

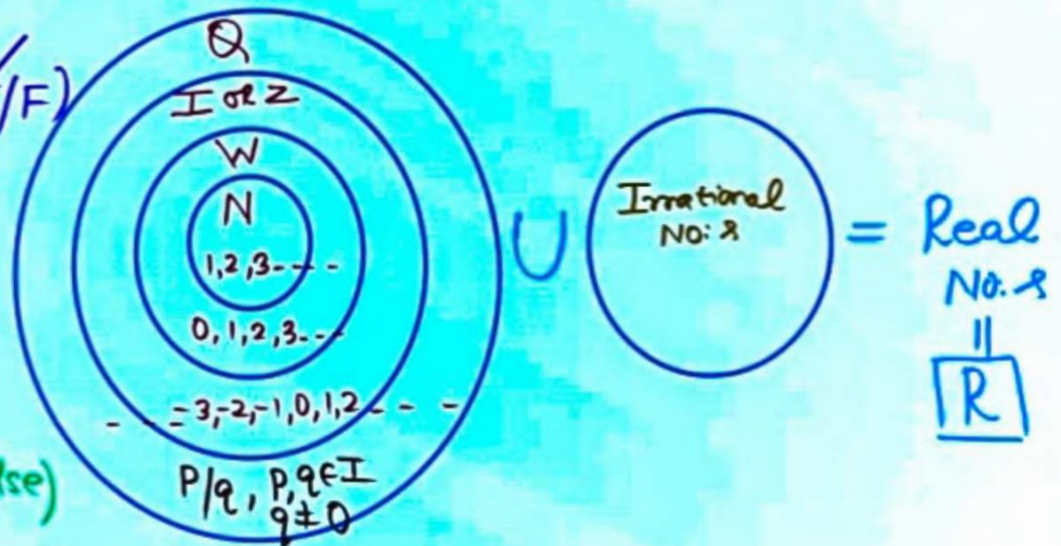


Number System

Basic Maths



- 1) 0 is a Rational (T/F)
- 2) Every Integer is Rational (T/F)
- 3) Every Rational is a whole NO: (False)



Rational No: $\frac{P}{Q}, P, Q \in I, Q \neq 0$

Terminating decimals

Ex: $\frac{9}{5} = 1.8$

$\frac{3}{4} = 0.75$

$\frac{1}{8} = 0.125$

Recurring/Repeating Decimals

Ex: $\frac{1}{3} = 0.33333... = 0.\overline{3}$

$\frac{1}{9} = 0.11111... = 0.\overline{1}$

$\frac{22}{7} = 3.\overline{142857}$



How know if a rational no: $\frac{p}{q}$, $p, q \in \mathbb{I}$, $q \neq 0$
is repeating or Terminating without dividing

Ex: $\frac{29}{25} \rightarrow 2 \cdot 5^2$
 Repeating ~~X~~
 Terminating ✓

Pechaan!!

q is of the form $2^m \cdot 5^n$
 where $m, n \in \mathbb{W}$.

Ex: $\frac{37}{18} \rightarrow 2 \cdot 3^2$
 Repeating ✓
 Terminating ~~X~~

Ex: $\frac{91}{14} \rightarrow 2 \cdot 7$
 Repeating ~~X~~
 Terminating ✓ $\Rightarrow \frac{91}{14} = \frac{13}{2} = 6.5$

KAAM KI BAAT: Every Integer
 is a Rational
 Number

B'wz: $2 = \frac{2}{1}$
 $-3 = \frac{-3}{1}$
 $7 = \frac{7}{1}$
 $0 = \frac{0}{1}$
 $\rightarrow \frac{p}{q}$ form $p, q \in \mathbb{I}, q \neq 0$.

IRRATIONAL NO:s

Neither Repeating nor Terminating Decimals.

Division By
 0 is not defined
 In Mathematics

Ex: $\sqrt{2} = 1.414 \dots$
 $\sqrt{3} = 1.73205 \dots$
 $\sqrt{5} = 2.236 \dots$
 $e \approx 2.718 \dots$
 $\pi \approx 3.14 \dots$

Remainder Theorem

Let $P(x)$ be a polynomial of degree ≥ 1 and ' a ' is any real number. If $P(x)$ is divided by $(x - a)$, then the remainder is $P(a)$.

$$\begin{array}{r}
 P(x) = x^3 - 3x^2 + 3x + 5 \\
 \begin{array}{r}
 x^2 - 2x + 1 \text{ --- Quotient} \\
 x-1 \overline{) x^3 - 3x^2 + 3x + 5} \\
 \underline{x^3 - x^2} \text{ --- Dividend} \\
 -2x^2 + 3x \\
 \underline{-2x^2 + 2x} \\
 x + 5 \\
 \underline{x - 1} \\
 6 \text{ --- Remainder}
 \end{array}
 \end{array}$$

by Remainder Theorem

$$\begin{aligned}
 P(x) &= x^3 - 3x^2 + 3x + 5 \\
 \text{Remainder when divided by } x-1 & \\
 P(1) &= 1 - 3 + 3 + 5 = 6
 \end{aligned}$$

$$\text{Dividend} = \text{Quotient} \times \text{Divisor} + \text{Remainder}$$

$$x^3 - 3x^2 + 3x + 5 = (x - 1)(x^2 - 2x + 1) + 6$$

$$\begin{aligned}
 x=0 & \\
 5 &= (-1) \cdot 1 + 6 = 5 \\
 x=1 & \quad 1 - 3 + 3 + 5 = 0 + 6 \\
 & \quad \quad \quad 6 = 6
 \end{aligned}$$

Trick

$$\begin{aligned}
 x - a &= 0 \\
 x &= a \\
 2x + 3 &= 0 \\
 x &= -3/2 \\
 x + a &= 0 \\
 x &= -a
 \end{aligned}$$

	Divisor	Remainder
$P(x)$	$x - a$	$P(a)$
	$x + a$	$P(-a)$
	$2x + 3$	$P(-3/2)$
	$3x - 5$	$P(5/3)$

Remark

- $p(-a)$ is remainder on dividing $p(x)$ by $(x + a)$ $[\because x + a = 0 \Rightarrow x = -a]$
- $p\left(\frac{b}{a}\right)$ is remainder on dividing $p(x)$ by $(ax - b)$ $[\because ax - b = 0 \Rightarrow x = \frac{b}{a}]$
- $p\left(-\frac{b}{a}\right)$ is remainder on dividing $p(x)$ by $(ax + b)$ $[\because ax + b = 0 \Rightarrow x = -\frac{b}{a}]$
- $p\left(\frac{b}{a}\right)$ is remainder on dividing $p(x)$ by $(b - ax)$ $[\because b - ax = 0 \Rightarrow x = \frac{b}{a}]$

FACTOR THM.

Remainder in Disguise.
Theorem.

Let $P(x)$ be a poly of degree ≥ 1 & if
 $P(a)=0 \Rightarrow x-a$ is a factor of $P(x)$

Conversely if $(x-a)$ is factor of $P(x)$ then $P(a)=0$.

Name of Exponent Rules		Rule
Zero Exponent Rule	$a^0 = 1$	(Where $a \neq 0$)
Identity Exponent Rule	$a^1 = a$	
Product Rule	$a^m \times a^n = a^{m+n}$	
Quotient Rule	$a^m / a^n = a^{m-n}$	
Negative Exponents Rule	$a^{-m} = 1/a^m, (a/b)^{-1} = (b/a)^m$	
Power of a Power Rule	$(a^m)^n = a^{mn}$	
Power of a Product Rule	$(ab)^m = a^m b^m, (a^p b^q)^\alpha = a^{p\alpha} b^{q\alpha}$	
Power of a Quotient Rule	$(a/b)^m = a^m / b^m$	
Fractional Rule	$a^{1/n} = \sqrt[n]{a}; a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m} = (a^{1/n})^m = (\sqrt[n]{a})^m$	

An Important Result

$$a^2 + b^2 + c^2 - ab - bc - ca \geq 0 \quad \forall a, b, c \in \mathbb{R}$$

equality holds i.e

$$a^2 + b^2 + c^2 - ab - bc - ca = 0$$

$$\text{If } a=b=c$$

An Important Result

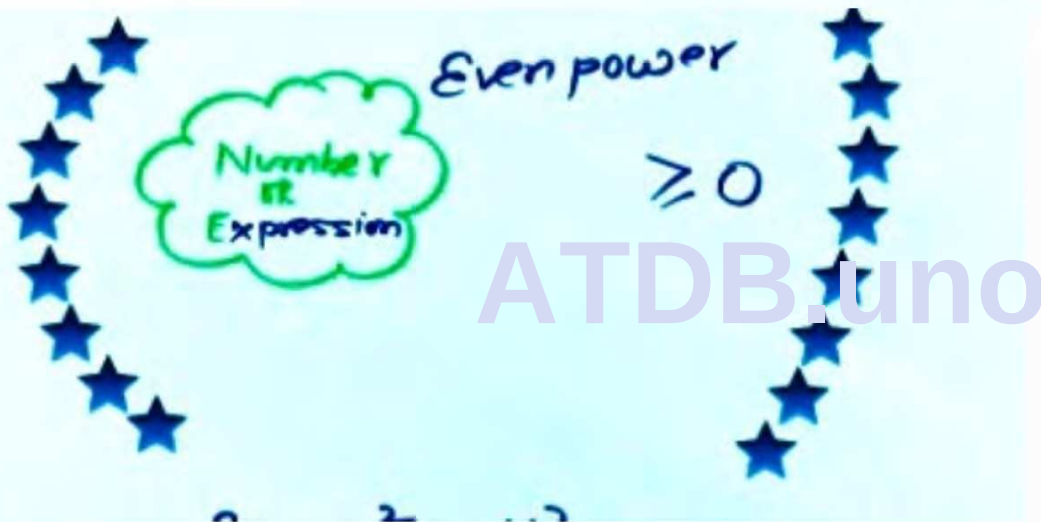
★ If $x, y \in \mathbb{R}$ & $x^2 + y^2 = 0 \Rightarrow x = 0$ & $y = 0$

Generalization:

If $a_1, a_2, \dots, a_n \in \mathbb{R}$ then $a_1^2 + a_2^2 + \dots + a_n^2 = 0$ then $a_1 = a_2 = \dots = a_n = 0$

Ex: find x & y $4x^2 + 4x + 1 + y^2 - 6y + 9 = 0, x, y \in \mathbb{R}$

$$(2x)^2 + 2 \cdot 2x \cdot 1 + 1^2 + y^2 - 2 \cdot 3 \cdot y + 3^2 = 0$$
$$\underbrace{(2x+1)^2}_{\geq 0} + \underbrace{(y-3)^2}_{\geq 0} = 0$$
$$2x+1 = 0 \text{ \& \> } y-3 = 0$$
$$x = -\frac{1}{2} \text{ \& \> } y = 3$$



(Any positive real number)

Any real power

> 0



Algebraic Identities

$I_1:$ $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

$I_2:$ $a^2 - b^2 = (a - b)(a + b)$

$I_3:$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$I_4:$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$I_5:$ $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$I_6:$ $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$I_7:$ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = a^2 + b^2 + c^2 + 2abc\left(\frac{a}{b} + \frac{b}{a} + \frac{c}{a} + \frac{a}{c} + \frac{b}{c} + \frac{c}{b}\right)$
 $= a^2 + b^2 + c^2 + 2abc\left(\frac{1}{b} + \frac{1}{a} + \frac{1}{c}\right)$

$I_8:$ $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a) = a^3 + b^3 + c^3 + 3abc\left(\frac{a}{b} + \frac{b}{a} + \frac{c}{a} + \frac{a}{c} + \frac{b}{c} + \frac{c}{b}\right)$

$I_9:$ $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Yaad Rakhe !!

★ $\sqrt{x^2} = |x|$

★ $\sqrt[3]{x^3} = x$

★ $\sqrt[2n]{x^{2n}} = |x|$

★ $\sqrt[2n+1]{x^{2n+1}} = x$

Some Golden Points

$\forall$ for all/ for every

1. If $a > 0$ and $D < 0$ then $y = ax^2 + bx + c > 0$ for all $x \in \mathbb{R}$

2. If $a < 0$ and $D < 0$ then $y = ax^2 + bx + c < 0$ for all $x \in \mathbb{R}$

Ex: $P(x) = x^2 + x + 3$

$$D = 1^2 - 4 \cdot 1 \cdot 3 = -11 < 0$$

$$a = 1 > 0$$

$$x^2 + x + 3 > 0 \quad \forall x \in \mathbb{R}$$

$$P(-2) = (-2)^2 + 2 + 3 = 5 > 0$$

$$P(0) = 0^2 + 0 + 3 = 3 > 0$$

for all or for every

Ex: $P(x) = -x^2 + x - 3$

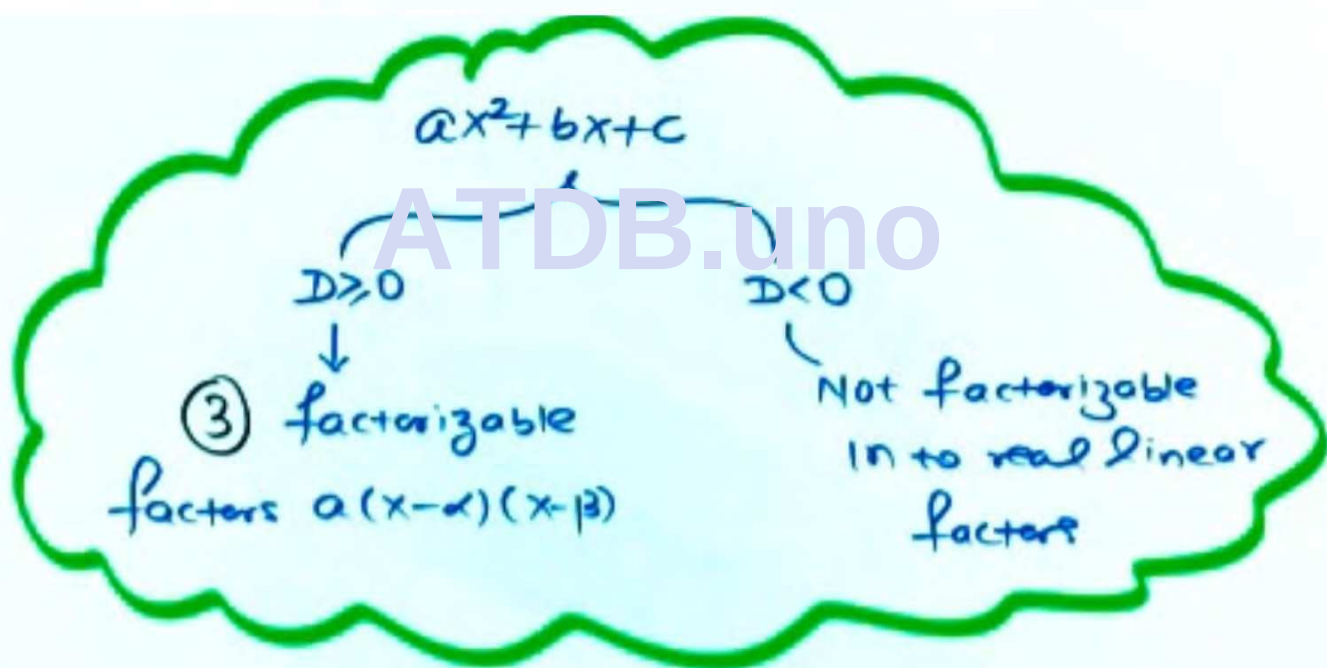
$$D = 1^2 - 4(-1)(-3) = 1 - 12 = -11 < 0$$

$$a = -1 < 0$$

$$\Rightarrow -x^2 + x - 3 < 0 \quad \forall x \in \mathbb{R}$$

$$P(2) = -(2)^2 + 2 - 3 = -4 + 2 - 3 = -5$$

$$P(0) = -0 + 0 - 3 = -3 < 0$$



Inequalities

B1: We can add (or subtract) any number 'k' on both sides of inequality. Doing this will not change the sign of inequality.

B2: We can multiply (or divide) any non-zero number 'k' on both sides of inequality and sign of inequality will change according to sign of 'k' that is

- If $k > 0$ then sign of inequality will remain same,
- If $k < 0$ then sign of inequality will get reversed.

B3 : Squaring (raising even power both side) is only allowed when both sides of inequality are non negative.

$$\begin{array}{l} 3 > 2 \\ 9 > 4 \end{array} \Bigg) \text{S.B.S.}$$

$$\begin{array}{l} \text{Ex: } -2 > -3 \\ 4 > 9 \end{array} \Bigg) \text{S.B.S.}$$

B4 : Raising both sides to odd power is fine.

$$\begin{array}{l} \text{Ex: } 3 > 2 \\ 27 > 8 \end{array} \Bigg) \text{C.B.S.}$$

$$\begin{array}{l} \text{Ex: } -2 > -3 \\ -8 > -27 \end{array} \Bigg) \text{C.B.S.}$$

$$\begin{array}{l} \text{Ex: } 5 > -2 \\ 125 > -8 \end{array} \Bigg) \text{C.B.S.}$$

$$\begin{array}{l} \text{Ex: } 3 > -7 \\ 27 > -343 \end{array} \Bigg) \text{C.B.S.}$$

$$\begin{array}{l} \text{Ex: } 5 > -2 \\ 25 > 4 \end{array} \Bigg) \text{S.B.S.}$$

$$\begin{array}{l} \text{Ex: } 3 > -7 \\ 9 > 49 \end{array} \Bigg) \text{S.B.S.}$$

Inequalities can be added provided they have same sign of inequality.
But inequalities can not be subtracted.

Inequalities can be multiplied provided both sides are positive and have same sign of inequality, but they can not be divided.

Method of Intervals

Steps Involved

1. Make one side of inequality 0. ✓✓
2. Factorize the non zero side in to linear factors ✓✓
3. Put each linear factor equal to zero & find value of x. ✓✓
4. Plot all values of x on a number line. ✓✓
5. Start with a positive sign on the extreme right part & then place negative, positive signs alternately.

Important Point to Note

1. Values of x corresponding to denominator are never included in answer.
2. Coefficient of x in every linear factor should be positive if not then make it positive.

Case of Repeated Factors

B₁ : Every odd integral power of a linear factor is treated as 1.

B₂ : In case of even power of any factor, first we assume that it is always positive. So we delete it from the inequality but in the end we make a direct check at that value of x where the deleted factor is zero.

In case if a factor is eliminated

In this case the factor that is cancelled in the Numerator & Denominator, it is put not equal to zero & it's roots are never included in answer

Ex: $\frac{x^2 - 5x + 6}{x^2 - 6x + 8} > 0$

$\frac{x^2 - 3x - 2x + 6}{x^2 - 2x - 4x + 8} > 0$

$\frac{(x-2)(x-3)}{(x-2)(x-4)} > 0$

$\frac{x-3}{x-4} > 0$

$x-2 \neq 0$
i.e. $x \neq 2$

$\frac{+}{-}$
 $\frac{-}{+}$
3 4

$x \in (-\infty, 3) \cup (4, \infty) - \{2\}$

$$* x + \frac{1}{x} \geq 2, x \in \mathbb{R}^+$$

$$* x + \frac{1}{x} \leq -2, x \in \mathbb{R}^-$$

$$* \rightarrow a^2 \geq 0, a \in \mathbb{R}$$

$$* \text{ If } a_1, a_2, \dots, a_n \in \mathbb{R} \text{ then } a_1^2 + a_2^2 + \dots + a_n^2 = 0 \text{ then } a_1 = a_2 = \dots = a_n = 0$$

$$* \text{ coeff of } x \text{ in each linear factor should +ve}$$

$$* ax^2 + bx + c$$

$$\rightarrow \text{ if } D < 0, a > 0 \Rightarrow ax^2 + bx + c > 0 \forall x \in \mathbb{R}$$

$$\rightarrow \text{ if } D < 0, a < 0 \Rightarrow ax^2 + bx + c < 0 \forall x \in \mathbb{R}$$

$$\rightarrow D \geq 0 \text{ then quad is factorizable into real linear factors}$$

$$ax^2 + bx + c = a(x - \alpha)(x - \beta)$$

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$* \sqrt{x^2} = |x|$$

$$* |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

consists of two or more variables try to make perfect squares

* In an inequality we can add or subtract any no. from both sides

* If we multiply or divide both sides by any +ve no: sign of inequality remains same

* If we multiply or divide both sides by any -ve no: sign of inequality reversed

$$* a^0 = 1 (a \neq 0) \quad | \quad 0^0 \rightarrow \text{not defined}$$

$$* 574 \xrightarrow{2^2} 25716$$

Agar dono sides non-tive ho then we can square

$$* (s-x)^2 = (x-3)^2$$

$$\text{but } (4-x)^3 \neq (x-4)^3$$

* $\log_a N$ denotes power to which should be raised in order to get

* $\log_a N$ is defined if $a > 0, a \neq 1, N > 0$

$$* a^{\log_a N} = N, \log_a a = 1, \log_a \frac{1}{a} = -1, \log_{\frac{1}{a}} a = -1$$

* $\log_a 1 = 0$ * $\log_1$ is not defined becoz it can not give definite value

$$* \log_a(mn) = \log_a(m) + \log_a(n)$$

$$* \log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$$

$$* \log_a m^x = x \cdot \log_a m$$

$$* \log_a b = \frac{\log_c b}{\log_c a}$$

$$* \log_a b = \frac{1}{\log_b a}$$

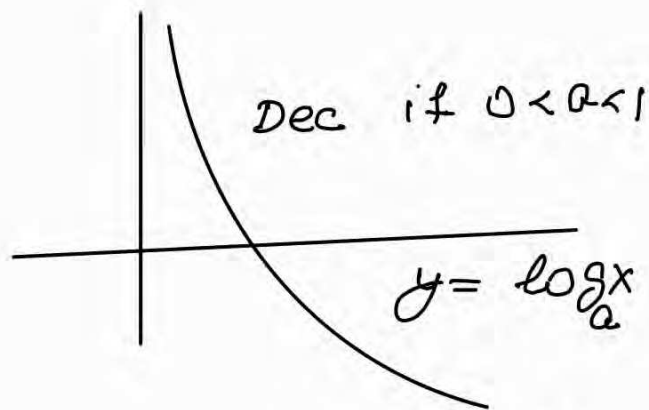
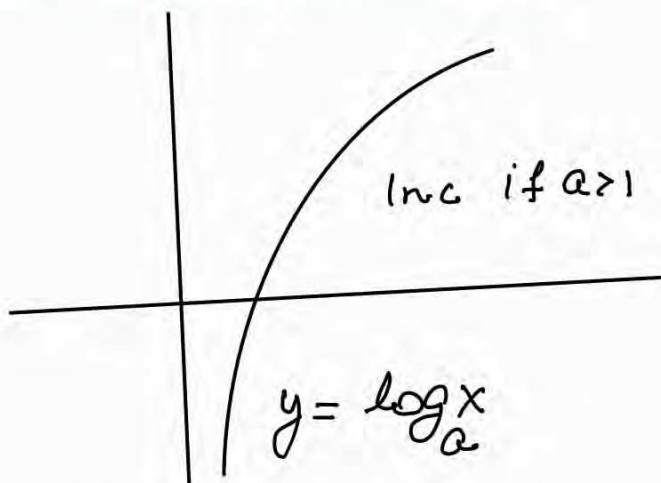
$$* a^{\log_c b} = b^{\log_c a}$$

$$* \log_a m^x = \frac{x}{y} \log_a m$$

$$* \log_{a_1} a_2 \cdot \log_{a_2} a_3 \cdot \log_{a_3} a_4 \dots$$

$$= -\log_{a_{n-1}} a_n = \log_{a_1} a_m$$

Logarithmic Inequalities

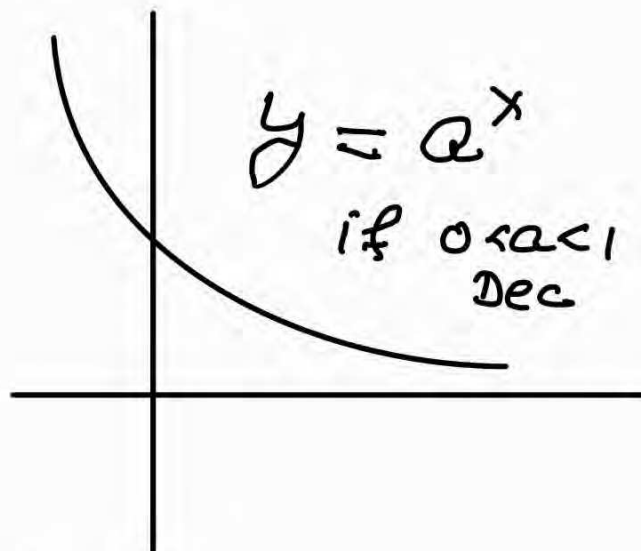
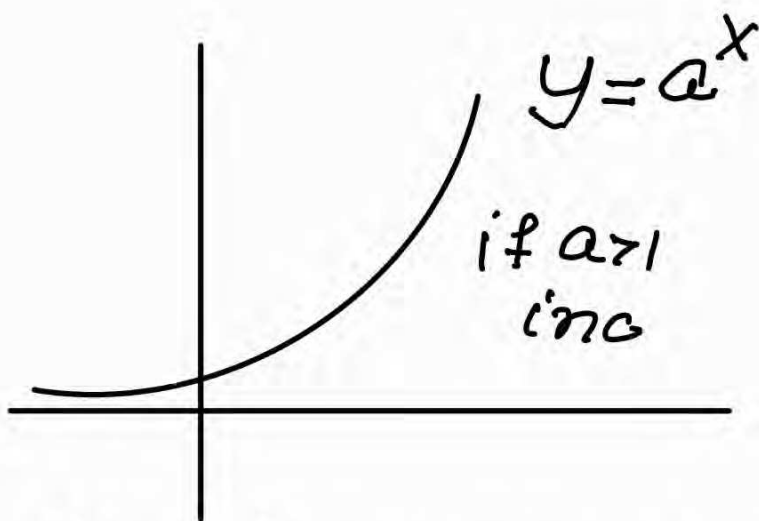


$\Rightarrow \log_a x > \log_a y$ where $a > 1$ then $x > y$

$\log_a x < \log_a y$ where $0 < a < 1$ then $x < y$

Exponential Inequalities

ATDB.uno



$a^x > a^y$ where $a > 1 \Rightarrow x > y$

$a^x > a^y$ where $0 < a < 1 \Rightarrow x < y$.

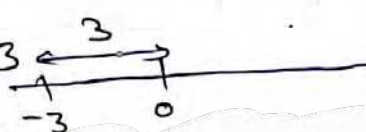
या हराओ को farak nahi Padtae

* Decrease Function kisi bhi Inequality mai lagao
या हराओ sign of inequality is reversed

Inc. Eg: a^x ($a > 1$), $\log_a(x)$ ($a > 0$)

Dec. Eg: a^x ($0 < a < 1$), $\log_a(x)$ ($0 < a < 1$)

* $|x|$ denotes distance b/w 0 & x on numberline

eg: $|1-3|=3$  $|x| \geq 0$ $\rightarrow |x| \in [0, \infty)$

* $|-x|=|x|$, $|x|^2=x^2$, $|xy|=|x|\cdot|y|$, $|\frac{x}{y}|=\frac{|x|}{|y|}$ $y \neq 0$, $\sqrt{x^2}=|x|$

* $\sqrt[2n]{x^{2n}}=|x|$, $\sqrt[2n+1]{x^{2n+1}}=x$ * $|x| \geq x$ $|x|=x \Leftrightarrow x \geq 0$
 $|x| > x \Leftrightarrow x < 0$

* $|x| \leq a$, $a \in \mathbb{R}^+ \Rightarrow x \in [-a, a]$ * $|x| = a$, $a \in \mathbb{R}^+$

* $a \leq |x| \leq b$, $a, b \in \mathbb{R}^+$ $\Rightarrow x \in (-\infty, -a] \cup [a, \infty)$

$\Rightarrow a \leq x \leq b$ OR $-b \leq x \leq -a$



Irrational Inequalities

$$\sqrt{f(x)} \geq g(x)$$

$$f(x) \geq 0 \rightarrow \text{cloud icon} \text{ (A)}$$

case ① if $g(x) \geq 0$ cloud icon

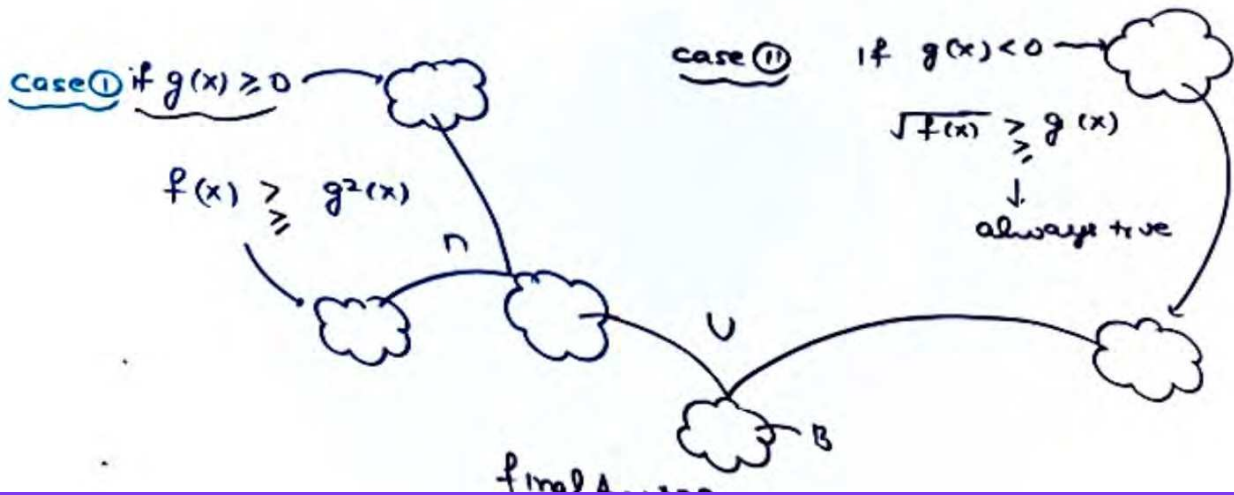
$$f(x) \geq g^2(x)$$

case ②

if $g(x) < 0$ cloud icon

$$\sqrt{f(x)} \geq g(x)$$

always true



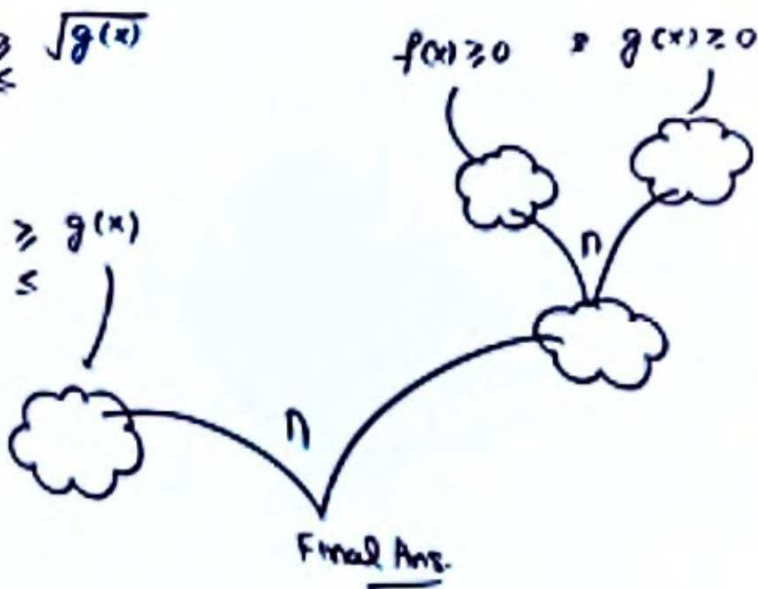


Irrational Inequalities

$$\sqrt{f(x)} \geq \sqrt{g(x)}$$

S.O.S

$$f(x) \geq g(x)$$



Irrational Inequalities

$$\sqrt{f(x)} < g(x)$$

$$f(x) \geq 0 \rightarrow \text{cloud} \text{ (A)}$$

case ① if $g(x) \geq 0$

$$f(x) < g^2(x)$$

case ②

if $g(x) < 0$

$$\sqrt{f(x)} < g(x)$$

↓

$x \in \phi$

$x \in \phi$

Final Ans: $A \cap B$



Using Triangle Inequality

P₉: $||a| - |b|| \leq |a + b| \leq |a| + |b|$

$|a+b| = ||a|-|b|| \Leftrightarrow ab \leq 0$

$|a+b| = |a|+|b| \Leftrightarrow ab \geq 0$

Ex: $||2|-|-3|| \leq |2+(-3)| \leq |2|+|-3|$

$1 \leq 1 \leq 5$

Ex: $||-5|-|-6|| \leq |-5-6| \leq |-5|+|-6|$
 $1 \leq 11 \leq 11$

b → -b

$||a|-|-b|| \leq |a-b| \leq |a|+|-b|$

P₁₀: $||a|-|b|| \leq |a-b| \leq |a|+|b|$

$|a-b| = |a|+|b| \Leftrightarrow a \cdot b \leq 0$

$|a|-|b| = |a-b| \Leftrightarrow ab \geq 0$

Very important points to Note

$||a|-|b|| = |a+b| \Leftrightarrow ab < 0$

$||a|-|b|| \leq |a+b| \leq |a|+|b|$

$|a+b| = |a|+|b| \Leftrightarrow ab \geq 0$

$|a+b| < |a|+|b| \Leftrightarrow ab < 0$

$||a|-|b|| < |a+b| \Leftrightarrow ab > 0$

$||a|-|b|| = |a-b|$
 $\Downarrow$
 $ab \geq 0$

★ $||a|-|b|| \leq |a-b| \leq |a|+|b|$

$|a-b| = |a|+|b| \Leftrightarrow ab \leq 0$

$||a|-|b|| < |a-b| \Leftrightarrow ab < 0$

$|a-b| < |a|+|b| \Leftrightarrow ab > 0$



Two Damdaar Properties



1. $|x + y| = |x| + |y| \Leftrightarrow x \cdot y \geq 0$

2. $|x - y| = |x| + |y| \Leftrightarrow x \cdot y \leq 0$

Ex: $|x^2 - 5x + 7| + |x^2 - 6x + 5| = |x + 2|$

$|a| + |b| = |a - b|$
 $|x^2 - 5x + 7| + |x^2 - 6x + 5| = |x^2 - 5x + 7 - (x^2 - 6x + 5)|$

$\Downarrow$
 $(x^2 - 5x + 7)(x^2 - 6x + 5) \leq 0$

$D = (-5)^2 - 4 \cdot 7 < 0$

$a = 1 > 0 \Downarrow$

always +ve

$\Rightarrow x^2 - 6x + 5 \leq 0$

$(x - 5)(x - 1) \leq 0$

Ex: Solve:

$|x - 3| + |2 - x| = 1$
 $|a| + |b| = |a + b|$

$\Downarrow$

$(x - 3)(2 - x) \geq 0$

$-(x - 3)(x - 2) \geq 0$

$(x - 3)(x - 2) \leq 0$

$\begin{matrix} + & - & + \\ & 2 & 3 \end{matrix}$

$x \in [2, 3]$

$x \in [1, 5]$

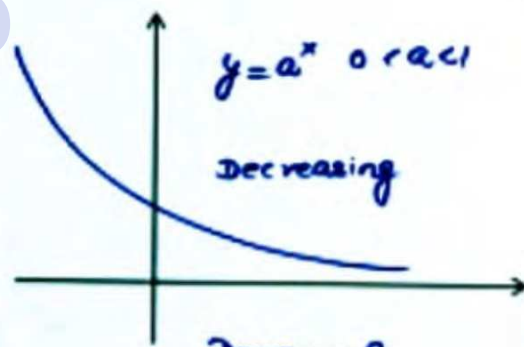


Exponential Inequality



Domain: $\mathbb{R}$

Range: $\mathbb{R}^+$



Domain: $\mathbb{R}$

Range: $\mathbb{R}^+$

Ex: $3^{x-2} > 3^{2x-3}$ find range of x

$x - 2 > 2x - 3$

$x < 1 \Rightarrow x \in (-\infty, 1)$

Ex: $(\frac{1}{2})^{x^2-2x} > (\frac{1}{2})^{4x-8}$ find range of x

$x^2 - 2x < 4x - 8 \Rightarrow x^2 - 6x + 8 < 0$

$(x - 2)(x - 4) < 0$
 $x \in (2, 4)$



Characteristic & Mantissa



logarithm of any no: to a given base always has two parts an integral part called characteristic and a fractional part called mantissa

Ex: $\log_2 16 = 4.0$

Integral part → characteristic = 4
fractional part → mantissa = 0

characteristic ∈ Integer.
Mantissa ∈ [0,1)

Ex: find characteristic of $\log_2 17$
clearly: $2^4 < 17 < 2^5 \rightarrow 4 < \log_2 17 < 5 \rightarrow \log_2 17 = 4.\text{something}$
characteristic = 4

$\log_{10} N$



$f \in [0,1)$

Ex: $\log_2 1 = 0, \log_{10} 1 = 0$
Ex: $\log_2 0$ is not defined in reals.

if $0 < N < 1$

$N = 0.689, 0.976 \rightarrow \frac{1}{10} \leq N < 1 \Rightarrow -1 \leq \log_{10} N < 0 \Rightarrow \log_{10} N = -1 + f$

$N = 0.078, 0.09705 \rightarrow \frac{1}{100} \leq N < \frac{1}{10} \Rightarrow -2 \leq \log_{10} N < -1 \Rightarrow \log_{10} N = -2 + f$

$N = 0.0078, 0.00965 \rightarrow \frac{1}{1000} \leq N < \frac{1}{100} \Rightarrow -3 \leq \log_{10} N < -2 \Rightarrow \log_{10} N = -3 + f$

characteristic

- 1
- 2
- 3

if $0 < N < 1$, $\log_{10} N$ has characteristic = - (No. of 0's immediately to right of decimal in N before a + 1 significant digit starts)

log₁₀ N KIKAHAAANI

$$f \in [0,1)$$

Ex: $\log_2 1 = 0$, $\log_{10} 1 = 0$
 Ex: $\log_2 0$ is not defined in reals

If $N \geq 1$		characteristic
$N = \overline{1.63}, \overline{9.85}$	$* 1 \leq N < 10 \Rightarrow 0 \leq \log_{10} N < 1 \Rightarrow \log_{10} N = 0 + f$	0
$N = \overline{95.62}, \overline{88.55}$	$* 10 \leq N < 100 \Rightarrow 1 \leq \log_{10} N < 2 \Rightarrow \log_{10} N = 1 + f$	1
$N = \overline{110.23}, \overline{999.25}$	$* 100 \leq N < 1000 \Rightarrow 2 \leq \log_{10} N < 3 \Rightarrow \log_{10} N = 2 + f$	2

If $N \geq 1$, $\log_{10} N$ has characteristic = $\left(\text{No. of significant digits to left of decimal in } N \right) - 1$

ATDB.uno

$* \log_a N = \underset{\substack{\downarrow \\ \text{Integer part} \\ \text{(characteristic)}}}{I} + \underset{\substack{\leftarrow \text{fractional part} \in [0,1) \\ \text{(mantissa)}}}{F}$

(mantissa is never negative)

if $N \geq 1$ characteristic of $\log_{10} N$
 = $\left(\text{No. of significant digits before decimal} \right) - 1$

if $0 < N < 1$
 characteristic of $\log_{10} N =$
 = $- \left(\text{No. of zeros immediately after decimal before significant digits starts} + 1 \right)$

$* \log_{10} 2 = 0.3010$
 $* \log_{10} 3 = 0.4771$
 $* \log_{10} 7 = 0.8451$

$\pi \approx 3.14$
 $\pi^2 \approx 9$
 $\pi/2 \approx 1.57$
 $3\pi/2 \approx 4.7$



$$* a \leq x < b \quad x, a, b \in I$$

No. of possible values of $x = 6 - a$

* $a < x \leq b$, $x, a, b \in \mathbb{I}$

No. of possible values of $x = 6 - a$

* $a \leq x \leq b, x, a, b \in \mathbb{I}$

No. of possible values of $x = b - a + 1$

* $a < x < b, a, b, x \in \mathbb{I}$

No. of values of $x = b - a - 1$

b-a se eak end
point answer mai
shamil hota hai

if $\frac{a}{b} = \frac{c}{d}$ then

* Compendio Dividendo

$$\frac{a+b}{a-b} = \frac{c+d}{c-d} \quad \text{or} \quad \frac{a-b}{a+b} = \frac{c-d}{c+d}$$

$$* \frac{a+b}{b} = \frac{c+d}{d} \quad \text{or} \quad \frac{a}{a+b} = \frac{c}{c+d}$$

Ex: find α, β if $3\alpha + 2\beta = 13$

$$3. \frac{\alpha-2}{1} = \frac{\beta-1}{2}$$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \dots$ then

$$* \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \dots = \frac{k_1 a_1 + k_2 b_1 + k_3 c_1 + \dots}{k_1 a_2 + k_2 b_2 + k_3 c_2 + \dots}$$

$$\frac{\alpha-2}{1} = \frac{\beta-1}{2} = \frac{3\alpha-6+2\beta-2}{3+4}$$

$$* \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \dots = \frac{a_1 + b_1 + c_1 + \dots}{a_2 + b_2 + c_2 + \dots}$$

$$\frac{\alpha-2}{1} = \frac{\beta-1}{2} = \frac{13-8}{1} = 5 \mid 7$$

$$\alpha = 19/7, \beta = 1 + 2/7 = 16/7$$

The End