

CHAPTER

13



Straight Lines

❖ **Distance Formula:** $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

❖ **Section Formula:** $x = \frac{mx_2 \pm nx_1}{m \pm n}$; $y = \frac{my_2 \pm ny_1}{m \pm n}$

❖ **Centroid, Incentre & Excentre:**

Centroid $G \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$,

Incentre $I \left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c} \right)$

Excentre $I_1 \left(\frac{-ax_1 + bx_2 + cx_3}{-a + b + c}, \frac{-ay_1 + by_2 + cy_3}{-a + b + c} \right)$

Remarks:

- (i) If the triangle is equilateral, then centroid, incentre, orthocenter, circumcenter, coincides.
- (ii) Orthocentre, centroid and circumcentre are always collinear and centroid divides the line joining. Orthocentre and circumcentre in the ratio 2 : 1.
- (ii) In a isosceles triangle centroid, orthocentre, incentre, circumcentre lies on the same line.

Area of Triangle

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of a triangle, then

$$\text{Area of } \triangle ABC = \frac{1}{2} \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$$

Equation of Straight Line

- (a) Equation of a line parallel to x-axis at a distance a is $y = a$ or $y = -a$.
- (b) Equation of x-axis is $y = 0$.
- (c) Equation of line parallel to y-axis at a distance b is $x = b$ or $x = -b$.
- (d) Equation of y-axis is $x = 0$.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ & $x_1 \neq x_2$ then slope of line $AB = \frac{y_2 - y_1}{x_2 - x_1}$.

Standard Forms of Equations of a Straight Line

- (a) **Slope Intercept form :** Let m be the slope of a line and c its intercept on y-axis, then the equation of this straight line is written as : $y = mx + c$.
- (b) **Point Slope form :** If m be the slope of a line and it passes through a point (x_1, y_1) , then its equation is written as : $y - y_1 = m(x - x_1)$.
- (c) **Two point form :** Equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is written as :

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ or } \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

- (d) **Intercept form :** If a and b are the intercepts made by a line on the axes of x and y , its equation is written as : $\frac{x}{a} + \frac{y}{b} = 1$.
- (e) **Normal form :** If p is the length of perpendicular on a line from the origin and α the angle which this perpendicular makes with positive x -axis, then the equation of this line is written as : $x \cos \alpha + y \sin \alpha = p$ (p is always positive), where $0 \leq \alpha < 2\pi$.

(f) **Parametric form :** $\frac{x - h}{\cos \theta} = \frac{y - k}{\sin \theta} = r$ is the equation.

(g) **General form :** We know that a first degree equation in x and y , $ax + by + c = 0$ always represents a straight line. This form is known as general form of straight line.

(i) Slope of this line = $\frac{-a}{b} = \frac{\text{coefficient of } x}{\text{coefficient of } y}$

(ii) Intercept by this line on x-axis = $-\frac{c}{a}$ and intercept by

this line on y-axis = $-\frac{c}{b}$.

(iii) To change the general form of a line to normal form, first take c to right hand side and make it positive, then divide the whole equation by $\sqrt{a^2 + b^2}$.

Angle Between Two Lines

(a) If θ be the angle between two lines : $y = m_1x + c_1$ and $y = m_2x + c_2$, then $\tan \theta = \pm \left(\frac{m_1 - m_2}{1 + m_1m_2} \right)$.

(b) If equation of lines are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then these line are-

(i) Parallel $\Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(ii) Perpendicular $\Leftrightarrow a_1a_2 + b_1b_2 = 0$

(iii) Coincident $\Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(iv) Intersecting $\Leftrightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Length of Perpendicular from a Point on a Line

Length of perpendicular from a point (x_1, y_1) on the line $ax + by + c = 0$ is

$$= \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

In particular the length of the perpendicular from origin with the

line $ax + by + c = 0$ is $P = \frac{|c|}{\sqrt{a^2 + b^2}}$.

Distance Between two Parallel Lines

(i) The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$.

(Note : The coefficients of x & y in both equations should be same).

(ii) The area of the parallelogram = $\frac{p_1 p_2}{\sin \theta}$, where p_1 & p_2 are distance between two pairs of opposite sides & θ is the

angle between any two adjacent sides. Note that area of the parallelogram bounded by the lines $y = m_1x + c_1, y = m_1x + c_2$

and $y = m_2x + d_1, y = m_2x + d_2$ is given $\left| \frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2} \right|$.

Equation of lines Parallel and Perpendicular to a Given Line

(i) Equation of line parallel to line $ax + by + c = 0$.

$$ax + by + \lambda = 0$$

(ii) Equation of line perpendicular to line $ax + by + c = 0$.

$$bx - ay + k = 0$$

Here λ, k , are parameters and their values are obtained with the help of additional information given in the problem.

Straight Line Making a given Angle with a Line

Equations of lines passing through a point (x_1, y_1) and making an angle α , with the line $y = mx + c$ is written as :

$$y - y_1 = \frac{m \pm \tan \alpha}{1 \mp m \tan \alpha} (x - x_1)$$

Position of Two Points with Respect to a Given Line

Let the given line be $ax + by + c = 0$ and $P(x_1, y_1), Q(x_2, y_2)$ be two points. If the quantities $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have the same signs, then both the points P and Q lie on the same side of the $ax + by + c = 0$. If the quantities $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have opposite signs, then they lie on the opposite sides of the line.

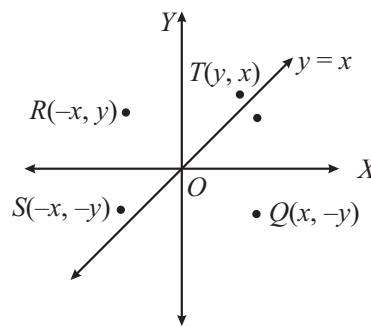
Concurrency of Lines

Three lines $a_1x + b_1y + c_1 = 0; a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y +$

$c_3 = 0$ are concurrent, if $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

Reflection of a Point

Let $P(x, y)$ be any point, then its image with respect to



(i) x -axis is $Q(x, -y)$

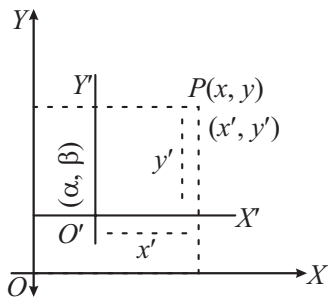
(ii) y -axis is $R(-x, y)$

(iii) origin is $S(-x, -y)$

(iv) line $y = x$ is $T(y, x)$

Transformation of Axes

(a) **Shifting of origin without rotation of axes :** If coordinates of any point $P(x, y)$ with respect to new origin (a, b) will be (x', y')

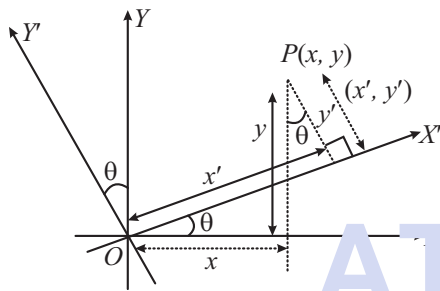


then $x = x' + \alpha, \quad y = y' + \beta$

or $x' = x - \alpha, \quad y' = y - \beta$

Thus if origin is shifted to point (α, β) without rotation of axes, then new equation of curve can be obtained by putting $x + \alpha$ in place of x and $y + \beta$ in place of y .

(b) Rotation of axes without shifting the origin : Let O be the origin. Let $P \equiv (x, y)$ with respect to axes OX and OY and let $P \equiv (x', y')$ with respect to axes OX' and OY' , where $\angle X'OX = \angle YOY' = \theta$



then $x = x' \cos \theta - y' \sin \theta$

$y = x' \sin \theta + y' \cos \theta$

or $y' = x \cos \theta + y \sin \theta$

$x' = -x \sin \theta + y \cos \theta$

The above relation between (x, y) and (x', y') can be easily obtained with the help of following table

New \ Old	$x \downarrow$	$y \downarrow$
$x' \rightarrow$	$\cos \theta$	$\sin \theta$
$y' \rightarrow$	$-\sin \theta$	$\cos \theta$

Equation of Bisectors of Angles between Two Lines

If equation of two intersecting lines are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then equation of bisectors of the angles between these lines are written are:

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}} \quad \dots(1)$$

(a) Equation of bisector of angle containing origin : If the equation of the lines are written with constant terms c_1 and c_2 positive, then the equation of bisectors of the angle containing the origin is obtained by taking sign in (1).

(b) Equation of bisector of acute/obtuse angles : See whether the constant terms c_1 and c_2 in the two equation are +ve or not. If not then multiply both sides of given equation by -1 to make the constant terms positive.

Determine the sign of $a_1a_2 + b_1b_2$

If sign of $a_1a_2 + b_1b_2$	For obtuse angle bisector	For acute angle bisector
+	use + sign in eq. (1)	use - sign in eq. (1)
-	use - sign in eq. (1)	use + sign in eq. (1)

i.e. if $a_1a_2 + b_1b_2 > 0$, then the bisector corresponding to + sign gives obtuse angle bisector

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Family of Lines

If equation of two lines be $P = a_1x + b_1y + c_1 = 0$ and $Q \equiv a_2x + b_2y + c_2 = 0$, then the equation of the lines passing through the point of intersection of these lines is : $P + \lambda Q = 0$ or $a_1x + b_1y + c_1 + \lambda(a_2x + b_2y + c_2) = 0$. The value of λ is obtained with the help of the additional information given in the problem.

General Equation and Homogeneous Equation of Second Degree

(a) A general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a pair of straight lines if $\Delta = abc + 2fgh - af^2 - bg^2 - c^2 = 0$ or $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

(b) If θ be the angle between the lines, then $\tan \theta = \pm \frac{2\sqrt{h^2 - ab}}{a + b}$.

Obviously these lines are

- (i)** Parallel, if $\Delta = 0, h^2 = ab$ or if $h^2 = ab$ and $bg^2 = af^2$.
- (ii)** Perpendicular, if $a + b = 0$ i.e. coeff. of $x^2 +$ coeff. of $y^2 = 0$.

(c) Homogeneous equation of 2nd degree $ax^2 + 2hxy + by^2 = 0$ always represent a pair of straight lines whose equations are

$$y = \left(\frac{-h \pm \sqrt{h^2 - ab}}{b} \right) x \equiv y = m_1x \text{ and } y = m_2x$$

and $m_1 + m_2 = -\frac{2h}{b}; m_1m_2 = \frac{a}{b}$

These straight lines passes through the origin and for finding the angle between these lines same formula as given for general equation is used.

The condition that these lines are :

- (i)** At right angles to each other is $a + b = 0$. i.e. co-efficient of $x^2 +$ co-efficient of $y^2 = 0$.

(ii) Coincident is $h^2 = ab$.

(iii) Equally inclined to the axis of x is $h = 0$. i.e. coefficient of $xy = 0$.

(d) The combined equation of angle bisectors between the lines represented by homogeneous equation of 2nd degree is given by

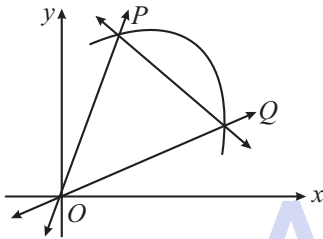
$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}, a \neq b, h \neq 0.$$

(e) Pair of straight lines perpendicular to the lines $ax^2 + 2hxy + by^2 = 0$ and through origin are given by $bx^2 - 2hxy + ay^2 = 0$.

(f) If lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ are parallel then distance between them is $= 2 \sqrt{\frac{g^2 - ac}{a(a + b)}}$.

Equations of Lines Joining the Points of Intersection of a Line and a Curve to the Origin

Let the equation of curve be:



$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \dots(i)$$

and straight line be

$$lx + my + n = 0 \quad \dots(ii)$$

$$ax^2 + 2hxy + by^2 + 2(gx + fy) \left(\frac{lx + my}{-n} \right) + c \left(\frac{lx + my}{-n} \right)^2 = 0$$

STANDARD RESULTS

(i) Area of rhombus formed by lines $a|x| + b|y| + c = 0$

$$\text{or } \pm ax \pm by + c = 0 \text{ is } \frac{2c^2}{|ab|}.$$

(ii) Area of triangle formed by line $ax + by + c = 0$ and axes is $\frac{c^2}{2|ab|}$.

(iii) Co-ordinate of foot of perpendicular (h, k) from (x_1, y_1) to the line $ax + by + c = 0$ is given by $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$.

(iv) Image of point (x_1, y_1) w.r. to the line $ax + by + c = 0$ is given by

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}.$$

Chart

