CHAPTER 6

Principle of Mathematical Induction

(i) $1 + 2 + 3 + ... + n = \Sigma n = \dfrac{n(n+1)}{2}$

(ii) $1^2 + 2^2 + 3^2 + ... + n^2 = \Sigma n^2 = \dfrac{n(n+1)(2n+1)}{6}$

(iii) $1^3 + 2^3 + 3^3 + ... + n^3 = \Sigma n^3 = (\Sigma n)^2 = \left\{\dfrac{n(n+1)}{2}\right\}^2$

(iv) $1^4 + 2^4 + 3^4 + ... + n^4 = \dfrac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

(v) $2 + 4 + 6 + ... + 2n = \Sigma 2n = n(n+1)$

(vi) $1 + 3 + 5 + ... + (2n-1) = \Sigma(2n-1) = n^2$

(vii) $x^n - y^n = (x - y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + ... + xy^{n-2} + y^{n-1})$, where $n \in N$

(viii) $x^n + y^n = (x + y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 + ... - xy^{n-2} + y^{n-1})$ where n is odd positive integer

(ix) $a + (a + d) + (a + 2d) + ... + (a + (n-1)d)$

$= \dfrac{n}{2}[2a + (n-1)d]$

(x) $a + ar + ar^2 + ... + ar^{n-1} = \dfrac{a(r^n - 1)}{r - 1}$, where $r \neq 1$

(xi) $(a + b)^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$

(xii) $\cos(a) \cdot \cos(2a) \cdot \cos(4a) ... \cos(2^{n-1}a) = \dfrac{\sin(2^n a)}{2^n \sin a}$

(xiii) $\sin(a) + \sin(a + b) + \sin(a + 2b) + ... + \sin(a + (n-1)b)$

$= \dfrac{\sin(nb/2)}{\sin(b/2)} \sin(a + (n-1)\dfrac{b}{2})$

(xiv) $\cos(a) + \cos(a + b) + \cos(a + 2b) + ... + \cos(a + (n-1)b)$

$= \dfrac{\sin(nb/2)}{\sin(b/2)} \cos(a + (n-1)\dfrac{b}{2})$