

PRAYAS

JEE 2025

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Lecture - 08

Physics

Oscillations

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Topics *to be covered*

1 Angular SHM

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2 Compound Pendulum & Torsional Pendulum

3 Phasor

4 Superposition of SHM

Linear SHM

$$\vec{a} = -\omega^2 \vec{x}$$

$$\vec{F}_{net} = -k\vec{x} \quad T = 2\pi \sqrt{\frac{m}{k}}$$

$$\vec{F}_{net} = m\vec{a} = -k\vec{x}$$

$$T = 2\pi \sqrt{\frac{m}{k}}, \quad \omega = \sqrt{\frac{k}{m}}$$

$$x = A \sin(\omega t + \phi)$$

$$v = \omega \sqrt{A^2 - x^2} \quad \text{Angular freq } \omega = \frac{2\pi}{T}$$

$$KE = \frac{1}{2}k(A^2 - x^2)$$

$$PE = \frac{1}{2}kx^2 + U_0$$

Angular SHM

$$\vec{\alpha} = -\omega^2 \vec{\theta}$$



$$\vec{\tau}_{net} = -k'\vec{\theta}$$

$$\vec{\tau} = I \vec{\alpha} = -k'\vec{\theta}$$

$$T = 2\pi \sqrt{\frac{I}{k'}}, \quad \omega = \sqrt{\frac{k'}{I}} = \frac{2\pi}{T}$$

$$\theta = \theta_0 \sin(\omega t + \phi)$$

$$\left(\frac{d\theta}{dt} = \omega\right), \quad \omega = \omega \sqrt{A^2 - \theta^2}$$

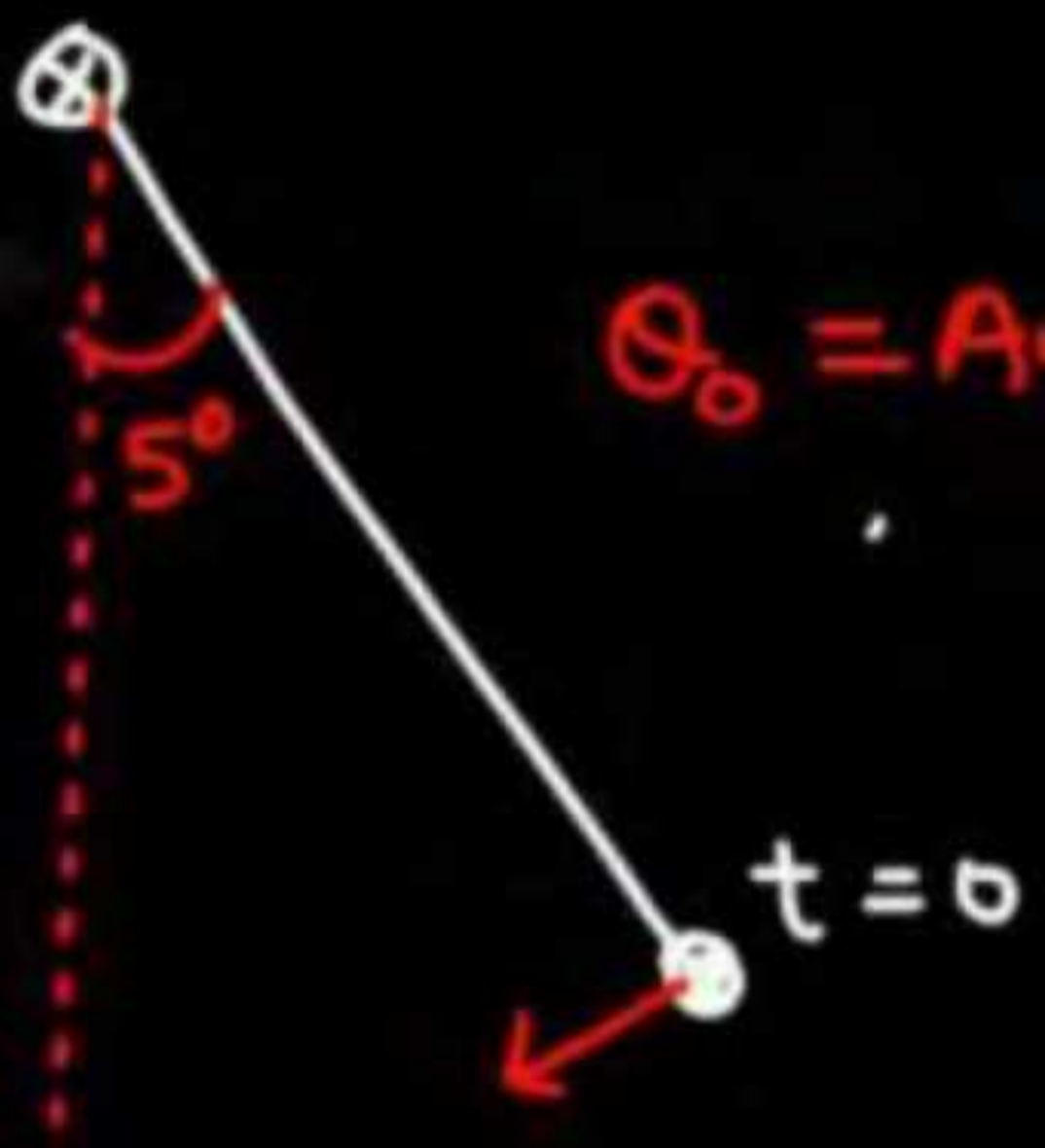
Angular velocity Angular freq.

$$KE = \frac{1}{2}k'(\theta_0^2 - \theta^2)$$

$$PE = \frac{1}{2}k'\theta^2 + U_0$$



Q Angular SHM



$\theta_0 = \text{Amplitude} = 10^\circ$

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$$\theta = 10^\circ \sin(\omega t + 150^\circ)$$

$$\omega = \frac{2\pi}{T}$$

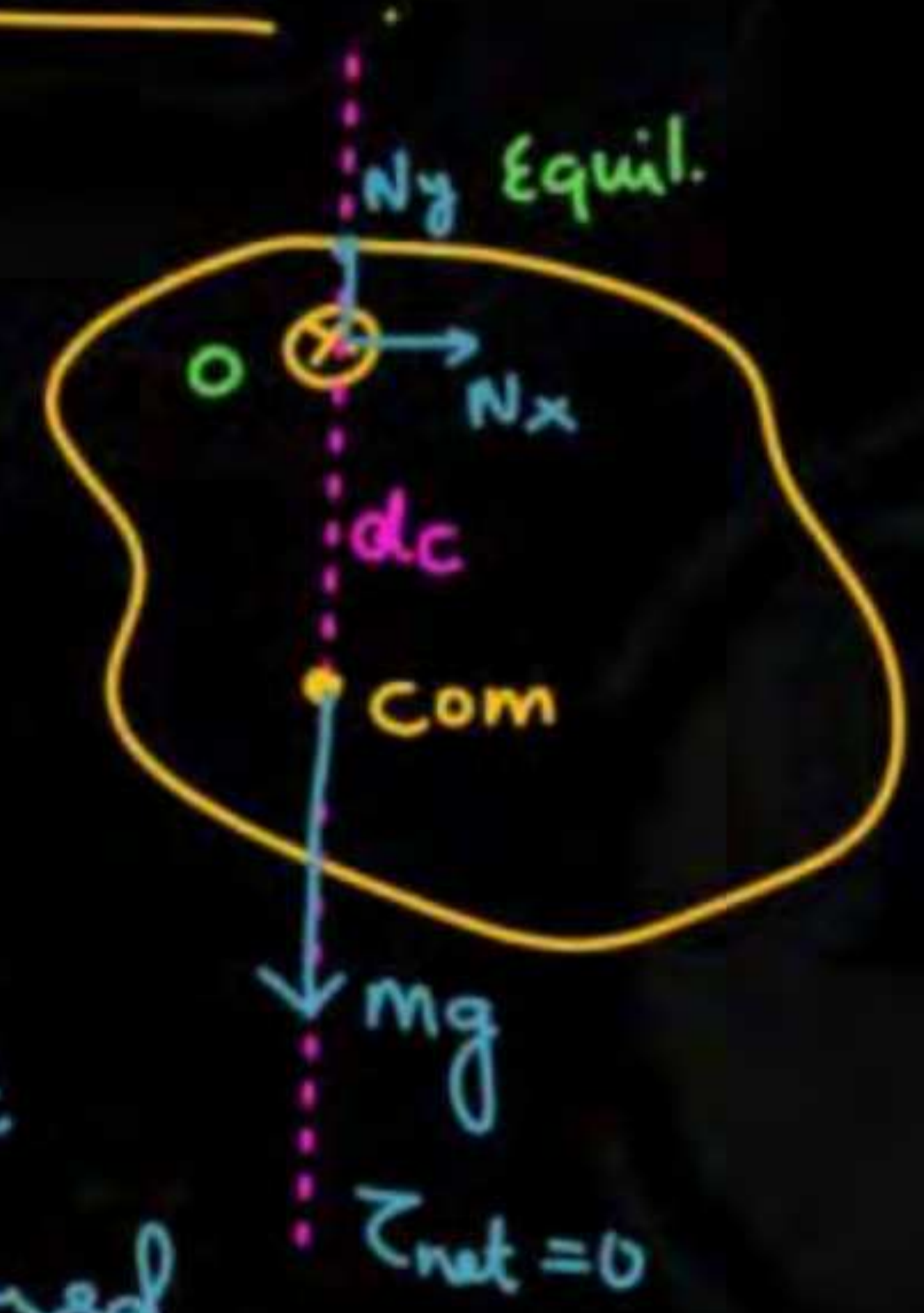


Compound Pendulum

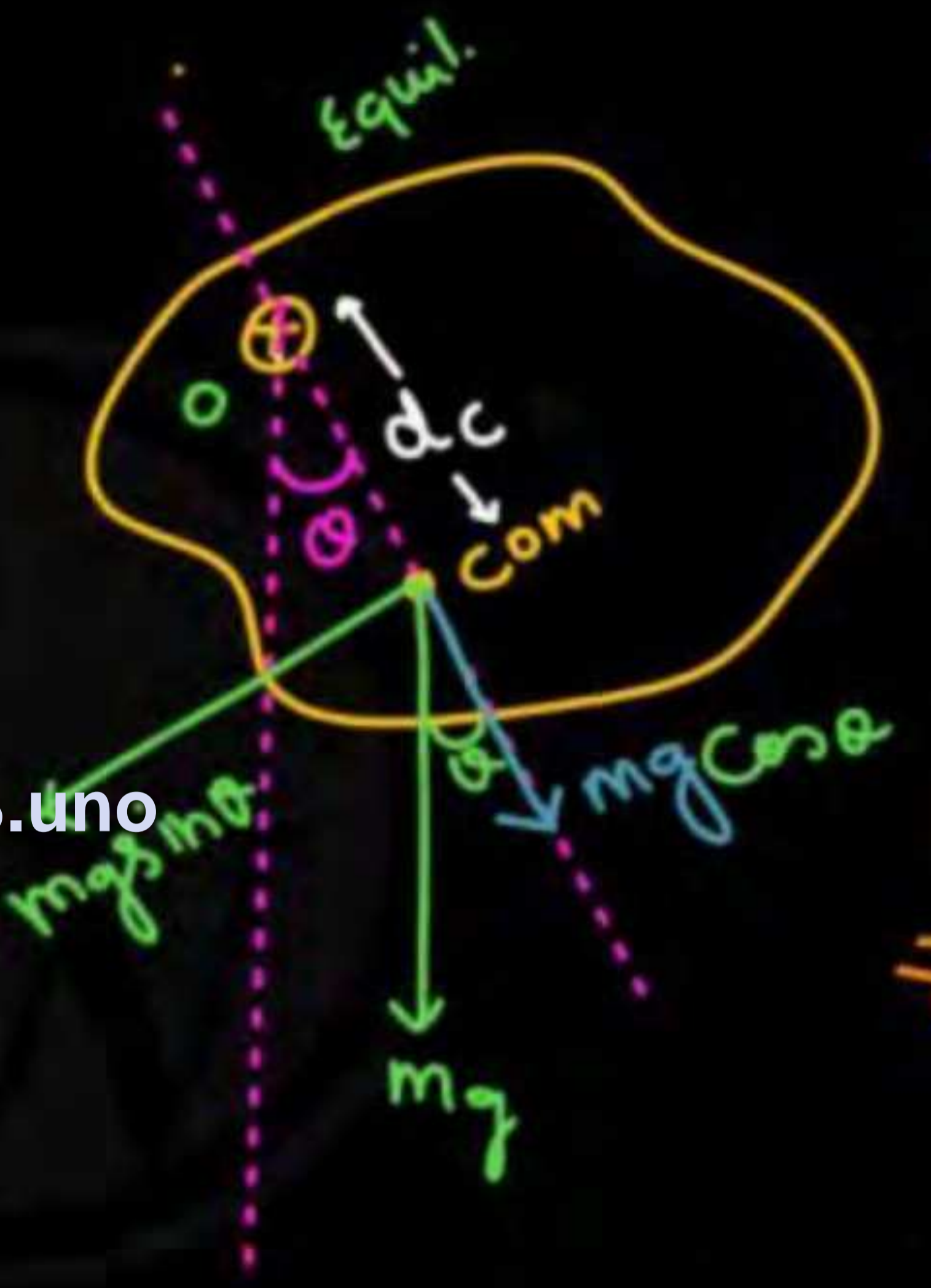
$$T = 2\pi \sqrt{\frac{I}{mgd_c}}$$

$I \rightarrow$ MOI abt hinged pt

$d_c \rightarrow$ com of hinged point at distance



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$$\tau_0 = mgsin\theta \times d_c$$

$$\tau = mgd_c \sin\theta$$

If θ is very small $\sin\theta \approx \theta$

$$\tau = mgd_c \theta$$

$$\tau = -mgd_c \theta$$

$$\tau_{net} = -K\theta$$

$$T = 2\pi \sqrt{\frac{I}{K}} = 2\pi \sqrt{\frac{I}{mgd_c}}$$



Q



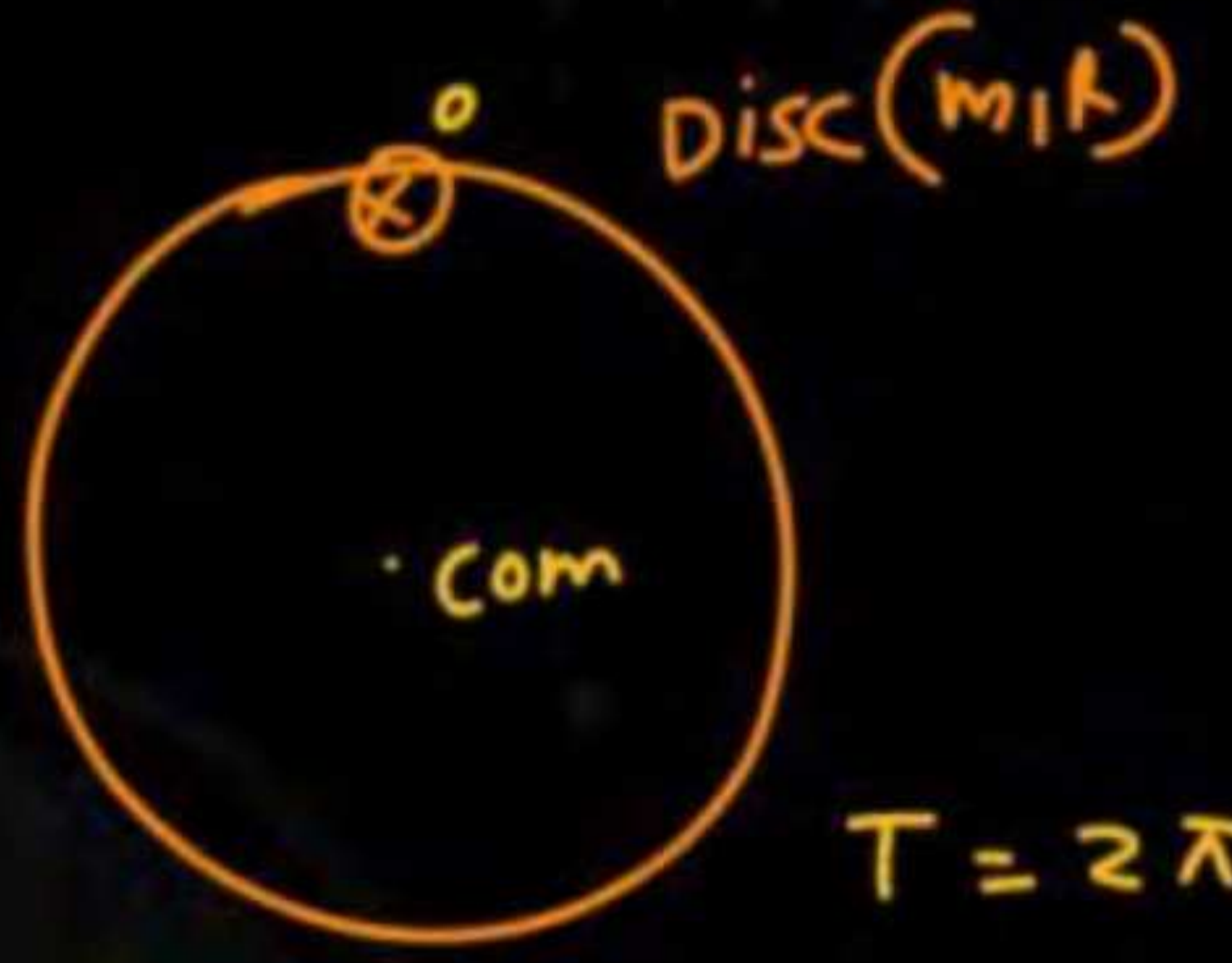
$$T = 2\pi \sqrt{\frac{I}{mgdc}}$$

$$T = 2\pi \sqrt{\frac{mL^2/3}{mg \cdot \frac{L}{2}}}$$

$$T = 2\pi \sqrt{\frac{2L}{3g}}$$

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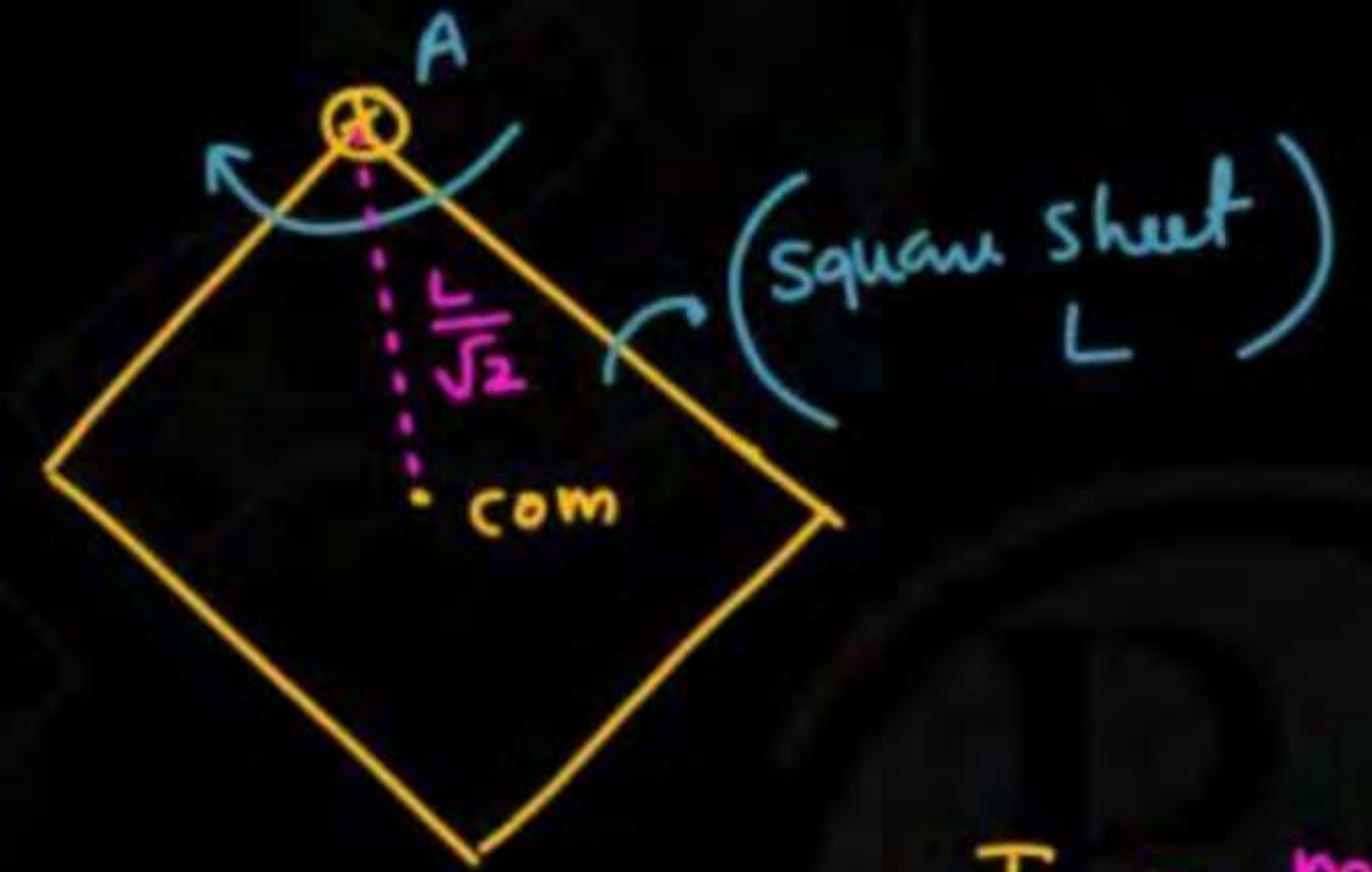
Q



$$T = 2\pi \sqrt{\frac{I}{mgdc}}$$

$$T = 2\pi \sqrt{\frac{MR^2/2 + mR^2}{mgR}}$$

Q



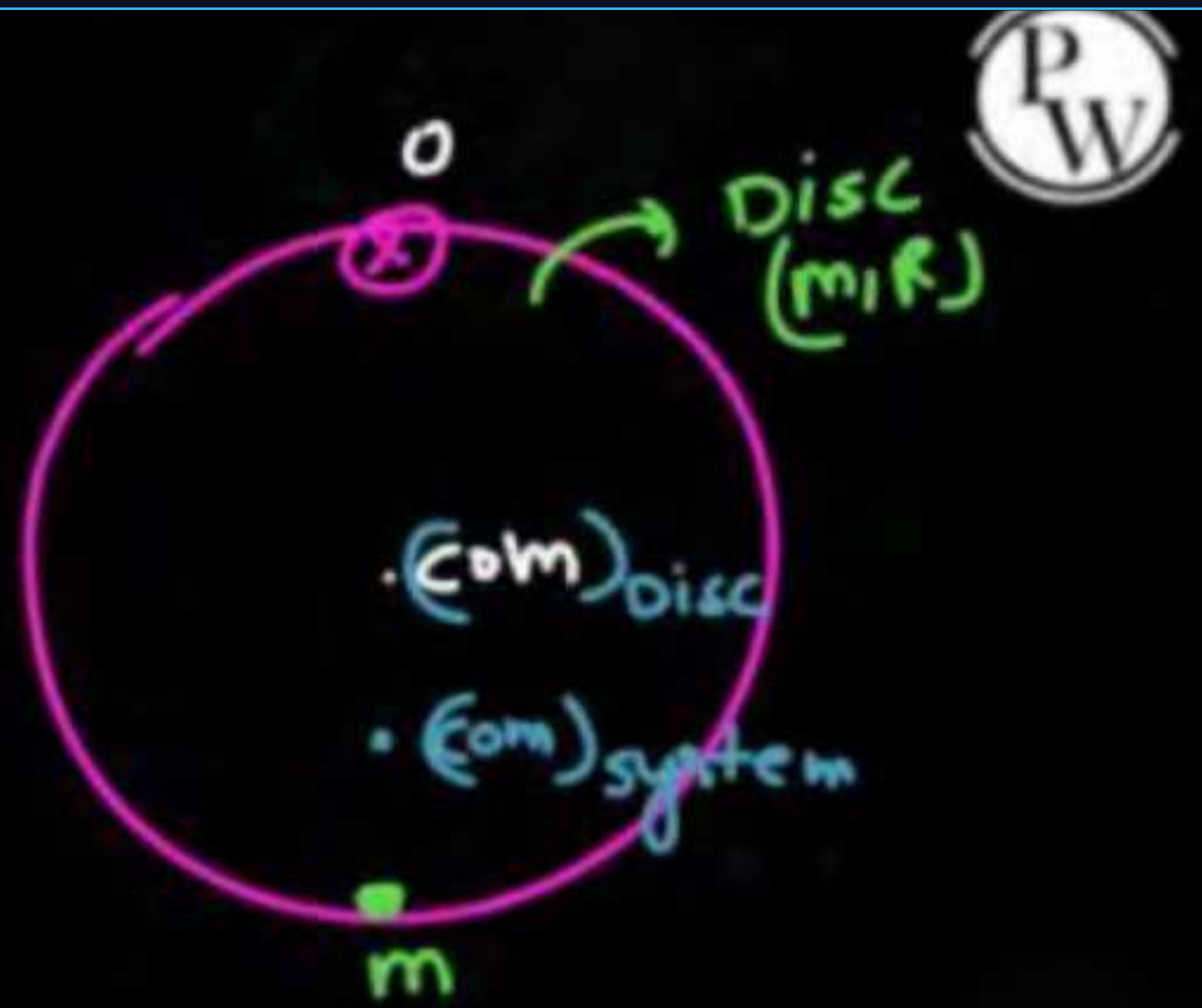
$$T = 2\pi \sqrt{\frac{I_A}{mgdc}}$$

$$I_A = \frac{mL^2}{12} + m\left(\frac{L}{\sqrt{2}}\right)^2$$

$$dc = \frac{L}{\sqrt{2}}$$

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Q



$$T = 2\pi \sqrt{\frac{I_0}{(2m)g\left(R + \frac{R}{2}\right)}}$$

$$I_0 = \frac{mR^2}{2} + mR^2 + m(2R)^2$$





Q2



$$\theta_0 = l_0$$

$$\phi = 90^\circ$$

$$T = 2\pi \sqrt{\frac{I}{mgd_c}} = 2\pi \sqrt{\frac{mL^2}{3mg} \frac{L}{2}} = \checkmark$$

$$\omega = \frac{2\pi}{T}$$

$$\theta = l_0 \sin(\omega t + 90^\circ)$$

Simple Pendulum



$$T = 2\pi \sqrt{\frac{I}{mgdc}}$$

$$T = 2\pi \sqrt{\frac{m l^2}{m g l}}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$



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$$T = 2\pi \sqrt{\frac{l}{g_{eff}}}$$

lift ✓ $g = g_{eff}$



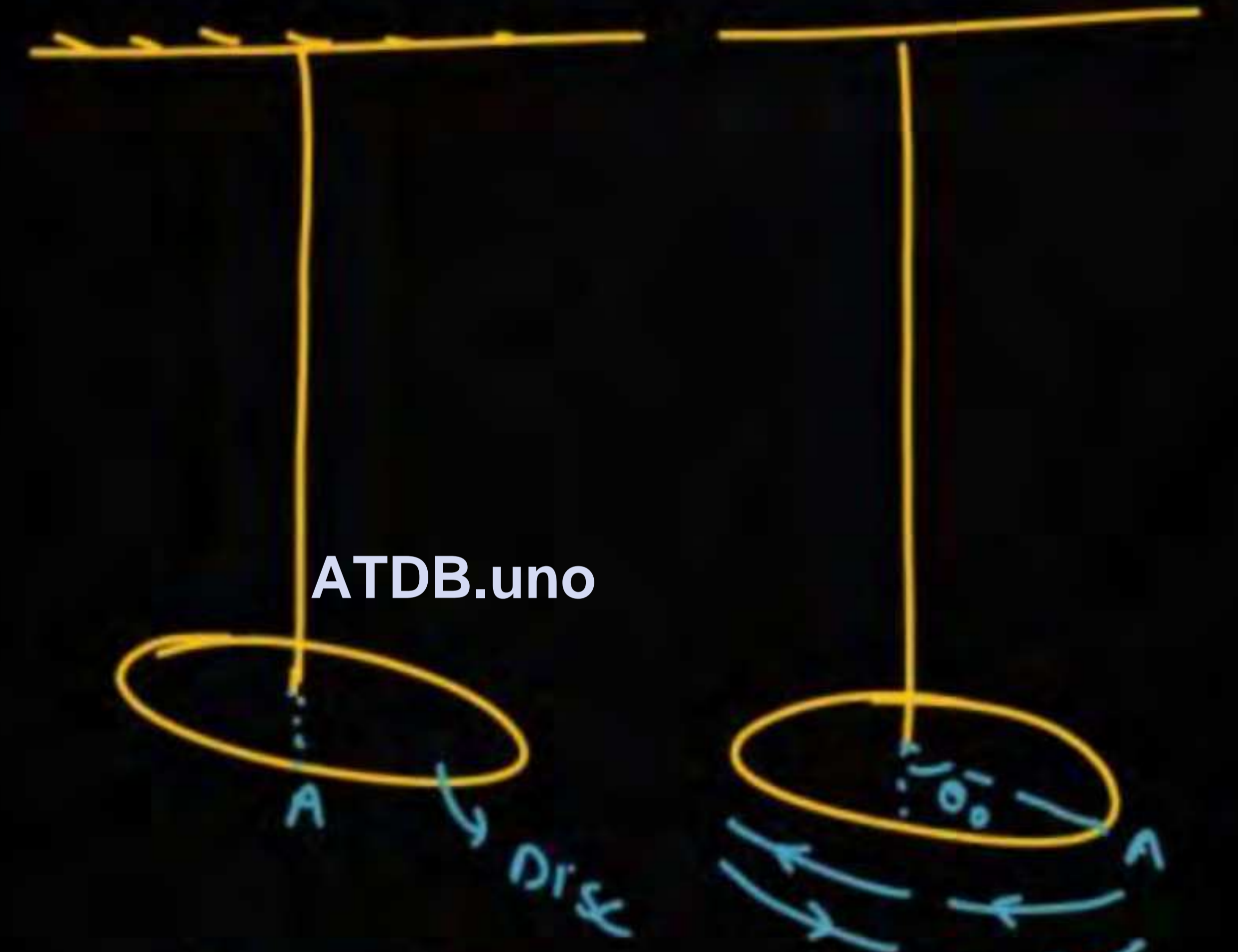
Torsional Pendulum

$$\vec{\tau}_{net} \propto -\vec{\theta}$$

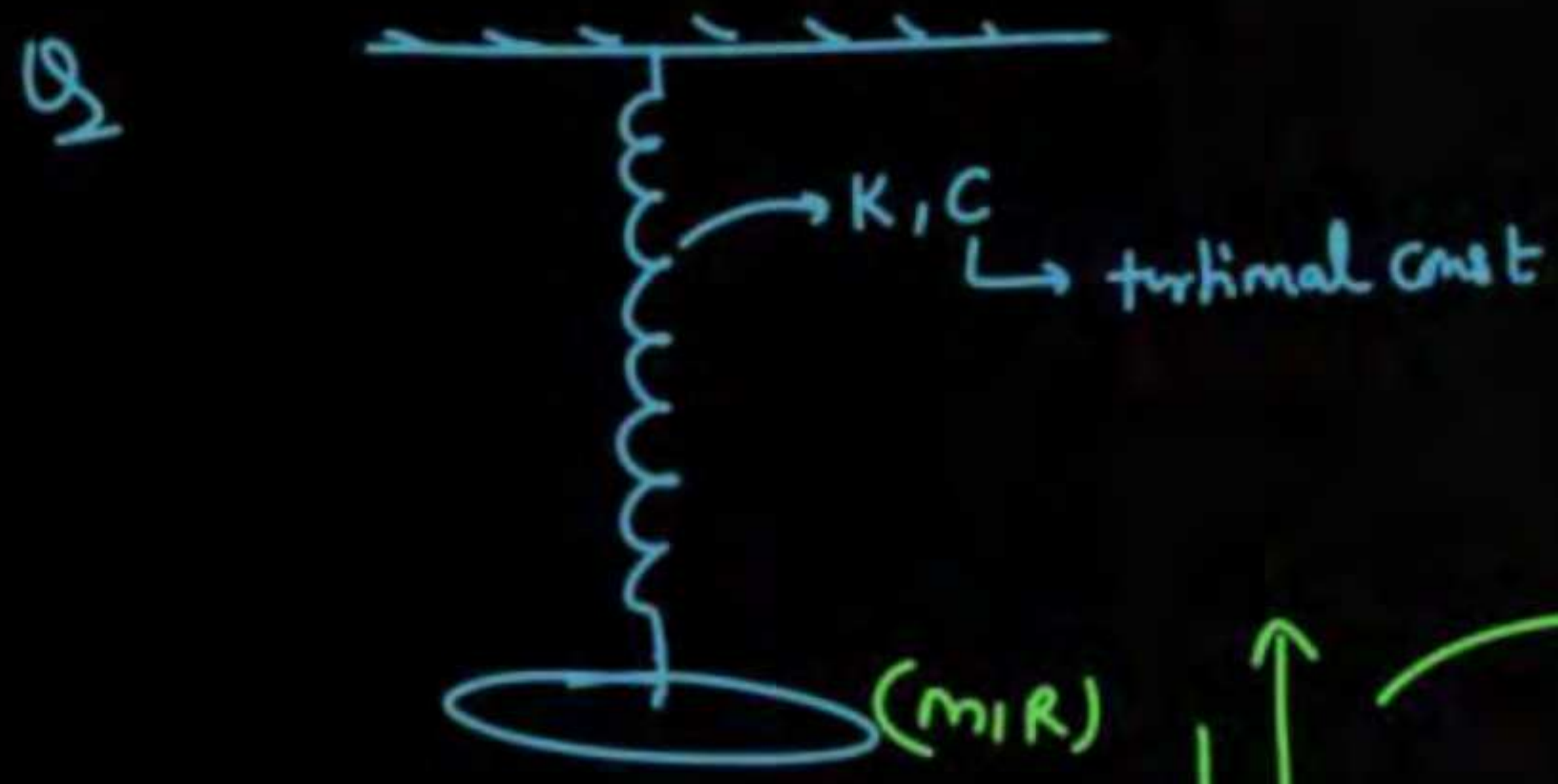
$$\vec{\tau}_{net} = -C\vec{\theta}$$

↙
Torsional
Cont

$$T = 2\pi \sqrt{\frac{I}{C}}$$



$$T = 2\pi \sqrt{\frac{MR^2/2}{C}}$$



$$\frac{T_1}{T_2} = ?$$

T_1 → Spring block
 T_2 → Torsional pend

$$T_1 = 2\pi \sqrt{\frac{m}{K}}$$

$$T_2 = 2\pi \sqrt{\frac{C}{H}}$$

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$$\frac{T_1}{T_2} = \sqrt{\frac{K}{C} \cdot \frac{m}{R}}$$

phasor

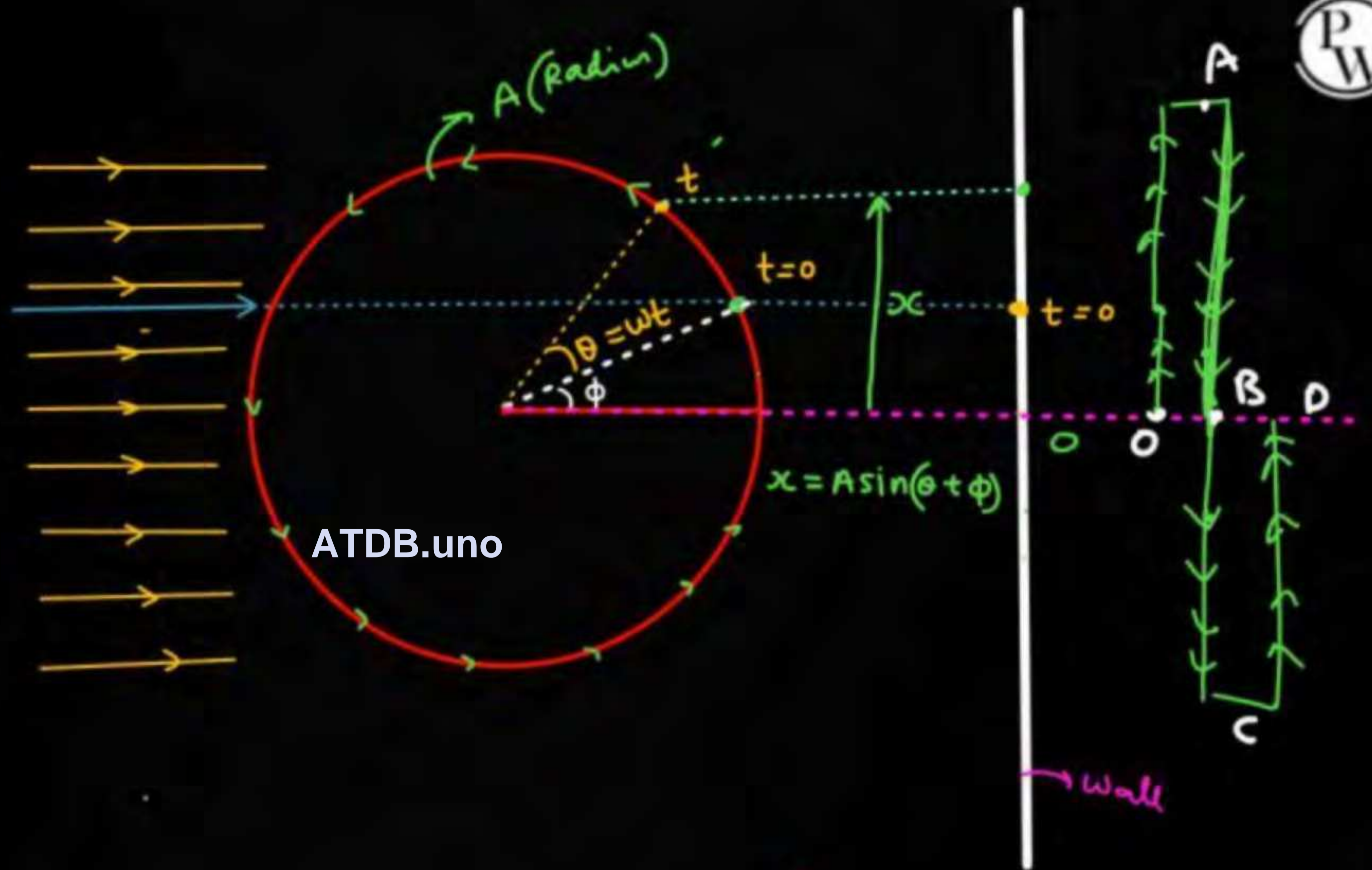
Uniform
Circular motion,
T, ω

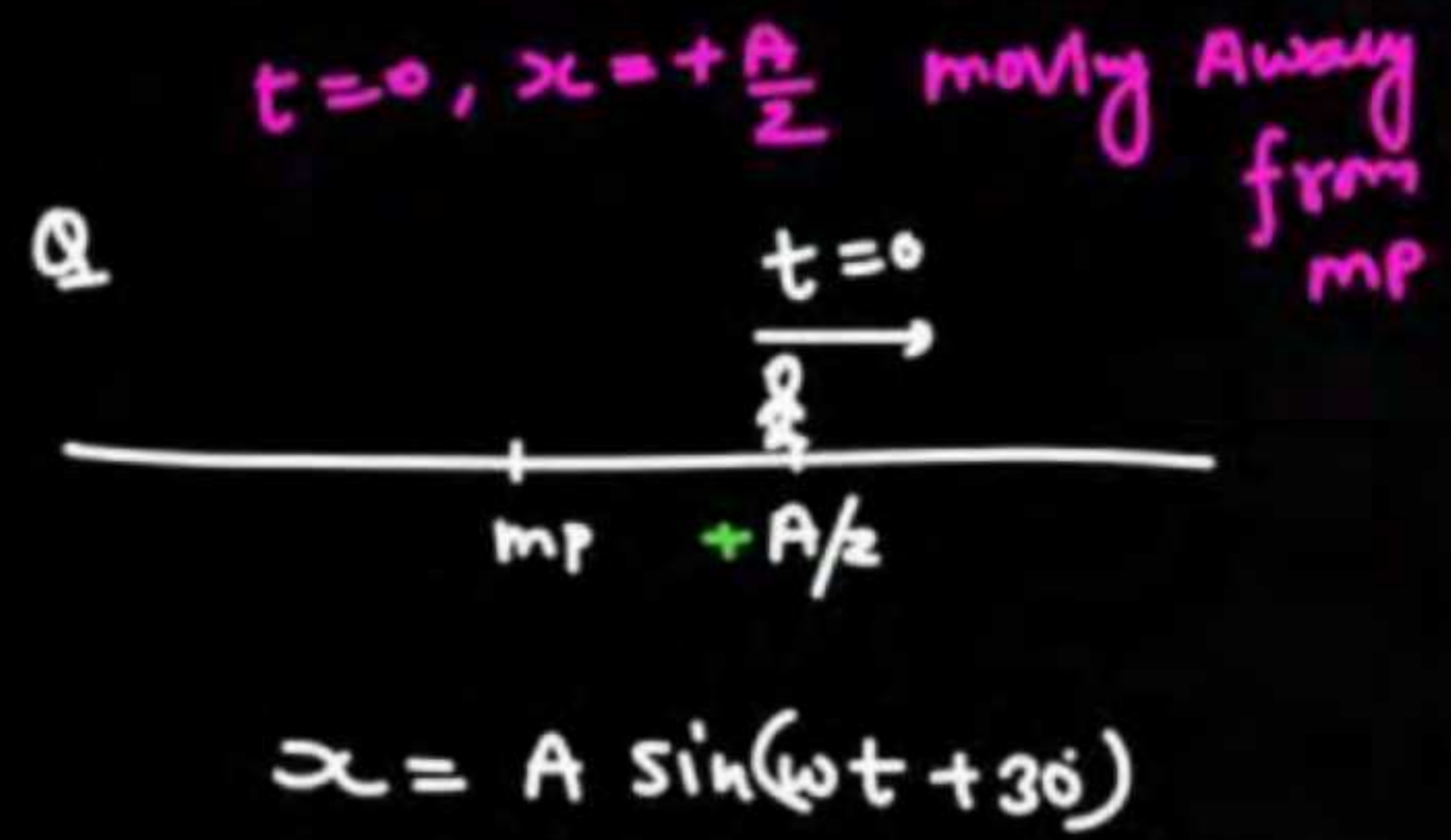
Shadow \rightarrow SHM.

$$x = A \sin(\theta + \phi)$$

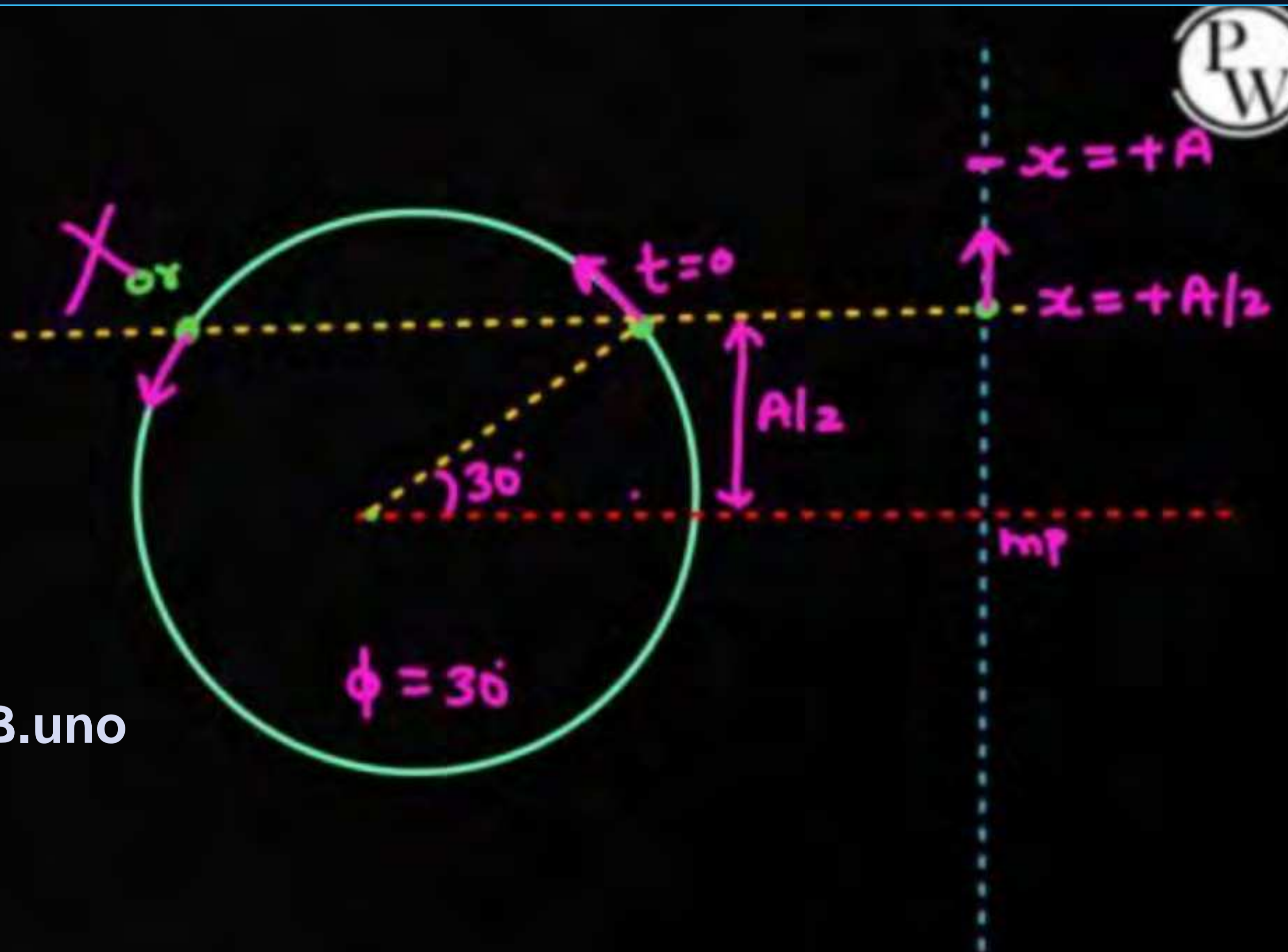
$$x = A \sin(\omega t + \phi)$$

SHM





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Q Two particles are performing SHM simultaneously in a single line, having same amplitude.

$$x_1 = A \sin \omega t \quad \text{--- (A)}$$

$$x_2 = A \sin(\omega t + 60^\circ) \quad \text{--- (B)} \quad (\omega, A \rightarrow \text{same})$$

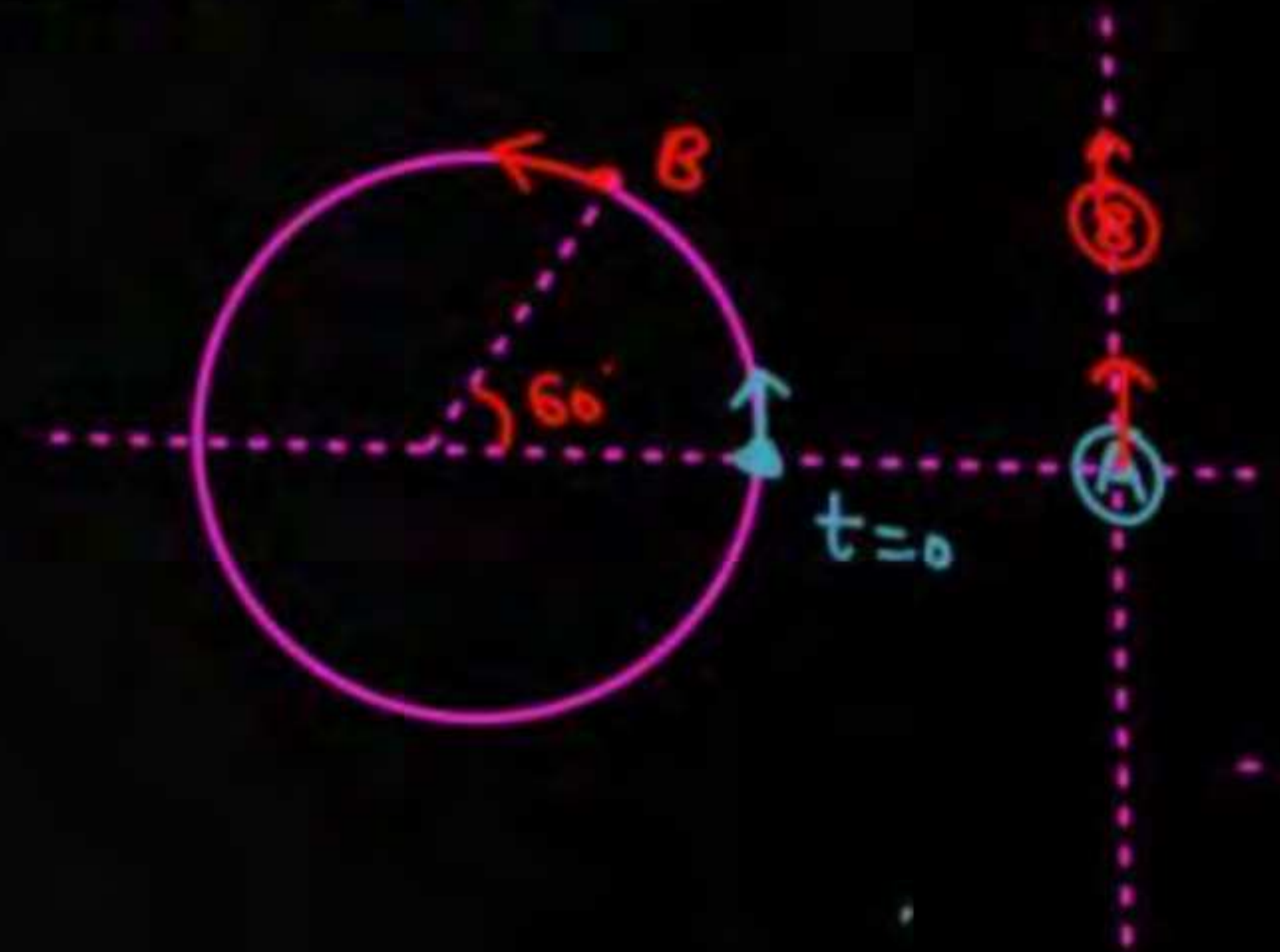
① find when they will meet

② find max sep^r b/w them & when it will be.



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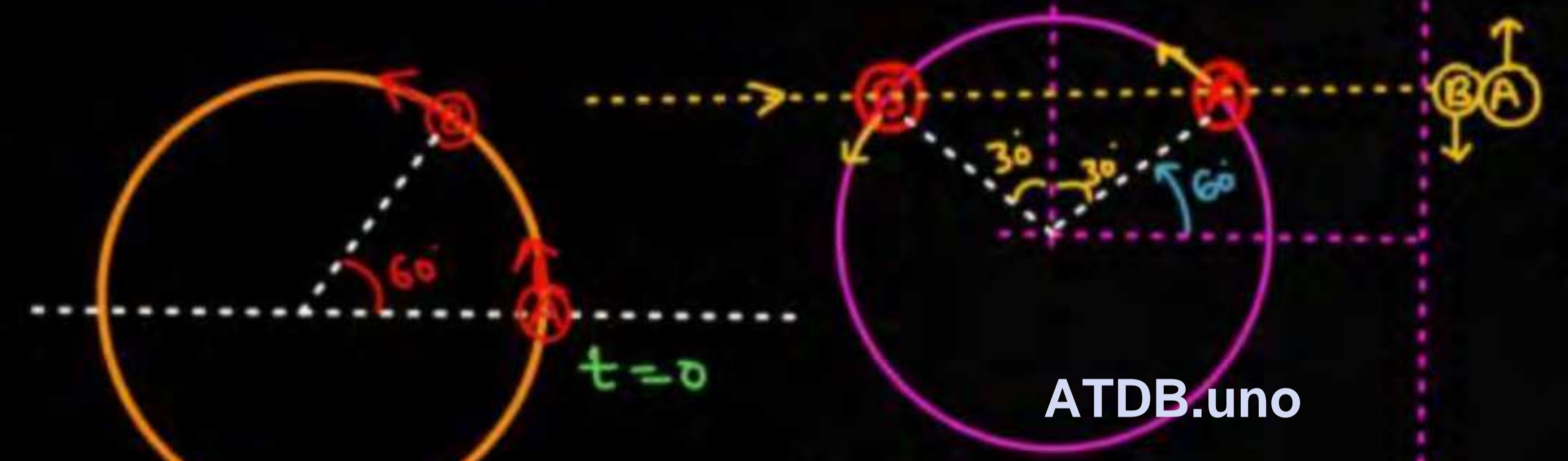
$t=0$



$t=0$

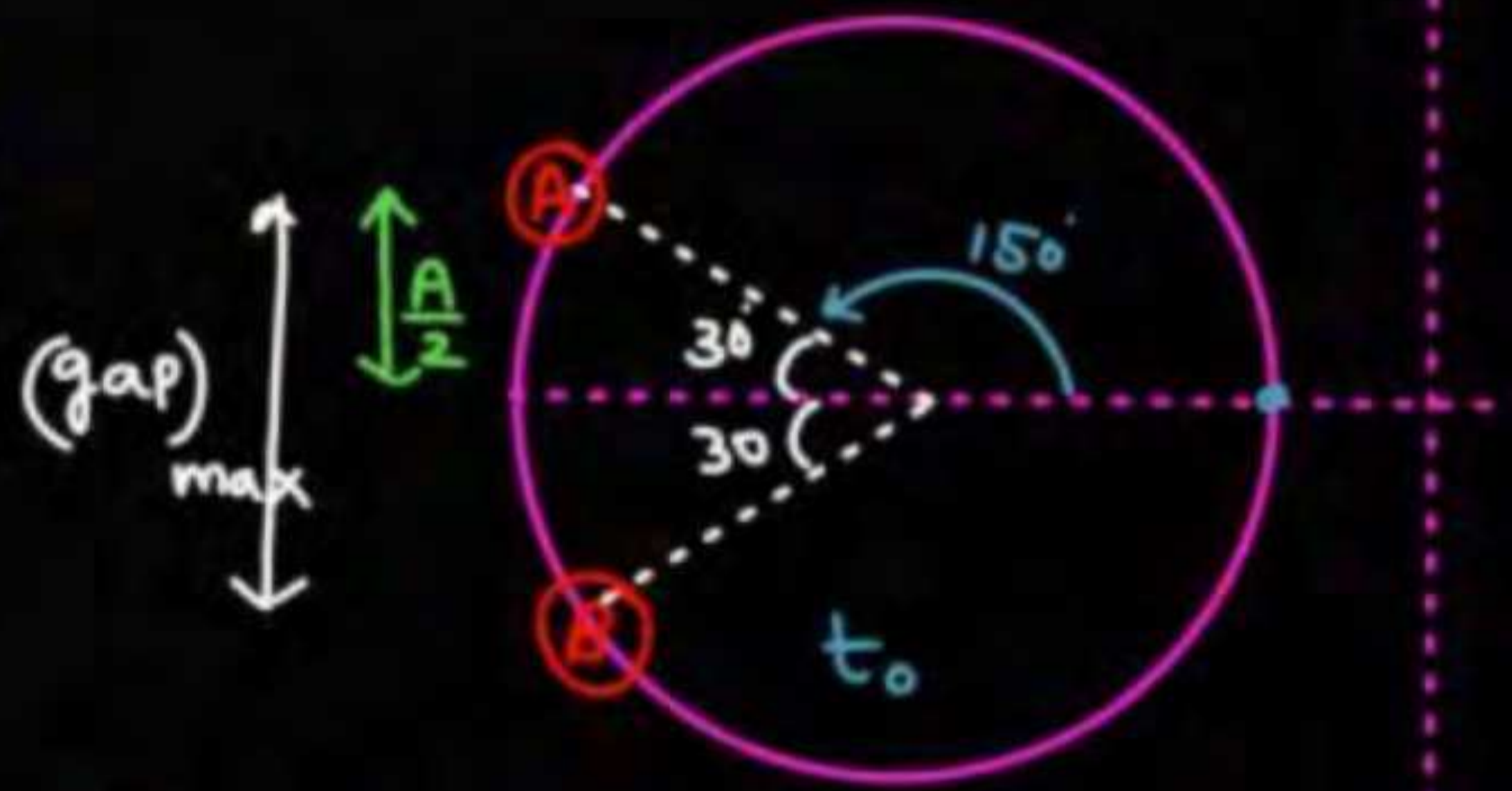
$x_1 = A \sin \omega t \Rightarrow A$
 $x_2 = A \sin(\omega t + 60) \Rightarrow B$

① when they meet



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② find max separation btw them



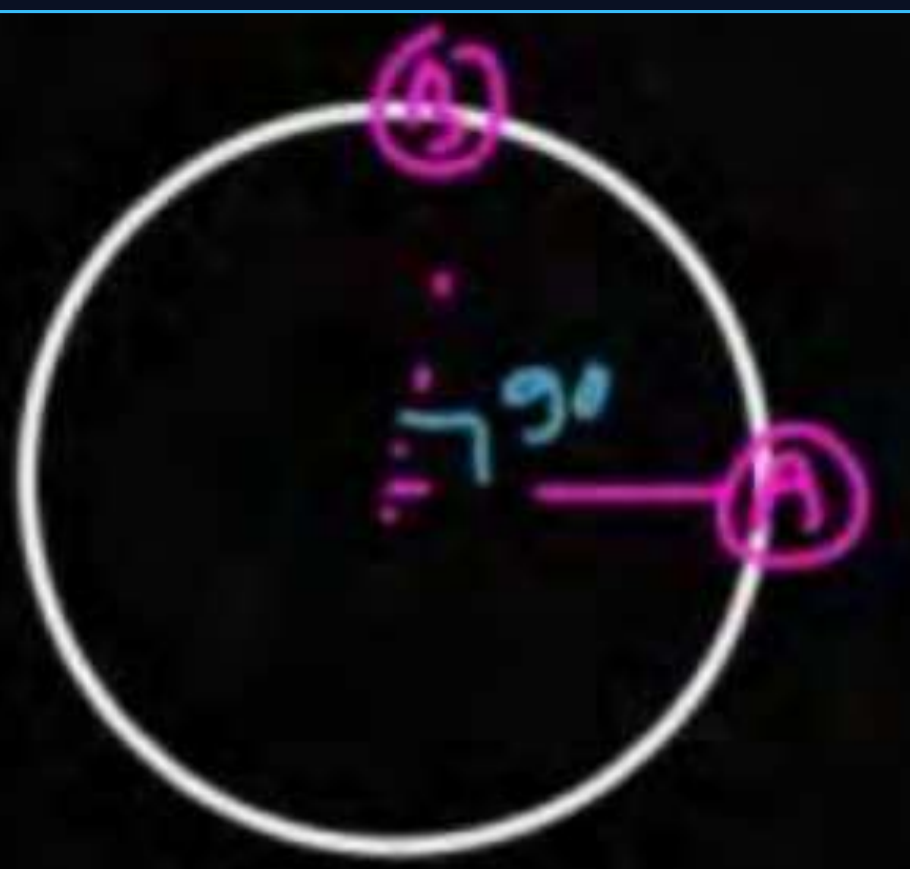
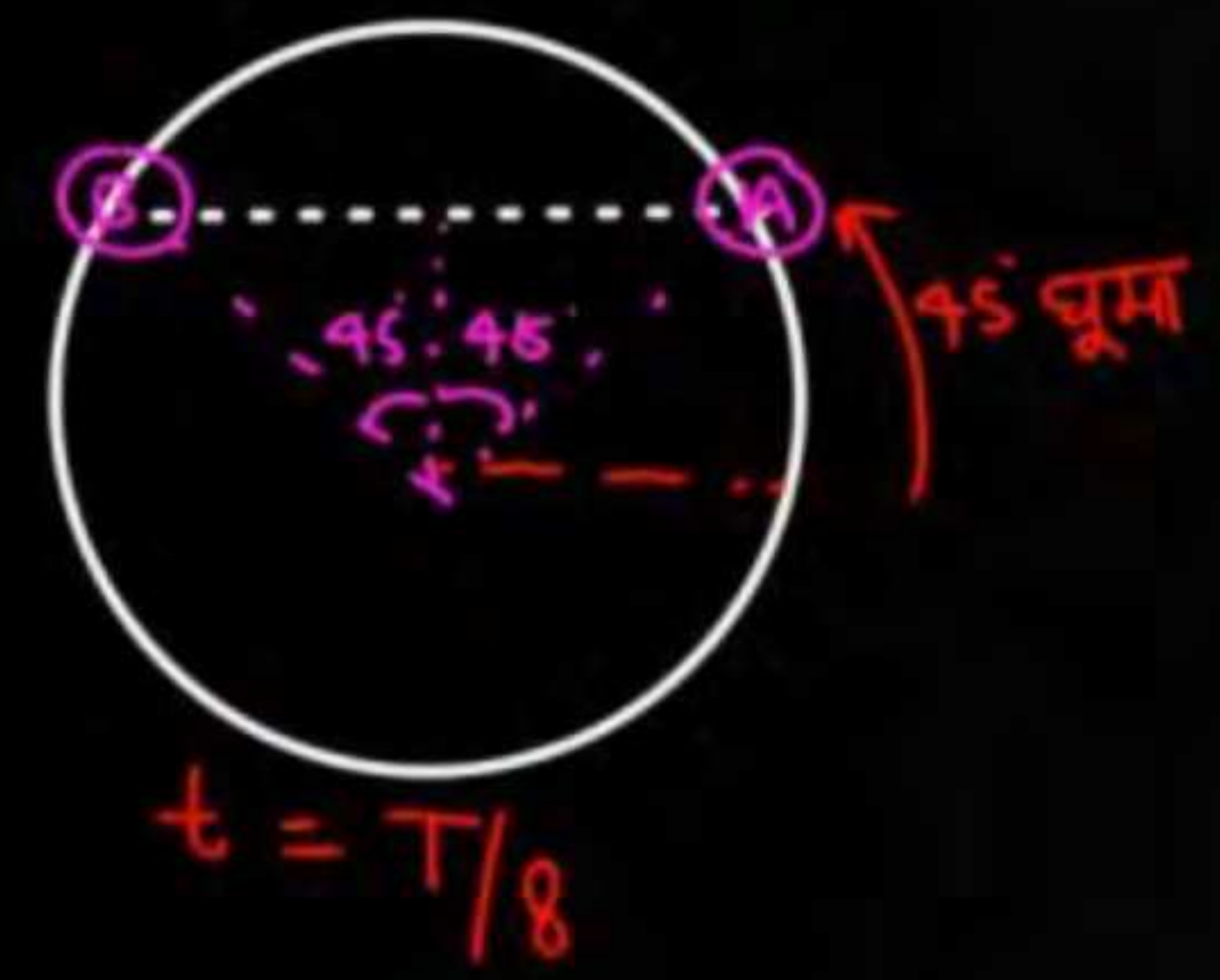
$A \equiv 60$ degrees
 $360 \rightarrow T$
 $60 \rightarrow T/6$
 Ann $T/6$

$gap = 2A \sin 30 = A$
 $360 \rightarrow T$
 $150 \rightarrow T/360 \times 150$



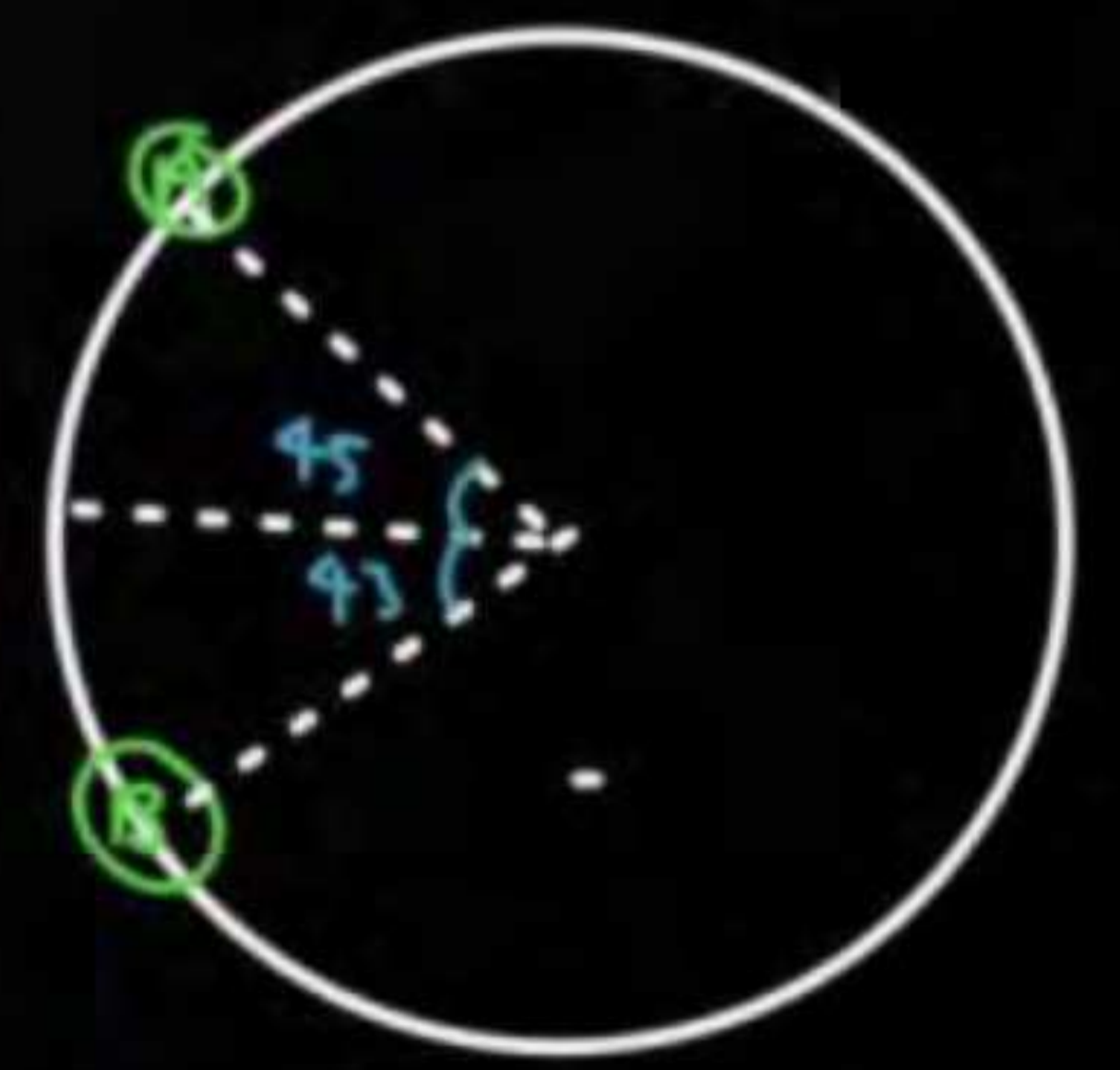
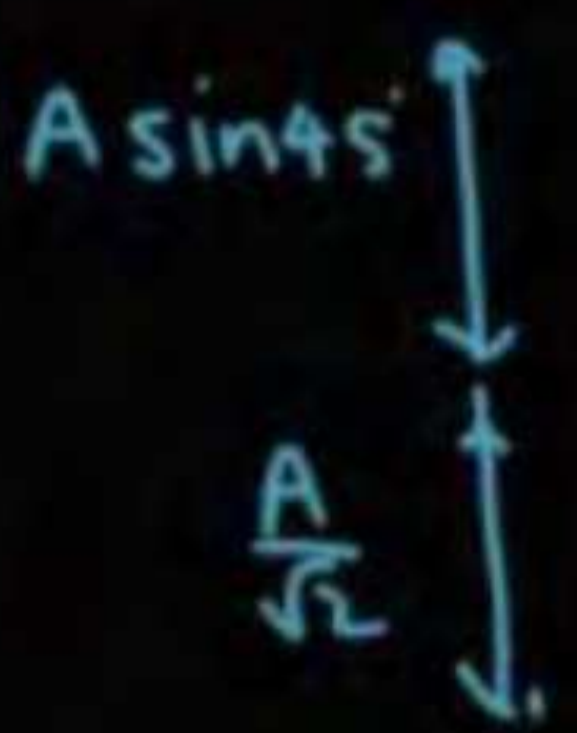
Q $x_1 = A \sin \omega t$ (A)
 $x_2 = A \sin(\omega t + 90)$ (B)

① When they will meet



② max sep^r

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(gap)_{max} = $\frac{A}{\sqrt{2}} + \frac{A}{\sqrt{2}} = A\sqrt{2}$



Superposition of SHM

$$\vec{F}_1 = -k\vec{x}_1 \longrightarrow x_1 = A_1 \sin \omega t$$

$$\vec{F}_2 = -k\vec{x}_2 \longrightarrow x_2 = A_2 \sin(\omega t + \phi)$$

$$x_{\text{net}} = A_{\text{net}} \sin(\omega t + \theta)$$

$$A_{\text{net}} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = -k(\vec{x}_1 + \vec{x}_2)$$

$$\boxed{\vec{F}_{\text{net}} = -k\vec{x}_{\text{net}}}$$

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you may skip this in notes

$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin(\omega t + \phi)$$

$$x_{net} = x_1 + x_2$$

$$x_{net} = A_{net} \sin(\omega t + \theta)$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$

phase diff.

write

maths

$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin(\omega t + \phi)$$

$$x_{net} = x_1 + x_2$$

$$x_{net} = A_1 \sin \omega t + A_2 \sin(\omega t + \phi)$$

$$= A_1 \sin \omega t + A_2 \sin \omega t \cos \phi + A_2 \cos \omega t \sin \phi$$

$$x_{net} = (A_1 + A_2 \cos \phi) \sin \omega t + A_2 \sin \phi \cos \omega t$$

$$x_{net} = a \sin \omega t + b \cos \omega t$$

$$x_{net} = \sqrt{a^2 + b^2} \left(\frac{a}{\sqrt{a^2 + b^2}} \sin \omega t + \frac{b}{\sqrt{a^2 + b^2}} \cos \omega t \right)$$

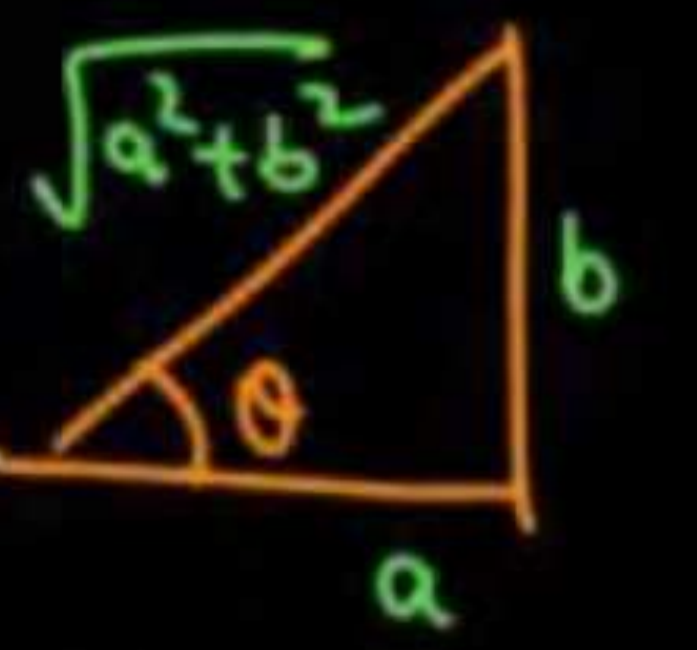
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$$x_{net} = \sqrt{a^2 + b^2} \sin(\omega t + \theta)$$

$$x_{net} = A_{net} \sin(\omega t + \theta)$$

$$\tan \theta = \frac{b}{a} = \frac{A_2 \sin \phi}{A_1 + A_2 \cos \phi}$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$





$$\begin{aligned}A_{\text{net}} &= \sqrt{a^2 + b^2} \\ &= \sqrt{(A_1 + A_2 \cos \phi)^2 + (A_2 \sin \phi)^2} \\ &= \sqrt{A_1^2 + \underline{A_2^2 \cos^2 \phi} + 2A_1 A_2 \cos \phi + \underline{A_2^2 \sin^2 \phi}}\end{aligned}$$

$$A_{\text{net}} = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos \phi}$$

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Q

$$x_1 = A \sin \omega t$$

$$x_2 = A \sin(\omega t + 60^\circ)$$

$$x_{\text{net}} = x_1 + x_2 = A_{\text{net}} \sin(\omega t + \theta)$$

$$A_{\text{net}} = \sqrt{A^2 + A^2 + 2 \cdot A \cdot A \cdot \cos 60^\circ}$$

$$= A\sqrt{3}$$

$$\tan \theta = \frac{A \sin 60^\circ}{A + A \cos 60^\circ} = \frac{\sqrt{3}/2}{1 + \frac{1}{2}} = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

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Q

$$x_1 = A \sin \omega t$$

$$x_2 = A\sqrt{3} \cos \omega t = A\sqrt{3} \sin(\omega t + 90^\circ)$$

$$x_{\text{net}} = ?$$

$$A_{\text{net}} = \sqrt{A^2 + (A\sqrt{3})^2 + 2 \cdot A \cdot A\sqrt{3} \cos 90^\circ}$$

$$= 2A$$

$$\tan \theta = \frac{A\sqrt{3} \sin 90^\circ}{A + A\sqrt{3} \cos 90^\circ} = \sqrt{3}$$

$$\theta = 60^\circ$$

$$A_{\text{net}} = 2A \sin(\omega t + 60^\circ)$$

Q

$$x_1 = A \sin \omega t$$

$$x_2 = A \sin(\omega t + 60)$$

$$x_{nt} = x_1 + x_2 = A_{nt} \sin(\omega t + \theta)$$



$$A_{nt} = \sqrt{A^2 + A^2 + 2A \cdot A \cos 60}$$

$$A_{nt} = A\sqrt{3}$$

$$x_{nt} = A\sqrt{3} \sin(\omega t + 30)$$

$$\cos \omega t = \sin(\omega t + 90)$$

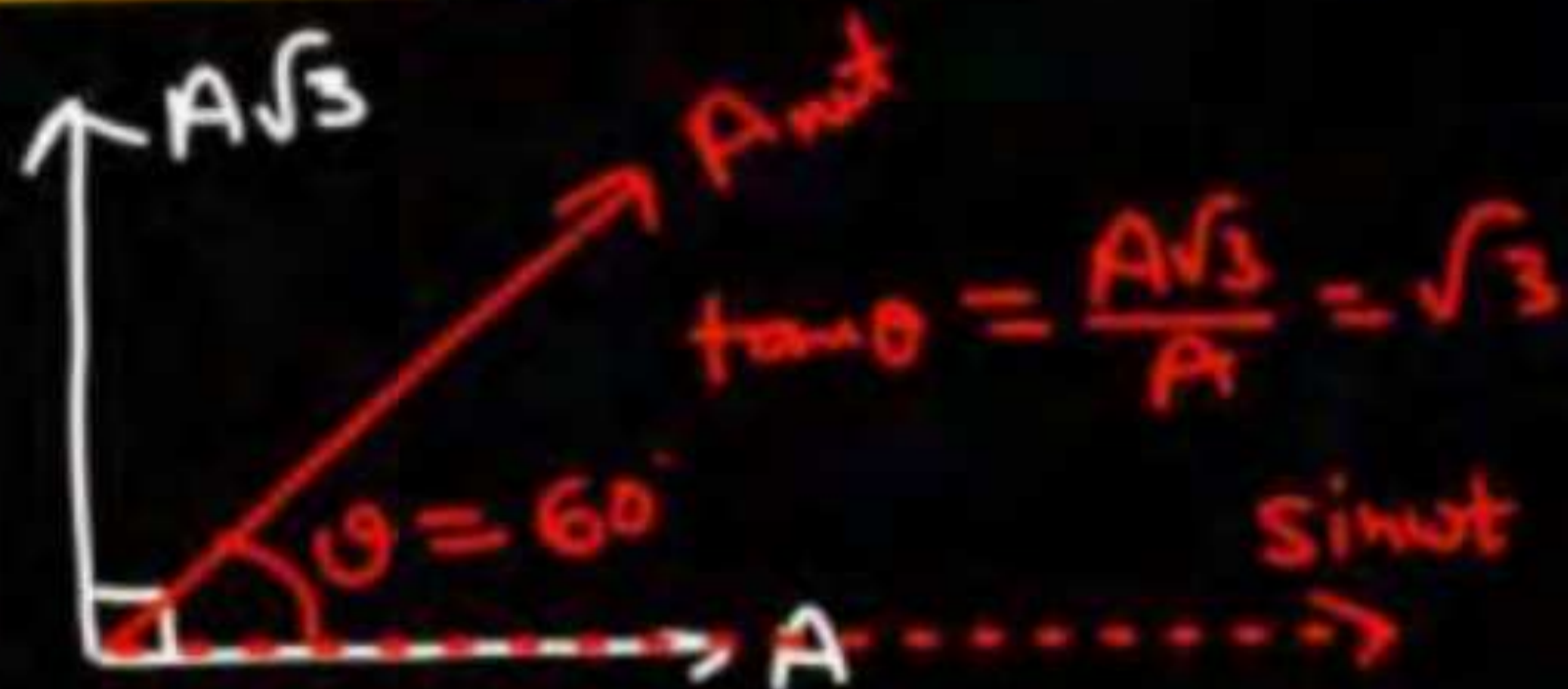
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Q

$$x_1 = A \sin \omega t$$

$$x_2 = A\sqrt{3} \cos \omega t = A\sqrt{3} \sin(\omega t + 90)$$

$$x_{nt} = ?$$



$$A_{nt} = \sqrt{A^2 + (A\sqrt{3})^2} = 2A$$

$$x_{nt} = 2A \sin(\omega t + 60)$$

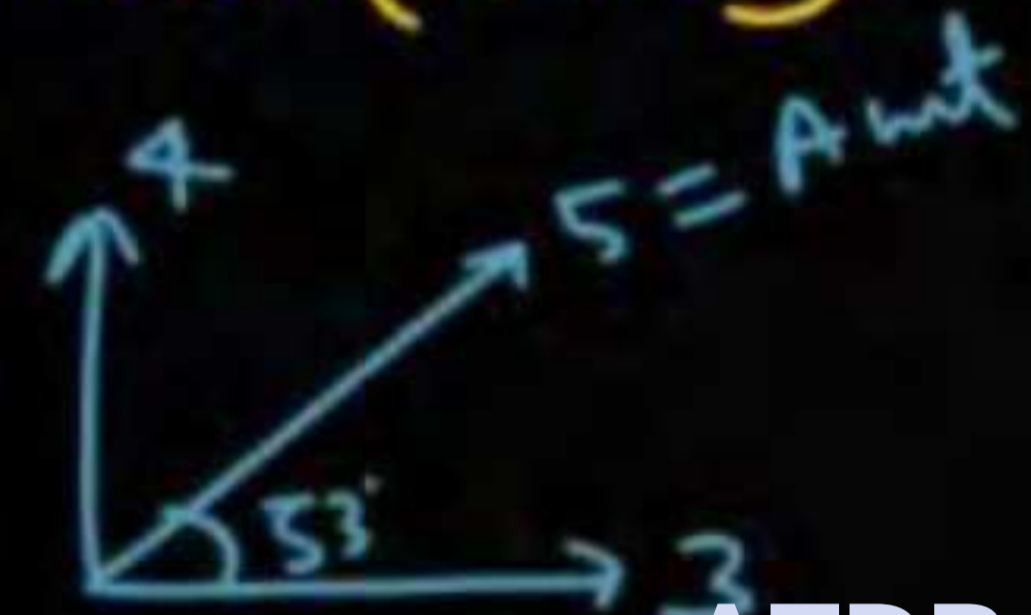




Q $x_1 = 3 \sin \omega t$
 $x_2 = 4 \sin(\omega t + 90^\circ)$

$x_{net} = 3 \sin \omega t + 4 \sin(\omega t + 90^\circ)$

$x_{net} = 5 \sin(\omega t + 53^\circ)$



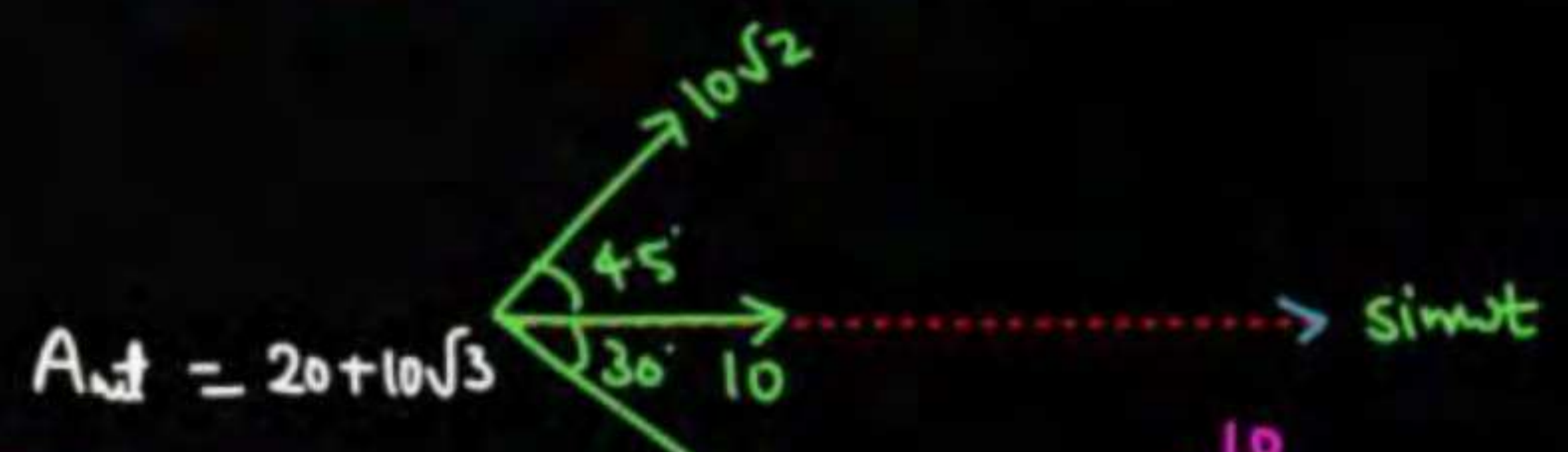
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Q $x_1 = 10\sqrt{2} \sin(\omega t + 45^\circ)$

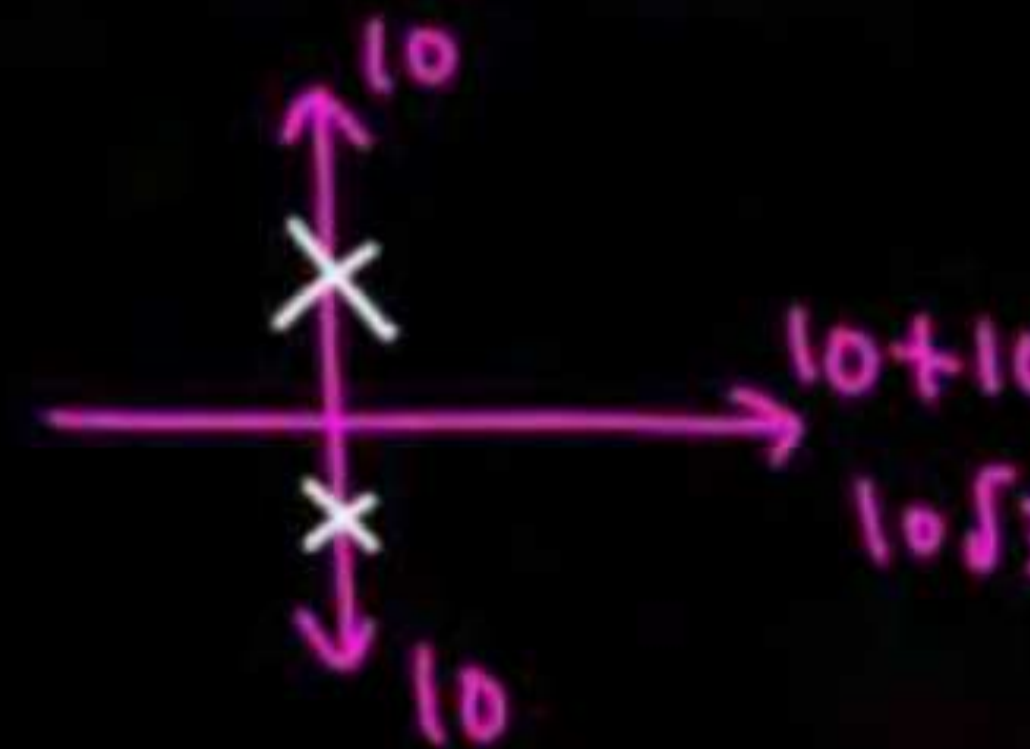
$x_2 = 10 \sin \omega t$

$x_3 = 20 \sin(\omega t - 30^\circ)$

find $x_1 + x_2 + x_3 = ?$



$x_{net} = (20 + 10\sqrt{3}) \sin(\omega t + 0)$



Q

$$x_1 = 10 \sin \omega t$$

$$x_2 = 20 \sin(\omega t + 90^\circ)$$

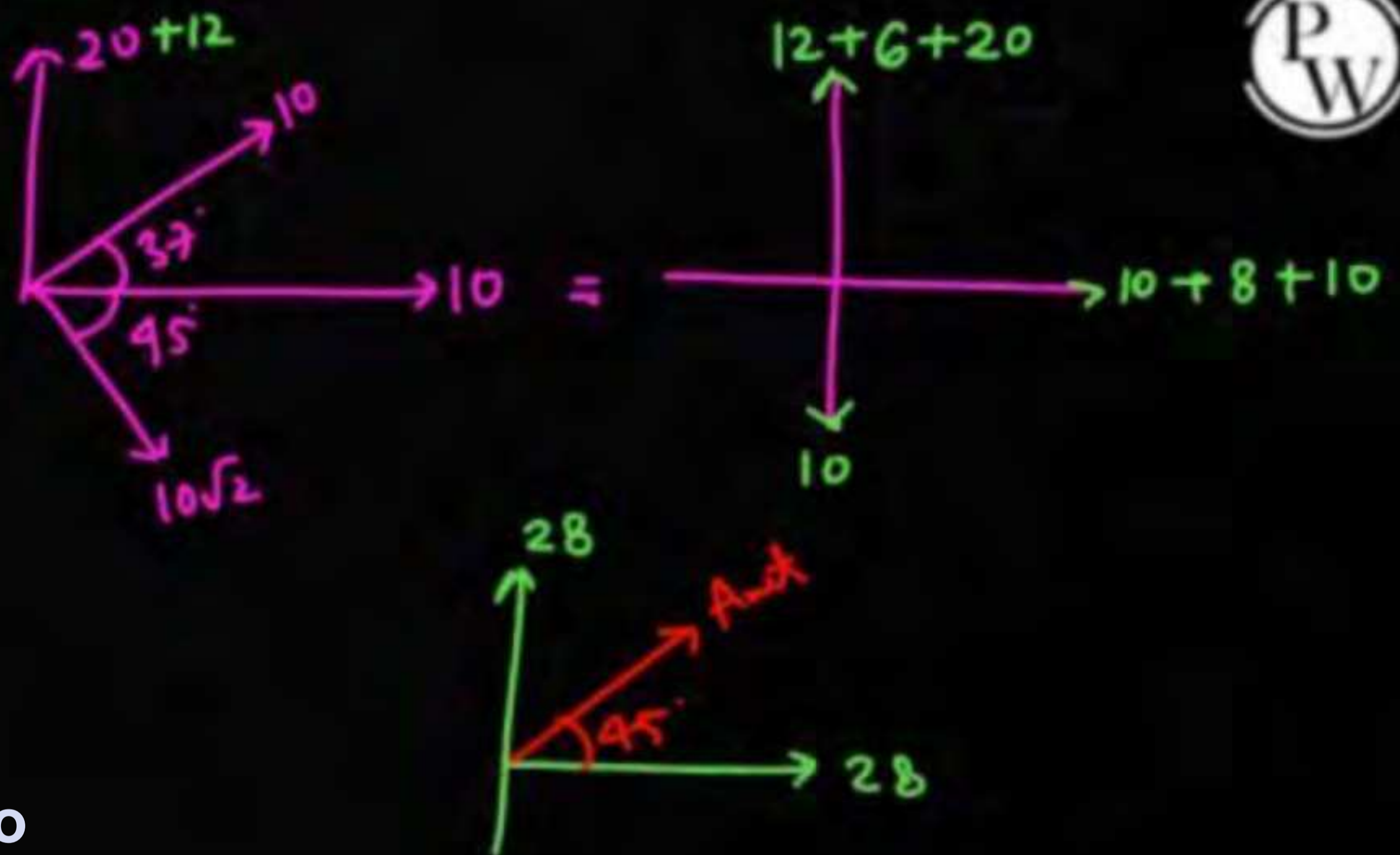
$$x_3 = 10 \sin(\omega t + 37^\circ)$$

$$x_4 = 10\sqrt{2} \sin(\omega t - 45^\circ)$$

$$x_5 = 12 \sin(\omega t + 90^\circ)$$

$$x_{\text{net}} = 28\sqrt{2} \sin(\omega t + 45^\circ)$$

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Q

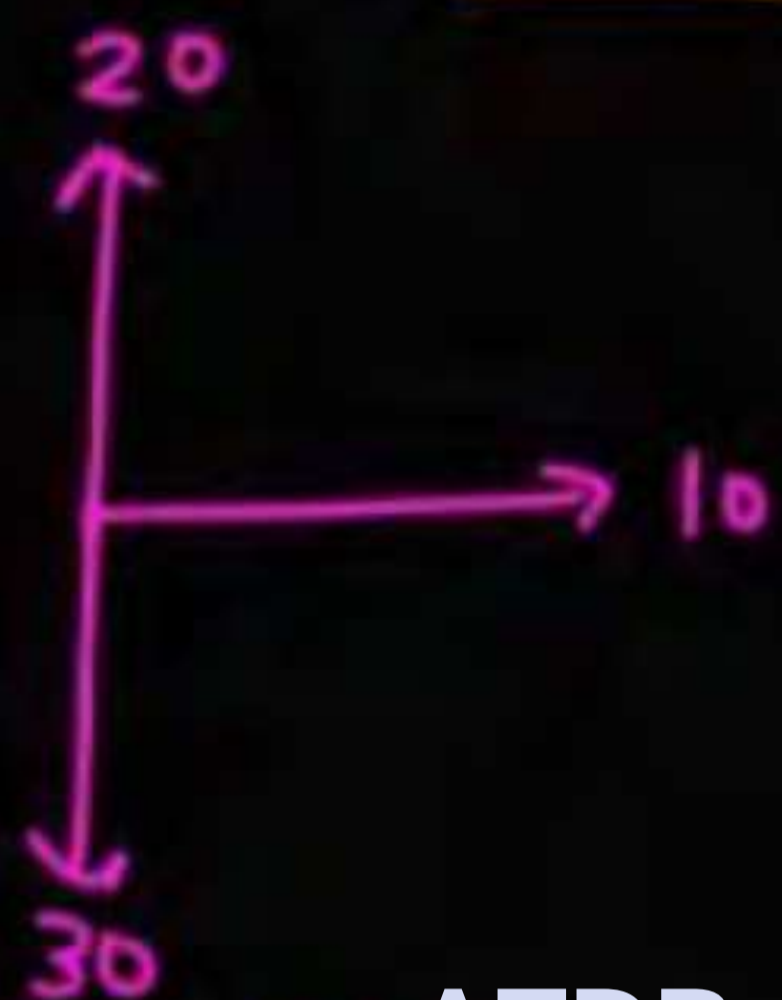
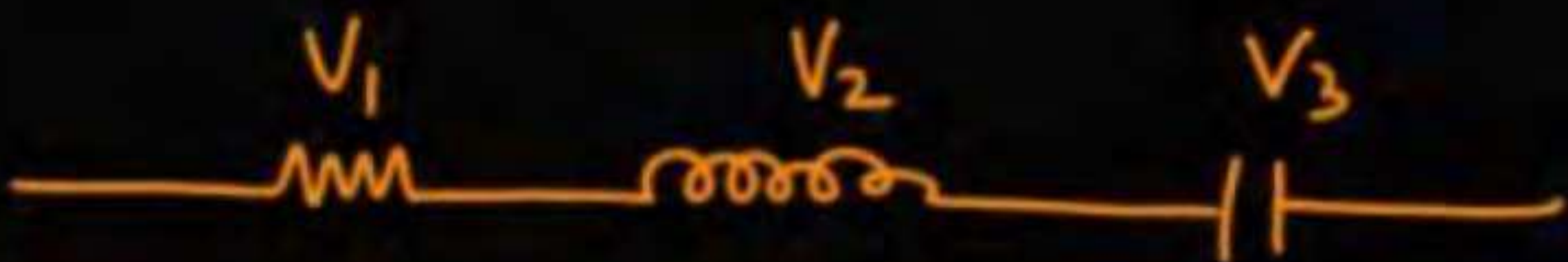
$$V_1 = 10 \sin \omega t$$

$$V_2 = 20 \sin(\omega t + 90^\circ)$$

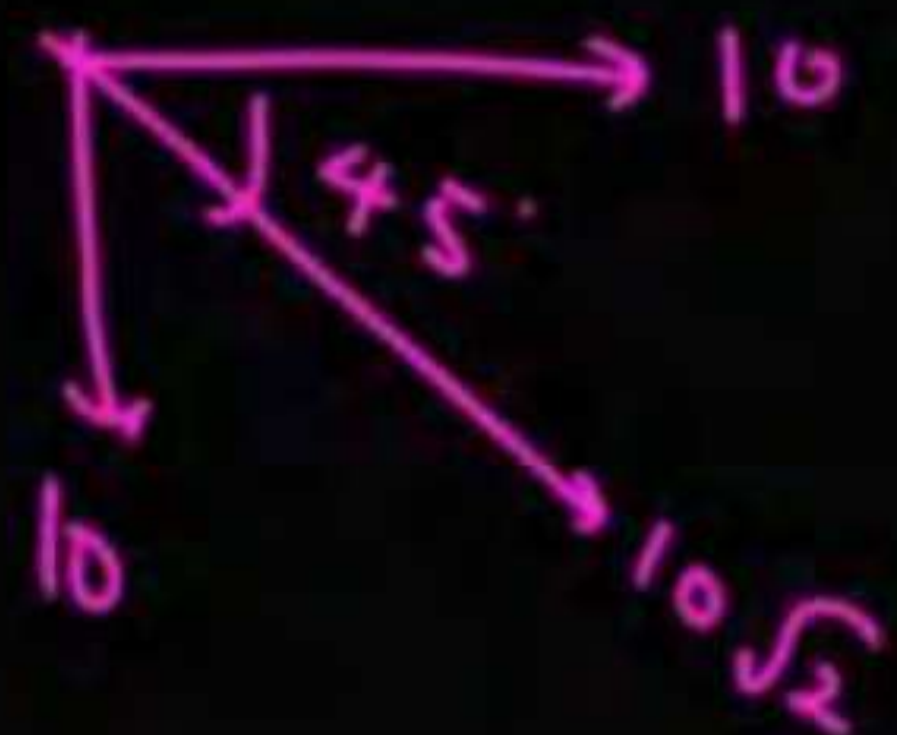
$$V_3 = 30 \sin(\omega t - 90^\circ)$$

$$V_1 + V_2 + V_3 =$$

$$= 10\sqrt{2} \sin(\omega t - 45^\circ)$$



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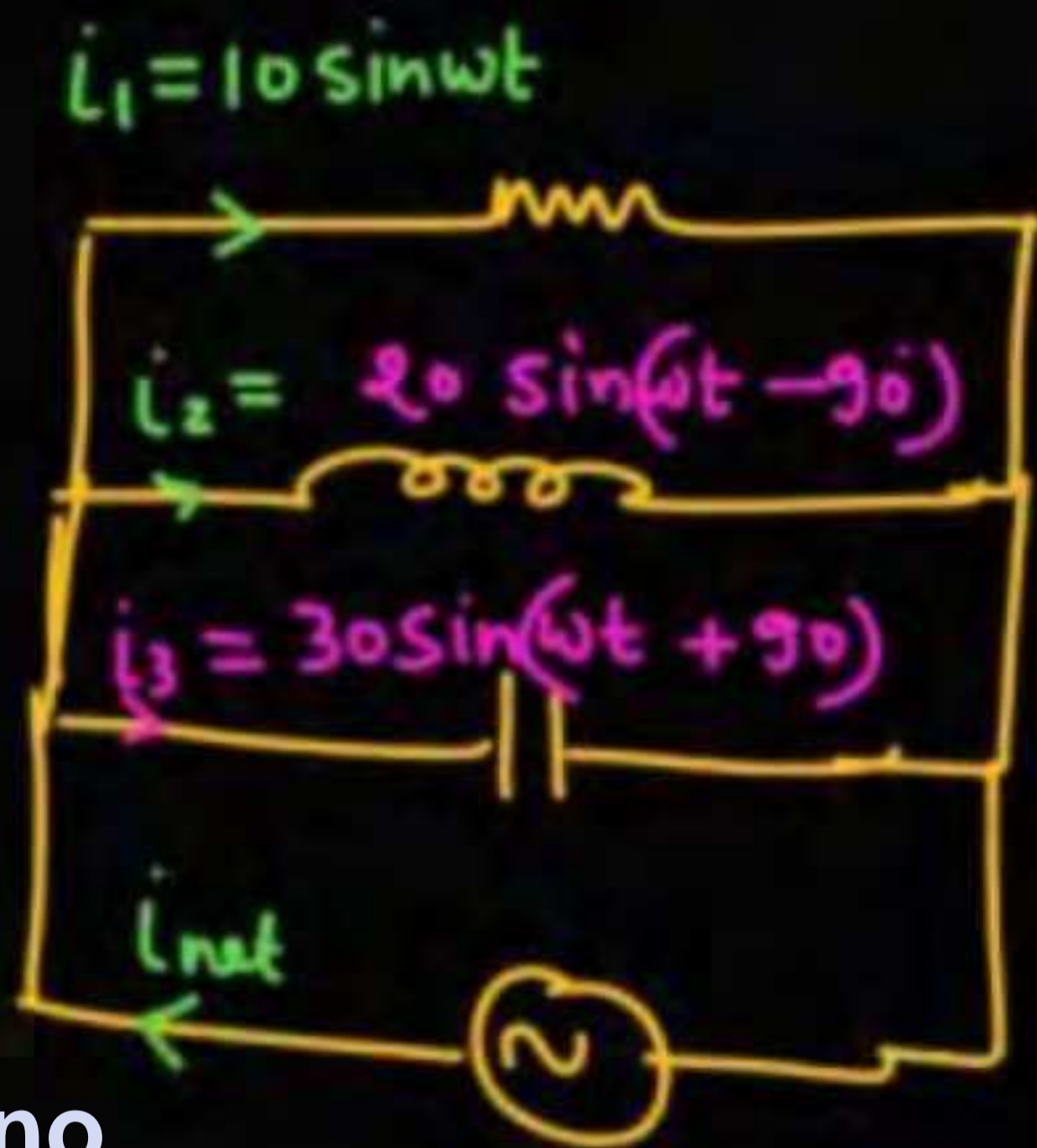




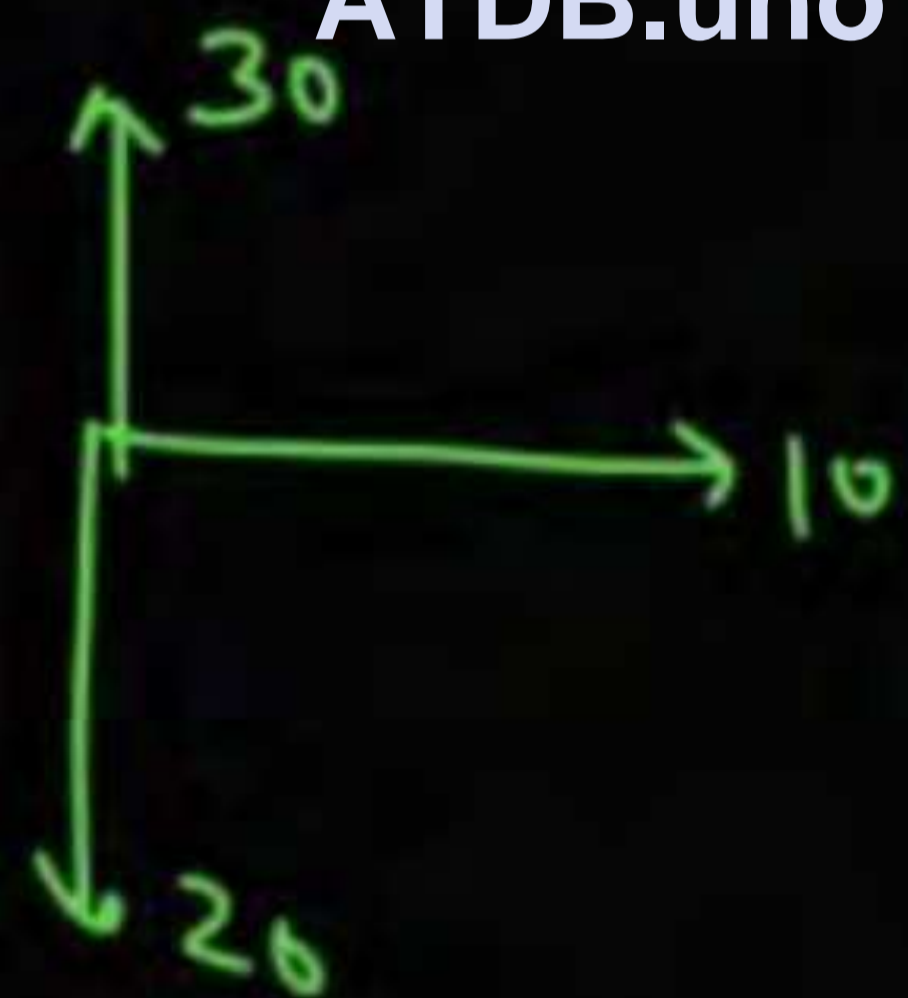
Q

$$\begin{aligned}i_{net} &= \bar{i}_1 + \bar{i}_2 + \bar{i}_3 \\ &= 10 \sin \omega t + 20 \sin(\omega t - 90^\circ) \\ &\quad + 30 \sin(\omega t + 90^\circ)\end{aligned}$$

$$= 10\sqrt{2} \sin(\omega t + 45^\circ)$$



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Homework

— Revise full STM

— module → Panikshit JA ⇒ 10, 17, 18, 19-23, 27,
PYQ → All

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THANK

YOU

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