \checkmark$$



Q. Find energy of photon required to excite electron of $z=1$ atom from first excited state to 3rd excited state.

Sol^n

$$n=2 \longrightarrow n=4$$

$$\Delta E = 13.6 \times 1^2\left(\frac{1}{2^2}-\frac{1}{4^2}\right)$$

Skip

* when external radiation is given to the hydrogen atom, electron in ground state jump to the higher energy state & atom is now called in excited state. Any excited state is unstable state & max lifeline of an excited state is of order of $10^{-8}$ seconds i.e. after the lifeline of the excited state $e^-$ jump to the ground state again directly or indirectly by emitting one or more electromagnetic Radiation. It may have many path to come to ground state



* Excitation of atom ≡ excitation of $e^-$

- when electron deexcite from $n = n$ to ground state

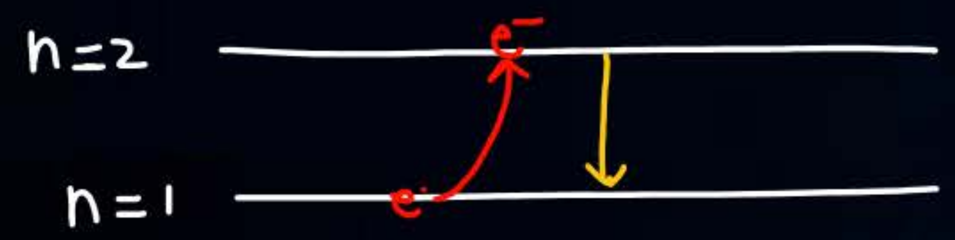
Total no. of way $= 2^{n-2}$

Total no. of different $\lambda$ of photon possible $\Rightarrow {}^{n}C_2$

## Dexcitation

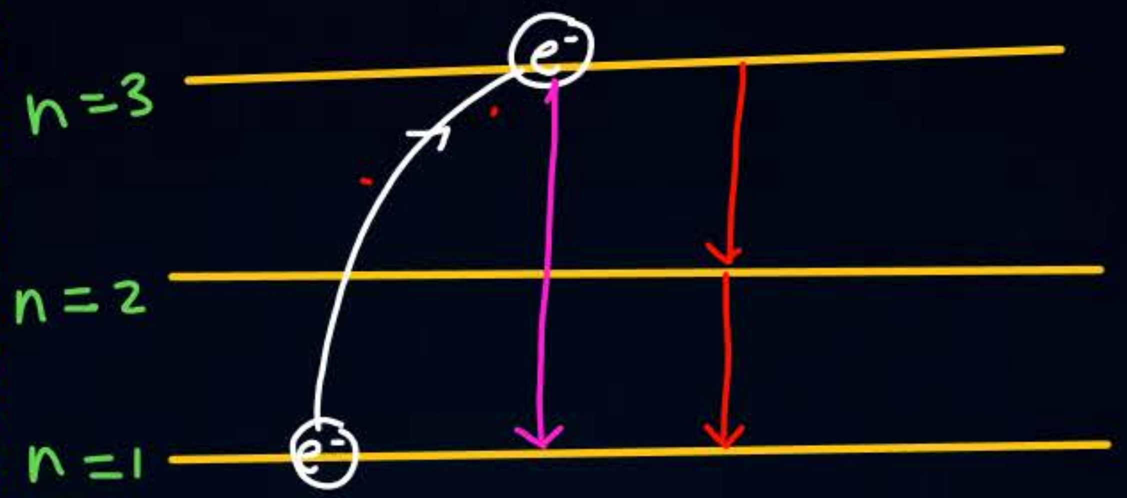
- An $e^-$ in higher energy state emit a photon of energy exactly equal to the energy difference b/w two corresponding state to return to a lower energy state.

$$\Delta E = 13.6 Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$




$^nC_2 \equiv$ Different $\lambda$

$2^{n-2} \equiv$ Diff. ways



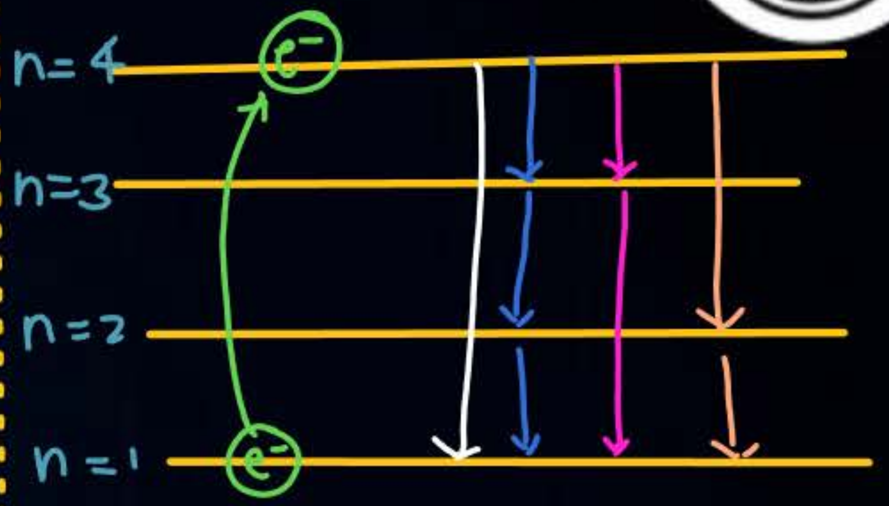

$3 \longrightarrow 1$

$3 \longrightarrow 2 \longrightarrow 1$

} 2 No. of ways



* 3 diff $\lambda$ of photon




* $4 \longrightarrow 1$

$4 \rightarrow 3 \rightarrow 2 \rightarrow 1$

$4 \rightarrow 3 \longrightarrow 1$

$4 \longrightarrow 2 \longrightarrow 1$

} 4 ways

* $\Delta E_{41}, \Delta E_{43}, \Delta E_{42}, \Delta E_{31}$, $\Delta E_{32}$

6 तरह के $\lambda$

Q If an $e^-$ deexcite from $n=3$ to $n=1$ of $Z=1$ atom directly. find

① Energy of photon release in eV

Joule में

$$\Delta E = 13.6 \times 1^2 \times \left(\frac{1}{1^2} - \frac{1}{3^2}\right) eV$$



② wavelength of photon released

$$13.6\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{1240}{\lambda}$$

③ Frequency of photon released.

$$c = f\lambda$$

$$E = h\nu$$

Joule

- $1^{st}$ line of Lyman series

  $2 \longrightarrow 1$

- $2^{nd}$ Line of Lyman Series

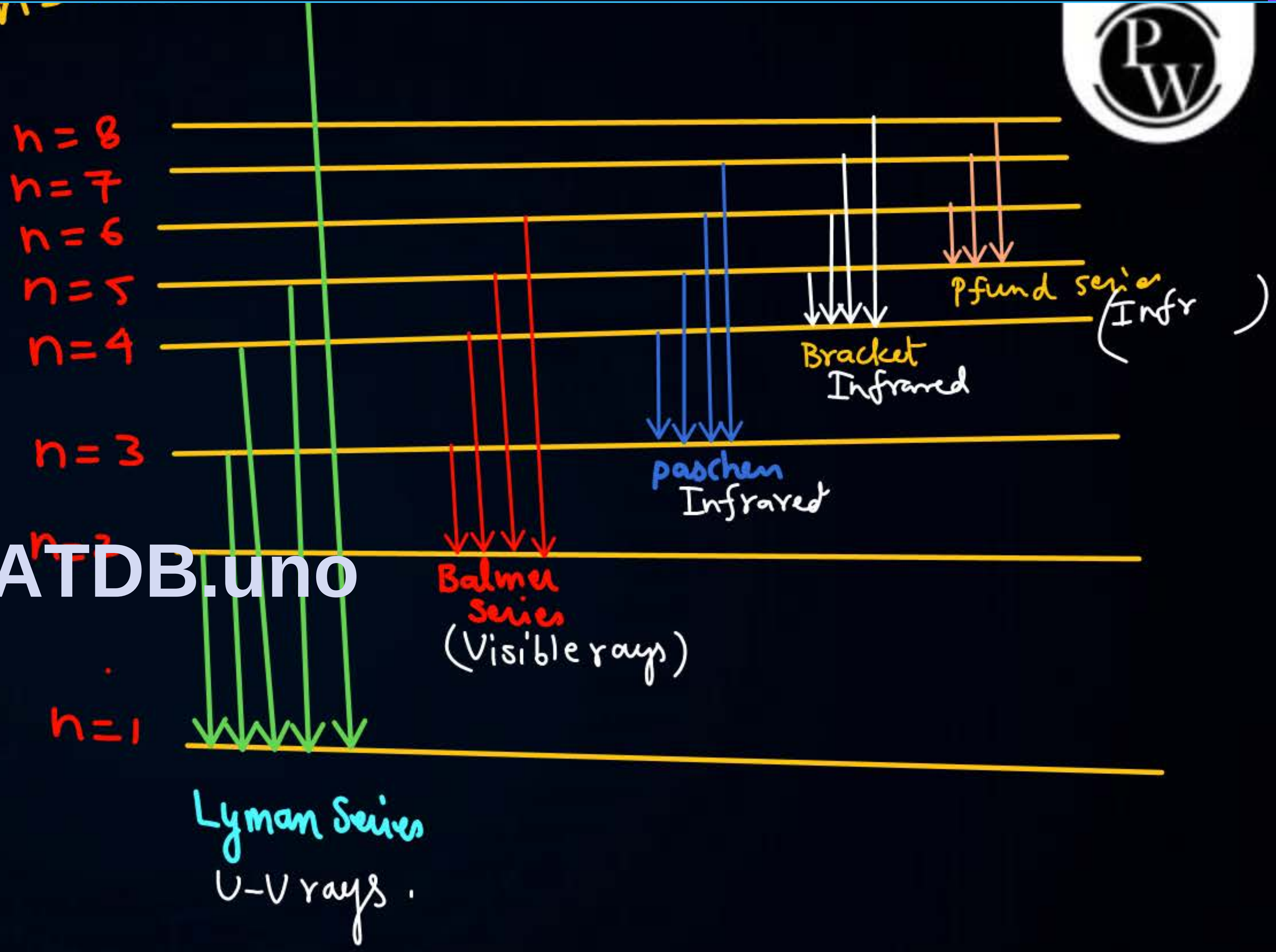
  $3 \longrightarrow 1$

- $5^{th}$ line of balmer Series

  $7 \longrightarrow 2$

- $2^{nd}$ line of paschen Series

  $5 \longrightarrow 3$



Q. find the wavelength of photon emitted in $2^{nd}$ line of balmer series for $z = 1$

Sol$^n$ $4 \longrightarrow 2$

$$\Delta E = 13.6\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{1240}{\lambda}$$

$\lambda \longrightarrow (nm)$

Q find the wavelength of photon emitted in $4^{th}$ line of paschen series for $Z = 1$

Sol^n

$$7 \longrightarrow 3$$

$$\Delta E = 13.6\left(\frac{1}{3^2} - \frac{1}{7^2}\right) = \frac{1240}{\lambda} \longrightarrow (nm)$$

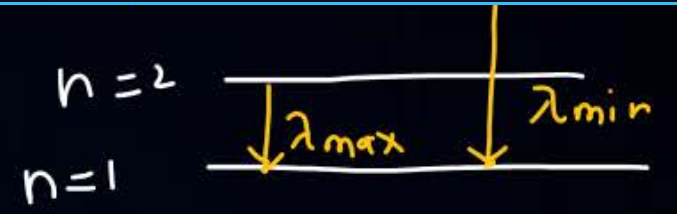
# $\Delta E = \frac{hc}{\lambda} = 13.6\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{1240}{\lambda_{nm}}$

$$\frac{1}{\lambda} = \frac{13.6}{hc} Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad R = \frac{13.6}{hc} = 1.097 \times 10^7\ m^{-1}$$

$R \longrightarrow$ Rydberg const.

$$\boxed{\frac{1}{\lambda} = \text{wavenumber}}$$




* **Lyman Series** $\Rrightarrow$

(Z = 1)

$\lambda_{min}$ when $\infty \longrightarrow 1$

$\lambda_{max}$ = Longest $\lambda$ possible $\Rrightarrow (\Delta E)_{min} \Rrightarrow n=2 \longrightarrow n=1$

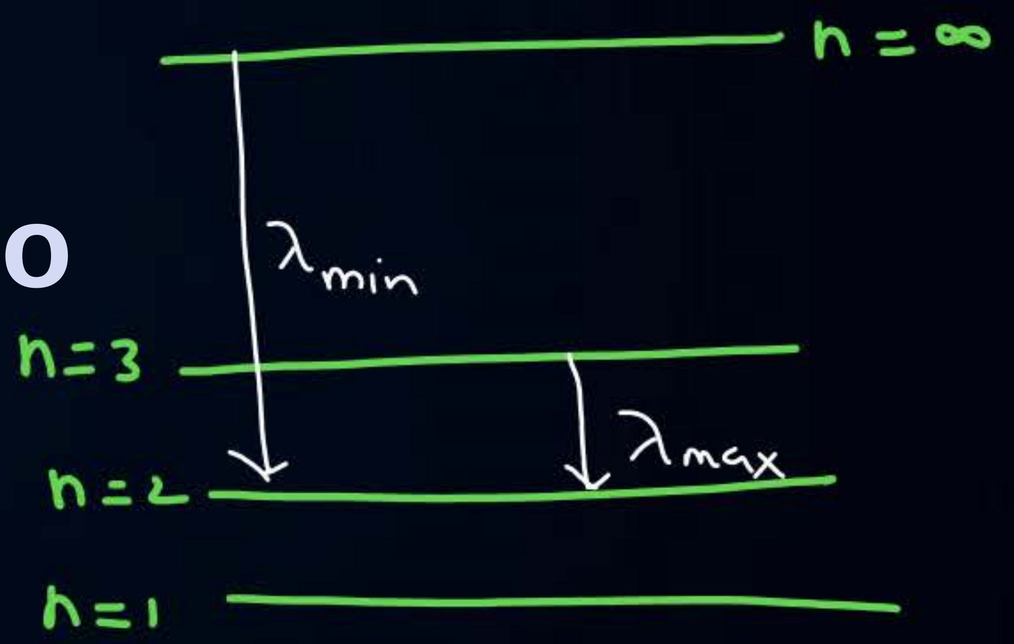
$\lambda \equiv$ (91.6 nm to 121.6 nm)
for Z = 1



* **Balmer Series**

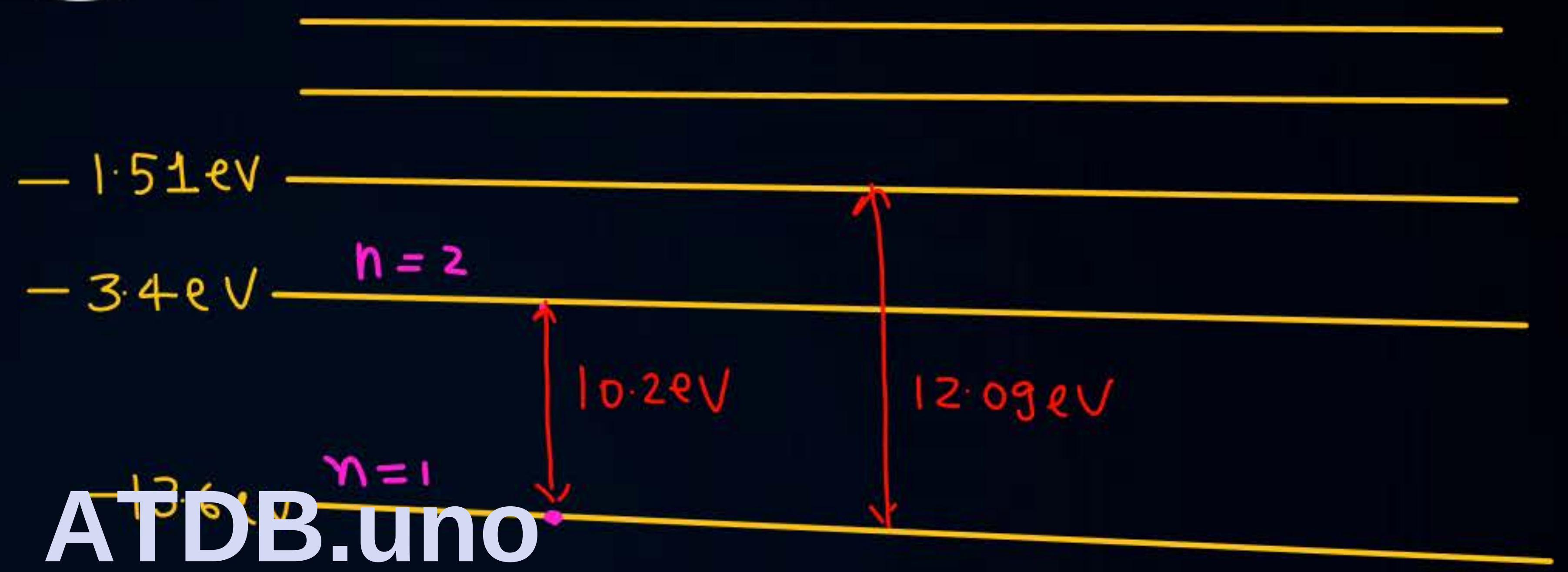
$\lambda_{min} \Rrightarrow \infty \longrightarrow 2$

$\lambda_{max} \Rrightarrow 3 \longrightarrow 2$



$$\frac{13.6 Z^2}{n^2}$$

$Z = 1$




* Ionisation energy for $n = 1 \Rightarrow 13.6\,eV$

* Ionisation potential for $n = 1 \Rightarrow 13.6$ Volt

* Excitation energy for $n = 1$ to $n = 2 \Rightarrow 10.2\,eV$

or $13.6 Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

just take reading

atom corresponds to infinite separation between electron and nucleus. Total positive energy implies that the atom is ionized and electron is in unbound (isolated) state moving with certain kinetic energy. Minimum energy required to move an electron from ground state to $n = \infty$ is called ionization energy of the atom or ion.

$$E_{\text{ionization}} = E_{\infty} - E_n = -E_n = \frac{13.6Z^2}{n^2}\ \text{eV}$$

Ionization energy of H atom = 13.6 eV

Ionization energy of $He^+$ ion = 54.4 eV

Ionization energy of $Li^{++}$ ion = 122.4 eV

**Ionization potential (IP):** Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionization energy of the atom is called ionization potential of the atom.

$$V_{\text{ionization}} = \frac{E_n}{e} = \frac{13.6Z^2}{n^2}\ \text{V}$$

IP of H atom = 13.6 V

IP of $He^+$ ion = 54.4 V


THANK
YOU