

CHAPTER

11



Binomial Theorem

Important terms in the binomial expansion are

(a) **General term:** The general term or the $(r + 1)^{th}$ term in the expansion of $(x + y)^n$ is given by

$$T_{r+1} = {}^n C_r \cdot x^{n-r} \cdot y^r$$

(b) **Middle term:** The middle term (s) is the expansion of $(x + y)^n$ is (are):

(i) If n is even, there is only one middle term which is given by

$$T_{(n+2)/2} = {}^n C_{n/2} \cdot x^{n/2} \cdot y^{n/2}$$

(ii) If n is odd, there are two middle terms which are

$$T_{(n+1)/2} \text{ and } T_{[(n+1)/2]+1}$$

(c) **Term independent of x :** Term independent of x contains no x ; Hence find the value of r for which the exponent of x is zero

If $(\sqrt{A} + B)^n = I + f$, where I & n are positive integers and $0 \leq f < 1$, then

(a) $(I + f) \cdot f = K^n$ if n is odd & $A - B^2 = K > 0$

(b) $(I + f)(1 - f) = k^n$ if n is even & $\sqrt{A} - B < 1$

Some results on binomial coefficients

(a) ${}^n C_x = {}^n C_y \Rightarrow x = y$ or $x + y = n$

(b) ${}^n C_{r-1} + {}^n C_r = {}^{n+1} C_r$

(c) $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$

(d) $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \dots + \frac{(-1)^n C_n}{n+1} = \frac{1}{n+1}$

(e) $C_0 + C_1 + C_2 + \dots = C_n = 2^n$

(f) $C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots = 2^{n-1}$

(g) $C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = 2^n C_n = \frac{(2n)!}{n!n!}$

(h) $C_0 \cdot C_r + C_1 \cdot C_{r+1} + C_2 \cdot C_{r+2} + \dots + C_{n-r} C_n = \frac{(2n)!}{(n+r)!(n-r)}$

Greatest coefficient and Greatest Term in Expansion of $(x + a)^n$

(a) If n is even greatest coefficient is ${}^n C_{n/2}$.

If n is odd greatest coefficient is ${}^n C_{(n-1)/2}$ or ${}^n C_{(n+1)/2}$

(b) **For greatest term:** Greatest term

$$= \begin{cases} T_p \text{ and } T_{p+1} & \text{if } \frac{n+1}{\left|\frac{x}{a}\right|+1} \text{ is an integer} \\ T_{q+1} & \text{if } \frac{n+1}{\left|\frac{x}{a}\right|+1} \text{ is non integer and } \in (q, q+1), q \in I \end{cases}$$

Multinomial Theorem

For any $n \in N$,

(i) $(x_1 + x_2 + \dots + x_k)^n = \sum_{r_1+r_2+\dots+r_k=n} \frac{n!}{r_1!r_2!\dots r_k!} x_1^{r_1} x_2^{r_2} \dots x_k^{r_k}$

(ii) The general term in the above expansion is

$$\frac{n!}{r_1!r_2!\dots r_k!} x_1^{r_1} x_2^{r_2} \dots x_k^{r_k}$$

Here total number of terms in the expansion = ${}^{n+k-1} C_{k-1}$

Binomial Theorem for Negative or Fractional Indices

If $n \in Q$, then $(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots \infty$ provided $|x| < 1$.

Notes

(i) $(1 - x)^{-1} = 1 + x + x^2 + x^3 + \dots \infty$

(ii) $(1 + x)^{-1} = 1 - x + x^2 - x^3 + \dots \infty$

(iii) $(1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots \infty$

(iv) $(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots \infty$

Exponential series

(a) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$; where x may be any real or

complex number and $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$.

(b) $a^x = 1 + \frac{x}{1!} \ln a + \frac{x^2}{2!} \ln^2 a + \frac{x^3}{3!} \ln^3 a + \dots \infty$, where $a > 0$.

Logarithmic Series

(a) $\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \infty$, where $-1 < x \leq 1$.

(b) $\ln(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots \infty$, where $-1 \leq x < 1$.

(c) $\ln \frac{(1+x)}{(1-x)} = 2 \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty \right)$, $|x| < 1$.