

CHAPTER

8

Indefinite Integration

1. If f & g are functions of x such that $g'(x) = f(x)$ then,

$$\int f(x)dx = g(x) + c \Leftrightarrow \frac{d}{dx}\{g(x) + c\} = f(x), \text{ where } c \text{ is called the constant of integration.}$$

2. Standard Formula:

$$(i) \int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c, n \neq -1$$

$$(ii) \int \frac{dx}{ax + b} = \frac{1}{a} \ln |ax + b| + c$$

$$(iii) \int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$$

$$(iv) \int a^{px+q} dx = \frac{1}{p} \frac{a^{px+q}}{\ln a} + c; a > 0$$

$$(v) \int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + c$$

$$(vi) \int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + c$$

$$(vii) \int \tan(ax + b) dx = \frac{1}{a} \ln |\sec(ax + b)| + c$$

$$(viii) \int \cot(ax + b) dx = \frac{1}{a} \ln |\sin(ax + b)| + c$$

$$(ix) \int \sec^2(ax + b) dx = \frac{1}{a} \tan(ax + b) + c$$

$$(x) \int \operatorname{cosec}^2(ax + b) dx = -\frac{1}{a} \cot(ax + b) + c$$

$$(xi) \int \sec x dx = \ln |(\sec x + \tan x)| + c$$

$$\text{Or } \ln \left| \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right| + c$$

$$(xii) \int \operatorname{cosec} x dx = \ln |(\operatorname{cosec} x - \cot x)| + c$$

$$\text{Or } \ln \left| \tan \frac{x}{2} \right| + c$$

$$\text{Or } -\ln |(\operatorname{cosec} x + \cot x)| + c$$

$$(xiii) \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$$

$$(xiv) \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$(xv) \int \frac{dx}{|x|\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} + c$$

$$(xvi) \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left| x + \sqrt{x^2 + a^2} \right| + c$$

$$(xvii) \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left| x + \sqrt{x^2 - a^2} \right| + c$$

$$(xviii) \int \frac{dx}{(a^2 - x^2)} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + c$$

$$(xix) \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$$

$$(xx) \int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$$

$$(xxi) \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \ln \left| \frac{x + \sqrt{x^2 + a^2}}{a} \right| + c$$

$$(xxii) \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln \left| \frac{x + \sqrt{x^2 - a^2}}{a} \right| + c$$

3. Integration by substitutions:

If we substitute $f(x) = t$, then $f'(x) dx = dt$

4. Integration by part:

$$\int (f(x)g(x))dx = f(x) \int (g(x))dx - \int \left(\frac{d}{dx} (f(x)) \int (g(x))dx \right) dx$$

5. Integration of type:

$$\int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}, \int \sqrt{ax^2 + bx + c} dx$$

Make the substitution $x + \frac{b}{2a} = t$

6. Integration of type:

$$\int \frac{px + q}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx, \int (px + q) \sqrt{ax^2 + bx + c} dx$$

Make the substitution $x + \frac{b}{2a} = t$, then split the integral as sum of two integrals one containing the linear term and the other containing constant term.

7. Integration of trigonometric functions:

$$(i) \int \frac{dx}{a + b \sin^2 x} \text{ Or, } \int \frac{dx}{a + b \cos^2 x} \text{ Or,}$$

$$\int \frac{dx}{a \sin^2 x + b \sin x \cos x + c \cos^2 x}, \text{ put } \tan x = t$$

$$(ii) \int \frac{dx}{a + b \sin x} \text{ Or, } \int \frac{dx}{a + b \cos x} \text{ Or}$$

$$\int \frac{dx}{a + b \sin x + c \cos x}, \text{ put } \tan \frac{x}{2} = t.$$

$$(iii) \int \frac{a \cos x + b \sin x + c}{l \cos x + m \sin x + n} dx. \text{ Express}$$

$$N^r \equiv A(D^r) + B \frac{d}{dx}(D^r) + c \text{ \& proceed.}$$

8. Integration of type:

$$\int \frac{x^2 \pm 1}{x^4 + Kx^2 + 1} dx \text{ where } K \text{ is any constant.}$$

$$\text{Divide } N^r \text{ \& } D^r \text{ by } x^2 \text{ \& put } x \mp \frac{1}{x} = t$$

9. Integration of type:

$$\int \frac{dx}{(ax+b)\sqrt{px+q}} \text{ Or } \int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$$

$$\text{put } px + q = t^2.$$

10. Integration of type:

$$\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}, \text{ put } ax+b = \frac{1}{t};$$

$$\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}} \text{ put } x = \frac{1}{t}$$

Some Standard Substitution

$$1. \int f(x)^n f'(x) dx \text{ Or } \int \frac{f'(x)}{[f(x)]^n} dx \text{ put } f(x) = t \text{ \& proceed.}$$

$$2. \int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c} dx$$

Express $ax^2 + bx + c$ in the form of perfect square & then apply the standard results.

$$3. \int \frac{(px+q)dx}{ax^2+bx+c}, \int \frac{(px+q)}{\sqrt{ax^2+bx+c}} dx$$

Express $px + q = A$ (differential coefficient of denominator) + B .

$$4. \int e^x [f(x) + f'(x)] dx = e^x \cdot f(x) + c$$

$$5. \int [f(x) + xf'(x)] dx = xf(x) + c$$

$$6. \int \frac{dx}{x(x^n+1)}, n \in N, \text{ take } x^n \text{ common \& put } 1+x^{-n} = t.$$

$$7. \int \frac{dx}{x^2(x^n+1)^{(n-1)/n}}, n \in N, \text{ take } x^n \text{ common \& put } 1+x^{-n} = t^n.$$

$$8. \int \frac{dx}{x^n(1-x^n)^{1/n}}, \text{ take } x^n \text{ common and put } 1+x^{-n} = t.$$

$$9. \int \frac{\sqrt{x-\alpha}}{\sqrt{\beta-x}} dx \text{ Or } \int \sqrt{(x-\alpha)(\beta-x)}; \text{ put } x = \alpha \cos^2 \theta + \beta \sin^2 \theta$$

$$\int \frac{\sqrt{x-\alpha}}{x-\beta} dx \text{ Or } \int \sqrt{(x-\alpha)(x-\beta)}; \text{ put } x = \alpha \sec^2 \theta - \beta \tan^2 \theta$$

$$\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}; \text{ put } x-\alpha = t^2 \text{ or } x-\beta = t^2.$$