

PRAAYAS

JEE 2026

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Mathematics

Basic Maths

Lecture - 11

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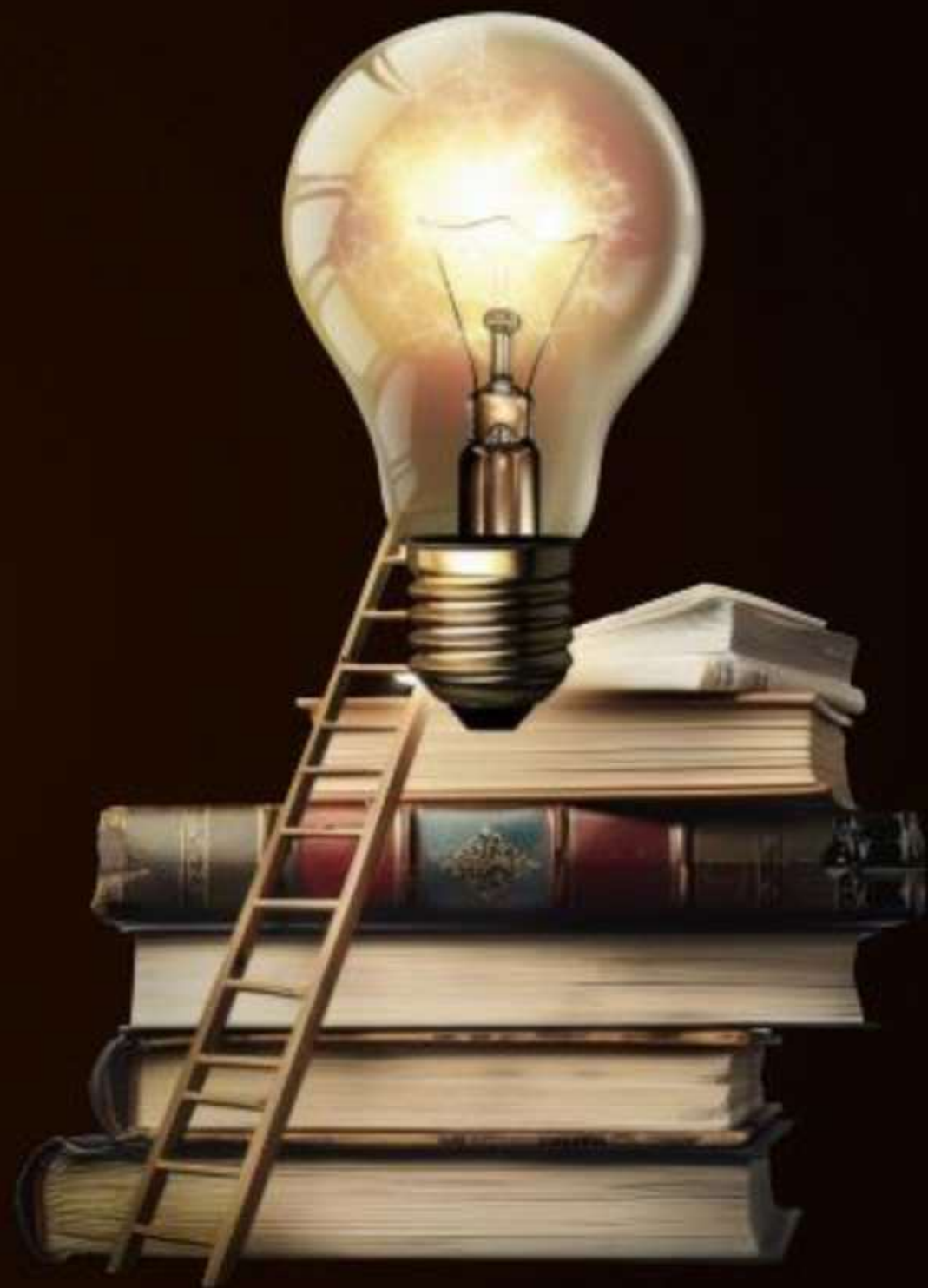


Topics *To be covered*



- A** Logarithm Equations involving Modulus
- B** Logarithmic Inequalities
- C** Problem Practice

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Homework Discussion

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QUESTION



Indicate all correct alternatives, where base of the log is 2.

The equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has :

A At least one real solution

B Exactly 3 real solutions

C Exactly one irrational solution

D Imaginary roots

$$\log_2 x \cdot \left(\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} \right) = \log_2 \sqrt{2}$$

$$\left(\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} \right) \cdot \log_2 x = \frac{1}{2}$$

let $\log_2 x = t$

$$\frac{3}{4}t^3 + t^2 - \frac{5}{4}t = \frac{1}{2}$$

$$3t^3 + 4t^2 - 5t - 2 = 0$$

$$3t^2(t-1) + 7t(t-1) + 2(t-1) = 0$$

$$(t-1)(3t^2 + 6t + t + 2) = 0$$

$$(t-1)(3t+1)(t+2) = 0$$



$$t = 1, -\frac{1}{3}, -2$$

$$\log_2 x = 1, -\frac{1}{3}, -2$$

$$x = 2, 2^{-\frac{1}{3}}, 2^{-2}$$

$$x = 2, \frac{1}{4}, \frac{1}{3\sqrt{2}}$$

irrational
root

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QUESTION



Every Real no. is a complex Number

The equation $x^{[(\log_3 x)^2 - \frac{9}{2} \log_3 x + 5]} = 3\sqrt{3}$ has

~~A~~ Exactly 3 real solutions

$$\log_3 \left(x^{(\log_3 x)^2 - \frac{9}{2} \log_3 x + 5} \right) = \log_3 3\sqrt{3}$$

~~B~~ At least one real solution

$$\left((\log_3 x)^2 - \frac{9}{2} \log_3 x + 5 \right) \cdot \log_3 x = \frac{3}{2}$$

~~C~~ Exactly one irrational solution

$$t^3 - \frac{9}{2}t^2 + 5t = \frac{3}{2}$$

~~D~~ Complex roots

$$2t^3 - 9t^2 + 10t - 3 = 0$$

$$2t^2(t-1) - 7t(t-1) + 3(t-1) = 0$$

$$(t-1)(2t^2 - 6t - t + 3) = 0$$

$$t=1, (2t-1)(t-3) = 0$$

$$t = \frac{1}{2}, 3$$

$x = 3, \sqrt{3}, 27$

QUESTION

(KTK 4)



Which of the following does not hold true for the expression

$$E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1}?$$

- A** $E = 2$ if $x \leq -1$
- B** $E = -2x$ if $-1 < x < 1$
- C** $E = -2$ if $x \geq 1$
- D** $E = -2$ for all x

$$E = |x-1| - |x+1|$$

$$\text{if } x \leq -1$$

$$E = -(x-1) + (x+1) = 2$$

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$$\text{if } -1 < x < 1$$

$$E = -(x-1) - (x+1) = -2x$$

$$\text{if } x \geq 1$$

$$E = x-1 - (x+1) = -2$$

Ans. D



Home Challenge-03



Positive integers a and b satisfy the condition $\log_2 [\underbrace{\log_2^a (\log_2^b (2^{1000}))}_t] = 0$. Then the possible values of $a + b$ is/are:

~~A~~ 501

~~B~~ 252

~~C~~ 128

D 66

$$\log_2 t = 0$$

$$t = 2^0 = 1$$

$$\Rightarrow \log_2^a (\log_2^b 2^{1000}) = 1$$

$$\frac{1}{a} \log_2 \left(\frac{1000}{b} \log_2 2 \right) = 1$$

$$\log_2 \left(\frac{1000}{b} \right) = a$$

$$\frac{1000}{b} = 2^a$$

$$2^a \cdot b = 1000$$

$$\begin{aligned} 1000 &= 2^1 \cdot 500 \\ &= 2^2 \cdot 250 \\ &= 2^3 \cdot 125. \end{aligned}$$

$$\begin{cases} a=1, b=500 \\ a=2, b=250 \\ a=3, b=125. \end{cases}$$



Aao Machaay Dhamaal Deh Swaal pe Deh Swaal

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If a certain term
occurs repeatedly in
a given Eqn/Expression
assume it to be t

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QUESTION

★★★KCLS★★★



Let $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192} + 16 - \log_2 12 \cdot \log_2 48 + 10$. Find $y \in \mathbb{N}$.

$$y = \sqrt{\log_2 3 (\log_2 4 + \log_2 3) (\log_2 16 + \log_2 3) (\log_2 64 + \log_2 3)} + 16 - (\log_2 4 + \log_2 3) (\log_2 16 + \log_2 3) + 10$$

$$= \sqrt{\log_2 3 (2 + \log_2 3) (4 + \log_2 3) (6 + \log_2 3)} + 16 - (2 + \log_2 3) (4 + \log_2 3) + 10$$

$$\text{let } \log_2 3 = t$$

$$y = \sqrt{t(2+t)(4+t)(6+t)} + 16 - (2+t)(4+t) + 10$$

$$y = \sqrt{(6t+t^2)(6t+t^2+8)} + 16 - (6t+t^2+8) + 10 \quad \text{let } m = t^2+6t$$

$$= \sqrt{m(m+8)} + 16 - (m+8) + 10$$

$$= \sqrt{m^2+8m+16} - m + 2$$



$$y = \sqrt{(m+4)^2} - m + 2$$

$$y = \underbrace{|m+4|}_{+ve} - m + 2$$

$$y = m + 4 - m + 2 = 6.$$

$$m = t^2 + 6t, \quad t = \log_2 3 > 0$$

\downarrow
 $m > 0$

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QUESTION



$3^{\log_3^2 x} + x^{\log_3 x} = 162$ then x is

- A 9
- B 1/9
- C 10
- D 1/10

$$3^{\log_3^2 x} + x^{\log_3 x} = 162$$

$$(3^{\log_3 x})^{\log_3 x} + x^{\log_3 x} = 162$$

$$x^{\log_3 x} + x^{\log_3 x} = 162$$

$$2 \cdot x^{\log_3 x} = 162$$

$$x^{\log_3 x} = 81$$

$$\log_3 x^{\log_3 x} = \log_3 81$$

$$\log_3 x \cdot \log_3 x = 4$$

$$\left(\log_3 x\right)^2 = 4 \Rightarrow \log_3 x = -2, 2$$

$$\therefore \log_3 3^4 = 4 \log_3 3 = 4$$

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$$x = 3^{-2}, 3^2$$

$$x = \frac{1}{9}, 9$$

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QUESTION



$$\text{Solve: } \log_{x^2+6x+8} \underbrace{(\log_{2x^2+2x+3} (x^2 - 2x))}_t = 0$$

$$\log_{x^2+6x+8} t = 0$$

$$t = (x^2+6x+8)^0$$

$$t = 1$$

$$\log_{2x^2+2x+3} (x^2 - 2x) = 1$$

$$x^2 - 2x = 2x^2 + 2x + 3$$

$$x^2 + 4x + 3 = 0$$

$$x = -1, -3$$

$$\text{@ } x = -1$$

$$\log_3 (\log_3 3) = 0$$

$$\text{@ } x = -3 \text{ (rejected)}$$

$\therefore x^2 + 6x + 8$ becomes -1
which is N.P

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QUESTION [JEE Advanced 2011]



Let (x_0, y_0) be the solution of the following equations

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$

$$3^{\ln x} = 2^{\ln y}$$

Then x_0 is

A $1/6$

B $1/3$

C $1/2$

D 6

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$

$$\ln(2x)^{\ln 2} = \ln(3y)^{\ln 3}$$

$$\ln 2 \cdot \ln 2x = \ln 3 \cdot \ln 3y$$

$$\ln 2 \cdot (\ln 2 + \ln x) = \ln 3 \cdot (\ln 3 + \ln y)$$

$$(\ln 2)^2 + \ln 2 \cdot \ln x = (\ln 3)^2 + \ln 3 \cdot \frac{\ln x \cdot \ln 3}{\ln 2}$$

$$(\ln 2)^2 - (\ln 3)^2 = \frac{\ln x \cdot (\ln 3)^2}{\ln 2} - \ln 2 \cdot \ln x$$

$$\cancel{(\ln 2)^2 - (\ln 3)^2} = \ln x \left(\frac{(\ln 3)^2}{\ln 2} - \ln 2 \right) = \ln x \left(\frac{(\ln 3)^2 - (\ln 2)^2}{\ln 2} \right)$$

$$3^{\ln x} = 2^{\ln y}$$

$$\ln(3^{\ln x}) = \ln(2^{\ln y})$$

$$\ln x \cdot \ln 3 = \ln y \cdot \ln 2$$

$$\ln y = \frac{\ln x \cdot \ln 3}{\ln 2}$$



$$-1 = \frac{\ln x}{\ln 2}$$

$$\ln x = -\ln 2$$

$$\ln x = \ln 2^{-1}$$

$$x = 2^{-1}$$

$$x = \frac{1}{2}$$

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QUESTION [JEE Advanced 2012]



The values of $6 + \log_{\frac{3}{2}} \left(\frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \dots}}}} \right)$ is

let E

$$\text{let } x = \sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \dots}}}}$$

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$$x = \sqrt{4 - \frac{1}{3\sqrt{2}} \cdot x}$$

$$x^2 = 4 - \frac{x}{3\sqrt{2}}$$

$$3\sqrt{2} x^2 = 12\sqrt{2} - x$$

$$3\sqrt{2} x^2 + x - 12\sqrt{2} = 0$$

$$3\sqrt{2} x^2 + 9x - 8x - 12\sqrt{2} = 0$$

$$3x(\sqrt{2}x + 3) - 4\sqrt{2}(\sqrt{2}x + 3) = 0$$

$$(3x - 4\sqrt{2})(\sqrt{2}x + 3) = 0$$

$$x = \frac{4\sqrt{2}}{3}, \quad \cancel{\frac{-3}{\sqrt{2}}}$$



Now from given Quest $x = \frac{4\sqrt{2}}{3}$

$$E = 6 + \log_{\frac{3}{2}} \left(\frac{1}{3\sqrt{2}} \cdot x \right)$$

$$= 6 + \log_{\frac{3}{2}} \left(\frac{1}{3\sqrt{2}} \cdot \frac{4\sqrt{2}}{3} \right)$$

$$= 6 + \log_{\frac{3}{2}} \left(\frac{4}{9} \right)$$

$$= 6 + \log_{\frac{3}{2}} \left(\frac{2}{3} \right)^2$$

$$= 6 + 2 \log_{\frac{3}{2}} \left(\frac{2}{3} \right)$$

$$= 6 - 2 = 4$$

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QUESTION



If $\log_{12} 27 = a$, then $\log_6 16$ is equal to

A $4 \left(\frac{3+a}{3-a} \right)$

~~**B** $4 \left(\frac{3-a}{3+a} \right)$~~

C $2 \left(\frac{3-a}{3+a} \right)$

D $2 \left(\frac{3+a}{3-a} \right)$

$\log_{12} 27 = a$

$\log_{12} 3^3 = a$

$3 \log_3 3 = a$

$3 \frac{\log_3 3}{\log_2 3} = a$

$\frac{3 \log_3 3}{\log_2 4 + \log_2 3} = a \Rightarrow$

$\log_6 16 = \log_6 2^4 = 4 \log_6 2 = \frac{4}{\log_2 6}$
 $= \frac{4}{\log_2 2 + \log_2 3} = \frac{4}{1 + \log_2 3}$

Let $\log_2 3 = t$
 $\log_6 16 = \frac{4}{1+t}$

$\frac{3t}{2+t} = a \Rightarrow 3t = a(2+t)$
 $3t = 2a + at$
 $t = \frac{2a}{3-a}$

$\log_6 16 = \frac{4}{1 + \frac{2a}{3-a}}$
 $= \frac{4(3-a)}{3-a+2a}$

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QUESTION

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Find the value of $\log_{54} (168)$ if $\log_7 12 = a$ and $\log_{12} 24 = b$.

$\log_7^{12} \cdot \log_{12}^{24} = ab$
 $\log_7^{24} = ab$

$$\log_{54} (168) = \frac{\log_7 168}{\log_7 54} = \frac{\log_7 7 + \log_7 24}{\log_7 3^3 \cdot 2} = \frac{1 + \log_7 24}{3 \log_7 3 + \log_7 2} = \frac{1 + ab}{3 \log_7 3 + \log_7 2}$$

$\log_7^{12} = a$, $a = \log_7 24$
 $\log_7 3 + 2 \log_7 2 = a$, $3 \log_7 2 + \log_7 3 = ab$

$\log_7 2 = ab - a$

$\log_7 3 + 2ab - 2a = a$
 $\log_7 3 = 3a - 2ab$

$$\log_{54} 168 = \frac{1 + ab}{9a - 6ab + ab - a} = \frac{1 + ab}{8a - 5ab} = \frac{1 + ab}{a(8 - 5b)}$$



$$\log_{a_1} a_2 \cdot \log_{a_2} a_3 \cdot \log_{a_3} a_4 \cdots \log_{a_{n-1}} a_n = \log_{a_1} a_n$$

$$\text{Ex: } \log_{12} 24 \cdot \log_{24} 7 = \log_{12} 7$$

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QUESTION



If $\log_{10} 5 = a$ and $\log_{10} 3 = b$, then :

Tahoi

- A** $\log_{30} 8 = \frac{3(1+a)}{b+1}$
- B** $\log_{30} 8 = \frac{3(1-a)}{b+1}$
- C** $\log_{243} (32) = \frac{(1-a)}{b}$
- D** $\log_{40} (15) = \frac{a+b}{3-2a}$

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Logarithmic Equations Involving Modulus



$$\star \log_a x^{2n} = 2n \log_a |x|$$

$$\star \log_a \sqrt{x^2} = \log_a |x|$$

$$\star |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

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QUESTION



$$\log_4(x^2 - 1) - \log_4(x - 1)^2 = \log_4 \sqrt{(4 - x)^2}$$

~~A~~ $3 + \sqrt{6}$

B $3 - \sqrt{6}$

C 5

D -1

$$\log_4(x^2 - 1) - \log_4(x - 1)^2 = \log_4 |4 - x|$$

$$\log_4 \frac{x^2 - 1}{(x - 1)^2} = \log_4 |4 - x|$$

$$\frac{(x - 1)(x + 1)}{(x - 1)^2} = |4 - x|$$

$$\frac{x + 1}{x - 1} = |x - 4|, x \neq 1$$

Case 1 if $x - 4 > 0$ i.e. $x > 4$

$$\frac{x + 1}{x - 1} = x - 4 \Rightarrow x + 1 = x^2 - 5x + 4 \Rightarrow x^2 - 6x + 3 = 0$$

$$x = \frac{6 \pm \sqrt{24}}{2} = 3 + \sqrt{6}, 3 - \sqrt{6}$$

$$* |a - b| = |b - a|$$

$$|-(b - a)| = |-1| |b - a| = |b - a|$$

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Case (i) If $x-4 < 0 \Rightarrow x < 4$

$$\frac{x+1}{x-1} = -(x-4)$$

$$x+1 = -(x^2-5x+4)$$

$$x^2-4x+5=0$$

$\Delta < 0$
(No real roots)

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QUESTION



$$2 \log_8(2x) + \log_8(x^2 + 1 - 2x) = \frac{4}{3}$$

A -1

~~**B** 2~~

C -2

D 1

$$2 \log_8 2x + \log_8 (x-1)^2 = \frac{4}{3}$$

$$2 \log_8 2x + 2 \log_8 |x-1| = \frac{4}{3}$$

$$\log_8 2x + \log_8 |x-1| = \frac{2}{3}$$

$$\log_8 (2x \cdot |x-1|) = \frac{2}{3}$$

$$2x \cdot |x-1| = 8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = 4$$

$$x \cdot |x-1| = 2$$

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Case (i) If $x-1 > 0$ i.e. $x \geq 1$

$$x(x-1) = 2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2, -1 \quad \text{rejected}$$

Case (ii) If $x-1 < 0 \Rightarrow x < 1$

$$-x(x-1) = 2$$

$$x^2 - x + 2 = 0$$

(No real roots)

QUESTION



$$2 \log_3(x - 2) + \log_3(x - 4)^2 = 0$$

Tah02

A $3 + \sqrt{2}$

B $3 - \sqrt{2}$

C 3

D -3

$$2 \log_3(x-2) + 2 \log_3|x-4| = 0$$

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QUESTION



$$|x - 1|^{\log_3 x^2 - 2\log_x 9} = (x - 1)^7$$

A 3

~~**B** 2~~

C 27

~~**D** 81~~

clearly $x > 1$

$$(x-1)^{\frac{\log x^2}{3} - 2 \log_x 9} = (x-1)^7$$

Case ① $x-1=1$
 $x=2$

Case ②

$$\log_3 x^2 - 2 \log_x 9 = 7$$

$$2 \log_3 x - 2 \cdot 2 \cdot \log_x 3 = 7$$

$$2 \log_3 x - \frac{4}{\log_3 x} = 7$$

$$2t - \frac{4}{t} = 7$$

$$2t^2 - 4 = 7t$$

$$2t^2 - 7t - 4 = 0$$

any real
 (any +ve real) $\in \mathbb{R}^+$

$$|x-1| > 0$$

$a^x = a^y$

- $a=1$
- $a=0, x, y > 0$
- $a=-1, (-1)^x = (-1)^y$
- $x=y$ & both sides are defined

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$$2t^2 - 8t + t - 4 = 0$$

$$(2t + 1)(t - 4) = 0$$

$$t = -\frac{1}{2}, 4$$

$$\log_3 x = -\frac{1}{2}, 4$$

$$x = 3^{-1/2}, 3^4$$

$$x = \frac{1}{\sqrt{3}}, 81$$

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QUESTION



$|x - 1|^{\log_3 x^2 - 2\log_x 9} = (x - 1)^7$

A 3

~~**B**~~ 2

C 27

~~**D**~~ 81

clearly $x > 1$

$(x-1)^{\frac{\log x^2}{3} - 2 \log_x 9} = (x-1)^7$

$\log_3 \left(\frac{(\log_3 x^2 - 2 \log_x 9)}{(x-1)} \right) = \log_3 (x-1)$

$(\log_3 x^2 - 2 \log_x 9) \cdot \log_3 (x-1) = 7 \log_3 (x-1)$

$(2 \log_3 x - 4 \log_3 3) \cdot \log_3 (x-1) - 7 \log_3 (x-1) = 0$

any real
(any +ve real) $\in \mathbb{R}^+$

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$\log_3 (x-1) \left(2 \log_3 x - \frac{4}{\log_3 x} - 7 \right) = 0$

$\log_3 (x-1) = 0$
 $x-1 = 3^0$
 $x = 2$

or $2 \log_3 x - \frac{4}{\log_3 x} - 7 = 0$

or $2t - \frac{4}{t} - 7 = 0$
 $2t^2 - 7t - 4 = 0$

$t = -1/2, 4$
 $x = \frac{1}{\sqrt{3}}, 81$

QUESTION



Tah 03

Solve the following logarithmic equations:

$$1. \log_3(x^2 - 3x - 5) = \log_3(7 - 2x)$$

$$2. x^{0.5 \log_{\sqrt{x}}(x^2 - x)} = 3^{\log_9 4}$$

$$3. 25^{\log_{10} x} = 5 + 4 \times \log_{10} 5$$

$$4. 1 + 2 \log_{x+2} 5 = \log_5(x + 2)$$

$$5. 2 \log_2(\log_2 x) + \log_{\frac{1}{2}}(\log_2(2\sqrt{2}x)) = 1$$

$$(x^2 - x)^{0.5 \log_{\sqrt{x}} x} = 4^{\log_9 3}$$

$$(x^2 - x)^{0.5 \times 2} = 4^{\frac{1}{2}}$$

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$$x^2 - x = 2$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \rightarrow x$$

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Today's BPP

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QUESTION



Solve the following equations :

(i) $\log_{x-1} 3 = 2$

(ii) $\log_4 \left(2 \log_3 \left(1 + \log_2 \left(1 + 3 \log_3 x \right) \right) \right) = \frac{1}{2}$

(iii) $\log_3 \left(1 + \log_3 \left(2^x - 7 \right) \right) = 1$ **ATDB.uno**

(iv) $\log_3 \left(3^x - 8 \right) = 2 - x$

(v) $\frac{\log_2 (9 - 2^x)}{3 - x} = 1$

QUESTION



Solve the following equations :

$$(vi) \log_{5-x}(x^2 - 2x + 65) = 2$$

$$(vii) \log_{10} 5 + \log_{10}(x + 10) - 1 = \log_{10}(21x - 20) - \log_{10}(2x - 1)$$

$$(viii) x^{1+\log_{10} x} = 10x$$

$$(ix) 2(\log_x \sqrt{5})^2 - 3\log_x \sqrt{5} + 1 = 0$$

$$(x) 3 + 2\log_{x+1} 3 = 2\log_3(x + 1)$$

Answers:

$$i. \{1 + \sqrt{3}\} \quad ii. \{3\} \quad iii. \{4\} \quad iv. \{2\}$$

$$v. \{0\} \quad vi. \{-5\} \quad vii. \{3/2, 10\} \quad viii. \{10^{-1}, 10\}$$

$$ix. \{\sqrt{5}, 5\} \quad x. \{-(3 - \sqrt{3})/3, 8\}$$



Solution to Previous TAH

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QUESTION

Prove that $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2} = 3$

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* TAH

TAH1) Prove that $\frac{\log_2 24}{\log_{46} 2} - \frac{\log_2 192}{\log_{12} 2} = 3$

$$\Rightarrow \log_2 24 \times \log_2 96 - \log_2 192 \times \log_2 12$$

$$\Rightarrow (\log_2 2^3 + \log_2 3) (\log_2 2^5 + \log_2 3) - (\log_2 2^6 + \log_2 3) (\log_2 2^2 + \log_2 3)$$

$$\Rightarrow (3 + \log_2 3) (5 + \log_2 3) - (6 + \log_2 3) (2 + \log_2 3)$$

$$= 15 + 3 \log_2 3 + 5 \log_2 3 + \log_2 3 \cdot \log_2 3 - 12 - 6 \log_2 3 - 2 \log_2 3 - \log_2 3 \cdot \log_2 3$$

$$= 15 + 8 \log_2 3 - 12 - 8 \log_2 3$$

$$= 15 - 12 = \underline{3}$$

$$= \text{RHS}$$

**Kriti Mathur
Raj.**

Home Prepared



$$\# \frac{\log_2 24}{\log_2 96} - \frac{\log_2 192}{\log_2 12} = 3 \quad \text{Tah-01}$$

$$\Rightarrow \log_2 24 \cdot \log_2 96 - \log_2 192 \cdot \log_2 12$$

$$\Rightarrow (\log_2 2 + \log_2 12) \cdot \log_2 96 - (\log_2 2 + \log_2 96) \log_2 12$$

$$\Rightarrow (1 + \log_2 12) \log_2 96 - (1 + \log_2 96) \log_2 12$$

$$\Rightarrow \log_2 96 + \log_2 12 \cdot \log_2 96 - \log_2 12 - \log_2 96 \cdot \log_2 12$$

$$\Rightarrow \log_2 96 - \log_2 12$$

$$\Rightarrow \log_2 \left(\frac{96}{12} \right)$$

$$\Rightarrow \log_2 8$$

$$\Rightarrow \log_2 2^3$$

$$\Rightarrow \underline{3}$$

proved

key concept.

$$\log_2 24 = \log_2 2 + \log_2 12$$

$$\log_2 192 = \log_2 2 + \log_2 96$$



Tah-01.

Prove that $\frac{\log_2 24}{\log_{36} 2} - \frac{\log_2 192}{\log_{12} 2} = 3$.

$\log_b a = \frac{1}{\log_a b}$ *

LHS.

$$\Rightarrow \log_2 36 \cdot \log_2 24 - \log_2 12 \cdot \log_2 192$$

$$\Rightarrow \log_2 (2^5 \times 3) \cdot \log_2 (2^3 \times 3) - \log_2 (2^2 \times 3) \cdot \log_2 (2^6 \times 3)$$

$$\Rightarrow (\log_2 2^5 + \log_2 3) \cdot (\log_2 2^3 + \log_2 3) - (\log_2 2^2 + \log_2 3) \cdot (\log_2 2^6 + \log_2 3)$$

Let: $\log_2 3 = t$

$$\Rightarrow (5+t)(3+t) - (2+t)(6+t)$$

$$\Rightarrow (15 + 5t + 3t + t^2) - (12 + 2t + 6t + t^2)$$

$$\Rightarrow \cancel{t^2} + 8t + 15 - \cancel{t^2} - 8t - 12$$

$$\Rightarrow \textcircled{3} = \text{RHS (proved)}$$

krish keshri
jharkhand

QUESTION



Simplify & compute :

$$\frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$$

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Tah-02.

Simplify & compute :

$$\Rightarrow \frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$$

$$\Rightarrow \log_5 50 \cdot \log_5 250 - \log_5 1250 \cdot \log_5 10$$

$$\Rightarrow \log_5 (5^2 \times 2) \cdot \log_5 (5^3 \times 2) - \log_5 (5^4 \times 2) \cdot \log_5 (5 \times 2)$$

$$\Rightarrow (\log_5 5^2 + \log_5 2) \cdot (\log_5 5^3 + \log_5 2) - (\log_5 5^4 + \log_5 2) \cdot (\log_5 5 + \log_5 2)$$

Let: $\log_5 2 = t$

$$\Rightarrow (2+t)(3+t) - (4+t)(1+t)$$

$$\Rightarrow (6+2t+3t+t^2) - (4+4t+t+t^2)$$

$$\Rightarrow t^2 + 5t + 6 - t^2 - 5t - 4$$

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$$\# \frac{\log_{\sqrt{5}} 250}{\log_{\sqrt{5}} 5} - \frac{\log_{\sqrt{5}} 10}{\log_{\sqrt{1250}} 5}$$

Tah - 02

$$\Rightarrow \log_{\sqrt{5}} 250 \cdot \log_{\sqrt{5}} 50 - \log_{\sqrt{5}} 10 \cdot \log_{\sqrt{5}} 1250$$

$$\Rightarrow \log_{\sqrt{5}} 250 \cdot (\log_{\sqrt{5}} 5 + \log_{\sqrt{5}} 10) - \log_{\sqrt{5}} 10 \cdot (\log_{\sqrt{5}} 5 + \log_{\sqrt{5}} 250)$$

$$\Rightarrow \log_{\sqrt{5}} 250 \cdot (1 + \log_{\sqrt{5}} 10) - \log_{\sqrt{5}} 10 (1 + \log_{\sqrt{5}} 250)$$

$$\Rightarrow \log_{\sqrt{5}} 250 + \frac{\log_{\sqrt{5}} 250 \cdot \log_{\sqrt{5}} 10}{\sqrt{5}} - \log_{\sqrt{5}} 10 - \frac{\log_{\sqrt{5}} 10 \cdot \log_{\sqrt{5}} 250}{\sqrt{5}}$$

$$\Rightarrow \log_{\sqrt{5}} 250 - \log_{\sqrt{5}} 10$$

$$\Rightarrow \log_{\sqrt{5}} \left(\frac{250}{10} \right)$$

$$\Rightarrow \log_{\sqrt{5}} 25$$

$$\Rightarrow \log_{\sqrt{5}} 5^2$$

$$\Rightarrow \underline{2} \text{ Ans}$$

QUESTION [JEE Advanced 2018]



The value of $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is

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Tah-03.

The value of $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is:

★
Let: $\log_2 9 = t$

★
 $\log_a b = \frac{1}{\log_b a}$

⇒ $(x^2)^{\frac{1}{\log_2 x}} \times (7)^{\frac{1}{2} \log_7 4}$

⇒ $(x)^2 \log_x 2 \times (7)^{\frac{1}{2} \log_7 4}$

★
 $(a)^{\log_a x} = x$

⇒ $x^{\log_x (2^2)} \times 7^{\log_7 (4)^{\frac{1}{2}}}$

⇒ $2^2 \times 2 = 2^3 = \textcircled{8} \text{ Ans.}$

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Tan 03

The value of $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is

$$\left((\log_2 9)^2 \right)^{\frac{1}{\log_2 \log_2 9}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$$

$$\left(\log_2 9 \right)^{2 \log_2 2} \times 7^{\frac{1}{2} \log_4 4}$$

$$\left(\log_2 9 \right)^{\log_2 4} \times 7^{\log_4 (4)^{1/2}}$$

$$4^{\log_2 \log_2 9} \times (4)^{1/2}$$

$$4 \times 4^{1/2} = 8$$

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QUESTION



Indicate all correct alternatives, where base of the log is 2.

The equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has :

- A** At least one real solution
- B** Exactly 3 real solutions
- C** Exactly one irrational solution
- D** Imaginary roots

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Tab-04

Indicate all correct alternatives, where base of the log is 2. The Equation :

$$\Rightarrow x^{\frac{3}{4}} (\log_2 x)^2 + \log_2 x - \frac{5}{4} = \sqrt{2} \text{ has :}$$

A) At least one real solⁿ. \Rightarrow Take log base 2 on both sides.

B) Exactly 3 real solⁿ. $\Rightarrow \log_2 x \left[\frac{3}{4} (\log_2 x)^2 + \log_2 x - \frac{5}{4} \right] =$

C) Exactly one irrational solⁿ.

D) Imaginary roots. \Rightarrow let $\log_2 x = t$ $\frac{1}{2} \log_2^2$

$$\Rightarrow t \left[\frac{3}{4} t^2 + t - \frac{5}{4} \right] = \frac{1}{2}$$

$$\Rightarrow \frac{3}{4} t^3 + t^2 - \frac{5}{4} t - \frac{1}{2} = 0$$

$$\Rightarrow \frac{3t^3 + 4t^2 - 5t - 2}{4} = 0$$

$$\Rightarrow 3t^3 + 4t^2 - 5t - 2 = 0 \quad \begin{matrix} \text{Put, } t=1 \\ = 3+4-5-2=0 \end{matrix}$$

$$\Rightarrow 3t^2(t-1) + 7t(t-1) + 2(t-1) = 0 \quad \text{--- } (t-1) \text{ is a factor.}$$

$$\Rightarrow (t-1)(3t^2 + 7t + 2) = 0$$

$$\Rightarrow (t-1)(3t+1)(t+2) = 0$$

$$\Rightarrow \boxed{t = 1, -\frac{1}{3}, -2}$$

$$1) \log_2 x = 1 \Rightarrow x = 2^1$$

$$2) \log_2 x = -\frac{1}{3} \Rightarrow x = 2^{-1/3} = \frac{1}{\sqrt[3]{2}}$$

$$3) \log_2 x = -2 \Rightarrow x = 2^{-2} = \frac{1}{4}$$

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TAH 4) Indicate all correct alternatives, where
is 2. The equation $\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} = \sqrt{2}$ has:

- A) At least one real solution
B) Exactly 3 real solutions
C) Exactly one irrational solution
D) Imaginary roots

$$\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} = \sqrt{2}$$

take log both sides with base 2

$$\left[\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} \right] \cdot \log_2 x = \log_2 \sqrt{2}$$

$$\left[\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} \right] \cdot \log_2 x = \frac{1}{2}$$

Let $\log_2 x = t$

$$\left[\frac{3}{4}t^2 + t - \frac{5}{4} \right] t = \frac{1}{2}$$

$$= (3t^2 + 4t - 5)t = 2$$

$$P = 3t^2 + 4t^2 - 5t - 2 = 0$$

$$\text{at } t = 1$$

$$P = 0 \therefore (t-1) \text{ is a factor}$$

$$3t^2(t-1) + 7t(t-1) + 2(t-1) = 0$$

$$(t-1)(3t^2 + 7t + 2) = 0$$

$$(t-1)(3t+1)(t+2) = 0$$

$$t = 1, \quad t = -\frac{1}{3}, \quad t = -2$$

$$\log_2 x = 1, \quad \log_2 x = -\frac{1}{3}, \quad \log_2 x = -2$$

$$x = 2, \quad x = \frac{1}{\sqrt[3]{2}}, \quad x = \frac{1}{4}$$

(A) (B) (C) (D) (E)

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Raj.

QUESTION



The equation $x^{[(\log_3 x)^2 - \frac{9}{2} \log_3 x + 5]} = 3\sqrt{3}$ has

- A** Exactly 3 real solutions
- B** At least one real solution
- C** Exactly one irrational solution
- D** Complex roots

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Tak-05

The equation $x \left[(\log_3 x)^2 - \frac{9}{2} \log_3 x + 5 \right] = 3\sqrt{3}$ has:

- A) Exactly 3 real solⁿ. \Rightarrow Taking log base 3 on both sides.
- B) At least one real solⁿ.
- C) Exactly one irrational solⁿ.
- D) Complex roots. $\Rightarrow \log_3 x \left[(\log_3 x)^2 - \frac{9}{2} \log_3 x + 5 \right] = \frac{3}{2} \log_3^3$

\Rightarrow let: $\log_3 x = t$

$\Rightarrow t \left[t^2 - \frac{9}{2} t + 5 \right] = \frac{3}{2}$

$\Rightarrow t^3 - \frac{9}{2} t^2 + 5t - \frac{3}{2} = 0$

$\Rightarrow \frac{2t^3 - 9t^2 + 10t - 3}{2} = 0$

$\Rightarrow 2t^3 - 9t^2 + 10t - 3 = 0$

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Put, $t = 1$
 $\Rightarrow 2 - 9 + 10 - 3 = 0$.
 (t-1) is a factor.

$\Rightarrow 2t^2(t-1) - 7t(t-1) + 3(t-1) = 0$

$\Rightarrow (t-1)(2t^2 - 7t + 3) = 0$

$\Rightarrow (t-1)(2t-1)(t-3) = 0$

$\Rightarrow t = 1, \frac{1}{2}, 3$

- 1) $\log_3 x = 1 \Rightarrow x = 3$.
- 2) $\log_3 x = \frac{1}{2} \Rightarrow x = 3^{1/2} = \sqrt{3}$.
- 3) $\log_3 x = 3 \Rightarrow x = 3^3 = 27$.



TAHS. The equation $x^{[(\log_3 x)^2 - \frac{9}{2} \log_3 x + 5]} = 3\sqrt{3}$

taking log both sides with base 3

$$\left[(\log_3 x)^2 - \frac{9}{2} \log_3 x + 5 \right] \cdot \log_3 x = \log_3 3\sqrt{3}$$

$$\left[(\log_3 x)^2 - \frac{9}{2} \log_3 x + 5 \right] \cdot \log_3 x = \frac{3}{2}$$

let $\log_3 x = t$

$$\left[t^2 - \frac{9}{2} t + 5 \right] t = \frac{3}{2}$$

Kriti Mathur

Raj.

$$(2t^2 - 9t + 10)t = 3$$

$$2t^3 - 9t^2 + 10t - 3 = 0$$

$$2t^2(t-1) - 7t(t-1) + 3(t-1) = 0$$

$$(t-1)(2t^2 - 7t + 3) = 0$$

$$(t-1)(2t^2 - 6t - t + 3) = 0$$

$$(t-1)[2t(t-3) - 1(t-3)]$$

$$(t-1)(2t-1)(t-3) = 0$$

$$t=1, \quad t=\frac{1}{2}, \quad t=3$$

$$\log_3 x = 1, \quad \log_3 x = \frac{1}{2}, \quad \log_3 x = 3$$

$$x = 3, \quad x = \sqrt{3}, \quad x = 27$$

- (A) exactly 3 real solutions
 (B) At least one real solution
 (C) exactly one irrational solution

- Ans



Solution to Previous KTKs

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QUESTION

(KTK 1)



Solve the following inequalities:

$$(1) \frac{x}{x-5} > \frac{1}{2}$$

$$(2) \frac{x-5}{x-9} \leq 0$$

$$(3) x \leq 3 - \frac{1}{x-1}$$

$$(4) \frac{1}{x} \leq 1$$

$$(5) \frac{5x}{3x-1} \leq 0$$

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Ans. (1) $x \in (-\infty, -5) \cup (5, \infty)$, (2) $x \in (-\infty, 0) \cup [1, \infty)$,
(3) $x \in [5, 9)$, (4) $x \in \left[0, \frac{1}{3}\right)$, (5) $x \in (-\infty, 1) \cup \{2\}$



Solve the following inequalities :

$$(1) \frac{x}{x-5} > \frac{1}{2}$$

$$\Rightarrow \frac{x}{x-5} - \frac{1}{2} > 0$$

$$\Rightarrow \frac{2x - x + 5}{2(x-5)} > 0$$

$$\Rightarrow \frac{x-5}{2x-10} > 0$$

$$\Rightarrow \begin{array}{c} \leftarrow + \quad - \quad + \rightarrow \\ | \quad | \\ -5 \quad 5 \end{array}$$

$$\Rightarrow \boxed{x \in (-\infty, -5) \cup (5, \infty)}$$

$$(3) x \leq 3 - \frac{1}{x-1}$$

$$\Rightarrow 3 - \frac{1}{x-1} - x \geq 0$$

$$\Rightarrow \frac{3x-3-1-x^2+x}{(x-1)} \geq 0$$

$$\Rightarrow \frac{-x^2+4x-4}{(x-1)} \geq 0$$

$$\Rightarrow \frac{x^2-4x+4}{(x-1)} \leq 0$$

$$\Rightarrow \frac{(x-2)^2}{(x-1)} \leq 0 ; \boxed{x=2} \text{ is possible.}$$

$$\Rightarrow \begin{array}{c} \leftarrow - \quad + \rightarrow \\ | \\ 1 \end{array}$$

$$\Rightarrow \boxed{x \in (-\infty, 1) \cup \{2\}}$$

$$(2) \frac{x-5}{x-9} \leq 0$$

$$\Rightarrow \begin{array}{c} \bullet \quad \circ \\ | \quad | \\ 5 \quad 9 \end{array}$$

$$\Rightarrow \boxed{x \in [5, 9)}$$

$$(4) \frac{1}{x} \leq 1$$

$$\Rightarrow \frac{1}{x} - 1 \leq 0$$

$$\Rightarrow \frac{1-x}{x} \leq 0$$

$$\Rightarrow \frac{x-1}{x} \geq 0$$

$$\Rightarrow \begin{array}{c} \bullet \quad \circ \\ | \quad | \\ 0 \quad 1 \end{array}$$

$$\Rightarrow \boxed{x \in (-\infty, 0) \cup [1, \infty)}$$

$$(5) \frac{5x}{3x-1} \leq 0$$

$$\Rightarrow \begin{array}{c} \bullet \quad \circ \\ | \quad | \\ 0 \quad \frac{1}{3} \end{array}$$

$$\Rightarrow \boxed{x \in [0, \frac{1}{3})}$$

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KTK - 01

$$1. \frac{x}{x-5} > \frac{1}{2}$$

$$\frac{x}{x-5} - \frac{1}{2} > 0$$

$$\frac{2x - (x-5)}{2(x-5)} > 0$$

$$\frac{x+5}{2x-10} > 0 \Rightarrow x \in (-\infty, -5) \cup (5, \infty)$$

$$2. \frac{x-5}{x-9} \leq 0$$

$$x \in [5, 9)$$

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$$\textcircled{iii}) \quad x \leq 3 - \frac{1}{x-1}$$

$$x + \frac{1}{x-1} - 3 \geq 0$$

$$x - 3 + \frac{1}{x-1} \leq 0$$

$$\frac{(x-3)(x-1)}{x-1} \leq 0$$

$$\frac{x^2 - 4x + 4}{x-1} \leq 0$$

$$\frac{(x-2)^2}{x-1} \leq 0$$

$$x \in (-\infty, 1) \cup \{2\}$$

$$\textcircled{iv}) \quad \frac{1}{x} \leq 1$$

$$\frac{x-1}{x} \geq 0$$

$$x \in (-\infty, 0) \cup (1, \infty)$$

$$\textcircled{v}) \quad \frac{5x}{3x-1} \leq 0$$

$$x \in (0, \frac{1}{3})$$

QUESTION

(KTK 2)



Complete solution set of inequality $\frac{(x+2)(x+3)}{(x-2)(x-3)} \leq 1$ is

- A** $(-\infty, 0]$
- B** $(-\infty, 0] \cup (2, 3)$
- C** $[2, 3]$
- D** $(-\infty, 2) \cup (3, \infty)$

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Ans. B

KTK-2

Q. Complete solution set of inequality $\frac{(x+2)(x+3)}{(x-2)(x-3)} \leq 1$.

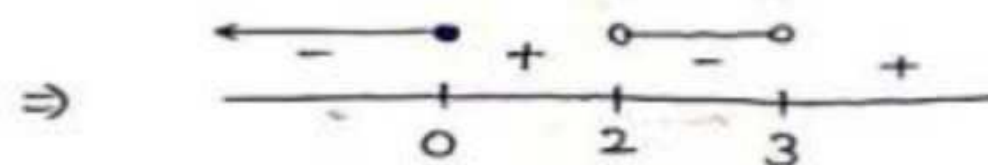
$$\Rightarrow \frac{(x+2)(x+3)}{(x-2)(x-3)} - 1 \leq 0$$

$$\Rightarrow \frac{(x+2)(x+3) - (x-2)(x-3)}{(x-2)(x-3)} \leq 0$$

$$\Rightarrow \frac{[x^2 + 5x + 6] - [x^2 - 5x + 6]}{(x-2)(x-3)} \leq 0$$

$$\Rightarrow \frac{\cancel{x^2} + 5x + \cancel{6} - \cancel{x^2} + 5x - \cancel{6}}{(x-2)(x-3)} \leq 0$$

$$\Rightarrow \frac{10x}{(x-2)(x-3)} \leq 0$$



$$\Rightarrow \boxed{x \in (-\infty, 0] \cup (2, 3)}$$

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Q complete soln set of inequality

- (A) $(-\infty, 0]$
- ~~(B) $(-\infty, 0] \cup (2, 3)$~~
- (C) $[2, 3]$
- (D) $(-\infty, 2) \cup (3, \infty)$

$$\frac{(x+2)(x+3)}{(x-2)(x-3)} \leq 1 \quad \text{if}$$

KTk-02

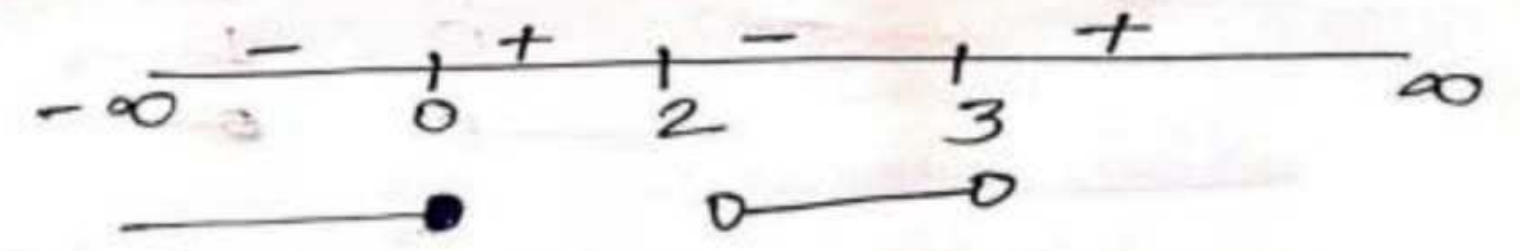
Soln

$$\Rightarrow \frac{(x+2)(x+3)}{(x-2)(x-3)} - 1 \leq 0$$

$$\Rightarrow \frac{(x+2)(x+3) - (x-2)(x-3)}{(x-2)(x-3)} \leq 0$$

$$\Rightarrow \frac{x^2 + 5x + 6 - (x^2 - 5x + 6)}{(x-2)(x-3)} \leq 0$$

$$\Rightarrow \frac{10x}{(x-2)(x-3)} \leq 0$$



$x \in (-\infty, 0] \cup (2, 3)$ Ans

Jiya Bihari May 3, 2025, 07:32



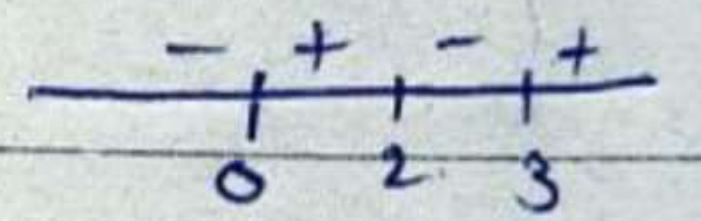
KTK 2

$$\frac{(x+2)(x+3)}{(x-2)(x-3)} \leq 1$$

$$\frac{(x+2)(x+3) - (x-2)(x-3)}{(x-2)(x-3)} \leq 0$$

$$\frac{x^2 + 5x + 6 - x^2 + 5x - 6}{(x-2)(x-3)} \leq 0$$

$$\frac{2(5x)}{(x-2)(x-3)} \leq 0$$



$$x \in (-\infty, 0] \cup (2, 3)$$

(B)

Sakshi

QUESTION**(KTK 3)**

Find sum of all integral values of x satisfying $\frac{x^2 - 5x + 6}{x^2 - x - 12} \leq 0$.

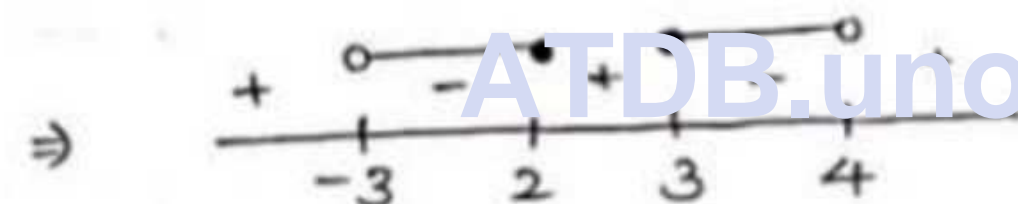
ATDB.uno**Ans. 3**

KTK-3

Q. Find the sum of all integral values of x satisfying

$$\frac{x^2 - 5x + 6}{x^2 - x - 12} \leq 0.$$

$$\Rightarrow \frac{(x-2)(x-3)}{(x-4)(x+3)} \leq 0$$



$$\Rightarrow x \in (-3, 2] \cup [3, 4).$$

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Sum of integral value :

$$\Rightarrow \cancel{(-2)} + \cancel{(-1)} + 0 + \cancel{1} + \cancel{2} + 3$$

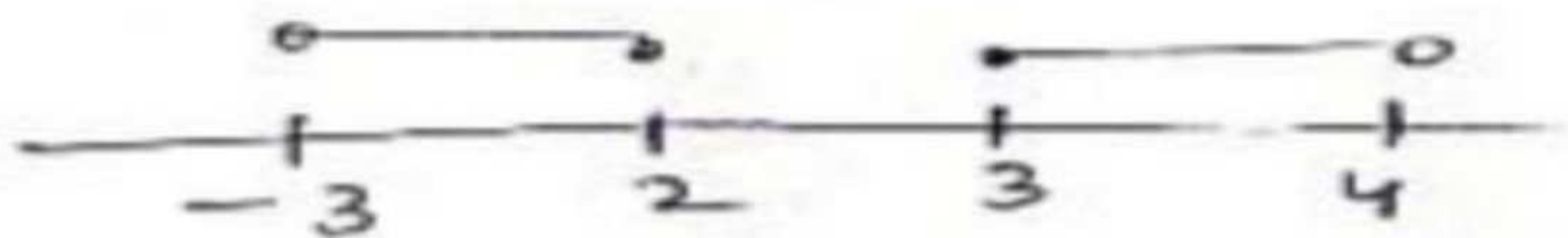
$$\Rightarrow \textcircled{3} \text{ Ans.}$$



KTK - 03

$$\frac{x^2 - 5x + 6}{x^2 - x - 12} \leq 0$$

$$\frac{-(x-3)(x-2)}{(x-4)(x+3)} \leq 0$$



$$(-3, 2] \cup [3, 4)$$

QUESTION

(KTK 4)



Which of the following does not hold true for the expression

$$E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1} ?$$

- A** $E = 2$ if $x \leq -1$
- B** $E = -2x$ if $-1 < x < 1$
- C** $E = -2$ if $x \geq 1$
- D** $E = -2$ for all x

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Ans. D

KTK-04

Q. Which of the following does not hold true for the expression $E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1}$?

A) $E = 2$ if $x \leq -1$. $\Rightarrow E = \sqrt{(x-1)^2} - \sqrt{(x+1)^2}$

B) $E = -2x$ if $-1 < x < 1$. $\Rightarrow E = |x-1| - |x+1|$ *

C) $E = -2$ if $x \geq 1$.

~~D) $E = -2$ for all x .~~

Option check :

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Ⓐ $x \leq -1$

$$\begin{aligned} E &= -(x-1) + (x+1) \\ &= \cancel{-x+1} + \cancel{x+1} \\ &= \textcircled{2} \checkmark \end{aligned}$$

Ⓑ $-1 < x < 1$

$$\begin{aligned} E &= -(x-1) - (x+1) \\ &= \cancel{-x+1} - \cancel{x-1} \\ &= -2x \checkmark \end{aligned}$$

Ⓒ $x \geq 1$

$$\begin{aligned} E &= (x-1) - (x+1) \\ &= \cancel{x-1} - \cancel{x-1} \\ &= -2 \checkmark \end{aligned}$$

Ⓓ For all x :

$E = -2$ for all x .

ये गलत है !

Ex : Option A & B.

Which of the following does not hold true for the expression.

$$E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1} = ?$$

- (A) $E = 2$ if $x \leq -1$
 (B) $E = -2x$ if $-1 < x < 1$
 (C) $E = -2$ if $x \geq 1$
 (D) $E = -2$ for all x .

KT K-04

Soln

$$E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1}$$

$$= \sqrt{x^2 - x - x + 1} - \sqrt{x^2 + x + x + 1}$$

$$= \sqrt{(x-1)(x-1)} - \sqrt{(x+1)(x+1)}$$

$$= \sqrt{(x-1)^2} - \sqrt{(x+1)^2}$$

$$= |x-1| - |x+1|$$

$x \geq 1$	$-1 < x < 1$	$x \leq -1$
$\Rightarrow x-1 - x-1$	$\Rightarrow -x+1 - x-1$	$\Rightarrow -(x-1) - \{-(x+1)\}$
$\Rightarrow -2$	$\Rightarrow -2x$	$\Rightarrow -x+1 + x+1$
		$\Rightarrow 2$

Ans \rightarrow (A), (B) & (C)



QUESTION



KTK 05

If $x \in [-5, 7]$, then number of integral values of x satisfying $\frac{2x+3}{x^2+x-12} < \frac{1}{2}$ is

A 5

B 6

C 7

D 8

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Ans. C

KTK-07.

Q. If $x \in [-5, 7]$, then number of integral value of x satisfying $\frac{2x+3}{x^2+x-12} < \frac{1}{2}$ is :

$$\Rightarrow \frac{2x+3}{x^2+x-12} - \frac{1}{2} < 0$$

$$\Rightarrow \frac{4x+6-x^2-x+12}{2(x^2+x-12)} < 0$$

$$\Rightarrow \frac{-x^2+3x+18}{x^2+x-12} < 0$$

$$\Rightarrow \frac{x^2-3x-18}{x^2+x-12} > 0$$

$$\Rightarrow \frac{(x-6)(x+3)}{(x+4)(x-3)} > 0 \Rightarrow \begin{array}{c} \leftarrow \text{---} \circ \text{---} \text{---} \circ \text{---} \text{---} \circ \text{---} \rightarrow \\ \text{+} \quad \text{---} \quad \text{---} \quad \text{+} \quad \text{---} \quad \text{---} \quad \text{+} \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ -4 \quad -3 \quad 3 \quad 6 \end{array}$$

$$\Rightarrow x \in (-\infty, -4) \cup (-3, 3) \cup (6, \infty)$$

$$\Rightarrow \begin{array}{ccccccccccccccc} \checkmark & \alpha & \alpha & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \alpha & \alpha & \alpha & \alpha & \checkmark \\ -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array}$$

$$\Rightarrow \textcircled{7} \text{ Ans.}$$



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if $x \in \mathbb{R}$, then integral values of x satisfying -

$$\frac{2x+3}{x^2+x-12} < \frac{1}{2} \quad \text{if}$$

Soln.

(A) 5

(B) 6

(C) 7

(D) 8

$$\Rightarrow \frac{2x+3}{x^2+x-12} < \frac{1}{2}$$

$$\Rightarrow \frac{2x+3}{x^2+x-12} - \frac{1}{2} < 0$$

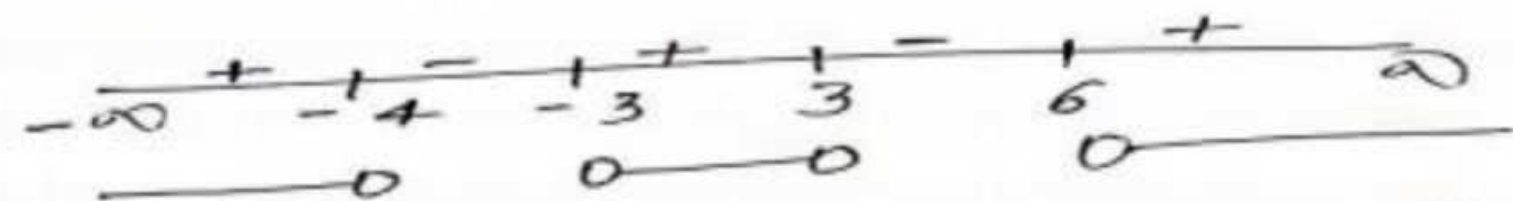
$$\Rightarrow \frac{4x+6 - x^2 - x + 12}{(x^2+x-12)2} < 0$$

$$\Rightarrow \frac{-x^2 + 3x + 18}{x^2+x-12} < 0$$

$$\Rightarrow \frac{x^2 - 3x - 18}{x^2+x-12} > 0$$

$$\Rightarrow \frac{x^2 - 6x + 3x - 18}{x^2 + 4x - 3x - 12} > 0$$

$$\Rightarrow \frac{(x-6)(x+3)}{(x+4)(x-3)} > 0$$



$$x \in (-\infty, -4) \cup (-3, 3) \cup (6, \infty)$$

given
of $[-5, 7]$

KTK-07



QUESTION

KTK 06



Solution set of the inequality $x + 1 \leq \frac{6}{x}$ is

- A** $(0, 2]$
- B** $[-3, 2)$
- C** $(-\infty, -3] \cup (0, 2]$
- D** $[-3, 0) \cup (2, \infty)$

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Ans. C



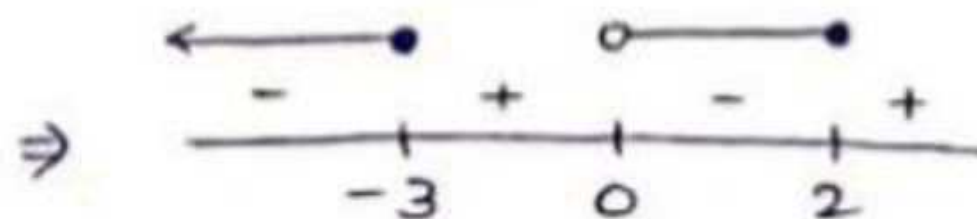
KTK-08

Q. Solution set of the inequality $x+1 \leq \frac{6}{x}$ is :

$$\Rightarrow x+1 - \frac{6}{x} \leq 0.$$

$$\Rightarrow \frac{x^2+x-6}{x} \leq 0.$$

$$\Rightarrow \frac{(x+3)(x-2)}{x} \leq 0.$$



$$\Rightarrow \boxed{x \in (-\infty, -3] \cup (0, 2]} \text{ Ans.}$$

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QUESTION

KTK 07



The set of all values of x for which $\frac{(x+1)(x-3)^2(x-5)(x-4)^3(x-2)}{x} < 0$ is

A $(-\infty, -1) \cup (0, 2) \cup (4, 5)$

B $(-1, 0) \cup (2, 4) \cup (5, \infty)$

C $(-1, 0) \cup (2, 3) \cup (4, 5)$

D $(-\infty, -1) \cup (0, 2) \cup [3, 5)$

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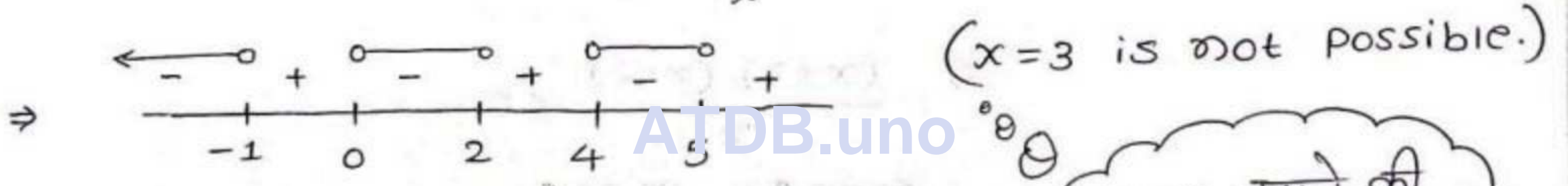
Ans. A



KTK-09.

Q. The set of all values of x for which

$$\Rightarrow \frac{(x+1)(x-3)^2(x-5)(x-4)^3(x-2)}{x} < 0 \text{ is :}$$



$$\Rightarrow x \in (-\infty, -1) \cup (0, 2) \cup (4, 5). \text{ Ans.}$$

x=3 हटाने की जरूरत ही नहीं है बिचारा को आ ही नहीं रहा है!

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—x—

QUESTION

KTK 8



Which of the following sets does not satisfy the inequality $\frac{1}{x-2} + \frac{1}{x-1} \geq \frac{1}{x}$?

A $(-\sqrt{2}, 0)$

B $(1, \sqrt{2})$

C $(2, \infty)$

D $(0, 1)$

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Ans. D



Q. Which of the following sets does not satisfy the inequality $\frac{1}{x-2} + \frac{1}{x-1} \geq \frac{1}{x}$?

A) $(-\sqrt{2}, 0)$

B) $(1, \sqrt{2})$

C) $(2, \infty)$

~~D) $(0, 1)$~~

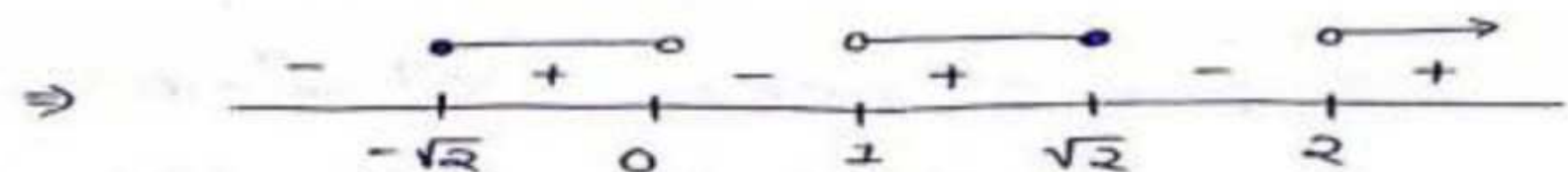
$$\Rightarrow \frac{1}{x-2} + \frac{1}{x-1} - \frac{1}{x} \geq 0.$$

$$\Rightarrow \frac{x(x-1) + x(x-2) - (x-1)(x-2)}{x(x-1)(x-2)} \geq 0.$$

$$\Rightarrow \frac{x^2 - x + x^2 - 2x - (x^2 - 2x - x + 2)}{x(x-1)(x-2)} \geq 0.$$

$$\Rightarrow \frac{\cancel{x^2} - \cancel{x} + \cancel{x} - \cancel{2x} - \cancel{x^2} + \cancel{2x} + \cancel{x} - 2}{x(x-1)(x-2)} \geq 0.$$

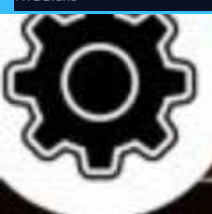
$$\Rightarrow \frac{x^2 - 2}{x(x-1)(x-2)} \geq 0.$$



$$\Rightarrow \boxed{x \in [-\sqrt{2}, 0) \cup (1, \sqrt{2}] \cup (2, \infty)}.$$

$\therefore (0, 1)$ does not satisfy the inequality.

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Home Challenge-03



Positive integers a and b satisfy the condition $\log_2 [\log_2^a (\log_2^b (2^{1000}))] = 0$. Then the possible values of $a + b$ is/are:

- A** 501
- B** 252
- C** 128
- D** 66

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Q. Positive integers a and b satisfy the condition $\log_2 [\log_2 a (\log_2 b (2^{1000}))] = 0$. Then the possible values of $a+b$ is/are.

~~A) 501.~~ $\Rightarrow \log_2 [\log_2 a (\log_2 b (2^{1000}))] = 0$.

~~B) 252.~~ $\Rightarrow \log_2 \left[\frac{1}{a} \log_2 \left(\frac{1000}{b} \log_2 2 \right) \right] = 0$.

~~C) 128.~~ $\Rightarrow \log_2 \left[\frac{1}{a} \log_2 \left(\frac{1000}{b} \right) \right] = 0$

D) 66.

$\Rightarrow \frac{1}{a} \log_2 \left(\frac{1000}{b} \right) = 1$.

$\Rightarrow \frac{1000}{b} = 2^a \Rightarrow \boxed{2^a \times b = 1000}$

① $2^a \times b = 2^1 \times 500$

$\Rightarrow a=1, b=500$

$\Rightarrow a+b=501$

② $2^a \times b = 2^2 \times 250$

$\Rightarrow a=2, b=250$

$\Rightarrow a+b=252$

③ $2^a \times b = 2^3 \times 125$

$\Rightarrow a=3, b=125$

$\Rightarrow a+b=128$

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THANK YOU

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