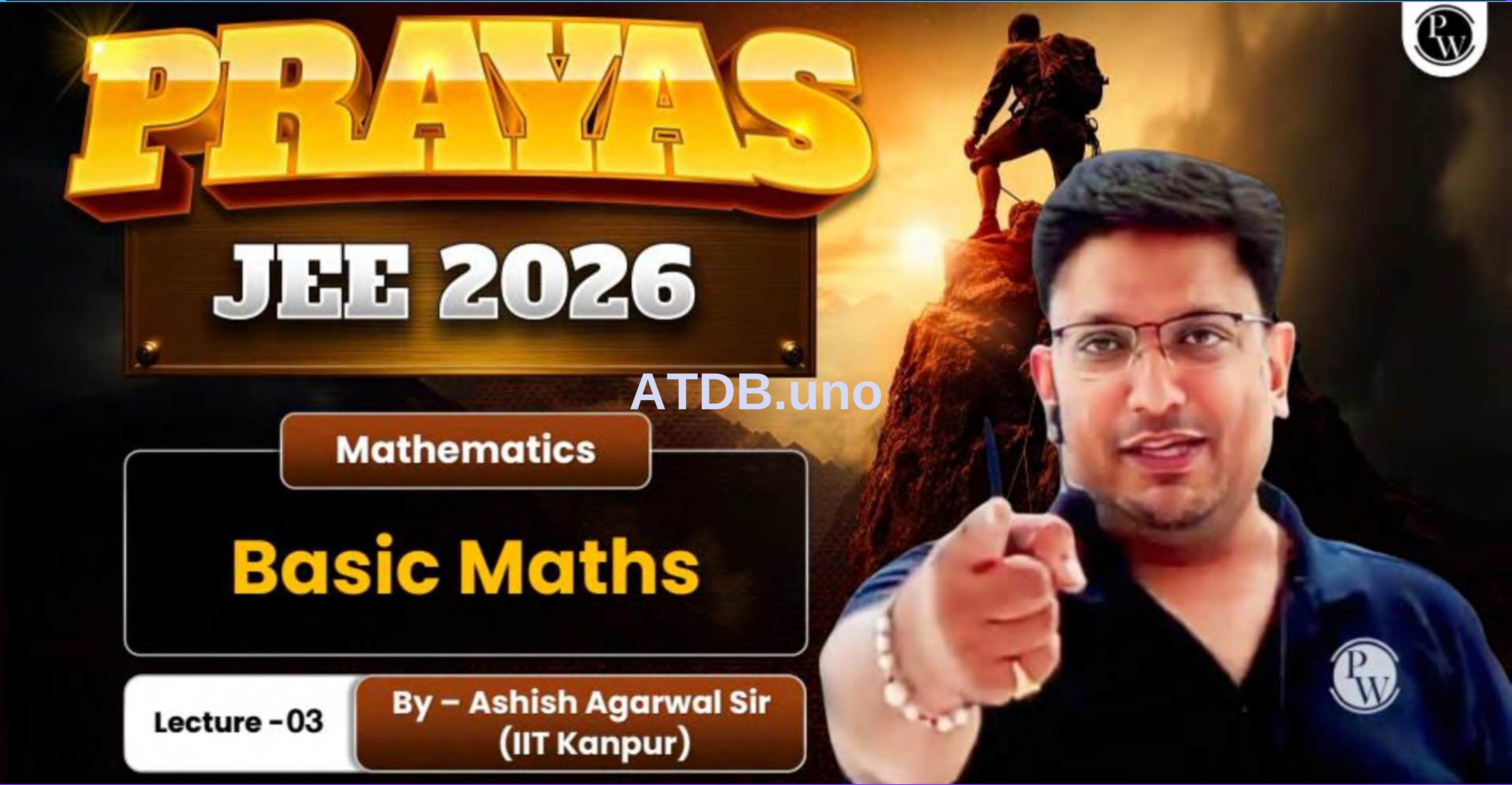
PRAYAS
JEE 2026
Mathematics
Basic Maths
Lecture -03
By – Ashish Agarwal Sir
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# Topics *to be covered*

**A** Divisibilty Rules

**B** Some Important Points



# Recap of previous lecture

2 is the only even prime

State True or False

1. Every prime except 2 is odd. (T)
2. Every prime $\geq 5$ is of type $6k \pm 1, k \in I^+$. (T)
3. Every number of type $6k \pm 1, k \in I^+$ is prime. (F)
4. Sum of two primes is also a prime. (F)
5. Every composite number has more than two positive factors. (T)
6. Every natural number is either prime or composite. (F)

1 is neither prime nor composite

# Recap of previous lecture

*State True or False*

7. 1 is the smallest prime. (F)

8. Every irrational number is real. (T)

9. $25.\overline{3}$ is a rational number but not a real number. (F)



10. If x is rational then $x^2$ is also rational. (T)

11. If x is irrational then $x^2$ is also irrational. (F)

## Kaam Ki Baat

Math में कोई भी Fact तभी सही होता है जब वह हर जगह सही हो एक भी जगह गलत होने पर उसे गलत ही कहते है



इसलिए अगर हमें कुछ सही prove करना है तो general proof देना पड़ेगा जबकि अगर किसी चीज को गलत proof करना है तो सिर्फ एक ही counter-example काफी है

## QUESTION

Fill in the Blanks :

1. Even integer ± Even integer = Even Integer
2. Even integer ± 1 = Odd Integer
3. Odd integer ± Odd integer = Even Integer
4. Odd integer ± Even integer = odd Integer
5. Odd integer ± 1 = Even.

same ± same = even

same ± opp = odd.

# Homework Discussion

## QUESTION

KTK 3

| Column-I | | Column-II | |
|---|---|---|---|
| (A) | A rectangular box has volume 48, and the sum of the length of the twelve edges of the box is 48. The largest integer that could be the length of an edge of the box, is | (P) | 1 |
| (B) | The number of zeroes at the end in the product of first 20 prime numbers, is $2 \cdot 3 \cdot 5 \cdot$ [illegible] $\Rightarrow$ only one zero at end | (Q) | 2 |
| (C) | The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is | (R) | 3 |
| | | (S) | 4 |
| | | (T) | 6 |

Ans. (A) T; (B) P; (C) P

(c) $2^{2x} - 3^{2y} = 55$

$2^{-1} = \frac{1}{2}$ , $3^{-1} = \frac{1}{3}$

$2^{-2} = \frac{1}{4}$ , $3^{-2} = \frac{1}{9}$

$(2^x - 3^y)(2^x + 3^y) = 55 = 11 \times 5 = 55 \times 1$

$x, y \in I^-$
(Not possible)

$(x, y \in I^+)$

$$2^x + 3^y = 11$$
$$2^x - 3^y = 5$$
$$\overline{2 \cdot 2^x = 16}$$
$$2^x = 8$$
$$x = 3$$

$$2^3 - 3^y = 5$$
$$2 \times 2 \times 2 - 3^y = 5$$
$$8 - 3^y = 5$$
$$3^y = 8 - 5$$
$$3^y = 3^1$$
$$y = 1$$

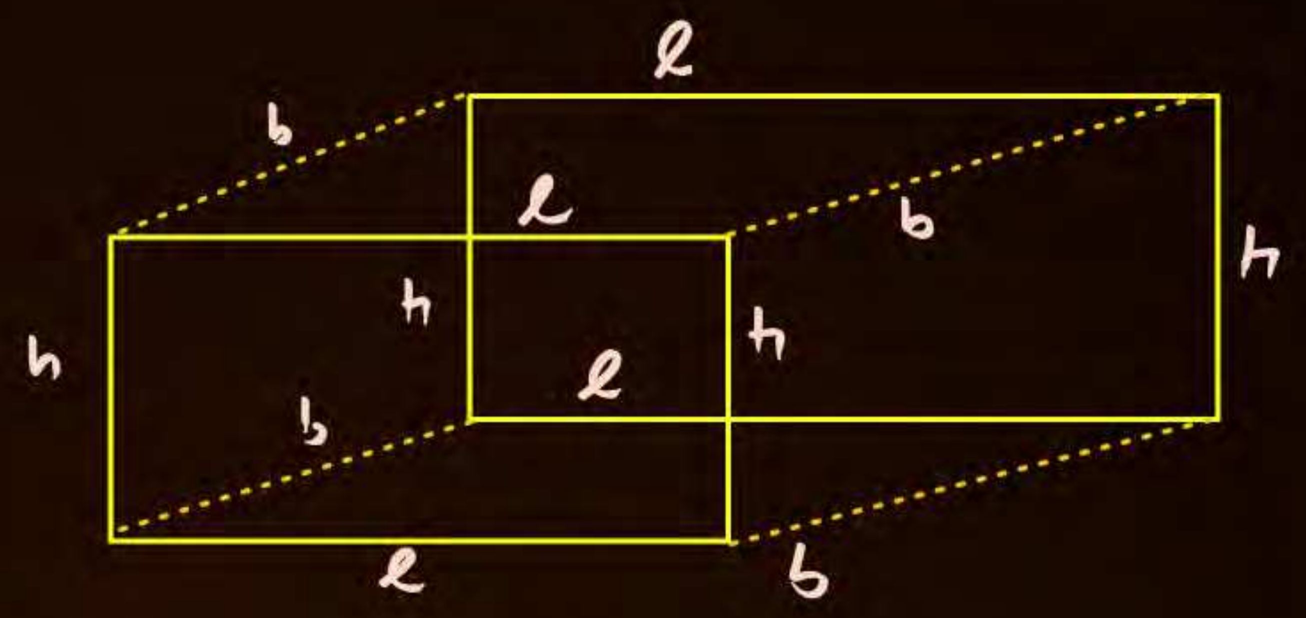
OR

$$2^x + 3^y = 55$$
$$2^x - 3^y = 1$$
$$\overline{2 \cdot 2^x = 56}$$

$2^x = 28$ N.P for any $x \in I^+$

$\Downarrow$

No integeral soln.




A $\quad V = l \cdot b \cdot h = 48$

$4(l + b + h) = 48$

$l + b + h = 12$

$lbh = 48$

$l = 6$

$b = 4$

$h = 2$

$l = 16 \times$

$l = 12 \times$

$l = 6$

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

## QUESTION

★★★★ KCLS ★★★★

For each positive number x, let $f(x) = \dfrac{\left(x+\frac{1}{x}\right)^6 - \left(x^6+\frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)}$. The minimum value of f(x) is

(A) 1

(B) 2

(C) 3

(D) 4

(E) 6

$x + \frac{1}{x} \geq 2,\ x \in R^+$

$3(x + 1/x) \geq 6 \Rightarrow E \geq 6$

$x + \frac{1}{x} = t \xrightarrow{CBS} x^3 + \frac{1}{x^3} + 3x\cdot\frac{1}{x}\left(x + \frac{1}{x}\right) = t^3$

$x^3 + \frac{1}{x^3} + 3t = t^3$

$x^3 + \frac{1}{x^3} = t^3 - 3t$

SBS

$x^6 + \frac{1}{x^6} + 2 = t^6 + 9t^2 - 6t^4$

$x^6 + \frac{1}{x^6} = t^6 + 9t^2 - 6t^4 - 2$

$E = \dfrac{\cancel{t^6} - (\cancel{t^6} + 9t^2 - 6t^4 \cancel{- 2}) \cancel{- 2}}{t^3 + t^3 - 3t}$

$= \dfrac{6t^4 - 9t^2}{2t^3 - 3t}$

$= \dfrac{3t^2 \cancel{(2t^2 - 3)}}{t \cancel{(2t^2 - 3)}} = 3t = 3(x + 1/x),\ x \in R^+$

$E = 3(x + 1/x) \geq 3\cdot 2 = 6$

$E_{min} = 6.$

$2t^2 - 3 \neq 0$ b'coz $2t^2 - 3 = 2\left(x + \frac{1}{x}\right)^2 - 3$

$= 2\left(x^2 + \frac{1}{x^2} + 2\right) - 3$

$= 2\left(x^2 + \frac{1}{x^2}\right) + 4 - 3$

$= 2\left(x^2 + \frac{1}{x^2}\right) + 1$ is +ve.

$2 - 8mx$

min value $= 2 - (-1) = 3$ ✗

⭐ $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$ $b \longrightarrow -b$

⭐ $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

QUESTION

If a, b, c are distinct real numbers such that $a^2 - b = b^2 - c = c^2 - a$, then $(a + b)(b + c)(c + a) =$ ______

Ta h 01

## Yaad Rakho

343 — last digit
Remaining No:
$34 - 2 \times 3 = 34 - 6 = 28$ divisible by 7

| DIVISIBILITY RULES | |
|---|---|
| 2 | If the last digit of a number is even, then the number is divisible by 2. |
| 3 | If the sum of all the digits in a number is divisible by 3, then the number is divisible by 3. |
| 4 | If the last two digits of a number are divisible by 4, then the number is divisible by 4. |
| 5 | If the last digit of a number is 0 or 5, then the number is divisible by 5. |
| 6 | If a number is divisible by both 2 and 3, then the number is divisible by 6. |
| 7 | If the last digit of a number is doubled and then subtracted from the rest of the number, and the answer is 0 or is divisible by 7, then the number is divisible by 7. |
| 8 | If the last three digits of a number are divisible by 8, then the number is divisible by 8. |
| 9 | If the sum of all the digits in a number is divisible by 9 , then the number is divisible by 9. |
| 10 | If the last digit of a number is 0, then the number is divisible by 10. |

Yaad Rakho

1331
1 2 3 4

3+1 − (1+3) = 0
divisible by 11

## DIVISIBILITY RULES

| | |
|---|---|
| 11 | To find out if a number is divisible by 11, add every even place digit, and call that sum 'x'. Add together the remaining digits, and call that sum 'y'. Take the positive difference of x and y. If the difference is zero or a multiple of eleven, then number is divisible by 11. Repeat the rule if necessary. |
| 13 | Delete the last digit from the number and then subtract 9 times the deleted digit from the remaining number. If what is left is divisible by 13, then number is divisible by 13. Repeat the rule if necessary. |



2197   219 − 9×7 = 219
− 63
156

15 − 6×9 = 15 − 54 = −39 divisible by 13.

## Yaad Rakho

For very large NO:s

### For 7, 11 & 13

If the positive difference of the last three digit and the rest of the digits is divisible by 7, 11, or 13, then the number is divisibly by 7, 11, or 13, respectively.

## QUESTION

Check which of the following is/are divisible by 7.

(A) ~~1078~~

(B) ~~3661~~

(C) ~~1257~~

(D) ~~6986~~

(A) 1078

$107 - 2 \times 8 = 107 - 16 = 107 - 16 = 91$ divisible by 7

(B) 3661

$366 - 2 \times 1 = 364$ Apply test again.

$36 - 2 \times 4 = 36 - 8 = 28$ divisible by 7

(D) 6986

$698 - 2 \times 6 = 698 - 12 = 686$ Apply test again

$68 - 12 = 56$ divisible by 7

(C) 1257

$125 - 2 \times 7 = 125 - 14 = 111$

$11 - 2 \times 1 = 9$ Not divisible by 7

## QUESTION

Check which of the following is/are divisible by 11

(A) 20449

(B) 39204

(C) 672535

(D) 35545444

20449
1 2 3 4 5

$2+4+9 = 15$
$0+4 = 4$
$15-4 = 11$

39204
1 2 3 4 5.

$3+2+4 = 9$
$9+0 = 9$
$9-9 = 0$

Divisible by 11

$7+5+5 = 17$
$6+2+3 = 12$
5

35545444
1 2 3 4 5 6 7 8

$5+4+4+4 = 17$
$3+5+5+4 = 17$
0

**QUESTION**

Check which of the following is/are divisible by 13

(A) 20449

(B) 24336

(C) 4225

(D) 492804

Test ① 49280(4)

$= 49280 - 4\times9$

$= 4924(4)$

$4924 - 9\times4$

$= 488(8)$

$488 - 9\times8$

$= 41(6)$

$41 - 9\times6$

$= -13$ divisible by 13.

4924
36
488

Test ②
804
492
312

$31 - 2\times9 = 31 - 18 = 13$

# Diamond Points to Note

**P₁** : $a^2 \geq 0, \qquad a \in R$

❖ Square of any Real number or an expression is "NEVER NEGATIVE"

Ex: $\underbrace{x^2}_{\geq 0} + 2 \Big|_{min} = 2$

Ex: $\underbrace{x^6}_{\geq 0} - 3 \Big|_{min} = -3$

Even NO:
* $(\text{Any real No:}) \geq 0.$

$a^{2n} \geq 0$

## Diamond Points to Note

**$P_2$:** If $x, y \in R$ & $x^2 + y^2 = 0$ $\Rightarrow x = 0$ & $y = 0$

$\underbrace{x^2}_{\geq 0} + \underbrace{y^2}_{\geq 0} = 0$ only possible if $x = 0, y = 0$

**Generalization:**



If $a_1 a_2 \ldots a_n \in R$ then $\underbrace{a_1^2}_{\geq 0} + \underbrace{a_2^2}_{\geq 0} + \cdots + \underbrace{a_n^2}_{\geq 0} = 0$ then $a_1 = a_2 = \cdots = a_n = 0$

Ex: $(x-1)^2 + (y-2)^2 + (z-3)^2 = 0$

find: $x + y \div 3$

Proof: $(x-1)^2 + (y-2)^2 + (z-3)^2 = 0$

$x - 1 = 0, \; y - 2 = 0, \; z - 3 = 0$

$x = 1, \; y = 2, \; z = 3$

(A) 1 — 67%

(B) 6 — 11%

(C) $5/3$ — 19%

(D) $4/3$ — 4.5%

$x=1,\ y=2,\ z=3$

$$x+y\div z = 1+2\div 3$$
$$= 3\div 3 = 1$$

Gadho/Gadhiyoo
aisaa naa karo

$$x+y\div z = 1+2\div 3$$
$$= 1+\frac{2}{3}$$
$$= 5/3$$

BODMAS

Bracket

Of

Divide

multiply

Add

subtract

# ASNC (Ashish Sir's Novel Concepts)

New

HSGH

"Whenever an equation consist of two or more variables always try to make perfect squares"

Ex: $x^2 + y^2 - 4x - 6y + 13 = 0$

find : $y^{x^{x^x}}$

(A) $3^8$ — 38%

(B) $3^4$ — 22%

(C) $3^{18}$ — 10%

(D) $3^{16}$ — 30%



Soln

$x^2 + y^2 - 4x - 6y + 13 = 0$

$x^2 - 4x + 4 + y^2 - 6y + 9 = 0$

$(x-2)^2 + (y-3)^2 = 0 \Rightarrow x = 2,\ y = 3$

$y^{x^{x^x}} = 3^{2^{2^2}} = 3^8$ Gadho/Gadhiyoo aisaa Na karo

$y^{x^{x^x}} = 3^{2^{2^2}} = 3^{2^4} = 3^{16}$

QUESTION

$x^2 - 2\cdot 3\cdot x$

$4z^2 - 4z + 1$

$(2z)^2 - 2\cdot 2z\cdot 1 + 1^2$

If $x^2 + y^2 + 4z^2 - 6x - 2y - 4z + 11 = 0$ then xyz equals

(A) 3/2

(B) 4

(C) 6

(D) 3

$x^2 - 6x + 9 + y^2 - 2y + 1 + 4z^2 - 4z + 1 = 0$

$(x-3)^2 + (y-1)^2 + (2z-1)^2 = 0$

$x = 3,\ y = 1,\ z = 1/2$

$xyz = 3/2$

QUESTION

If x, y & z are three real numbers such that $x^2 + 4y^2 + 9z^2 - 2x - 4y - 6z + 3 = 0$ then find the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$.

Tah 02

QUESTION

Let a, b, c are real numbers and satisfy $a = 8 - b$ and $c^2 = ab - 16$, then $\frac{a}{b}$ is equal to

Tah03

## QUESTION

Solve in real numbers the system of equations

$$\begin{cases} y^2+u^2+v^2+w^2=4x-1 \\ x^2+u^2+v^2+w^2=4y-1 \\ x^2+y^2+v^2+w^2=4u-1 \\ x^2+y^2+u^2+w^2=4v-1 \\ x^2+y^2+u^2+v^2=4w-1 \end{cases}$$

$+$

$4x^2+4y^2+4u^2+4v^2+4w^2=4x+4y+4u+4v+4w-5$

$4x^2-4x+1+4y^2-4y+1+4v^2-4v+1+4u^2-4u+1+4w^2-4w+1=0$



$(2x-1)^2+(2y-1)^2+(2v-1)^2+(2u-1)^2+(2w-1)^2=0$

$2x-1=0$
$2y-1=0$
$2v-1=0$
$2u-1=0$
$2w-1=0$

$x=y=v=u=w=\frac{1}{2}$

# Diamond Points to Note

$P_3$ : $k^4 + k^2 + 1 = (k^2+k+1)(k^2-k+1)$

$$k^4+k^2+1 = k^4+2k^2+1-k^2$$
$$= (k^2+1)^2 - k^2$$
$$= (k^2+1+k)(k^2+1-k)$$
$$= (k^2-k+1)(k^2+k+1)$$

QUESTION

If $a \in I$ and $a^4 + a^2 + 1$ is prime. The number of possible values of a is

(A) 0 — 28 %

(B) 1 — 34 %

(C) 2 — 29 %

(D) 3 8 %

$a^4 + a^2 + 1 = (a^2 - a + 1)(a^2 + a + 1) \in$ prime

$a^2 - a + 1 = 1$ & $a^2 + a + 1 =$ prime

$a^2 - a = 0$ @ $a = 0$ ✗ $a^2 + a + 1 = 1 \notin$ prime

$a = 0, 1$ @ $a = 1$ $a^2 + a + 1 = 3 \in$ prime



$a^2 + a + 1 = 1$ & $a^2 - a + 1 =$ prime

$a = 0, -1$ @ $a = 0$ $a^2 - a + 1 = 1 \notin$ prime

@ $a = -1$ $a^2 - a + 1 = 3 \in$ prime

$a = 1, -1$

Don’t Forget to



Retry all the class illustrations

# Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

(KTK 1)

a, b, c are reals such that $a + b + c = 3$ and $\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} = \frac{10}{3}$.
The value $E = \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$ is

A 9

B 7

C 5

D 3

QUESTION

(KTK 2)

Solve the equations : $\begin{cases} 2^x + 3^y = 41 \\ 2^{x+2} + 3^{y+2} = 209 \end{cases}$



Ans. x = 5 and y = 2

QUESTION

(KTK 3)

What is the area of an equilateral triangle inscribed in a circle of radius 4 cm?

(A) $12 \text{ cm}^2$

(B) $9\sqrt{3} \text{ cm}^2$

(C) $8\sqrt{3} \text{ cm}^2$

(D) $12\sqrt{3} \text{ cm}^2$



Ans. D

# Solution to Previous TAH

## QUESTION

Indicate which numbers in the given sets are (a) Natural numbers (b) Whole numbers (c) Integers (d) Rational numbers (e) Irrational numbers.

(i) $\left\{-10, -\sqrt{2}, -\frac{3}{4}, 0, \frac{4}{5}, \sqrt{4}, \pi, 7, \frac{18}{2}, 100\right\}$

(ii) $\left\{-\sqrt[3]{8}, \frac{0}{3}, \sqrt[3]{7}, \sqrt{\frac{4}{9}}, 1.\overline{126}\right\}$

TAH – 01

(ii) $\left\{-\sqrt[3]{8},\ \frac{0}{3},\ \sqrt[3]{7},\ \sqrt{\frac{4}{9}},\ 1.\overline{126}\right\}$

$-\sqrt[3]{8} \in I, Q$

$\frac{0}{3} \in W, Q, I$

$\sqrt[3]{7} \in R - Q$

$\sqrt{\frac{4}{9}} \in Q$

$1.\overline{126} \in Q.$

QUESTION

If a, b, c are non-zero real numbers, then the minimum value of expression

$$\left(\frac{(a^4+3a^2+1)(b^4+5b^2+1)(c^4+7c^2+1)}{a^2b^2c^2}\right) \text{ is}$$



Ans. 315

Que $a, b, c$ non-zero real no.'s,
min value of $\left(\frac{(a^4+3a^2+1)(b^4+5b^2+1)(c^4+7c^2+1)}{a^2b^2c^2}\right)$

$a^4+3a^2+1 \Rightarrow a^2\left(a^2+3+\frac{1}{a^2}\right) \Rightarrow a^2\left(a^2+\frac{1}{a^2}+3\right)$

$b^4+5b^2+1 \Rightarrow b^2\left(b^2+\frac{1}{b^2}+5\right)$

$a, b, c \in \mathbb{R}$

$\Rightarrow a^2, b^2, c^2 \geq 0$

$x^2+\frac{1}{x^2} \geq 2$

$c^4+7c^2+1 \Rightarrow c^2\left(c^2+\frac{1}{c^2}+7\right)$



$$\frac{\cancel{a^2b^2c^2}\left(a^2+\frac{1}{a^2}+3\right)\left(b^2+\frac{1}{b^2}+5\right)\left(c^2+\frac{1}{c^2}+7\right)}{\cancel{a^2b^2c^2}}$$

$$\left(a^2+\frac{1}{a^2}+3\right)\left(b^2+\frac{1}{b^2}+5\right)\left(c^2+\frac{1}{c^2}+7\right)$$

$(2+3)(2+5)(2+7)$

$5\times7\times9 \Rightarrow 35\times9$

$\boxed{315}$

Tahoz

if $a, b, c$ are non-zero real no. then the minimum value of exp.

$$\left(\frac{(a^4+3a^2+1)(b^4+5b^2+1)(c^4+7c^2+1)}{a^2b^2c^2}\right) \text{ is}$$

$\because$

$$\left(\frac{(a^4+3a^2+1)}{a^2}\right)\left(\frac{b^4+5b^2+1}{b^2}\right)\left(\frac{c^4+7c^2+1}{c^2}\right)$$



$$= \left(a^2+\frac{1}{a^2}+3\right)\left(b^2+\frac{1}{b^2}+5\right)\left(c^2+\frac{1}{c^2}+7\right)$$

min = 2 min = 2 min = 2

$a, b, c \in \{R-0\}$

$\therefore x^2+\frac{1}{x^2} \geq 2$

$= (5)(7)(9)$

$= 315$

# Solution to Previous KTKs

QUESTION

KTK 1

The equation $\frac{2x^2}{x-1}-\frac{2x+7}{3}+\frac{4-6x}{x-1}+1=0$ has the roots-

(A) 4 and 1

(B) only 1

(C) only 4

(D) neither 4 nor 1



Ans. C

$$\frac{2x^2}{x-1} - \frac{2x+7}{3} + \frac{4-6x}{x-1} + 1 = 0$$

$D^r \neq 0$

$$\frac{2x^2-6x+4}{x-1} - \frac{2x+7}{3} + 1 = 0$$

$x-1 \neq 0$

$x \neq 1$

$$\frac{2(x^2-3x+2)}{(x-1)} - \frac{2x+7}{3} + 1 = 0$$

$$\frac{2(x-2)\cancel{(x-1)}}{\cancel{(x-1)}} - \frac{2x+7}{3} + 1 = 0$$



$$2x-4+1-\frac{2x+7}{3} = 0$$

$$\frac{6x-9-7-2x}{3} = 0$$

$$4x = 16$$

$$x = 4$$

Ans - Only 4 ✓.

QUESTION

KTK 2

Which one of the following does not reduce to sin x for every x, wherever defined, is

(A) $\frac{\tan x}{\sec x}$

(B) $\frac{\sin x}{\sec^2 x - \tan^2 x}$

(C) $\frac{\sin^2 x \sec x}{\tan x}$

(D) All reduce to sin x



Ans. D

KTK
2) Which one of following does not reduce to $\sin x$ for ey $x$, wherever defined, is

(a) $\dfrac{\tan x}{\sec x}$ : $\dfrac{\sin x}{\cos x} \cdot \cos x = \sin x$ (✓)



(b) $\dfrac{\sin x}{\sec^2 x - \tan^2 x}$ : $\dfrac{\sin x}{1} = \sin x$ (✓)

(c) $\dfrac{\sin^2 x \sec x}{\tan x}$ : $\dfrac{\sin^2 x \cancel{\sec x}}{\sin x \cdot \cancel{\sec x}} = \sin x$ (✓)

(d) All reduce to $\sin x$

$\therefore$ All reduce to $\sin x$ (✓)

**QUESTION**

KTK 3

| | Column-I | | Column-II |
|---|---|---|---|
| (A) | A rectangular box has volume 48, and the sum of the length of the twelve edges of the box is 48. The largest integer that could be the length of an edge of the box, is | (P) | 1 |
| (B) | The number of zeroes at the end in the product of first 20 prime numbers, is | (Q) | 2 |
| (C) | The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is | (R) | 3 |
| | | (S) | 4 |
| | | (T) | 6 |



Ans. (A) T; (B) P; (C) P

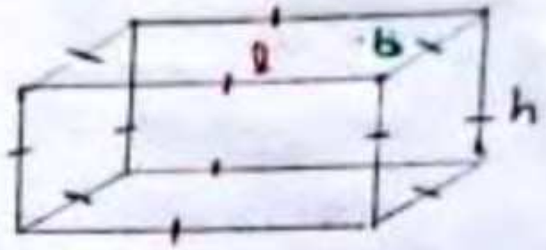
Q (A) A rectangular box has volume 48, and the sum of the length of the twelve edges of the box is 48. The largest integer that could be the length of an edges of the box, is

Given, $V = \boxed{l \times b \times h = 48}$

Sum of ~~the~~ length of the twelve edges of the box

$4(l+b+h) = 48$

$\boxed{l+b+h = 12}$




$48 = 2\times2\times2\times2\times3$

$l = 2\times2 = 4$

$b = 2\times3 = 6$

$h = 2 = 2$

should be → 12

∴ Given, The largest integer that could be the length of an edges of the box

∴ $\boxed{l = 6}$ Ans

(B) The number of zero's at the end in the product of first 20 Prime number, is

First Prime No. → 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71

Take Product me zero are ke liy find How many times comes 2 & 5 ]] or multiple of 2 & 5

∵ The number 2 appears once

The number 5 appears once.

Hence There is only one Pair of 2 and 5.

∴ No. of zero = Multiple of 2 × Multiple of 5

∴ Number of zero is 1 Ans

... of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is

$(2^x)^2 - (3^y)^2 = 55$

$(2^x+3^y)(2^x-3^y) = 55 \rightarrow 5\times11$

The integer factor Pairs of 55 are :- (5,11), (11,5), (1,55), (55,1)

Case-I: $2^x+3^y=5$ and $2^x-3^y=11$

adding the equation

$2\cdot 2^x = 16$

$2^{x+1}=16$

$2^{x+1}=2^4$

$x+1=4$

$\boxed{x=3}$, $\boxed{3^y=-3}$

Hence, There is no integer solution for y in this case.

Case-II: $2^x+3^y=11$ and $2^x-3^y=5$

adding the equation

$2^{x+1} = 16$ | $8+3^y=11$

$2^{x+1} = 2^4$ | $3^y = 3$

$x+1=4$ | $\boxed{y=1}$

$\boxed{x=3}$

~~Hence~~ Accepted

Case-III: ...

Adding the equation

$2\cdot 2^x = 56$

$2^{1+x} = 56$

Since 56 is not a Power of 2, there is no integer solution for x in this case.

Case-IV: $2^x+3^y=55$ and $2^x-3^y=1$

Adding the equation

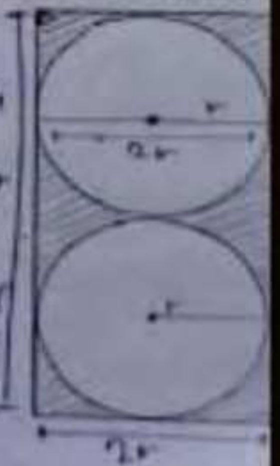
$2\cdot 2^x = 56$

Since 56 is not a Power of 2, there is no integer solution for x in this case.

The only integer solution is $(x,y) = (3,1)$ Ans.

KTK:-04

Q. The ratio of total area of the rectangle to the total shaded area.




Length of rectangle → $4r$

Width of rectangle → $2r$

Shaded Area = Area of rectangle − 2(Area of circle)

$= 8r^2 - 2\pi r^2$

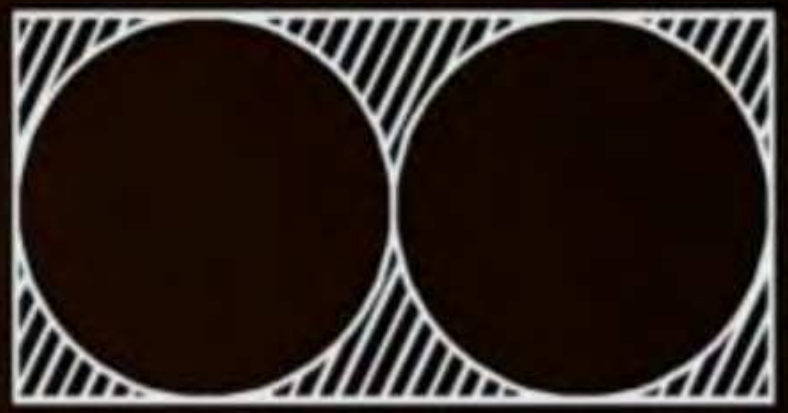
$= 2r^2(4-\pi)$

∴ $8r^2$ ... = 4 ... is Ratio of total area ...

QUESTION

KTK 4

The ratio of total area of the rectangle to the total shaded area



A $\frac{2}{\pi}$

B $\frac{4}{4-\pi}$

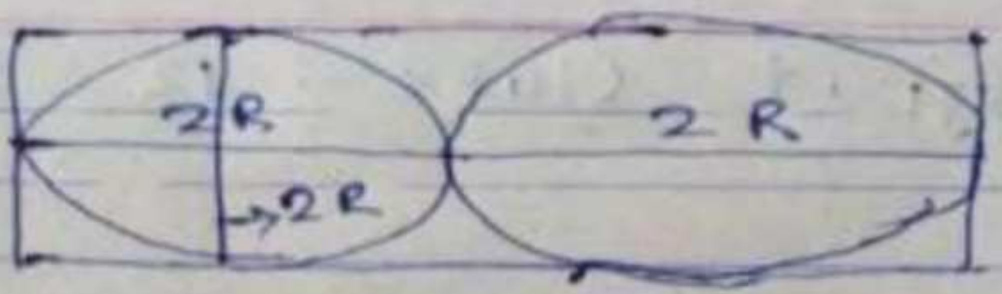
C $\frac{4-\pi}{\pi}$

D $\frac{\pi}{4}$



Ans. B

Question—




$l = 4R, \quad b = 2R$

Area of rectangle $= 4R \times 2R$

$= 8R^2$



shaded area $= 8R^2 - 2\pi R^2$

$= 2R^2(4-\pi)$

Ratio $= \dfrac{8R^2}{2R^2(4-\pi)}$

$$\boxed{\text{Ratio} = \frac{4}{(4-\pi)}}$$


THANK
YOU