CHAPTER 2

Sets

Set

A set is a well-defined collection of objects. A Collection is said to be well-defined when there is no ambiguity regarding inclusion and exclusion of the object and all objects have same common properties. Each object of a set is called an element of a set.

Methods of Representing a Set

(*i*) Roster or Tabular Form: In this form, a set is described by listing elements, separated by commas, within braces {}.

(*ii*) Set-builder Form: In this form, a set is described by a characterizing property $P(x)$ of its elements x. In such a case, the set is described by $\{x: P(x) \text{ holds}\}$, which is read as 'the set of all x such that $P(x)$ holds'.

Types of Sets

(*i*) Empty Set: A set having no element in it is called an empty set.

(*ii*) Singleton Set : A set Containing one element is called a singleton set.

(*iii*) Finite Set : A set having fixed no. of elements is called a finite set.

(*iv*) Infinite Set : A set that is not finite is infinite set.

(*v*) Equal sets: Two sets A and B are said to be equal if every element of A is a member of B and Vice-Versa.

Subsets

A set A is said to be a subset of a set B if every element of A is also an element of B. i.e., $A \subset B$ if $a \in A \Rightarrow a \in B$

Note that:

(*i*) Every set is a subset of itself.

(*ii*) Empty set ϕ is a subset of every set.

Intervals as Subsets of R

Let $a, b \in R$ and $a < b$, then

(*i*) Closed Interval

$[a, b] = \{x \in R : a \leq x \leq b\}$

(*ii*) Open Interval

$(a, b) = \{x \in R : a < x < b\}$

(*iii*) Semi-open or Semi-closed Interval

$(a, b] = \{x \in R : a < x \leq b\}$ and $[a, b) = \{x \in R : a \leq x < b\}$

Power Set

The collection of all subsets of set A is called the power set of A. It is denoted by $P(A)$. Every element in $P(A)$ is a set. Note that if A is a finite set having n elements, then $P(A)$ has 2^n elements.

Universal Set

It is a set which includes all the elements of the sets under consideration. It is denoted by U. Eg., if $A = \{1, 2, 3\}$, $B = \{3, 4, 7\}$ and $C = \{2, 8, 9\}$, then $U = \{1, 2, 3, 4, 7, 8, 9\}$

Venn Diagrams

Most of the relationships between sets can be represented by means of diagrams which are known as Venn diagrams.

Operations on Sets

Union of Sets: The union of two sets A and B is the set of all those elements which are either in A or in B. It is denoted by $A \cup B$.

Properties of the Operation of Union

(*i*) $A \cup B = B \cup A$ (Commutative Law)

(*ii*) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative Law)

(*iii*) $A \cup \phi = A$ (Law of identity element, ϕ is the identity of U)

(*iv*) $A \cup A = A$ (Idempotent Law)

(*v*) $U \cup A = U$ (Law of U)

Intersection of Sets

The intersection of two sets A and B is the set of all the elements which are common. It is denoted by $A \cap B$.

Properties of the Operation of Intersection

(*i*) $A \cap B = B \cap A$ (Commutative Law)

(*ii*) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative Law)

(*iii*) $\phi \cap A = \phi$, $U \cap A = A$ (Law of ϕ and $\cup$)

(*iv*) $A \cap A = A$ (Idempotent Law)

(*v*) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive Law)

Difference of Sets

The difference of two sets A and B i.e., $A–B$, is the set of all those elements of A which do not belong to B.

Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$

Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$

Some Important Results on Number of Elements in Sets

(i) If A and B are finite sets such that $A \cap B = \phi$, then
$n(A \cup B) = n(A) + n(B)$

(ii) If $A \cup B \neq \phi$, then

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

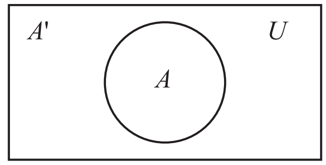
(iii) If A, B and C are finite sets, then
$(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Complement of a Set

Let U be the universal set and let A be a set such that $A \subset U$. Then, the complement of A with respect to U is denoted by A^c or A' or U – A and is defined as the set of all those elements of U which are not in A.

Therefore, $A' = \{x \in U : x \notin A\}$

Clearly, $x \in A' \Leftrightarrow x \notin A$



Properties of Complement Sets

(1) **Complement Laws**

(i) $A \cup A' = U$ (ii) $A \cap A' = \phi$

(2) **De Morgan's Law**

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

(3) **Law of Double Complementation**

$(A')' = A$

(4) **Laws of ϕ and U**

$\phi' = U$ and $U' = \phi$