PRAYAS

JEE 2025

Lecture - 04 | **Physics**

Oscillations

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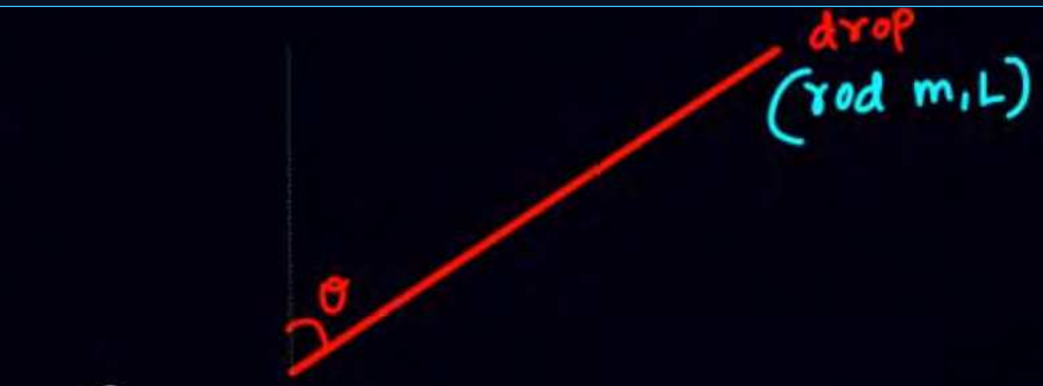
Topics *to be covered*

1. Kinematics & Graph in SHM

2. Relation between x, v, a, KE, PE
3.
4.

HW

Q



$m = 10\,kg$

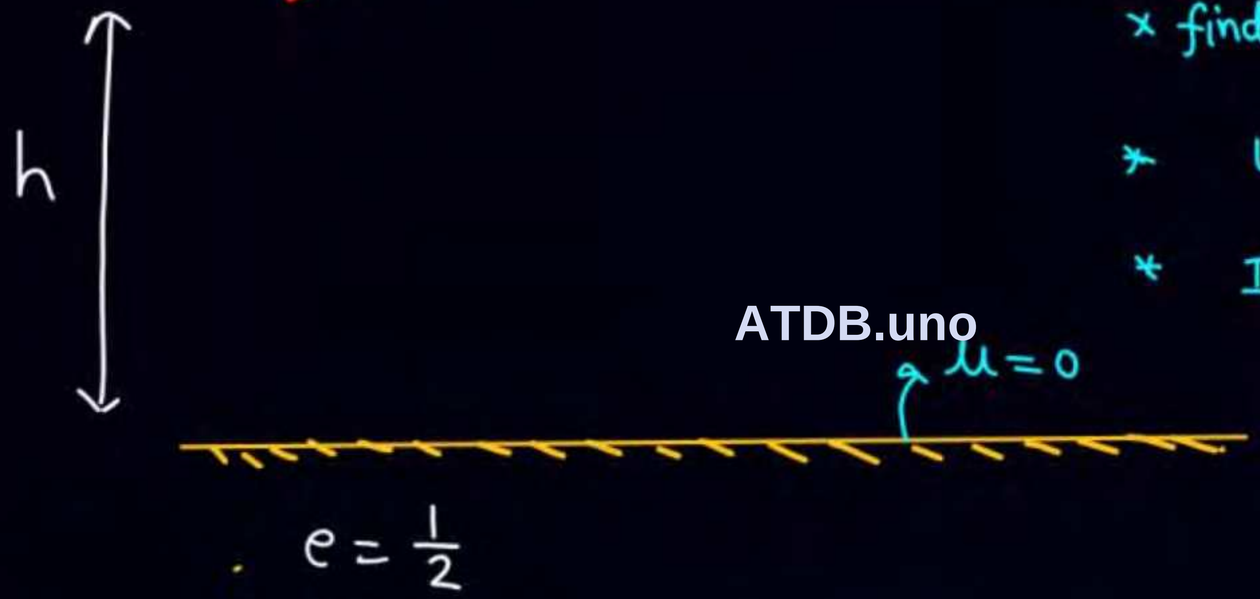
$L = 2\,m$

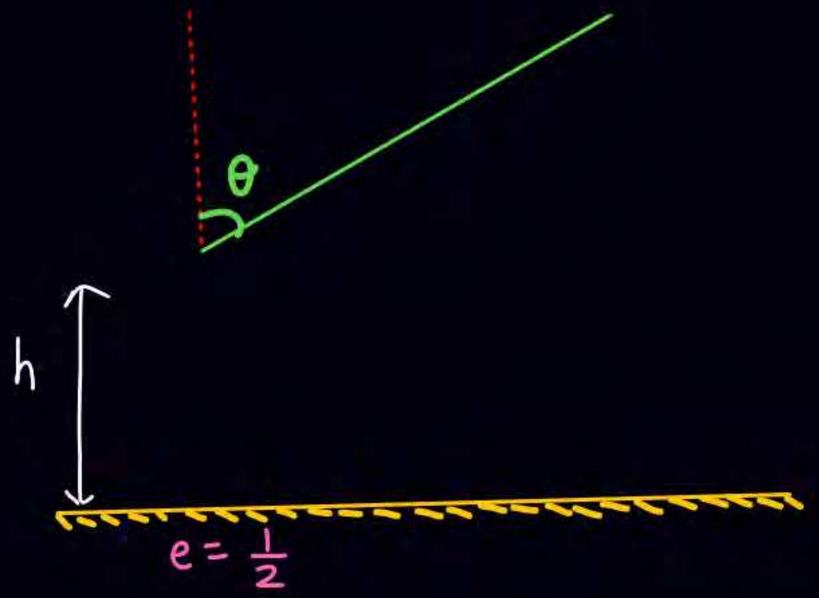
$\theta = 37^\circ$, $h = 20\,m$

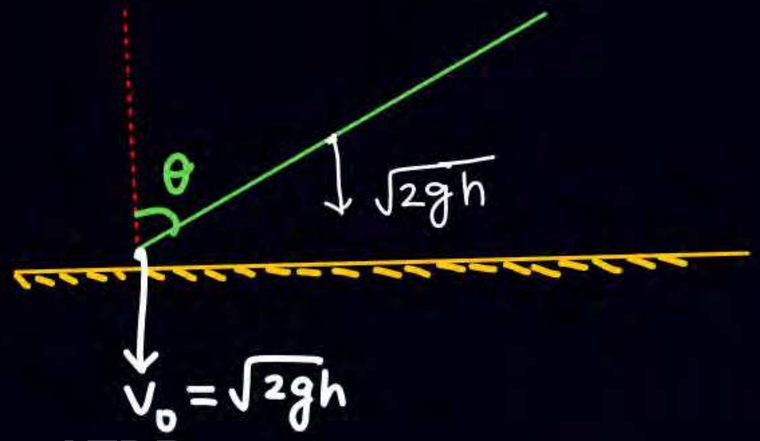
* find max. height attain by COM

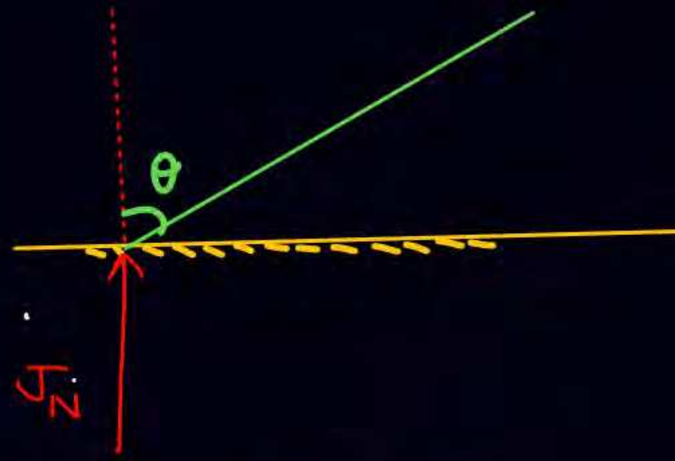
* v_{com} & ω just after collision

* Impulse imparted by ground





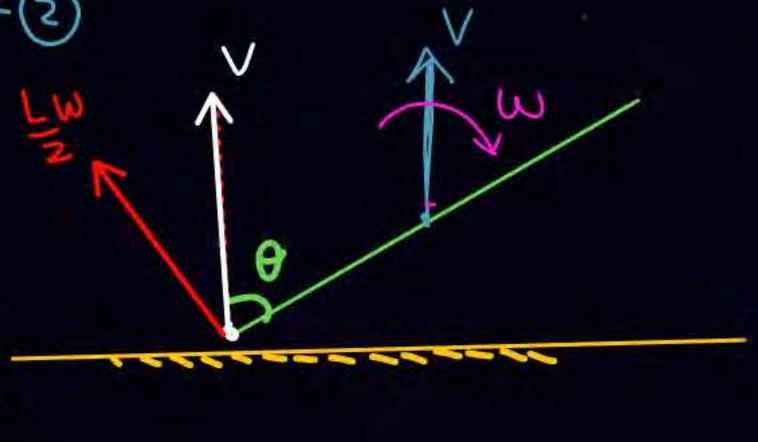




$$J_N \sin\theta \frac{L}{2} = \frac{mL^2}{12}\omega \quad \text{——} (2)$$

$$e = \frac{1}{2} = \frac{V + \frac{L\omega}{2}\sin\theta - 0}{\sqrt{2gh}}$$

—— (3)



$$J_N = mV - \left(m(-V_0)\right)$$

$$J_N = m\left(V + \sqrt{2gh}\right)$$

HW

Q

$x = A\sin(\omega t + 60^\circ)$

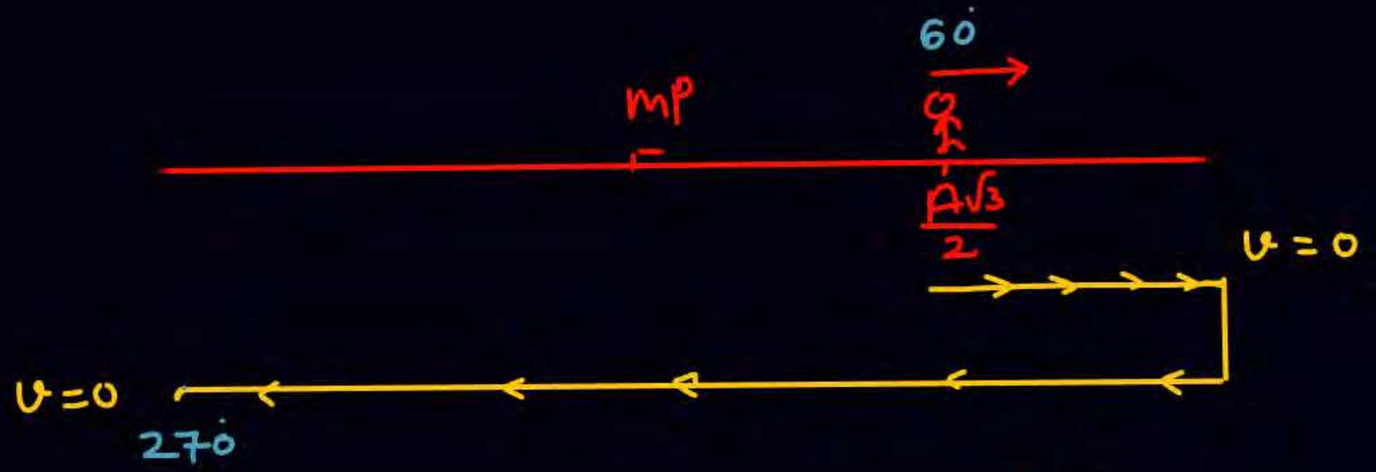
① find avg velocity from $t=0$ to particle comes to at rest second time.

" Speed

②

" " " " acc become zero second time

(1)



$$t = \frac{T}{360} \times 210$$

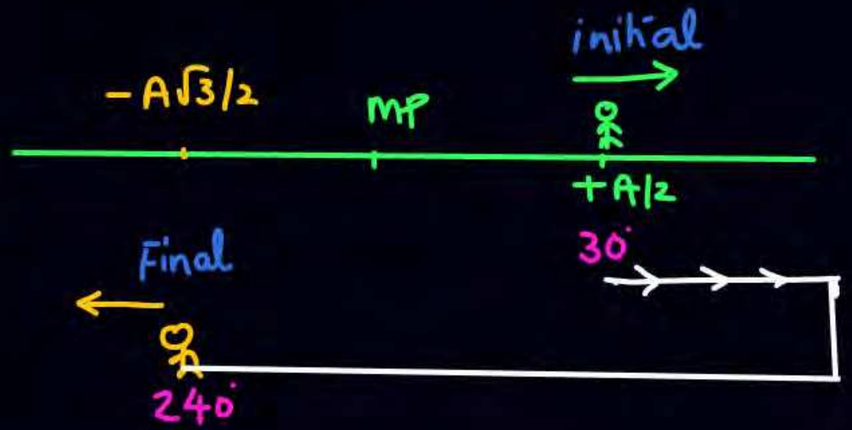
$$\text{Avrg velocity} = \frac{\left(\frac{A\sqrt{3}}{2} + A\right)}{\frac{T}{360} \times 210} (-\hat{i})$$

$$\text{Avrg Speed} = \frac{A - \frac{A\sqrt{3}}{2} + 2A}{\frac{T}{360} \times 210}$$

Q find min time taken by particle performing SHM in following case (A, ω, T..)

① Initially $x = +\frac{A}{2}$ moving Away MP

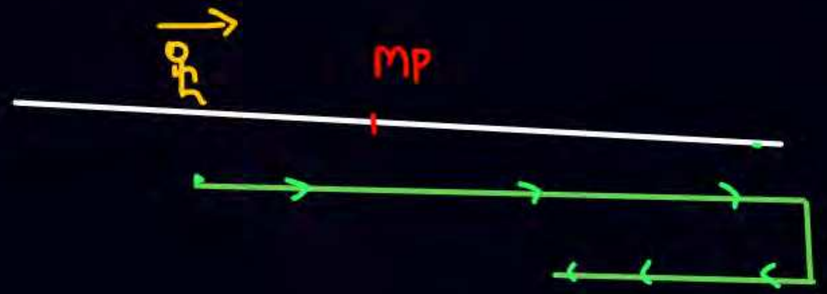
finally $x = -\frac{A\sqrt{3}}{2}$ moving Away M.P.



$$\frac{T}{360} \times 210$$

Q Initially $x = -\frac{A\sqrt{3}}{2}$ moving towards MP

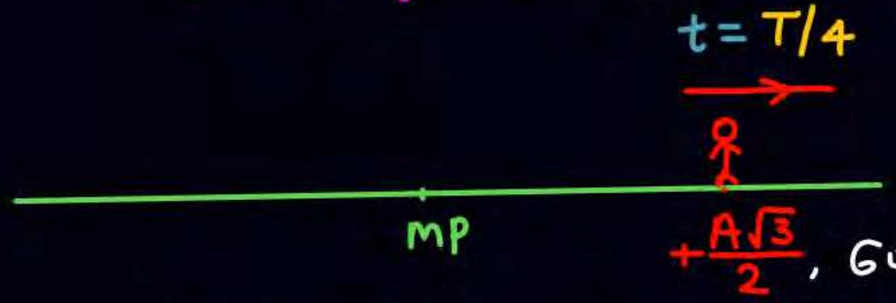
finally $x = +A/2$ " towards M.P



$$\frac{T}{360} \times 210$$

$$60 + 150 \equiv 210^\circ$$

find eqⁿ of SHM



$x = A\sin(\omega t + \phi)$

$\frac{A\sqrt{3}}{2} = A\sin\left(\frac{2\pi}{T}\frac{T}{4} + \phi\right)$

$\frac{\sqrt{3}}{2} = \sin(90 + \phi)$

$90 + \phi \rightarrow 60°, 120°$

$v = A\omega\cos(\omega t + \phi)$

$t = T/4 \quad v > 0$

$A\omega\cos\left(\frac{2\pi}{T}\cdot\frac{T}{4} + \phi\right) > 0$

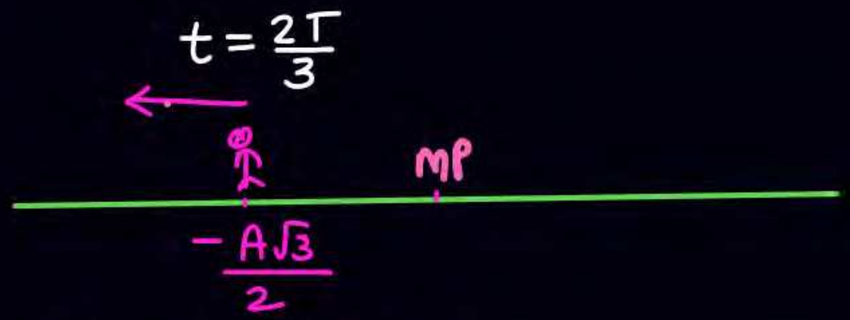
$\cos(90 + \phi) > 0$

$\phi \rightarrow -30°, 30°$

$\phi = -30$ ✓

$x = A\sin(\omega t - 30)$

Q

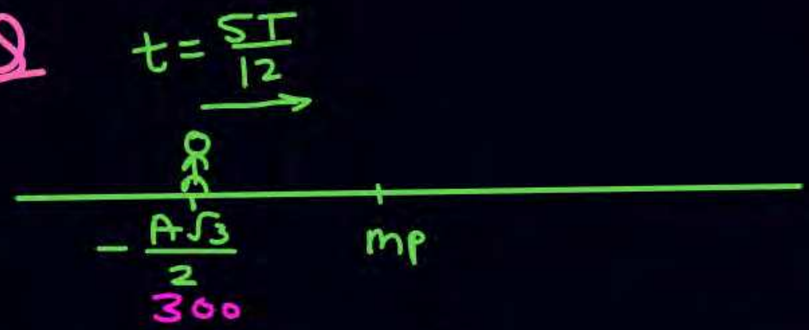


find eqn of SHM.

$$x = A\sin(\omega t + 240 - 240)$$

$$\boxed{x = A\sin\omega t}$$

Q



$$x = A\sin(\omega t + 300 - 150)$$

$x = A\sin(\omega t + \phi)$

$v = A\omega\cos(\omega t + \phi)$

$v_{max} = A\omega$

$a = -A\omega^2\sin(\omega t + \phi)$

$a = -A\sin(\omega t + \phi)\,\omega^2$

$a = -x\omega^2$

$a_{max} = A\omega^2$

$v = A\omega\cos(\omega t + \phi)$

$v = A\omega\sqrt{1 - \sin^2(\omega t + \phi)}$

$v = \omega\sqrt{A^2 - A^2\sin^2(\omega t + \phi)}$

$v = \omega\sqrt{A^2 - x^2}$

$x = 0$, MP $\Rightarrow v = A\omega = v_{max}$

$x = \pm A$ EP $\Rightarrow v = 0$

$v_{max} = A\omega$

$a_{max} = A\omega^2$ (magnitude)

$a = -x\omega^2$

$v = \omega\sqrt{A^2 - x^2}$

v → Speed

$\omega = \sqrt{\frac{k}{m}}$

$\omega^2 = \frac{k}{m}$ $\boxed{k = m\omega^2}$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$

$PE = \frac{1}{2}kx^2$ (Assume $U_0 = 0$)

$PE = \frac{1}{2}m\omega^2x^2$

Total Energy $= KE + P.E.$

$= \frac{1}{2}m\omega^2A^2 = \frac{1}{2}kA^2$

Not imp

$dU = -F\,dx$

$\int_{U_0}^{U} dU = -\int_{0}^{x}(-kx)\,dx$

$U - U_0 = \frac{1}{2}kx^2$

$\boxed{U = \frac{1}{2}kx^2 + U_0}$

If at $x = 0$, MP $U_0 = 0$

$U = \frac{1}{2}kx^2$

$$U - U_0 = \frac{1}{2}kx^2$$

$$U = \frac{1}{2}kx^2 + U_0$$

$$k = m\omega^2$$

$$TE = KE + PE$$

$$= \frac{1}{2}m\omega^2\left(A^2 - x^2\right) + \frac{1}{2}m\omega^2 x^2 + U_0$$

$$TE = \frac{1}{2}m\omega^2 A^2 + U_0 = \frac{1}{2}KA^2 + U_0$$

काम का सेवा

- $K = m\omega^2$
- $v = \omega\sqrt{A^2 - x^2}$, $v_{max} = A\omega$
- $a = -x\omega^2$ $\quad a_{max} = A\omega^2$
- $KE = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}K(A^2 - x^2)$
- $PE = \frac{1}{2}Kx^2 + U_0$
- Total Energy $= KE + PE = \frac{1}{2}KA^2 + U_0 = \text{Const}$

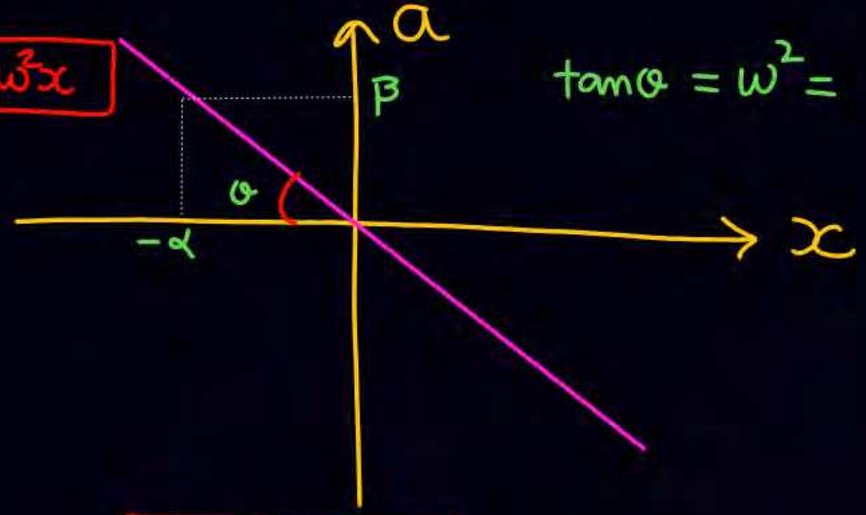
* If $U_0 = 0$, $KE = \frac{1}{2}K(A^2 - x^2)$

$PE = \frac{1}{2}Kx^2$

$TE = \frac{1}{2}KA^2 = \text{Const}$

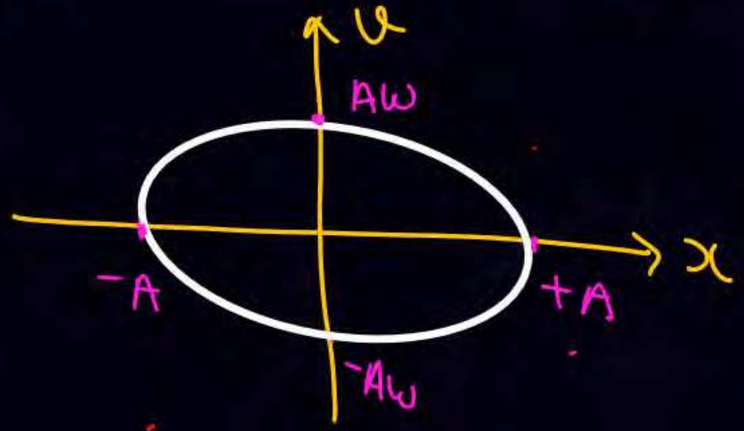
Graph

$a = -\omega^2 x$

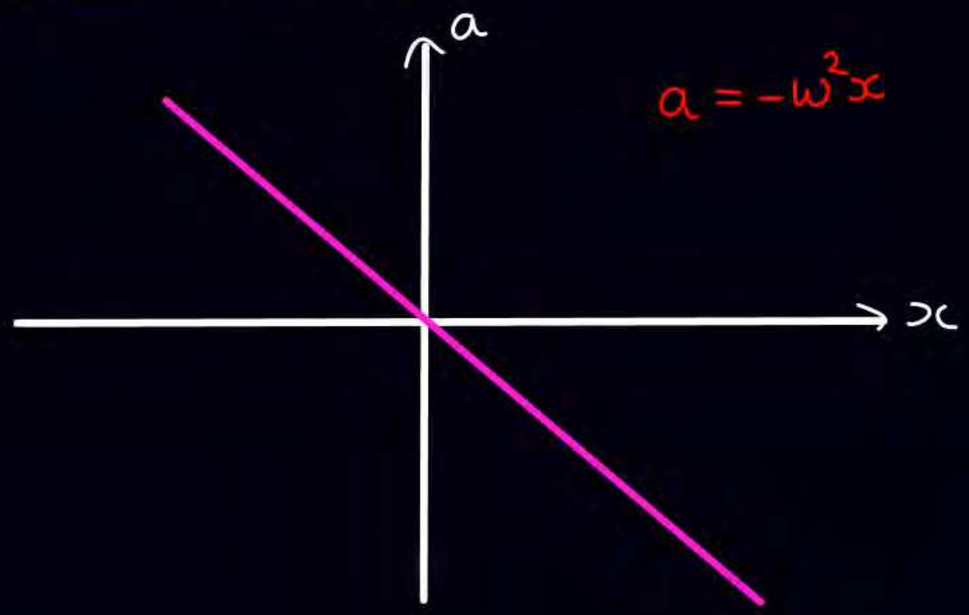


$\tan\theta = \omega^2 = \beta/\alpha$

$v - V_s\ x$ ≡ Ellipse.

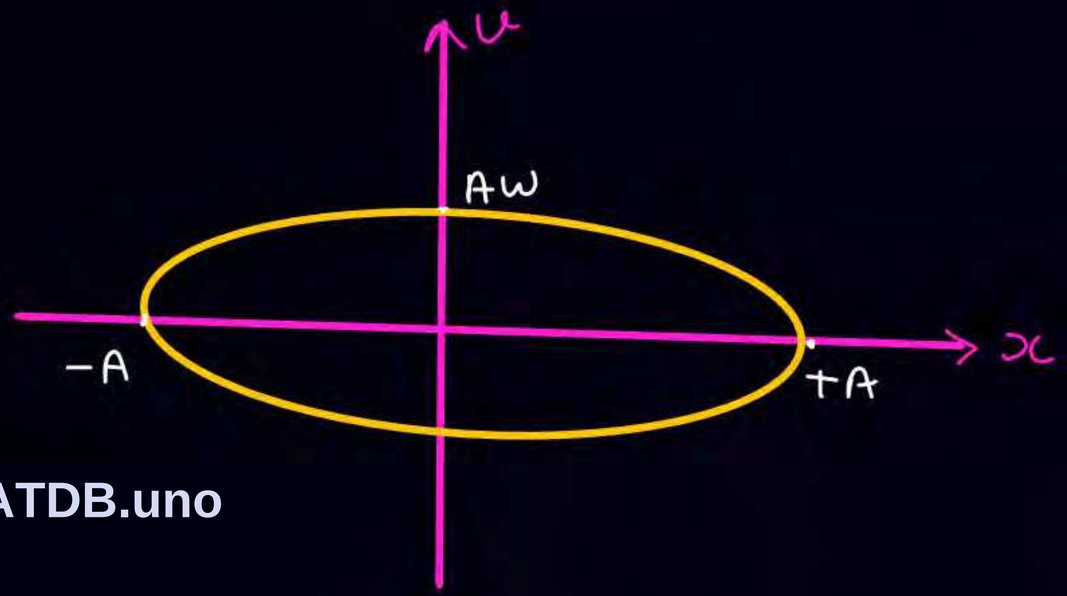


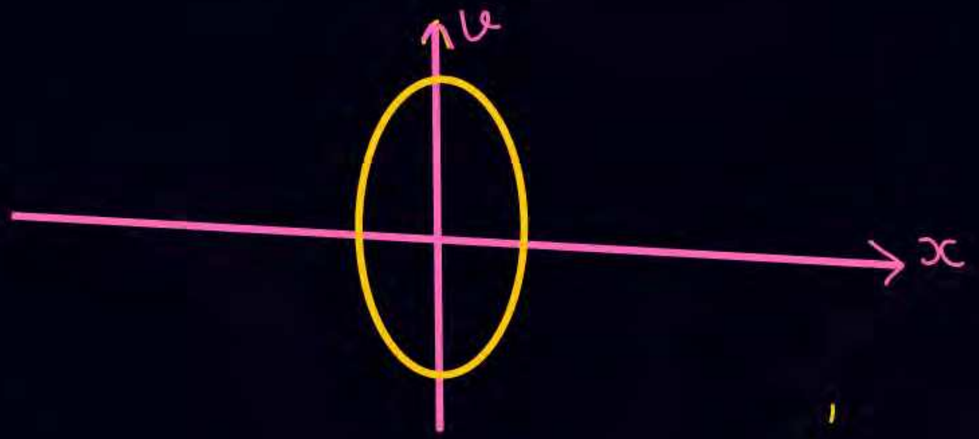
Graph b/w acc & x



$$a = -\omega^2 x$$

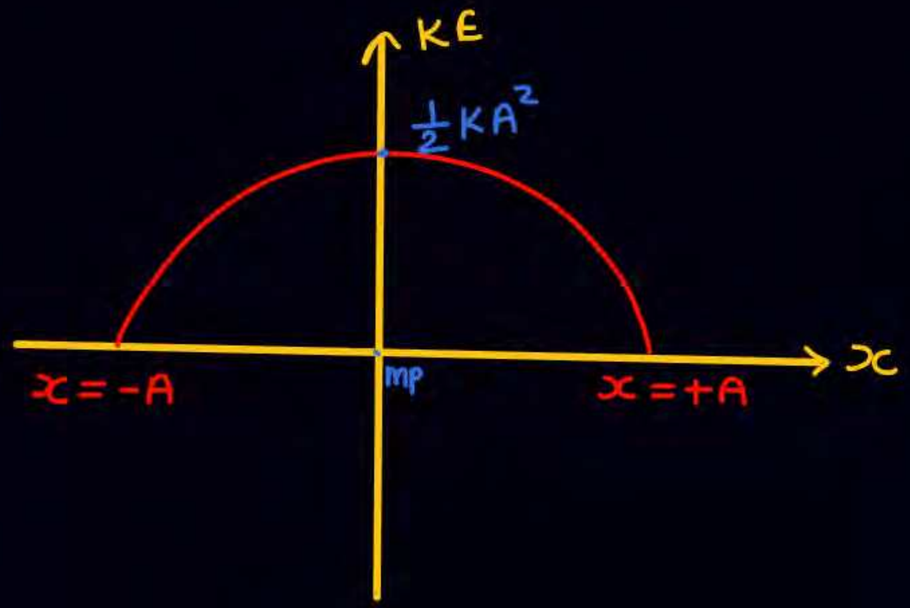
Graph b/w v vs x (Ellipse)



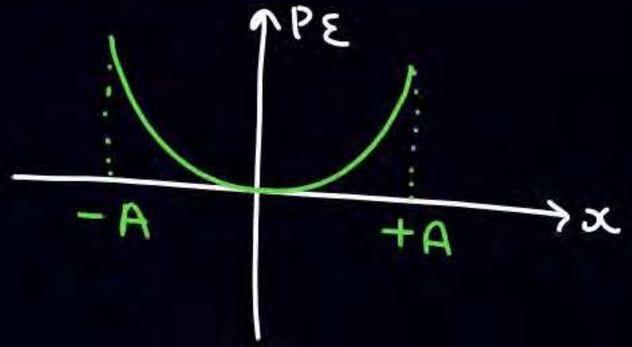


If $U_0 = 0$

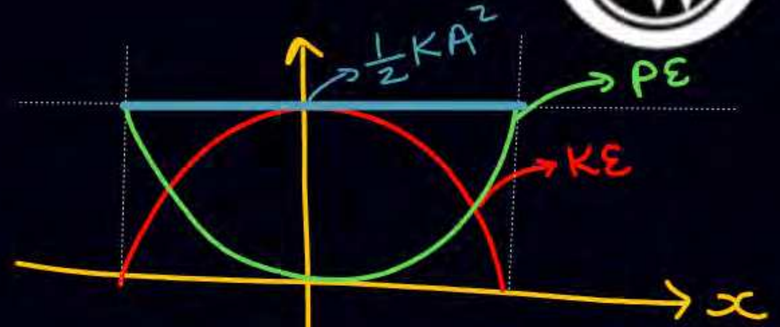
$KE = \frac{1}{2}K(A^2 - x^2)$



$PE = \frac{1}{2}Kx^2$



Total Energy $= \frac{1}{2}KA^2 = \text{Const}$



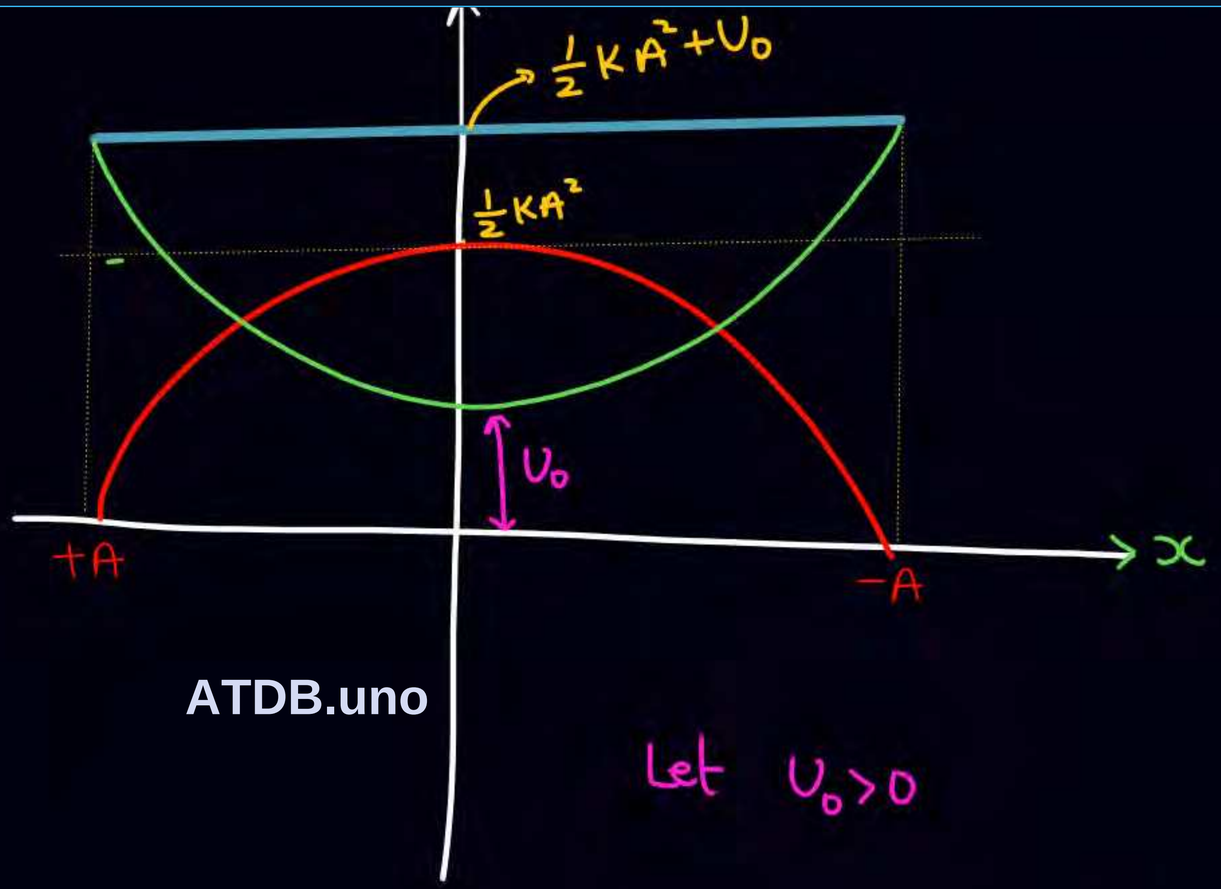
$U_0 = 0$

If $U_0 \neq 0$

$$KE = \frac{1}{2}k(A^2 - x^2)$$

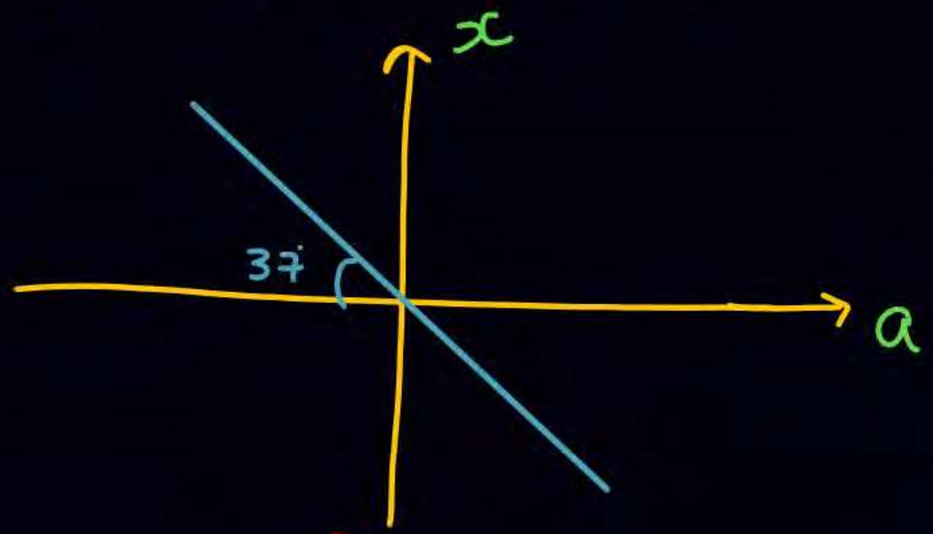
$$PE = \frac{1}{2}kx^2 + U_0$$

$$T.E = \frac{1}{2}kA^2 + U_0$$



Let $U_0 > 0$

Q find Time period



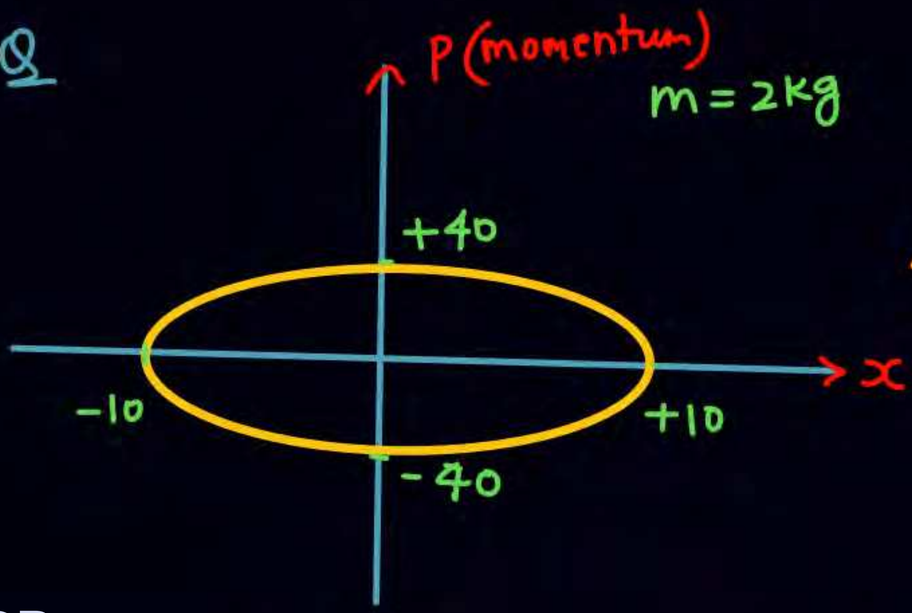
$a = -\omega^2 x$

$x = -\frac{1}{\omega^2} a$

$\frac{1}{\omega^2} = \tan 37°$

$\omega = \sqrt{\frac{4}{3}} = \frac{2\pi}{T}$

Q



$P = mV$

$40 = 2 \times A\omega$

$20 = A\omega$

$20 = 10\omega$

$A = 10$

$\omega = 2$

$T = \frac{2\pi}{\omega} = \pi$

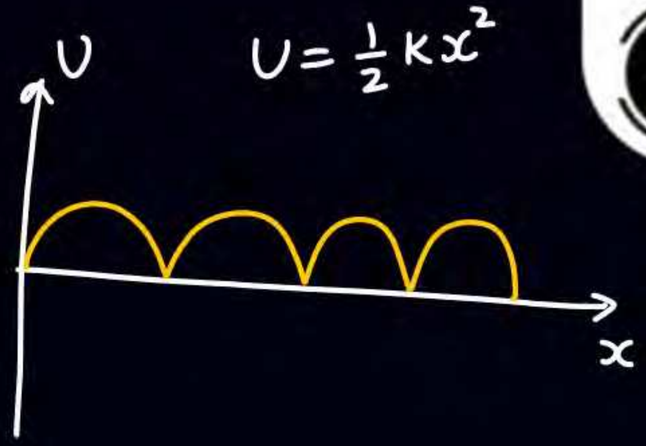
$V_{max} = A\omega = 20$

$K_{max} = \frac{1}{2} m V_{max}^2$

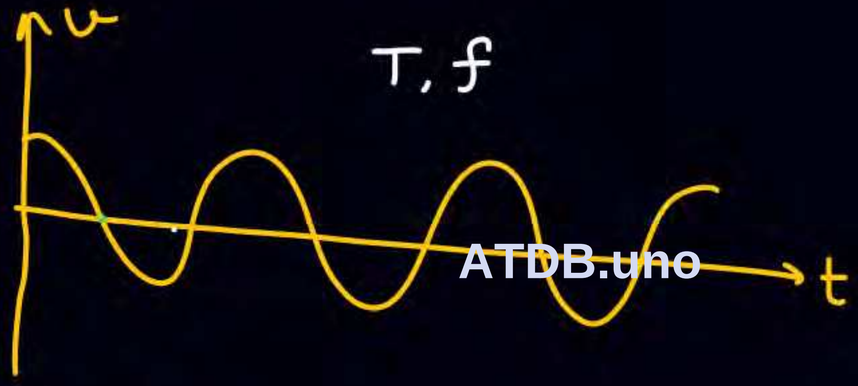
$= \frac{1}{2} \times 2 \times 20^2$

$= 400$

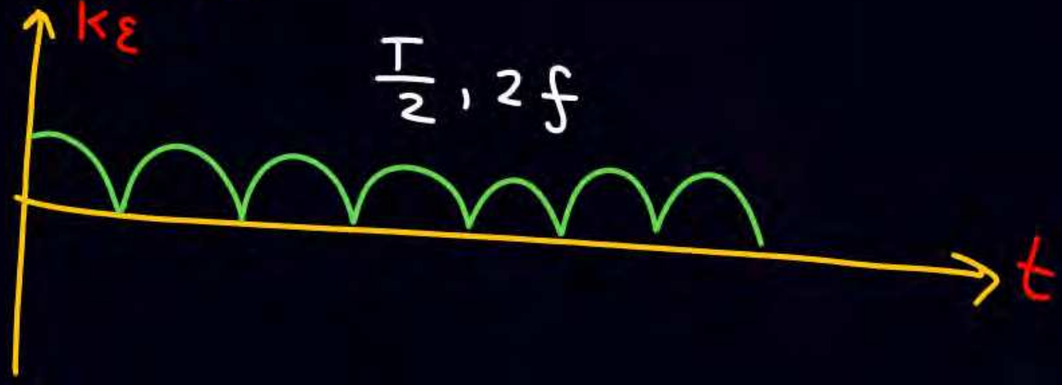
$x = A\sin\omega t$



$v = A\omega\cos\omega t$



$K\varepsilon = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2\omega t$



Q If max speed of particle is 20 m/s. find speed of particle when it is at a distance of $\frac{A\sqrt{3}}{2}$ from m.p. (SHM)

Soln

$V_{max} = A\omega = 20$

$v = \omega\sqrt{A^2 - x^2}$

$v = \omega\sqrt{A^2 - \left(\frac{A\sqrt{3}}{2}\right)^2} = \frac{A\omega}{2} = \frac{20}{2} = 10$

Q If total energy of a particle (SHM) is E. find KE of particle when particle is at a distance of half of amplitude from m.p.

$E = \frac{1}{2}kA^2$,

$KE = \frac{1}{2}k(A^2 - x^2) = \frac{1}{2}k\left[A^2 - \left(\frac{A}{2}\right)^2\right] = \frac{1}{2}kA^2 \times \frac{3}{4}$

$KE = \frac{3E}{4}$

$PE = E/4$

$$v = \omega\sqrt{A^2 - x^2}$$

$$\frac{v^2}{\omega^2} = A^2 - x^2$$

$$\frac{v^2}{\omega^2} + \frac{x^2}{1} = A^2$$

$$\frac{v^2}{A^2\omega^2} + \frac{x^2}{A^2} = 1$$

ellipse

ATDB.uno

QUESTION

In a linear Simple Harmonic Motion (SHM)

(A) Restoring force is directly proportional to the displacement.

(B) The acceleration and displacement are opposite in direction.

(C) The velocity is maximum at mean position.

(D) The acceleration is minimum at extreme points.

Choose the correct answer from the options given below:

[15 April 2023 - Shift 1]

1. (C) and (D) only
2. (A), (C) and (D) only
3. (A), (B) and (C) only
4. (A), (B) and (D) only

Ans. (3)

QUESTION

A particle executes simple harmonic motion between $x = -A$ and $x = +A$. If time taken by particle to go from $x = 0$ to $\frac{A}{2}$ is 2 s; then time taken by particle in going from $x = \frac{A}{2}$ to A is:

[25 January 2023 - Shift 2]

1. 3 s
2. 2 s
3. 1.5 s
4. 4 s

Ans. (4)

QUESTION

For a periodic motion represented by the equation $y = \sin\omega t + \cos\omega t$ the amplitude of the motion is _____.

[10 April 2023 - Shift 2]

1. 1
2. 0.5
3. 2
4. $\sqrt{2}$

Ans. (4)

QUESTION

A particle executes S.H.M. of amplitude A along x-axis. At t = 0, the position of the particle is $x = \frac{A}{2}$ and it moves along positive x-axis. The displacement of particle in time t is $x = A\sin(\omega t + \delta)$, then the value δ will be _____.

[10 April 2023 - Shift 1]

1. $\frac{\pi}{2}$
2. $\frac{\pi}{6}$
3. $\frac{\pi}{3}$
4. $\frac{\pi}{4}$

Ans. (2)

QUESTION

The velocity of a particle executing SHM varies with displacement (x) as $4v^2 = 50 - x^2$.

The time period of oscillations is $\frac{x}{7}s$. The value of x is _____.

$\left(\text{Take } \pi = \frac{22}{7}\right)$

[30 January 2023 - Shift 2]

Ans. (88)

QUESTION

A particle of mass 250 g executes a simple harmonic motion under a periodic force $F = (-25x)N$. The particle attains a maximum speed of 4 m/s during its oscillation. The amplitude of the motion is _____cm.

[29 January 2023 - Shift 2]

Ans. (40)

QUESTION

When a particle executes simple Harmonic motion, the nature of graph of velocity as function of displacement will be:

[26 July, 2022 - Shift 1]

1. Circular
2. Elliptical
3. Sinusoidal
4. Straight line

Ans. (2)

QUESTION

The displacement of simple harmonic oscillator after 3 seconds starting from its mean position is equal to half of its amplitude. The time period of harmonic motion is:

[27 June, 2022 - Shift 1]

1. 6 s
2. 8 s
3. 12 s
4. 36 s

Ans. (4)

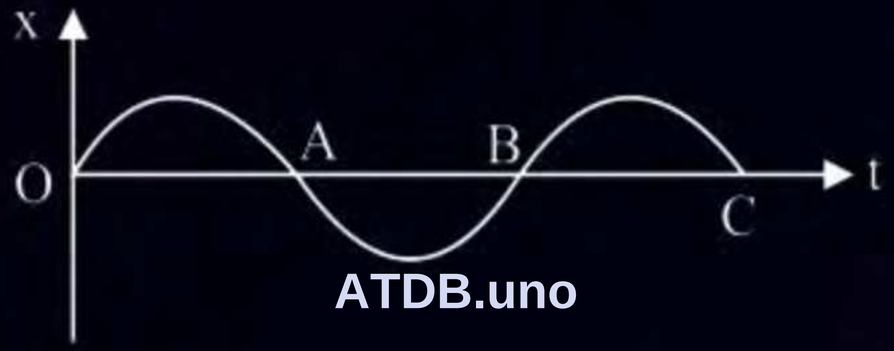
QUESTION

A particle executives simple harmonic motion. Its amplitude is 8 cm and time period is 6s. The time it will take to travel from its position of maximum displacement to the point corresponding to half of its amplitude, is _____s. **[27 June, 2022 - Shift 2]**

Ans. (1)

QUESTION

The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure.

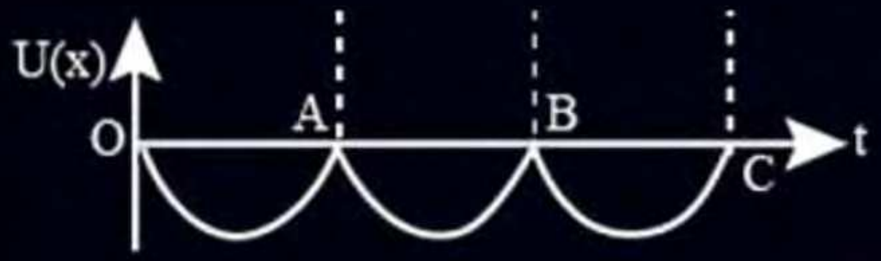


The potential energy U(x) versus time (t) plot of the particle is correctly shown in figure:

[27 Aug, 2021 - Shift 1]

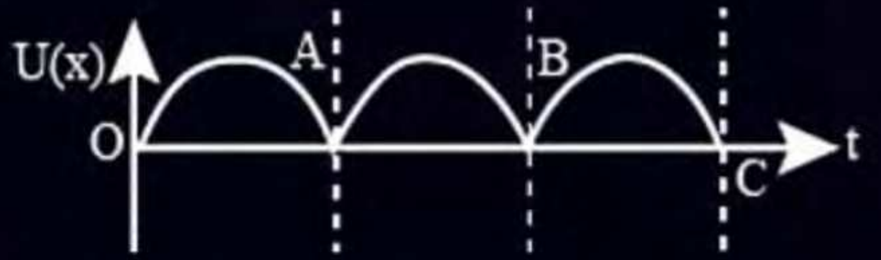
1

2



3

4



Ans. (4)

QUESTION

A particle executes S.H.M., the graph of velocity as a function of displacement is:

[26 Feb, 2021 - Shift 2]

1. A circle
2. A parabola
3. An ellipse
4. A helix

Ans. (3)

QUESTION

When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is:

[24 Feb, 2021 - Shift 2]

1. Circular
2. Straight line
3. Parabolic
4. Elliptical

Ans. (4)

QUESTION

$Y = A\sin(\omega t + \phi_0)$ is the time-displacement equation of a SHM. At $t = 0$ the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then the initial phase angle ϕ_0 will be:

[25 Feb, 2021 - Shift 2]

1. $\frac{\pi}{6}$
2. $\frac{\pi}{3}$
3. $\frac{5\pi}{6}$
4. $\frac{2\pi}{3}$

Ans. (3)

QUESTION

A particle executes S.H.M. with amplitude 'a' and time period 'T'. The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{x}a}{2}$. The value of x is _____.

[26 Feb, 2021 - Shift 2]

Ans. (3)

QUESTION

A particle performs simple harmonic motion with a period of 2 second. The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is $\frac{1}{a}$ s. The value of 'a' to the nearest integer is _____. **[18 March, 2021 - Shift 1]**

Ans. (6)

QUESTION

A particle executes simple harmonic motion represented by displacement function as $x(t) = A \sin(\omega t + \phi)$

If the position and velocity of the particle at $t = 0$ s are 2 cm and 2ω cm s^{-1} respectively, then its amplitude is $x\sqrt{2}$ cm where the value of x is _____. **[27 July, 2021 – Shift 2]**

Ans. (2)

QUESTION

The displacement time graph of a particle executing S.H.M. is given in figure: (sketch is schematic and not to scale)

Witch of the following statement is/are true for this motion?

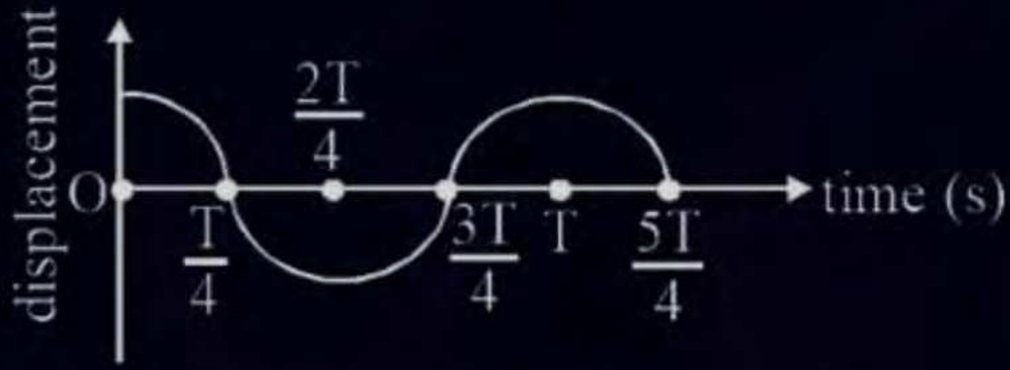
[2 Sep, 2020 - Shift 2]

(A) The force is zero $t = \frac{3T}{4}$

(B) The acceleration is maximum at $t = T$

(C) The speed is maximum at $t = \frac{T}{4}$

(D) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$



1. (A), (B) and (D)
2. (A) and (D)
3. (B), (C) and (D)
4. (A), (B) and (C)

Ans. (4)

QUESTION

A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A \sin \frac{\pi t}{90}$. The ratio of kinetic to potential energy of the particle at t = 210 s will be:

[11 Jan, 2019 - Shift 1]

1. 1/9
2. 1
3. 2
4. 1/3

Ans. (4)

QUESTION

A particle executes SHM of amplitude A. The distance from the mean position when its kinetic energy becomes equal to its potential energy is: **[13 April 2023 - Shift 2]**

1. $\frac{1}{\sqrt{2}}A$
2. $2A$
3. $\sqrt{2A}$
4. $\frac{1}{2}A$

Ans. (1)

QUESTION

The maximum potential energy of a block executing simple harmonic motion is 25 J. A is amplitude of oscillation. At A/2, the kinetic energy of the block is:

[31 January 2023 - Shift 1]

1. 37.5 J
2. 9.75 J
3. 18.75 J
4. 12.5 J

Ans. (3)

QUESTION

A particle is executing simple harmonic motion (SHM). The ratio of potential energy and kinetic energy of the particle when its displacement is half of its amplitude will be _____.

[12 April 2023 - Shift 1]

1. 1 : 1
2. 1 : 3
3. 2 : 1
4. 1 : 4

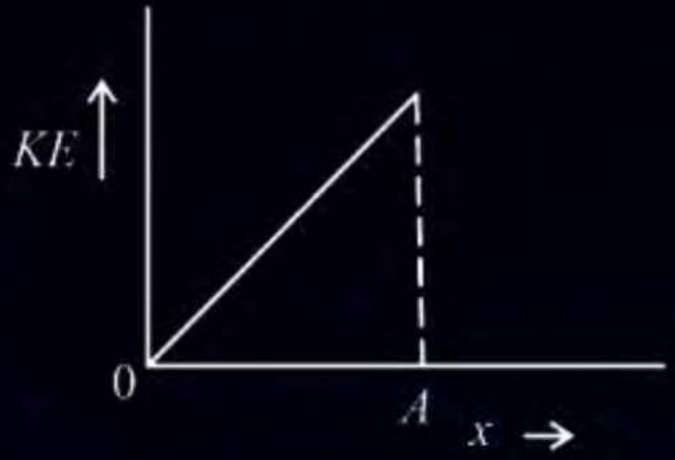
Ans. (2)

QUESTION

The variation of kinetic energy (KE) of a particle executing simple harmonic motion with the displacement (x) starting from mean position to extreme position (A) is given by.

[11 April 2023 - Shift 1]

1



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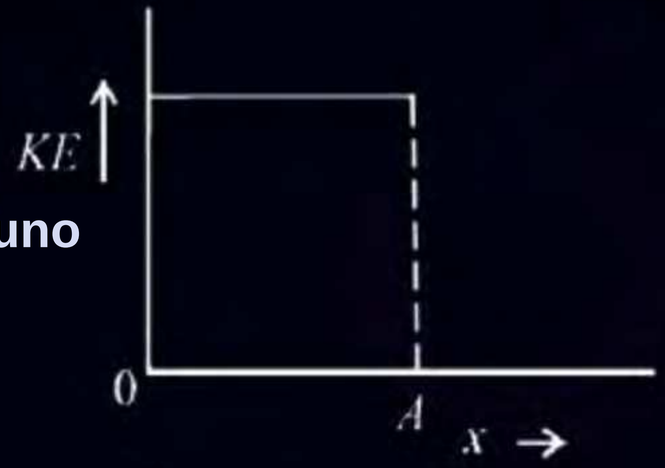

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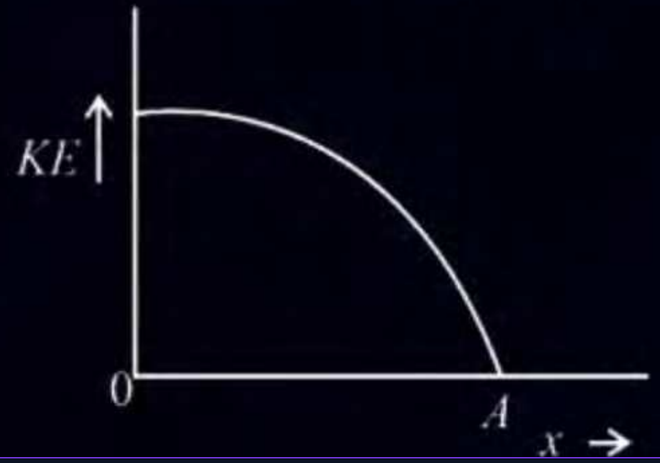
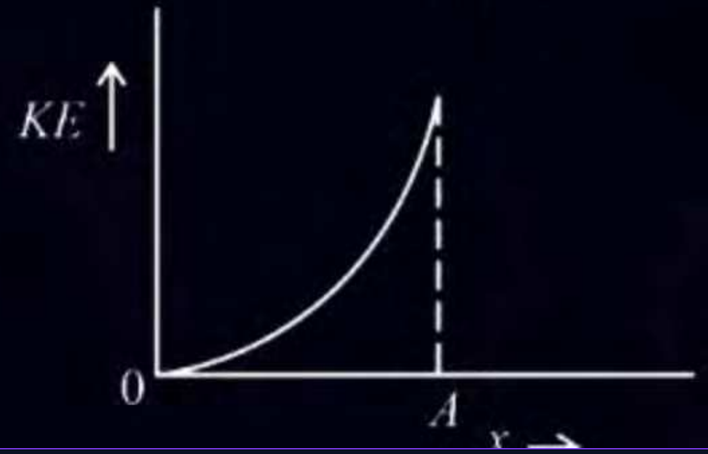


4

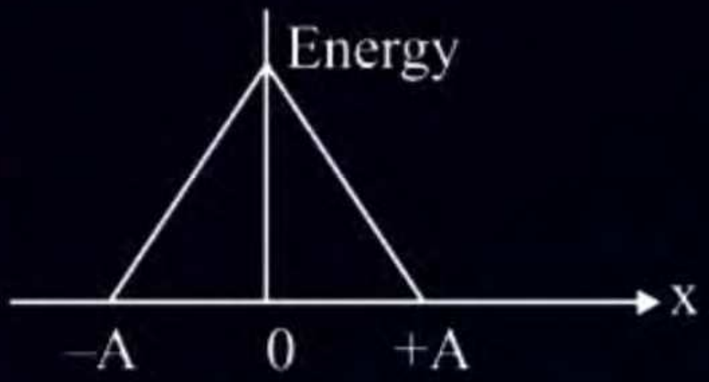


Ans. (3)

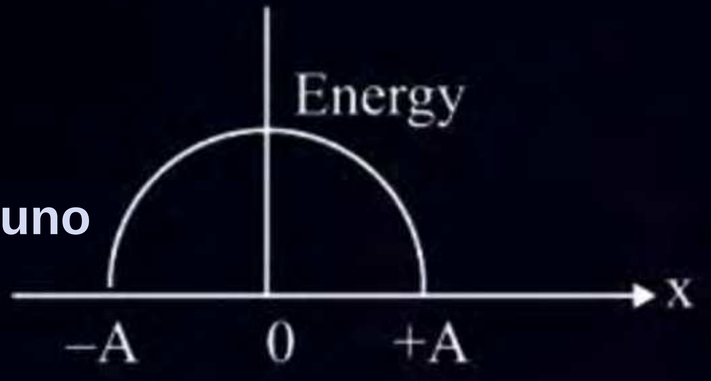
QUESTION

Which graph represents the difference between total energy and potential energy of a particle executing SHM vs its distance from mean position? **[13 April 2023 - Shift 1]**

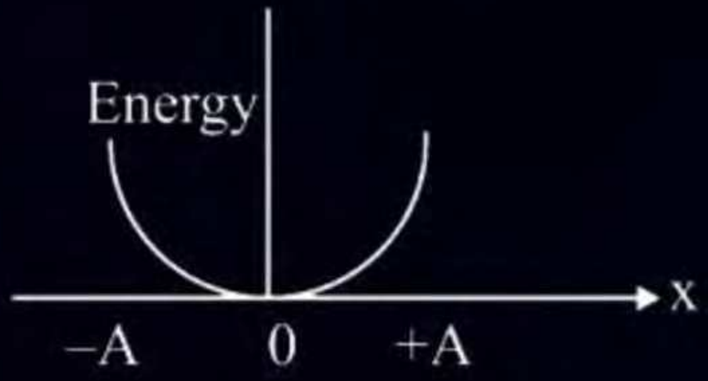
1



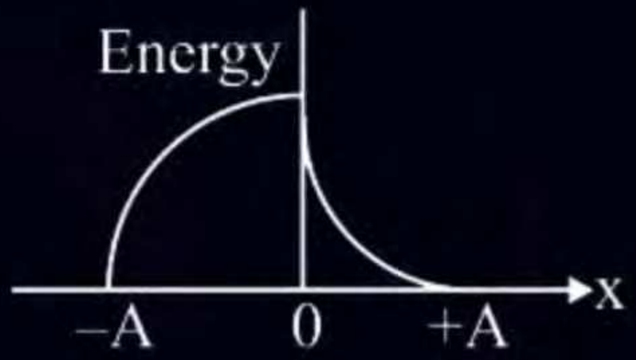
2



3



4



Ans. (2)

QUESTION

The general displacement of a simple harmonic oscillator is $x = A \sin\omega t$. Let T be its time period. The slope of its potential energy (U) – time (t) curve will be maximum when $t = \frac{T}{\beta}$. The value of β is:

[30 January 2023 - Shift 1]

Ans. (8)

QUESTION

At a given point of time the value of displacement of a simple harmonic oscillator is given as $y = A\cos(30°)$. If amplitude is 40 cm and kinetic energy at that time is 200 J, the value of force constant 1.0×10^{x} Nm^{-1}. The value of x is _____.

[13 April 2023 - Shift 1]

Ans. (4)

QUESTION

The amplitude of a particle executing SHM is 3 cm. The displacement at which its kinetic energy will be 25 % more than the potential energy is _____cm.

[01 February 2023 - Shift 1]

Ans. (2)

QUESTION

For a body executing S.H.M.:

(a) Potential energy is always equal to its K.E.

(b) Average potential and kinetic energy over any given time interval are always equal.

(c) Sum of the kinetic and potential energy at any point of time is constant.

(d) Average K.E. in one time period is equal to average potential energy in one time period.

Choose the most appropriate option from the options given below:

[31 Aug, 2021 – Shift 2]

1. (c) and (d)
2. (b) and (c)
3. only (c)
4. only (b)

Ans. (1)

QUESTION

An object of mass 0.5 kg is executing simple harmonic motion. It amplitude is 5 cm and time period (T) is 0.2 s. What will be the potential energy of the object at an instant $t = \frac{T}{4}$ s starting from mean position. Assume that the initial phase of the oscillation is zero.

[27 July, 2021 - Shift 2]

1. 0.62 J
2. 6.2×10^{-3} J
3. 1.2×10^{3} J
4. 6.2×10^{3} J

Ans. (1)

QUESTION

A particle starts executing simple harmonic motion (SHM) of amplitude 'a' and total energy E. At any instant, its kinetic energy is $\frac{3E}{4}$ then its displacement 'y' is given by:

[27 July, 2021 - Shift 1]

1. $y = a$
2. $y = \frac{a}{\sqrt{2}}$
3. $y = \frac{a\sqrt{3}}{2}$
4. $y = \frac{a}{2}$

Ans. (4)

QUESTION

(a) Formula of total energy of a particle executing simple 20. In a simple harmonic oscillation, what fraction of total mechanical energy is in the form of kinetic energy, when the particle is midway between mean and extreme position?

[25 July, 2021 - Shift 2]

1. 1/2
2. 3/4
3. 1/3
4. 1/4

Ans. (2)

Home Work

- DPP-03
- Jm P40 Attached 30 ques must try ≡ 30 mint
- Module → Jm P40 till taught

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