

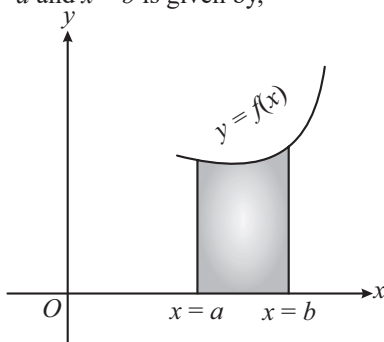
CHAPTER

10



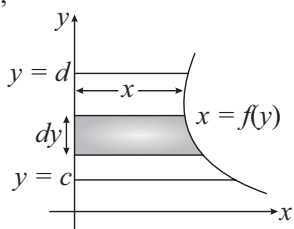
Application of Integrals

1. The area bounded by the curve $y = f(x)$, the x -axis and the ordinates $x = a$ and $x = b$ is given by,



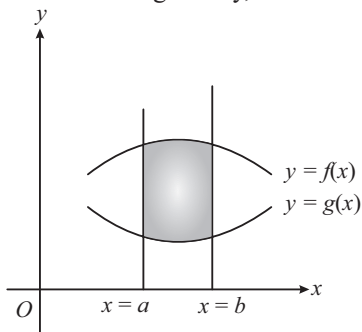
$$A = \int_a^b f(x) dx = \int_a^b y dx$$

2. If the area is below the x -axis, the area is negative. The convention is to consider the magnitude only i.e. $A = \left| \int_a^b y dx \right|$ in this case.
3. The area bounded by the curve $x = f(y)$, y -axis and abscissa $y = c$, $y = d$ is given by,



$$\text{Area} = \int_c^d x dy = \int_c^d f(y) dy$$

4. Area between the curves $y = f(x)$ and $y = g(x)$ between the ordinates $x = a$ and $x = b$ is given by,



$$A = \int_a^b f(x) dx - \int_a^b g(x) dx$$

$$= \int_a^b [f(x) - g(x)] dx$$

5. Average value of a function $y = f(x)$ w.r.t. x over an interval $a \leq x \leq b$ is defined as:

$$y_{av} = \frac{1}{b-a} \int_a^b f(x) dx.$$

6. **Curve Tracing:**

The following outline procedure is to be applied in Sketching the graph of a function $y = f(x)$ which in turn will be extremely useful to quickly and correctly evaluate the area under the curves.

(a) **Symmetry:** The symmetry of the curve is judged as follows

- (i) If all the powers of y in the equation are even then the curve is symmetrical about the axis of x .
- (ii) If all the powers of x are even, the curve is symmetrical about the axis of y .
- (iii) If powers of x and y both are even, the curve is symmetrical about the axis of x as well as y .
- (iv) If the equation of the curve remains unchanged on interchanging x and y , then the curve is symmetrical about $y = x$.
- (v) If on interchanging the signs of x and y both the equation of the curve is unaltered then there is symmetry in opposite quadrants.

- (b) Find dy/dx and equate it to zero to find the points on the curve where you have horizontal tangents.
- (c) Find the points where the curve crosses the x -axis and also the y -axis.
- (d) Examine if possible the intervals when $f(x)$ is increasing or decreasing. Examine what happens to 'y' when $x \rightarrow \infty$ or $-\infty$.

7. **Useful Results:**

- (a) Whole area of the ellipse, $x^2/a^2 + y^2/b^2 = 1$ is πab .
- (b) Area enclosed between the parabolas $y^2 = 4ax$ and $x^2 = 4by$ is $16ab/3$.
- (c) Area included between the parabola $y^2 = 4ax$ and the line $y = mx$ is $8 a^2/3 m^3$.