CHAPTER

20 Limits and Derivatives

Limit

- The expected value of the function as dictated by the points to the left of a point defines the left hand limit of the function at that point. Similarly the right hand limit.
- Limit of a function at a point is the common value of the left and right hand limits, if they coincide.
- For a function f and a real number a, $\lim_{x\to a} f(x)$ and $f(a)$ may not be same (In fact, one may be defined and not the other one).

Fundamental Theorems on Limits

- For functions f and g if $\lim_{x\to a} f(x)$ & $\lim_{x\to a} g(x)$ exists, then the following holds:

$$\lim_{x\to a}[f(x)\pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x).g(x)] = \lim_{x\to a} f(x).\lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \quad g(x)\neq 0 \;\&\; \lim_{x\to a} g(x)\neq 0$$

Theorem

- Following are some of the standard limits

$$\lim_{x\to a}\frac{x^n - a^n}{x-a} = na^{n-1}, \text{for any } +ve \text{ integer}$$

$$\lim_{x\to 0}\frac{\log(1+x)}{x} = 1$$

$$\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{\tan x}{x} = 1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x} = 0$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

$$\lim_{x\to 0}\frac{e^x - 1}{x} = 1$$

$$\lim_{x\to 0}\frac{a^x - 1}{x} = \log_e a$$

L'hopital's Rule

If $\lim_{x\to a}\frac{f(x)}{g(x)}$ is of form $\frac{0}{0}$

Then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

Derivatives

- Derivative of a function f at any point x is defined by

$$f'(x) = \frac{df(x)}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

- For functions u and v the following holds:

$(u \pm v)' = u' \pm v'$

$(uv)' = u'v + uv'$

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ provided all are defined.

- Following are some of the standard derivatives.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \log a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$