CHAPTER

12 Sequences and Series

An arithmetic progression (A.P.) : a, $a + d$, $a + 2d$..........$a + (n - 1)d$ is an A.P.

Let a be the first term and d be the common difference of an A.P., then n^{th} term $= t_n = a + (n - 1)\, d$

The sum of first n terms of are A.P.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}\left[a + \ell\right]$$

r^{th} term of an A.P. when sum of first r terms is given is $t_r = S_r - S_{r-1}$.

Properties of A.P.

(i) If a, b, c are in A.P

$\Rightarrow 2b = a + c$ & if a, b, c, d are in A.P.

$\Rightarrow a + d = b + c$.

(ii) Three numbers in A.P. can be taken as $a - d, a, a + d$; four numbers in A.P. can be taken as $a - 3d, a - d, a + d, a + 3d$ five numbers in A.P. are $a - 2d, a - d, a, a + d, a + 2d$ & six terms in A.P. are $a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$ etc.

(iii) Sum of the terms of an A.P. equidistant from the beginning & end = sum of first & last term.

Arithmetic Mean (Mean or Average) (A.M.):

If three terms are in A.P. then the middle term is called the A M. between the other two, so if a, b, c are in A.P., b isA.M. of a & c.

n–Arithmetic Means Between Two Numbers:

If a, b are any two given numbers & a, $A_1\, A_2 .....A_n$, b are in A.P. then $A_1\, A_2,...\, A_n$ are the A.M .

$$A_1 = a + \frac{b-a}{n+1},\ A_2 = a + \frac{2(b-a)}{n+1}, ..., A_n = a + \frac{n(b-a)}{n+1}$$

$\sum_{r=1} A_r = nA$ where A is the single A M. between a & b.

Geometric Progression: a, ar, ar^2, ar^3, ar^4.....with a as the first term & r as common ratio.

(i) n^{th} terms $= ar^{n-1}$

(ii) Sum of the first n terms i.e. $S_n = \begin{cases} \dfrac{a(r^n - \ )}{r-1}, & r \neq 1 \\ na & r = 1 \end{cases}$

Geometric Means (Mean Proportional) (GM.):

If $a.\ b,\ c > 0$ are in G P . b is the G M between a & c, then $b^2 = ac$

***n* Geometric Means Between Positive Number *a*, *b*:** If a, b are two given numbers & a. $G_1, G_2 ....,G_3$ b are in G.P Then $G_1, G_2, G_3 ...., G_n$ are n GM.s between a & b. G_2 = a $(b/a)^{2/n+1}..G_n = a(b/a)^{n/n+1}$

Harmonic Mean (H.M.):

If a, b, c are in HP., b is the H.M. between a & c, then $b = \dfrac{2ac}{a+c}$

H.M, H of $a_1, a_2,..a_n$ is given by $\dfrac{1}{H} = \dfrac{1}{n}\left[\dfrac{1}{a_1} + \dfrac{1}{a_2} + ... + \dfrac{1}{a_n}\right]$

Relation between Means:

$G^2 = AH$ A.M. $\geq$ G.M $\geq$ H.M. and A.M. = G.M. = H.M. if $a_1 = a_2 = a_3 = ... = a_n$

Important Results

(i) $\displaystyle\sum_{r=1}^{n}(a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$

(ii) $\displaystyle\sum_{r=1}^{n} ka_r = k\sum_{r=1}^{n} a_r$

(iii) $\displaystyle\sum_{r=1}^{n} k = nk$ where k is a constant,

(iv) $\displaystyle\sum_{r=1}^{n} r = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$

(v) $\displaystyle\sum_{r=1}^{n} r^2 = 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

(vi) $\displaystyle\sum_{=1} r^3 = 1^3 + 2^3 + 3^3 + ... + n^3 = \frac{n^2(n+1)^2}{4}$

(vii) $\displaystyle\sum_{i<j=1}^{n} a_i a_j = (a_1 + a_2 + ... + a_n)^2 - (a_1{}^2 + a_2{}^2 + ... + a_n^2)$