# PRAYAS

## JEE 2026



### Mathematics

# Basic Maths

Lecture - 16

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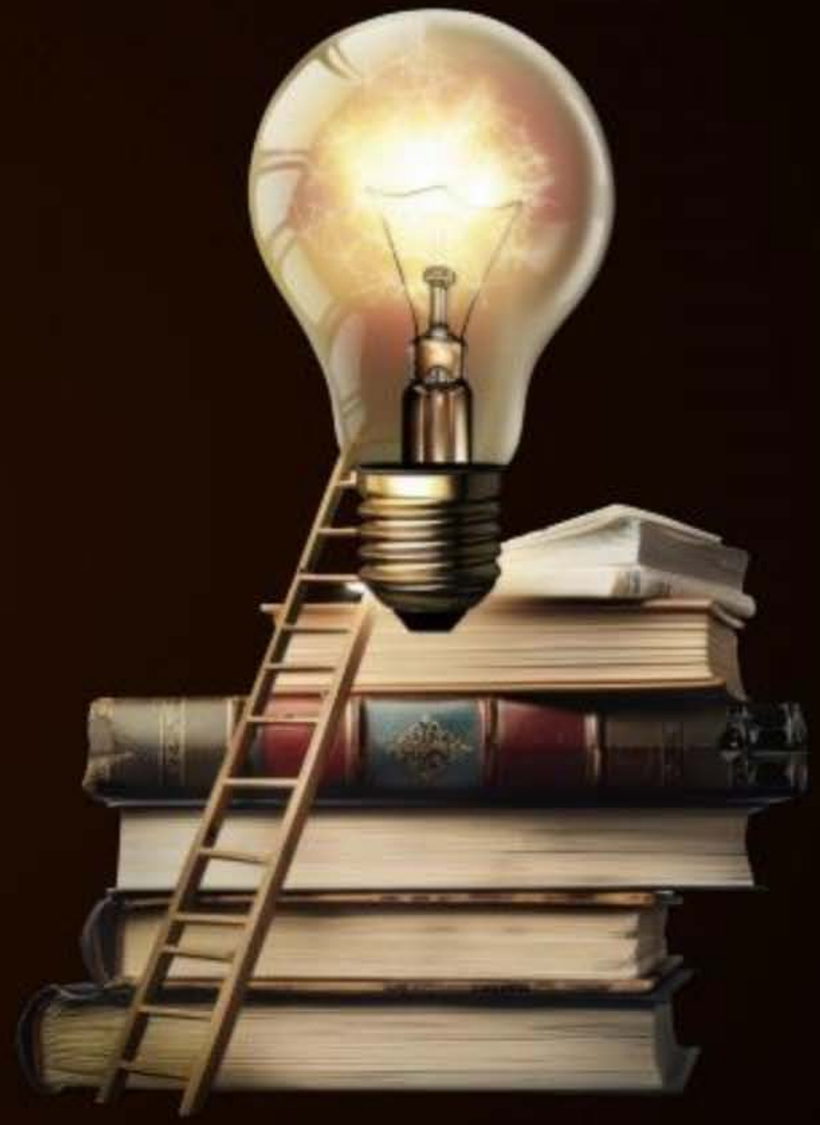
# Topics *to be covered*

A Graphs involving Modulus

B Irrational Inequalities

C Problem Practice

ATDB.uno



# Homework Discussion

# QUESTION

(KTK 2)

$$\frac{1}{x-1}-\frac{4}{x-2}+\frac{4}{x-3}-\frac{1}{x-4}<\frac{1}{30}$$

$$\frac{1}{x-1}-\frac{1}{x-4}+\frac{4}{x-3}-\frac{4}{x-2}<\frac{1}{30}$$

$$\frac{x-4-x+1}{(x-1)(x-4)}+\frac{4x-8-4x+12}{(x-2)(x-3)}<\frac{1}{30}$$

$$\frac{-3}{x^2-5x+4}+\frac{4}{x^2-5x+6}<\frac{1}{30}$$

Let $x^2-5x=t$

$$\frac{4}{t+6}-\frac{3}{t+4}<\frac{1}{30}$$

$$\frac{4t+16-3t-18}{(t+4)(t+6)}<\frac{1}{30}$$

$$\frac{(t-2)}{t^2+10t+24}-\frac{1}{30}<0$$

$$\frac{30t-60-t^2-10t-24}{30(t+4)(t+6)}<0.$$

$$\frac{-t^2+20t-84}{(t+4)(t+6)}<0.$$

$$\frac{t^2-20t+84}{(t+4)(t+6)}>0$$

$$\frac{(t-6)(t-14)}{(t+4)(t+6)}>0$$

$$\frac{(x^2-5x-6)(x^2-5x-14)}{(x^2-5x+4)(x^2-5x+6)}>0$$

$$\frac{(x-6)(x+1)(x-7)(x+2)}{(x-4)(x-1)(x-2)(x-3)}>0$$

Ans. $x \in (-\infty,-2)\cup(-1,1)\cup(2,3)\cup(4,6)\cup(7,\infty)$

# QUESTION

(KTK 4)

For the equation $\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$, which of the following do not hold good?

A. no real solution

B. one prime solution

C. one integral solutions

D. no irrational solution

$$\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$$

$$\frac{\log_3 x}{\log_3 3\sqrt{x}} + \frac{\log_3 \sqrt{x}}{\log_3 3x} = 0$$

$$\frac{\log_3 x}{1+\frac{1}{2}\log_3 x} + \frac{\frac{1}{2}\log_3 x}{1+\log_3 x} = 0 \quad \text{let } \log_3 x = t$$

$$t\left(\frac{1}{1+t/2} + \frac{1}{2(1+t)}\right) = 0$$

$t = 0$ or $\frac{2}{2+t} + \frac{1}{2(1+t)} = 0$

$x = 1$

$4 + 4t + 2 + t = 0$

$t = -6/5$

$x = 3^{-6/5} = \frac{1}{3^{6/5}} = \frac{1}{3\sqrt[5]{3}}$

$3^{6/5} = 3^{\frac{5+1}{5}} = 3^{1+\frac{1}{5}} = 3 \cdot 3^{1/5} = 3\sqrt[5]{3}$.

Ans. A, B, D

# Aao Machaay Dhamaal
# Deh Swaal pe Deh Swaal

Lecture = 15

Ans.

**Tahol.** Solve $|x|_{T_1} + |x-1|_{T_2} \geq 7$.

??

$T_1$: ⊖ (0) ⊕ (1) ⊕

$T_2$: ⊖ (0) ⊖ (1) ⊕

=> Case 01 $x \geq 1$

=> $x + x - 1 \geq 7$.

=> $2x - 8 \geq 0$.

=> $x - 4 \geq 0$.

$\cap$ ↝ $x \geq 1$

galti Batao

Case 02. $0 \leq x < 1$

=> $x - x + 1 \geq 7$

=> $1 \geq 7$ (Not possible)

$\cap$ => $x \in \phi$

ATDB.uno

??

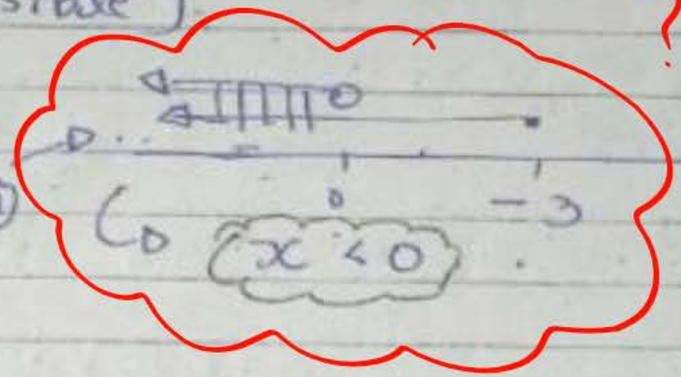
Case 03. $x < 0$

=> $-x - x + 1 \geq 7$.

=> $-2x - 6 \geq 0$

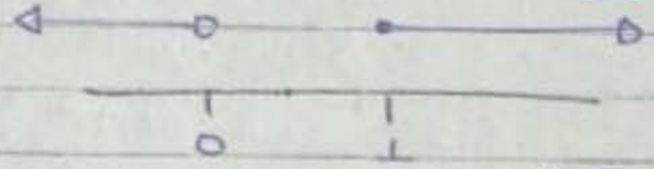
=> $x + 3 \leq 0$.

$\cap$




↳ $x < 0$

=> Union of all cases




↳ $x \in (-\infty, 0) \cup [1, \infty)$

KTK-6

(A) $x^{\log_{10} x} = 100x$

$\log_{10} x^{\log_{10} x} = \log_{10} 100x$

$\log_{10} x \log_{10} x = \log_{10} 100x$

$(\log_{10} x)^2 = \log_{10} 100x$

$2\log_{10} x = \log_{10} 100x$ ??



$\log_{10} x^2 = \log_{10} 100x$

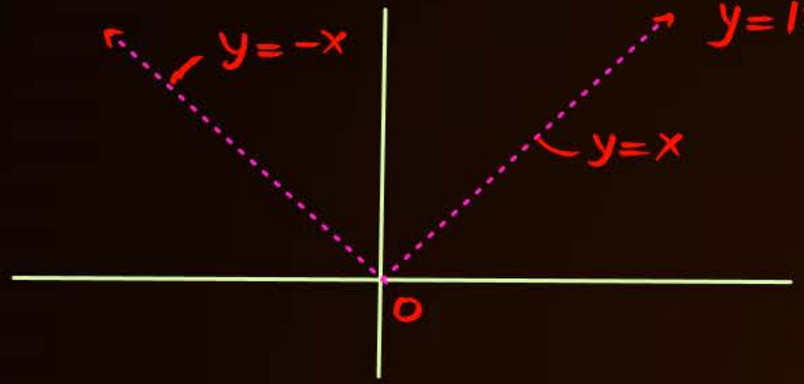
$x^2 = 100x$

$\boxed{x = 100}$

→ rational

composite

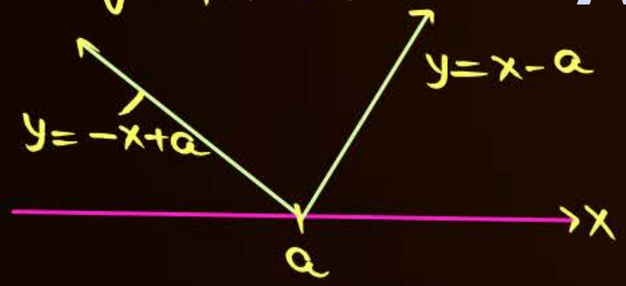
# Problems Involving Graph of Modulus Function



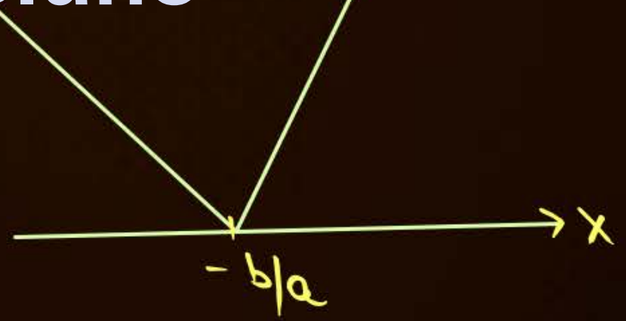

$$y=|x|=\begin{cases} x & x\geq 0 \\ -x & x<0 \end{cases}=\begin{cases} x & x>0 \\ -x & x\leq 0 \end{cases}$$



$y=|x-a|$




$y=|ax+b|$




$ax+b=0$

$x=-\frac{b}{a}$

Ex: $y = |3x-2|$

slope -ve

$y = -3x+2$

(0,2)

slope +ve

$y = 3x-2$

$\frac{2}{3}$

Ex: $y = |2x+3|$

$m = \text{slope} = 3$

$L_2: y = -2x-3$

$L_1: y = 2x+3$

$\frac{-3}{2}$



Ex: $y = |x-1| + |x-2|$

-ve | +ve | +ve

-ve 1 -ve 2 +ve

$$y = \begin{cases} -x+1-x+2 & x \le 1 \\ x-1+2-x & 1 < x < 2 \\ x-1+x-2 & x \ge 2 \end{cases} = \begin{cases} 3-2x & x \le 1 \\ 1 & 1 < x < 2 \\ 2x-3 & x \ge 2 \end{cases}$$

$y = 3-2x$

$y = 2x-3$

$y = 1$

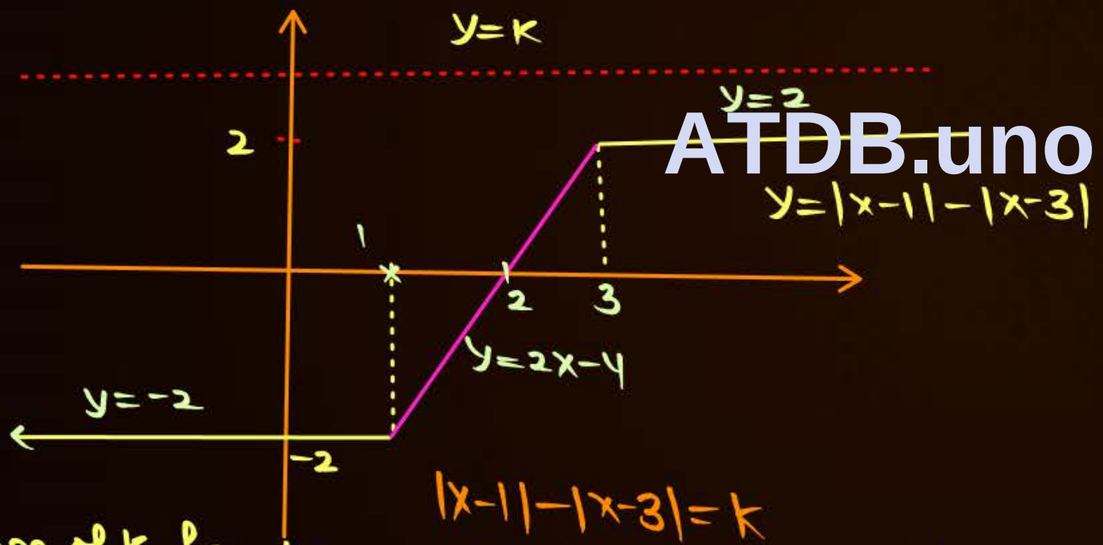
1

1

2

$y = |x-1| - |x-3|$


$$y = \begin{cases} -x+1+x-3 & x \le 1 \\ x-1+x-3 & 1 < x < 3 \\ x-1-(x-3) & x \ge 3 \end{cases} = \begin{cases} -2 & x \le 1 \\ 2x-4 & 1 < x < 3 \\ 2 & x \ge 3 \end{cases}$$




$|x-1|-|x-3| = k$

find Range of k for

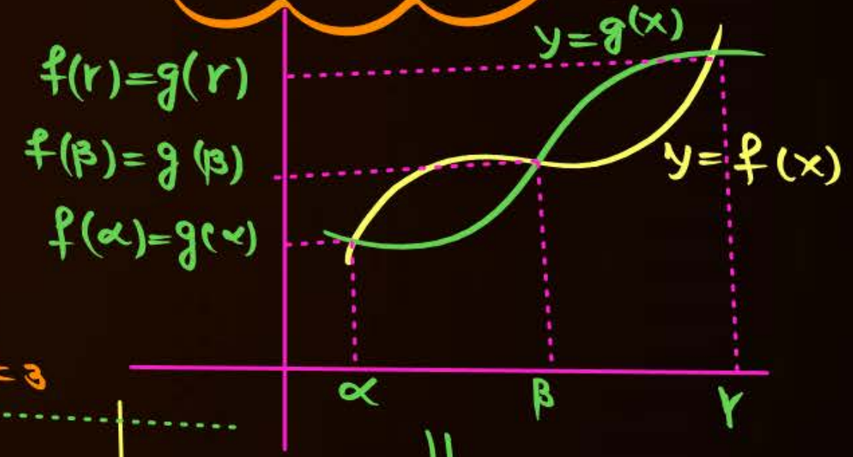
(a) no real soln $\rightsquigarrow k \in (-\infty, -2) \cup (2, \infty)$

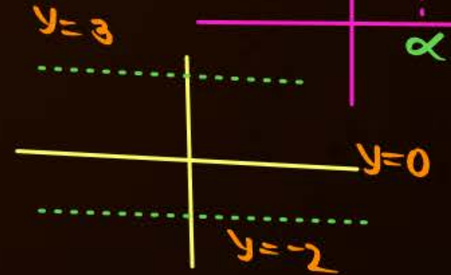
(b) Exactly one real soln $\rightsquigarrow k \in (-2, 2)$

(c) $\infty$ many soln $\rightsquigarrow k = 2, -2$

y=a is a line parallel to x axis

No: of real solns of $f(x) = g(x)$ is obtained by no: of points of Intersection of graphs $y = f(x)$ & $y = g(x)$




No: of real soln = 3

QUESTION

Draw graph of $y = |x - 2|$.

# QUESTION

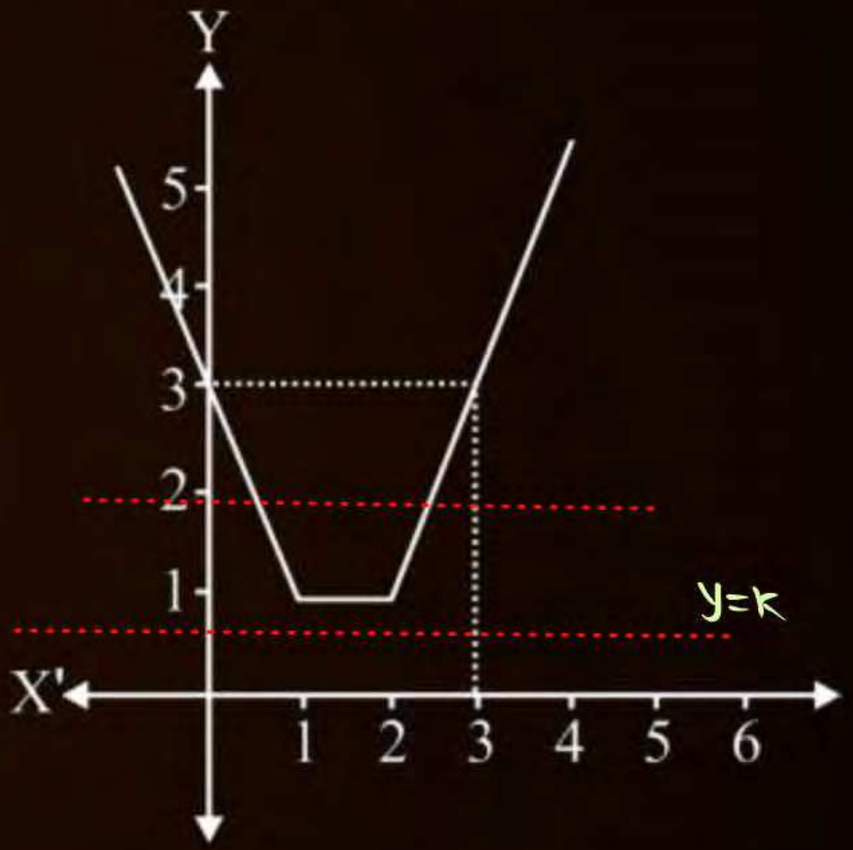
$f|x-1|+|x-2|=k$ then find range of k for which the equation has

(a) No solution ⟶ $k \in (-\infty, 1)$

(b) Two solution only ⟶ $k \in (1, \infty)$

(c) Infinitely many solution ⟶ $k = 1$.

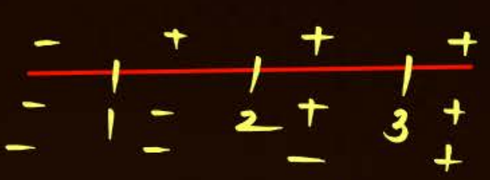




# QUESTION

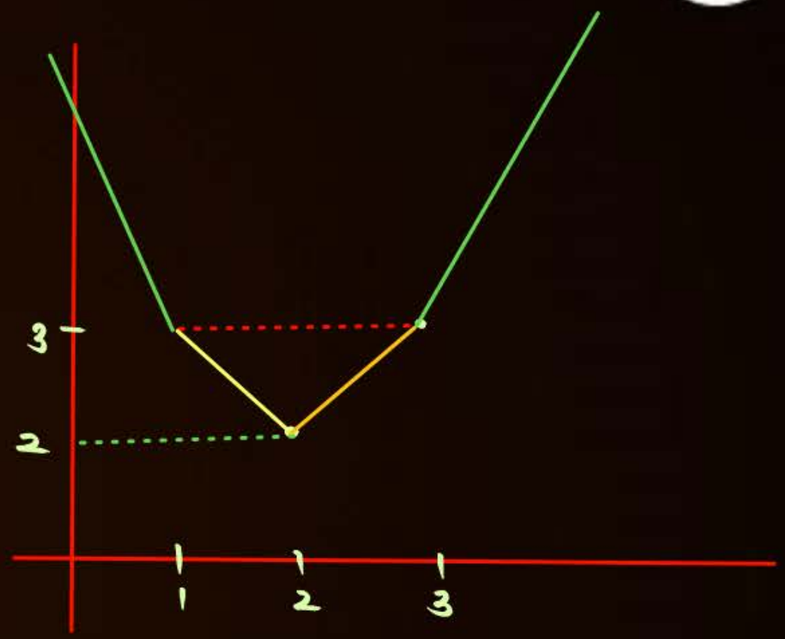
Draw graph of $y = |x - 1| + |x - 2| + |x - 3|$.

($T_1$, $T_2$, $T_3$)




$$y = \begin{cases} -x+1-x+2-x+3, & x \le 1 \\ x-1-x+2-x+3, & 1 < x < 2 \\ x-1+x-2-x+3, & 2 \le x \le 3 \\ x-1+x-2+x-3, & x > 3 \end{cases}$$

$$= \begin{cases} 6-3x & x \le 1 \\ 4-x & 1 < x < 2 \\ x & 2 \le x \le 3 \\ 3x-6 & x > 3 \end{cases}$$



# QUESTION

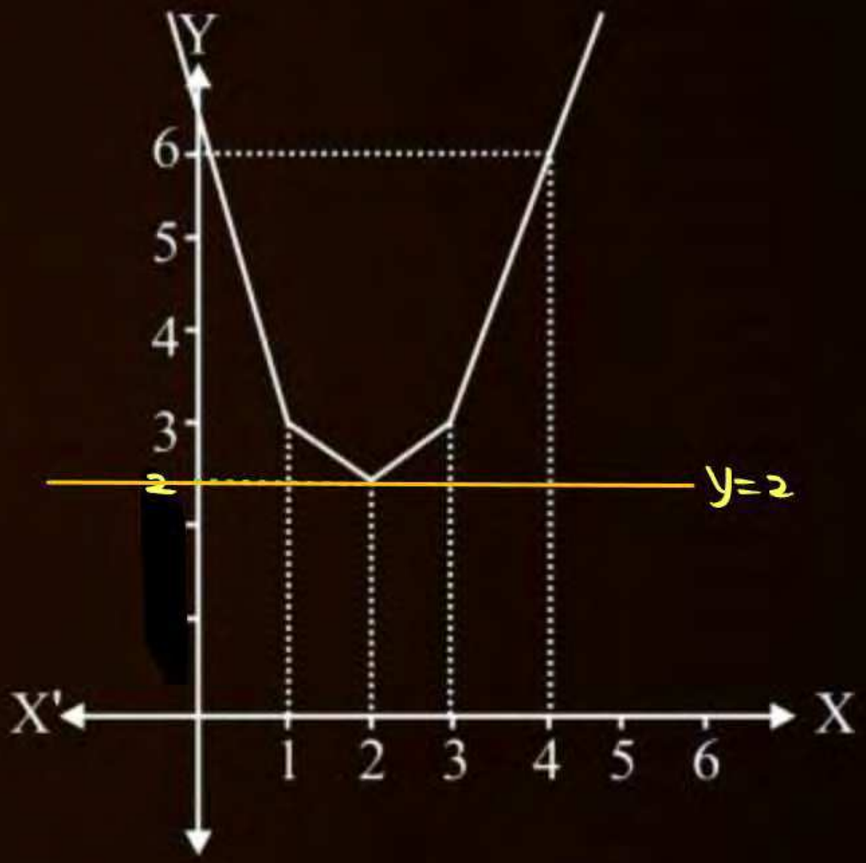
If $|x-1|+|x-2|+|x-3| = k$ then find range of k for which the equation has

(a) No solution $\rightarrow k \in (-\infty, 2)$

(b) Only one solution $\rightarrow k = 2$

(c) Two solution $\rightarrow k \in (2, \infty)$





# QUESTION

Tahol

If $|x - 5| - |x - 7| = k$ then for

(A) K = 2 the equation has infinitely many solution

(B) K = –2 the equation has infinitely many solution



(C) $K \in (2, \infty)$ the equation has no solution

(D) K ∞ (–2, 2) the equation has exactly one solution

# QUESTION


Draw graph of $y = |x| + |x - 2|$ and answer the following.

Find the range of k for which the equation $|x| + |x - 2| = k$ has

A. No Solution

B. Infinitely Many Solutions

C. Only two solutions

## QUESTION


Draw the graph of $y = |x - 4| - |x + 3|$ and answer the following.

Find the range of k for which the equation $|x - 4| - |x + 3| = k$ has

A. No Solution

B. Infinitely Many Solutions

C. Only two solutions

# QUESTION

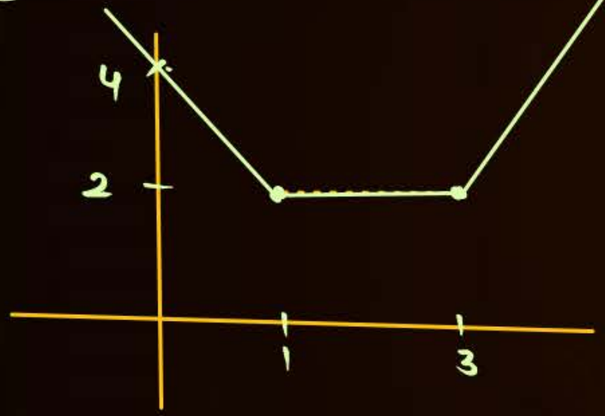
Tanoy

Consider the function $f(x) = |x - 1| - 2|x + 2| + |x + 3|$

| Column-I | Column-II |
|---|---|
| (A) If $f(x) = k$ has no solution, then $k \in$ | (p) $(2, 4)$ |
| (B) If $f(x) = k$ has one solution, then $k \in$ | (q) $(-\infty, -2) \cup (4, \infty)$ |
| (C) If $f(x) = k$ has two solution, then $k \in$ | (r) $(-2, 2) \cup \{4\}$ |
| (D) If $f(x) = k$ has more then two solution, then $k \in$ | (s) $\{-2, 2\}$ |

Ex: $y = |x-1| + |x-3|$


$$y = \begin{cases} -x+1-x+3 & x<1 \\ x-1-x+3 & 1\le x\le 3 \\ x-1+x-3 & x>3 \end{cases} = \begin{cases} 4-2x & x<1 \\ 2 & 1\le x\le 3 \\ 2x-4 & x>3 \end{cases}$$




$f(x) = |x-1| + |x-3|$

Range of $f(x) = [2, \infty)$

y axis pe Range milti hai

$x=1 < x=2 < x=3$

$y = |x-1| + |x-2| + |x-3|$

$y_{min}$ @ $x=2$

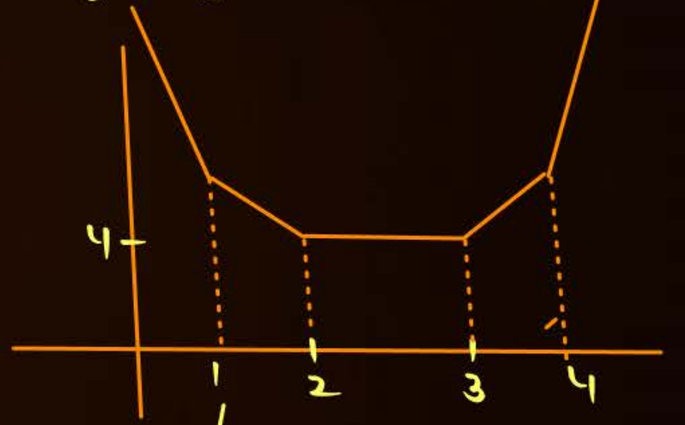
Range $[1+0+1, \infty) = [2, \infty)$

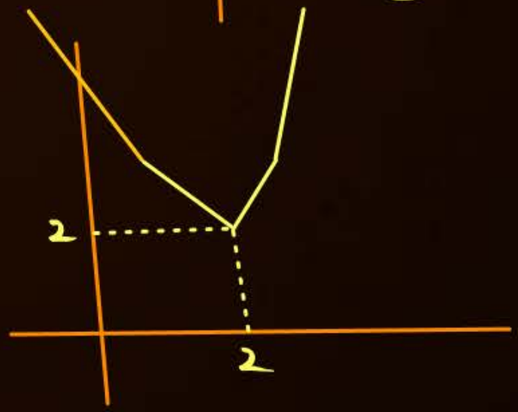
$x=1 < x=2 < x=3 < x=4$

$y = |x-1| + |x-2| + |x-3| + |x-4|$

$y_{min}$ @ $x \in [2,3]$

Range : $[1+0+1+2, \infty) = [4, \infty)$



# QUESTION

$y = |x-1| + |x-2| + |x-3|$ and find $y_{min}$.

$y_{min}$ @ $x = 2$

$y_{min} = 1 + 0 + 1 = 2$

Ex: $y = |x+1| + |x-3| + |x+5| + |x-4| + |x-7|$

$x = -5 < x = -1 < x = 3 < x = 4 < x = 7$

$y = |x+5| + |x+1| + |x-3| + |x-4| + |x-7|$

$y_{min}$ at $x = 3$.

Range $[8+4+0+1+4, \infty) = [17, \infty)$

# QUESTION

Given, $f(x) = |x| + 2|x-1| + |x-2| + |x-4| + |x-6| + 2|x-10|$ where $x \in R$. Find the minimum value of f.

$x=0 < x=1 \leq x=1 < x=2 < x=4 < x=6 < x=10 \leq x=10$

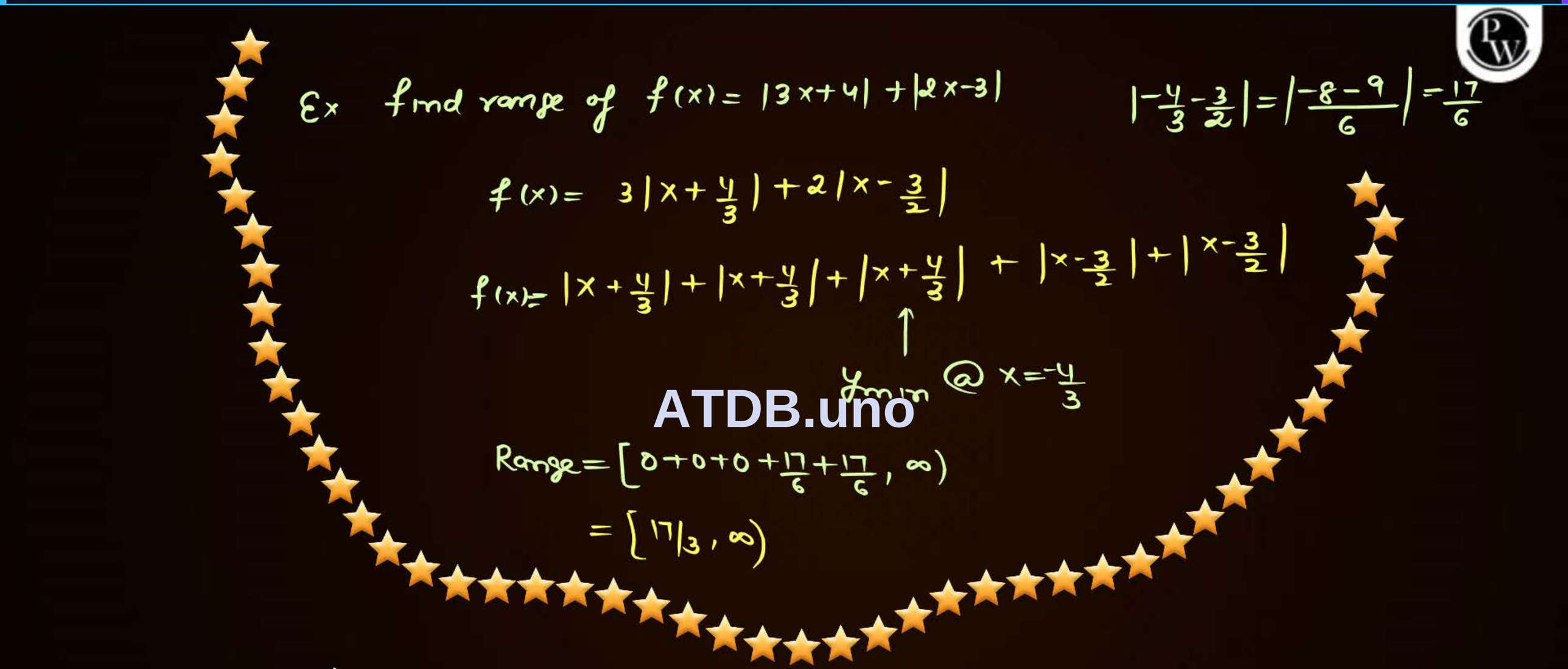
$y = |x| + |x-1| + |x-1| + |x-2| + |x-4| + |x-6| + |x-10| + |x-10|$

$y_{min}$ @ $x \in [2,4]$

$y_{min} = 2 + 1 + 0 + 2 + 4 + 8 + 8 = 26$



Range $f(x) = [26, \infty)$



Ex find range of $f(x) = |3x+4| + |2x-3|$

$\left|-\frac{4}{3}-\frac{3}{2}\right| = \left|\frac{-8-9}{6}\right| = \frac{17}{6}$

$f(x) = 3\left|x+\frac{4}{3}\right| + 2\left|x-\frac{3}{2}\right|$

$f(x) = \left|x+\frac{4}{3}\right| + \left|x+\frac{4}{3}\right| + \left|x+\frac{4}{3}\right| + \left|x-\frac{3}{2}\right| + \left|x-\frac{3}{2}\right|$

$y_{min}$ @ $x = -\frac{4}{3}$



$\text{Range} = \left[0+0+0+\frac{17}{6}+\frac{17}{6}, \infty\right)$

$= [17/3, \infty)$

$$y = |3x+4| + |2x-3|$$

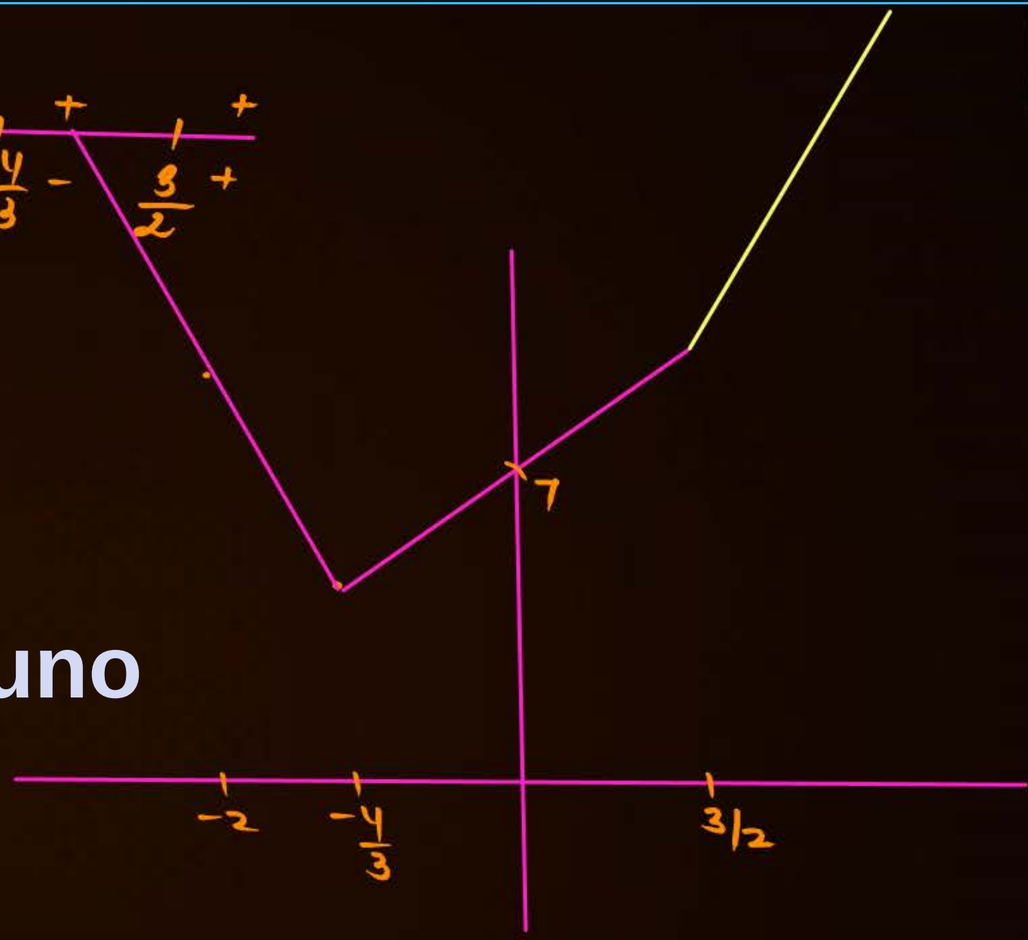

$$y = \begin{cases} -3x-4-2x+3, & x \le -\frac{4}{3} \\ 3x+4-2x+3, & -\frac{4}{3} < x < \frac{3}{2} \\ 3x+4+2x-3, & x \ge \frac{3}{2} \end{cases}$$



$$y = \begin{cases} -5x-1, & x \le -\frac{4}{3} \\ x+7, & -\frac{4}{3} < x < \frac{3}{2} \\ 5x+1, & x \ge \frac{3}{2} \end{cases}$$

| x | $-4/3$ | $-2$ |
|---|---|---|
| y | $17/3$ | 9 |

| x | $-4/3$ | 0 |
|---|---|---|
| y | $17/3$ | 7 |



# Irrational Inequalities

$f(x) \geq 0$ ⟶ (A)

$\sqrt{f(x)} < g(x)$

$\leq$

Case (I) if $g(x) \geq 0$ ⟶

$f(x) < g^2(x)$

$\leq$

$\cap$

B

Case (II) if $g(x) < 0$ ⟶

$\sqrt{f(x)} < g(x)$

↓

$x \in \phi$

$x \in \phi$

final Ans: $A \cap B$

# Irrational Inequalities

$f(x) \geq 0$ — A

$\sqrt{f(x)} \geq g(x)$

Case ① if $g(x) \geq 0$

$f(x) \geq g^2(x)$

∩

Case ⑪ if $g(x) < 0$

$\sqrt{f(x)} \geq g(x)$

↓

always true

∪

B

final Ans : A ∩ B

# Irrational Inequalities

$\sqrt{f(x)} \gtrless \sqrt{g(x)}$

S·B·S

$f(x) \gtrless g(x)$

$f(x) \geq 0$ & $g(x) \geq 0$

# QUESTION

Solve for x if, $4 - x < \sqrt{x^2 - 2x}$

$x^2 - 2x \geq 0$

$x(x-2) \geq 0$

non-ve

$x \in (-\infty, 0] \cup [2, \infty)$ — (A)

Ineq mai dono side hamesha odd power toh laga sakte hai but Even power ke liyay dono sides negative nahi honi chahiyay

**case (I)** if $4 - x \geq 0 \rightarrow x \leq 4$

$4 - x < \sqrt{x^2 - 2x}$

non-ve, non-ve

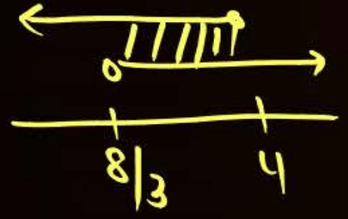
S.B.S

$(4-x)^2 < x^2 - 2x$

$16 + x^2 - 8x < x^2 - 2x$

$6x > 16$

$x > 8/3$




$\cap$

$x \in (8/3, 4]$

**case (II)** if $4 - x < 0 \Rightarrow x > 4$

$4 - x < \sqrt{x^2 - 2x}$

-ve, non-ve

always true.

$x \in (4, \infty)$

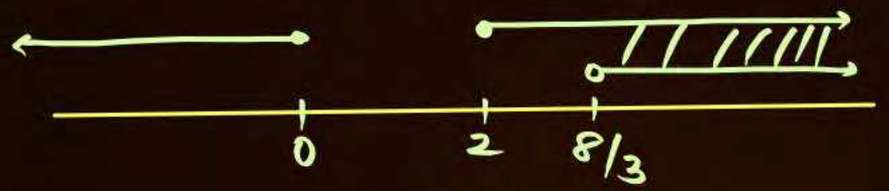
UNION

$x \in (8/3, 4] \cup (4, \infty) = (8/3, \infty)$ — (B)

$A \cap B$

$x \in (-\infty, 0] \cup [2, \infty)$

$x \in (8/3, \infty)$






Ans: $(8/3, \infty)$

# QUESTION

Solve for x, if $\sqrt{x^2 - 3x + 2} > x - 2$

$x^2 - 3x + 2 \geq 0$

$(x-1)(x-2) \geq 0$

$x \in (-\infty, 1] \cup [2, \infty)$ — (A)

**Case (I)** If $x - 2 \geq 0 \Rightarrow x \geq 2$

non-ve ← $\sqrt{x^2 - 3x + 2} > x - 2$ → non-ve

S.B.S $\quad x^2 - 3x + 2 > x^2 - 4x + 4$

$x > 2$

$\cap$

$x \in (2, \infty)$

**Case (II)** If $x - 2 < 0 \Rightarrow x < 2$

$\sqrt{x^2 - 3x + 2} > x - 2$

$\sqrt{\quad}$: non-ve ; $x - 2$: -ve

non-ve

always true

$x < 2$

UNION

$x \in (-\infty, \infty) - \{2\}$ — (B)

Ans: $A \cap B = (-\infty, 1] \cup (2, \infty)$ Ans

# QUESTION

Solution of the inequality, $x - 3 < \sqrt{x^2 + 4x - 5}$


A $(-\infty, -5] \cup [1, \infty)$

B $(-\infty, -5] \cup (7/5, \infty)$

C $(-5,3]$

D $(7/5, \infty)$

# QUESTION

Solve following inequalities

(i) $\frac{\sqrt{2x-1}}{x-2} < 1$ — Ans. $\left[\frac{1}{2}, 2\right) \cup (5, \infty)$

(ii) $x - \sqrt{1-|x|} < 0$ — Ans. $[-1, (\sqrt{5}-1)/2)$

(iii) $\sqrt{x^2-x-6} < 2x-3$ — Ans. $x \in [3, \infty)$

(iv) $\sqrt{x^2-6x+8} \le \sqrt{x+1}$ — Ans. $x \in \left[\frac{7-\sqrt{21}}{2}, 2\right] \cup \left[4, \frac{7+\sqrt{21}}{2}\right]$

(iii) $x^2-x-6 \ge 0 \Rightarrow (x-3)(x+2) \ge 0$

$x \in (-\infty, -2] \cup [3, \infty)$ — (A)

Tah 06

SBS

Case (I) if $2x-3 \ge 0 \Rightarrow x \ge \frac{3}{2}$

$\sqrt{x^2-x-6} < 2x-3$ — non-ve

non-ve

$x^2-x-6 < 4x^2+9-12x$

$3x^2-11x+15 > 0$

$D < 0$, $a > 0$

always +ve i.e $x \in R$

$x \in [3/2, \infty)$

Case (II) if $2x-3 < 0 \Rightarrow x < 3/2$

$\sqrt{x^2-x-6} < 2x-3$

non-ve — -ve

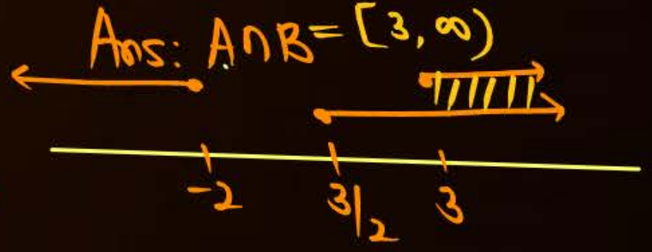
Not possible

$x \in \phi$

U

$x \in [3/2, \infty)$ — (B)

Ans: $A \cap B = [3, \infty)$



(iv) $\sqrt{x^2-6x+8} \le \sqrt{x+1}$

$x^2-6x+8 \ge 0$ & $x+1 \ge 0$

$(x-2)(x-4) \ge 0$ & $x \ge -1$

$x \in (-\infty, 2] \cup [4, \infty)$

SBS $\quad x^2-6x+8 \le x+1$

$x^2-7x+7 \le 0$

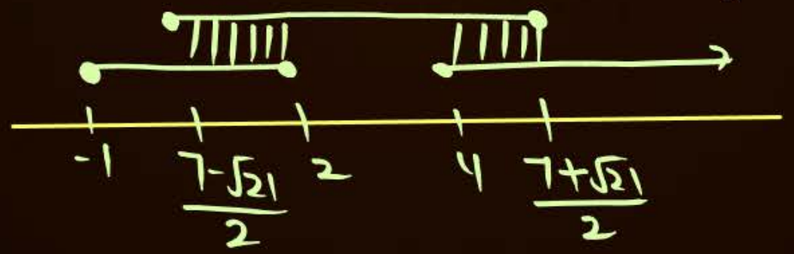
$x = \frac{7 \pm \sqrt{21}}{2}$

$\left(x - \frac{7+\sqrt{21}}{2}\right)\left(x - \frac{7-\sqrt{21}}{2}\right) \le 0$

$x \in \left[\frac{7-\sqrt{21}}{2}, \frac{7+\sqrt{21}}{2}\right]$

$x \in [-1, 2] \cup [4, \infty)$






$x \in \left[\frac{7-\sqrt{21}}{2}, 2\right] \cup \left[4, \frac{7+\sqrt{21}}{2}\right]$

# QUESTION [JEE Mains 2023 (1 Feb)]

Let $S = \{x: x \in \mathbb{R} \text{ and } (\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10\}$. Then n(S) is equal to

A. 6

B. 4

C. 0

D. 2

$$\log_{10}\left((\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4}\right) = \log_{10} 10$$

$$\log_{10}(\sqrt{3}+\sqrt{2})^{x^2-4} + \log_{10}(\sqrt{3}-\sqrt{2})^{x^2-4} = \log_{10} 10$$

Gadho / Gadhiyoo aisaa naa karo



$$\log_a(b+c) \neq \log_a b + \log_a c$$

Ex: $\log_2(8+4) \neq \log_2 8 + \log_2 4 = 3+2=5$.

$$\log_2 12$$

Ans. B

# QUESTION [JEE Mains 2023 (1 Feb)]

Let $S = \{x: x \in \mathbb{R} \text{ and } (\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10\}$. Then n(S) is equal to

A. 6

B. 4

C. 0

D. 2

$(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2}) = 3-2 = 1$

$\sqrt{3}-\sqrt{2} = \frac{1}{\sqrt{3}+\sqrt{2}}$

$(\sqrt{3}+\sqrt{2})^{x^2-4} + \left(\frac{1}{\sqrt{3}+\sqrt{2}}\right)^{x^2-4} = 10$

let $(\sqrt{3}+\sqrt{2})^{x^2-4} = t$

$t + \frac{1}{t} = 10$

$t^2 - 10t + 1 = 0$

$t = \frac{10 \pm \sqrt{96}}{2} = \frac{10 \pm 4\sqrt{6}}{2}$

$t = 5 \pm 2\sqrt{6} = (\sqrt{3}\pm\sqrt{2})^2$

$(\sqrt{3}+\sqrt{2})^{x^2-4} + \frac{1}{(\sqrt{3}+\sqrt{2})^{x^2-4}} = 10$

$(\sqrt{3}+\sqrt{2})^{x^2-4} + \frac{1}{(\sqrt{3}+\sqrt{2})^{x^2-4}} = 10$

$\log_{10}(\sqrt{3}+\sqrt{2})^{x^2-4} + \log_{10}(\sqrt{3}-\sqrt{2})^{x^2-4} = \log_{10} 100$

Gadho / Gadhiyon aisaa naa karo



Ans. B

$$(\sqrt{3}+\sqrt{2})^{x^2-4} = (\sqrt{3}+\sqrt{2})^2 \text{ or } (\sqrt{3}-\sqrt{2})^2$$

$$x^2-4=2 \quad \text{or} \quad (\sqrt{3}+\sqrt{2})^{x^2-4} = (\sqrt{3}-\sqrt{2})^2 = \frac{1}{(\sqrt{3}+\sqrt{2})^2} = (\sqrt{3}+\sqrt{2})^{-2}$$

$$x = \pm\sqrt{6} \qquad x^2-4=-2$$

$$x^2=2$$

$$x=\pm\sqrt{2}$$



Ans: $S=\{\sqrt{6},-\sqrt{6},\sqrt{2},-\sqrt{2}\}$

$n(S)=4$

# QUESTION [JEE Mains 2024 (1 Feb)]

Let $S = \{x \in R: (\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10\}$. Then the number of elements in S is :

A 4

B 0

C 2

D 1



Ans. C

# QUESTION [JEE Mains 2023 (31 Jan)]

Tan08

The number of real roots of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$, is

A 0

B 1

C 3

D 2



Ans. B

QUESTION

Tan 09

Solve: $\sqrt{x^2 - 6x + 9} + \sqrt{x^2 + 8x + 16} = 7$

# QUESTION [JEE Mains 2019 (8 April)]

Tan 08

The sum of the solutions of the equation $|\sqrt{x}-2|+\sqrt{x}(\sqrt{x}-4)+2=0\ (x>0)$ is equal to:

A 9

B 12

C 4

D 10



Ans. D

## Sabse Important Baat

# Sabhi Class Illustrations Retry Karnay hai...

# Home Challenge-08

Let $f(x) = e^{\left(\frac{2^{-x}}{1+2^{-x}}\right)}$. If $\prod_{r=-10}^{10} f(\ln 2^r) = e^{\lambda}$, then find the value of $(2\lambda - 15)$. [Ans. 6]

# Bumper Practice Problems

1. $\sqrt[3]{\frac{x-2}{x-1}} < \sqrt[3]{\frac{1}{x-1}}$

2. $\sqrt{3x-2} < \sqrt{x+4}$

3. $\sqrt{4x-3} < \sqrt{2x+5}$

4. $\sqrt[3]{(3x-5)} < (x-1)$

5. $\sqrt[3]{3x-2} < x$

6. $\sqrt{(x+14)} < (x+2)$



7. $\sqrt{2x-2} < x-1$

8. $\sqrt{-x^2+4x-3} > (6-2x)$

9. $\sqrt{x-2} + \sqrt{x-1} > 2$

10. $7\sqrt{x} + 8\sqrt{-x} + \frac{15}{x^3} = 98$

11. $\sqrt{2x-4} - \sqrt{x+5} = 1$

12. $\sqrt{x-1} + \sqrt{2x+6} = 6$

# Answers

1. $x \in (1, 3)$

2. $x \in \left[\frac{2}{3}, 6\right)$

3. $x \in \left[\frac{3}{4}, 1\right)$

4. $x \in (-1, \infty) - \{2\}$

5. $x \in (-2, 1) \cup (1, \infty)$

6. $x \in (2, \infty)$



7. $x \in (3, \infty)$

8. $x \in \left(\frac{13}{5}, 3\right)$

9. $x \in \left(\frac{41}{16}, \infty\right)$

# Homework From Module

**Prarambh (Topicwise)** : Q1 to Q25

**Prabal (JEE Main Level)** : Q1 to Q33

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# Solution to Previous TAH

## ATDB.uno

QUESTION

Solve: $|x| + |x - 1| \geq 7$

# Lecture 15 : 08/05/2025

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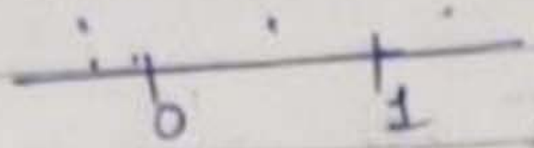
## # IAH-01 :




(i) If $x \leq 0$,

$-x - x + 1 \geq 7$

$-2x \geq 6$

$x \leq -3$

$\Rightarrow \boxed{x \in (-\infty, -3)}$

(ii) If $0 < x < 1$,

$+x - x + 1 \geq 7$

$1 \geq 7$

(Not possible)

$\Rightarrow x \in \phi$

(iii) If $x \geq 1$

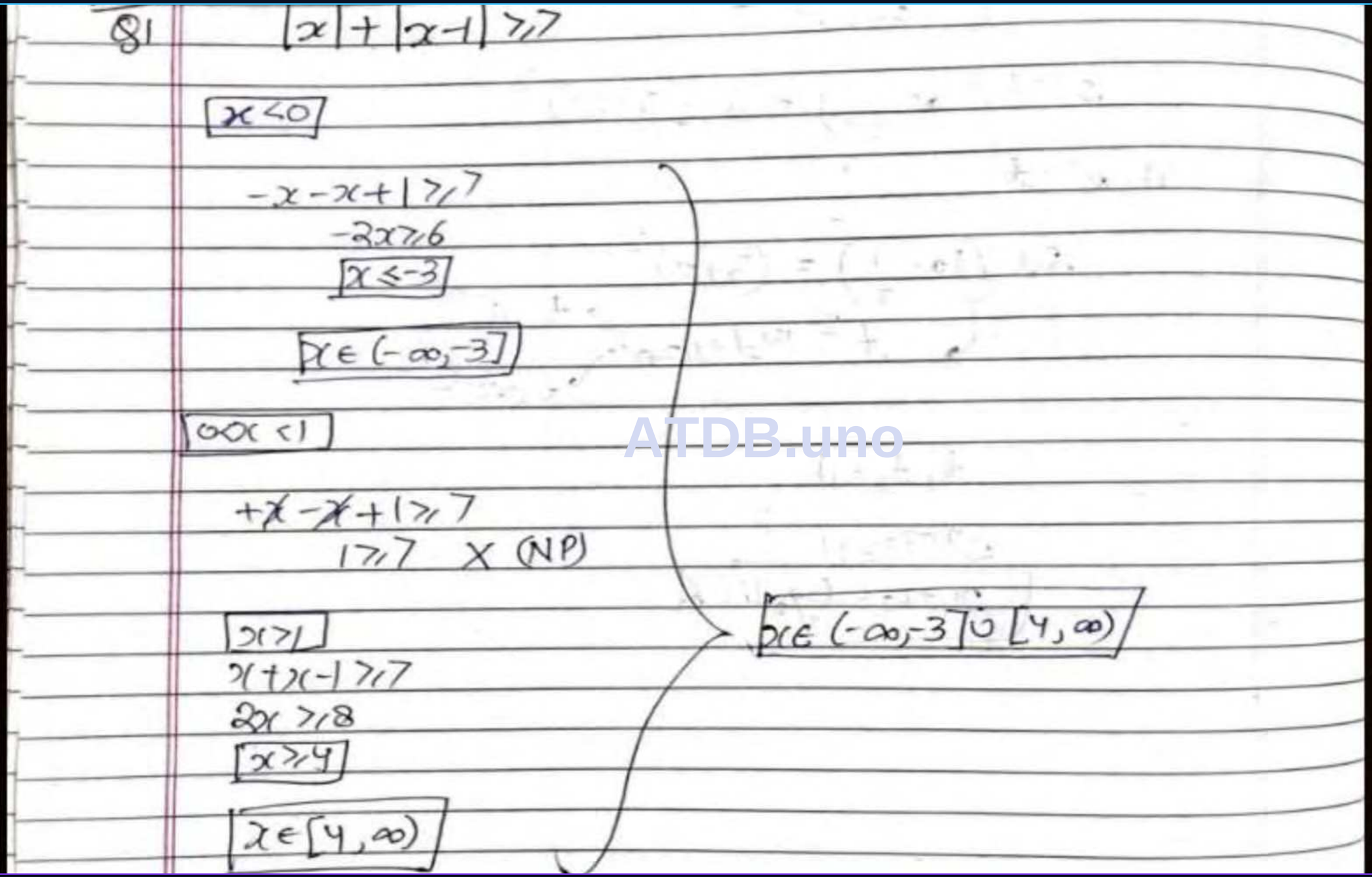
$x + x - 1 \geq 7$

$2x \geq 8$

$x \geq 4$

$\Rightarrow x \in (4, \infty)$

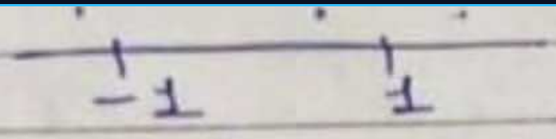
$x \in (-\infty, -3) \cup (4, \infty)$ Ans.



QUESTION

Solve: $|x + 1| + |x - 1| > 2$

# IAH-02 :




(i) If $x \leq -1$,

$-x-1-x+1 > 2$

$-2x > 2$

$x < -1$

$\Rightarrow x \in (-\infty, -1)$

(ii) If $-1 < x < 1$

$x+1-x+1 > 2$

$2 > 2$

(Not possible

$\Rightarrow x \in \phi$

(iii) If $x \geq 1$

$x+1+x-1 > 2$

$2x > 2$

$x > 1$

$\Rightarrow x \in (1, \infty)$

$\{x \in (-\infty, -1) \cup (1, \infty)\}$ Ans



Method-2 : Using triangle inequality.

$|x+1| + |x-1| > 2$

$|a| + |b| > |a-b| \iff ab > 0$

$\Rightarrow (x+1)\cdot(x-1) > 0$

$\Rightarrow \{x \in (-\infty, -1) \cup (1, \infty)\}$ Ans

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$T_1$ $T_2$

**Tah.02** $|x+1| + |x-1| > 2$

| | | | |
|---|---|---|---|
| $T_1$ | -ve | +ve | +ve |
| $T_2$ | -ve | -ve | +ve |
| | | -1 | 1 |

**Case I →** $x \leq -1$

$-(x+1) - (x-1) > 2$

$-x-1-x+1 > 2$

$-2x > 2$

$-2x-2 > 0$

$-2(x+1) > 0$

$x+1 < 0$

$x \in (-\infty, -1)$

**Case II** $-1 < x < 1$

$x+1-(x-1) > 2$

$x+1-x+1 > 2$

$2 > 2$

No.

**Case III →** $x \geq 1$

$(x+1)+(x-1) > 2$

$x+1+x-1 > 2$

$2x > 2$

$x > 1$

$x \in (1, \infty)$

$x \in (-\infty, -1) \cup (1, \infty)$

QUESTION

Solve: $|x - 6| > |x^2 - 5x + 9|$

Tan-05

8 $|x-6| > |x^2-5x+9|$

$(x-6)^2 > (x^2-5x+9)^2$

$(x-6+x^2-5x+9)(x-6-x^2+5x-9) > 0$

$(x^2-4x+3)(-x^2+6x-15) > 0$

$(x^2-4x+3)(x^2-6x+15) < 0$

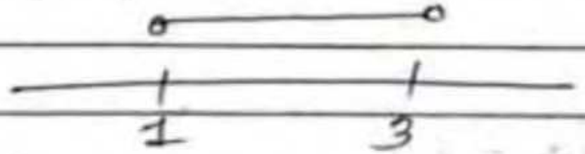
$D < 0$



$(x^2-3x-x+3)(x^2-6x+15) < 0$

$x(x-3)-1(x-3) < 0$

$(x-1)(x-3) < 0$




$x \in (1,3)$

# TAH-03:

$$(x-6)^2 > (x^2-5x+9)^2$$

$$(x-6+x^2-5x+9)(x-6-x^2+5x-9) > 0$$

$$(x^2-4x+3)\cdot(-x^2+6x-15) > 0$$

$$(x-3)(x-1)(x^2-6x+15) < 0 \Rightarrow (x-3)\cdot(x-1) < 0$$

↳ $D<0, a>0$ (always +ve)

$x \in (1,3)$ Ans

QUESTION

Solve: $(|x-1|-3)(|x+2|-5)<0$

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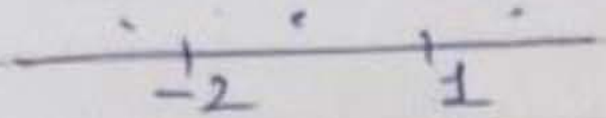
# TAH-04 :




(i) If $x \leq -2$,

$(-x+1-3)\cdot(-x-2-5)<0$

$(x+2)\cdot(x+7)<0$

$x \in (-7,-2)$

$\Rightarrow \boxed{x \in (-7,-2)}$

(ii) If $-2<x<1$

$(-x+1-3)(x+2-5)<0$

$(x+2)\cdot(x-3)>0$

$x \in (-\infty,-2) \cup (3,\infty)$

$\Rightarrow x \in \phi$

(iii) If $x \geq 1$

$(x-1-3)\cdot(x+2-5)<0$

$(x-4)(x-3)<0$

$x \in (3,4)$

$\Rightarrow \boxed{x \in (3,4)}$

$\{x \in (-7,-2) \cup (3,4)\}$ Ans

$(|x-1|-3)(|x+2|-5)<0$.

Either $|x-1|-3>0$ & $|x+2|-5$ X This method was already told by sir...

(H-7)

Case (1) when $x \leq -2$.

$(-x+1-3)(-x-2-5)<0$.

$(x+2)(x+7)<0$.

$(-7,-2)$

When $-2<x\leq 1$

$(-x+1-3)(x+2+5)<0$.

$(x+2)(x-3)>0$.

$(-\infty,-2) \cup (3,\infty)$. Case Rejected.

When $x \geq 1$.

$(x-4)(x-3)<0$.

$x \in (3,4)$

$(-7,-2) \cup (3,4)$ Ans

# QUESTION

(a) $|x^2 - 9| + |16 - x^2| = 7$

Ans: $x \in [-4, -3] \cup [3, 4]$

(b) Solve: $|(x^2 + 2x + 2) + (3x + 7)| < |x^2 + 2x + 2| + |3x + 7|$

Ans: $x < -7/3$

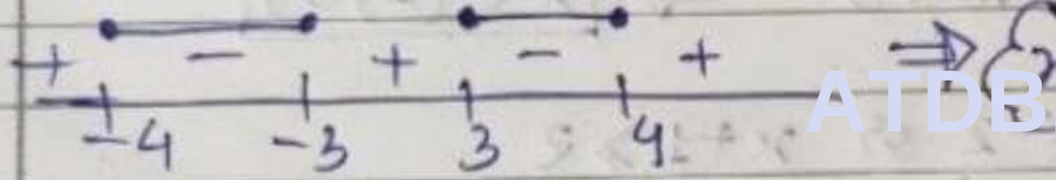
(a) $|x^2-9|+|16-x^2|=7$

$|a| + |b| = |a+b| \iff ab \geq 0$

$(x^2-9).(16-x^2) \geq 0$

$(x^2-9)(x^2-16) \leq 0$

$(x-3)(x+3)(x-4)(x+4) \leq 0$




$\Rightarrow x \in [-4,-3] \cup [3,4]$ Ans.

(b) $|(x^2+2x+2)+(3x+7)| < |x^2+2x+2| + |3x+7|$

$|a+b| < |a| + |b| \iff ab < 0$

$(x^2+2x+2).(3x+7) < 0 \Rightarrow 3x+7 < 0$

$(D<0, a>0)$

$x < -\frac{7}{3}$ Ans.

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Tanos(0) $|x^2-9| + |16-x^2| = 7$

$|a| + |b| \geq |a+b|$

$|x^2-9| + |16-x^2| \geq |x^2-9+16-x^2| = 7$

holds equality when —

$ab \geq 0$

$(x^2-9)(16-x^2) \geq 0$

$(x-3)(x+3)(x^2-16) \leq 0$

$(x-3)(x+3)(x-4)(x+4) \leq 0$

$x \in [-4,-3] \cup [3,4]$

(b) $|(x^2+2x+2) + (3x+7)| < |x^2+2x+2| + |3x+7|$

$|a+b| < |a| + |b|$

strictly increase when $ab < 0$

Therefore —

$(x^2+2x+2)(3x+7) < 0$

$D < 0$

$a > 1$ (always +ve)

$(3x+7) < 0$

$x < -\frac{7}{3}$

$x \in \left(-\infty, -\frac{7}{3}\right)$

# Solution to Previous KTKs

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# QUESTION

(KTK 1)

| Column-I | Column-II |
|---|---|
| (A) $\frac{\log_2 32}{\log_3 \sqrt{243}}$ | (P) positive integer |
| (B) $\frac{2\log 6}{\log 12+\log 3}$ | (Q) negative integer |
| (C) $\log_{1/4}\left(\frac{1}{16}\right)^{-2}$ | (R) rational but not integer |
| (D) $\frac{\log_5 16-\log_5 4}{\log_5 128}$ | (S) Prime |



Ans. A → P, S; B → P; C → Q; D → R

A.) $\log_{10}^{x} \cdot \log_{10}^{x} = \log_{10}^{100x}$

$\log_{10}^{2} x = 2 + \log_{10}^{x}$

$t^2 - t - 2 = 0$

$t^2 - 2t + t - 2 = 0$

$(t-2)(t+1) = 0$

$\log_{10} x = 2$ or $\log_{10} x = -1$

$x = 100$ or $\frac{1}{10}$

$x_1 x_2 = 10$

Q15

B.) $9 - 9^x = 2^{3-x}$

$9 - 9^x = \frac{9^3}{9^x}$

$9 - t = \frac{8}{t}$

$9t - t^2 = 8$

$0 = t^2 - 9t + 8$

$0 = t^2 - 8t - t + 8$

$0 = (t-8)(t-1)$

$9^x = 9^3$ or $9^x = 9^0$

$x = 3$ or $0$

$x_1, x_2 = 0$

Q

C.) $\log_{\frac{1}{8}}\left(\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right)\right) = \frac{1}{3}$

$\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right) = \frac{1}{2}$

$\log_{\frac{1}{2}} x = \frac{1}{2}$

$x = \frac{1}{\sqrt{2}}$ (P)

D.) $\log_b a = 3$, $\log_b c = -4$, $a^{3x} = c^{x-1}$

$a = b^3$, $c = b^{-4}$

$b^{9x} = b^{4-4x}$

$b^{9x} = b^4 \cdot b^{4x}$

$b^{13x} = b^4$

$13x = 4$

$x = \frac{4}{13}$, $p + q = 17$ (Q, R)

(A) $\dfrac{\log_2 2^5}{\log_3 3^{5/2}} \Rightarrow \dfrac{5}{5} \times 2 \Rightarrow 2 \Rightarrow$ P, S

(B) $\dfrac{2\log 6}{\log 12 + \log 3} \Rightarrow \dfrac{\log 6^2}{\log 6^2} \Rightarrow 1 \Rightarrow$ P

(C) $\log_{\frac{1}{4}} (16)^2 \Rightarrow \log_{4^{-1}} (4)^4 \Rightarrow -4 \Rightarrow$ Q

(D) $\dfrac{\log_5 4^4}{\log_5 2^7} \Rightarrow \dfrac{2\log_5 2^2}{7\log_5 2} \Rightarrow \dfrac{2}{7} \Rightarrow$ R

Sol:

(A) $\frac{\log_2 32}{\log_{\sqrt{3}} \sqrt{243}} = \frac{\log_2 (2)^5}{\log_{\sqrt{3}} \sqrt{(3)^5}} = \frac{5}{\frac{5}{2}} = 2$

→ (P) (S)

(B) $\frac{2\log 6}{\log 12 + \log 3} = \frac{2\log 6}{\log 36} = \frac{2\log 6}{\log (6)^2} = \frac{2\log 6}{2\log 6} = 1$

→ (P)

(C) $\log_{1/9} (1/16)^{-2} = \log_{1/3} \left(\frac{1}{4}\right)^{-2} = \log_{1/4} (1/2)^{-4} = -4$

→ (Q)

(D) $\frac{\log_5 16 - \log_5 4}{\log_5 128} = \frac{\log_5 4}{\log_5 (2)^7} = \frac{\log_5 (2)^2}{\log_5 (2)^7} = \frac{2 \log_5 2}{7 \log_5 2} = \frac{2}{7}$

→ (R)

# QUESTION

(KTK 2)

$$\frac{1}{x-1}-\frac{4}{x-2}+\frac{4}{x-3}-\frac{1}{x-4}<\frac{1}{30}$$



Ans. $x \in (-\infty,-2) \cup (-1,1) \cup (2,3) \cup (4,6) \cup (7,\infty)$

$$\frac{1}{x-1}-\frac{4}{x-2}+\frac{4}{x-3}-\frac{1}{x-4}<\frac{1}{30}$$

$$\Rightarrow \frac{1}{x-1}-\frac{1}{x-4}+\frac{4}{x-3}-\frac{4}{x-2}<\frac{1}{30}$$

$$\frac{(x-4)-(x-1)}{(x-1)(x-4)}+4\left[\frac{(x-2)-(x-3)}{(x-3)(x-2)}\right]<\frac{1}{30}$$

$$\frac{-3}{(x-1)(x-4)}+\frac{4}{(x-3)(x-2)}<\frac{1}{30}$$

$$\frac{-3}{x^2-5x+4}+\frac{4}{x^2-5x+6}<\frac{1}{30}$$

$x^2-5x=t$

$$\frac{-3}{t+4}+\frac{4}{t+6}-\frac{1}{30}<0$$

$$\frac{-3(30)(t+6)+4(30)(t+4)-(t+4)(t+6)}{(t+4)(t+6)(30)}<0$$

$$\frac{-90t-540+120t+480-t^2-10t-24}{(t+4)(t+6)}<0$$

Ans. $x \in (-\infty,-2)\cup(-1,1)\cup(2,3)\cup(4,6)\cup(7,\infty)$

$$\frac{-t^2+20t-84}{(t+4)(t+6)} < 0$$

$$\Rightarrow \frac{t^2-20t+84}{(t+4)(t+6)} > 0$$

$$\Rightarrow \frac{t^2-14t-6t+84}{(t+4)(t+6)} > 0$$

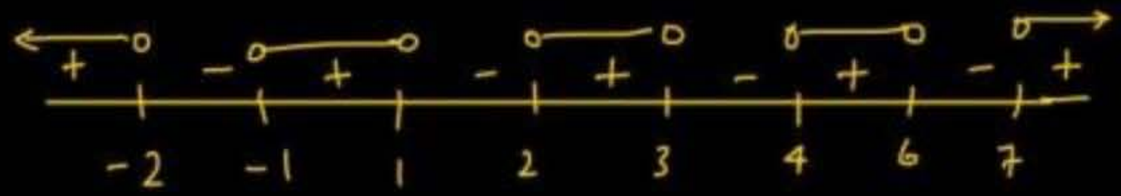
$$\Rightarrow \frac{(t-14)(t-6)}{(t+4)(t+6)} > 0$$

$$\frac{(x^2-5x-14)(x^2-5x-6)}{(x^2-5x+4)(x^2-5x+6)} > 0$$

$$\frac{(x^2-7x+2x-14)(x^2-6x+x-6)}{(x^2-5x+4)(x^2-5x+6)} > 0$$

$$\frac{(x-7)(x+2)(x-6)(x+1)}{(x-1)(x-4)(x-3)(x-2)} > 0$$






$$x \in (-\infty,-2) \cup (-1,1) \cup (2,3) \cup (4,6) \cup (7,\infty)$$

# QUESTION

(KTK 3)

The equation $\frac{\log_8\left(\frac{8}{x^2}\right)}{(\log_8 x)^2} = 3$ has

A no integral solution

B one natural solution

C two real solutions

D one irrational solution



Ans. B, C

KTK-03 The eq$^n$ $\dfrac{\log_8(8/x^2)}{(\log_8 x)^2} = 3$

$\Rightarrow$ $\dfrac{1-\log_8 x^2}{(\log_8 x)^2} = 3$

$\Rightarrow 1 - 2\log_8 x = 3(\log_8 x)^2$

let $\log_8 x = t$

$1 - 2t = 3t^2$

$3t^2 + 2t - 1 = 0$

$3t^2 + 3t - t - 1 = 0$

$3t(t+1) - 1(t+1) = 0$

$(3t-1)(t+1) = 0$

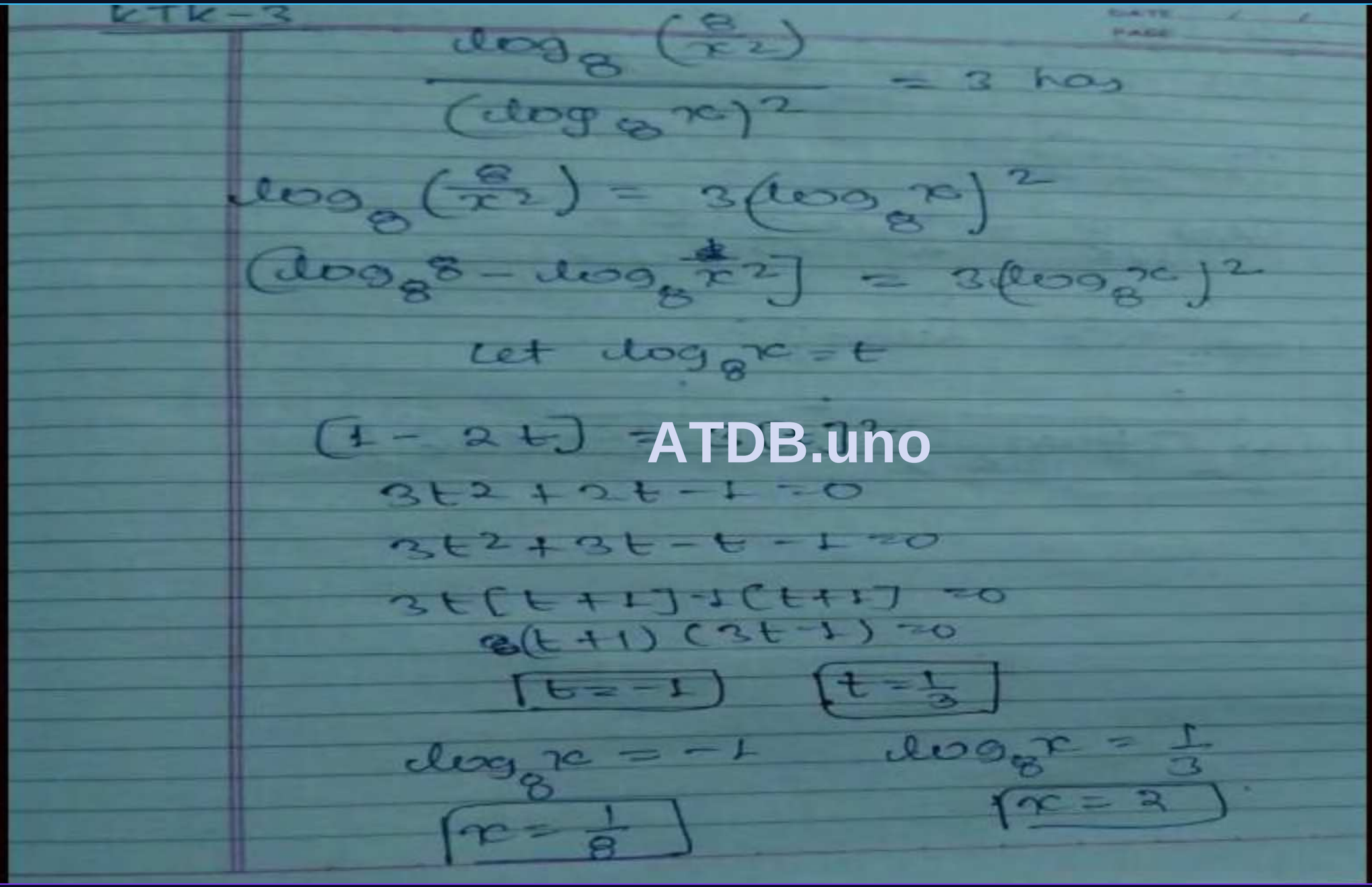
$t = \dfrac{1}{3}, -1$

$\log_8 x = \dfrac{1}{3}$ $\qquad$ $\log_8 x = -1$

$x = 2$ $\qquad$ $x = \dfrac{1}{8}$

B) one natural sol$^n$

C) 2 real sol$^n$



# QUESTION

(KTK 4)

For the equation $\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$, which of the following do not hold good?

A no real solution

B one prime solution

C one integral solutions

D no irrational solution



Ans. A, B, D

$$\frac{\log_3 x}{\log_3 3\sqrt{x}} + \frac{\log_3 \sqrt{x}}{\log_3 3x} = 0$$

$$\frac{\log_3 x}{1+\log_3 \sqrt{x}} + \frac{\log_3 \sqrt{x}}{1+\log_3 x} = 0$$

$$\frac{\log_3 x}{1+\frac{1}{2}\log_3 x} + \frac{\frac{1}{2}\log_3 x}{1+\log_3 x} = 0$$

$$\frac{2t}{2+t} + \frac{t}{2(1+t)} = 0$$

$$\frac{2t\cdot(2)(1+t)+t(2+t)}{2(1+t)(2+t)} = 0$$

$$\frac{4t+4t^2+2t+t^2}{2(1+t)(2+t)} = 0$$

$$5t^2+6t = 0$$

$$t(5t+6) = 0$$

$$t = 0, \frac{-6}{5}$$

$$\log_3 x = 0, \frac{-6}{5} \Rightarrow x = 1, (3)^{-6/5}.$$

Ans: (a), (b), (d)

$$\boxed{x = 1, \frac{1}{3^{6/5}}}$$

# QUESTION

(KTK 5)

If $\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4(x^2) + \log_8(x^3) + \log_{16}(x^4) = 40$ then x is equal to



Ans. 256

KTK-05 If $\log_{\sqrt{2}}\sqrt{x} + \log_2 x + \log_4(x^2) + \log_8(x^3) + \log_{16}(x^4) = 40$, then $x$ is equal to



$\Rightarrow \log_2 x + \log_2 x + \log_2 x + \log_2 x + \log_2 x = 40$

$5\log_2 x = 40$

$x = 2^8$

$= 256$

KTK-05

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$\log_{\sqrt{2}}$

$\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4 x^2 + \log_8 x^3 + \log_{16} x^4 = 40$

$2\log_2 \sqrt{x} + \log_2 x + \frac{1}{2}\log_2 x^2 + \frac{1}{3}\log_2 x^3 + \frac{1}{4}\log_2 x^4 = 40$

$\log_2 x + \log_3 x + \log_2 x + \log_2 x + \log_2 x = 40$

$5t = 40$

$t = \frac{40}{5} = 8$

$\log_2 x = 8$

$\boxed{x = 2^8 = 256}$

# QUESTION

**(KTK 6)**

| Column-I | | Column-II | |
|---|---|---|---|
| (A) | If $x_1$ and $x_2$ satisfy the equation $x^{\log_{10} x} = 100x$ then the value of $x_1 x_2$ equals | (P) | irrational |
| (B) | Sum of the squares of the roots of the equation $\log_2(9 - 2^x) = 3 - x$ is | (Q) | rational |
| (C) | If $\log_{\frac{1}{8}}\left(\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right)\right) = \frac{1}{3}$ then x is | (R) | prime |
| (D) | Let $\log_b a = 3, \log_b c = -4$. If the value of x satisfying the equation $a^{3x} = c^{x-1}$ is expressed in the form p/q, where p and q are relatively prime then $p + q$ is | (S) | composite |



Ans. A → Q, S; B → Q, S; C → P; D → QR

[FTK-06]

(a) $x^{\log_{10} x} = 100x$

Taking log both sides

$\log_{10} x \log_{10} x = \log_{10} 100x$

$(\log_{10} x)^2 = \log_{10} 100 + \log_{10} x$

$(\log_{10} x)^2 = 2 + \log_{10} x$

Let $(\log_{10} x) = t$

$t^2 = 2 + t$

$t^2 - t - 2 = 0$

$t^2 - 2t + t - 2 = 0$

$(t-2)(t+1) = 0$

$t = +2, -1$

$\log_{10} x = 2$

$\boxed{x_1 = 100}$

$\log_{10} x = -1$

$\boxed{x_2 = \frac{1}{10}}$

$x_1 \cdot x_2 = 100 \times \frac{1}{10} = 10$ // A → S, Q

(b) $\log_2 (9 - 2^x) = 3 - x$

$2^{(3-x)} = 9 - 2^x$

$2^3 = 9 - 2^x \Rightarrow 8 = 9 \cdot 2^x - 2^{2x}$

$2^x = t$

$t^2 - 9t + 8 = 0$

$t^2 - 8t - t + 8 = 0$

$(t-8)(t-1) = 0$

$\{t = 8, 1\}$

$2^x = 8$

$[x = 3]$

$2^x = 1$

$[x = 0]$

$[B \rightarrow Q, S]$

$3^2 + 0^2 = 9$

(c) $\log_{\frac{1}{8}}\left(\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right)\right) = \frac{1}{3}$

$\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right) = \left(\frac{1}{8}\right)^{\frac{1}{3}}$

$\log_{\frac{1}{2}} x = \frac{1}{4}\left(\frac{1}{2}\right)$

$\log_{\frac{1}{2}} x = \frac{1}{2}$

$x = \left(\frac{1}{2}\right)^{\frac{1}{2}}$

$\left[x = \frac{1}{\sqrt{2}}\right]$ $[C \rightarrow P]$

$\log_b a = 3$ $\log_b c = -4$, $a^{3x} = c^{x-1}$

$a^{3x} = c^{x-1}$

Taking log both sides

$3x \log_b a = (x-1)\log_b c$

$3x(3) = (x-1)(-4)$

$9x = -4x + 4$

$13x = 4$

$x = \frac{4}{13}$

$4 + 13$

$(17)$

$[D \rightarrow R]$

RTH06

Q

A $x^{\log_{10} x} = 100x$

$(\log_{10} x)^2 = \log_{10} 10^2 + \log_{10} x$

$t^2 = 2t + t$

$t^2 - t - 2 = 0$

$t = -1$ $\quad t = 2$

$x = \frac{1}{10}$ $\quad x_2 = 100$

$x_1 x_2 = 10$

(B) $\log_2 (9 - 2^x) = 3 - x$

$9 - 2^x = \frac{8}{2^x}$

$9 - t = \frac{8}{t}$

$9t - t^2 = 8$

$t^2 - 9t + 8 = 0$

$t^2 - 8t - t + 8 = 0$

$t(t-8) - 1(t-8) = 0$

$t = 1$ $\quad t = 8$

$x = 0$ $\quad x = 3$

(C)

$$\log_{\frac{1}{4}}(\log_5 x) = 2^{-2\times\frac{1}{2}}$$

$$\log_{\frac{1}{2}} x = 2^{-1}$$

$$\log_{\frac{1}{2}} x = \frac{1}{2}$$

$$x = \left(\frac{1}{2}\right)^{\frac{1}{2}}$$

$$\boxed{x = \frac{1}{\sqrt{2}}}$$

(D)

$$\log_b a = 3 \quad ; \quad \log_b c = -4$$

$$\boxed{a = b^3} \qquad \boxed{c = b^{-4}}$$



$$a^{3x} = c^{x-1}$$

$$(a)^{3x} = \frac{(c)^x}{c}$$

$$(b^3)^{3x} = \frac{(b^{-4})^x}{b^{-4}}$$

$$b^{9x} = \frac{b^4}{b^{4x}}$$

$$b^{9x} = b^{4-4x}$$

$$9x = 4 - 4x$$

$$13x = 4 \qquad \boxed{x = \frac{4}{13}} \qquad \boxed{p = 4} \qquad \boxed{q = 13}$$

# Home Challenge-07

Find the sum of all the integral solution(s) of the equation $3^{|x|} = \left(\frac{3}{(\sqrt{3})^{|x-2|}}\right)^2$. [Ans. 3]

Home Challenge - 07

Find the sum of all integral sol$^{n}$(s) of the eq$^{n}$

$$3^{|x|} = \left(\frac{3}{(\sqrt{3})^{|x-2|}}\right)^2$$

$$\Rightarrow 3^{|x|} = 3^{2-|x-2|}$$

bases same, powers can be compared

$$|x| = 2 - |x-2| \Rightarrow |x| + |x-2| - 2 = 0$$




(I) $x \le 0$

$-x - x + 2 - 2 = 0$

$-2x = 0$

$x = 0$

(II) $0 < x < 2$

$x - x + x - 2 = 0$

$\forall\ x \in R$

$\cap$ $x \in (0, 2)$

(III) $x \ge 2$

$x + x - 2 - 2 = 0$

$2x = 4$

$x = 2$

$\cup$

$x \in [0, 2]$

$\therefore$ integral values of $x = 0, 1, 2$

Sum $= 0 + 1 + 2 = 3$

Home Challenge-01

8

$$3^{|x|} = \left(\frac{3}{(\sqrt{3})^{|x-2|}}\right)^2$$

$$3^{|x|} = 3^{2-|x-2|}$$

$$|x| = 2-|x-2|$$

**$x \le 0$**

$-x = 2 + x - 2$

$2x = 0$

$x = 0$ ✓

**$0 < x \le 2$**

$x = 2 + x - 2$

$x = x$

$x \in (0, 2]$ ✓

**$x > 2$**

$x = 2 - x - 2$

$2x = 0$

$x = 0$ X

integral sol$^{n}$ $x = \{1, 0, 2\}$

sum = 3 A

# Solution to Previous BPPs

## ATDB.uno

# Bumper Practice Problems

Solve the following inequality for x.

(a) $|x| > 2$

(b) $|x - 1| > 3$

(c) $|x - 2| < 1$

(d) $|x + 1| \geq 2$

(e) $|x - 1| \leq 5$

(f) $|2x - 3| > 7$

(g) $|3x + 5| < 2$

(h) $|4x + 6| > 5$

(i) $|2x - 3| > -2$



(j) $|4x - 9| \leq 7$

(k) $|3x + 5| \geq 2$

(l) $|2x + 3| \geq 0$

(m) $|4 - 3x| < -2$

(n) $|5 - 3x| \leq 0$

(o) $\left|\frac{x-3}{x-5}\right| > 1$

(p) $\left|\frac{x+4}{x+2}\right| \leq 1$

(q) $|x^2 - 4x| < 5$

(r) $|x - 3| > -1$

(s) $|3x - 2.5| \geq 2$

(t) $|x - 2| \leq |x + 4|$

# Answers

(a) $(-\infty, -2) \cup (2, \infty)$

(b) $(-\infty, -2) \cup (4, \infty)$

(c) $x \in (1, 3)$

(d) $x \in (-\infty, -3] \cup [1, \infty)$

(e) $x \in (-4, 6)$

(f) $(-\infty, -2) \cup (5, \infty)$

(g) $\left(-\frac{7}{3}, -1\right)$

(h) $x \in \left(-\infty, \frac{-11}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)$

(i) $x \in (-\infty, \infty)$



(j) $x \in \left[\frac{1}{2}, 4\right]$

(k) $x \in \left(-\infty, \frac{-2}{3}\right] \cup [-1, \infty)$

(l) $x \in \mathbb{R}$

(m) $x \in \phi$

(n) $x \in \left\{\frac{5}{3}\right\}$

(o) $(4, \infty) - \{5\}$

(p) $(-\infty, -3]$

(q) $(-1, 5)$

(r) $x \in (-\infty, \infty)$

(s) $x \in \left(-\infty, \frac{1}{6}\right] \cup \left[\frac{3}{2}, \infty\right)$

(t) $x \in [-1, \infty)$

# QUESTION

Solve the following equations

(a) $|x-1|+|x-3|=2$

(b) $|x|+|x+5|=5$

(c) $|x-1|+|x-4|=2$

(d) $|x^2-2x|+|x-4|=|x^2-3x+4|$



## Answers:

(a) $[1,3]$ (b) $[-5,0]$ (c) $\{\}$ (d) $x \in (-\infty,0] \cup [2,4]$

# (B.P.P.)

(a) $|x| > 2 \Rightarrow \boxed{x \in (-\infty, -2) \cup (2, \infty)}$

(b) $|x-1| > 3 \Rightarrow x-1 < -3$ or $x-1 > 3$

$x < -2$ or $x > 4$

$\Rightarrow \boxed{x \in (-\infty, -2) \cup (4, \infty)}$

(c) $|x-2| < 1 \Rightarrow -1 < x-2 < 1$

$1 < x < 3 \Rightarrow \boxed{x \in (1, 3)}$

(d) $|x+1| \geq 2 \Rightarrow x+1 \leq -2$ or $x+1 \geq 2$

$x \leq -3$ or $x \geq 1$

$\Rightarrow \boxed{x \in (-\infty, -3] \cup [1, \infty)}$

(e) $|x-1| \leq 5 \Rightarrow -5 \leq x-1 \leq 5$

$-4 \leq x \leq 6 \Rightarrow \boxed{x \in [-4, 6]}$

(f) $|2x-3| > 7 \Rightarrow 2x-3 < -7$ or $2x-3 > 7$

$2x < -4$ or $2x > 10$

$x < -2$ or $x > 5$

$\Rightarrow \boxed{x \in (-\infty, -2) \cup (5, \infty)}$

(g) $|3x+5| < 2 \Rightarrow -2 < 3x+5 < 2$

$-7 < 3x < -3$

$-\frac{7}{3} < x < -1 \Rightarrow \boxed{x \in \left(-\frac{7}{3}, -1\right)}$

(h) $|4x+6| > 5 \Rightarrow 4x+6 < -5$ or $4x+6 > 5$

$4x < -11$ $\quad$ $4x > -1$

$x < -\frac{11}{4}$ $\quad$ $x > -\frac{1}{4}$

$\Rightarrow \boxed{x \in \left(-\infty, -\frac{11}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)}$

(i) $|2x-3| > -2 \Rightarrow \boxed{x \in R}$

(j) $|4x-9| \leq 7 \Rightarrow -7 \leq 4x-9 \leq 7$

$2 \leq 4x \leq 16$

$\frac{1}{2} \leq x \leq 4 \Rightarrow \boxed{x \in \left[\frac{1}{2}, 4\right]}$

(k) $|3x+5| \geq 2 \Rightarrow 3x+5 \leq -2$ or $3x+5 \geq 2$

$3x \leq -7$ $\quad$ $3x \geq -3$

$x \leq -\frac{7}{3}$ $\quad$ $x \geq -1$

$\Rightarrow \boxed{x \in \left(-\infty, -\frac{7}{3}\right] \cup [-1, \infty)}$

(l) $|2x+3| \geq 0 \Rightarrow \boxed{x \in R}$

(m) $|4-3x| < -2 \Rightarrow \boxed{x \in \phi}$

(n) $|5-3x| \leq 0 \Rightarrow |5-3x| = 0$

$5-3x = 0$

$3x = 5 \Rightarrow \boxed{x = \frac{5}{3}}$

(o) $\left|\frac{x-3}{x-5}\right| > 1 \Rightarrow \frac{x-3}{x-5} < -1$ or $\frac{x-3}{x-5} > 1$

$\frac{x-3}{x-5} + 1 < 0$ or $\frac{x-3}{x-5} - 1 > 0$

$\frac{x-3+x-5}{x-5} < 0$ or $\frac{x-3-x+5}{x-5} > 0$

$\frac{2x-8}{x-5} < 0$ or $\frac{2}{x-5} > 0$

$\frac{x-4}{x-5} < 0$ or $\frac{1}{x-5} > 0$

$x \in (4,5)$ or $x-5>0 \Rightarrow x>5$

$\Rightarrow \boxed{x \in (4,5) \cup (5,\infty)}$

(p) $\left|\frac{x+4}{x+2}\right| \leq 1 \Rightarrow -1 \leq \frac{x+4}{x+2} \leq 1$

$\frac{x+4}{x+2} \geq -1$ & $\frac{x+4}{x+2} \leq 1$

$\frac{x+4}{x+2} + 1 \geq 0$ & $\frac{x+4}{x+2} - 1 \leq 0$

$\frac{2x+6}{x+2} \geq 0$ & $\frac{x+4-x-2}{x+2} \leq 0$

$\frac{x+3}{x+2} \geq 0$ & $\frac{2}{x+2} \leq 0$

$x \in (-\infty,-3] \cup (-2,\infty)$ & $x+2<0 \Rightarrow x<-2$

$\Downarrow$

$\{x \in (-\infty,-3]\}$ Ans

(q) $|x^2-4x| < 5 \Rightarrow -5 < x^2-4x < 5$

$x^2-4x+5>0$ & $x^2-4x-5<0$

$(D<0)$ & $(x-5)(x+1)<0$

$x \in R$ & $x \in (-1,5)$

$\Downarrow$

$\{x \in (-1,5)\}$ Ans

(r) $|x-3| > -1 \Rightarrow \boxed{x \in R}$

(s) $|3x-2.5| \geq 2 \Rightarrow 3x-2.5 \leq -2$ or $3x-2.5 \geq 2$

$3x \leq 0.5$ $\quad 3x \geq 4.5$

$x \leq \frac{0.5}{3}$ $\quad x \geq \frac{4.5}{3} \Rightarrow x \geq 1.5$

$\Rightarrow \boxed{x \in (-\infty, \frac{0.5}{3}] \cup [1.5,\infty)}$

(t) $|x-2| \leq |x+4| \Rightarrow (x-2)^2-(x+4)^2 \leq 0$

$(x-2+x+4)\cdot(x-2-x-4)\cdot \leq 0$

$\cdot(2x+2)\cdot(-6) \leq 0$

$x+1 \geq 0$

$x \geq -1 \Rightarrow \boxed{x \in [-1,\infty)}$

Q) Solve the following questions :

(a) $|x-1|+|x-3|=2$

$|a|+|b| = |a-b|$

$\Leftrightarrow ab \leq 0$

$\Rightarrow (x-1)\cdot(x-3) \leq 0$

$\Rightarrow \boxed{x \in [1,3]}$ Ans

(b) $|x|+|x+5|=5$

$|a|+|b| = |a-b| \Leftrightarrow ab \leq 0$

$\Rightarrow x\cdot(x+5) \leq 0 \Rightarrow \boxed{x \in [-5,0]}$ Ans

(c) $|x-1|+|x-4|=2$ Ans: $x \in \phi$

(i) If $x \leq 1$, $-x+1-x+4=2$, $3=2x$, $x=3/2$ ✗ (reject)

(ii) If $1<x<4$, $x-1-x+4=2$, $3=2$ (Not possible), $x \in \phi$

(iii) If $x \geq 4$, $x-1+x-4=2$, $2x=7$, $x=7/2$ (reject)

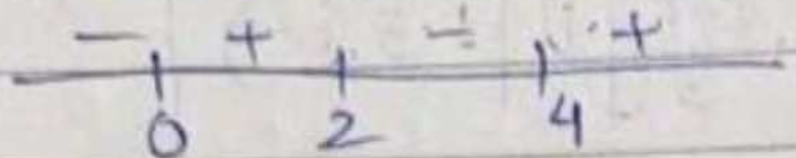
Date:

(d) $|x^2-2x|+|x-4|=|x^2-3x+4|$

$|a|+|b|=|a-b| \iff ab \le 0$

$(x^2-2x)\cdot(x-4)\le 0$

$x(x-2)(x-4)\le 0$ $\Rightarrow x \in [0,2] \cup [4,\infty)$




$\{x \in (-\infty, 0] \cup [2, 4]\}$

Ans.