PRAYAS

JEE 2026

Mathematics

Basic Maths

Lecture - 10

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Topics *to be covered*

A Logarithm and its Properties

B Problem Practice

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Homework Discussion

QUESTION (Home Challenge-02)

Solve in real numbers the equation

$$\sqrt{x_1-1}+2\sqrt{x_2-4}+\cdots+n\sqrt{x_n-n^2}=\frac{1}{2}(x_1+x_2+\cdots+x_n) \text{ for } x_1, x_2, x_3 \ldots, x_{n-1}, x_n.$$

$$2\sqrt{x_1-1}+4\sqrt{x_2-4}+---+2n\sqrt{x_n-n^2}=x_1+x_2+--+x_n$$

$$\sqrt{x_1-1}^2-2\sqrt{x_1-1}+1+\sqrt{x_2-4}^2-4\sqrt{x_2-4}+4+\sqrt{x_3-9}^2-6\sqrt{x_3-9}+9+---$$

$$--+\sqrt{x_n-n^2}^2-2n\sqrt{x_n-n^2}+n^2=0$$

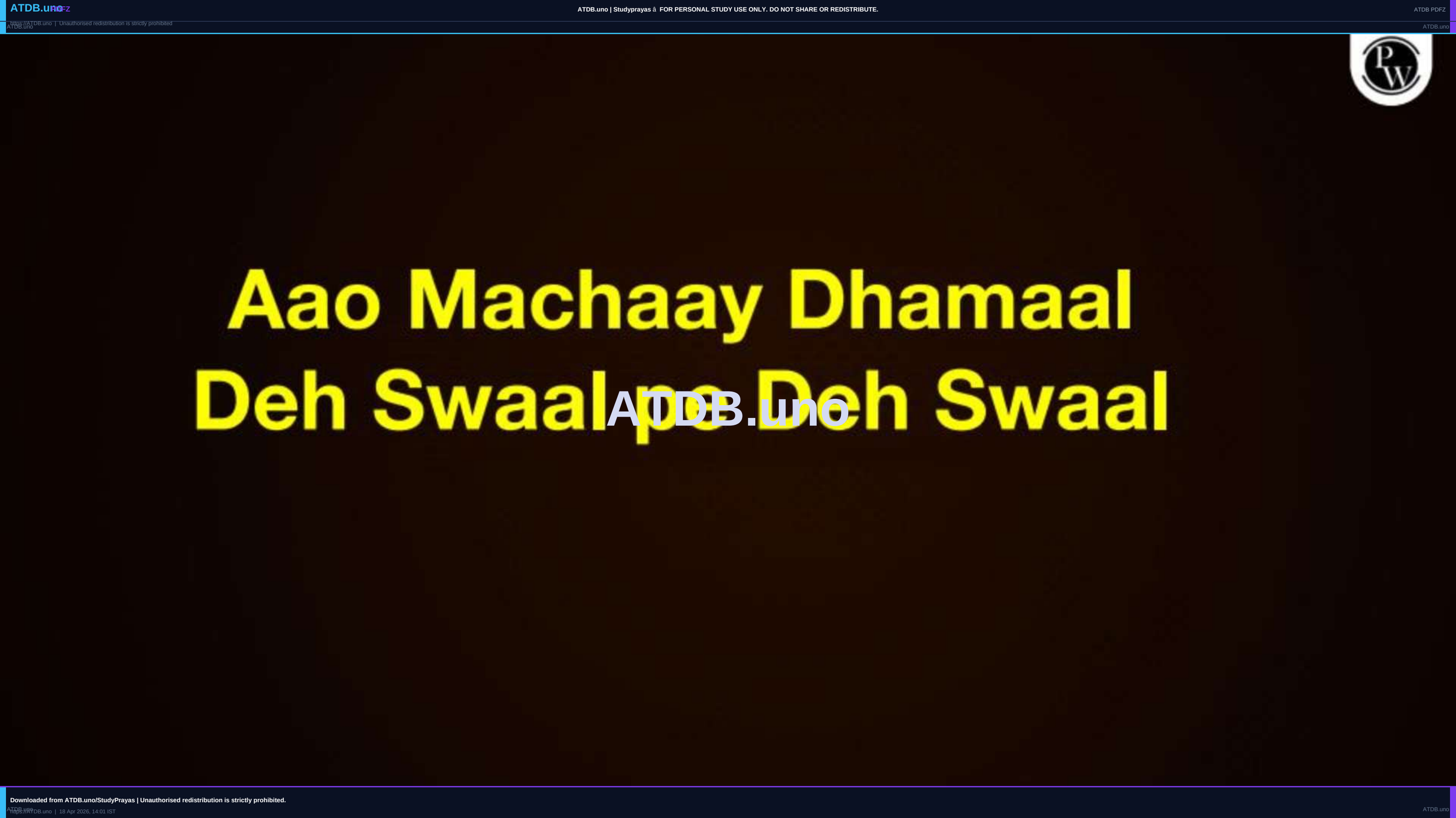
$$\Rightarrow\left(\sqrt{x_1-1}-1\right)^2+\left(\sqrt{x_2-4}-2\right)^2+\left(\sqrt{x_3-9}-3\right)^2+---+\left(\sqrt{x_n-n^2}-n\right)^2=0$$

$\sqrt{x_1-1}=1 \Rightarrow x_1=2$

$\sqrt{x_2-4}=2 \Rightarrow x_2=8$

$\sqrt{x_3-9}=3 \Rightarrow x_3=18$

$\sqrt{x_n-n^2}=n \Rightarrow x_n=2n^2$



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$$D_3: \log_{a_1} a_2 \cdot \log_{a_2} a_3 \cdot \log_{a_3} a_4 \ldots\ldots \log_{a_{n-1}} a_n = \log_{a_1} a_n$$

Proof: $\log_{a_1} a_2 \cdot \log_{a_2} a_3 \log_{a_3} a_4 - - - \log_{a_{n-1}} a_n$

$$= \frac{\log_{10} a_2}{\log_{10} a_1} \cdot \frac{\log_{10} a_3}{\log_{10} a_2} \cdot \frac{\log_{10} a_4}{\log_{10} a_3} - - - \frac{\log_{10} a_n}{\log_{10} a_{n-1}}$$

$$= \frac{\log_{10} a_n}{\log_{10} a_1} = \log_{a_1} a_n = \underline{\text{RHS}}$$

Ex: $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 - - - \log_{31} 32 = ?$

$\log_2 32 = \log_2 2^5 = 5\log_2 2 = 5.$

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D_4 : $\log_{a^y} m^x = \frac{x}{y}\log_a m$ where $a, m > 0,\ a \neq 1, x, y \in R, y \neq 0$

Proof: $\log_{a^y} m^x = \frac{\log_a m^x}{\log_a a^y} = \frac{x \log_a m}{y \log_a a} = \frac{x \log_a m}{y} = \frac{x}{y}\log_a m$

Ex: $\log_{16} 32 = \log_{2^4} 2^5 = \frac{5}{4}\log_2 2 = 5/4$

Ex: $\log_{3\sqrt{9}} \sqrt[4]{3} = \log_{3^{2/3}} 3^{1/4} = \frac{1/4}{2/3}\log_3 3 = \frac{3}{8}$ Ans

$$D_5: \quad a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$$

LHS $\quad a^{\sqrt{\log_a^b}} = a^{\frac{\log_a^b}{\sqrt{\log_a^b}}} = \left(a^{\log_a^b}\right)^{\frac{1}{\sqrt{\log_a^b}}}$ $\quad \left(\because a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}\right)$

$= b^{\sqrt{\log_b a}} = \text{RHS}$ $\quad \left(\because \sqrt{x} = \frac{x}{\sqrt{x}}\right)$ $\quad x \in R^+$

Ex: $2^{\sqrt{\log_2^3}} - 3^{\sqrt{\log_3^2}} = ?$

$3^{\sqrt{\log_3^2}} - 3^{\sqrt{\log_3^2}} = 0$

Kaam Ki Baat

Common base : $\log x = \log_{10} x$

Natural base : $\log_e x = \ln(x)$

(where $e \cong 2.718$ & $e^2 \cong 7.389$)

 # In Calculus : log x is considered as log with natural base.

$\log_e 10 \cong 2.303$

$\log_{10} x = \frac{\ln x}{2.303}$

HOW??

- $\ln x + \ln y = \ln(xy)$
- $\ln x^n = n \ln x$

$\log_{10} x = \frac{\log_e x}{\log_e 10} = \frac{\ln x}{2.303}$

$\ln 2 \cong 0.693,$

$\ln 3 \cong 1.098,$

$\ln 5 \cong 1.609,$

$\ln 7 \cong 1.945,$

QUESTION

Column 1		Column 2	
(a)	$\frac{1}{9}^{\log_3 7}$	(P)	1/27
(b)	$2^{-\log_{1/2} 7}$	(Q)	1/2
(c)	$8^{-\frac{1}{\log_3 2}}$	(R)	1/49
(d)	$\left(\frac{1}{4}\right)^{\log_2 6}$	(S)	7
(e)	$3^{-\log_3 2}$	(T)	1/36

(a) $\left(\frac{1}{9}\right)^{\log_3 7} = 7^{\log_3 \frac{1}{9}}$ (R)

$= 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

(b) $2^{-\log_{\frac{1}{2}} 7} = \left(2^{\log_{\frac{1}{2}} 7}\right)^{-1}$

$= \left(7^{\log_{\frac{1}{2}} 2}\right)^{-1}$

$= (7^{-1})^{-1} = 7^1 = 7$ (S)

(c) $8^{-\frac{1}{\log_3 2}} = 8^{-\log_2 3}$

$= \left(8^{\log_2 3}\right)^{-1} = \left(3^{\log_2 8}\right)^{-1}$

$= (3^3)^{-1} = 3^{-3} = \frac{1}{27}$ (P)

(d) $\left(\frac{1}{4}\right)^{\log_2 6} = 6^{\log_2 \frac{1}{4}} = 6^{-2} = \frac{1}{36}$ (T)

(e) $3^{-\log_3 2} = \left(2^{\log_3 3}\right)^{-1}$

$= 2^{-1} = \frac{1}{2}$ (Q)

QUESTION

$A = \log_{11}(11^{\log_{11}1331})$;

$B = \log_{385}5 + \log_{385}11 + \log_{385}7$.

$C = \log_4(\log_2(\log_5 625))$;

$D = 10^{\log_{100}16}$

Find the value of $A \times B \div C - D$

$A = \log_{11}1331 = 3$

$B = \log_{385}(5\times11\times7) = \log_{385}385 = 1$

$C = \log_4(\log_2 4) = \log_4^2 = 1/2$

$D = 10^{\log_{100}16} = 16^{\log_{100}10} = 16^{\log_{10^2}10} = 16^{\frac{1}{2}\log_{10}10} = 16^{1/2} = \sqrt{16} = 4$

$A \times B \div C - D$

$3 \times 1 \div 1/2 - 4$

$3 \times 1 \times 2 - 4$

$= 6 - 4 = 2$ Ans

QUESTION

Find the value of $\frac{1}{\log_3 2} + \frac{2}{\log_9 4} - \frac{3}{\log_{27} 8} =$

$$= \log_2 3 + 2\log_4 9 - 3\log_8 27$$

$$= \log_2 3 + 2 \cdot \log_{2^2} 3^2 - 3\log_{2^3} 3^3$$

$$= \log_2 3 + 2 \cdot \frac{2}{2}\log_2 3 - 3 \cdot \frac{3}{3}\log_2 3$$

$$= \log_2 3 + 2\log_2 3 - 3\log_2 3$$

$$= 0$$

QUESTION

Find the value of $\log_2 10 - \log_8 125$.

$\log_2 10 - \log_2 5^3$

$\log_2 10 - \frac{3}{3}\log_2 5$

$\log_2 10 - \log_2 5$

$= \log_2 (10/5) = \log_2 2 = 1$

QUESTION

If $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \ldots\ldots \log_n (n+1) = 10$ find n.

$\log_2 (n+1) = 10$

$n+1 = 2^{10}$

$n = 1023$ Ans

QUESTION

Simplify : $\dfrac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409} \cdot \left((\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6} \right)$

$$\frac{81^{\log_9 5} + 3^{3 \cdot \log_3 \sqrt{6}}}{409} \left(\sqrt{7}^{\,2 \log_7 25} - 125^{\log_{25} 6} \right)$$

$$= \frac{5^{\log_9 81} + \sqrt{6}^{\,3 \log_3 3}}{409} \left(25^{2 \log_7 \sqrt{7}} - 6^{\log_{25} 125} \right)$$

$$= \frac{(5^2 + \sqrt{6}^3)}{409} \left(25^{2 \cdot 1/2} - 6^{\log_{5^2} 5^3} \right)$$

$$= \frac{(25 + 6\sqrt{6}) \cdot (25 - 6^{3/2})}{409} = \frac{(25 + 6\sqrt{6})(25 - 6\sqrt{6})}{409} = \frac{625 - 216}{409} = 1$$

$a^{m \log_c b} = b^{m \log_c a}$

$(a^{\log_c b})^m$

$(b^{\log_c a})^m$

$= b^{m \log_c a}$

$6^{3/2} = 6^{1 + 1/2}$

$= 6^1 \cdot 6^{1/2}$

$= 6\sqrt{6}$.

QUESTION

Simplify : $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

$5^{\log_3 7} + 7^{\log_5 3} - 5^{\log_3 7} - 7^{\log_5 3} = 0$

QUESTION

Prove that $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2} = 3$

QUESTION

Simplify & compute :

$$\frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$$

Tan02

QUESTION [JEE Advanced 2018]

The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is

QUESTION [JEE Advanced 2013]

If $3^x = 4^{x-1}$, then x =

(A) $\dfrac{2\log_3 2}{2\log_3 2 - 1}$

(B) $\dfrac{2}{2 - \log_2 3}$

(C) $\dfrac{1}{1 - \log_4 3}$

(D) $\dfrac{2\log_2 3}{2\log_2 3 - 1}$

$a = b \iff \log_c a = \log_c b$, $a, b \in R^+$, $c > 0, c \neq 1$

$3^x = 4^{x-1}$

$\log_2 3^x = \log_2 4^{x-1}$

$x \log_2 3 = (x-1)\log_2 4$

$x \log_2 3 = (x-1)\cdot 2 = 2x - 2$

$2 = 2x - x\log_2 3$

$x(2 - \log_2 3) = 2$

$x = \dfrac{2}{2 - \log_2 3}$ (B)

$x = \dfrac{2}{2 - \log_2 3} = \dfrac{2}{2 - \frac{1}{\log_3 2}}$

$x = \dfrac{2\log_3 2}{2\log_3 2 - 1}$ (A)

$x = \dfrac{2}{2 - \log_2 3} = \dfrac{2}{2 - \log_{4^{1/2}} 3}$

$= \dfrac{2}{2 - 2\log_4 3} = \dfrac{2}{2(1 - \log_4 3)} = \dfrac{1}{1 - \log_4 3}$ (C)

Logarithmic Equations

$a^{m+n} = a^m \cdot a^n$

$a^{\log_a b} = b$

$5^2 = 5 \times 5 = 25$

$5^{2+\log_5 x}$

$= 5^2 \cdot 5^{\log_5 x}$

$25 \cdot x$

$25x$.

* Base of logarithm should be same throughout

* Try Simplify each term if possible. Ex: $5^{2+\log_5 x}$

$= 5^2 \cdot 5^{\log_5 x}$

$= 25x$

* If you take log on both sides of Eqn then take it to base already given in Question.

* Verify All answers from original Eqn and check if any term becomes undefined or imaginary

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$n \geq 2, n \in N$

$2 = \sqrt{4} = 4^{1/2}$

QUESTION

★★ASRQ★★

$$\frac{\log_2(9-2^x)}{x-3} = 1.$$ Then number of solution is

(A) 3

(B) 2

(C) 1

(D) 0

$\log_2(9-2^x) = x-3$

$9-2^x = 2^{x-3}$

$9-2^x = 2^x \cdot 2^{-3}$

$9-2^x = \frac{2^x}{2^3}$) let $2^x = a$

$9-a = \frac{a}{8}$

$72-8a = a$

$9a = 72$

$a = 8$

$2^x = 8$

$2^x = 2^3 \Rightarrow x = 3$ - (rejected)

QUESTION

$\log_{x-1} 4 = 1 + \log_2(x-1)$ then x is

A) 3

B) 2

C) 5/4

D) 4/5

$\log_{x-1} 2^2 = 1 + \log_2(x-1)$

$2\log_{x-1} 2 = 1 + \log_2(x-1)$

$\frac{2}{\log_2(x-1)} = 1 + \log_2(x-1)$ → let $\log_2(x-1) = t$

$\frac{2}{t} = 1 + t$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$t = -2, 1$

$\log_2(x-1) = -2, 1$

$x - 1 = 2^{-2}$ or $x - 1 = 2^1$

$x = 1 + \frac{1}{2^2} = 5/4, x = 3$

$x = 5/4, 3$

QUESTION

$(x+1)^{\log_{10}(x+1)} = 100(x+1)$ then x is

A 99

B −0.9

C 0.9

D 10

$a^x = N \Leftrightarrow \log_a N = x$

↳ Taking log on both sides to base 10

$\log_{10}(x+1)^{\log(x+1)} = \log_{10}(100\cdot(x+1))$

$\log_{10}(x+1)\cdot\log_{10}(x+1) = \log_{10}100 + \log_{10}(x+1)$ → let $\log_{10}(x+1) = t$

$t^2 = 2+t$

$t^2 - t - 2 = 0$

$(t-2)(t+1) = 0$

$t = 2, -1$

$\log_{10}(x+1) = 2$ or $\log_{10}(x+1) = -1$

$x+1 = 10^2$ or $x+1 = 10^{-1}$

$x = 99$, $x = \frac{1}{10} - 1 = -\frac{9}{10} = -0.9$

QUESTION

★★KCLS★★

$\log_5\left(\sqrt[x]{5}+125\right)=\log_5 6+1+\frac{1}{2x}$ then x is

$a^{m+n+p}=a^m.a^n.a^p$

(A) -1

(B) 1/2

(C) 1/16

(D) 1/4

$\log_5\left(5^{1/x}+125\right)=\log_5 6+1+\frac{1}{2x}$

$5^{\frac{1}{x}}+125=5^{\log_5 6+1+\frac{1}{2x}}=5^{\log_5 6}.5^1.5^{\frac{1}{2x}}$

$5^{\frac{1}{x}}+125=6.5.5^{\frac{1}{2x}}$

$5^{1/x}+125=30.5^{\frac{1}{2x}}$

$t^2+125=30t$

$t^2-30t+125=0$

$t=5,25.$

$5^{1/x}=t$

$5^{1/2x}=t^2$

let $5^{1/2x}=t$

$\left(5^{\frac{1}{2x}}\right)^2=t2$

$5^{1/x}=t2$

Gadho/Gadhiyaa aisaa naa karo!!

QUESTION

$$\log_5\left(5^{1/x}+125\right)=\log_5 6+1+\frac{1}{2x}\text{ then x is}$$

A. −1

B. 1/2

C. 1/16

D. 1/4

$\sqrt[x]{5}=5^{1/x}$, $x\in N$

$5^{1/x}=\sqrt[x]{5}$ only if $x\in N$ $x\geq 2$

$5^{3/4}\neq\sqrt[3]{5}$.

QUESTION

$$5^{1+\log_4 x} + 5^{\log_{\frac{1}{4}} x - 1} = \frac{26}{5} \text{ then x is}$$

(A) 1

(B) 1/16

(C) 1/2

(D) 16

$a^{m-n} = \frac{a^m}{a^n} = a^m \cdot a^{-n}$

* $a^{-m} = \frac{1}{a^m}$
* $\log_2 4 - 1 = 2 - 1 = 1$
* $\log_3(28-1) = \log_3 27 = 3$

Gadho/Gadhiyoo aisaa maa karo $5 \cdot 5^{1/x} = 25^{1/x}$

$5^1 \cdot 5^{\log_4 x} + 5^{\log_{\frac{1}{4}} x} \cdot 5^{-1} = \frac{26}{5}$

$5 \cdot 5^{\log_4 x} + 5^{\log_{4^{-1}} x} = 26$

$5 \cdot 5^{\log_4 x} + \frac{5^{-\log_4 x}}{5} = \frac{26}{5}$

$5 \cdot 5^{\log_4 x} + \frac{1}{5^{\log_4 x} \cdot 5^1} = \frac{26}{5}$

let $5^{\log_4 x} = t$

$5t + \frac{1}{5t} = \frac{26}{5}$

$25t^2+1=26t$

$25t^2-26t+1=0$

$25t^2-25t-t+1=0$

$(25t-1)(t-1)=0$

$t=\frac{1}{25}, 1$

$5^{\log_4 x}=5^{-2}$ or $5^{\log_4 x}=5^0$

$\log_4 x=-2$ or $\log_4 x=0$

$x=4^{-2}$, $x=4^0$

$x=1/16$, 1

QUESTION

ASRQ

Find the square of the sum of the roots of the equation

$$\log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x + \log_4 x \cdot \log_5 x + \log_5 x \cdot \log_3 x.$$

Case ① If $\log_3^x \cdot \log_4^x \cdot \log_5^x \neq 0$

$$1 = \frac{1}{\log_5^x} + \frac{1}{\log_3^x} + \frac{1}{\log_4^x}$$

$$1 = \log_x^5 + \log_x^3 + \log_x^4$$

$$1 = \log_x(5 \times 3 \times 4)$$

$$1 = \log_x 60$$

$x^1 = 60$

Case ② If $\log_3^x \cdot \log_4^x \cdot \log_5^x = 0$

i.e if $x = 1$

LHS = 0 = RHS

$\Downarrow$

$x = 1$ is also a soln

Ans: $(60+1)^2 = 3721$

Gadho/Gadhiyoo aisaa naa karo

QUESTION

★★KCLS★★

Solve for x : $\log^2(4-x) + \log(4-x)\cdot\log\left(x+\frac{1}{2}\right) - 2\log^2\left(x+\frac{1}{2}\right) = 0.$

Let $\log(4-x) = a$

$\log\left(x+\frac{1}{2}\right) = b$

$a^2 + ab - 2b^2 = 0$

$a^2 + 2ab - ab - 2b^2 = 0$

$a(a+2b) - b(a+2b) = 0$

$(a+2b)(a-b) = 0$

$a = b, -2b.$

$\log(4-x) = \log(x+1/2)$ or $\log(4-x) = -2\log(x+\frac{1}{2})$

$4-x = x+1/2$

$x = 7/4$

$\log(4-x) = \log\left(\frac{2x+1}{2}\right)^{-2}$

$4-x = \frac{4}{(2x+1)^2}$

$A\cdot f^{2n}(x) + B\cdot f^{n}(x)\cdot g^{m}(x) + C\cdot g^{2m}(x) = 0$

Try splitting middle term as in quad

Ex: $x^4 + 5x^2y^2 + 6y^4$

$x^4 + 3x^2y^2 + 2x^2y^2 + 6y^4$

$(x^2+2y^2)(x^2+3y^2)$

$$(4-x)(2x+1)^2 \geq 4$$

$$(4-x)(4x^2+1+4x) = 4$$

$$16x^2 + 4 + 16x - 4x^3 - x - 4x^2 = 4$$

$$-4x^3 + 12x^2 + 15x = 0$$

$$-x(4x^2 - 12x - 15) = 0$$

$$x = 0, \quad 4x^2 - 12x - 15 = 0$$

$$x = \frac{12 \pm \sqrt{144+240}}{8}$$

$$x = \frac{12 \pm \sqrt{4 \times 16 \times 6}}{8}$$

$$x = \frac{3 \pm 2\sqrt{6}}{2}$$

$$x = 0, -1/4, \frac{3+2\sqrt{6}}{2}, \frac{3-2\sqrt{6}}{2}$$

$x + 1/2 < 0$

$$x + \frac{1}{2} = \frac{3-2\sqrt{6}}{2} + \frac{1}{2}$$

$$= \frac{4-2\sqrt{6}}{2} = -ve$$

384

96.

QUESTION

Indicate all correct alternatives, where base of the log is 2.

The equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}} = \sqrt{2}$ has :

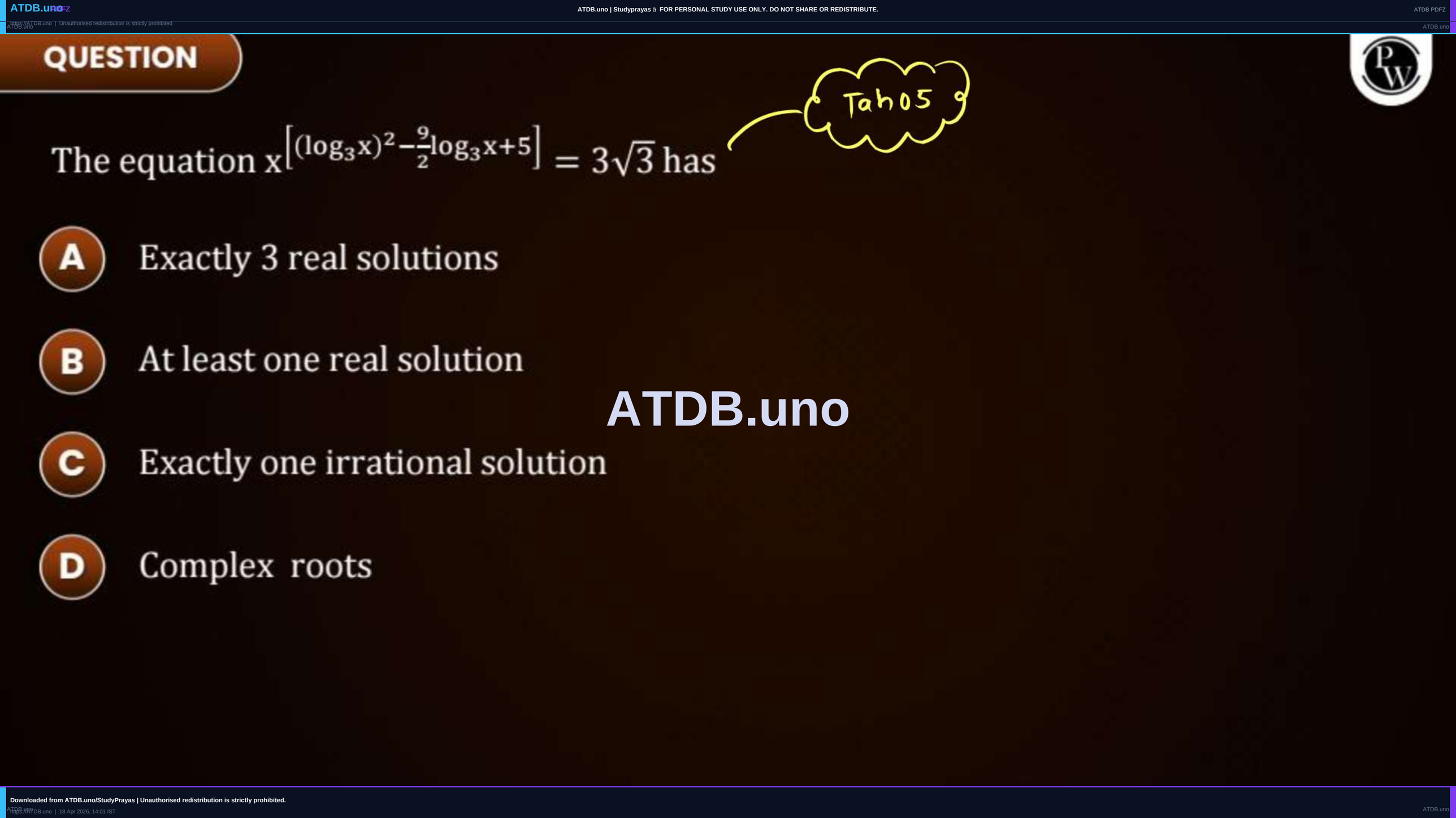
A. At least one real solution

B. Exactly 3 real solutions

C. Exactly one irrational solution

D. Imaginary roots

QUESTION



The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has

A Exactly 3 real solutions

B At least one real solution

C Exactly one irrational solution

D Complex roots

Saari Class Illustrations

Retry karni Hai

Home Challenge-03

Positive integers a and b satisfy the condition $\log_2\left[\log_{2^a}\left(\log_{2^b}\left(2^{1000}\right)\right)\right] = 0$. Then the possible values of a + b is/are:

A 501

B 252

C 128

D 66

Today's KTK

No Selection → TRISHUL Apnao IIT Jao → Selection with Good Rank

QUESTION

(KTK 1)

Solve the following inequalities:

(1) $\frac{x}{x-5} > \frac{1}{2}$

(2) $\frac{x-5}{|x-9|} \leq 0$

(3) $x \leq 3 - \frac{1}{x-1}$

(4) $\frac{1}{x} \leq 1$

(5) $\frac{5x}{3x-1} \leq 0$

Ans. (1) $x \in (-\infty, -5) \cup (5, \infty)$, (2) $x \in (-\infty, 0) \cup [1, \infty)$,
(3) $x \in [5, 9)$, (4) $x \in \left[0, \frac{1}{3}\right)$, (5) $x \in (-\infty, 1) \cup \{2\}$

QUESTION

(KTK 2)

Complete solution set of inequality $\frac{(x+2)(x+3)}{(x-2)(x-3)} \leq 1$ is

A $(-\infty, 0]$

B $(-\infty, 0] \cup (2, 3)$

C $[2, 3]$

D $(-\infty, 2) \cup (3, \infty)$

Ans. B

QUESTION

(KTK 3)

Find sum of all integral values of x satisfying $\frac{x^2-5x+6}{x^2-x-12} \leq 0$.

Ans. 3

QUESTION

(KTK 4)

Which of the following does not hold true for the expression

$E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1}$?

A $E = 2$ if $x \leq -1$

B $E = -2x$ if $-1 < x < 1$

C $E = -2$ if $x \geq 1$

D $E = -2$ for all x

Ans. D

QUESTION

KTK 07

If $x \in [-5, 7]$, then number of integral values of x satisfying $\frac{2x+3}{x^2+x-12} < \frac{1}{2}$ is

A 5

B 6

C 7

D 8

Ans. C

QUESTION

KTK 08

Solution set of the inequality $x + 1 \leq \frac{6}{x}$ is

A $(0, 2]$

B $[-3, 2)$

C $(-\infty, -3] \cup (0, 2]$

D $[-3, 0) \cup (2, \infty)$

Ans. C

QUESTION

KTK 09

The set of all values of x for which $\frac{(x+1)(x-3)^2(x-5)(x-4)^3(x-2)}{x} < 0$ is

A $(-\infty, -1) \cup (0, 2) \cup (4, 5)$

B $(-1, 0) \cup (2, 4) \cup (5, \infty)$

C $(-1, 0) \cup (2, 3) \cup (4, 5)$

D $(-\infty, -1) \cup (0, 2) \cup [3, 5)$

Ans. A

QUESTION

KTK 10

Which of the following sets does not satisfy the inequality $\frac{1}{x-2}+\frac{1}{x-1}\geq\frac{1}{x}$?

A $(-\sqrt{2},0)$

B $(1,\sqrt{2})$

C $(2,\infty)$

D $(0,1)$

Ans. D

Solution to Previous TAH

ATDB.uno

QUESTION

Solve: $(x^2 - x - 1)(x^2 - x - 7) < -5$

Ans. $x \in (-2, -1) \cup (2, 3)$

$(x^2-x-1)(x^2-x-7) < -5$

Sol

Let, $x^2-x=t$

Tah-01

$(t-1)(t-7) < -5$

$t^2-7t-t+7 < -5$

$t^2-8t+7 < -5$

$t^2-8t+12 < 0$

$t^2-6t-2t+12 < 0$

$t(t-6)-2(t-6) < 0$

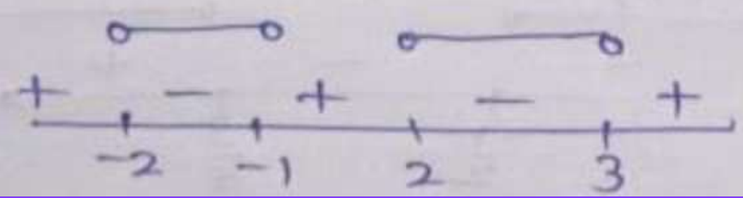
$(t-2)(t-6) < 0$

$(x^2-x-2)(x^2-x-6) < 0$

$(x^2-2x+x-2)(x^2-3x+2x-6) > 0$

$x(x-2)+1(x-2)$ $x(x-3)+2(x-3)$

$(x+1)(x-2)(x+2)(x-3) < 0$



$x \in (-2,-1) \cup (2,3)$

• **Q(TAH)-1:** Solve : $(x^2-x-1)(x^2-x-7) < -5$.

TAH 1

Soln: let $x^2-x = t$.

$(t-1)(t-7) < -5$

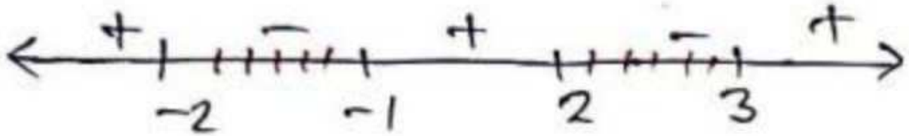
or, $t^2-7t-t+7 < -5$

or, $t^2-8t+12 < 0$

or, $(t-6)(t-2) < 0$

or $(x^2-x-6)(x^2-x-2) < 0$

or $(x-3)(x+2)(x-2)(x+1) < 0$



$\therefore x \in (-2,-1) \cup (2,3)$

QUESTION

If $\frac{x^3(x-1)^2(x+4)}{(x+1)(x-3)} \geq 0$, then $x \in$

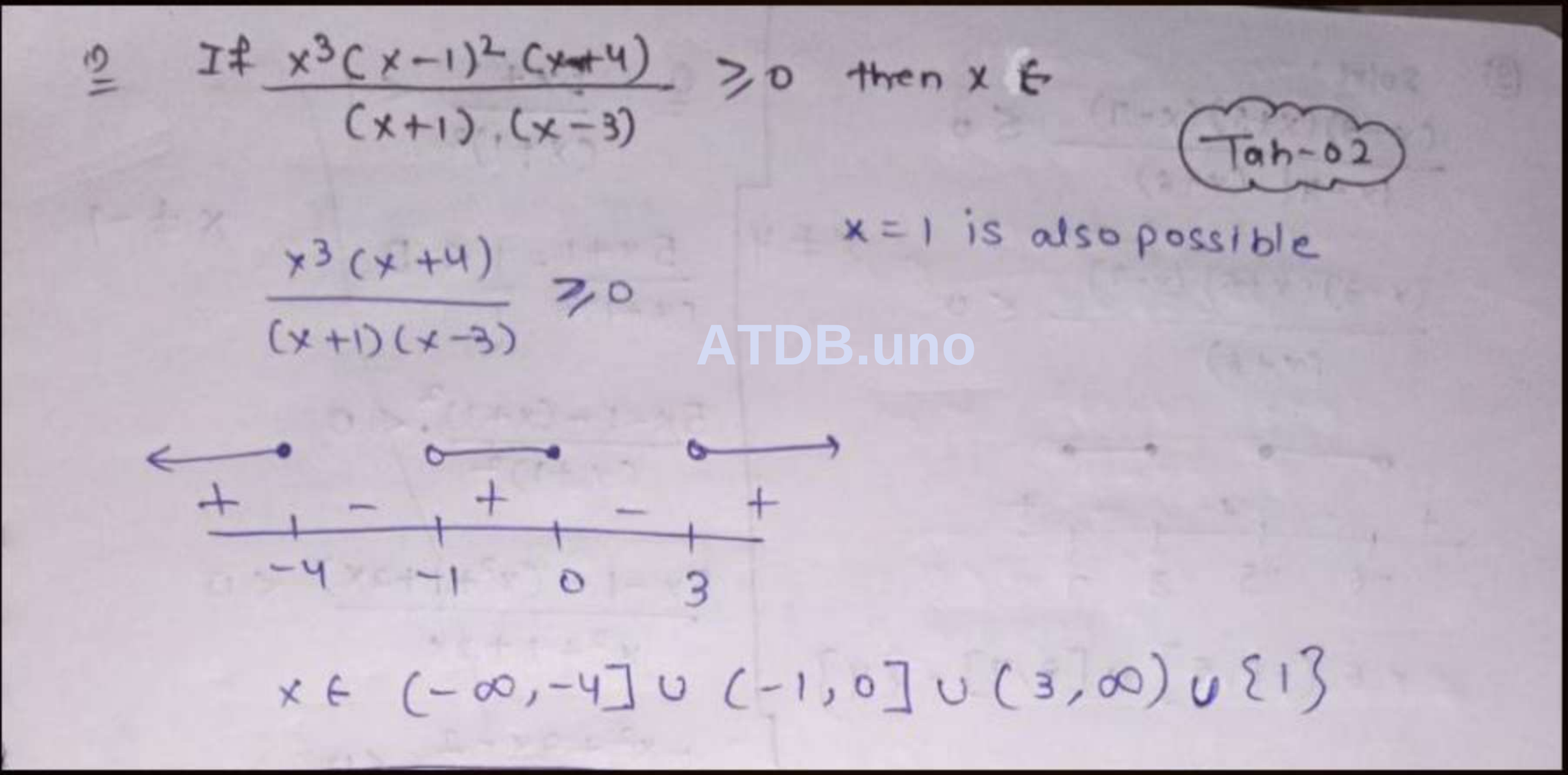
A $(-\infty, -4]$

B $(-1, 0]$

C $(3, \infty)$

D $\{1\}$

Ans. A, B, C, D

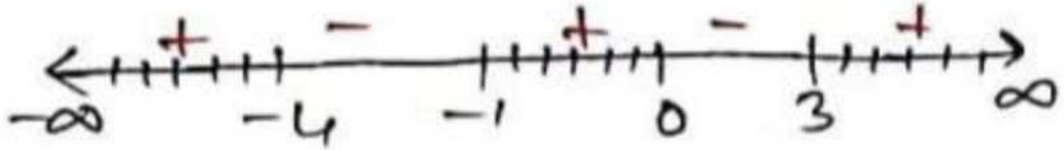


TAH 2

TAH-2: $\frac{x^3(x-1)^2(x+4)}{(x+1)(x-3)} \geq 0$; $x \in ?$

Soln: $\frac{x^3(x-1)^2(x+4)}{(x+1)(x-3)} \geq 0.$

$\Rightarrow \frac{x(x+4)}{(x+1)(x-3)} \geq 0$; $x = 1$ is also possible.



$\therefore x \in (-\infty, -4] \cup (-1, 0] \cup (3, \infty) \cup \{1\}$

QUESTION

TAH 03

1. Solve $\frac{x(3-4x)(x+1)}{(2x-5)} < 0$ [Ans. $x \in (-\infty, -1) \cup (0, 3/4) \cup (5/2, \infty)$]

2. Solve $\frac{(2x+3)(4-3x)^3(x-4)}{(x-2)^2x^5} \leq 0$ [Ans. $x \in (-\infty, -3/2) \cup (0, 4/3] \cup [4, \infty)$]

3. Solve $\frac{(x-3)(x+5)(x-7)}{|x-4|(x+6)} \leq 0$ [Ans. $x \in (-6, -5] \cup [3, 4) \cup (4, 7]$]

4. Solve $\frac{5x+1}{(x+1)^2} < 1$ [Ans. $x < 0$ or $x > 3, x \neq -1$]

5. Solve $\frac{x^4}{(x-2)^2} > 0$ [Ans. $x \in R - \{0, 2\}$]

6. Solve $\frac{6x^2-5x-3}{x^2-2x+6} \leq 4$ [Ans. $-\frac{9}{2} \leq x \leq 3$]

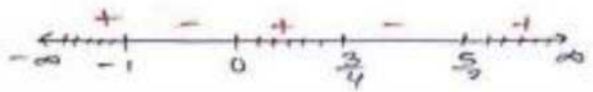
7. Solve $\frac{(x+2)(x^2-2x+1)}{-4+3x-x^2} \geq 0$ [Ans. $x \in (-\infty, -2] \cup \{1\}$]

TAH 3 by Reed from WB

TA H-3: Solve the inequalities

Soln:- (i) $\frac{x(3-4x)(x+1)}{(2x-5)} \cdot < 0$

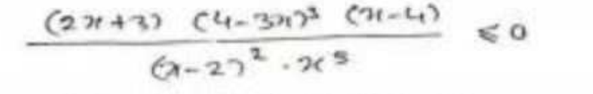
$\Rightarrow \frac{x(4x-3)(x+1)}{(2x-5)} > 0$



$\therefore x \in (-\infty, -1) \cup (0, \frac{3}{4}) \cup (\frac{5}{2}, \infty)$ Ans.

(ii) $\frac{(2x+3)(4-3x)^3(x-4)}{(x-2)^2 \cdot x^5} \leq 0$

$\Rightarrow \frac{(2x+3)(3x-4)(x-4)}{x} \geq 0 \; ; \; (x \neq 2)$



$x \in (-\infty, -\frac{3}{2}] \cup (0, \frac{4}{3}] \cup [4, \infty)$ Ans.

(iv) $\frac{5x+1}{(x+1)^2} < 1$

$\Rightarrow \frac{5x+1-x^2-2x-1}{(x+1)^2} < 0$

$\Rightarrow \frac{-x^2+3x}{(x+1)^2} < 0$

$\Rightarrow \frac{x(x-3)}{(x+1)^2} > 0$

$\Rightarrow x(x-3) > 0 \; ; \; x \neq -1$

$\therefore x \in (-\infty, 0) \cup (3, \infty) - \{-1\}$

(v) $\frac{x^4}{(x-2)^2} > 0$ $\quad [x \neq 0]$

$\Rightarrow \frac{x^4}{(x-2)^2}$ → always +ve for $\forall x \in R-\{0\}$ → $\Rightarrow x \in R-\{2,0\}$ Ans

always +ve for $\forall x \in R-\{2\}$

$[x \neq 2]$

(vi) $\frac{6x^2-5x-3}{x^2-2x+6} \leq 4$

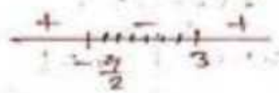
$\Rightarrow \frac{6x^2-5x-3-4(x^2-2x+6)}{x^2-2x+6} \leq 0$

→ $D<0, a>0 \Rightarrow$ always +ve

$\Rightarrow 2x^2+3x-27 \leq 0$

$\Rightarrow 2x^2+9x-6x-27 \leq 0$

$\Rightarrow (2x+9)(x-3) \leq 0.$



$x \in [-\frac{9}{2}, 3]$.

TAH 3(part2) By Reed from WB

(vii) $\frac{(x+2)(x^2-2x+1)}{-4+3x-x^2} \geq 0$

$\Rightarrow \frac{(x+2)(x-1)^2}{(x^2-3x+4)} \leq 0$

→ $D<0, a>0$ always +ve.

& $x = 1$ is also possible.

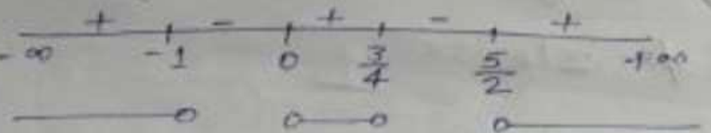
$\Rightarrow \frac{(x+2)}{A} \leq 0$

$\Rightarrow \boxed{x \leq -2}$.

$x \in (-\infty, -2] \cup \{1\}$

(i) $\frac{x(3-4x)(x+1)}{(2x-5)} < 0$

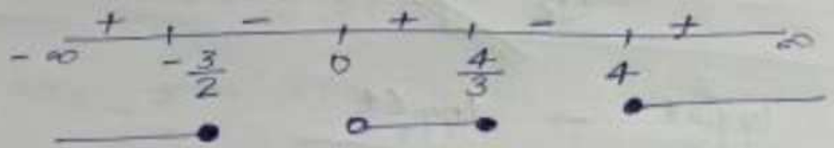
$\Rightarrow \frac{x(4x-3)(x+1)}{(2x-5)} > 0$ → +ve



$x \in (-\infty, -1) \cup (0, \frac{3}{4}) (\frac{5}{2}, \infty)$ Ans.

(ii) $\frac{(2x+3)(4-3x)^3(x-4)}{(x-2)^2 \; x^5} \leq 0$

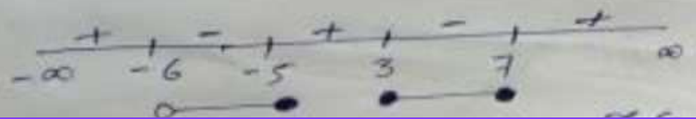
$\Rightarrow \frac{(2x+3)(3x-4)^3(x-4)}{x^5} \geq 0 \quad x \neq 2$ (+ve)



$x \in (-\infty, \frac{-3}{2}] \cup (0, \frac{4}{3}] \cup [4, \infty)$ Ans.

(iii) $\frac{(x-3)(x+5)(x-7)}{|x-4|(x+6)} \leq 0$

$\Rightarrow \frac{(x-3)(x+5)(x-7)}{(x+6)} \leq 0 \quad x \neq 4$ (-ve)



(4) $\frac{5x+1}{(x+1)^2} < 1$

$\Rightarrow \frac{5x+1}{(x+1)^2} - 1 < 0$

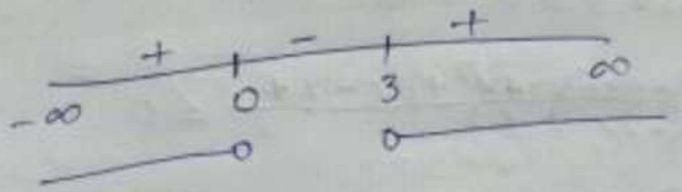
$\Rightarrow \frac{5x+1-(x+1)^2}{(x+1)^2} < 0$

$\Rightarrow \frac{5x+1-x^2-2x-1}{(x+1)^2} < 0$

$\Rightarrow \frac{-x^2+3x}{(x+1)^2} < 0$

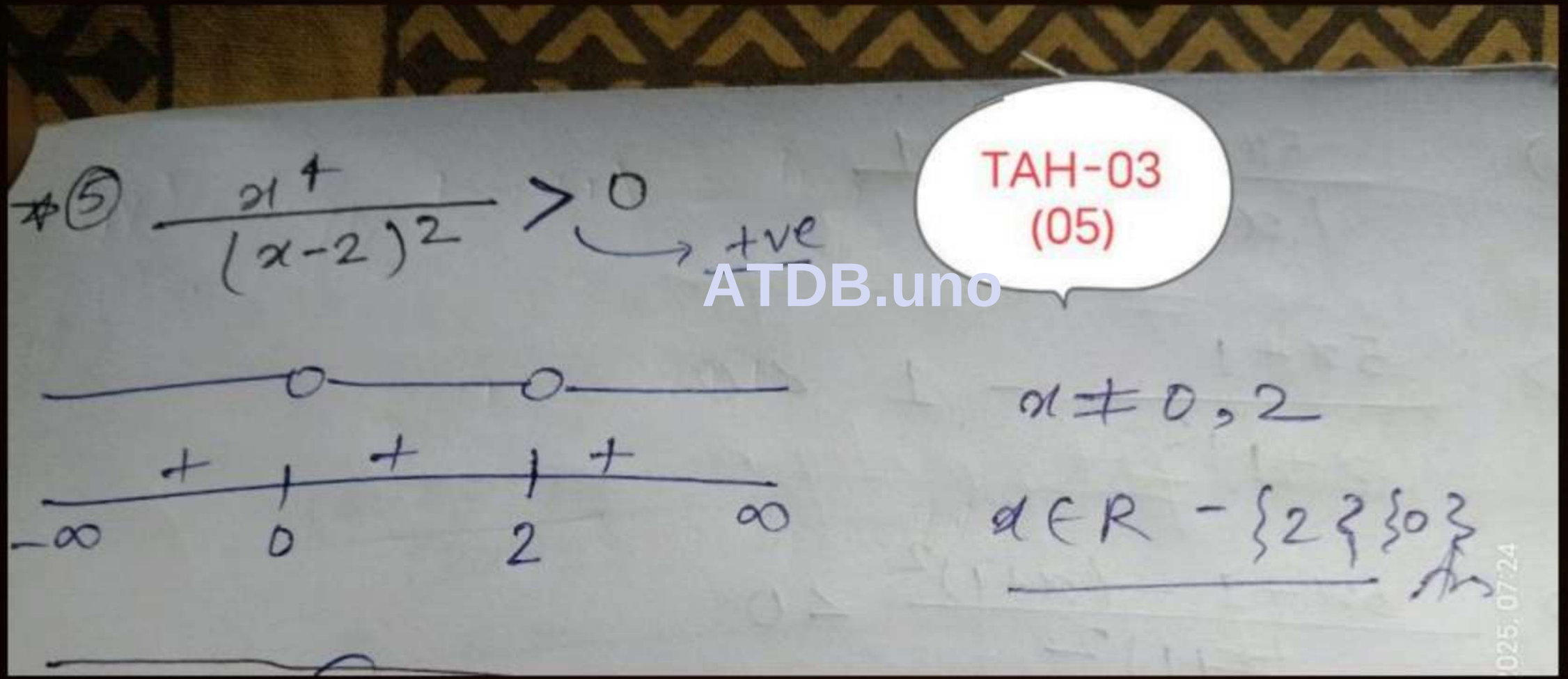
$\Rightarrow \frac{-x(x-3)}{(x+1)^2} < 0$

$\Rightarrow \frac{x(x-3)}{(x+1)^2} > 0 \Rightarrow x(x-3) > 0 \quad x \neq 1$ → +ve.



$x \in (-\infty, 0) \cup (3, \infty)$ Ans

TAH-03 (04)



(96) $\dfrac{6x^2-5x-3}{x^2-2x+6} - 4 \leq 0$

$\Rightarrow \dfrac{6x^2-5x-3-4x^2+8x-24}{(x^2-2x+6)} \leq 0$

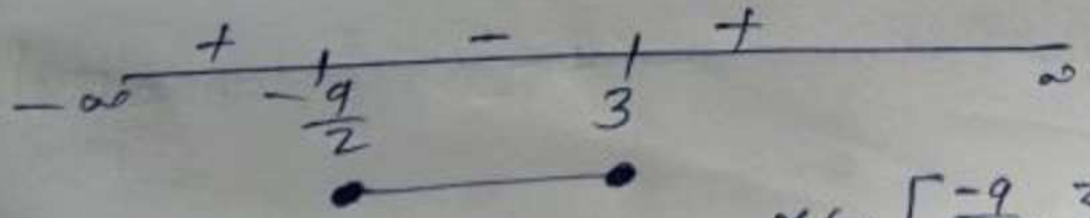
$\Rightarrow \dfrac{2x^2+3x-27}{x^2-2x+6} \leq 0$

$\Rightarrow \dfrac{2x^2+9x-6x-27}{(x^2-2x+6)} \leq 0$

$\Rightarrow \dfrac{(2x+9)(x-3)}{(x^2-2x+6)} \leq 0$

Tah-03
(6)

$D<0$
$a>0$
x^2-2x+6
always +ve



$x \in \left[\frac{-9}{2}, 3\right]$

(7) $\frac{-(x+2)(x^2-2x+1)}{-4+3x-x^2} \geq 0$

TAH-03 (07)

$\Rightarrow \frac{(x+2)(x^2-2x+1)}{x^2-3x+4} \leq 0$

$D < 0$

$a > 0$

$\Rightarrow \frac{(x+2)(x-1)(x-1)}{(x-1)(x-4)} \leq 0$

$x \neq 1$

$-\infty \quad -2 \quad \infty$ (− | + | +)

$x \in (-\infty, -2]$

QUESTION

Find all values of x for which the following equalities hold true.

(i) $\log_2 x^2 = 1$

(ii) $\log_3 x = \log_3 (2 - x)$

(iii) $\log_4 x^2 = \log_4 x$

(iv) $\log_{1/2}(2x + 1) = \log_{1/2}(x + 1)$

(v) $\log_{1/3}(x^2 + 8) = -2$

TAH-4: Find all values of x for which the following equalities hold true:

(i) $\log_2 x^2 = 1$

(ii) $\log_4 x^2 = \log_4 x$

(iii) $\log_{1/3}(x^2+8) = -2$

(iv) $\log_3 x = \log_3(2-x)$

(v) $\log_{1/2}(2x+1) = \log_{1/2}(x+1)$

Soln: (i) $\log_2 x^2 = 1$

$\Rightarrow x^2 = 2^1$

$\Rightarrow \boxed{x = \pm\sqrt{2}}$

$\therefore$ Equality holds at $x = \sqrt{2}, -\sqrt{2}$

(ii) $\log_4 x^2 = \log_4 x$

$\Rightarrow x^2 = x$

$\Rightarrow x(x-1) = 0$

$x = 0$ ✓ N.P. | $x = 1$ ✓ (F.M.E)

$\therefore$ Equality holds at $x = 1$.

(iii) $\log_{\frac{1}{3}}(x^2+8) = -2$

$\Rightarrow x^2 + 8 = \left(\frac{1}{3}\right)^{-2} = 9$

$\Rightarrow x^2 = 1$

$\Rightarrow \boxed{x = \pm 1}$ ✓

(iv) $\log_3 x = \log_3(2-x)$

$\Rightarrow x = 2 - x$

$\Rightarrow 2x = 2$

$\Rightarrow \boxed{x = 1}$

$\therefore$ Equality holds at $x = 1$.

(v) $\log_{\frac{1}{2}}(2x+1) = \log_{\frac{1}{2}}(x+1)$

$\Rightarrow 2x+1 = x+1$

$\Rightarrow \boxed{x = 0}$ ✓ (valid)

$\therefore$ Equality holds at $x = 0$.

$\therefore$ Equality holds at $x = 1, -1$.