

PRAYAS

JEE 2025

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Lecture - 09

Physics

Oscillations

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Topics *to be covered*

Time period of STM In Various Cases.

1

2

3

4

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Simple pendulum

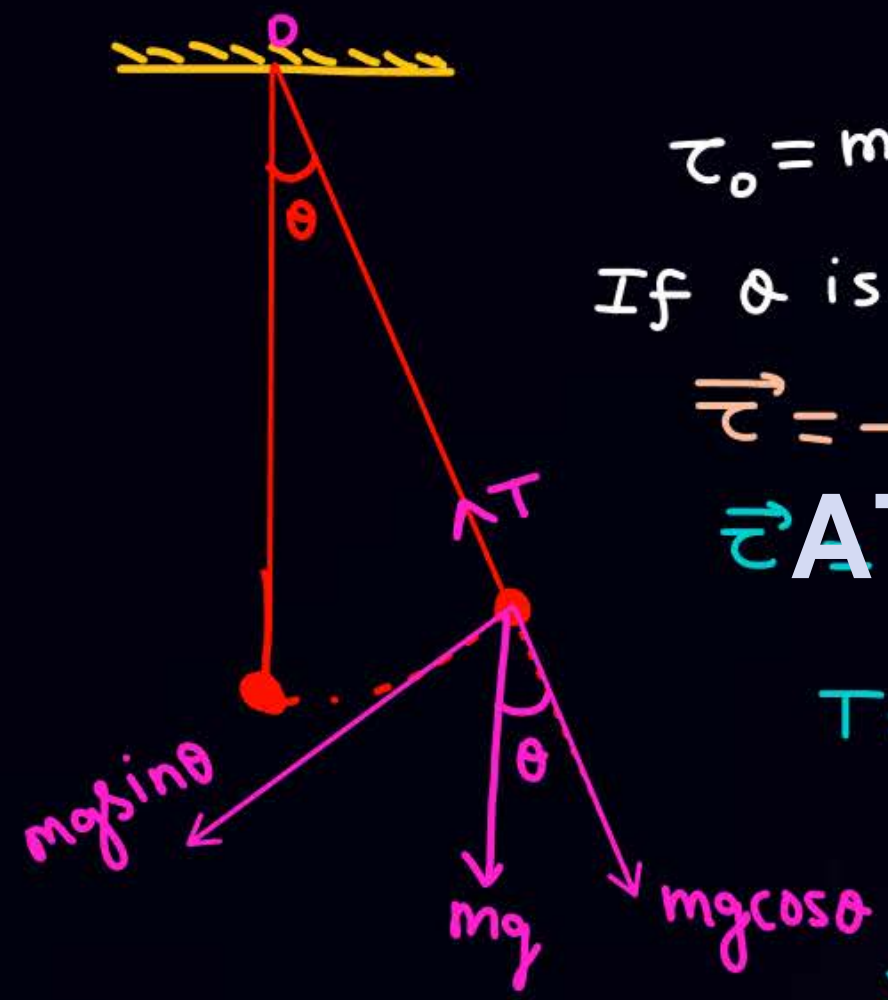
$$T = 2\pi \sqrt{\frac{l}{g_{eff}}}$$

second pendulum $T = 2 \text{ sec}$
 $l = 1 \text{ m}$

$$T = 2\pi \sqrt{\frac{I}{mgd_c}}$$

$$T = 2\pi \sqrt{\frac{mL^2}{mgL}}$$

$$T = 2\pi \sqrt{\frac{L}{g}}$$



$$\tau_o = mgsin\theta \cdot L$$

If θ is very small $sin\theta \approx \theta$

$$\vec{\tau} = -mgL \vec{\theta}$$

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$$T = 2\pi \sqrt{\frac{I}{K}} = 2\pi \sqrt{\frac{mL^2}{mgL}}$$

$$T = 2\pi \sqrt{\frac{L}{g}}$$



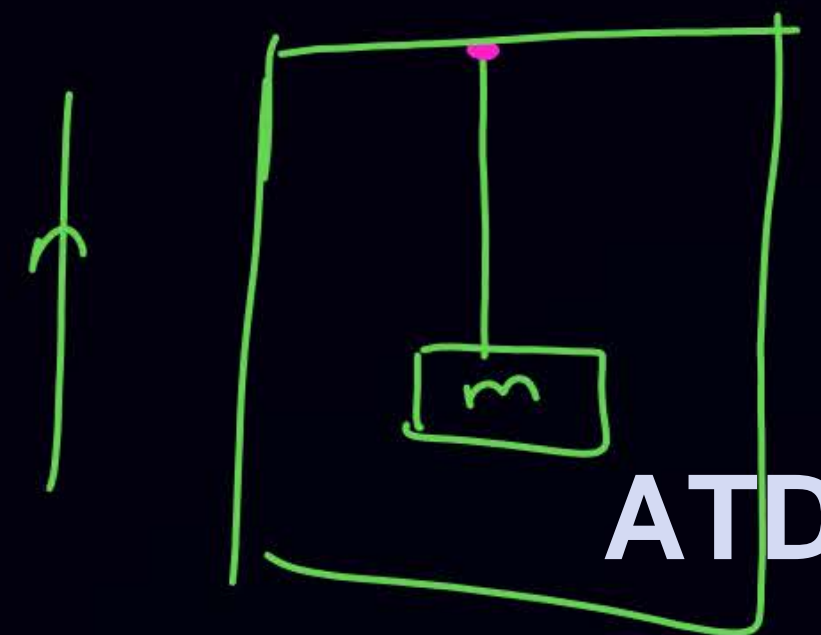
$$sin\theta \approx tan\theta \approx \frac{x}{L}$$

$$F_{net} = mgsin\theta \approx mg \frac{x}{L}$$

$$\vec{F}_{net} = -\frac{mg}{L} \vec{x}$$

$$T = 2\pi \sqrt{\frac{m}{mg/L}} = 2\pi \sqrt{\frac{L}{g}}$$



$a = 6$ 

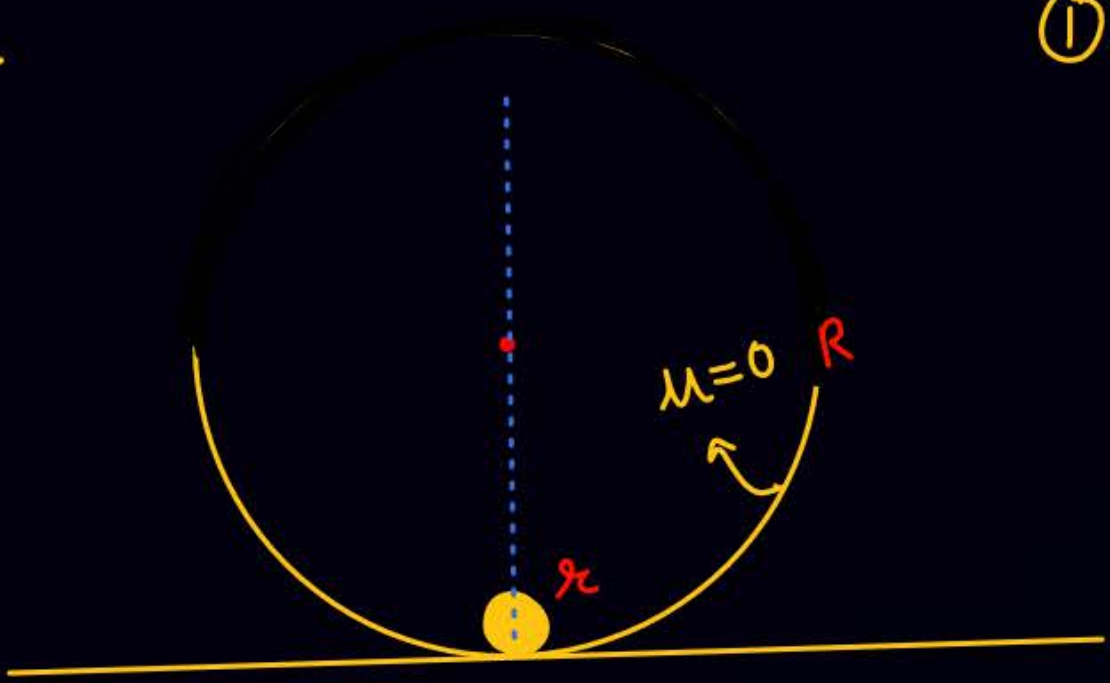
$y = 3t^2$
 $v_y = 6t$
 $a_y = 6$

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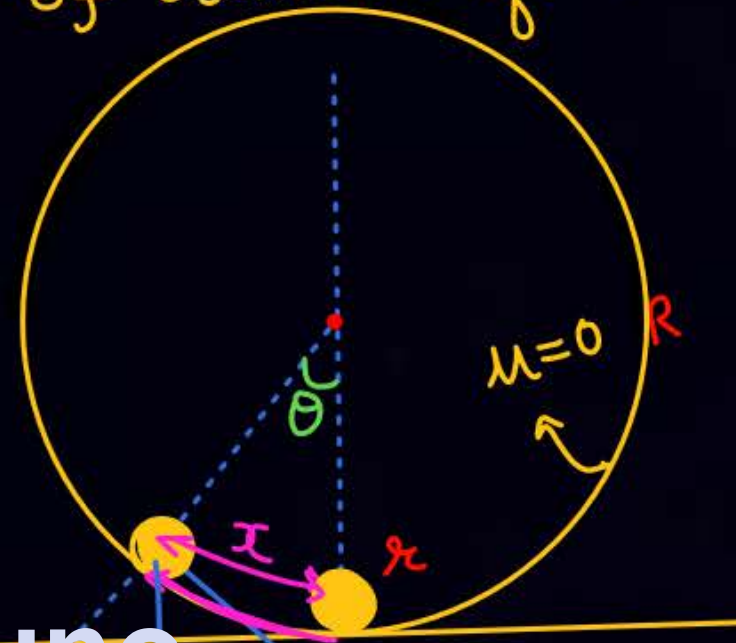
$T = 2\pi \sqrt{\frac{l}{g+a}} = 2\pi \sqrt{\frac{l}{16}}$



Q



① Inner surface of hemisphere is smooth
find T of oscillation of ball. (solid sphere) (m, r)



$$a = g \sin \theta$$

$$a = g \cdot \frac{x}{R-r}$$

$$\vec{a} = -\frac{g}{R-r} \vec{x}$$

$$\omega^2 = \frac{g}{R-r}$$

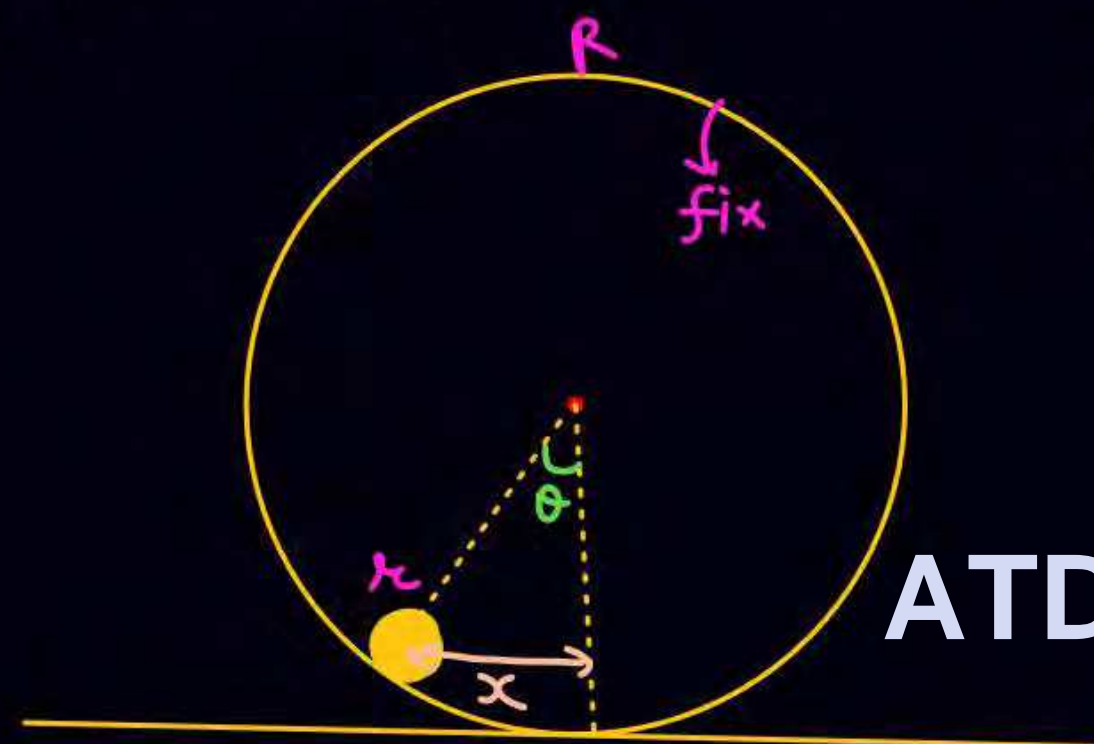
$$T = 2\pi \sqrt{\frac{R-r}{g}}$$

If ball is very small
 $r \ll R$

$$T = 2\pi \sqrt{\frac{R}{g}}$$

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⑥ $\mu \neq 0$, inner surface have sufficient friction for rolling. Repeat the above prob.



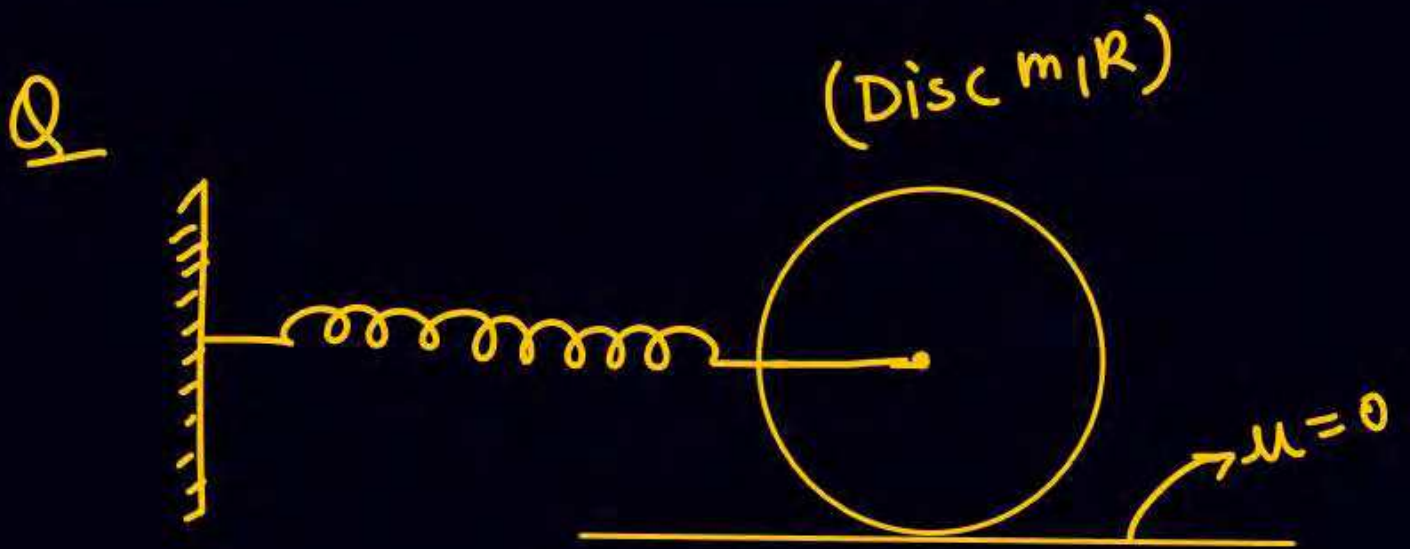
$$a = \frac{g \sin \theta}{1 + \frac{I}{mR^2}} = \frac{g \sin \theta}{1 + \frac{2mR^2}{5mR^2}} = \frac{5}{7} g \sin \theta$$

$$a = \frac{5}{7} g \frac{x}{R-r}$$

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$$\vec{a} = -\frac{5g}{7(R-r)} \vec{x}$$

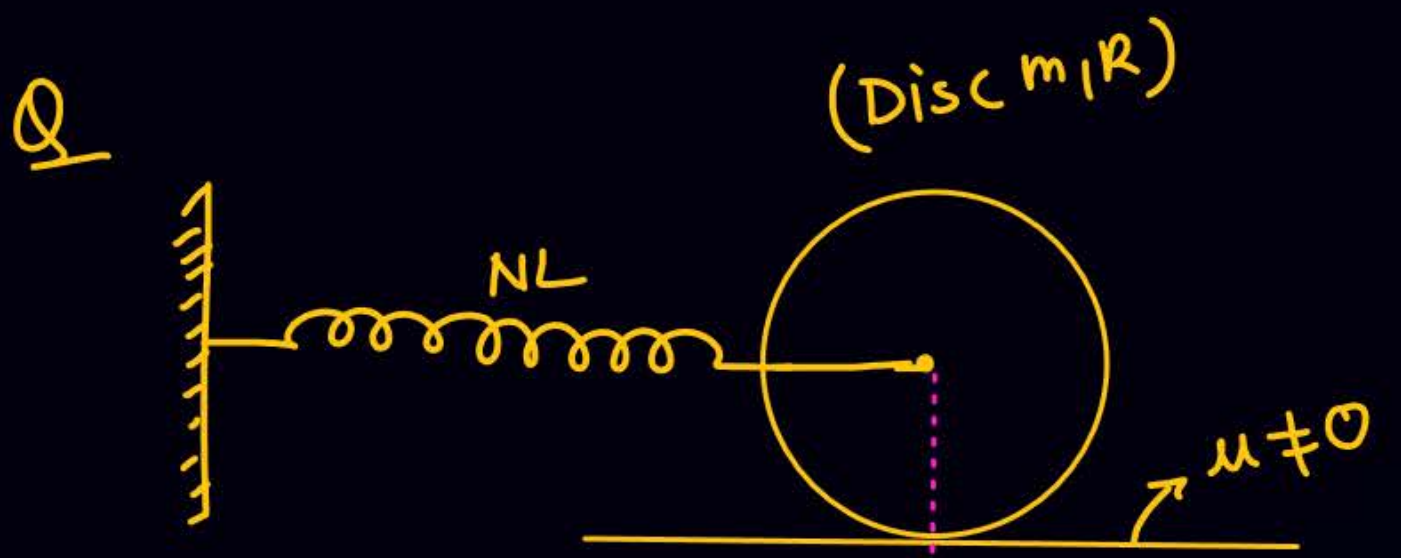
$$T = 2\pi \sqrt{\frac{7(R-r)}{5g}}$$



$$T = 2\pi \sqrt{\frac{m}{k}}$$

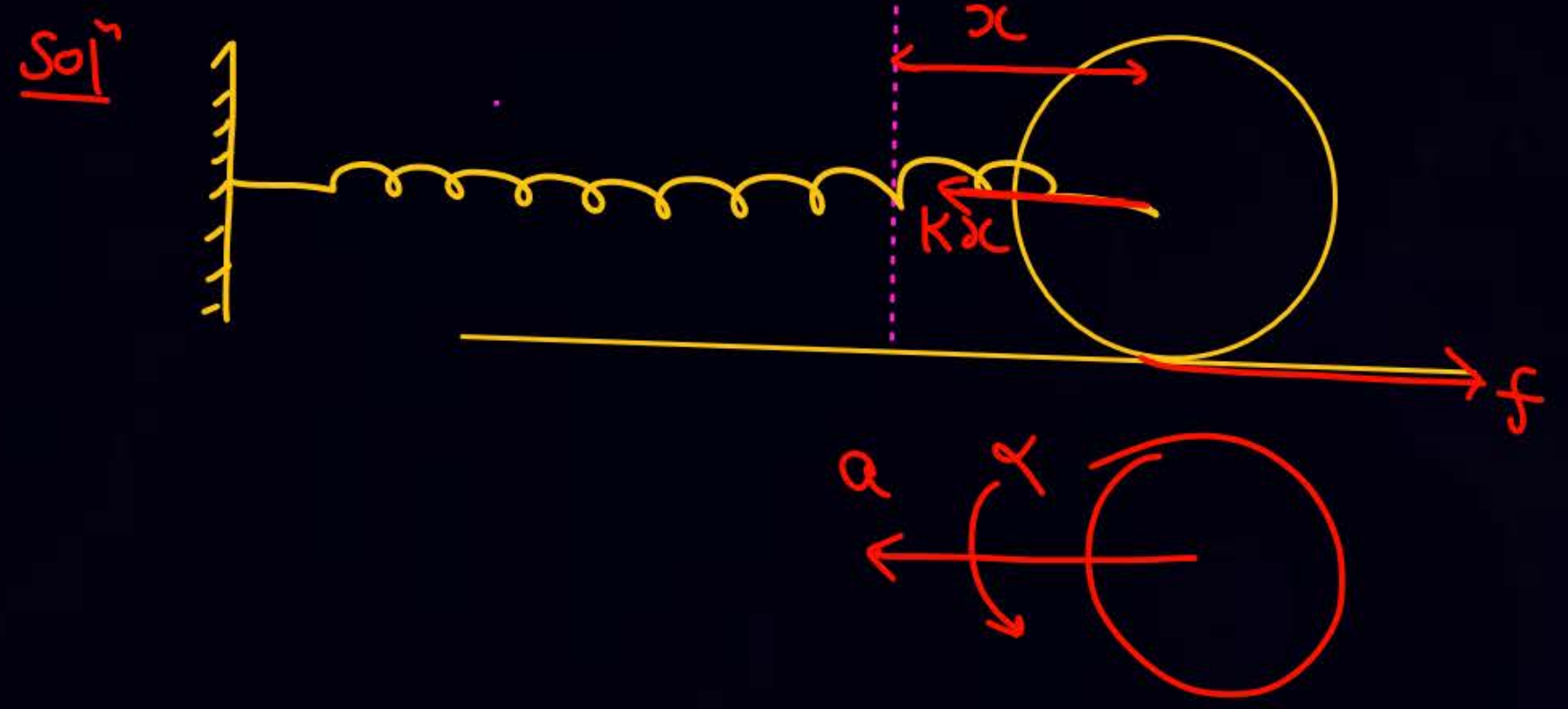
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friction is sufficient for pure rolling
find T of oscillation for small displacement.

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$$kx - f = ma$$

$$f \cdot R = I\alpha = \frac{mR^2}{2} \frac{a}{R}$$

$$f = \frac{ma}{2}$$

$$kx - \frac{ma}{2} = ma$$

$$kx = \frac{3ma}{2}$$

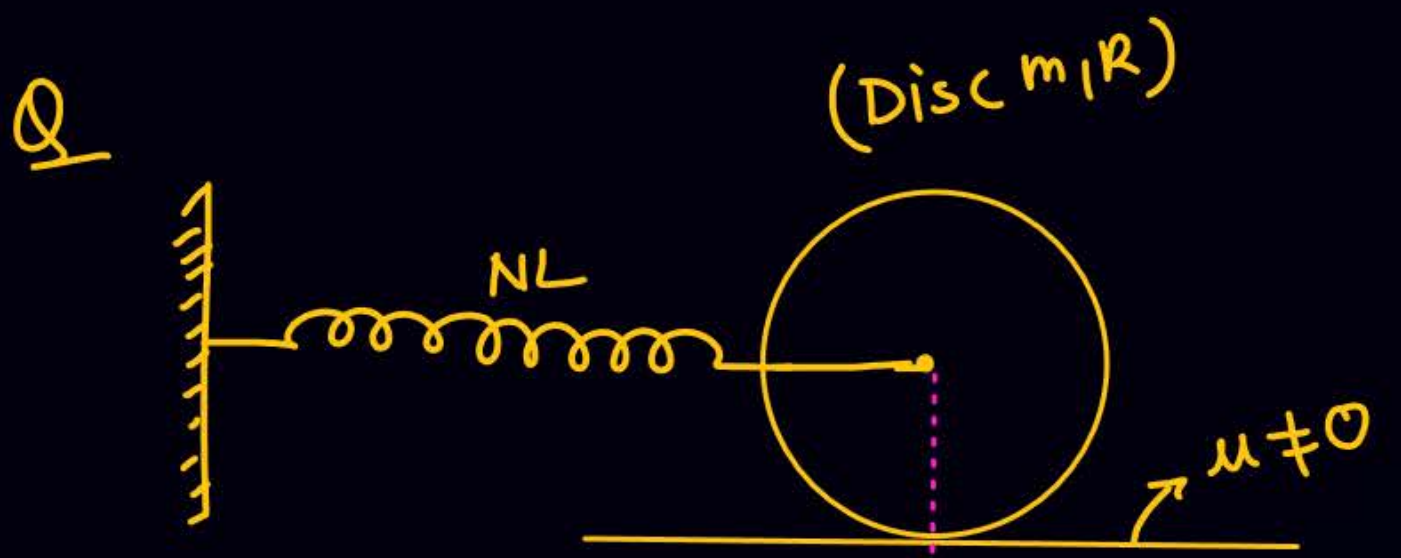
$$a = \frac{2k}{3m} x$$

$$a = -\frac{2k}{3m} x$$

$$\omega^2 = \frac{2k}{3m}$$

$$\omega = \sqrt{\frac{2k}{3m}}$$

$$T = 2\pi \sqrt{\frac{3m}{2k}}$$



$$\frac{d}{dx} \left(\frac{1}{2} kx^2 + \frac{1}{2} mv^2 + \frac{1}{2} \cdot \frac{mR^2}{2} \cdot \frac{v^2}{R^2} \right) = 0$$

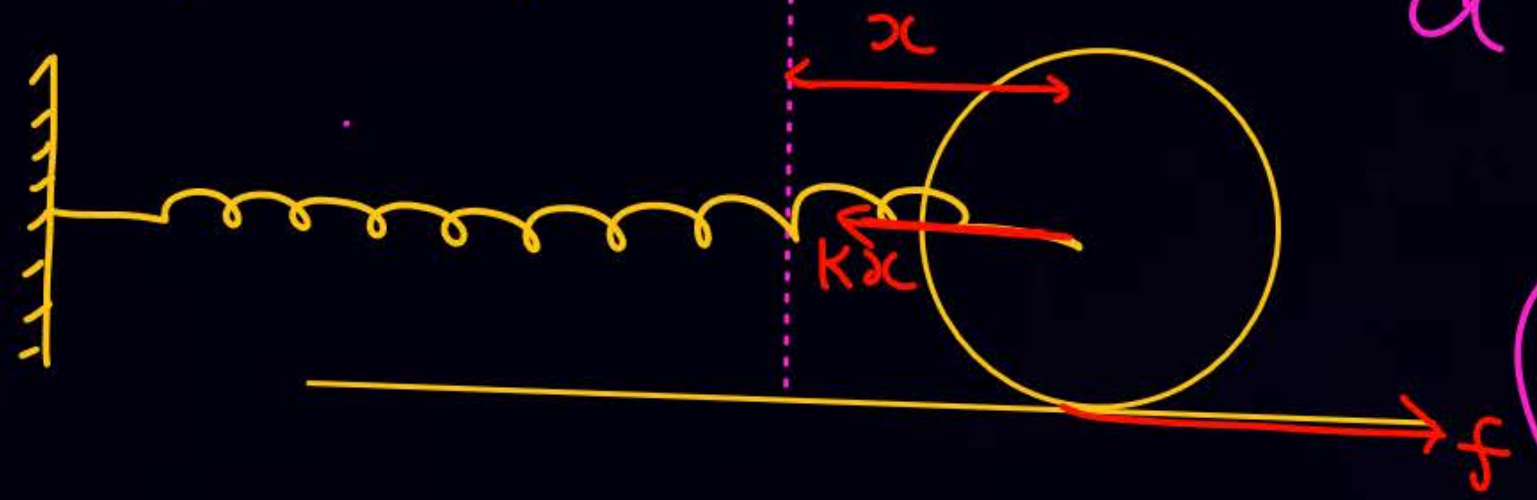
$$\frac{k}{2} 2x + \frac{3}{4} m \left(2v \frac{dv}{dx} \right) = 0$$

$$kx = -\frac{3}{2} ma$$

friction is sufficient for pure rolling
find T of oscillation for small displacement.

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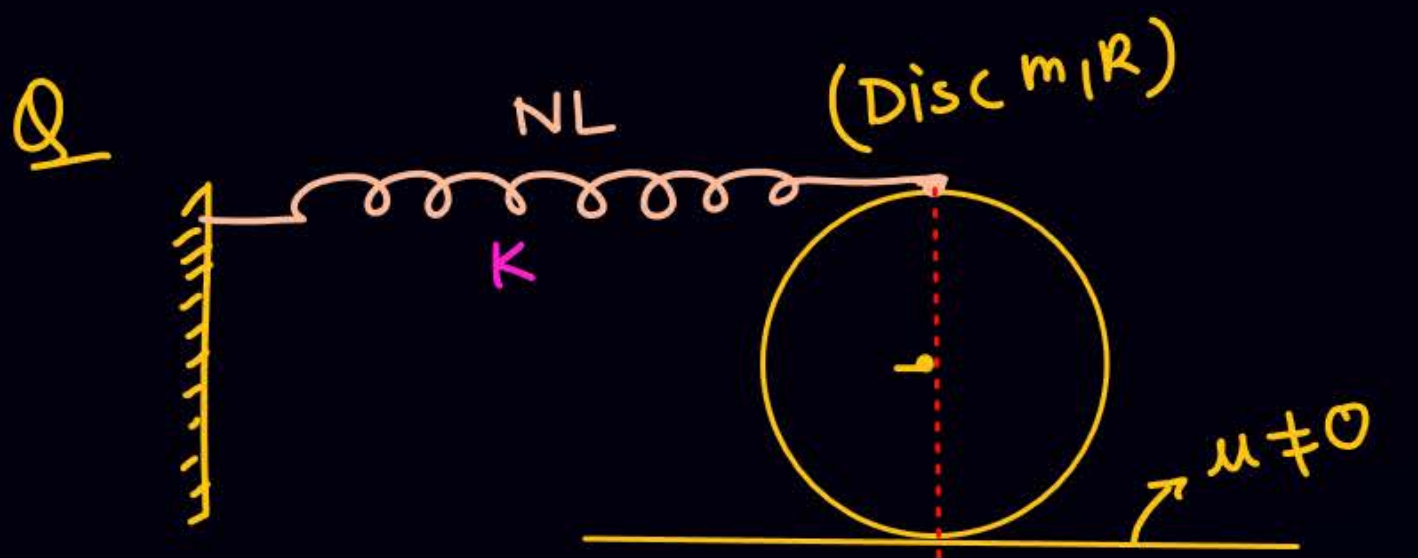
Solⁿ



$$a = -\frac{2kx}{3m}$$

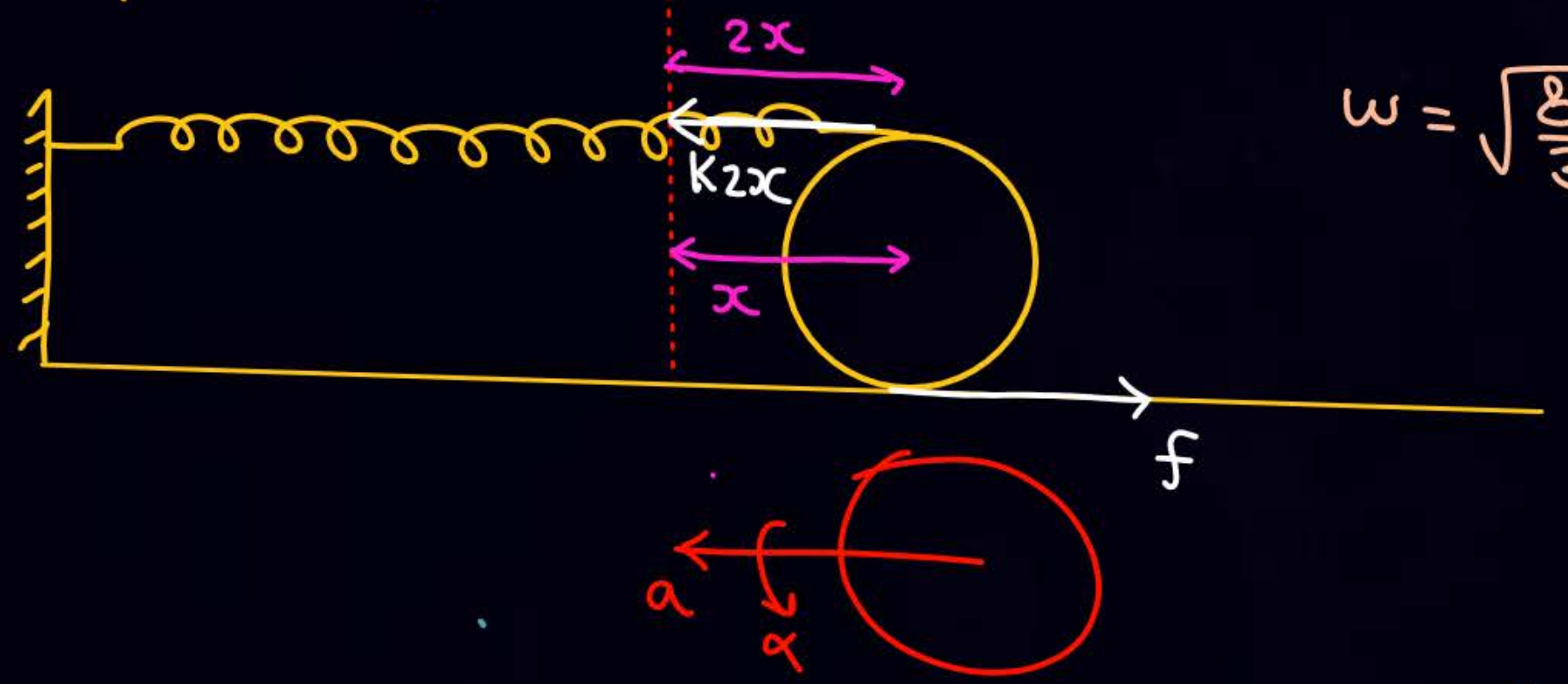
$$T = 2\pi \sqrt{\frac{3m}{2k}}$$

$$T = 2\pi \sqrt{\frac{3m}{2k}}$$



friction is sufficient for pure rolling
find T of oscillation for small displacement.

Solⁿ



$$2Kx - f = ma \quad \text{--- (1)}$$

$$2KxR + fR = I\alpha = \frac{mR^2}{2} \cdot \frac{a}{R}$$

$$2Kx + f = \frac{ma}{2} \quad \text{--- (2)}$$

$$4Kx = \frac{3ma}{2}$$

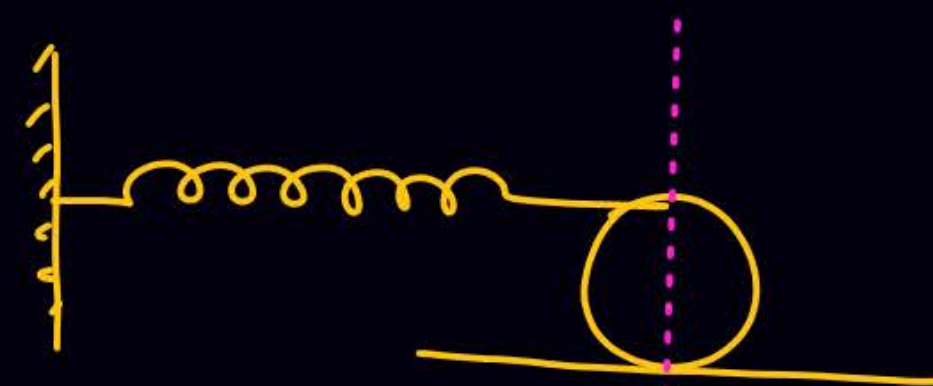
$$\vec{a} = -\frac{8K}{3m} \vec{x}$$

$$\omega = \sqrt{\frac{8K}{3m}}$$

$$T = 2\pi \sqrt{\frac{3m}{8K}}$$

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*
M-2



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$$T.E = \frac{1}{2} k (2x)^2 + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$T.E = 2Kx^2 + \frac{1}{2} m v^2 + \frac{1}{2} \cdot \frac{mR^2}{2} \cdot \frac{v^2}{R^2}$$

$$T.E = 2Kx^2 + \frac{3}{4} m v^2 = \text{const}$$

$$\frac{d(T.E)}{dx} = 2K \cdot 2x + \frac{3}{4} m \cdot 2v \frac{dv}{dx} = 0$$

$$4Kx + \frac{3}{2} m a = 0$$

$$\vec{a} = -\frac{8K}{3m} \vec{x}$$

$$\omega = \sqrt{\frac{8K}{3m}}$$

$$T = 2\pi \sqrt{\frac{3m}{8K}}$$

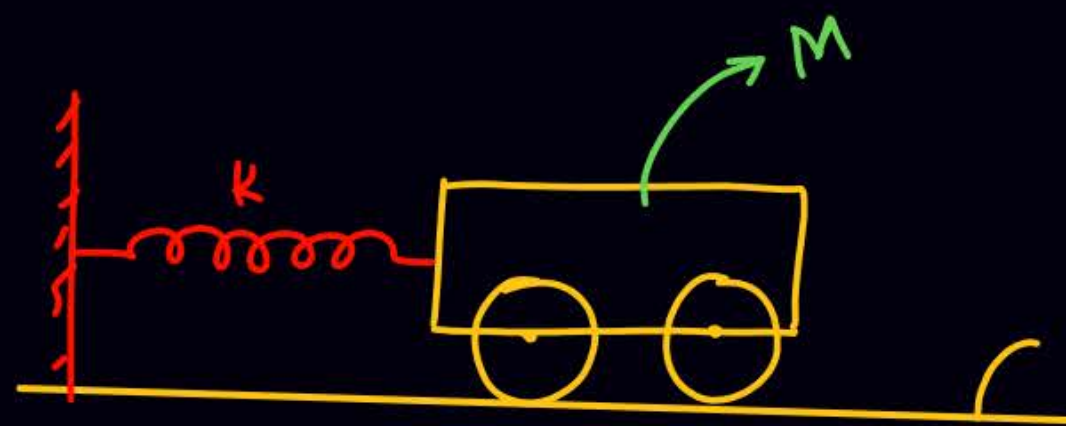
$$\frac{d(T.E)}{dt} = 2K \cdot 2x \frac{dx}{dt} + \frac{3}{4} m \cdot 2v \frac{dv}{dt} = 0$$

$$4Kx = -\frac{3}{2} m a$$

$$\vec{a} = -\frac{8K}{3m} \vec{x}$$



Q



friction is sufficient for pure rolling

$M \rightarrow$ mass of box
 wheel \Rightarrow Disc $\equiv (m, R)$

find time period for small oscillation.

Solⁿ

$$T.E = \frac{1}{2} K x^2 + \frac{1}{2} M V^2 + \left(\frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \right) \times 4$$

$$= \frac{1}{2} K x^2 + \frac{1}{2} M V^2 + \left(\frac{1}{2} m v^2 + \frac{1}{2} \frac{m R^2}{2} \frac{V^2}{R^2} \right) \times 4$$

$$T.E = \frac{1}{2} K x^2 + \frac{1}{2} M V^2 + 3 m v^2$$

$$\frac{d(T.E)}{dx} = 0 = \frac{k}{2} 2x + \frac{1}{2} M 2v \cdot \frac{dv}{dx} + 3m 2v \frac{dv}{dx} = 0$$

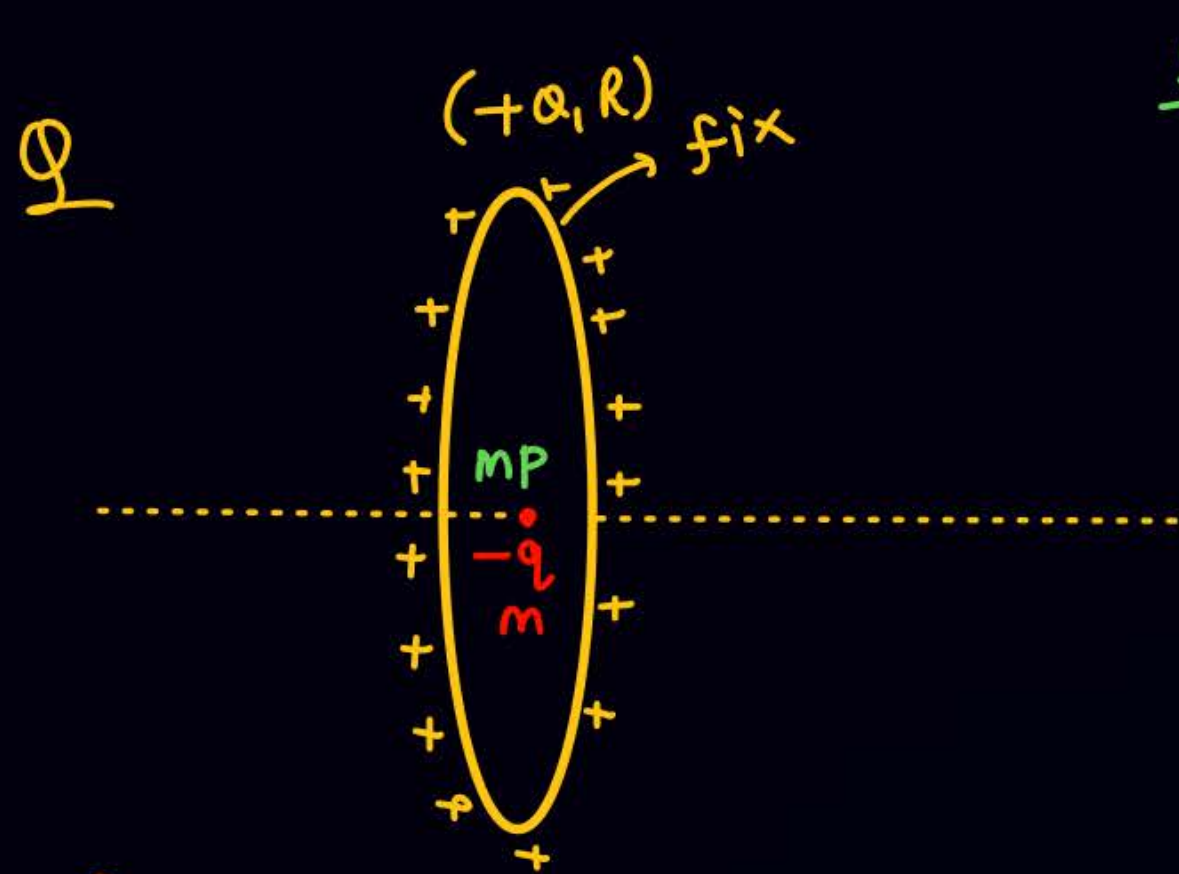
$$kx + M a + 6 m a = 0$$

$$a(M + 6m) = -kx$$

$$\vec{a} = \left(\frac{-K}{M + 6m} \right) \vec{x}$$

$$\omega = \sqrt{\frac{K}{M + 6m}}$$

$$T = 2\pi \sqrt{\frac{M + 6m}{K}}$$



Solⁿ



$$\vec{F} = q\vec{E}$$

$$E = \frac{kQx}{(R^2+x^2)^{3/2}}$$

for small oscillation of $(-q, m)$
find time period

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$$F = qE = \frac{qkQx}{(R^2+x^2)^{3/2}}$$

$$\vec{F} = - \frac{kQq \vec{x}}{(R^2+x^2)^{3/2}}$$

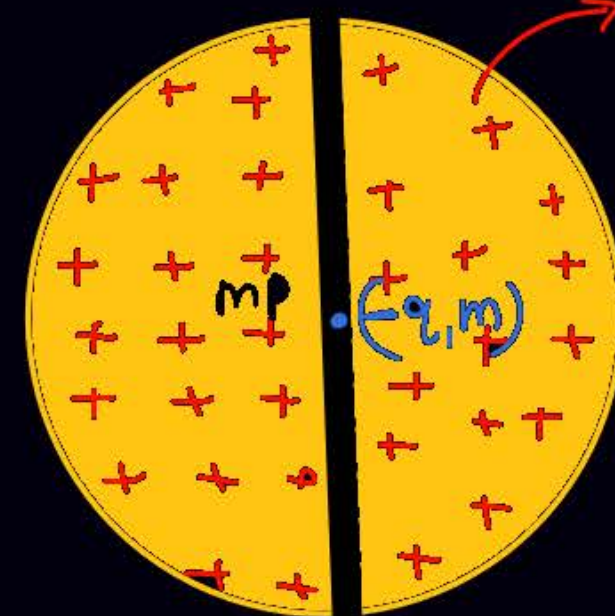
IF $x \ll R$

$$\vec{F} = - \frac{kQq \vec{x}}{R^3}$$

$$T = 2\pi \sqrt{\frac{mR^3}{kQq}}$$



Q



(P, R)
fix

Sol

$$F_{net} = q E$$

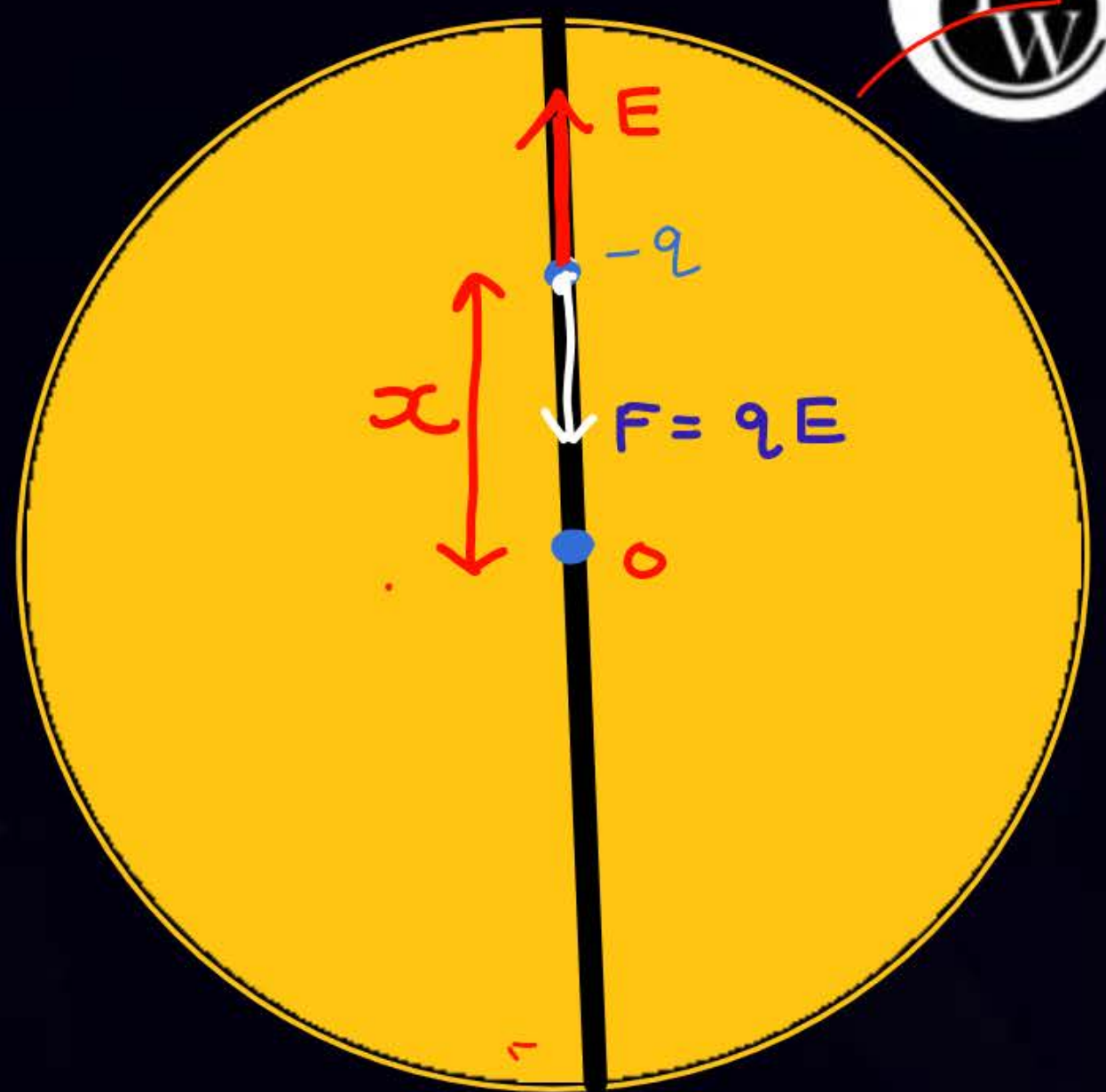
$$\vec{E} = \rho \frac{\vec{r}}{3\epsilon_0}$$

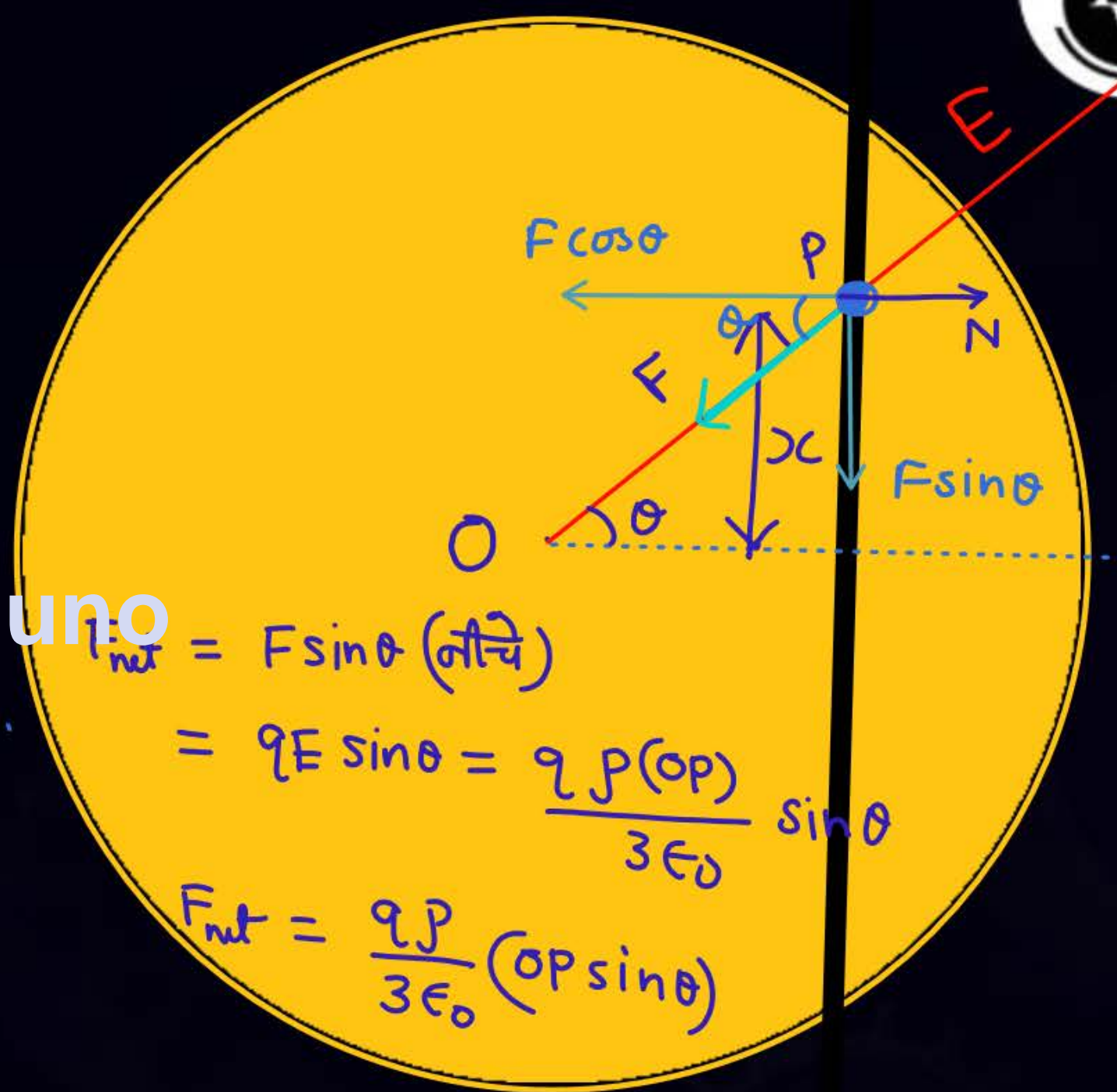
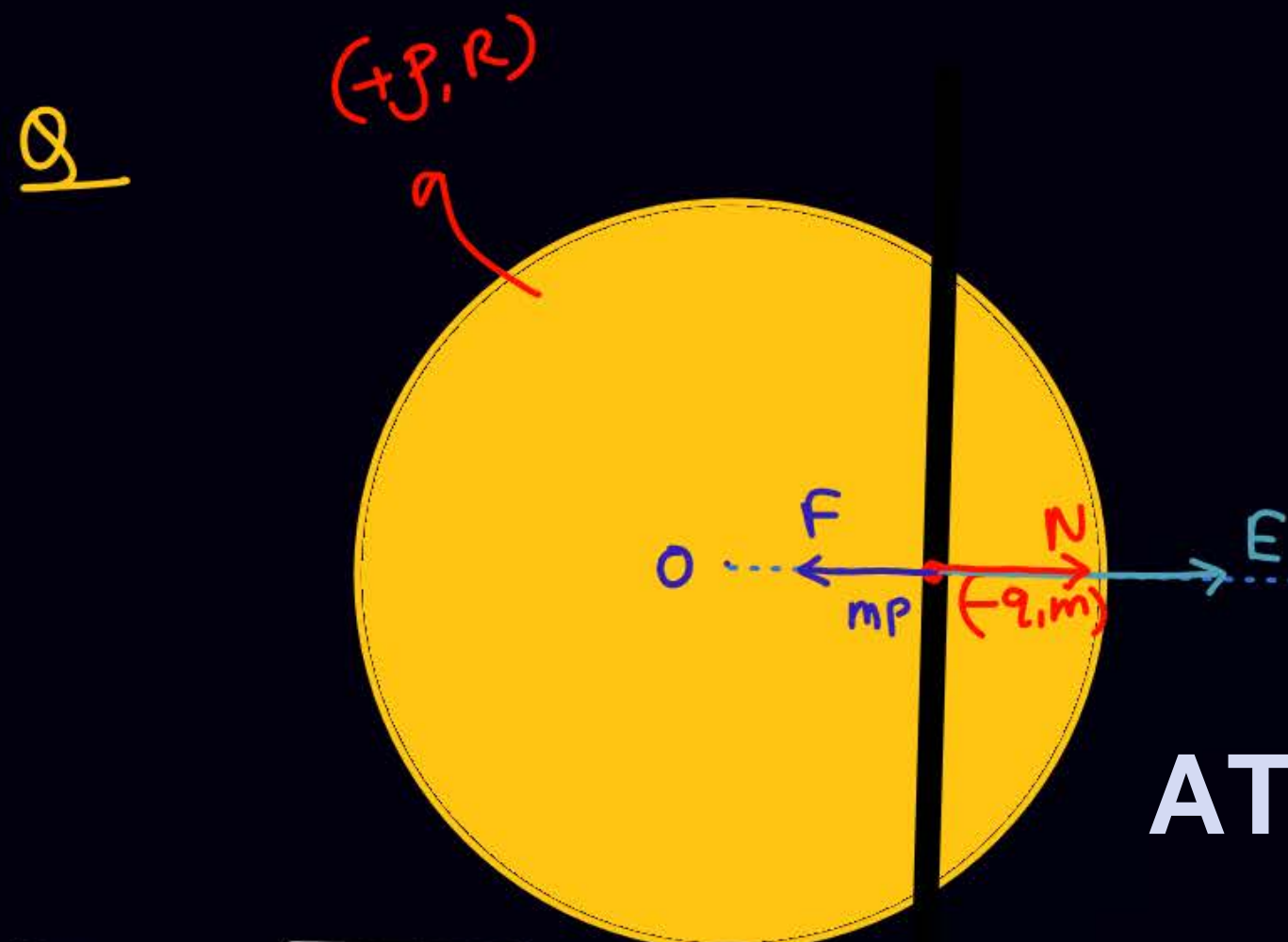
$$\vec{F} = -q \rho \vec{x}$$

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find T of oscillation
if $(-q, m)$ is displaced

$$T = 2\pi \sqrt{\frac{m 3\epsilon_0}{q \rho}}$$





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$$F_{net} = F \sin \theta \text{ (नीचे)}$$

$$= qE \sin \theta = \frac{qP(OP)}{3\epsilon_0} \sin \theta$$

$$F_{net} = \frac{qP}{3\epsilon_0} (OP \sin \theta)$$

$$T = 2\pi \sqrt{\frac{m \cdot 3\epsilon_0}{qP}}$$

$$F_{net} = \frac{qP}{3\epsilon_0} x$$

$$F_{net} = - \frac{qP}{3\epsilon_0} x$$



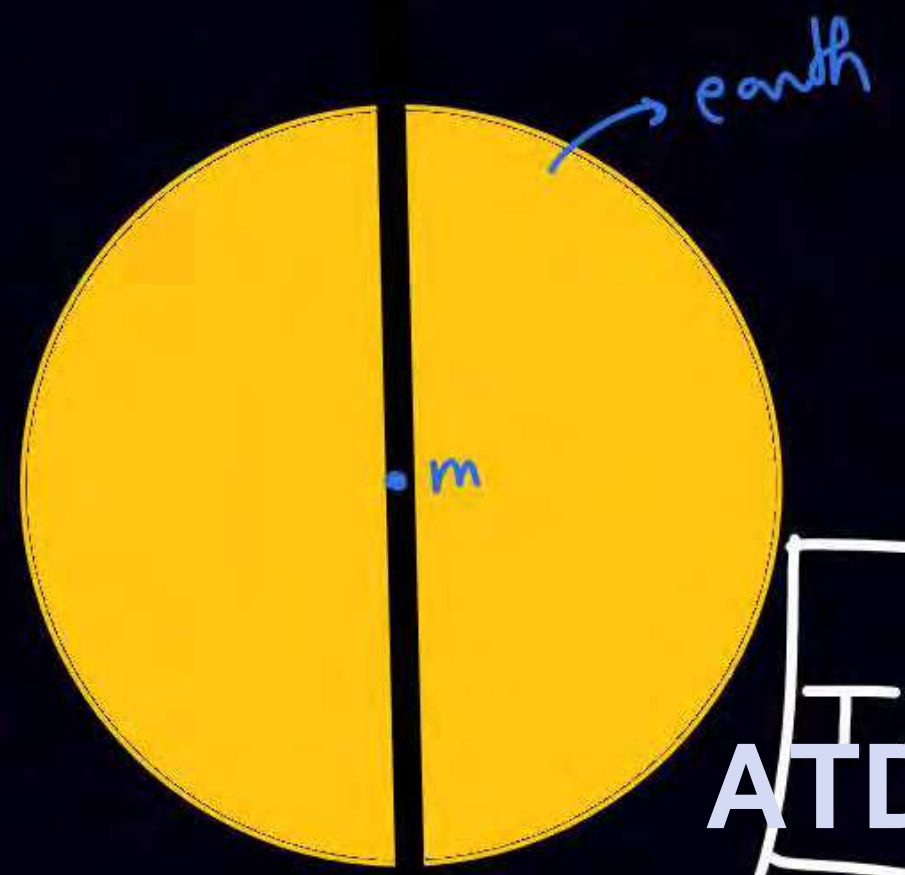
$$T = 2\pi \sqrt{\frac{R}{g}}$$

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$$g = \frac{GM}{R^2}$$

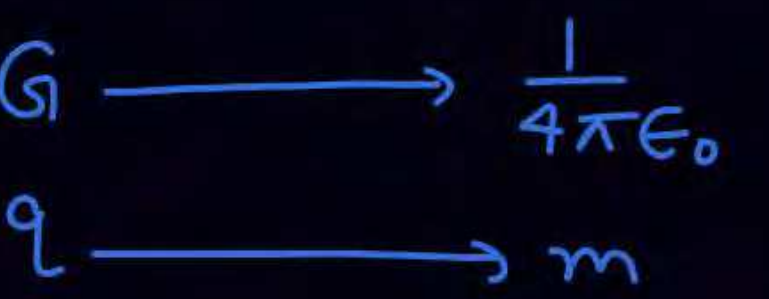


Q



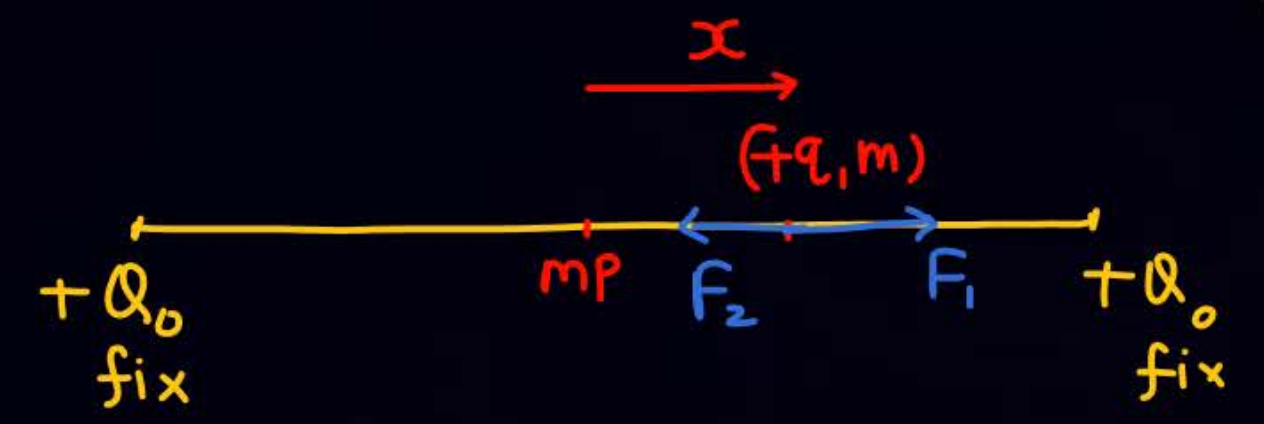
T = ?

$$T = 2\pi \sqrt{\frac{R}{g}}$$





$$T = 2\pi \sqrt{\frac{m}{4kQ_0q}}$$



find T of oscillation if $(+q, m)$ is slightly displaced along x .

$$F_{net} = \frac{kQ_0q}{(l+x)^2} - \frac{kQ_0q}{(l-x)^2}$$

$$F_{net} = -\frac{4kQ_0q l x}{(l^2 - x^2)^2}$$

~~SHM~~
 $x \ll l$

$$\vec{F} = -\frac{4kQ_0q}{l^3} \vec{x}$$

$$\begin{aligned} \vec{F}_{net} &= kQ_0q \left[\frac{(l-x)^2 - (l+x)^2}{(l+x)^2 (l-x)^2} \right] \\ &= kQ_0q \left[\frac{2l \cdot (-2x)}{(l^2 - x^2)^2} \right] \end{aligned}$$



$$\frac{\omega_{\text{Axis}}^2}{\omega_{\text{equit.}}^2} = \frac{4KQ_0^2}{m l^3} \quad \left(\frac{T_{\text{equi.}}}{T_{\text{Axis}}} = \sqrt{2} \right)$$

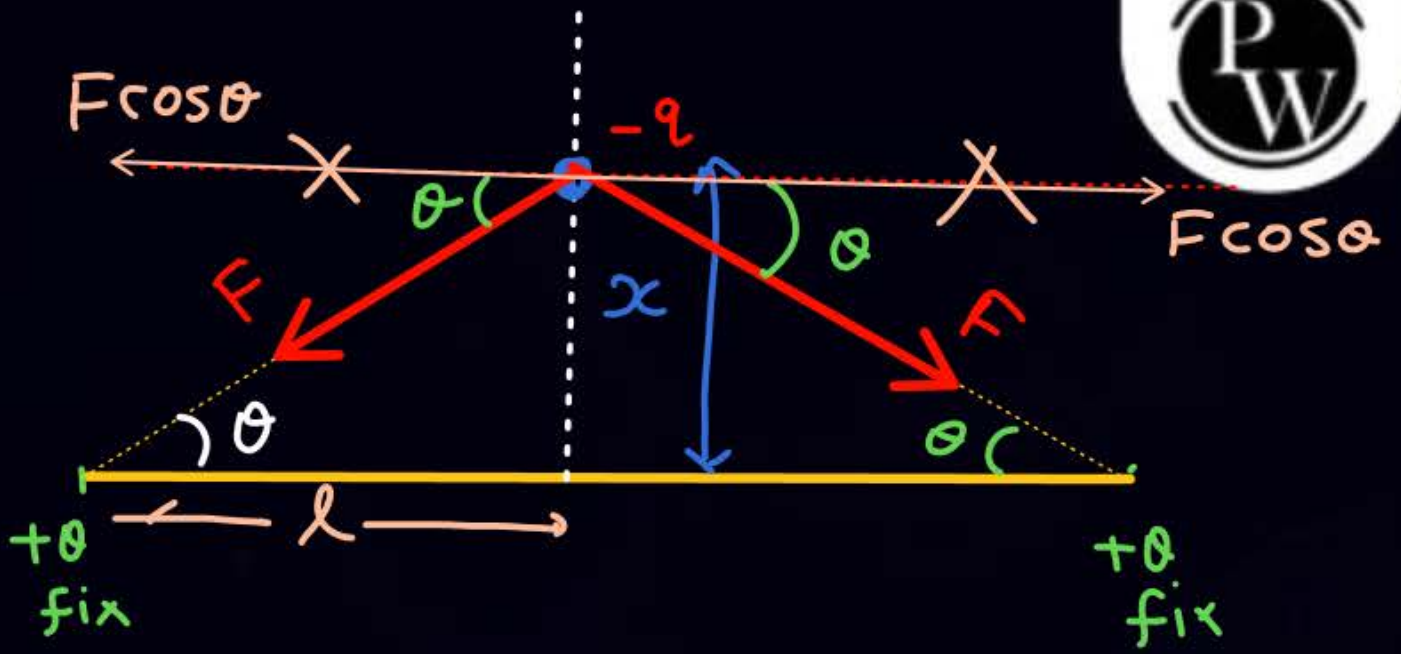
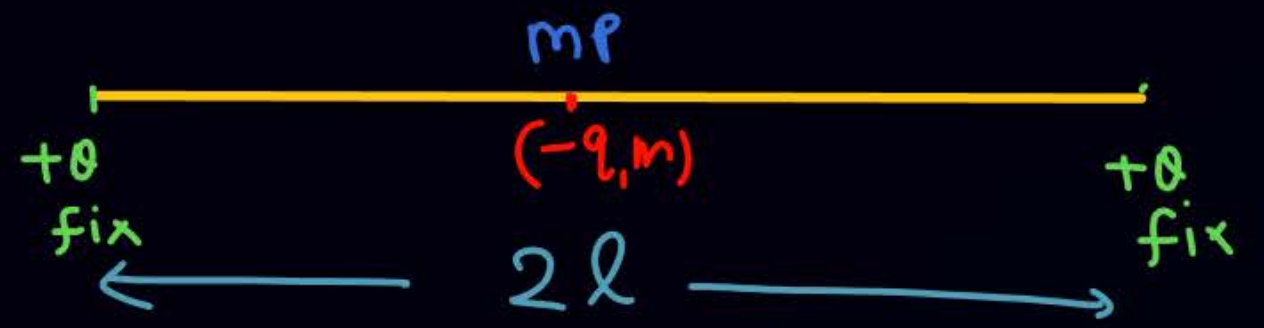
$$\omega_{\text{equit.}}^2 = \frac{2KQ_0^2}{m l^3}$$

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$$\frac{\omega_{\text{Axis}}}{\omega_{\text{equi}}} = \sqrt{2}$$



Q



find T if $(-q, m)$ is displaced slightly
along y-axis

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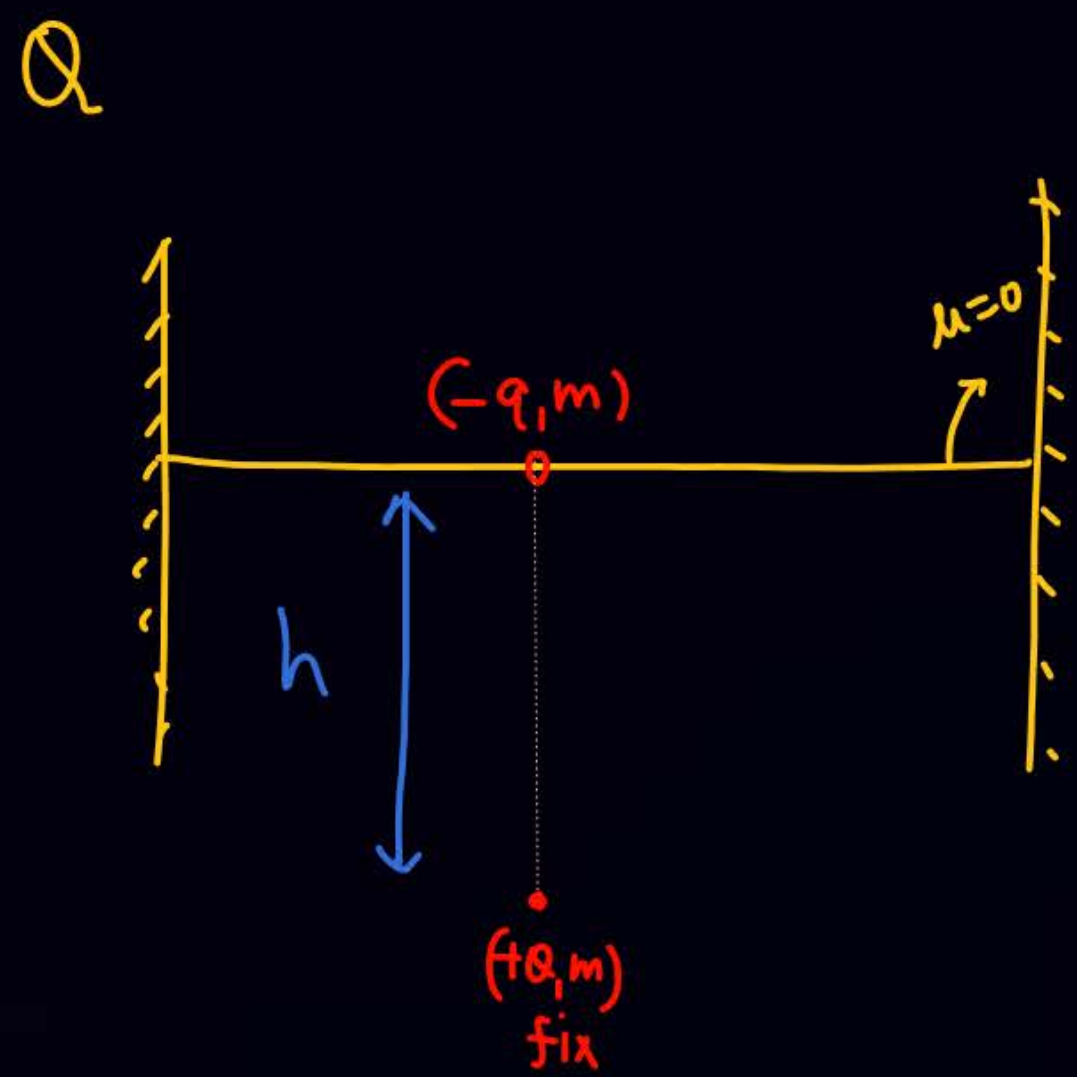
$$F_{net} = 2F \sin \theta \quad (\uparrow \hat{y})$$

$$= 2 \frac{k \alpha q}{(l^2 + x^2)} \times \frac{x}{\sqrt{l^2 + x^2}} = \frac{2k \alpha q x}{(l^2 + x^2)^{3/2}}$$

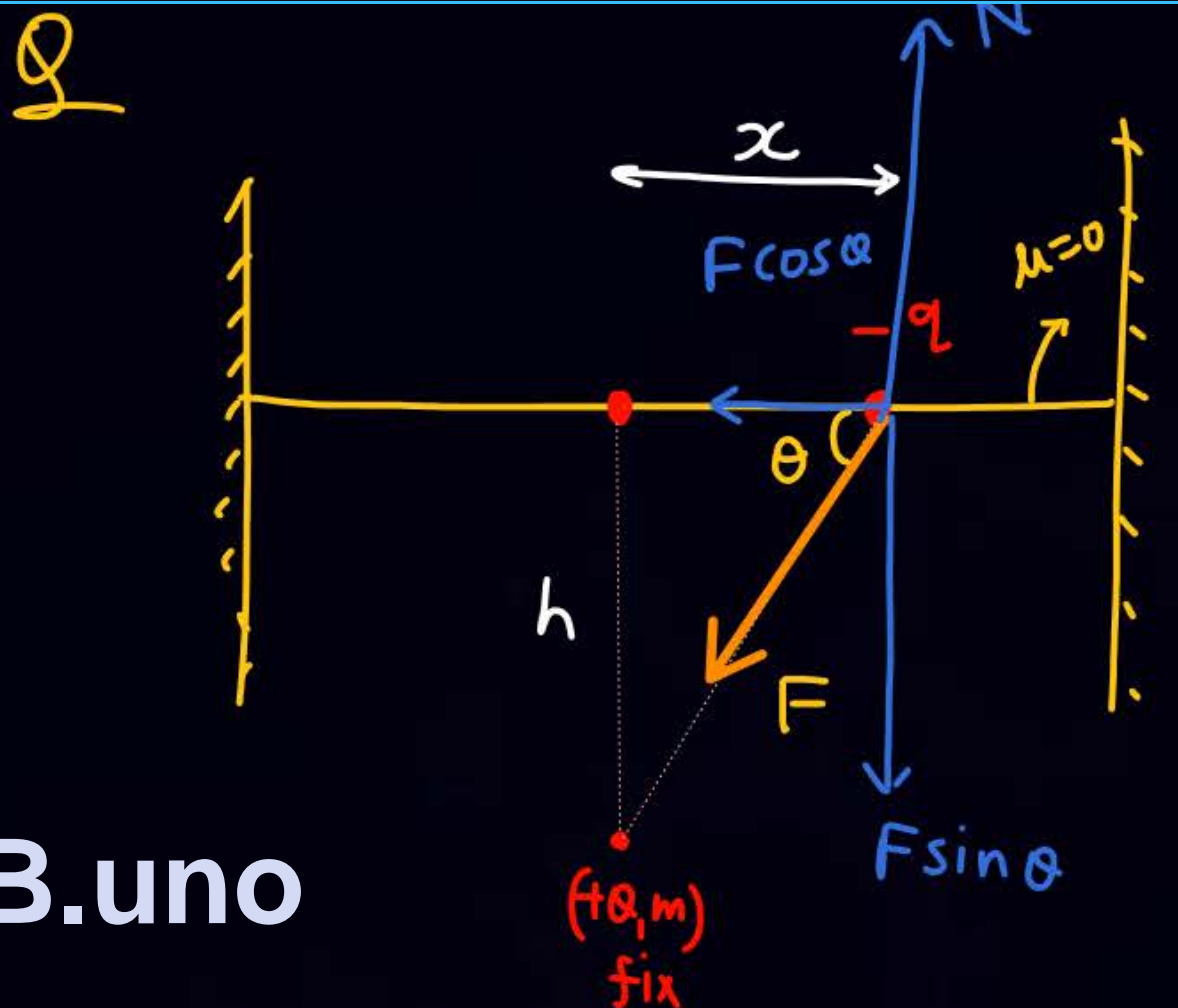
$(x \ll l)$

$$F_{net} = - \left(\frac{2k \alpha q}{l^3} \right) x$$

$$T = 2\pi \sqrt{\frac{m l^3}{2k \alpha q}}$$



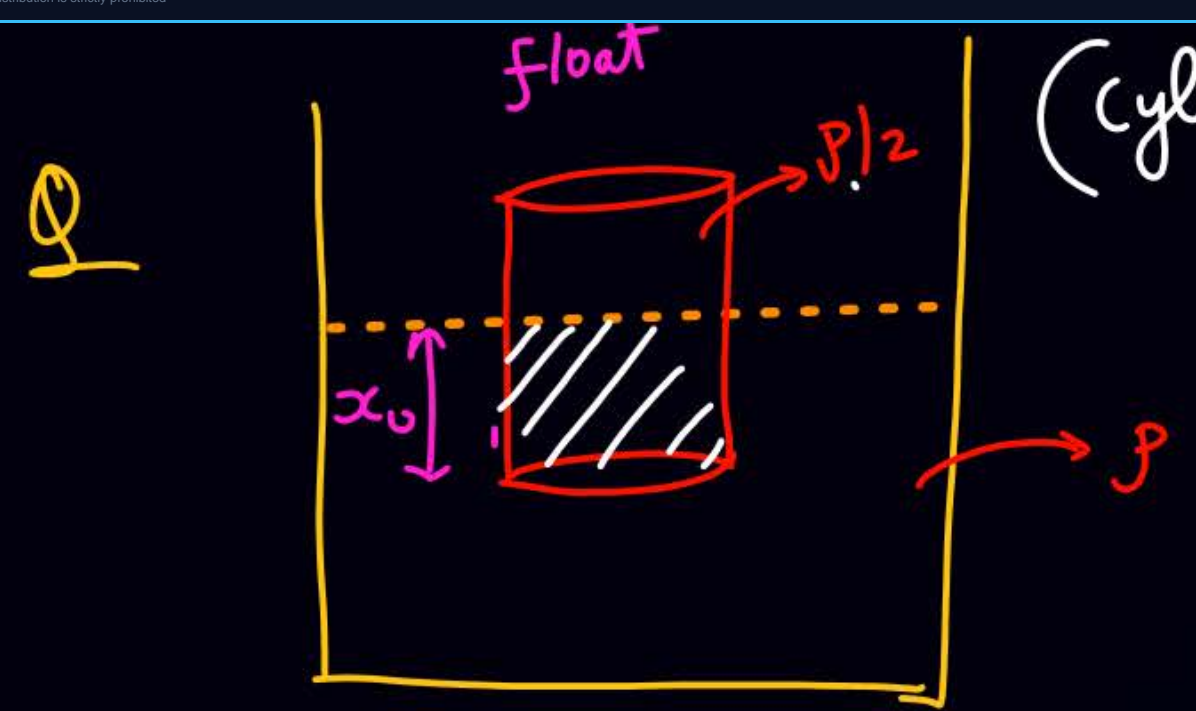
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$$F_{\text{int}} = F \cos \theta = \frac{kQq}{(\sqrt{h^2+x^2})^2} \cdot \frac{x}{\sqrt{h^2+x^2}}$$

$$F = \frac{kQq}{(h^2+x^2)^{3/2}} x$$

If $x \ll h$ $\Rightarrow F = -\frac{kQq}{h^3} x$

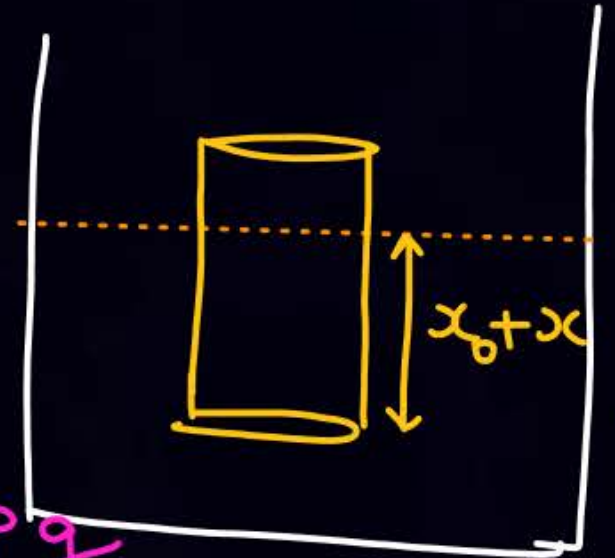


(cylinder $\rho/2, L,$)

If cylinder is pushed down by x & release
 $T = ?$

$$B = V_d \rho_l g$$

$$m_{body} = \rho_{body} V_{body}$$



$$F_{net} = 0$$

$$mg = B = V_d \cdot \rho_l g$$

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$$F_{net} = mg - B_{TF}$$

$$F_{net} = mg - A(x_0 + x)\rho_l g$$

$$mg = AL \frac{\rho}{2} g = Ax_0 \rho_l g$$

$$x_0 = L/2$$

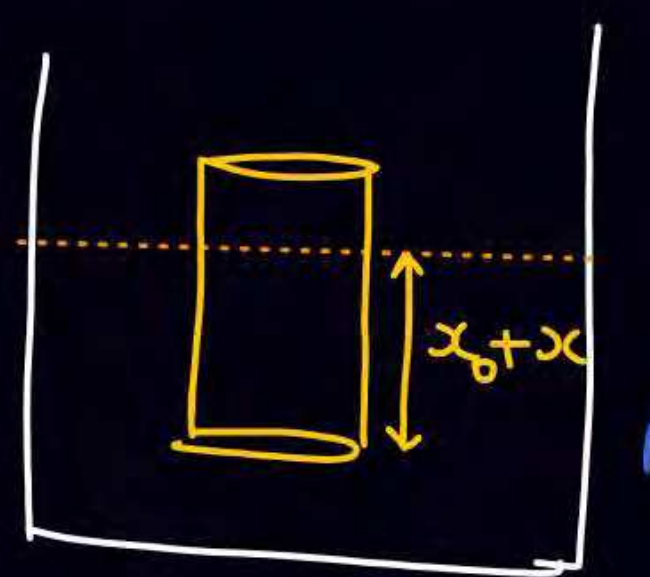
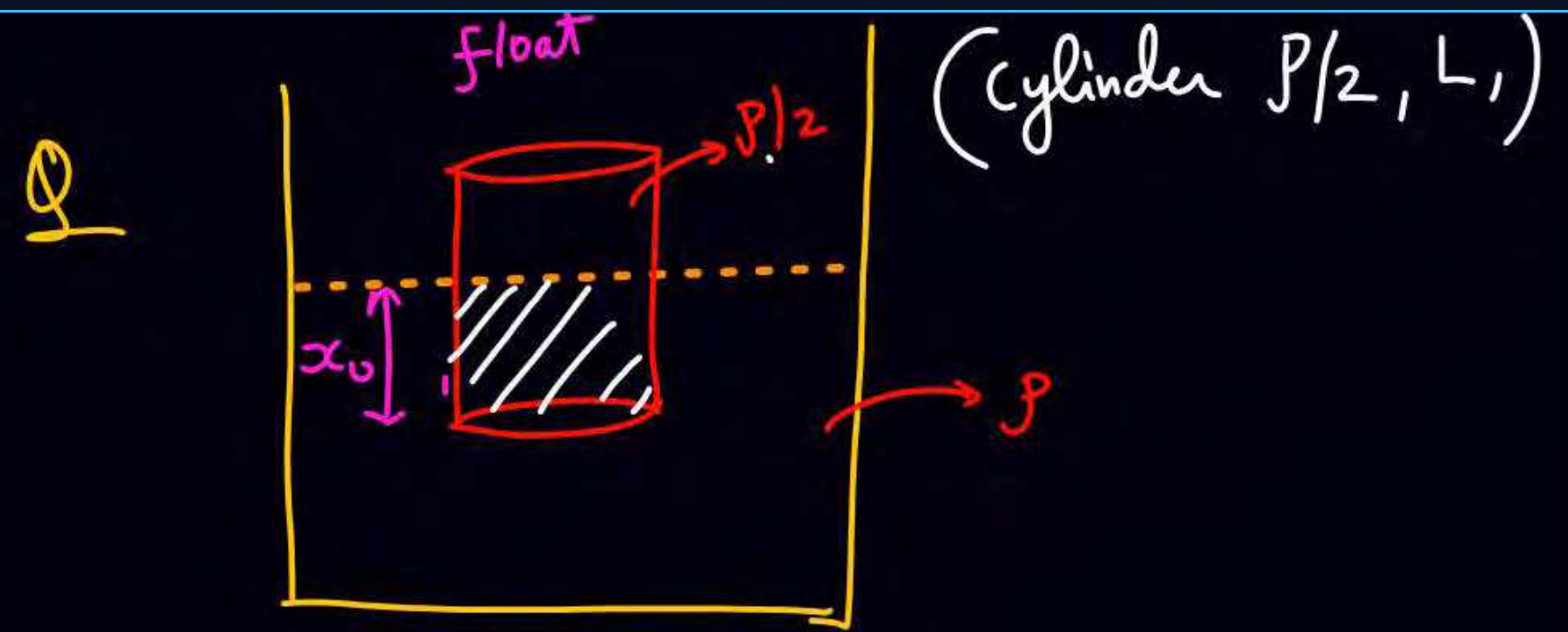
$$F_{net} = mg - Ax_0 \rho_l g - Ax \rho_l g$$

$$F_{net} = -A \rho_l g x$$

$$T = 2\pi \sqrt{\frac{m}{A \rho_l g}}$$

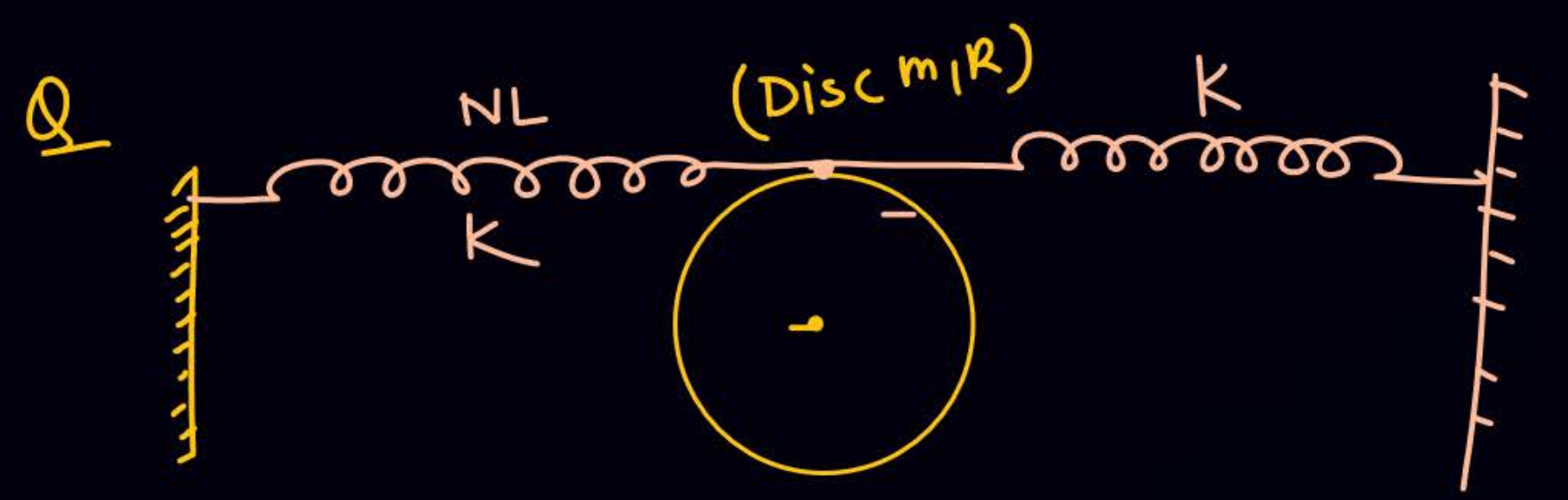
$$T = 2\pi \sqrt{\frac{AL \rho_{body}}{A \rho_{liq} g}}$$

$$T = 2\pi \sqrt{\frac{L}{g} \frac{\rho_{body}}{\rho_{liq}}}$$



$$F_{net} = \rho_{extra} V g$$

$$F_{net} = A x \rho_l g$$



$K_{eq} = 2K$

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$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$1 = \pi^2 \frac{l}{g}$$

$$g = \pi^2 l$$

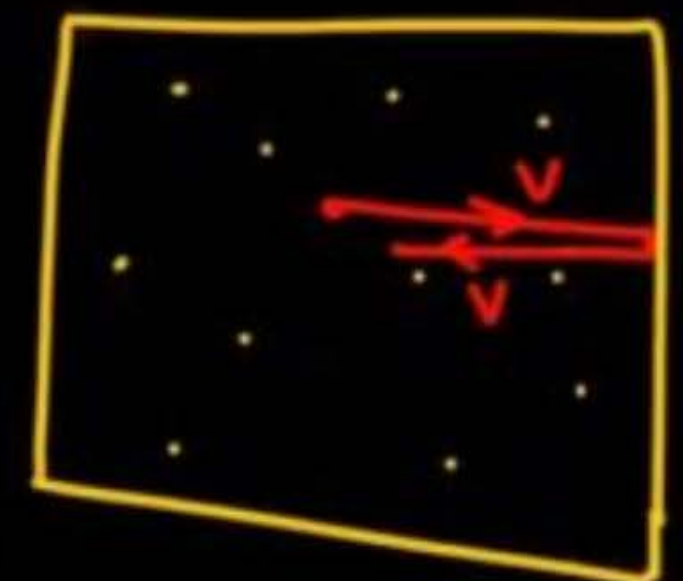
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KTG & Thermodynamics

Assumption

- Random
- All the collision are elastic.
- Effect of gravity neglected
- Size of molecule is very small as compare to volⁿ of container.
- No interaction
- P.E. = 0, Internal Energy \Rightarrow In the form of K.E.
- Real gas behave like ideal gas at high tem & low. pressure.



$$\Delta P = 2mv$$

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A gas which follows all gas laws and gas equation at every possible temperature and pressure is known as ideal or perfect gas.

Volume of gas molecules is negligible as compared to volume of container so volume of gas = volume of container (Except 0K)

No intermolecular force act between gas molecules.

Potential energy of ideal gas is zero so internal energy of ideal gas is perfectly translational K.E. of gas. It is directly proportional to absolute temperature.

So, internal energy depends only and only on its temperature.

$$E_{\text{trans}} \propto T$$

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All real gases behaves as ideal gas at high temperature and low pressure.

Gas molecule have point mass and negligible volume and velocity is very high (10^7 cm/s). That's why there is no effect of gravity on them.

Very-Very-Very Imp

Proof Not important



EE main
24

Average Velocity = $\frac{\vec{V}_1 + \vec{V}_2 + \vec{V}_3 + \dots}{n} = 0$

Remi

V_{rms} = Root mean Square Velocity = $\sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + \dots}{n}} = \sqrt{\frac{3RT}{M}} \propto \sqrt{T}$

Avg Speed = $\frac{V_1 + V_2 + V_3 + \dots}{n} = \sqrt{\frac{8RT}{\pi M}} \propto \sqrt{T}$

most probable speed = $V_{mp} = \sqrt{\frac{2RT}{M}} \propto \sqrt{T}$

$R = \frac{25}{3} \frac{J}{Kmol}$
 $R = 8.33 \frac{J}{Kmol}$

$T \rightarrow$ kelvin
 $M \rightarrow$ molar mass

$V_{rms} > Avg\ speed > V_{mp}$

DIFFERENT SPEEDS OF GAS MOLECULES

- Average velocity**

Because molecules are in random motion in all possible direction in all possible velocity. Therefore,

the average velocity of the gas in molecules in container is zero. $\langle \vec{v} \rangle = \frac{\vec{v}_1 + \vec{v}_2 + \dots + \vec{v}_N}{N} = 0$

rms speed of molecules $v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3kT}{m}} = 1.73 \sqrt{\frac{kT}{m}}$

Mean speed of molecules :By maxwell's velocity distribution law v_M or $\langle |\vec{v}| \rangle = v_{mean}$

$$\langle |\vec{v}| \rangle = v_{mean} = \frac{|\vec{v}_1| + |\vec{v}_2| + \dots + |\vec{v}_n|}{N} = \sqrt{\frac{8P}{\pi\rho}} = \sqrt{\frac{8RT}{\pi M_w}} = \sqrt{\frac{8kT}{\pi m}} = 1.59 \sqrt{\frac{kT}{m}}$$

Most probable speed of molecules (v_{mp})

At a given temperature, the speed to which maximum number of molecules belongs is called as most

probable speed (v_{mp}) $v_{mp} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2kT}{m}} = 1.41 \sqrt{\frac{kT}{m}}$

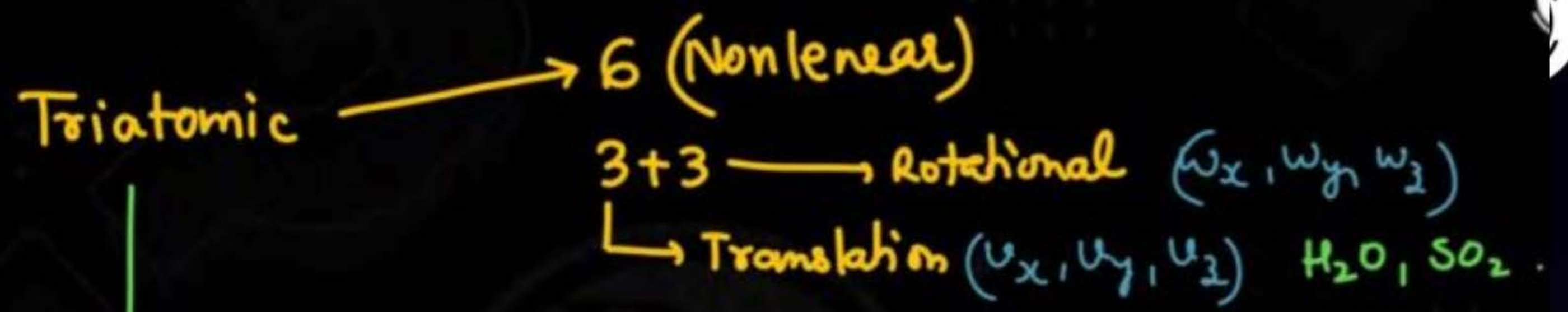
Q Speed of ten gas particles are given as 1, 0, 2, 4, 6, 3, 6, 8, 6, 4

$$\text{Avg speed} = \frac{1+0+2+4+6+3+6+8+6+4}{10} = 4$$

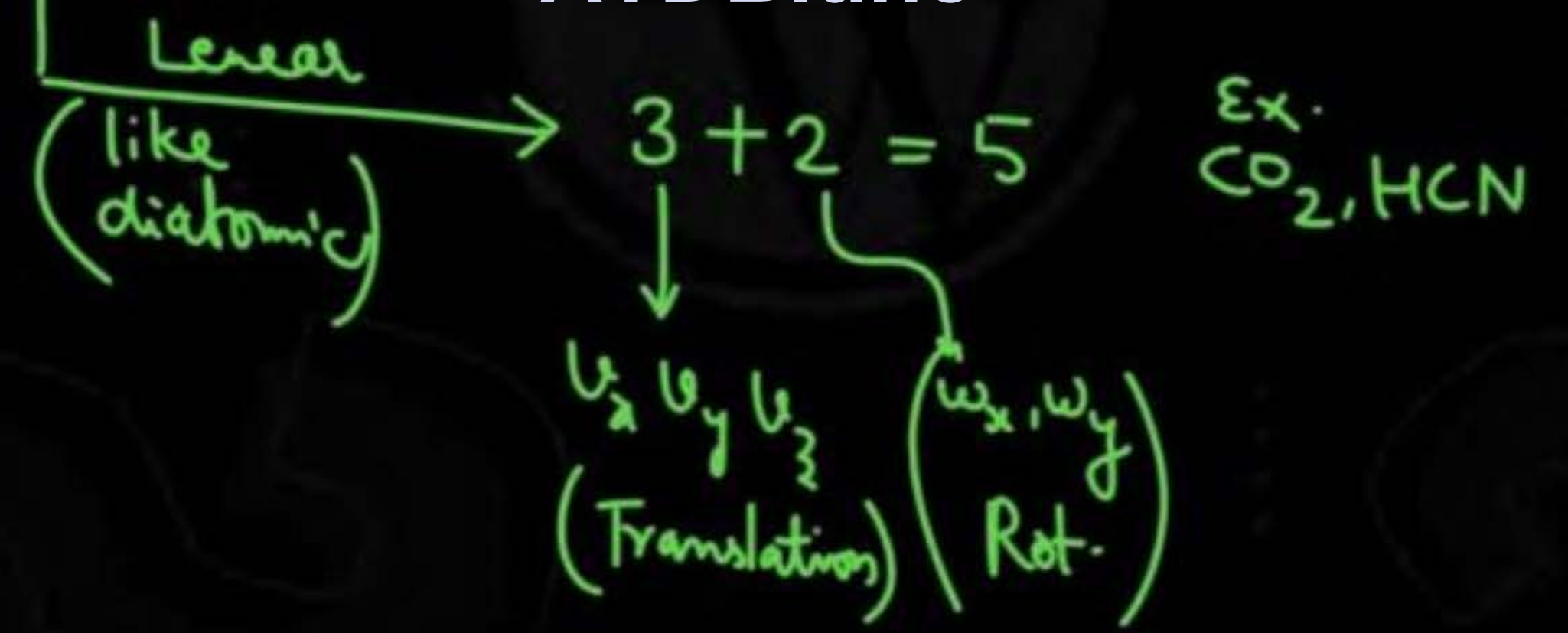
$$V_{\text{rms}} = \sqrt{\frac{1^2+0^2+2^2+4^2+6^2+3^2+6^2+8^2+6^2+4^2}{10}}$$

$$V_{\text{most prob. speed}} = 6$$

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maxwell equipartition Law of Energy.

Energy corresponding to each dof of an ideal gas molecule = $\frac{K T}{2}$

\Rightarrow Energy of one molecule of a gas with dof f is equal to = $f \frac{K T}{2}$

$$K = \frac{R}{N_A}$$

$$U = \frac{f K T}{2} = \frac{R T}{N_A} \frac{f T}{2} \longrightarrow \text{1 molecule}$$

$$\text{So for 1 mole } \Rightarrow U = N_A \cdot \left(\frac{R f}{N_A} \frac{T}{2} \right) = \frac{R f T}{2}$$

$$\text{for } n \text{ mole } U = n \left(\frac{R f T}{2} \right)$$

degree of freedom

$$U = \frac{nfRT}{2} \Rightarrow \begin{matrix} T \uparrow, U \uparrow, \Delta U > 0 \\ T \downarrow, U \downarrow, \Delta U < 0 \end{matrix}$$
$$\Delta U = \frac{nfR \Delta T}{2}$$

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Home work

- DPP
- module PYQ (solve)

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THANK YOU

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