

PRAYAS

JEE 2026

Mathematics

Trigonometric
Functions

Lecture - 01

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Topics *to be covered*



- A** Exponential & Logarithmic Series
- B** Some special Telescoping series
- C** Introduction to Trigonometry

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Recap of previous lecture

1. If $a_1, a_2, \dots, a_n \in \mathbb{R}$ then $RMS = \sqrt{\frac{a_1^2 + a_2^2 + \dots + a_n^2}{n}}$ & AM $\frac{a_1 + a_2 + \dots + a_n}{n}$ also we have $\underline{RMS} \geq \underline{A.M}$ where equality holds if $\underline{a_1 = a_2 = \dots = a_n}$

2. If $a_1, a_2, a_3, \dots, a_n \in \mathbb{R}^+$ then arrange their arithmetic, geometric, harmonic means and RMS respectively, A, G, H, RMS in descending order $\underline{RMS \geq A \geq G \geq H}$

3. $8 \cos^2 x + 2 \sec^2 x$ has minimum value of 8

$$\frac{8 \cos^2 x + 2 \sec^2 x}{2} \geq \sqrt{8 \cos^2 x \cdot 2 \sec^2 x}$$

$$8 \cos^2 x + 2 \sec^2 x \geq 8$$

4. $x^2 + \frac{1}{x^2 + 1}$ has minimum value = 1

at $8 \cos^2 x = 2 \sec^2 x$

$$\cos^4 x = 1/4$$

$$\cos^2 x = 1/2$$

$\cos x = \pm \frac{1}{\sqrt{2}}$

@ $x = \pi/4, 3\pi/4, \dots$

$$y = x^2 + \frac{1}{x^2+1} = \underbrace{x^2+1 + \frac{1}{x^2+1}}_{\geq 2} - 1$$

$y_{\min} = 2 - 1 = 1$

@ $x^2+1=1$
 $x^2=0$
 $x=0$

if $x \in \mathbb{R}$

$x + \frac{1}{x} \geq 2$ min

@ $x=1$

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Recap

of previous lecture



5.

If $a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n \in \mathbb{R}$ then by C-S inequality we have _____

$(a_1b_1 + a_2b_2 + \dots + a_nb_n)^2 \leq (a_1^2 + a_2^2 + \dots + a_n^2) \cdot (b_1^2 + b_2^2 + \dots + b_n^2)$

6.

$\sum_{n=r}^{\infty} \frac{1}{(n-r)!} =$ _____

e

7.

$e^x =$ _____

$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

8.

$e^{-x} =$ _____

$1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$

9.

$e =$ _____

$1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$

10.

$e^{-1} =$ _____

$1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots$



Recap of previous lecture

$$11. \left(1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots \infty\right) = \frac{e+e^{-1}}{2}$$

$$12. \left(1 + \frac{1}{3!} + \frac{1}{5!} + \dots \infty\right) = \frac{e-e^{-1}}{2}$$

$$13. \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} = \left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots \infty\right) = \frac{e-e^{-1}}{2}$$

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Homework Discussion

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Exponential & Logarithmic Series

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QUESTION [JEE Mains 2023 (1 Feb)]



The sum $\sum_{n=1}^{\infty} \frac{2n^2 + 3n + 4}{(2n)!}$ is equal to :

- A

$\frac{11e}{2} + \frac{7}{2e}$
- ~~B~~

$\frac{13e}{4} + \frac{5}{4e} - 4$
- C

$\frac{11e}{2} + \frac{7}{2e} - 4$
- D

$\frac{13e}{4} + \frac{5}{4e}$

$(2n)! = 2n \cdot (2n-1)!$
 $(2n-1)! = (2n-1)(2n-2)!$

$T_n = \frac{2n^2 + 3n + 4}{(2n)!} = \frac{2n \cdot n}{(2n)!} + \frac{3}{2} \cdot \frac{2n}{(2n)!} + \frac{4}{(2n)!}$

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$= \frac{n}{(2n-1)!} + \frac{3}{2} \cdot \frac{1}{(2n-1)!} + \frac{4}{(2n)!}$

$= \frac{2n-1+1}{2(2n-1)!} + \frac{3}{2(2n-1)!} + \frac{4}{(2n)!}$

$= \frac{1}{2(2n-2)!} + \frac{1}{2(2n-1)!} + \frac{3}{2(2n-1)!} + \frac{4}{(2n)!}$

$= \frac{1}{2(2n-2)!} + 2 \cdot \frac{1}{(2n-1)!} + \frac{4}{(2n)!}$

$S = \sum_{n=1}^{\infty} \frac{1}{2(2n-2)!} + 2 \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} + 4 \sum_{n=1}^{\infty} \frac{1}{(2n)!}$



$$S = \frac{1}{2} \left(\frac{1}{0!} + \frac{1}{2!} + \frac{1}{4!} + \dots \infty \right) + 2 \left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots \infty \right) + 4 \left(\frac{1}{2!} + \frac{1}{4!} + \dots \infty \right)$$

$$= \frac{1}{2} \left(1 + \frac{1}{2!} + \frac{1}{4!} + \dots \infty \right) + 2 \left(1 + \frac{1}{3!} + \frac{1}{5!} + \dots \infty \right) + 4 \left(\frac{1}{2!} + \frac{1}{4!} + \dots \infty \right)$$

$$= \frac{e + e^{-1}}{4} + \frac{2(e - e^{-1})}{2} + 4 \left(\frac{e + e^{-1}}{2} - 1 \right)$$

$$= \frac{e}{4} + \frac{e^{-1}}{4} + \underline{e - e^{-1}} + \underline{2(e + e^{-1}) - 4}$$

$$= \frac{13e}{4} + \frac{5e^{-1}}{4} - 4$$

$$= \frac{13e}{4} + \frac{5}{4e} - 4$$

QUESTION [JEE Mains 2021 (Aug)]*Tahoi*

Let $S_n = 1 \cdot (n - 1) + 2 \cdot (n - 2) + 3 \cdot (n - 3) + \dots + (n - 1) \cdot 1, n \geq 4$.

The sum $\sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!} \right)$ is equal to:

A $\frac{e-1}{3}$

B $\frac{e-2}{6}$

C $\frac{e}{3}$

D $\frac{e}{6}$

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Logarithm Series

$$\star \log_e (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots - \infty, \quad -1 < x \leq 1$$

$$\star \log_e (1-x) = - \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots \infty \right) \quad -1 \leq x < 1$$

$$\star \log_e (1-x^2) = - \left(x^2 + \frac{x^4}{2} + \frac{x^6}{3} + \frac{x^8}{4} + \dots \infty \right) = -2 \left(\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \frac{x^8}{8} + \dots \infty \right)$$

$$\star \log_e \left(\frac{1+x}{1-x} \right) = 2 \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty \right) \quad (-1 < x < 1)$$



$$\log_e^2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots - \infty$$

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QUESTION [JEE Mains 2021 (Aug)]



If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 \dots$, is equal to :

- ☐ A $x \left(\frac{1+x}{1-x} \right) + \log_e(1-x)$
- ☒ B $x \left(\frac{1-x}{1+x} \right) + \log_e(1-x)$
- ☐ C $\frac{1-x}{1+x} + \log_e(1-x)$
- ☐ D $\frac{1+x}{1-x} + \log_e(1-x)$

$$T_n = \frac{(2n+1)x^{n+1}}{n+1}$$

$$= \frac{(2n+2-1)x^{n+1}}{n+1}$$

$$= \left(2 - \frac{1}{n+1} \right) x^{n+1} = 2 \cdot x^{n+1} - \frac{x^{n+1}}{n+1}$$

$$S = 2 \sum_{n=1}^{\infty} x^{n+1} - \sum_{n=1}^{\infty} \frac{x^{n+1}}{n+1}$$

$$= 2 \left(x^2 + x^3 + x^4 + \dots - \infty \right) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots - \infty \right)$$

$$= 2 \frac{x^2}{1-x} - \left(\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots - \infty \right) - x \right)$$



$$= \frac{2x^2}{1-x} - (-\log_e(1-x) - x)$$

$$= \frac{2x^2}{1-x} + x + \log_e(1-x)$$

$$= \frac{2x^2 + x - x^2}{1-x} + \log_e(1-x)$$

$$= \frac{x^2 + x}{1-x} + \log_e(1-x)$$

$$= x \frac{(1+x)}{1-x} + \log_e(1-x)$$

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QUESTION



Evaluate the sum of the series $\frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} - \frac{1}{4 \cdot 5} + \dots \infty$

$$T_n = \frac{(-1)^{n+1}}{n(n+1)}$$

$$T_n = (-1)^{n+1} \left(\frac{n+1-n}{(n+1)n} \right)$$

$$T_n = (-1)^{n+1} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

$$T_n = \frac{(-1)^{n+1}}{n} - \frac{(-1)^{n+1}}{n+1}$$

$$S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} - \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$$

$$= \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \infty \right) - \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \dots \infty \right)$$

$$= \log_e 2 + (-1/2 + 1/3 - 1/4 + 1/5 - 1/6 + \dots \infty)$$



$$S = \log_e 2 + \left(\underbrace{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots}_{\log_e 2} - 1 \right)$$

$$S = \log_e 2 + \log_e 2 - 1$$

$$S = 2\log_e 2 - 1$$

$$S = \log_e 4 - 1 = \log_e 4 - \log_e e$$

$$S = \log_e \left(\frac{4}{e} \right)$$

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QUESTION [JEE Mains 2024 (5 April)]

$$(\sqrt{3}-\sqrt{2})^3 = 3\sqrt{3}-2\sqrt{2}-3\cdot 3\cdot \sqrt{2}+3\cdot \sqrt{3} = 9\sqrt{3}-11\sqrt{2}$$



If $1 + \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}} + \frac{5-2\sqrt{6}}{18} + \frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}} + \frac{49-20\sqrt{6}}{180} + \dots$ upto $\infty = 2 + \left(\sqrt{\frac{b}{a}} + 1\right) \log_e \left(\frac{a}{b}\right)$, where a and b are integers with $\gcd(a, b) = 1$, then $11a + 18b$ is equal to

$$S = 1 + \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}} + \frac{(\sqrt{3}-\sqrt{2})^2}{18} + \frac{(\sqrt{3}-\sqrt{2})^3}{36\sqrt{3}} + \frac{(\sqrt{3}-\sqrt{2})^4}{180} + \dots \infty$$

$$= 1 + \frac{\sqrt{3}(1-\sqrt{\frac{2}{3}})}{2\sqrt{3}} + \frac{3(1-\sqrt{\frac{2}{3}})^2}{18} + \frac{3\sqrt{3}(1-\sqrt{\frac{2}{3}})^3}{36\sqrt{3}} + \frac{9(1-\sqrt{\frac{2}{3}})^4}{180} + \dots \infty$$

$$\text{Let } 1 - \sqrt{\frac{2}{3}} = x \Rightarrow 1 - x = \sqrt{\frac{2}{3}}$$

$$S = 1 + \frac{x}{2} + \frac{x^2}{6} + \frac{x^3}{12} + \frac{x^4}{20} + \dots \infty$$

$$= 1 + \left(\frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \frac{x^3}{3 \cdot 4} + \frac{x^4}{4 \cdot 5} + \dots \infty\right)$$

$$= 1 + S'$$

$$T_n = \frac{x^n}{n(n+1)}$$

Ans. 76



$$T_n = \frac{x^n}{n(n+1)} = x^n \left(\frac{n+1-n}{n(n+1)} \right)$$

$$T_n = x^n \left(\frac{1}{n} - \frac{1}{n+1} \right) = \frac{x^n}{n} - \frac{x^n}{n+1}$$

$$S' = \sum_{n=1}^{\infty} \frac{x^n}{n} - \sum_{n=1}^{\infty} \frac{x^n}{n+1}$$

$$= \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots - \infty \right) - \left(\frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + \dots - \infty \right)$$

$$= -\log_e(1-x) - \frac{1}{x} \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots - \infty \right)$$

$$= -\log_e(1-x) - \frac{1}{x} \left(\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots - \infty \right) - x \right)$$

$$= -\log_e(1-x) - \frac{1}{x} (-\log_e(1-x) - x)$$



$$s' = -\log_e(1-x) + \frac{1}{x} \cdot \log_e(1-x) + 1$$

$$= (\log_e(1-x)) \left(\frac{1}{x} - 1 \right) + 1$$

$$S = 1 + S'$$

$$S = 2 + \left(\frac{1}{x} - 1 \right) \log_e(1-x)$$

$$S = 2 + \frac{1-x}{x} \log_e(1-x)$$

$$= 2 + \frac{\sqrt{\frac{2}{3}}}{1-\sqrt{\frac{2}{3}}} \cdot \log_e\left(\sqrt{\frac{2}{3}}\right)$$

$$= 2 + \frac{1}{2} \left(\sqrt{\frac{2}{3}} \right) \left(\frac{1+\sqrt{2/3}}{1-2/3} \right) \log_e\left(\frac{2}{3}\right)$$

$$S = 2 + \frac{3}{2} \left(\frac{2}{3} + \sqrt{\frac{2}{3}} \right) \log_e \frac{2}{3}$$

$$= 2 + \left(1 + \sqrt{\frac{3}{2}} \right) \log_e\left(\frac{2}{3}\right)$$

$$a=2, b=3$$

$$11a + 18b$$

$$22 + 54 = 76 \text{ Ans}$$



Some More Problem Practice

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Kaam ki Baatien



let a, b be two +ve Reals.

$a, A_1, A_2, \dots, A_n, b$ are in A.P

$a, G_1, G_2, \dots, G_n, b$ are in G.P then ① $A_1 \geq G_1 \geq H_1$
 $A_2 \geq G_2 \geq H_2$

$a, H_1, H_2, \dots, H_n, b$ are in H.P.

$A_n \geq G_n \geq H_n$

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$$A_p = a + pd \quad T_{n+2} = b.$$

$$a + (n+1)d = b.$$

$$\textcircled{2} \quad ab = A_1 H_n = A_2 H_{n-1} = \dots = A_n H_1$$

$$A_p = a + \frac{(b-a)p}{n+1} \quad d = \frac{b-a}{n+1}$$

$$A_p = \frac{a(n+1) + (b-a)p}{(n+1)} = \frac{a(n+1-p) + pb}{n+1} \quad \textcircled{1}$$



$a, H_1, H_2, \dots, H_n, b$ are in H.P

$\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \dots, \frac{1}{H_n}, \frac{1}{b}$ are in A.P

$$T_{n+2} = \frac{1}{b} = \frac{1}{a} + (n+1)d'$$

$$\frac{1}{H_2} = \frac{1}{a} + 2 \cdot d'$$

$$\frac{1}{b} - \frac{1}{a} = (n+1)d'$$

$$= \frac{1}{a} + \frac{2 \cdot (a-b)}{ab(n+1)}$$

$$\frac{a-b}{ab(n+1)} = d'$$

$$= \frac{b(n+1) + 2a - 2b}{ab(n+1)}$$

$$\frac{1}{H_2} = \frac{b(n+1-2) + 2a}{ab(n+1)}$$

$$H_2 = \frac{ab(n+1)}{b(n+1-2) + 2a}$$

$$A_p H_q = \frac{(n+1-p)a + pb}{(n+1)} \cdot \frac{ab(n+1)}{(n+1-2)b + 2a}$$

$$A_1 H_n = \frac{na+b}{n+1} \cdot \frac{ab(n+1)}{b+na} = ab$$

QUESTION



Let n harmonic means $H_1, H_2, H_3, \dots, H_n$ and n arithmetic means $A_1, A_2, \dots, A_n$ be inserted between 25 & 40 such that

$$\sum_{i=1}^n \log_{10}(A_i H_i) = 2019. \text{ Find the value of } n.$$

$$25 \times 40 = A_1 H_n = A_2 H_{n-1} = \dots = A_n H_1$$

$$25, A_1, A_2, \dots, A_n, 40 \text{ are in A.P.}$$

$$25, H_1, H_2, \dots, H_n, 40 \text{ are in H.P.}$$

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$$\sum_{i=1}^n \log_{10}(A_i H_i) = 2019$$

$$\log_{10}(A_1 H_1) + \log_{10}(A_2 H_2) + \dots + \log_{10}(A_n H_n) = 2019$$

$$\log_{10}(A_1 H_1 A_2 H_2 \dots A_n H_n) = 2019$$

$$\underbrace{(A_1 H_1)}_{1000} \underbrace{(A_2 H_2)}_{1000} \dots \underbrace{(A_n H_n)}_{1000} = 10^{2019} \Rightarrow (1000)^n = 10^{2019} \Rightarrow 3n = 2019$$

$$n = 673.$$

Ans. 673

QUESTION

Tah02



Let $f(x) = (a^2 + b^2 - 4a - 6b + 13)(2x^2 - 4x + 5)$, $a, b, x \in \mathbb{R}$ such that $f(0) = f(1) = f(2)$.
If $a, A_1, A_2, \dots, A_{10}, b$ is an arithmetic progression and $a, H_1, H_2, \dots, H_{10}, b$ is harmonic progression then the value of

$$\frac{1}{10} \left(\sum_{i=4}^8 A_i H_{11-i} \right) \text{ is equal to}$$

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Ans. 3

QUESTION [JEE Advanced 2016 (Paper 2)]



Let $b_i > 1$ for $i = 1, 2, \dots, 101$. Suppose $\log_e b_1, \log_e b_2, \dots, \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, \dots, a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + \dots + b_{51}$ and $s = a_1 + a_2 + \dots + a_{51}$, then

- ☒ A $s > t$ and $a_{101} > b_{101}$
- ☒ B $s > t$ and $a_{101} < b_{101}$
- ☐ C $s < t$ and $a_{101} > b_{101}$
- ☐ D $s < t$ and $a_{101} < b_{101}$

$a_1, a_2, \dots, a_{50}, a_{51}, a_{52}, \dots, a_{101}$ A.P.
 $b_1, b_2, b_3, \dots, b_{50}, b_{51}, b_{52}, \dots, b_{101}$ G.P CR=2

$t = b_1 + b_2 + \dots + b_{50} + b_{51}$
 $s = a_1 + a_2 + \dots + a_{50} + a_{51}$
49 A.M
 $a_2 > b_2$
 $a_3 > b_3$
 $\vdots$
 $a_{50} > b_{50}$
 $\} s > t$

$a_1 + a_{101} = a_{51} + a_{51} = 2a_{51}$
 $b_1 \cdot b_{101} = b_{51} \cdot b_{51} = b_{51}^2$
 $b_{51} = G.M. b_1, b_{101}$
 $a_{51} = A.M. a_1, a_{101}$

Ans. B

QUESTION [IIT-JEE 2007]



a, b are +ve no.s
 $G^2 = AH$

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$. Let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

Which one of the following statements is correct?

- A

B

C

D

$A_1 > A_2 > A_3 > \dots$

$A_1 < A_2 < A_3 < \dots$

$A_1 > A_2 > A_3 > \dots$ and $A_1 < A_2 < A_3 < \dots$

$A_1 < A_2 < A_3 < \dots$ and $A_1 > A_2 > A_3 > \dots$

$A, P, Q, A, , b$
 $G, P, Q, G, , b$
 $H, P, Q, H, , b$

A_2, G_2, H_2 are A.M, G.M, H.M of $A, \& H,$

A_3, G_3, H_3 are A.M G.M & H.M of A_2, H_2

A_4, G_4, H_4 are A.M, G.M & H.M of A_3, H_3

Ans. A

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$a < b$

$a < H_1 < G_1 < A_1 < b$

$a > b$

$a > A_1 > G_1 > H_1 > b$

$H_1 < H_2 < G_2 < A_2 < A_1$
 $H_2 < H_3 < G_3 < A_3 < A_2$
 $H_3 < H_4 < G_4 < A_4 < A_3$

$A_1 > A_2 > G_2 > H_2 > H_1$
 $A_2 > A_3 > G_3 > H_3 > H_2$
 $A_3 > A_4 > G_4 > H_4 > H_3$

$A_1 > A_2 > A_3 \text{ --- } -$
 $H_1 < H_2 < H_3 \text{ --- } -$

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a, G_1, b
 a, A_1, b
 a, H_1, b

$G_1^2 = ab$
 $G_1^2 = A_1 H_1$
 $G_1^2 = A_1 H_1 = ab.$

A_1, G_2, H_1
 A_1, A_2, H_1
 A_1, H_2, H_1

$G_2^2 = A_1 H_1$
 $G_2^2 = A_2 H_2$
 $G_2^2 = A_1 H_1 = A_2 H_2 = ab.$

A_2, G_3, H_2
 A_2, A_3, H_2
 A_2, H_3, H_2

$G_3^2 = A_2 H_2$
 $G_3^2 = A_3 H_3$
 $G_3^2 = A_2 H_2 = ab = A_3 H_3$

QUESTION [IIT-JEE 2007]



Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$. Let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

Which one of the following statements is correct?

- A

$G_1 > G_2 > G_3 > \dots$
- B

$G_1 < G_2 < G_3 < \dots$
- C

$G_1 = G_2 = G_3 = \dots$
- D

$G_1 < G_2 < G_3 < \dots$ and $G_1 > G_2 > G_3 > \dots$

a, G_1, b
 $a, A_1 < b$
 $a, H_1 < b$
 $G^2 = A_1 H_1 = ab.$

A_1, G_2, H_1
 A_1, A_2, H_1
 A_1, H_2, H_1
 $G^2_2 = A_1 H_1 = G^2_1 = ab.$
 $G^2_2 = A_2 H_2$

A_2, G_3, H_2
 A_2, A_3, H_2
 $A_2, H_3 < H_2$
 $G^2_3 = A_2 H_2 = G^2_2$
 $G_1 = G_2 = G_3 = \dots$

Ans. C

QUESTION [IIT-JEE 2007]

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$. Let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

Which one of the following statements is correct?

A $H_1 > H_2 > H_3 > \dots$

~~**B** $H_1 < H_2 < H_3 < \dots$~~

C $H_1 > H_2 > H_3 > \dots$ and $H_1 < H_2 < H_3 < \dots$

D $H_1 < H_2 < H_3 < \dots$ and $H_1 > H_2 > H_3 > \dots$

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Ans. B

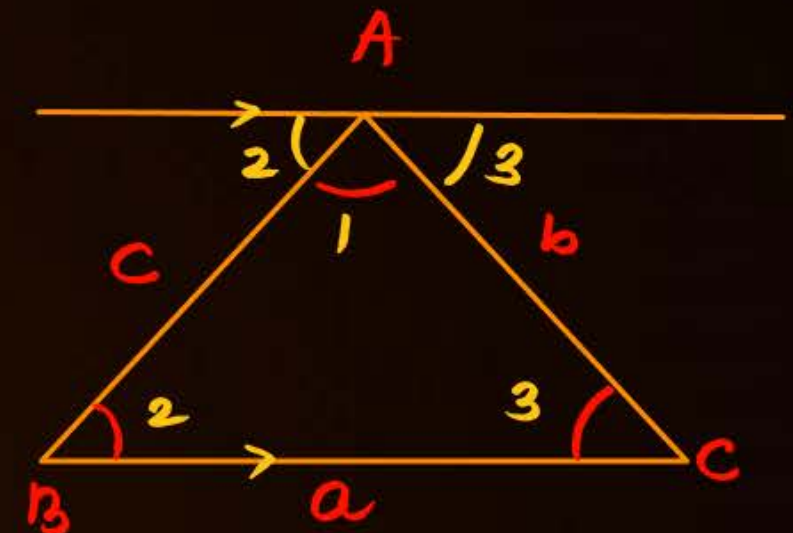
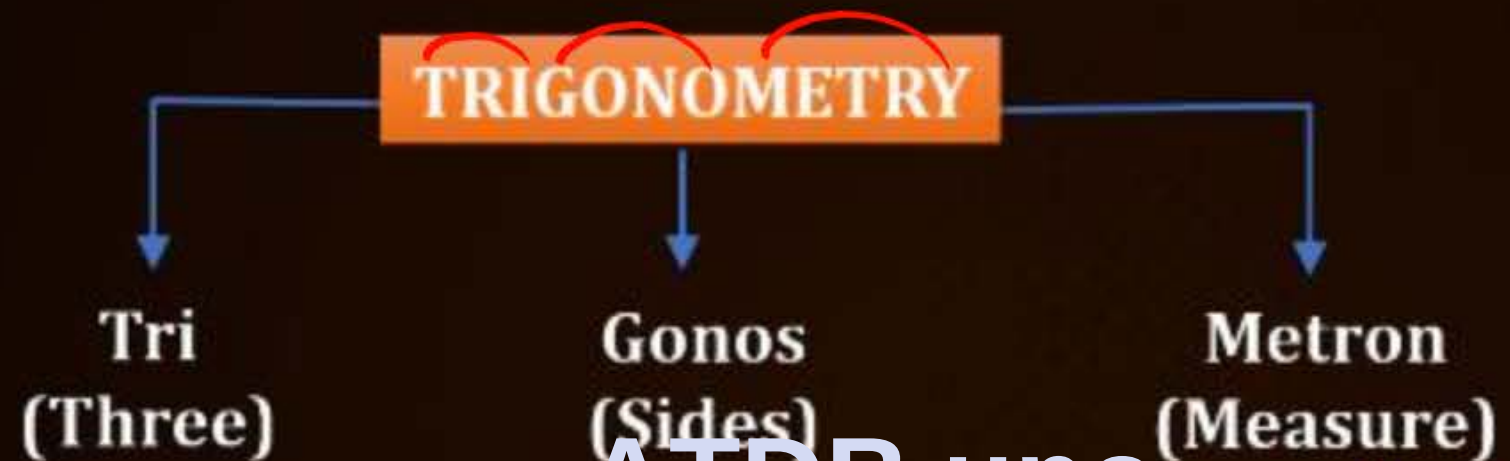


Trigonometry



Trigonometry

Trigonometry



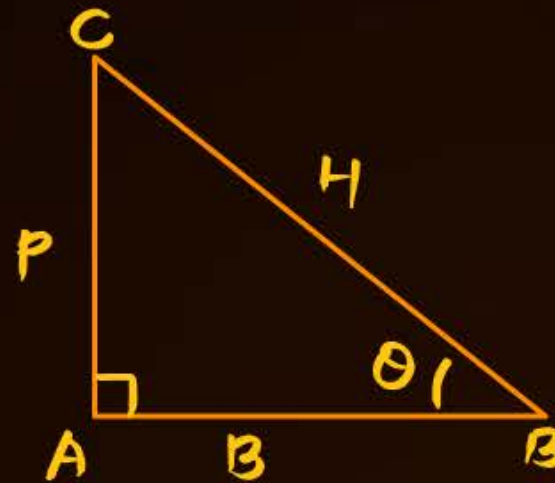
$$\angle A + \angle B + \angle C = 180^\circ$$

proof: $\angle 1 + \angle 2 + \angle 3 = 180^\circ$ (st. line)

Trigonometry is the branch of mathematics in which we study about triangle. Basically these are six parameters in a triangle (three sides & angles)



Bachpan KI Yaadien



S	C	T
P	B	P
H	H	B

Basic T-Ratios

$\sin \theta = \frac{P}{H}$

$\cos \theta = \frac{B}{H}$

$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{P}{B}$

$\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{B}{P}$

$\sec \theta = \frac{1}{\cos \theta} = \frac{H}{B}$

$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{H}{P}$

Derived T-Ratios

$\sin^2 \theta + \cos^2 \theta = \frac{P^2 + B^2}{H^2} = \frac{H^2}{H^2} = 1$

$1 + \tan^2 \theta = \sec^2 \theta$

$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$



$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\Rightarrow \sec^2 \theta - \tan^2 \theta = 1$$

$$(\sec \theta - \tan \theta)(\sec \theta + \tan \theta) = 1$$

$\Rightarrow \sec \theta - \tan \theta$ & $\sec \theta + \tan \theta$ are reciprocal of each other.

lly $\csc \theta - \cot \theta$ & $\csc \theta + \cot \theta$ are reciprocal of each other

QUESTION



Asking : Which of the following reduces to unity¹ for $0 < A < 90^\circ$

 $\tan^2 A$

~~A~~ $(\sec^2 A - 1) \cot^2 A$

~~B~~ $\cos A \operatorname{cosec} A \sqrt{\sec^2 A - 1} = \cos A \cdot \frac{1}{\sin A} \cdot \tan A = \cos A \cdot \frac{1}{\sin A} \cdot \frac{\sin A}{\cos A}$

~~C~~ $(\operatorname{cosec}^2 A - 1) \tan^2 A$

~~D~~ $(1 - \cos^2 A)(1 + \cot^2 A)$
 $\sin^2 A \cdot \operatorname{cosec}^2 A$

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QUESTION



If $(\sec \alpha + \tan \alpha)(\sec \beta + \tan \beta)(\sec \gamma + \tan \gamma) = \tan \alpha \tan \beta \tan \gamma$. Then value of $(\sec \alpha - \tan \alpha)(\sec \beta - \tan \beta)(\sec \gamma - \tan \gamma)$ equals to

- A

$\tan \alpha \tan \beta \tan \gamma$

$\frac{1}{(\sec \alpha + \tan \alpha)(\sec \beta + \tan \beta)(\sec \gamma + \tan \gamma)}$
- ~~B~~

$\cot \alpha \cot \beta \cot \gamma$

$= \frac{1}{\tan \alpha \cdot \tan \beta \cdot \tan \gamma}$
 $= \cot \alpha \cdot \cot \beta \cdot \cot \gamma$
- C

$\tan \alpha + \tan \beta + \tan \gamma$
- D

$\cot \alpha + \cot \beta + \cot \gamma$

QUESTION



The value of $\log_{\sin^2 x + \cos^4 x + 2}(\cos^2 x + \sin^4 x + 2)$ is equal to

~~A~~ 1

$$\log_{\sin^2 x + \cos^4 x + 2} (1 - \sin^2 x + (1 - \cos^2 x)^2 + 2)$$

B -1

$$= \log_{\sin^2 x + \cos^4 x + 2} (1 - \sin^2 x + 1 + \cos^4 x - 2 \cos^2 x + 2)$$

C 0

$$= \log_{\sin^2 x + \cos^4 x + 2} (4 + \cos^4 x - \sin^2 x - 2 \cos^2 x)$$

D 2

$$= \log_{\sin^2 x + \cos^4 x + 2} (2 + \cos^4 x + \sin^2 x)$$
$$= 1$$

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Values of T – Rations of some standard angles



	0°	30°	45°	60°	90°
sin θ	0	1/2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	Not Defined
cot θ	Not defined	√3	1	1/√3	0
sec θ	1	2/√3	√2	2	Not defined
cosec θ	Not defined	2	√2	2/√3	1

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Sabse Important Baat



Sabhi Class Illustrations Retry Karnay hai...

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QUESTION



Let $N = \frac{\overbrace{999\dots9}^{16 \text{ digits}} - \overbrace{5555\dots5}^{8 \text{ digits}}}{\underbrace{9000\dots04}_{9 \text{ digits}}}$, then sum of digits of N is

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Today's KTK



No Selection TRISHUL Selection with Good Rank
Apnao IIT Jao



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QUESTION**KTk 01**

The dimensions of a Cuboid are $a > b > c$. The volume = 216 and the total outer surface area = 252. If a, b, c are in G.P., then $c =$

A 3

B 1

C 5

D 2

ATDB.uno**Ans. A**

QUESTION

KTk 02



A ball falls from a height of 100 m on a floor. If in each rebound, it describes $(4/5)^{\text{th}}$ height of the previous falling height, then the total distance travelled by the ball before it comes to rest is?

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Ans. 900 m

QUESTION [JEE Mains 2020 (Jan)]

KTk 03



The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \dots$ to ∞ is equal to

A $2^{\frac{1}{4}}$

B $2^{\frac{1}{2}}$

C 1

D 2

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Ans. B

QUESTION [JEE Mains 2020 (Jan)]

KTK 04



If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n} \theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n} \theta$, for $0 < \theta < \frac{\pi}{4}$, then:

- A** $x(1 + y) = 1$
- B** $y(1 - x) = 1$
- C** $y(1 + x) = 1$
- D** $x(1 - y) = 1$

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Ans. B

QUESTION [JEE Mains 2024 (1 Feb)]

KTK 05



Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is $15 : 7$, then $S_{15} - S_5$ is equal to:

- A 800
- B 890
- C 790
- D 690

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Ans. C



Revision Practice Problems (RPP)

QUESTION

(RPP 1)



Paragraph

Consider the quadratic equation $2x^2 - (4m + 2)x + m^2 + m = 0, m \in \mathbb{R}$

1. The number of positive integer values of 'm' such that the equation has exactly one root in (2, 3) is

(A) 3

(B) 4

(C) 5

(D) 6

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2. The number of negative integral values of 'm' such that $m > -10$ and at least one root of the equation is smaller than '2' is

(A) 8

(B) 9

(C) 6

(D) 4

Ans. (1) B, (2) B

QUESTION

(Challenger Problem (Answers Sahi hai))



If $ax^2 + bx + c = 0, a \neq 0, a, b, c \in \mathbb{R}$ has two distinct real roots in $(1, 2)$ then

RPP 02

- A** (a) $(5a + 2b + c) > 0$
- B** (a) $(5a + 2b + c) < 0$
- C** $2a + b > 0$
- D** (a) $(4a + 2b + c) > 0$

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Ans. A, D



Solution to Previous TAH

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QUESTION [JEE Mains 2023 (11 April)]



Let a, b, c and d be positive real numbers such that $a + b + c + d = 11$. If the maximum value of $a^5 b^3 c^2 d$ is 3750β , then the value of β is

- A** 110
- B** 108
- C** 90
- D** 55

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Ans. C

Tan-01
[Mains-23]

$a+b+c+d=11$
 $a^5b^3c^2d=3750\beta$

$\frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2} + d$
11

$\frac{a}{5} \cdot \frac{a}{5} \cdot \frac{a}{5} \cdot \frac{a}{5} \cdot \frac{a}{5} \cdot \frac{b}{3} \cdot \frac{b}{3} \cdot \frac{b}{3} \cdot \frac{c}{2} \cdot \frac{c}{2} \cdot d$
GM

AM $\Rightarrow \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2} + d \Rightarrow \frac{11}{11} \Rightarrow 1$

GM $\Rightarrow \frac{a^5b^3c^2d}{(5)^5(3)^3(2)^2}$

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AM $\geq$ GM

$1 \geq \frac{a^5b^3c^2d}{(5)^5(3)^3(2)^2}$

Richathakur

$(5^5)(3)^3(2)^2 \geq 3750\beta$
 $5^2 \cdot 5^3 (3)^3 (2)^2 \geq 3750\beta$
 $\beta \leq \frac{5^5 \times 3^3 \times 2^2}{30 \times 10 \times 2}$

$\beta \leq 90$ (c)



Tah-01.

$$a+b+c+d=11$$

$$(a^5 b^3 c^2 d)_{\max} = 3750 \beta$$

Lucky Kumari

$$\Rightarrow \frac{\overset{AM \geq GM}{\frac{\frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2} + d}{11}}}{11} \geq \left(\frac{a^5 b^3 c^2 d}{5^5 3^3 2^2} \right)^{\frac{1}{11}}$$

$$\Rightarrow \frac{a+b+c+d}{11} \geq \left(\frac{a^5 b^3 c^2 d}{5^5 3^3 2^2} \right)^{\frac{1}{11}}$$

$$\Rightarrow \frac{11}{11} \geq \left(\frac{a^5 b^3 c^2 d}{5^5 3^3 2^2} \right)^{\frac{1}{11}}$$

$$\Rightarrow 1 \geq \frac{a^5 b^3 c^2 d}{5^5 3^3 2^2}$$

$$\Rightarrow a^5 b^3 c^2 d \leq 5^5 3^3 2^2$$

Now,

$$(a^5 b^3 c^2 d)_{\max} = 3750 \beta$$

$$\Rightarrow 5^5 (3^3)(2^2) = \cancel{5^4} \times \cancel{3^2} \times \cancel{2} \beta$$

$$\Rightarrow \beta = 3^2 \times 2 \times 5$$

$$\Rightarrow \beta = 90.$$

QUESTION



(a) If $x, y, z > 0$ and $x + y + z = 1$, prove that:

(i) $x^2yz \leq \frac{1}{64}$

(ii) $x^2 + y^2 + z^2 \geq \frac{1}{3}$

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Tah-02

Prove that $x^2yz \leq \frac{1}{64}$

$$x, y, z > 0$$

$$x + y + z = 1$$

AM

$$\left(\frac{x \cdot x \cdot y \cdot z}{2^2} \right)^{\frac{1}{4}}$$

$$\frac{\frac{x}{2} + \frac{x}{2} + y + z}{4} = \frac{1}{4}$$

$$AM = \frac{1}{4}$$

$$AM \geq GM$$

$$\frac{1}{4} \geq \left(\frac{x^2yz}{4} \right)^{\frac{1}{4}}$$

$$\left(\frac{1}{4} \right)^4 \geq \frac{x^2yz}{4}$$

$$\frac{1}{256} \geq \frac{x^2yz}{4}$$

$$\frac{1}{64} \geq x^2yz \quad \text{proved}$$

Richathakur



Tah-02

$$x, y, z > 0$$

$$x + y + z = 1$$

$$(i) \quad x^2 y z \leq \frac{1}{64}$$

$$\frac{x}{2} \quad AM \geq GM$$

$$\Rightarrow \frac{\frac{x}{2} + \frac{x}{2} + y + z}{4} \geq \left(\frac{x}{2} \cdot \frac{x}{2} \cdot y \cdot z \right)^{\frac{1}{4}}$$

$$\Rightarrow \frac{x + y + z}{4} \geq \left(\frac{x^2 y z}{4} \right)^{\frac{1}{4}}$$

$$\Rightarrow \left(\frac{1}{4} \right)^4 \geq \left(\frac{x^2 y z}{4} \right)^{\frac{1}{4} \times 4}$$

$$\Rightarrow \frac{1}{4^4} \geq \frac{x^2 y z}{4}$$

$$\Rightarrow x^2 y z \leq \frac{1}{4^3}$$

$$\Rightarrow x^2 y z \leq \frac{1}{64}$$

Lucky kumari



TAH $\rightarrow$ 02

$$x, y, z > 0 \quad \& \quad x + y + z = 1$$

AM $\geq$ GM

$$\frac{\frac{x}{2} + \frac{x}{2} + y + z}{4} \geq (x_{1/2} \cdot x_{1/2} \cdot y \cdot z)^{1/4}$$

$$\frac{1}{4} \geq (x_{1/4} y z)^{1/4}$$

$$\frac{x y z}{4} \leq \frac{1}{4^4}$$

$$x y z \leq \frac{1}{4^3}$$

$$x y z \leq \frac{1}{64} \quad (\text{Proved})$$



Tah-o2: $x, y, z > 0$, $x + y + z = 1$, Prove that:

$$x^2 y z \leq \frac{1}{64}.$$

$$\Rightarrow \frac{\frac{x}{2} + \frac{x}{2} + y + z}{4} \geq \left(\frac{x}{2} \cdot \frac{x}{2} \cdot y \cdot z \right)^{1/4}$$

$$\Rightarrow \frac{1}{4} \geq \left(\frac{x^2 y z}{4} \right)^{1/4}$$

$$\Rightarrow x^2 y z \leq \left(\frac{1}{4} \right)^4 \cdot 4$$

$$\Rightarrow x^2 y z \leq \frac{1}{4^4} \cdot 4$$

$$\Rightarrow x^2 y z \leq \frac{1}{64} \quad \text{Hence proved.}$$

krish

QUESTION



(b) If $a + b + c = 3$ and a, b, c are positive, then prove that $a^2b^3c^2 \frac{3^{10} \cdot 2^4}{7^7}$.

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12h-03

$$a+b+c=3$$

a, b, c are +ve

$$a^2 b^3 c^2 \leq \frac{3^{10} \cdot 2^4}{7^7}$$

$$\frac{\frac{a}{2} + \frac{a}{2} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2}}{7} = 3$$

7

$$AM = \frac{3}{7}$$

$$GM = \left[\frac{a^2 b^3 c^2}{2^2 \cdot 3^3 \cdot 2^2} \right]^{\frac{1}{7}}$$

$$AM \geq GM$$

$$\frac{3}{7} \geq \left[\frac{a^2 b^3 c^2}{2^2 \cdot 3^3 \cdot 2^4} \right]^{\frac{1}{7}}$$

$$\left(\frac{3}{7} \right)^7 \geq \frac{a^2 b^3 c^2}{2^4 \cdot 3^3}$$

$$\frac{3^7 \cdot 2^4 \cdot 3^3}{7^7} \geq a^2 b^3 c^3$$

$$\frac{3^{10} \cdot 2^4}{7^7} \geq a^2 b^3 c^3$$

proved

Richathakur



TAH $\Rightarrow$ 03

$$a, b, c > 0$$

$$a + b + c = 3$$

AM $\geq$ GM

$$\frac{\frac{a}{2} + \frac{a}{2} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2}}{7} \geq (a^{1/2} b^{3/3} c^{1/2})^{1/7}$$

$$\frac{3}{7} \geq \left(\frac{a^{1/2} b^{3/3} c^{1/2}}{2^4 3^3} \right)^{1/7}$$

$$\frac{3^7}{7^7} \geq \frac{a^{1/2} b^{3/3} c^{1/2}}{2^4 3^3}$$

$$a^{1/2} b^{3/3} c^{1/2} \leq \frac{3^{10} 2^4}{7^7}$$

(Proved)

1ah-03

Lucky Kumari



$$a+b+c=3 \quad ; (a,b,c) > 0$$

To prove, $a^2 b^3 c^2 \leq \frac{3^{10} \cdot 2^4}{7^7}$

$$\Rightarrow \frac{\frac{a}{2} + \frac{a}{2} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2}}{7} \geq \left(\frac{a^2 b^3 c^2}{2^4 3^3} \right)^{1/7}$$

$$\Rightarrow \left(\frac{a+b+c}{7} \right)^7 \geq \left(\frac{a^2 b^3 c^2}{2^4 3^3} \right)^{1/7 \times 7}$$

$$\Rightarrow \left(\frac{3}{7} \right)^7 \geq \frac{a^2 b^3 c^2}{2^4 3^3}$$

$$\Rightarrow a^2 b^3 c^2 \leq \frac{3^9 \cdot 2 \cdot 3}{7^7}$$

$$\Rightarrow a^2 b^3 c^2 \leq \underline{\underline{\frac{3^{10} \cdot 2}{7^7}}}$$

QUESTION



If $a_i < 0$ for all $i = 1, 2, \dots, n$ prove that

(ii) $(1 - a_1 + a_1^2)(1 - a_2 + a_2^2) \cdots (1 - a_n + a_n^2) \geq 3^n(a_1 a_2 \dots a_n)$ (where n is even)

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Tāh-04

$a_i < 0$; all $i = 1, 2, \dots, n$

$n \rightarrow \text{even}$

Prove that

$$(1 - a_1 + a_1^2)(1 - a_2 + a_2^2) \dots (1 - a_n + a_n^2) \geq 3^n (a_1 a_2 \dots a_n)$$

$$\frac{1 + (-a_1) + a_1^2}{3} \geq [(1(-a_1)(a_1)^2)]^{1/3}$$

Richathakur

$$\frac{1 - a_1 + a_1^2}{3} \geq -a_1$$

$$[-a_1^3]^{1/3}$$

$$i=1 \Rightarrow \frac{1 - a_1 + a_1^2}{3} \geq -a_1$$

$$i=2 \Rightarrow \frac{1 - a_2 + a_2^2}{3} \geq -a_2$$

$$i=3 \Rightarrow \frac{1 - a_3 + a_3^2}{3} \geq -a_3$$

$$\vdots$$

$$i=n \Rightarrow \frac{1 - a_n + a_n^2}{3} \geq -a_n$$

$$\frac{(1 - a_1 + a_1^2)(1 - a_2 + a_2^2) \dots (1 - a_n + a_n^2)}{3^n} \geq (-1)^n (a_1 a_2 \dots a_n)$$

$$(1 - a_1 + a_1^2)(1 - a_2 + a_2^2) \dots (1 - a_n + a_n^2) \geq 3^n (a_1 a_2 \dots a_n)$$

Proved



Tah-04: If $a_i < 0$ for all $i = 1, 2, \dots, n$ prove that
 $(1 - a_1 + a_1^2)(1 - a_2 + a_2^2) \dots (1 - a_n + a_n^2) \geq 3^n (a_1 a_2 \dots a_n)$
 (where n is even.)

$$\Rightarrow \frac{1 + (-a_i) + a_i^2}{3} \geq (1 \cdot (-a_i) \cdot a_i^2)^{1/3}$$

$$\text{"} \geq ((-a_i)^3)^{1/3} = -a_i$$

$$\Rightarrow i=1; \quad \frac{1 - a_1 + a_1^2}{3} \geq -a_1$$

$$\Rightarrow i=2; \quad \frac{1 - a_2 + a_2^2}{3} \geq -a_2$$

$\vdots$

$$\Rightarrow i=n; \quad \frac{1 - a_n + a_n^2}{3} \geq -a_n$$

n is even

$$\Rightarrow (1 - a_1 + a_1^2)(1 - a_2 + a_2^2) \dots (1 - a_n + a_n^2) \geq (-1)^n 3^n (a_1 a_2 \dots a_n)$$

$$\Rightarrow (1 - a_1 + a_1^2)(1 - a_2 + a_2^2) \dots (1 - a_n + a_n^2) \geq 3^n (a_1 a_2 \dots a_n)$$

Hence proved.

krish



$$a_i < 0 \Rightarrow -a_i > 0$$

$$AM \geq GM$$

$$\Rightarrow \frac{1 + (-a_1) + (-a_1)^2}{3} \geq (1 \cdot (-a_1) \cdot (-a_1)^2)^{1/3}$$

$$\Rightarrow 1 - a_1 + a_1^2 \geq 3(-a_1)^{3 \times 1/3}$$

$$\Rightarrow 1 - a_1 + a_1^2 \geq -3a_1 \text{ --- (I)}$$

Similarly,

$$\Rightarrow 1 - a_2 + a_2^2 \geq -3a_2 \text{ --- (II)}$$

$$\Rightarrow 1 - a_3 + a_3^2 \geq -3a_3 \text{ --- (III)}$$

$$\vdots$$

$$\Rightarrow 1 - a_n + a_n^2 \geq -3a_n \text{ --- (IV)}$$

Multiply all the equations :-

$$\Rightarrow (1 - a_1 + a_1^2) \cdot (1 - a_2 + a_2^2) \cdot (1 - a_3 + a_3^2) \cdot \dots \cdot (1 - a_n + a_n^2)$$

$$\geq +3^n (a_1 \cdot a_2 \cdot a_3 \cdot \dots \cdot a_n)$$

(where, n is even).

$$\Rightarrow (1 - a_1 + a_1^2) \cdot (1 - a_2 + a_2^2) \cdot (1 - a_3 + a_3^2) \cdot \dots \cdot (1 - a_n + a_n^2)$$

$$\geq 3^n (a_1 \cdot a_2 \cdot a_3 \cdot \dots \cdot a_n)$$

Proved.

Lucky kumari

QUESTION

Find the sum of following series :

$$(i) \quad S = \sum_{n=1}^{100} n(n!)$$

$$(ii) \quad S = \sum_{n=1}^{50} \frac{n}{(n+1)!}$$

$$(iii) \quad S = \sum_{r=1}^{100} (r^2 + 1) \cdot r!$$

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Tan-05

$$S = \sum_{r=1}^{100} (r^2 + 1) \cdot r!$$

$$\downarrow$$

$$[r(r+1) - (r-1)] \cdot r!$$

$$r(r+1)! - (r-1)r!$$

$$r(r+1)! - (r-1)r!$$

$$T_1 = 1 \cdot 2! - 0 \cdot 1!$$

$$T_2 = 2 \cdot 3! - 1 \cdot 2!$$

$$T_3 = 3 \cdot 4! - 2 \cdot 3!$$

$\vdots$

$$T_{100} = 100 \cdot 101! - 99 \cdot 100!$$

$$S = 100 \cdot 101!$$

$$S = 100 \cdot 101! - 0$$

Richathakur

144-05

$$\begin{aligned} \text{iii)} \quad S &= \sum_{r=1}^{100} (r^2 + 1) \cdot r! \\ &= \sum_{r=1}^{100} (r^2 + 2r + 1 - 2r) r! \\ &= \sum_{r=1}^{100} \{(r+1)^2 - 2r\} r! \\ &= \sum_{r=1}^{100} \{(r+1)^2 r! - 2r r!\} \\ &= \sum_{r=1}^{100} \left[\{(r+1)(r+1)!\} - 2\{(r+1-1)r!\} \right] \\ &= \sum_{r=1}^{100} \left(\{(r+2-1)(r+1)!\} - 2\{(r+1)!\} - r! \right) \\ &= \sum_{r=1}^{100} \left(\{(r+2)!\} - (r+1)!\} - 2\{(r+1)!\} - r! \right) \\ &= \sum_{r=1}^{100} \left((r+2)!\} - (r+1)!\} \right) - \sum_{r=1}^{100} 2((r+1)!\} - r! \\ &= \begin{aligned} & \left(\begin{aligned} & (3! - 2!) \\ & + (4! - 3!) \\ & + (5! - 4!) \\ & \vdots \\ & + (102! - 101!) \end{aligned} \right) \\ & - 2 \left(\begin{aligned} & (2! - 1!) \\ & + (3! - 2!) \\ & + (4! - 3!) \\ & \vdots \\ & + (101! - 100!) \end{aligned} \right) \end{aligned} \\ &= (102! - 2!) - 2(101! - 1!) = 100(101!) \\ &= 102! - 2! - 2(101)! + 2! = 102! - 2(101)! \end{aligned}$$

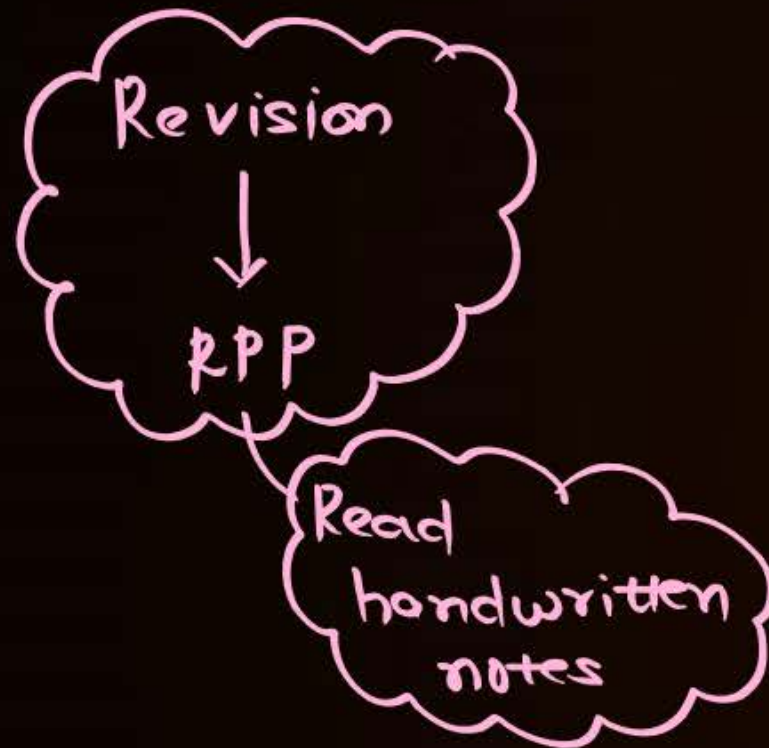
144-05

Lucky kumari

$$\begin{aligned} S &= \sum_{r=1}^{100} (r^2 + 1)(r!) \\ \Rightarrow T_r &= (r^2 + 1)(r!) \\ \Rightarrow T_r &= ((r+1)^2 - 2r)(r!) \\ \Rightarrow T_r &= (r+1)^2 r! - 2r r! \\ \Rightarrow T_r &= (r+1)(r+1)(r!) - r r! - r r! \\ \Rightarrow T_r &= (r+1)(r+1)! - r(r!) - ((r+1)-1)r! \\ \Rightarrow T_r &= \{(r+1)(r+1)! - r(r!)\} - \{(r+1)r! - r!\} \\ \Rightarrow T_r &= \{(r+1)(r+1)! - r(r!)\} - \{(r+1)! - r!\} \end{aligned}$$
$$\begin{aligned} T_1 &= \{2 \cdot 2! - 1 \cdot 1!\} - \{2! - 1!\} \\ T_2 &= \{3 \cdot 3! - 2 \cdot 2!\} - \{3! - 2!\} \\ T_3 &= \{4 \cdot 4! - 3 \cdot 3!\} - \{4! - 3!\} \\ &\vdots \\ T_r &= \{(r+1)(r+1)! - r r!\} - \{(r+1)! - r!\} \end{aligned}$$
$$\begin{aligned} \Rightarrow T_1 + T_2 + T_3 + \dots + T_r &= \{(r+1)(r+1)! - r!\} - \{(r+1)! - r!\} \\ \Rightarrow T_1 + T_2 + T_3 + \dots + T_{100} &= \{(101)(101)! - (101)!\} \\ \Rightarrow S &= (101)!(101-1) \\ \Rightarrow S &= (101)!(100) \\ \Rightarrow S &= 100(101!) \end{aligned}$$



मन की बात Ashish Sir के साथ



Question solve nahi hotay??

- Class karayay hue swaal
- KTK / TAH solutions se HINT lekar

ATDB.uno

Backlog Kaa soln??

- Study from class notes.
- Don't miss Live lectures
- Isi chapter mai Beech mai mat kudo!!



THANK
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YOU