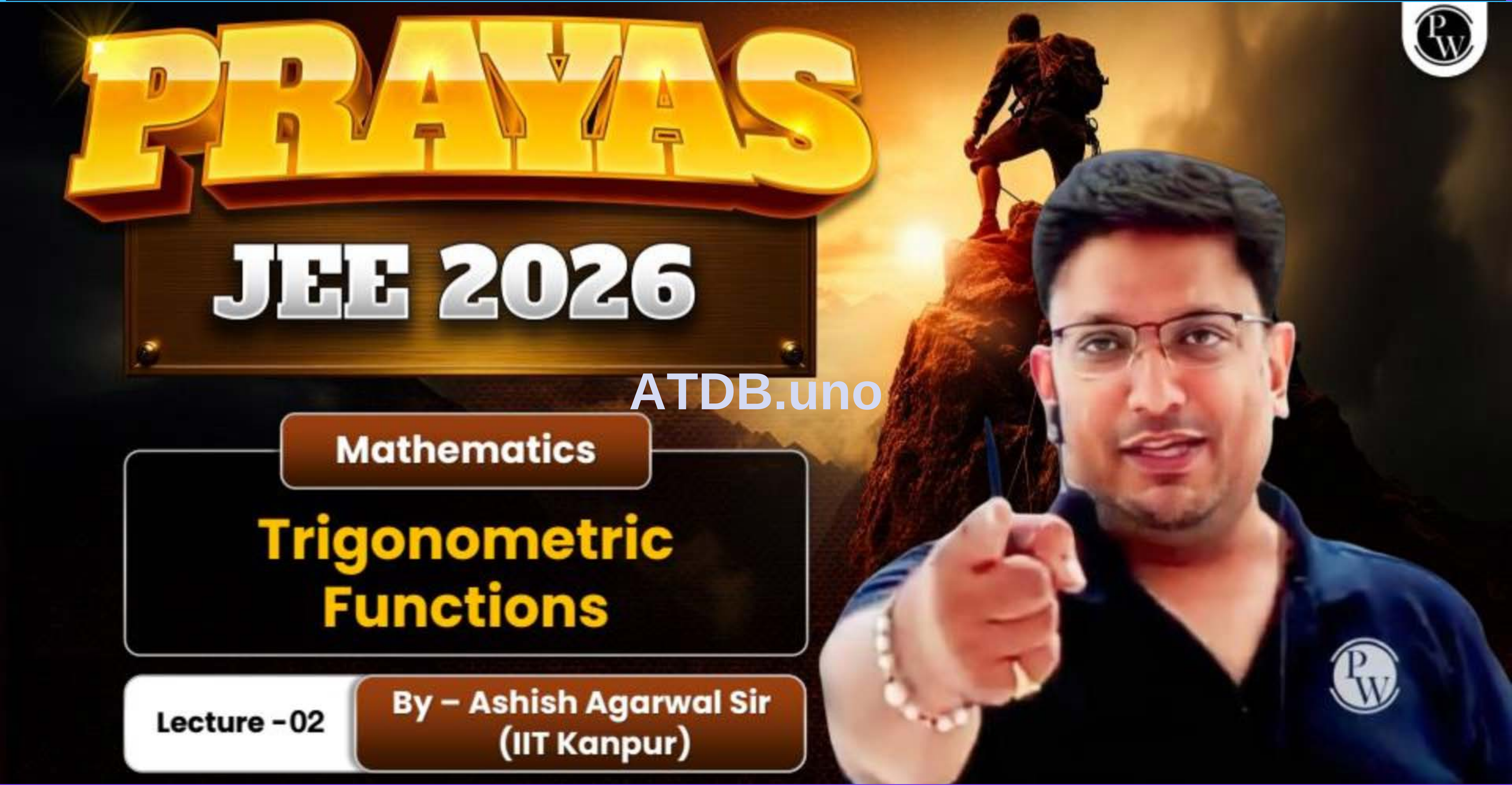
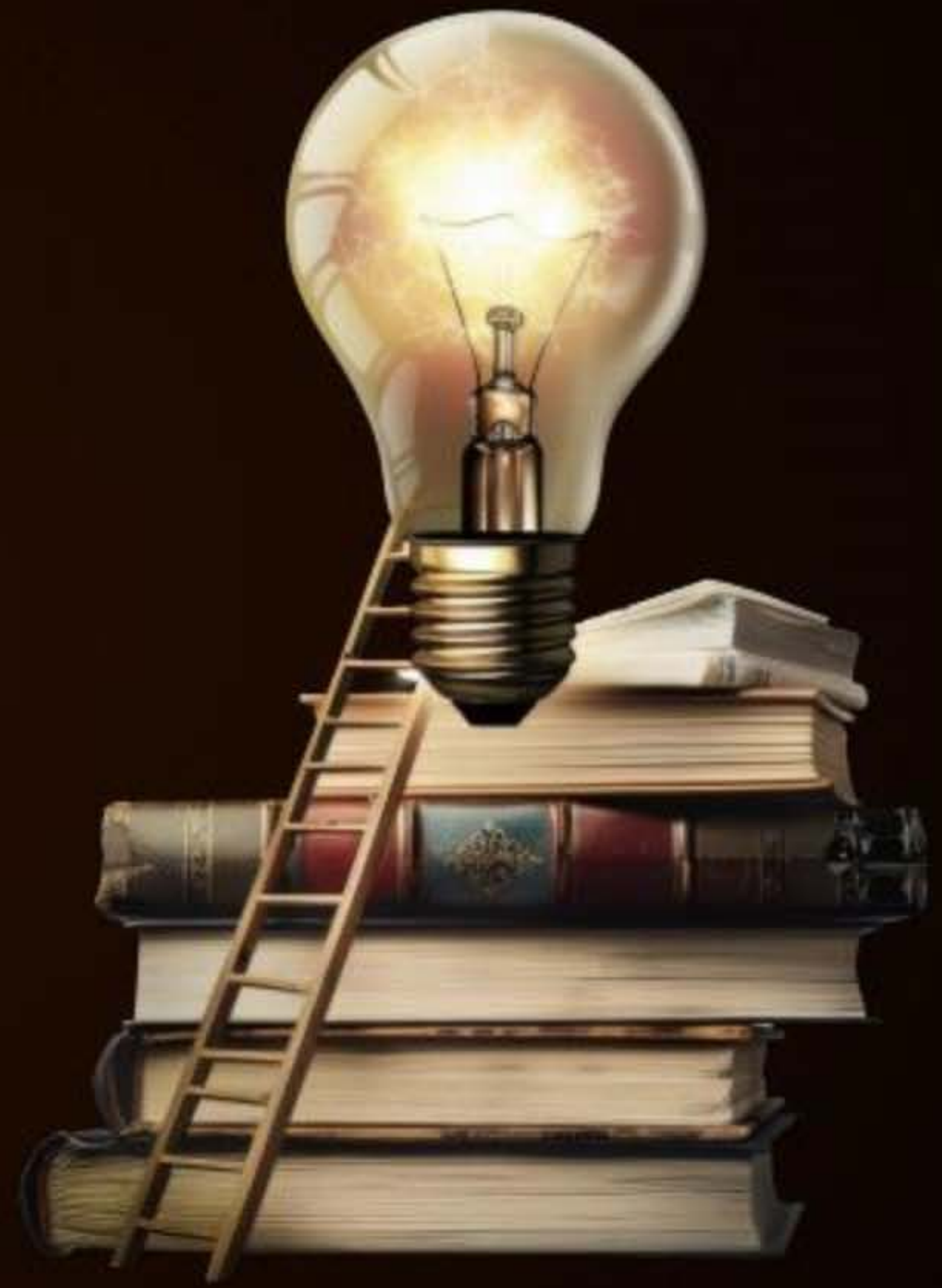

PRAYAS
JEE 2026
ATDB.uno
Mathematics
Trigonometric Functions
Lecture -02
By – Ashish Agarwal Sir
(IIT Kanpur)

Topics *to be covered*

A Reduction Formula

[redacted]



Recap *of previous lecture*

1. If for two positive numbers a & b we have
   a, $A_1, A_2, \dots A_n$, b are in A.P.
   a, $G_1, G_2, \dots G_n$, b are in G.P.
   a, $H_1, H_2, \dots H_n$, b ore in H.P.
   then $A_1 \geq G_1 \geq H_1$, $A_2 \geq G_2 \geq H_2$, ... $A_n \geq G_n \geq H_n$

   also $A_1 H_n = A_2 H_{n-1} = A_3 H_{n-2} = \dots\dots = A_n H_1 = ab$

2. $2x + 3y = 5$ then minimum value of $x^2 + y^2 =$ $25/13$.

3. $a + b + c + d = 1$ then $\sqrt{a} + \sqrt{b} + \sqrt{c} + \sqrt{d} \leq$ 2

② 2, 3, $x, y \in R$

$\underbrace{\text{By CS ineq}}$

$(2x+3y)^2 \leq (2^2+3^2)(x^2+y^2)$

$25 \leq 13(x^2+y^2)$

$x^2+y^2 \geq \frac{25}{13}$.

$a+b+c+d=1$ $\quad$ $1, 1, 1, 1, \sqrt{a}, \sqrt{b}, \sqrt{c}, \sqrt{d} \in R$

M ①

$$\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d}\Big|_{max} = 2$$

By CS inequality

$$\left(1\cdot\sqrt{a}+1\cdot\sqrt{b}+1\cdot\sqrt{c}+1\cdot\sqrt{d}\right)^2 \le \left(1^2+1^2+1^2+1^2\right)\left(\sqrt{a}^2+\sqrt{b}^2+\sqrt{c}^2+\sqrt{d}^2\right)$$

$$\left(\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d}\right)^2 \le 4\cdot 1$$

$$\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d} \le 2.$$

M ②

using RMS $\ge$ A.M

$$\sqrt{\frac{\sqrt{a}^2+\sqrt{b}^2+\sqrt{c}^2+\sqrt{d}^2}{4}} \ge \frac{\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d}}{4}$$

$$\frac{1}{2} = \sqrt{\frac{a+b+c+d}{4}} \ge \frac{\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d}}{4}$$

$$\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d} \le 2 \text{ Ans}$$

Recap of previous lecture

Tan !@

4. If $a + b + c = 1$ & $a, b, c > 0$ then the minimum value of $a^2 + 2b^2 + c^2$ is ______

5. $\sec x - \tan x$ & $\sec x + \tan x$ are ___reciprocal___ of each other.

6. $\operatorname{cosec} 30° + \sec 60° + \tan\frac{\pi}{4} + \cot^2\frac{\pi}{3} =$ ___ $2+2+1+\frac{1}{3} = 16/3$ ___

7. $\tan 90°$ is ___N.D___ $\cot 90°$ is ___0___

$\operatorname{cosec} 0°$ is ___N.D___ $\cot 0°$ is ___N.D___

$\sec 90°$ is ___N.D___

Homework Discussion

QUESTION

(RPP 1)

Paragraph

Consider the quadratic equation $2x^2 - (4m + 2)x + m^2 + m = 0, m \in R$

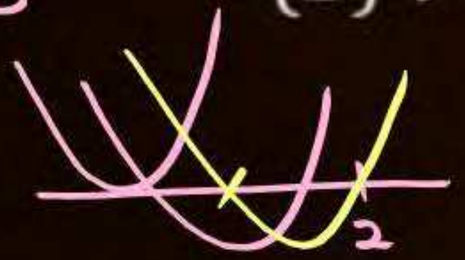
1. The number of positive integer values of 'm' such that the equation has exactly one root in $(2, 3)$ is

(A) 3 (B) 4 (C) 5 (D) 6

2. The number of negative integral values of 'm' such that $m > -10$ and atleast one root of the equation is smaller than '2' is

(A) 8 (B) 9 (C) 6 (D) 4

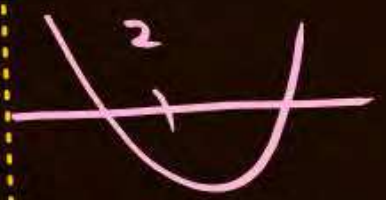
case(i)



$-\frac{b}{2a} < 2$, $D \geq 0$

$f(2) > 0$

case(ii)



$f(2) < 0$

Ans. (1) B, (2) B

Q1. case (1)

$f(2) \cdot f(3) < 0$

case (ii)

case (iii)

QUESTION

(Challenger Problem (Answers Sahi hai)

If $ax^2 + bx + c = 0, a \neq 0, a, b, c \in R$ has two distinct real roots in $(1, 2)$ then

RPP 02

A. (a) $(5a + 2b + c) > 0$

B. (a) $(5a + 2b + c) < 0$

C. $2a + b > 0$

D. (a) $(4a + 2b + c) > 0$

(c)

$1 < -\frac{b}{2a} < 2$

$1 + \frac{b}{2a} < 0$

$\frac{2a+b}{2a} < 0.$

$a > 0$: $2a + b < 0$

$a < 0$: $2a + b > 0$

$a f(1) > 0$

$a f(2) > 0 - a(4a + 2b + c) > 0$

Now $a(5a + 2b + c)$

$= a(a + 4a + 2b + c)$

$= \underbrace{a^2}_{>0} + \underbrace{a(4a + 2b + c)}_{>0} > 0$

Ans. A, D

$1 < \alpha, \beta < 2$

$\alpha + \beta > 2 \qquad \alpha + \beta < 4$

$\downarrow$

$-\frac{b}{a} > 2$

$2 + \frac{b}{a} < 0$

$\frac{2a+b}{a} < 0$

$a < 0 \qquad a > 0$

$2a+b > 0 \qquad 2a+b < 0.$

Trigonometry

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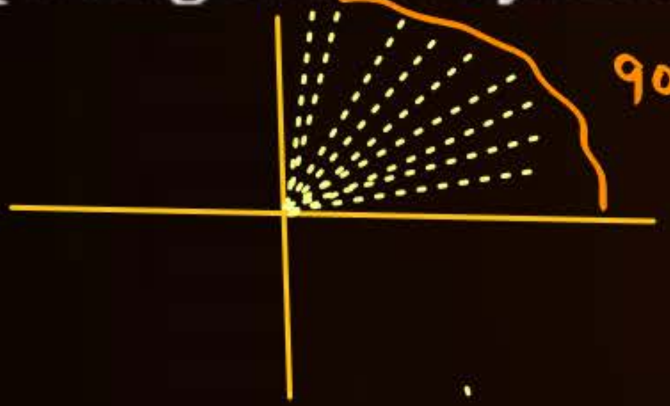
Trigonometry

System of measurement of angle

Degree (Sexagesimal system)

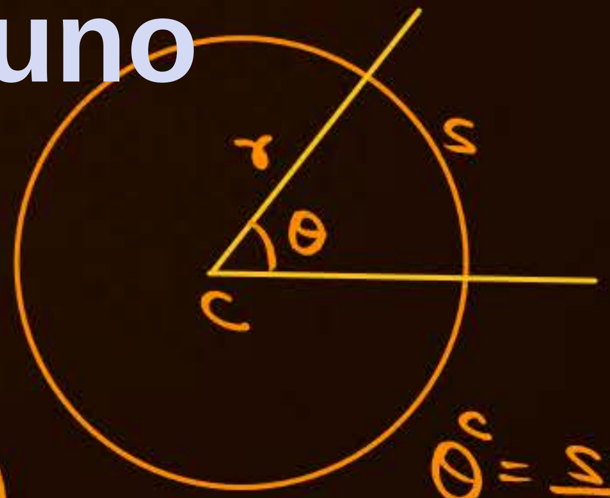


90 equal parts

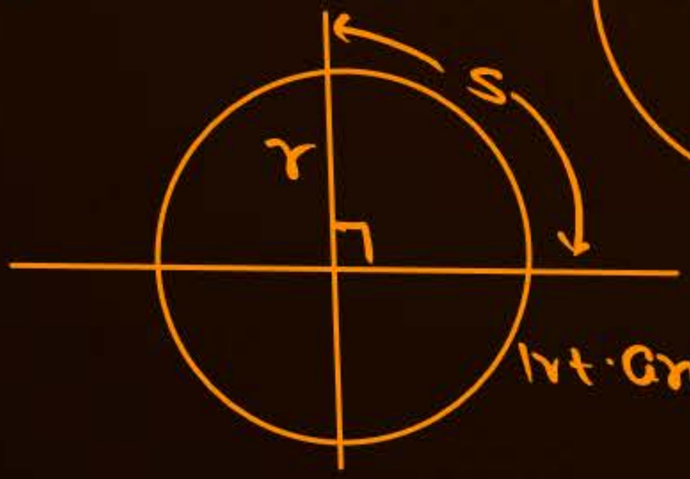
↓

each part is called 1°

Radian (Circular system)



$\theta^c = \frac{S}{r}$ > dimensionless.



$1 \text{rt} \cdot \text{angle} = \frac{S}{r} = \frac{\frac{2\pi r}{4}}{r} = \pi/2$

French (not in used)

100 equal parts

each part is called 1 grade

$1 \text{rt} \cdot \text{angle} = 100^g$

1 rt angle $= 90^\circ = \left(\frac{\pi}{2}\right)^c = 100^g$.

$90^\circ = \left(\frac{\pi}{2}\right)^c$

$1^\circ = \left(\frac{\pi}{180}\right)^c = 0.0175^c$

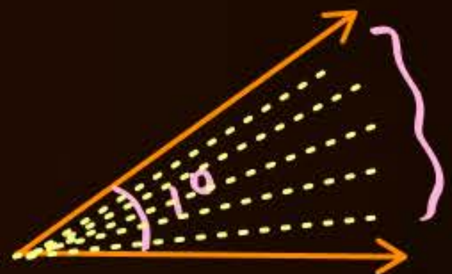
$1^c = \left(\frac{180}{\pi}\right)^\circ = 57.3^\circ$

$\theta^\circ = \left(\frac{\pi}{180}\cdot\theta\right)^c$

$\theta^c = \left(\frac{180}{\pi}\theta\right)^\circ$

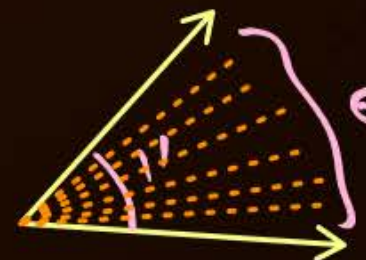
Yoad Rakho

- $\frac{\pi}{6} = 30^\circ$
- $\frac{\pi}{4} = 45^\circ$
- $\frac{\pi}{12} = 15^\circ$
- $\frac{\pi}{15} = 12^\circ$
- $\frac{\pi}{10} = 18^\circ$
- $\frac{\pi}{3} = 60^\circ$
- $\frac{\pi}{2} = 90^\circ$
- $\frac{2\pi}{3} = 120^\circ$
- $\frac{3\pi}{4} = 135^\circ$
- $\frac{5\pi}{6} = 150^\circ$
- $\frac{7\pi}{6} = 210^\circ$
- $\frac{4\pi}{3} = 240^\circ$
- $\frac{5\pi}{4} = 225^\circ$
- $\frac{3\pi}{2} = 270^\circ$
- $\frac{11\pi}{6} = 330^\circ$
- $\frac{5\pi}{3} = 300^\circ$
- $\frac{7\pi}{4} = 315^\circ$
- $2\pi = 360^\circ$
- $\pi/8 = 22.5^\circ$
- $\frac{3\pi}{8} = 67.5^\circ$
- $\frac{5\pi}{12} = 75^\circ$



60 equal parts each part is called one minute (1′)

$1^\circ = 60'$

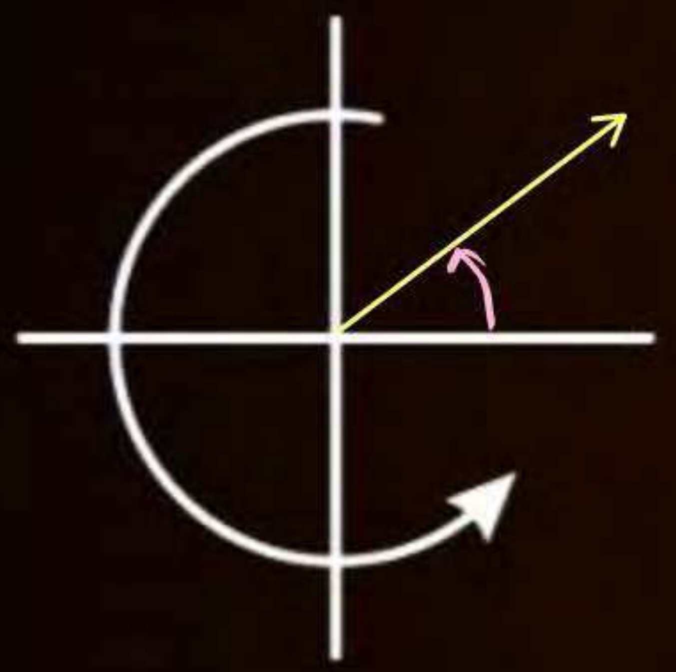


60 equal parts each part is called One second (1″)

$1' = 60''$

Measurement of Angle

Sign Convention:



Anticlockwise direction will indicate +ve angle

Clockwise direction will indicate –ve angle

Angles at Boundaries of Quadrants

$5+(n-1)4 = 4n+1$

T_{-2}, T_{-1}, T_0, T_1, T_2

$- - - -\frac{7\pi}{2}, -\frac{3\pi}{2}, \pi/2, \frac{5\pi}{2}, \frac{9\pi}{2} - - - \quad (4n+1)\frac{\pi}{2}$

$(2n+1)\pi$

$- - -3\pi, -\pi, \pi, 3\pi, 5\pi - - -$

(odd π)

$- - -4\pi, -2\pi, 0, 2\pi, 4\pi, 6\pi - - - - \quad (2n\pi)$

(Even pi)

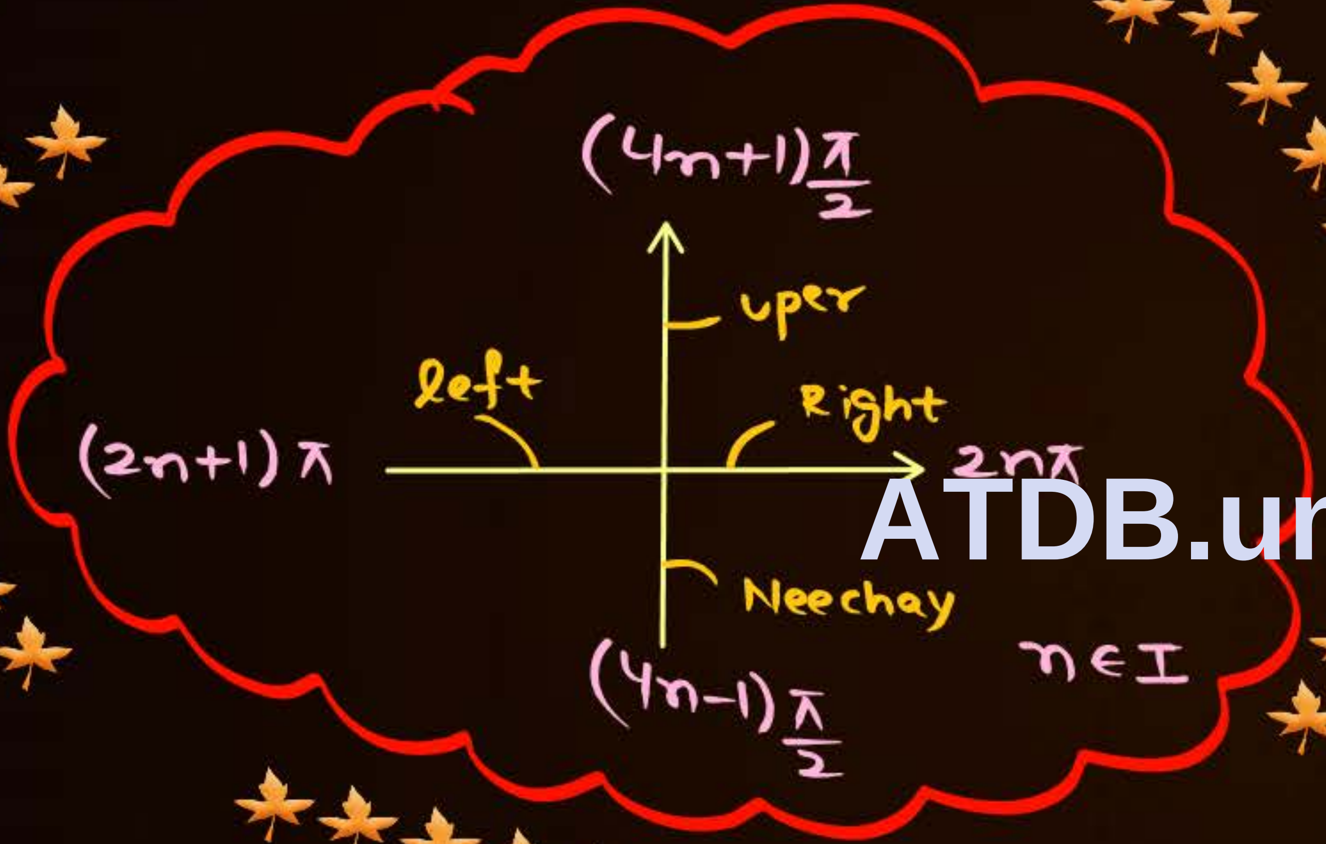
$- - - -\frac{5\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2} - - - \quad (4n-1)\pi/2$

$n \in I$

$$4\,\overline{)\,25\,}\quad 6,\quad 24,\quad 1$$

$65 = 4\times16+1$

$27 = 6\times4+3$
$= 6\times4+4-1$
$= 7\times4-1$



★ $2025\frac{\pi}{2}$ — $(4n+1)\frac{\pi}{2}$ type
↓
uper

★ $2027\frac{\pi}{2}$ → $(4n-1)\frac{\pi}{2}$ type
↓
Neechay.

$$4\,\overline{)\,27\,}\quad 6,\quad 24,\quad 3$$

$27 = 4\cdot7-1$

$47 = 4\times12-1$

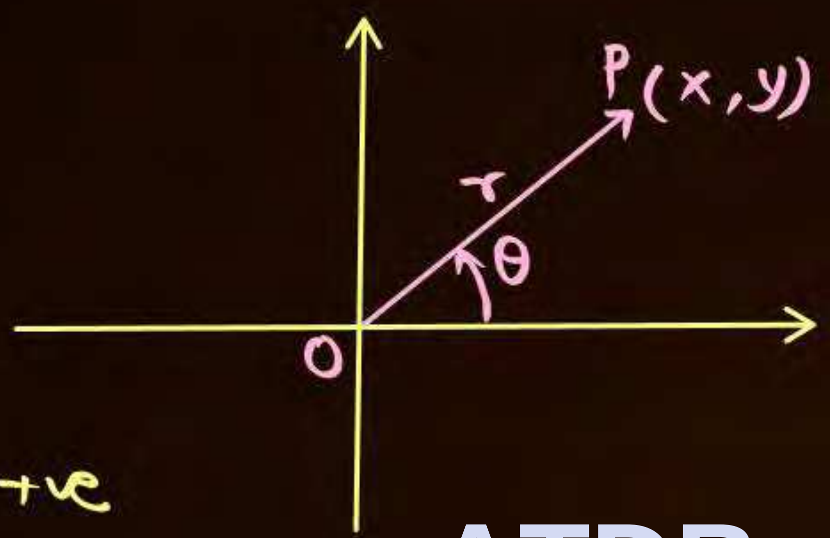
★ $5047\frac{\pi}{2}$ — $(4n-1)\frac{\pi}{2}$ (Neechay)

★ $2025\pi \rightarrow$ left

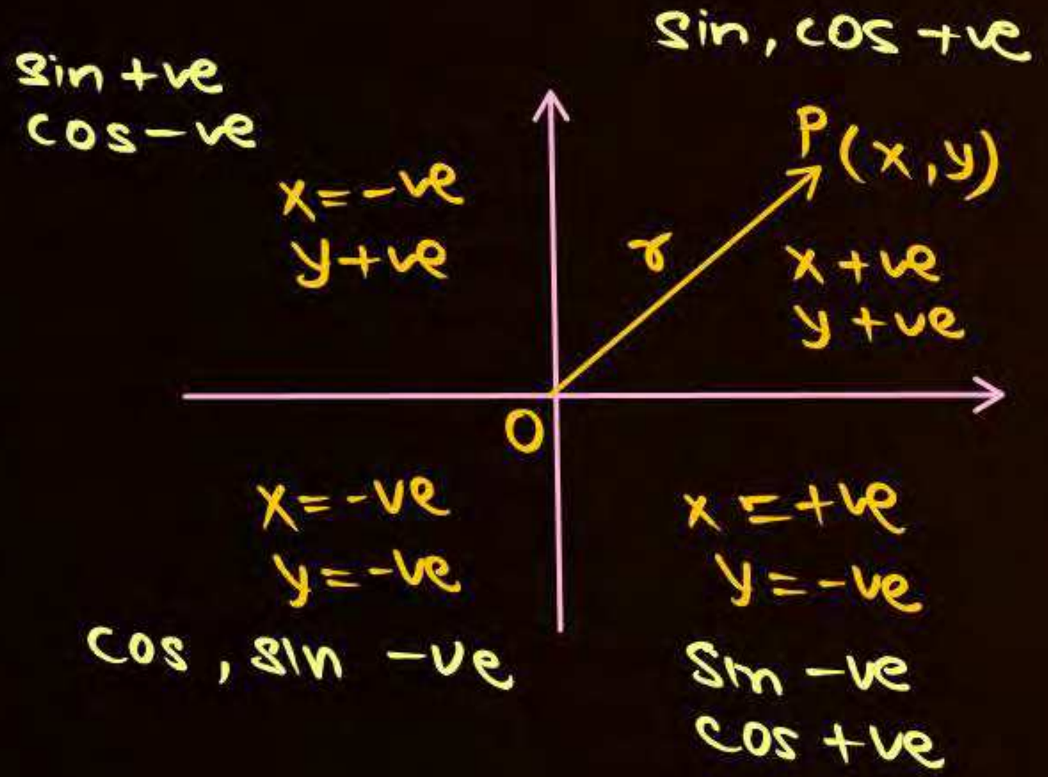
★ $7065\frac{\pi}{2}$ — $(4n+1)\frac{\pi}{2}$ (uper)

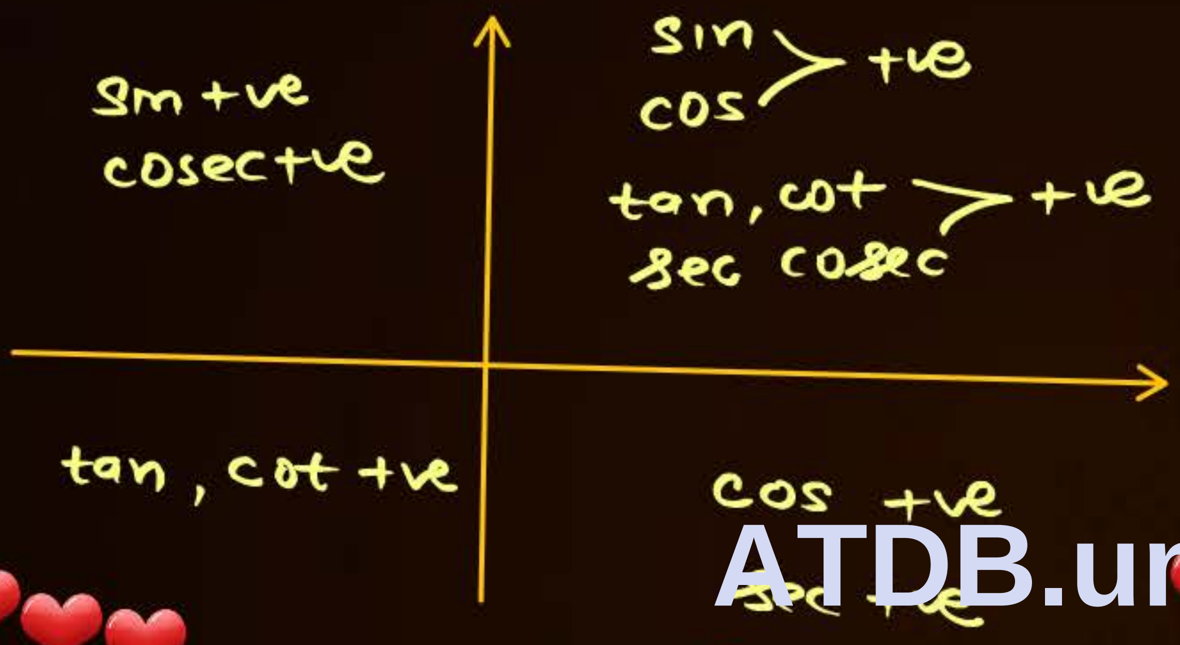
$47 = 4\times11+3$
$= 4\times11+4-1$
$= 4\times(11+1)-1$
$= 4\times12-1$

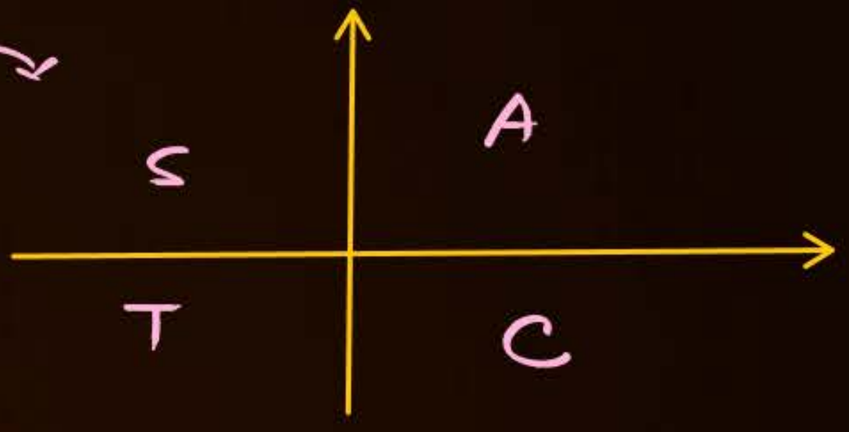
Real definition of 2 basic functions (Sine & Cosine)



- $\sin\theta = \dfrac{y \text{ coord of } P}{\text{length of } OP} = \dfrac{y}{r}$
- $\cos\theta = \dfrac{x \text{ coord of } P}{\text{length of } OP} = \dfrac{x}{r}$
- $\tan\theta = \dfrac{y}{x}$ $\cot\theta = \dfrac{x}{y}$
- $\sec\theta = \dfrac{r}{x}$ $\mathrm{cosec}\,\theta = \dfrac{r}{y}$







★ $2n\pi \cup (2n+1)\pi = n\pi$

★ $(4n+1)\frac{\pi}{2} \cup (4n-1)\frac{\pi}{2} = (2n+1)\pi/2$

$\dots -\frac{3\pi}{2}, -\frac{\pi}{2}, \pi/2, 3\pi/2, 5\pi/2, 7\pi/2 \dots$

$\dots -\frac{7\pi}{2}, -\frac{3\pi}{2}, \pi/2, 5\pi/2, 9\pi/2 \dots \cup \dots, -\pi/2, 3\pi/2, 7\pi/2 \dots$

$n\pi \cup (2n+1)\pi/2$
$=$
$\frac{n\pi}{2}$

Value of T-Ratios at Boundaries of Quadrants

$\sin(2n+1)\pi = 0$

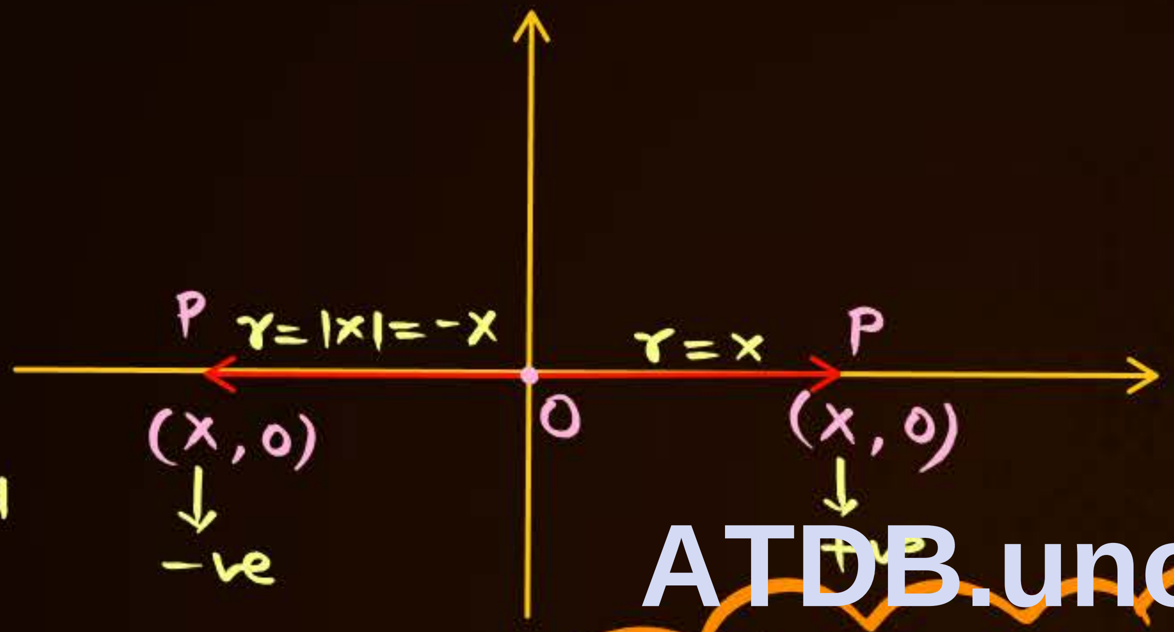
$\cos(2n+1)\pi = \frac{x}{r} = -1$

$\tan(2n+1)\pi = 0$

$\sec(2n+1)\pi = -1$

$\cot(2n+1)\pi$ N·D

$\text{cosec}(2n+1)\pi$ N·D.



$\sin 2n\pi = \frac{0}{r} = 0$

$\cos 2n\pi = \frac{x}{r} = 1$

$\tan 2n\pi = 0$

$\sec 2n\pi = 1$

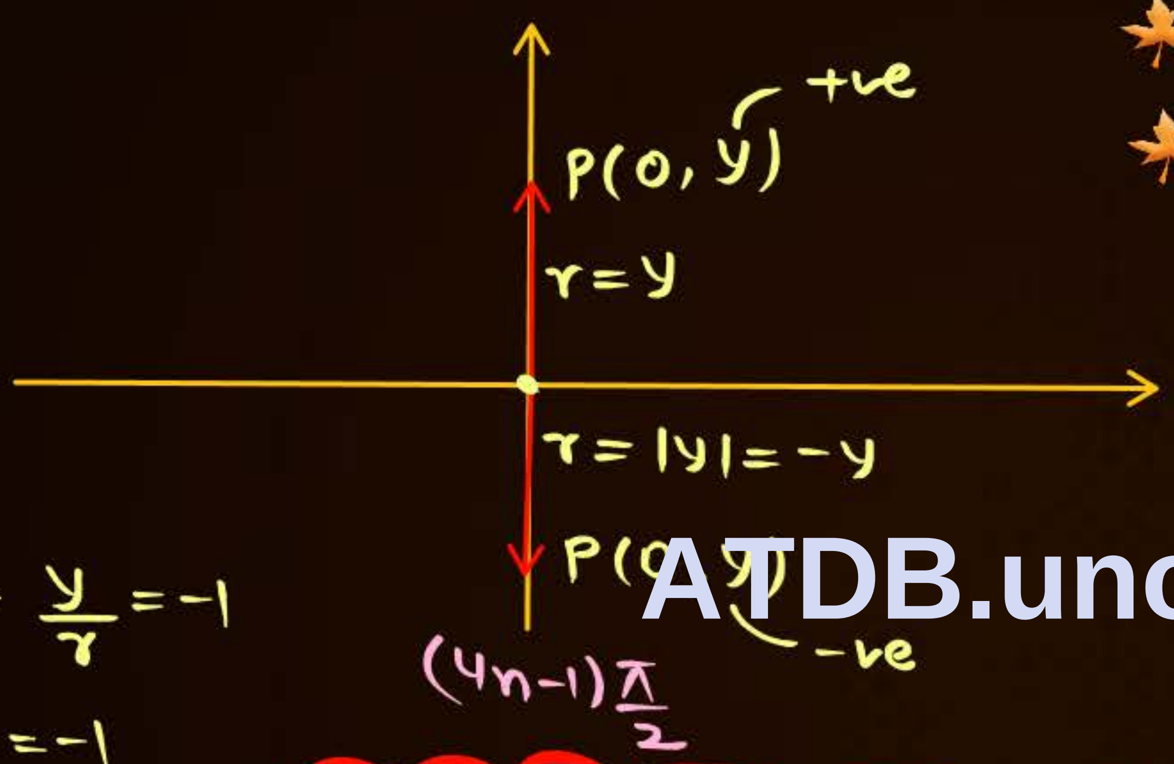
$\text{cosec}\, 2n\pi$ N·D

$\cot 2n\pi$ N·D.

- $\sin\theta = 0 \Leftrightarrow \theta = n\pi$
- $\tan\theta = 0 \Leftrightarrow \theta = n\pi$
- $\sec\theta, \cos\theta = 1 \Leftrightarrow \theta = 2n\pi$
- $\sec\theta, \cos\theta = -1 \Leftrightarrow \theta = (2n+1)\pi$

- $\cot\theta$, $\text{cosec}\,\theta$ are not defined for $n\pi$

Value of T-Ratios at Boundaries of Quadrants



- $\sin(4n+1)\frac{\pi}{2} = \frac{y}{r} = 1$
- $\cos(4n+1)\frac{\pi}{2} = \frac{0}{r} = 0$

$\tan(4n+1)\frac{\pi}{2} = N.D$

$\cot(4n+1)\pi/2 = 0$

$\sec(4n+1)\pi/2 = N.D$

$\operatorname{cosec}(4n+1)\pi/2 = 1$

$\sin(4n-1)\frac{\pi}{2} = \frac{y}{r} = -1$

$\operatorname{cosec}(4n-1)\pi/2 = -1$

$\cos(4n-1)\pi/2 = \frac{0}{r} = 0$

$\sec(4n-1)\pi/2 = N.D.$

$\tan(4n-1)\pi/2 = N.D.$

$\cot(4n-1)\pi/2 = 0$

- $\operatorname{cosec}\theta, \sin\theta = 1 \Leftrightarrow \theta = (4n+1)\pi/2$
- $\operatorname{cosec}\theta, \sin\theta = -1 \Leftrightarrow \theta = (4n-1)\pi/2$
- $\cos\theta = 0 \Leftrightarrow \theta = (2n+1)\pi/2$
- $\tan\theta, \sec\theta$ are N.D at $\theta = (2n+1)\pi/2$
- $\cot\theta = 0 \Leftrightarrow \theta = (2n+1)\pi/2$

- $\tan\theta, \sin\theta = 0 \iff \theta = n\pi$
- $\sec\theta, \cos\theta = 1 \iff \theta = 2n\pi$
- $\sec\theta, \cos\theta = -1 \iff \theta = (2n+1)\pi$

$\cos\theta = \pm 1 \iff \theta = n\pi$

- $\cot\theta, \operatorname{cosec}\theta$ are not defined at $\theta = n\pi$

- $\sin\theta, \operatorname{cosec}\theta = 1 \iff \theta = (4n+1)\frac{\pi}{2}$
- $\sin\theta, \operatorname{cosec}\theta = -1 \iff \theta = (4n-1)\frac{\pi}{2}$
- $\cos\theta = 0 \iff \theta = (2n+1)\pi/2$
- $\tan\theta, \sec\theta$ are not defined at $\theta = (2n+1)\pi/2$
- $\cot\theta = 0 \iff \theta = (2n+1)\pi/2$

QUESTION

If $a = \cos(2012\pi)$, $b = \sec(2013\pi)$ and $c = \tan(2014\pi)$ then

(A) $a < b < c$

(B) $b < c < a$

(C) $c < b < a$

(D) $a = b < c$

$a = \cos(2012\pi) = 1$

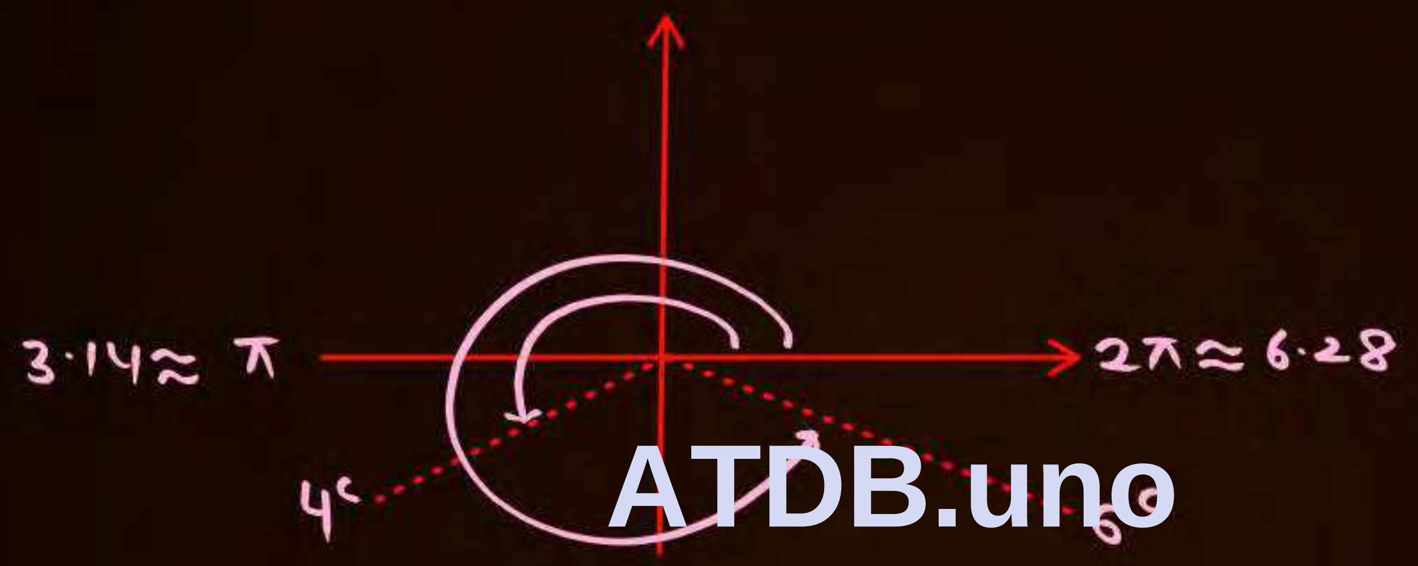
$b = \sec(2013\pi) = -1$

$c = \tan(2014\pi) = 0$

$a > c > b$.

QUESTION

$\dfrac{\sin 4}{\sin 6}$ is negative. T/F

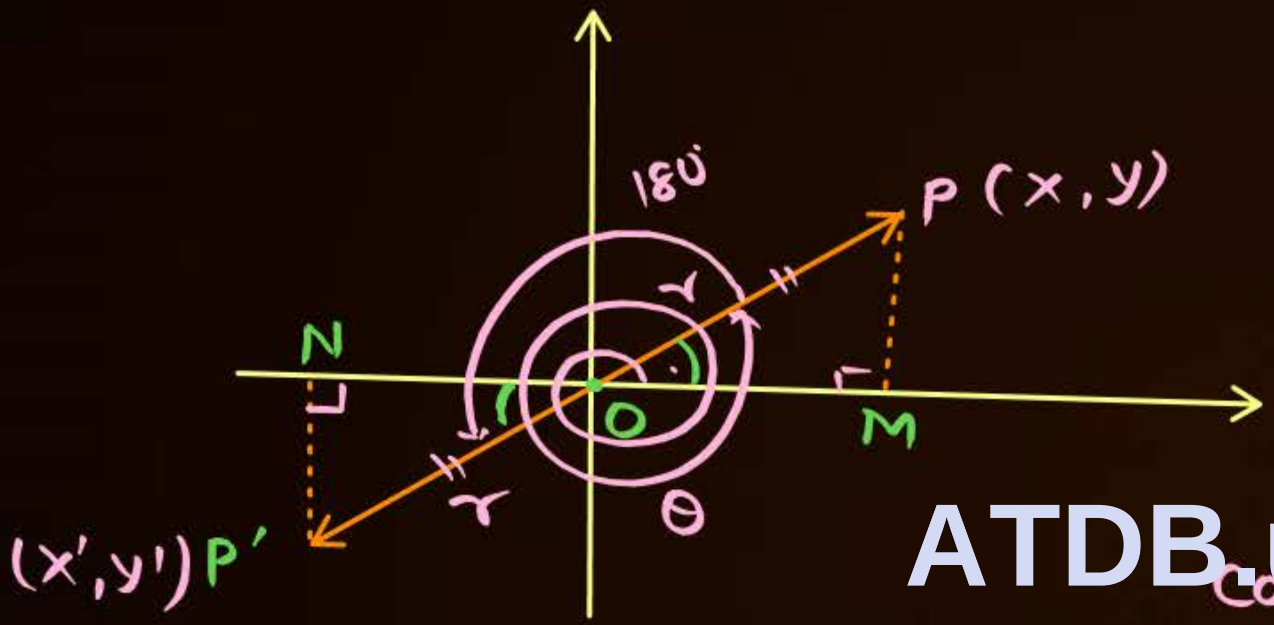


$\sin 4 < 0$

$\sin 6 < 0$

Reduction Formulae

used for finding T-ratios of larger Angles



$\Delta POM \cong \Delta P'ON$

$NP' = MP$
$ON = OM$ (cpct)

$\sin(180° + \theta) = \frac{y'}{r} = -\frac{NP'}{r} = -\frac{MP}{r} = \frac{-y}{r}$

$\sin(180° + \theta) = -\frac{y}{r} = -\sin\theta$

$\cos(180° + \theta) = \frac{x'}{r} = -\frac{ON}{r} = -\frac{OM}{r} = -\frac{x}{r} = -\cos\theta$

- $\sin(180° + \theta) = -\sin\theta$
- $\cos(180° + \theta) = -\cos\theta$
- $\tan(180° + \theta) = \tan\theta$
- $\cot(180° + \theta) = \cot\theta$
- $\operatorname{cosec}(180° + \theta) = -\operatorname{cosec}\theta$
- $\sec(180° + \theta) = -\sec\theta$

Reduction Formulae

used for finding T-ratios of larger Angles

$25 = 6\times4+1$

$5 = 4\times1+1$

$27 = 4\times7-1$

change
$\sin \leftrightarrow \cos$
$\tan \leftrightarrow \cot$
$\sec \leftrightarrow \text{cosec}$

No change

No change

change
$\sin \longleftrightarrow \cos$
$\tan \longleftrightarrow \cot$
$\sec \longleftrightarrow \text{cosec}$.

★ $\sin\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta$

★ $\cos\left(2025\frac{\pi}{2}+\theta\right) = -\sin\theta$ (5th)

$(4n+1)\frac{\pi}{2}$ type

★ $\tan(5\pi/2-\theta) = \cot\theta$

★ $\cot\left(2027\frac{\pi}{2}+\theta\right) = -\tan\theta$

$\sin(5000\pi+\theta) = +\sin\theta$

$\cos(2521\pi-\theta) = -\cos\theta$

★ $\sin(\pi/2+\theta) = +\cos\theta$ (5th)

★ $\tan(\pi/2+\theta) = -\cot\theta$

★ $\sin(\pi-\theta) = +\sin\theta$ (5th)

★ $\sin(2\pi-\theta) = -\sin\theta$

Reduction Formulae

x	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$
$\sin x$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$
$\cos x$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos\alpha$
$\tan x$	$\cot\alpha$	$-\cot\alpha$	$-\tan\alpha$	$\tan\alpha$	$\cot\alpha$	$-\cot\alpha$	$-\tan\alpha$
$\cot x$	$\tan\alpha$	$-\tan\alpha$	$-\cot\alpha$	$\cot\alpha$	$\tan\alpha$	$-\tan\alpha$	$-\cot\alpha$

Reduction Formulae for −θ

$\cos(-\theta) = \cos(2\pi + (-\theta))$
$= \cos(2\pi - \theta)$
$= \cos\theta$

$\sin(-\theta) = \sin(2\pi + (-\theta))$
$= \sin(2\pi - \theta)$
$= -\sin\theta$

$\sin(2\pi + \theta) = \sin\theta$
$\cos(2\pi + \theta) = \cos\theta$
$\tan(2\pi + \theta) = \tan\theta$
$\vdots$

- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$
- $\cot(-\theta) = -\cot\theta$
- $\mathrm{cosec}(-\theta) = -\mathrm{cosec}\,\theta$
- $\sec(-\theta) = \sec\theta$

QUESTION

Find the value of

(i) $\sin\left(\frac{25\pi}{3}\right)$ (ii) $\cos\left(\frac{41\pi}{4}\right)$ (iii) $\tan\left(\frac{-16\pi}{3}\right)$ (iv) $\cot\left(\frac{29\pi}{4}\right)$

$\sin\left(8\pi + \frac{\pi}{3}\right) = +\sin\pi/3 = \frac{\sqrt{3}}{2}$

$\cos\left(10\pi + \frac{\pi}{4}\right) = \cos\pi/4 = \frac{1}{\sqrt{2}}$

$-\tan\frac{16\pi}{3} = -\tan\left(5\pi + \frac{\pi}{3}\right) = -\tan\pi/3 = -\sqrt{3}$

$\cot\left(7\pi + \frac{\pi}{4}\right) = \cot\pi/4 = 1$

QUESTION

$+360°$

Find the value of

(i) sin(405°)

$\sin(360°+45°) = \sin 45° = \frac{1}{\sqrt{2}}$

(ii) sec (–1470°)

$\sec(1470°) = \sec(4\times 360° + 30°)$

$4 \times 360° \rightarrow 2\pi$

$= \sec(8\pi + 30°)$

$= \sec 30° = \frac{2}{\sqrt{3}}$

(iii) tan (–300°)

(iv) cot (585°)

TAH 1A

$$\begin{array}{r} 4 \\ 360° \overline{)\,1470°} \\ 1440 \\ \hline 30° \end{array}$$

QUESTION

Express in terms of ratios of a positive angle, which is less than 45°, the quantities.

(i) cos 1410° (ii) cot (–1054°) (iii) cosec (–756°)

TAH 1B

$360° \overline{)1410}$ quotient 3; $1410 - 1080 = 330$

$\cos(360° \times 3 + 330°) = \cos(6\pi + 330°) = \cos 330°$

$\cos(360° - 30°)$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\cos(4 \times 360° - 30°)$

$\cos(8\pi - 30°)$

$= \cos 30° = \frac{\sqrt{3}}{2}$

QUESTION

TAH 2

Fill in the rest of the blanks yourself.

$\angle A$	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1												
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0												
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND												
cosec A	ND	2	$\sqrt{2}$	$2\sqrt{3}$	1												
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND												
cot A	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0												

$\theta \in (0, \pi/4)$
- $\sin\theta < \cos\theta$
- $\tan\theta < \cot\theta$

$\theta \in (\pi/4, \pi/2)$
- $\cos\theta < \sin\theta$
- $\cot\theta < \tan\theta$

QUESTION

What sign has sin A – cos A for the following values of A?

(i) 215°

(ii) –634°

$$\begin{array}{r} 720 \\ 634 \\ \hline 86 \\ \hline \end{array}$$

$\sin 215^\circ - \cos 215^\circ = \sin(180^\circ + 35^\circ) - \cos(180^\circ + 35^\circ)$

$= -\sin 35^\circ + \cos 35^\circ$

$= \cos 35^\circ - \sin 35^\circ$

$\cos 35^\circ > \sin 35^\circ$

$\sin(-634^\circ) - \cos(-634^\circ)$

$= -\sin 634^\circ - \cos 634^\circ$

$= -\sin(\underset{\downarrow \atop 4\pi}{720^\circ} - 86^\circ) - \cos(\underset{\downarrow \atop 4\pi}{720^\circ} - 86^\circ)$

$= -\sin(4\pi - 86^\circ) - \cos(4\pi - 86^\circ)$

$= -(-\sin 86^\circ) - (+\cos 86^\circ)$

$= \sin 86^\circ - \cos 86^\circ = +ve$

QUESTION

TAH 3

What sign has sin A + cos A for the following values of A?

(i) $278°$

(ii) $-1125°$

QUESTION

TAH 4A

Find the value of

(i) $\operatorname{cosec}(-750°)$

(ii) $\cos(-2220°)$

QUESTION

TAH 4B

$\sin 420° \cos 390° + \cos(-300°) \sin(-330°)$ equals

(A) 0

(B) 1

(C) -1

(D) 2

QUESTION

TAH 4C

Find the value of

(i) $\sec\left(-\frac{19\pi}{3}\right)$

(ii) $\operatorname{cosec}\left(-\frac{33\pi}{4}\right)$

ASNC (Nayi Soch)

$\sin(\pi-\theta) = \sin\theta$ $\quad$ $\tan(\pi-\theta) = -\tan\theta$

$\cos(\pi-\theta) = -\cos\theta$ $\quad$ $\cot(\pi-\theta) = -\cot\theta$

$\text{cosec}(\pi-\theta) = \text{cosec}\,\theta$ $\quad$ $\sec(\pi-\theta) = -\sec\theta$

$\sin(180^\circ - 5000^\circ)$ (5th)

$= +\sin 5000$

$\alpha+\beta=\pi$ i.e α, β are supplementary

$\sin\alpha = \sin\beta$, $\text{cosec}\,\alpha = \text{cosec}\,\beta$

$\cos\alpha = -\cos\beta$ $\quad$ $\cot\alpha = -\cot\beta$

$\tan\alpha = -\tan\beta$ $\quad$ $\sec\alpha = -\sec\beta$

b'coz $\sin\alpha = \sin(\pi-\beta) = \sin\beta$

ASNC (Nayi Soch)

if $\alpha + \beta = \frac{\pi}{2}$

- $\sin\alpha = \cos\beta$
- $\tan\alpha = \cot\beta$
- $\sec\alpha = \operatorname{cosec}\beta$

QUESTION

$\frac{11\pi}{17} + \frac{6\pi}{17} = \pi$

Evaluate :

(i) $\dfrac{\sin\frac{11\pi}{17}\cos\frac{10\pi}{13}\tan\frac{\pi}{7}}{\cos\frac{3\pi}{13}\sin\frac{6\pi}{17}\tan\frac{6\pi}{7}} = 1$

$-1 \quad -1$

(ii) $\dfrac{\sin\frac{\pi}{5}\cos\frac{7\pi}{9}\tan\frac{6\pi}{11}}{\cos\frac{2\pi}{9}\sin\frac{4\pi}{5}\tan\frac{5\pi}{11}} = 1$

$1 \quad -1 \quad -1$

$\frac{10\pi}{13} + \frac{3\pi}{13} = \pi \qquad \pi/7 + \frac{6\pi}{7} = \pi$

$\cos\frac{10\pi}{13} = -\cos\frac{3\pi}{13}$

QUESTION

Evaluate :

(iii) $$\frac{\overset{1}{\sin\frac{\pi}{7}}\,\overset{1}{\cos\frac{5\pi}{11}}\,\overset{-1}{\tan\frac{3\pi}{7}}}{\cos\frac{5\pi}{14}\,\sin\frac{\pi}{22}\,\tan\frac{4\pi}{7}} = -1$$

$$\frac{\pi}{7}+\frac{5\pi}{14}=\frac{7\pi}{14}=\pi/2$$

$$\frac{5\pi}{11}+\frac{\pi}{22}=\frac{11\pi}{22}=\pi/2$$

$$\frac{3\pi}{7}+\frac{4\pi}{7}=\pi$$

QUESTION

$$\tan\frac{\pi}{11}+\tan\frac{2\pi}{11}+\tan\frac{4\pi}{11}+\tan\frac{7\pi}{11}+\tan\frac{9\pi}{11}+\tan\frac{10\pi}{11}\text{ equals}$$

$\frac{\pi}{11}+\frac{10\pi}{11}=\pi$

(A) 1

(B) –1

(C) 0

(D) 2

QUESTION

$$\sin^2\frac{\pi}{18}+\sin^2\frac{\pi}{9}+\sin^2\frac{7\pi}{18}+\sin^2\frac{4\pi}{9} \text{ equals}$$

$$\cos^2\frac{\pi}{9}+\cos^2\frac{\pi}{18}$$

$$\frac{\pi}{18}+\frac{4\pi}{9}=\frac{9\pi}{18}=\frac{\pi}{2}$$

$$\frac{\pi}{9}+\frac{7\pi}{18}=\frac{2\pi+7\pi}{18}=\frac{\pi}{2}$$

(A) 1

(B) –1

(C) 0

(D) 2

QUESTION [JEE Mains 2024 (8 April)]

If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to

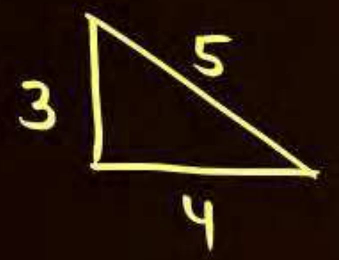
(A) 109

(B) 108

(C) 19

(D) 18

$\sin x = -\frac{3}{5}$ — sign bhool jao



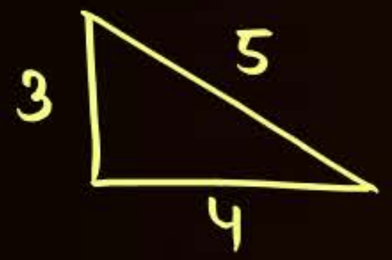
$\tan x = +\frac{3}{4}$ — $\tan^2 x = \frac{9}{16}$

$\cos x = -\frac{4}{5}$ — $-\cos x = \frac{4}{5}$

$\tan^2 x - \cos x = \frac{9}{16} + \frac{4}{5} = \frac{45+64}{80}$

$80(\tan^2 x - \cos x) = 109$.

$\cos x = -\frac{4}{5}$ — $x \in$ quad II or quad III



$\tan x = -\frac{3}{4}$ OR $\tan x = \frac{3}{4}$

$\sin x = \frac{3}{5}$ $\qquad$ $\sin x = -\frac{3}{5}$

$\downarrow$ $\qquad$ $\downarrow$

$x \in$ quad II $\qquad$ $x \in$ quad III

$\cos\theta = -\frac{4}{5} = \frac{x \text{ coor}}{r}$

$x = -4$ $\qquad$ $\theta \in \text{II}, \text{III}$

$r = 5$

$r^2 = x^2 + y^2$

$25 = 16 + y^2$

$y = \pm 3$

$\sin\theta = \frac{y}{r} = \frac{3}{5}$ or $-\frac{3}{5}$.

$\tan\theta = \frac{y}{x} = \frac{3}{-4}, \frac{-3}{-4}$

quad II $\qquad$ quad III

Sabse Important Baat

Sabhi Class illustrations Retry Karni hai

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

KTK 1

$$\frac{\tan(90°-\theta)\sec(180°-\theta)\sin(-\theta)}{\sin(180°+\theta)\cot(360°-\theta)\operatorname{cosc}(90°-\theta)} \text{ equals}$$

(A) 1

(B) –1

(C) 0

(D) 2

Ans. A

QUESTION

KTK 2

If ABCD is a quadrilateral, then show that

(a) $\cos\frac{B+C}{2}+\cos\frac{A+D}{2}=0$

(b) $\tan\frac{A+C}{4}=\cot\frac{B+D}{4}$

QUESTION

KTK 3

The value of $\cos^2\frac{\pi}{16}+\cos^2\frac{3\pi}{16}+\cos^2\frac{5\pi}{16}+\cos^2\frac{7\pi}{16}$ is

(A) 2

(B) 1

(C) 0

(D) None of these

Ans. A

QUESTION [JEE Mains 2019]

KTK 4

Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for $k = 1,2,3, ...$

Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to

(A) $\frac{5}{12}$

(B) $\frac{-1}{12}$

(C) $\frac{1}{4}$

(D) $\frac{1}{12}$

Ans. D

QUESTION

KTK 5

If θ is the each interior angle of a regular dodecagon then the value of $\sin\theta + \cos\theta + \tan\theta + \cot\theta + \sec\theta + \operatorname{cosec}\theta$, is

(A) positive

(B) negative and less than (–1)

(C) zero

(D) negative and less than (–2)

Ans. B

QUESTION

KTK 6

The value of $\sum_{n=0}^{1947} \frac{1}{2^n + \sqrt{2^{1947}}}$ is equal to

(A) $\frac{487}{\sqrt{2^{1945}}}$

(B) $\frac{1946}{\sqrt{2^{1947}}}$

(C) $\frac{1947}{\sqrt{2^{1947}}}$

(D) $\frac{1948}{\sqrt{2^{1947}}}$

Ans. A

QUESTION

KTK 7

If $a, b, c \in R^+$ then the minimum value of $a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2)$ is equal to

(A) abc

(B) 2abc

(C) 3abc

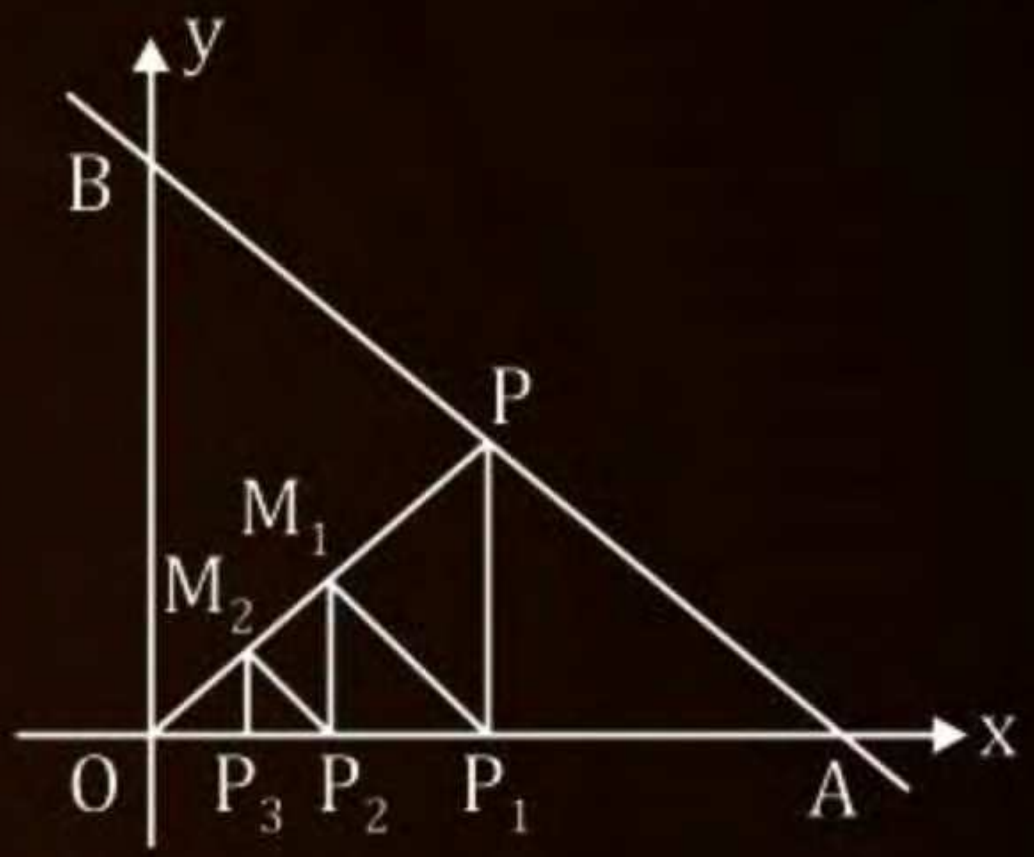
(D) 6abc

Ans. D

QUESTION

KTK 8

The line $x + y = 1$ meets x-axis at A and y-axis at B, P is the mid-point of AB;
P is the foot of die perpendicular from P to OA;
M_1 is that of P_1 from OP;
P_2 is that of M_1 from OA;
M_3 is that of P_2 from OP;
P_3 is that of M_2. from OA; and so on.
If P_n denotes the n^{th} foot of the perpendicular on OA; then OP is



(A) $\left(\frac{1}{2}\right)^{n-1}$

(B) $\left(\frac{1}{2}\right)^{n}$

(C) $\left(\frac{1}{2}\right)^{n+1}$

(D) None of these

Ans. B

Homework From Module

Sequence Series:

Prarambh (Topicwise) : Complete

Prabal (JEE Main Level) : Complete

Parikshit (JEE Advanced Level) : Complete

Revision Practice Problems (RPP)

QUESTION

RPP 01

If a, b are the roots of $x^2 + px + 1 = 0$ and c, d are the roots of $x^2 + qx + 1 = 0$. Then find the value of $(a - c)(b - c)\,(a + d)(b + d)/(q^2 - p^2)$.

Ans. 1

QUESTION

RPP 02

The least prime integral value of '2a' such that the roots α, β of the equation $2x^2 + 6x + a = 0$ satisfy the inequality $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} < 2$ is

Ans. 11

Solution to Previous TAH

QUESTION [JEE Mains 2021 (Aug)]

Let $S_n = 1\cdot(n-1) + 2\cdot(n-2) + 3\cdot(n-3) + \cdots + (n-1)\cdot 1, n \geq 4.$

The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ is equal to:

(A) $\frac{e-1}{3}$

(B) $\frac{e-2}{6}$

(C) $\frac{e}{3}$

(D) $\frac{e}{6}$

$(n-1)\cdot 1$, $n \geq 4$.

The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ is equal.

$\Rightarrow T_r = r(n-r) = rn - r^2$.

$\Rightarrow S_n = \sum_{r=1}^{n} rn - \sum_{r=1}^{n} r^2$.

$= n\cdot\frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$.

$= \frac{n(n+1)}{2}\left[n - \frac{(2n+1)}{3}\right]$

$= \frac{n(n+1)}{2}\left(\frac{n-1}{3}\right) = \frac{n(n+1)(n-1)}{6}$.

$= \frac{n^3-n}{6}$.

\# $\sum_{n=4}^{\infty} \frac{2S_n}{n!} - \frac{1}{(n-2)!}$.

$\Rightarrow \sum_{n=4}^{\infty} \frac{n^3-n}{3(n)!} - \frac{1}{(n-2)!} \Rightarrow \sum_{n=4}^{\infty} \frac{n^2-1^2}{3(n-1)!} - \frac{1}{(n-2)!}$

$\Rightarrow \sum_{n=4}^{\infty} \frac{(n+1)}{3(n-2)!} - \frac{1}{(n-2)!} \Rightarrow \sum_{n=4}^{\infty} \frac{(n-2)}{3(n-2)!}$

$\Rightarrow \sum_{n=4}^{\infty} \frac{1}{3(n-3)!}$

$\Rightarrow \frac{1}{3}\left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \infty\right)$

$\Rightarrow \frac{1}{3}(e-1) \Rightarrow \frac{e-1}{3}$ Ans.

krish

Soln: $S_n = 1.(n-1) + 2(n-2) + 3(n-3) + \ldots (n-1).1 \geq 4$

$T_K = K(n-K)$

$= nK - K^2$

$S_n = \sum_{K=1}^{n} T_K = n\sum_{K=1}^{n} K - \sum_{K=1}^{n} K^2$

$S_n = \frac{n(n)(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow S_n = \frac{n(n+1)\{3n - 2n - 1\}}{6}$

$\Rightarrow S_n = \frac{n(n+1)(n-1)}{6}$

RASIDUL

$\sum_{n=4}^{\infty}\left\{\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right\}$

$= \sum_{n=4}^{\infty}\left\{\frac{2n(n+1)(n-1)}{6\,n!} - \frac{1}{(n-2)!}\right\}$

$= \sum_{n=4}^{\infty}\left\{\frac{2(n+1)}{6(n-2)!} - \frac{1}{(n-2)!}\right\}$

$= \sum_{n=4}^{\infty}\left\{\frac{n+1-3}{3(n-2)!}\right\}$.

$= \sum_{n=4}^{\infty}\left\{\frac{1(n-2)}{3(n-2)!}\right\}$

$= \frac{1}{3}\sum_{n=4}^{\infty} \frac{1}{(n-3)!}$

$= \frac{1}{3}\left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \infty\right)$

$= \frac{1}{3}\left(\left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \infty\right) - 1\right)$

$= \frac{e-1}{3}$.

Ans: (A) $\frac{e-1}{3}$

TAH-01 $S_n = 1\cdot(n-1) + 2\cdot(n-2) + 3\cdot(n-3) + \cdots (n-1)\cdot 1$

$$\Rightarrow S_n = 1(n-1) + 2(n-2) + 3\cdot(n-3) + \cdots + (n-1)(n-(n-1))$$

$$\Rightarrow S_n = [n+2n+3n+\cdots+(n-1)n] - [1+4+9+16+\cdots+(n-1)^2]$$

$$\Rightarrow S_n = n\left[\frac{(n-1)n}{2}\right] - \frac{(n-1)n(2n-1)}{6}$$

Kritisha

$$\Rightarrow 2S_n = n^2(n-1) - \frac{n(n-1)(2n-1)}{3}$$

$$\Rightarrow \sum_{n=4}^{\infty}\left(\frac{n^2(n-1) - \frac{n(n-1)(2n-1)}{3}}{n!}\right) - \sum_{n=4}^{\infty}\frac{1}{(n-2)!}$$

$$\Rightarrow \sum_{n=4}^{\infty}\frac{n(n-1)}{(n-1)!} - \sum_{n=4}^{\infty}\frac{n(n-1)(2n-1)}{3\,n!} - \sum_{n=4}^{\infty}\frac{1}{(n-2)!}$$

$$\Rightarrow \sum_{n=4}^{\infty}\frac{n}{(n-2)!} - \frac{1}{3}\sum_{n=4}^{\infty}\frac{2n-1}{(n-2)!} - \sum_{n=4}^{\infty}\frac{1}{(n-2)!}$$

$$\Rightarrow \sum_{n=4}^{\infty}\frac{n-2+2}{(n-2)!} - \frac{1}{3}\sum_{n=4}^{\infty}\frac{2n-4+3}{(n-2)!} - \sum_{n=4}^{\infty}\frac{1}{(n-2)!}$$

$$\Rightarrow \sum_{n=4}^{\infty}\frac{1}{(n-3)!} + 2\sum_{n=4}^{\infty}\cancel{\frac{1}{(n-2)!}} - \frac{2}{3}\sum_{n=4}^{\infty}\frac{1}{(n-3)!} - \cancel{\sum_{n=4}^{\infty}\frac{1}{(n-2)!}} - \cancel{\sum_{n=4}^{\infty}\frac{1}{(n-2)!}}$$

$$\Rightarrow \frac{1}{3}\sum_{n=4}^{\infty}\frac{1}{(n-3)!} = \frac{1}{3}\left(\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots\right)$$

$$= \frac{1}{3}(e-1)$$

$$= \frac{e-1}{3}$$ (A) Ans.

QUESTION

Let $f(x) = (a^2 + b^2 - 4a - 6b + 13)(2x^2 - 4x + 5)$, $a, b, x \in R$ such that $f(0) = f(1) = f(2)$. If $a, A_1, A_2, \ldots\ldots A_{10}, b$ is an arithmetic progression and $a, H_1, H_2, \ldots\ldots, H_{10}, b$ is harmonic progression then the value of

$$\frac{1}{10}\left(\sum_{i=4}^{8} A_i H_{11-i}\right)$$ is equal to

Ans. 3

Tah-02 .

LUCKY KUMARI

$$f(x) = (a^2+b^2-4a-6b+13)(2x^2-4x+5)$$

$$f(0) = f(1)$$

$$(a^2+b^2-4a-6b+13)(5) = (a^2+b^2-4a-6b+13)(3)$$

$$5a^2+5b^2-20a-30b+65 = 3a^2+3b^2-12a-18b+39$$

$$2a^2+2b^2-8a-12b+26=0$$

$$a^2+b^2-4a-6b+13=0$$

$$(a^2-4a+4)+(b^2-6b+9)=0$$

$$\underbrace{(a-2)^2}_{\geq 0}+\underbrace{(b-3)^2}_{\geq 0}=0$$

$$a=2 \quad \& \quad b=3.$$

$$\left.a, A_1, A_2, \text{------}, A_{10}, b\right\} AP$$

$$\left.a, H_1, H_2, \text{-------}, H_{10}, b\right\} HP$$

So, $\quad ab = A_1H_{10} = A_2H_9 = \text{......} = A_{10}H_1$.

$$\frac{1}{10}\sum_{i=4}^{8} A_iH_{11-i} = \frac{1}{10}\left(\underbrace{A_4H_7}_{ab}+\underbrace{A_5H_6}_{ab}+\underbrace{A_6H_5}_{ab}+\underbrace{A_7H_4}_{ab}+\underbrace{A_8H_3}_{ab}\right)$$

$$= \frac{ab\times 5}{10}$$

$$= \frac{2(3)(5)}{10} = 3.$$

Solution to Previous KTKs

QUESTION

KTK 01

The dimensions of a Cuboid are $a > b > c$. The volume $= 216$ and the total outer surface area $= 252$. If a, b, c are in G.P., then $c =$

(A) 3

(B) 1

(C) 5

(D) 2

Ans. A

$a > b > c \longrightarrow a, b, c$ are in G.P

vol. $\Rightarrow \boxed{abc = 216}$

$2(ab + bc + ca) = 252$

$\boxed{ab + bc + ca = 126}$

Let $a = \frac{p}{r}$

$b = p$

$c = pr$

$abc = p^3 = 216$

$\boxed{p = 6}$

Kritisha (W.B)

$ab + bc + ca = \frac{p^2}{r} + p^2 r + p^2$

$= p^2\left(1 + r + \frac{1}{r}\right) = 126$

$1 + r + \frac{1}{r} = \frac{\cancel{126}\ \cancel{21}\ 7}{\cancel{36}\ \cancel{6}\ 2}$

$r + \frac{1}{r} = \frac{7-2}{2} = \frac{5}{2}$

$r + \frac{1}{r} = \frac{5}{2}$

$\Rightarrow r^2 + 1 - \frac{5}{2}r = 0 \Rightarrow \boxed{2r^2 - 5r + 2 = 0}$

$2r^2 - 5r + 2 = 0$

$2r^2 - 4r - r + 2 = 0$

$\boxed{(2r-1)(r-2) = 0} \longrightarrow r = 1/2$ as, $a > b > c$

Thus, $c = pr = \frac{6}{2} = 3$ (A)

Ans.

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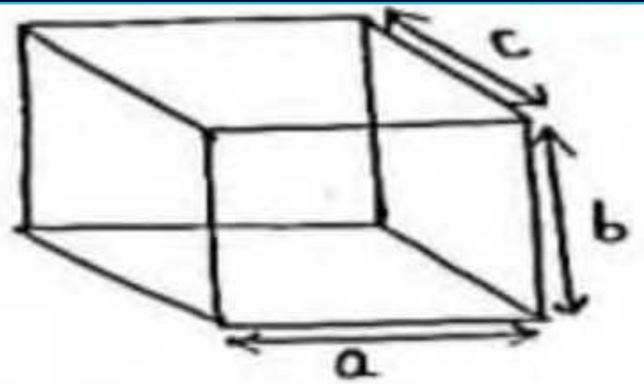


a, b, c } GP

$b^2 = ac$

$\boxed{ac = 36}$

$\Rightarrow$ Volume $= 216$

$a \times b \times c = 216$

$b(b^2) = 216$

$\boxed{b = 6}$

$\Rightarrow$ Total outer S.A $= 252$

$2(ab + bc + ca) = 252$

$(ab + bc + b^2) = 126$

$(6a + 6c + 36) = 126$

$6(a + c) = 90$

$a + c = 15$

$a + \frac{36}{a} = 15$

$a^2 + 36 = 15a$

$a^2 - 12a - 3a + 36 = 0$

$(a - 12)(a - 3) = 0$

$\boxed{\begin{matrix} a = 12 \\ c = 3 \end{matrix}}$ & $a = 3$ & $c = 12$ (But $a > b > c$ $\Rightarrow a > c$)

Hence, $\boxed{c = 3}$

KTK-01

$a > b > c$,

Volume of a cuboid $= abc = 216$

Outer surface area of a cuboid
$= 2(ab + bc + ca) = 252$

$\Rightarrow ab + bc + ca = 126$

$a, b, c \longrightarrow$ in G.P

Let $C.R = r$

$\Rightarrow 0 < r < 1$ because, $a > b > c$ (given)
[$r < 0$ not possible because sides are never negative]

$b = ar$

$c = ar^2$

$\therefore abc = 216$

$a(ar)(ar^2) = 216$

$a^3r^3 = 216$

$ar = 6 \longrightarrow a = 6/r$

$ab + bc + ca = 126$

$a(ar) + ar(ar^2) + ar^2 \cdot a = 126$

$a^2(r + r^3 + r^2) = 126$

$\frac{36}{r^2}(r^3 + r^2 + r) = 126$

$\frac{36}{r}(r^2 + r + 1) = 126$ (6 2 21 7)

$2r^2 + 2r + 2 = 7r$

$2r^2 - 5r + 2 = 0$

$2r^2 - 4r - r + 2 = 0$

$(2r - 1)(r - 2) = 0$

$r = 1/2$, $r = 2$ × rejected [$0 < r < 1$]

Ans (A)

★ KTK-1. The dimension of a cuboid are a, b, c. The volume $= 216$ and the total outer surface area $= 252$. If a, b, c are in G.P., then $c =$

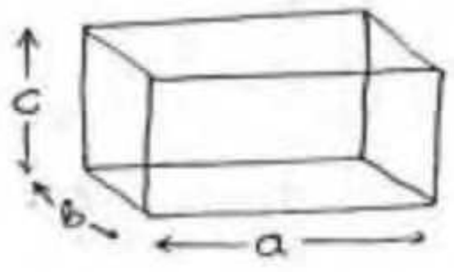
\# Volume $= 216$

$abc = 216$ —— ①

\# T.S.A $= 252$

$2(ab + bc + ca) = 252$

$ab + bc + ca = 126$ — ②



Cuboid.

\# If a, b, c are in G.P.

then: $\boxed{b^2 = ac} \Rightarrow a = \frac{b^2}{c} = \frac{36}{c}$.

\# Put in eq ①

$b^3 = 216$

$b = 6$

\# $6a + 6c + 36 = 126$

$6(a + c) = 90$

$a + c = 15$

$\Rightarrow \frac{36}{c} + c = 15$

$\Rightarrow 36 + c^2 = 15c$

$\Rightarrow c^2 - 15c + 36 = 0$

$\Rightarrow (c - 12)(c - 3) = 0$

$\Rightarrow c = 12$ ×, $c = 3$ ✓

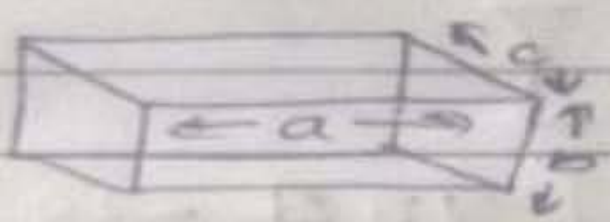
$\Rightarrow$ bcz given: $a > b > c$ in question & $\boxed{b = 6}$ then

$\boxed{c = 3}$ only

KTK01:

Ankush

Sol^n:



Volume = 216

$a \cdot b \cdot c = 216$ —— (i)

$\therefore$ $a, b, c \to GP$

$b = ar, \quad c = ar^2$

Total outer surface area = 252

$2(ab+bc+ca) = 252$

$ab+bc+ca = 126$ —— (ii)

Then, $a \cdot b \cdot c = 216$

$a \cdot ar \cdot ar^2 = 216$

$a^3 r^3 = 216$

$\boxed{ar = 6}$

$\therefore$ $ab+bc+ca = 126$

$a \cdot ar + ar \cdot ar^2 + ar^2 \cdot a = 126$

$a^2 r + a^2 r^3 + a^2 r^2 = 126$

$a^2 r + a^2 r^3 = 126 - 36$

$\boxed{a^2 r + a^2 r^3 = 90}$

Now $\dfrac{a^2 r + a^2 r^3}{ar} = \dfrac{90}{6}$

$a + ar^2 = 15$

$a(1+r^2) = 15 \Rightarrow \dfrac{6}{r}(1+r^2) = 15$

$6 + 6r^2 = 15r \Rightarrow 2r^2 - 5r + 2 = 0$

$(2r-1)(r-2) \Rightarrow r = \dfrac{1}{2},\ r \neq 2 \because 0<r<1$

$\therefore$ a, b, c are GP

$b^2 = ac$

$c = \dfrac{b^2}{a} = \dfrac{a^2 r^2}{a} = \dfrac{36 \times r}{6} = 6 \times \dfrac{1}{2} = 3$

$\boxed{c = 3}$

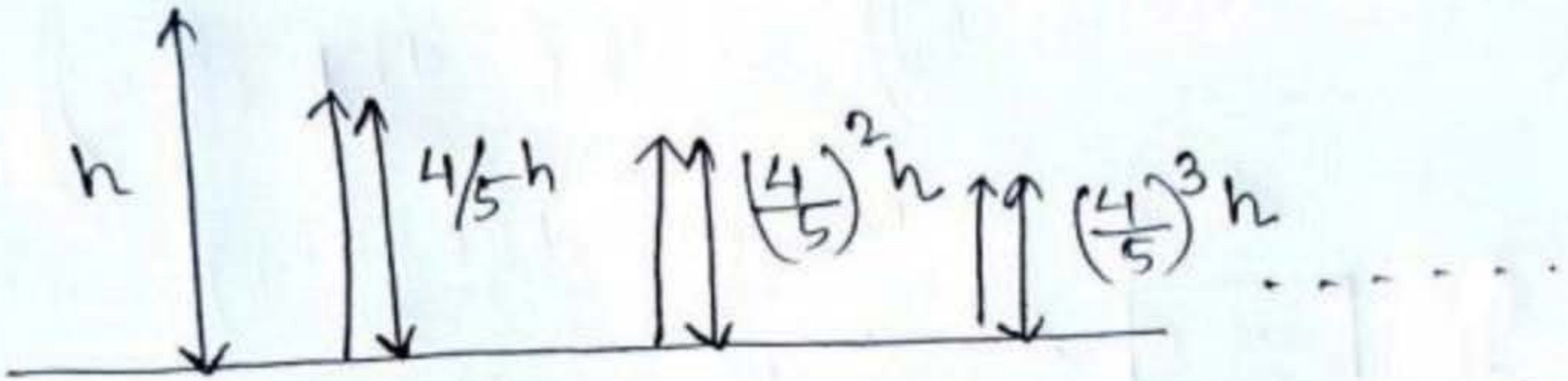
QUESTION

KTK 02

A ball falls from a height of 100 m on a floor. If in each rebound, it describes $(4/5)^{th}$ height of the previous falling height, then the total distance travelled by the ball before it comes to rest is?

Ans. 900 m

KTK-02



total distance travelled $\left(h + 2\left(\frac{4}{5}h + \left(\frac{4}{5}\right)^2 h + \left(\frac{4}{5}\right)^3 + \text{------}\right)\right)$

$$= h + 2h\left(\frac{\frac{4}{5}}{1-\frac{4}{5}}\right)$$

Kritisha (W.B)

$$= h + 2h\left(\frac{4}{1}\right)$$

$$= 9h = \boxed{900\,m} \text{ (Ans)}$$

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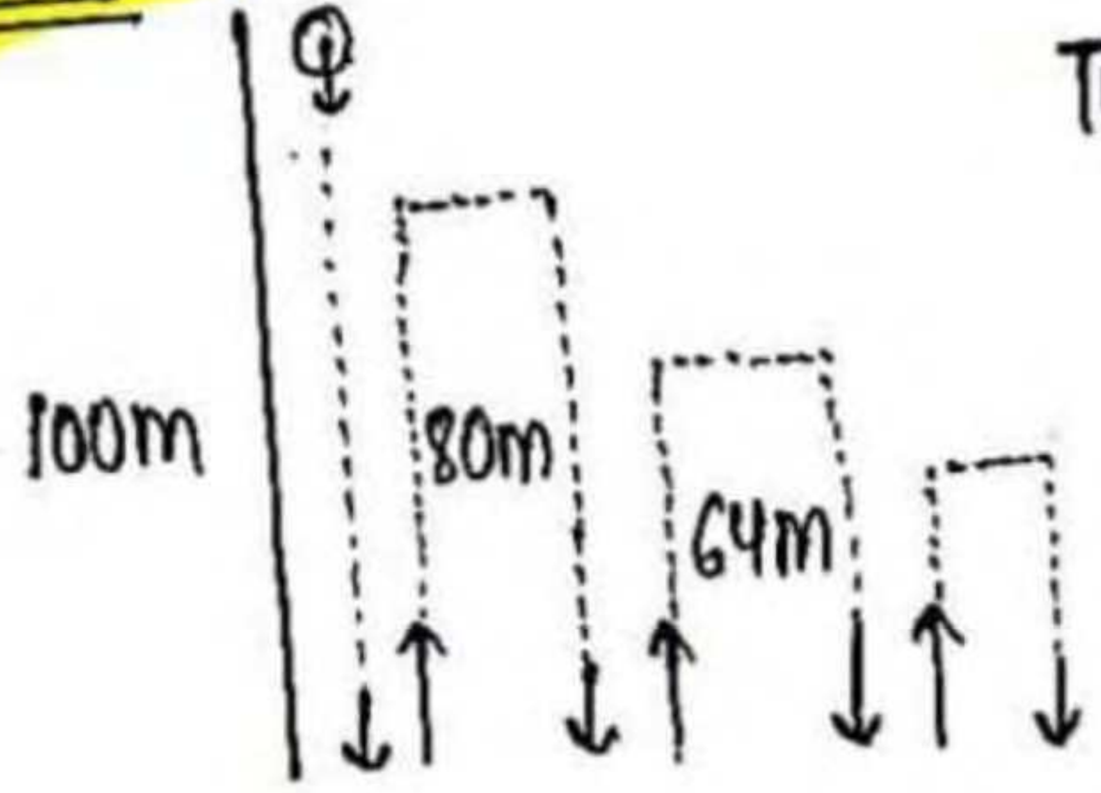
KIK-02



Total dist. travelled

$$= 100 + 2\left(\frac{4}{5}(100) + \frac{4}{5}\left(\frac{4}{5}\right)(100) + \ldots\ldots\infty\right)$$

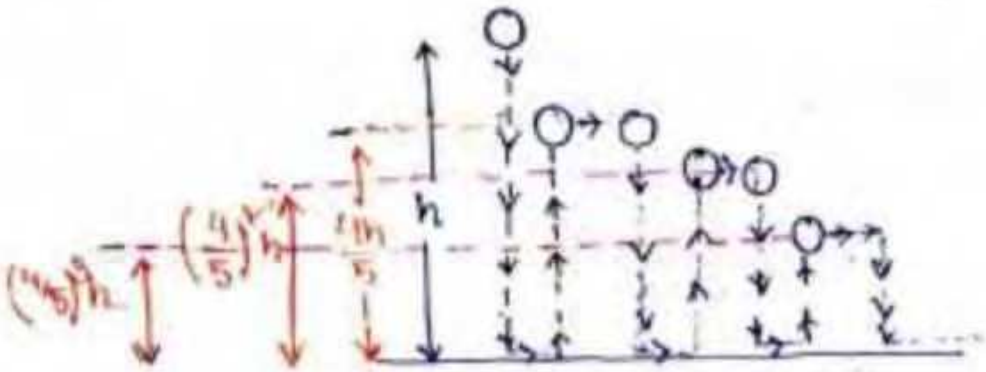
$$= 100 + 2(100)\left(\frac{4}{5} + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + \ldots\ldots\infty\right)$$

$$= 100 + 200\left(\frac{4/5}{1-4/5}\right)$$

$$= 100 + 200(4)$$

$$= (100 + 800)\text{m} = 900\text{ m.}$$

KTK-02



Total distance travelled by the ball

$$= h + \left(\frac{4h}{5} + \frac{4h}{5}\right) + \left(\left(\frac{4}{5}\right)^2 h + \left(\frac{4}{5}\right)^2 h\right) + \left(\left(\frac{4}{5}\right)^3 h + \left(\frac{4}{5}\right)^3 h\right) + \ldots \infty$$

$$= h + 2\left[\frac{4h}{5} + \left(\frac{4}{5}\right)^2 h + \left(\frac{4}{5}\right)^3 h + \left(\frac{4}{5}\right)^4 h + \ldots \infty\right]$$

$$= h + 2h\left[\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + \left(\frac{4}{5}\right)^4 + \ldots \infty\right]$$

$$= h + 2h\left(\frac{4/5}{1-4/5}\right)$$

$$= h + 2h\,\frac{4/\cancel{5}}{1/\cancel{5}}$$

$$= h + 8h$$

$$= 9h \quad [h = 100\text{m given}]$$

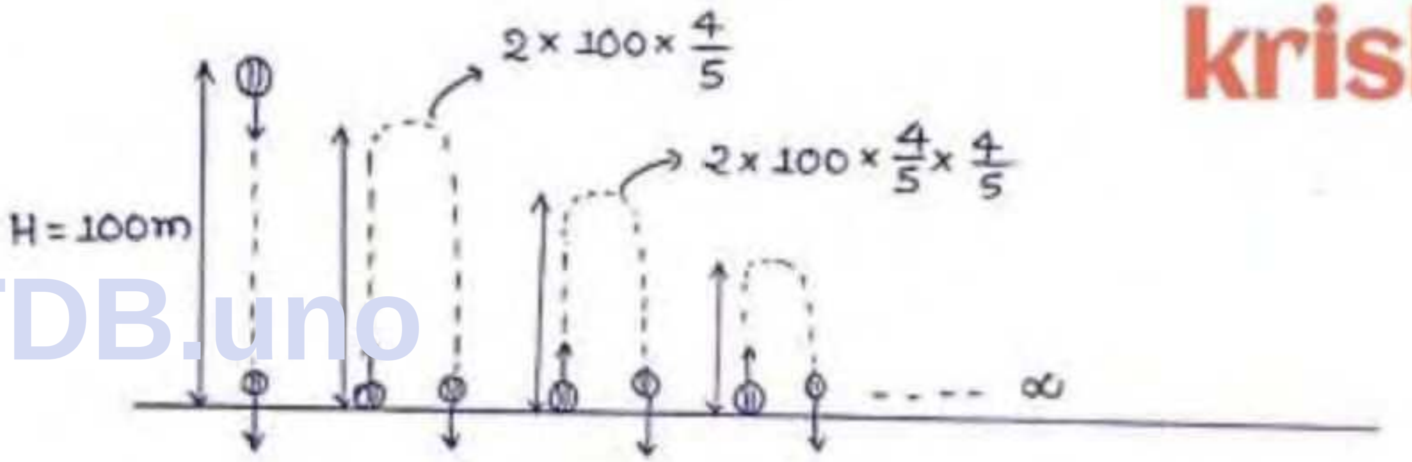
$$= 9 \times 100$$

$$= 900\text{m}$$

Ans

KTK-2. A ball falls from a height 100m on a floor. If in each rebound, it describes $(4/5)^{th}$ height of previous falling height, then the total distance travelled by the ball before it comes to rest is?

krish



$$S = 100 + 2 \times 100 \times \frac{4}{5} + 2 \times 100\left(\frac{4}{5}\right)^2 + \ldots \infty$$

$$= 100 + 2 \times \overset{20}{\cancel{100}} \times \frac{4}{\cancel{5}}\left[1 + \frac{4}{5} + \left(\frac{4}{5}\right)^2 + \ldots \infty\right]$$

$$= 100 + 160\left[\frac{1}{1-4/5}\right]$$

$$= 100 + 160 \times 5$$

$$= 100 + 800 = 900\text{ m} \quad \text{Ans.}$$

QUESTION [JEE Mains 2020 (Jan)]

KTK 03

The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}}$ ...... to ∞ is equal to

(A) $2^{\frac{1}{4}}$

(B) $2^{\frac{1}{2}}$

(C) 1

(D) 2

Ans. B

KTK-03

The product $2^{1/4} \cdot 4^{1/16} \cdot 8^{1/48} \cdot 16^{\frac{1}{128}} \ldots$ to ∞ is equal to

$$\Rightarrow 2^{\frac{1}{4}} \cdot 2^{\frac{2}{16}} \cdot 2^{\frac{3}{48}} \cdot 2^{\frac{4}{128}} \ldots \infty$$

$$= 2^{\frac{1}{4}} \cdot 2^{\frac{1}{8}} \cdot 2^{\frac{1}{16}} \cdot 2^{\frac{1}{32}} \cdots$$

$$= 2^{\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\right)}$$

$$= 2^{1/2} \text{ (B)}$$

Ans.

$$\left[\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\right.$$

$$= \frac{\frac{1}{4}}{1-\frac{1}{2}} = \frac{\frac{1}{4}}{\frac{1}{2}}$$

$$\left.= \frac{1}{2}\right]$$

KTK-03. Lucky kumari

$$2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdots\cdots \infty$$

$$\Rightarrow 2^{1/4} \cdot 2^{\frac{2}{16}} \cdot 2^{\frac{3}{48}} \cdot 2^{\frac{4}{128}} \cdots\cdots \infty$$

$$\Rightarrow 2^{\frac{1}{4}} \cdot 2^{\frac{1}{8}} \cdot 2^{\frac{1}{16}} \cdot 2^{\frac{1}{32}} \cdots\cdots \infty$$

$$\Rightarrow 2^{\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots\cdots \infty\right)}$$

$$\Rightarrow 2^{\left(\frac{\frac{1}{4}}{1-1/2}\right)} = 2^{\left(\frac{1}{4}\right)\left(\frac{2}{1}\right)} = 2^{1/2}.$$

KTK-3. The product of $2^{1/4} \cdot 4^{1/16} \cdot 8^{1/48} \cdot 16^{1/128} \ldots$ to ∞ is equal to :

$$\Rightarrow (2)^{1/4} \cdot (2)^{2 \times 1/16} \cdot (2)^{3 \times 1/48} \cdot (2)^{4 \times 1/128} \cdots \infty.$$

$$\Rightarrow 2^{1/4} \cdot 2^{1/8} \cdot 2^{1/16} \cdot 2^{1/32} \cdots \infty$$

$$\Rightarrow 2^{\left(1/4 + 1/8 + 1/16 + 1/32 + \cdots \infty\right)}.$$

$$\Rightarrow 2^{\left(\frac{1/4}{1 - 1/2}\right)} \Rightarrow 2^{\left(\frac{1/4}{1/2}\right)} \Rightarrow 2^{1/2} \text{ Ans.}$$

QUESTION [JEE Mains 2020 (Jan)]

KTK 04

If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$, for $0 < \theta < \frac{\pi}{4}$, then:

(A) $x(1 + y) = 1$

(B) $y(1 - x) = 1$

(C) $y(1 + x) = 1$

(D) $x(1 - y) = 1$

Ans. B

$$x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$$

$$= 1 - \tan^2\theta + \tan^4\theta - \tan^6\theta + \tan^8\theta - \tan^{10}\theta + \tan^{12}\theta - \tan^{14}\theta + \cdots$$

$$= (1-\tan^2\theta) + \tan^4\theta(1-\tan^2\theta) + \tan^8\theta(1-\tan^2\theta) + \tan^{12}\theta(1-\tan^2\theta) + \cdots$$

$$= (1-\tan^2\theta)(1+\tan^4\theta+\tan^8\theta+\tan^{12}\theta+\cdots)$$

$$= (1-\tan^2\theta)\left(\frac{1}{1-\tan^4\theta}\right)$$

$$x = \frac{1}{1+\tan^2\theta} = \frac{1}{\sec^2\theta} = \cos^2\theta$$

$$y = \sum_{n=0}^{\infty} \cos^{2n}\theta = 1 + \cos^2\theta + \cos^4\theta + \cos^6\theta + \cdots$$

$$= \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}$$

thus, $x(1+y) = \cos^2\theta\left(1+\frac{1}{\sin^2\theta}\right) \neq 1$

$$y(1-x) = \frac{1}{\sin^2\theta}(1-\cos^2\theta) = \frac{\sin^2\theta}{\sin^2\theta} = 1$$

(B) Ans.

Kritisha (W.B)

KTK-04

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$$x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta \quad \text{and } y = \sum_{n=0}^{\infty} \cos^{2n}\theta$$

$$\Rightarrow x = 1 + (-1)\tan^2\theta + (1)\tan^4\theta + (-1)\tan^6\theta + (1)\tan^8\theta + \ldots\ldots \infty$$

$$\Rightarrow x = 1 - \tan^2\theta + \tan^4\theta - \tan^6\theta + \tan^8\theta + \ldots\ldots \infty$$

$$\Rightarrow x = \frac{1}{1-(-\tan^2\theta)} = \frac{1}{1+\tan^2\theta} = \frac{1}{\sec^2\theta} = \cos^2\theta.$$

$$\Rightarrow y = 1 + \cos^2\theta + \cos^4\theta + \cos^6\theta + \ldots\ldots \infty$$

$$\Rightarrow y = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta} = \operatorname{cosec}^2\theta.$$

option (A). $x(1+y) = \cos^2\theta(1+\operatorname{cosec}^2\theta)$

$$= \frac{\cos^2\theta}{\sin^2\theta}(\sin^2\theta+1)$$

$$= \cot^2\theta(\sin^2\theta+1)$$

$$\neq \text{RHS}.$$

option (B) $y(1-x) = \operatorname{cosec}^2\theta(1-\cos^2\theta)$

$$= \operatorname{cosec}^2\theta(\sin^2\theta)$$

$$= \frac{1}{\sin^2\theta}\cdot\sin^2\theta$$

$$= 1 = \text{RHS}.$$

If $x = \sum_{n=0} (-1)^n \tan^{2n}\theta$ & $y = \sum_{n=0} \cos^{2n}\theta$,

for $0 < \theta < \frac{\pi}{4}$ then :

\# $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$

$$x = 1 - \tan^2\theta + \tan^4\theta \quad \tan^6\theta + --\cdot \infty$$

$$= \frac{1}{1-(-\tan^2\theta)} = \frac{1}{1+\tan^2\theta}$$

$$= \frac{1}{\sec^2\theta} = \cos^2\theta .$$

\# $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$

$$y = 1 + \cos^2\theta + \cos\theta + ----+ \infty$$

$$= \frac{1}{1-\cos^2\theta}$$

$$\Rightarrow \quad y = \frac{1}{1-x} \qquad \because \; x = \cos^2\theta$$

$$\Rightarrow \quad y - xy = 1$$

$$\Rightarrow \quad y(1-x) = 1 \qquad \text{Ans.}$$

QUESTION [JEE Mains 2024 (1 Feb)]

KTK 05

Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to:

(A) 800

(B) 890

(C) 790

(D) 690

Ans. C

$$S_{10} = 5(2a+9d) = 390$$

$$\boxed{2a+9d = 78}$$

Kritisha (W.B)

$$\frac{a_{10}}{a_5} = \frac{15}{7}$$

$$\frac{a+9d}{a+4d} = \frac{15}{7} \Rightarrow \left(\frac{a+9d+a+4d}{5d} = \frac{22}{8}\right)$$

$$\Rightarrow \frac{78+4d}{5d} = \frac{11}{4}$$

$$\Rightarrow 4(78+4d) = 55d$$

$$\Rightarrow 312 = (55-16)d$$

$$\Rightarrow \boxed{d = \frac{312}{39} = 8} \; ; \; 2a = 78-72$$

$$\boxed{a=3}$$

$$S_{15} - S_5 = \frac{15}{2}(2a+14d) - \frac{5}{2}(2a+4d)$$

$$= \frac{15}{2}(6+112) - \frac{5}{2}(6+32)$$

$$= \frac{15}{2}(118) - \frac{5}{2}(38)$$

$$= (15 \times 59) - (5 \times 19)$$

$$= 885 - 95 = 790 \text{ (c)}$$

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PW

KTK-05.

$S_{10} = 390$

$\Rightarrow \cancel{\frac{10}{2}}^{5}[2a+9d] = \cancel{390}\ 78$

$\Rightarrow 2a+9d = 78$ ——— (I)

$\frac{T_{10}}{T_5} = \frac{15}{7}$

$\Rightarrow \frac{a+9d}{a+4d} = \frac{15}{7}$

$\Rightarrow 7a+63d = 15a+60d$

$\Rightarrow 3d = 8a$

$\Rightarrow \boxed{2a = \frac{3d}{4}}$ ——— (II)

from (I) & (II)

$\frac{3d}{4} + 9d = 78$

$\cancel{39}d = \cancel{78}^{2}(4)$

$\boxed{d=8}$

$\boxed{a=3}$

Now, $S_{15} - S_5$

$\frac{15}{2}[2a+14d] - \frac{5}{2}[2a+4d]$

$\Rightarrow 15(a+7d) - 5(a+2d)$

$\Rightarrow 10a + 95d$

$\Rightarrow 10(3) + 95(8)$

$= 30 + 760 = 790.$

KTK-5. Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{10} = 390$ & the ratio of the tenth and the fifth term is 15 : 7, then $S_{15} - S_5$ is equal to :

$\Rightarrow S_{10} = 390$

$\Rightarrow \frac{10}{2}[2a + 9d] = 390$ (78)

$\Rightarrow 2a + 9d = 78$ — ①

$\Rightarrow \frac{a_{10}}{a_5} = \frac{15}{7}$

$\Rightarrow \frac{a+9d}{a+4d} = \frac{15}{7}$

$\Rightarrow 7a + 63d = 15a + 60d$

$\Rightarrow 8a = 3d \Rightarrow a = \frac{3d}{8}$.

$\Rightarrow 2\left(\frac{3d}{8}\right) + 9d = 78$

$\Rightarrow 3d + 36d = 78 \times 4$

$\Rightarrow 39d = 78 \times 4$

$d = 8$

$\Rightarrow a = \frac{3 \times 8}{8} \Rightarrow a = 3$

find : $S_{15} - S_5$

$\Rightarrow \frac{15}{2}[2a + 14d] - \frac{5}{2}[2a + 4d]$

$\Rightarrow 15a + 105d - 5a - 10d$

$\Rightarrow 10a + 95d$

$\Rightarrow 30 + 760$

$\Rightarrow 790$ Ans.

Solution to Previous RPPs

QUESTION

(RPP 1)

Paragraph

Consider the quadratic equation $2x^2 - (4m+2)x + m^2 + m = 0, m \in R$

1. The number of positive integer values of 'm' such that the equation has exactly one root in $(2, 3)$ is

(A) 3 (B) 4 (C) 5 (D) 6

2. The number of negative integral values of 'm' such that $m > -10$ and atleast one root of the equation is smaller than '2' is

(A) 8 (B) 9 (C) 6 (D) 4

Ans. (1) B, (2) B

QUESTION

(Challenger Problem (Answers Sahi hai)

If $ax^2 + bx + c = 0, a \neq 0, a, b, c \in R$ has two distinct real roots in $(1, 2)$ then

RPP 02

A (a) $(5a + 2b + c) > 0$

B (a) $(5a + 2b + c) < 0$

C $2a + b > 0$

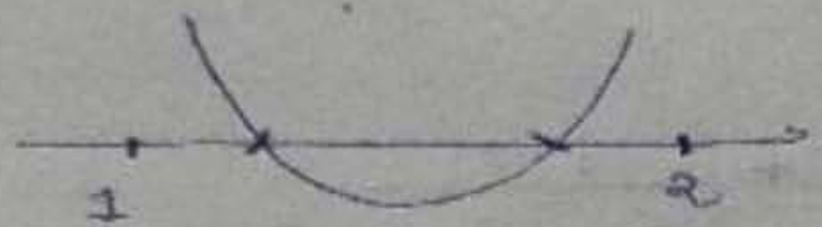
D (a) $(4a + 2b + c) > 0$

Ans. A, D

RPP-1

If $ax^2+bx+c=0$, $a \neq 0$, $a, b, c \in R$ has two distinct roots in $(1, 2)$ then

Sol^n



$1 < -\frac{b}{2a} < 2$ — (iii)

$a \cdot f(1) > 0$ — (i)

$a \cdot f(2) > 0$ — (ii)

$a \cdot f(2) > 0$

$a \cdot (4a+2b+c) > 0 \Rightarrow$ (D) True

$1 < \frac{-b}{2a} < 2$

$\Rightarrow 1 < -\frac{b}{2a}$ | $-\frac{b}{2a} < 2$

$2a+b < 0$ | $0 < 4a+b$

In option-A

(a) $(a+4a+2b+c)$

$a^2 + a(4a+2b+c)$

≥ 0 $\quad$ > 0

always ≥ 0

(A) & (D)


THANK
YOU