PRAYAS

JEE 2026

Mathematics

Basic Maths

Lecture – 02

By – Ashish Agarwal Sir
(IIT Kanpur)

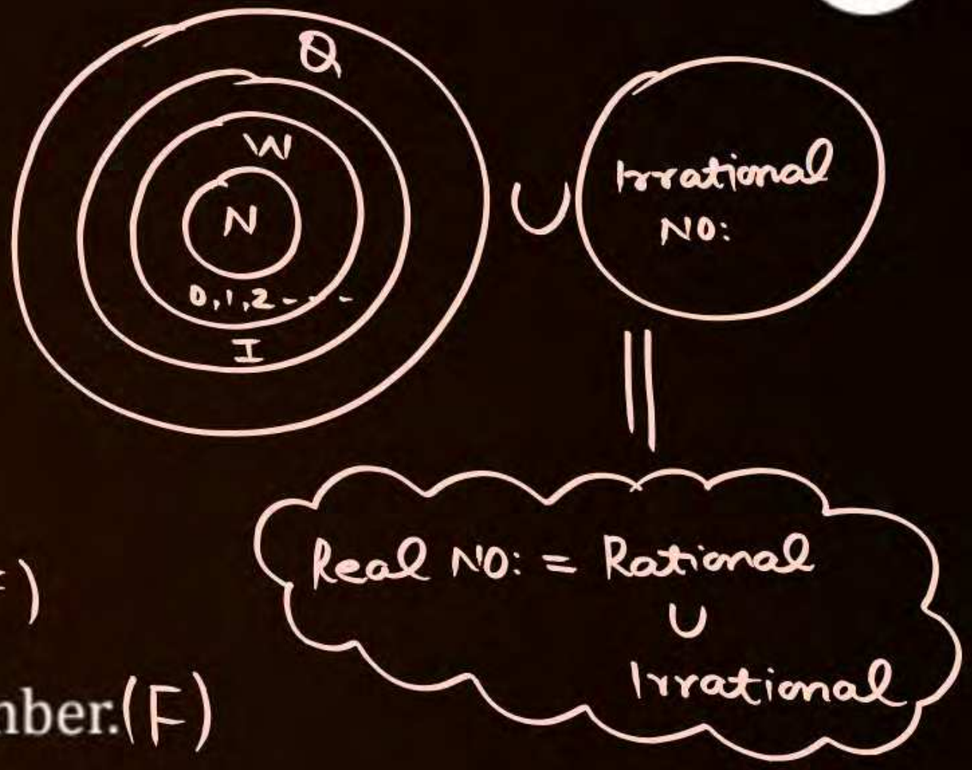
Topics *to be covered*

- **A** Important Notations
- **B** Types of Integers & Natural Numbers
- **C** Two Golden Inequalities
- **D** Some Important Points

Recap *of previous lecture*

State True or False

1. Every Natural number is an integer. (T)
2. Every integer is a rational number. (T)
3. 0 is a rational number. (T)
4. Every rational number is a terminating decimal. (F)
5. $\sqrt{2} + \sqrt{3}$ is an irrational number but not a real number. (F)
6. Every real number is a complex number. (T)
7. 5 is a natural number but not an integer. (F)
8. $\frac{343}{240}$ is a terminating decimal. (F) $\quad \frac{343}{240} = \frac{343}{3 \times 2^3 \times 2 \times 5}$



Recap of previous lecture

State True or False

9. $\frac{547}{2500}$ is a terminating decimal. (T)

$2500 = 5^2 \times 5^2 \times 2^2 = 5^4 2^2$

10. If x^3 & x^7 are both non zero rational then x should also be rational. (T)

$x^3 \in Q$

$x^3 \cdot x^3 \in Q$

$x^6 \in Q$

Now $x^7 \in Q$

$\frac{x^7}{x^6} \in Q$

$x \in Q$

Fill in the blank

a. 

Fill in the blanks with N, I, Q.

Recap of previous lecture

b. Rational numbers $\cap$ irrational numbers = ϕ

c. Rational numbers $\cup$ irrational numbers = Real NO:s

d. $N \cap W = N$, $N \cup Q = Q$

e. $I \cap Q = I$, $I \cup Q = \emptyset$

f. $0.52\overline{3} = \frac{157}{300}$

g. $\frac{\text{Any number}}{0}$ = Not defined

h. $x - \frac{2}{x+1} = 1 - \frac{2(x-1)}{x^2-1}$ then number of solutions = 0

$x - \frac{2}{x+1} = 1 - \frac{2}{x+1}$ $x \neq 1$

$x = 1$

A ke saarray elements B mai hai

A is subset of B then

* $A \cap B = A$
* $A \cup B = B$

$x = 0.52333\ldots$

$100x = 52.3333\ldots$ ~ (1)

$1000x = 523.3333\ldots$ ~ (11)

(11) - (1)

$900x = 471 \Rightarrow x = \frac{471}{900} = \frac{157}{300}$

Ex: $x = 0.3\overline{4}$

$x = 0.3444\ldots$

$10x = 3.4444\ldots$

$100x = 34.444\ldots$

$90x = 31$

$x = \frac{31}{90}$

Ex: $x = 2.\overline{32}$

$x = 2.323232\ldots$

$100x = 232.323232\ldots$

$99x = 230$

$x = \frac{230}{99}$ Ans

Important Notations

0 is neither +ve nor -ve

(i) R^+ : Set of all positive real numbers i.e. all real numbers greater than 0 i.e.
$R^+ = (0, \infty)$

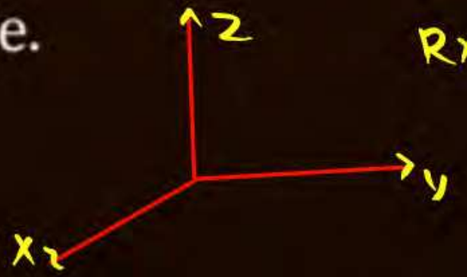
(ii) R^- : Set of all negative real numbers i.e. all real numbers less than 0 i.e.
$R^- = (-\infty, 0)$

(iii) R_0 : Set all real numbers except zero, $R_0 = (-\infty, \infty) - \{0\}$.

(iv) $R^2 = R \times R$ = All points In 2-D plane

$R \times R = R^2 = \{(x, y) \mid x, y \in R\}$

(v) $R^3 = R \times R \times R$ = All points in 3-D space.



$R \times R \times R = R^3 = \{(x, y, z) \mid x, y, z \in R\}$

A close look at Set of Integers

Positive integers : {1, 2, 3, 4 - - -}

Negative integers : {-1, -2, -3 - - -}

Non Negative integers : {0, 1, 2, 3, 4 - - -}

None Positive integers : {0, -1, -2, -3 - - -}

QUESTION

Indicate which numbers in the given sets are (a) Natural numbers (b) Whole numbers (c) Integers (d) Rational numbers (e) Irrational numbers.

(i) $\left\{-10, -\sqrt{2}, -\frac{3}{4}, 0, \frac{4}{5}, \sqrt{4}, \pi, 7, \frac{18}{2}, 100\right\}$

W, N, I, Q

I, Q R-Q Q I, W, Q R-Q Q

$7 \in$ I, N, W, Q

$\frac{18}{2} \in$ I, N, W, Q

$100 \in$ I, N, W, Q

(ii) $\left\{-\sqrt[3]{8}, \frac{0}{3}, \sqrt[3]{7}, \sqrt{\frac{4}{9}}, 1.\overline{126}\right\}$

Tah o1(A)

* $\sqrt{4} = 2$

principal root of 4

$\sqrt{4} \notin$ R-Q Gadho/Gadhiyoo Dhyaan se Dekho

* $\sqrt{16} = 4$
* $\sqrt{9} = 3$
* $\sqrt{0} = 0$

Ex: $x^2 = 4$

$x = \pm 2$

But $\sqrt{4} = 2$

$\sqrt{a}$, $a \geq 0$ aisaa non-ve NO: jiskaa square 'a' ho

QUESTION

The greatest integer lying between −10 and −15 is

A −10

B −11

C −15

D −14

Ans. B

QUESTION

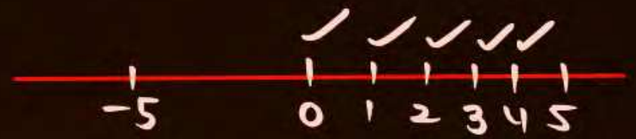
Number of whole numbers lying between −5 and 5

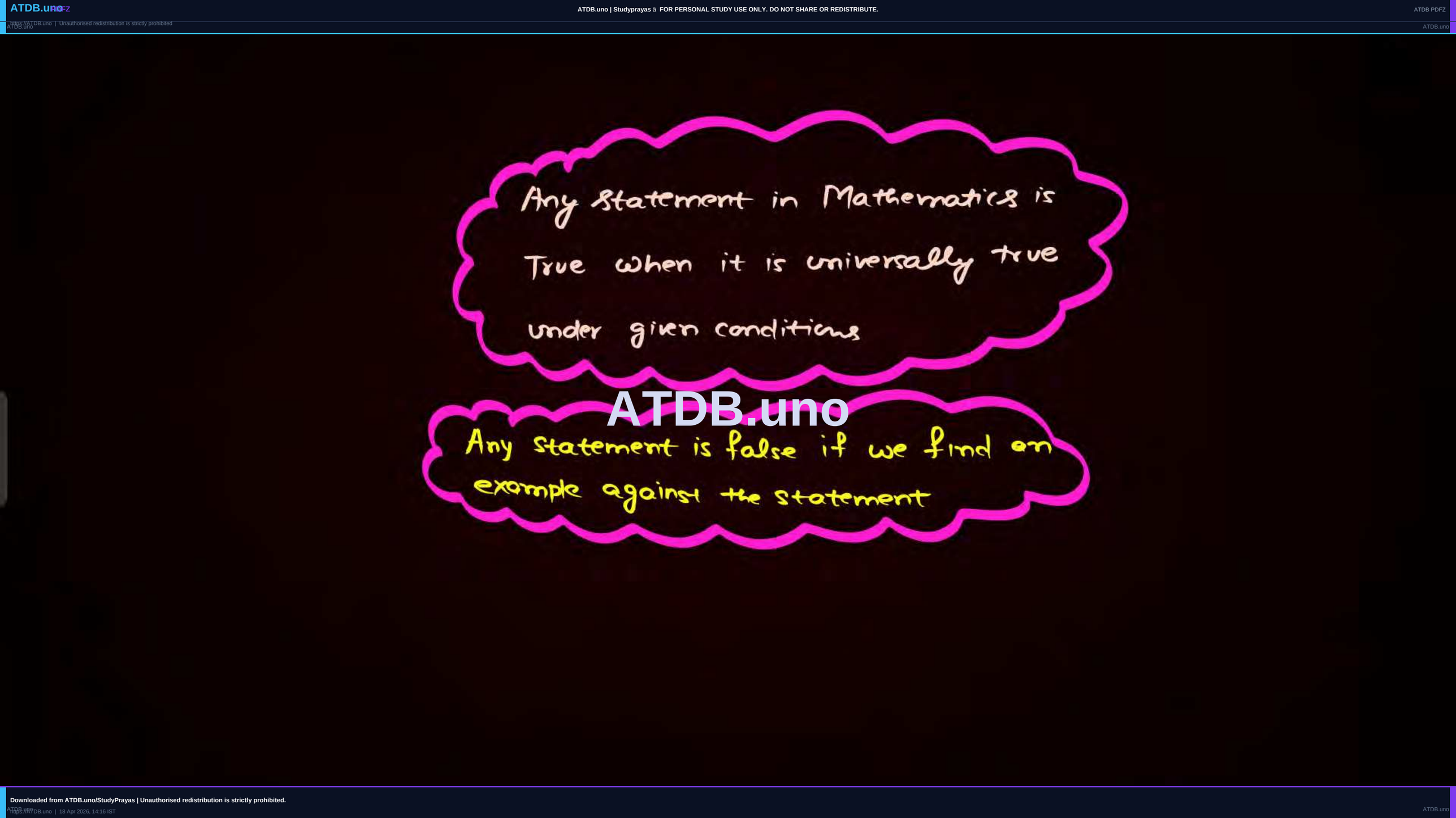
A 10

B 3

C 4

D 5





QUESTION

Which of the following conditions imply that the real number x is rational ?

I. $x^{1/2}$ is rational (given)

II. x^2 and x^5 are rational

III. x^2 and x^4 are rational

$x^{1/2} \in Q \Rightarrow x^{1/2} \cdot x^{1/2} \in Q \Rightarrow x \in Q$ (given)

$x^2, x^5 \in Q$ (given) $\Rightarrow x^2 \cdot x^2 \in Q$ i.e $x^4 \in Q$, also $x^5 \in Q$

$\frac{x^5}{x^4} = x \in Q$, $x \neq 0$

If $x = 0 \Leftrightarrow x^2, x^5 \in Q$

$\sqrt{2}^2, \sqrt{2}^4 = 2, 4 \in Q$ But $\sqrt{2} \notin Q$

(A) I and II only

(B) I and III only

(C) II and III only

(D) I, II, and III

Ans. A

QUESTION

Let a, b, c be positive integer S.t. $\frac{a\sqrt{2}+b}{b\sqrt{2}+c}$ is a rational number, then which of the following values of a, b, c is/are possible?

(A) a = 1, b = 2, c = 4

(B) a = 4, b = 6, c = 9

(C) a = 2, b = 5, c = 8

(D) a = 3, b = 9, c = 27

$\frac{a\sqrt{2}+b}{b\sqrt{2}+c} \in Q$ (given)

$a, b, c \in I^+$

$b^2 - ac \in I$

$b^2 - ac \neq \sqrt{2}$

$\frac{a\sqrt{2}+b}{b\sqrt{2}+c} \cdot \frac{b\sqrt{2}-c}{b\sqrt{2}-c} \in Q$

$\frac{2ab - ac\sqrt{2} + b\sqrt{2} - bc}{2b^2 - c^2} \in Q$

$\frac{2ab - bc + \sqrt{2}(b^2 - ac)}{2b^2 - c^2} \in Q$

$b^2 - ac = 0$

$b^2 = ac$

Types of Integers and Natural Numbers

1. **Even Integers:** $2n, n \in I = \{....., -6, -4, -2, 0, 2, 4, 6 .....\}$

2. **Odd Integers:** $2n + 1$ or $2n - 1, n \in I = \{....., -5, -3, -1, 1, 3, 5 .....\}$

3. **Prime Numbers:** 2, 3, 5, 7, 11, 13 ..... All natural numbers with only two divisors 1 & itself.

4. **Composite Numbers:** All natural numbers with more than two distinct positive divisors. Ex 4 — Divisor : 2, 4, 1
   6 → Divisors : 2, 3, 6, 1

5. **Relatively Prime / Coprime:** Two natural numbers whose HCF is one. Ex : 2 & 3
   8 & 21
   7 & 16

6. **Twin Prime Numbers:** Two prime numbers with a difference of two are called twin primes. Ex: 3 & 5, 5 & 7, 17 & 19

1 is neither prime nor Composite Number

N − prime − {1} = Composite NO:
NO: 8

Every prime NO: ≥ 5 is of type $6K \pm 1$, $K \in N$

$5 = 6 \cdot 1 - 1$

$7 = 6 \cdot 1 + 1$

$11 = 6 \cdot 2 - 1$

$13 = 6 \cdot 2 + 1$

$17 = 6 \cdot 3 - 1$

$23 = 6 \cdot 4 - 1$

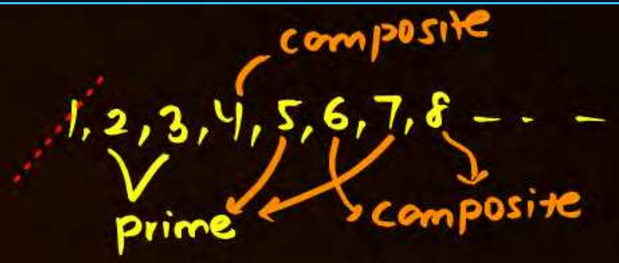
$19 = 6 \cdot 3 + 1$

But Every NO: of form $6K \pm 1$ is not prime

$6 \cdot 4 + 1 = 25$ ∉ prime

$6 \cdot 6 - 1 = 35$ ∉ prime.

Kaam ki Baatien



1. 2 is smallest prime.

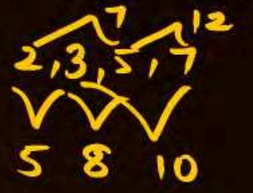
2. 4 is the smallest composite number.

3. 1 is neither prime nor composite.

4. Every prime ≥ 5 is of type _____ where _____ but every number of form _____ is not _____.

QUESTION

If 'a' and 'b' are two distinct prime numbers lying between 1 and 10, which of the following can be the sum of 'a' and 'b' -



A 5

B 6

C 7

D 8

Ans. A, C, D

Two Golden Inequalities

$G_1: x + \frac{1}{x} \geq 2, \quad x \in R^+$

$G_2: x + \frac{1}{x} \leq -2, \quad x \in R^-$

★ any +ve Real + its reciprocal ≥ 2

★ any -ve Real + its reciprocal ≤ -2

$x + \frac{1}{x} = 2, x \in R^+ \iff x = 1$

$x + \frac{1}{x} = -2, x \in R^- \iff x = -1$

If $x \in R$ then $x^2 \geq 0$

Ex: $(-\sqrt{2})^2 = 2$

$(-5)^2 = 25$

$(3)^2 = 9$

$0^2 = 0$

Proof: Since $x \in R^+ \Rightarrow \sqrt{x} \in R^+$, $\frac{1}{\sqrt{x}} \in R^+$

(i) $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 \geq 0$

$x + \frac{1}{x} - 2\sqrt{x} \cdot \frac{1}{\sqrt{x}} \geq 0$

$x + 1/x \geq 2$. (Hence proved)

(ii) Since $x \in R^-$ let $x = -t$ ($\Downarrow$ $t = -x$), $t > 0 \Rightarrow t + \frac{1}{t} \geq 2$ from (i)

$-x - \frac{1}{x} \geq 2 \Rightarrow x + 1/x \leq -2$

$f(x) = x + \frac{1}{x}$ Range : $(-\infty, -2] \cup [2, \infty)$

$f(x) = k$ has a real soln
if $k \in$ Range of $f(x)$

Ex: $\sin x = 2 \longrightarrow$ No soln

Ex: $\cos x = -3 \rightsquigarrow$ No soln.

QUESTION [JEE Mains 2023 (24 Jan)]

The number of real solutions of the equation $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$,is

A 3

B 4

C 0

D 2

$x+\frac{1}{x}=t$

S·B·S $x^2+\frac{1}{x^2}+2x\cdot\frac{1}{x}=t^2$

$x^2+\frac{1}{x^2}=t^2-2$

$3(t^2-2)-2t+5=0$

$3t^2-2t-1=0$

$3t^2-3t+t-1=0$

$(3t+1)(t-1)=0$

$t=-1/3, 1$

$x+1/x=-1/3$ or $x+1/x=1$

No real soln

No real soln ∉ Range of $x+1/x$

bcoz $-1/3$ does not lie in

Ans. C

QUESTION

Consider the expressions

$$E_1 = x + \frac{1}{x}, x \in R^-; \quad E_2 = y^2 + \frac{1}{y^2}, y \in R_0; \quad E_3 = z^2 + \frac{1}{z^2+1} + \underbrace{1+1}_{2}, z \in R$$

If $\alpha = \max E_1, \beta = \min E_2, \gamma = \min E_3$ then $\alpha + \beta + \gamma$ is

(A) 3

(B) 5

(C) 2

(D) 7

$E_1 = x + \frac{1}{x}$ $x \in R^- \Rightarrow E_1 = x + \frac{1}{x} \le -2$, $E_2 = y^2 + \frac{1}{y^2}$, $y \in R_0 \Rightarrow$ clearly $y^2 > 0$ i.e $y^2 \in R^+$

$\alpha = \max E_1 = -2$ @ $x = -1$

$\Rightarrow E_2 = y^2 + \frac{1}{y^2} \ge 2$

$\beta = \min E_2 = 2$ @ $y^2 = 1 \Rightarrow y = \pm 1$

$E_3 = z^2 + 1 + \frac{1}{z^2+1} + 1$, $z \in R$

$z^2 \ge 0 \Rightarrow 1 + z^2 \ge 1$

$1 + z^2 \in R^+$

$(z^2+1) + \frac{1}{z^2+1} \ge 2$

$(z^2+1) + \frac{1}{z^2+1} + 1 \ge 3$

$\gamma = \min E_3 = 3$ @ $z^2 + 1 = 1 \Rightarrow z = 0$

$\alpha + \beta + \gamma = -2 + 2 + 3 = 3$.

Ans. A

QUESTION

Tah 02

If a, b, c are non-zero real numbers, then the minimum value of expression

$$\left(\frac{(a^4+3a^2+1)(b^4+5b^2+1)(c^4+7c^2+1)}{a^2b^2c^2}\right) \text{ is}$$

Ans. 315

Yaad Karnay hai

$\sqrt{1} = 1$	$\sqrt{11} = 3.3166$
$\sqrt{2} = 1.4142$	$\sqrt{12} = 3.4641$
$\sqrt{3} = 1.732$	$\sqrt{13} = 3.6055$
$\sqrt{4} = 2$	$\sqrt{14} = 3.7416$
$\sqrt{5} = 2.236$	$\sqrt{15} = 3.8729$
$\sqrt{6} = 2.4494$	$\sqrt{16} = 4$
$\sqrt{7} = 2.6457$	$\sqrt{17} = 4.1231$
$\sqrt{8} = 2.8284$	$\sqrt{18} = 4.2426$
$\sqrt{9} = 3$	$\sqrt{19} = 4.3588$
$\sqrt{10} = 3.1622$	$\sqrt{20} = 4.4721$

Yaad Karnay hai

$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
$6^2 = 36$	$7^2 = 49$	$8^2 = 64$	$9^2 = 81$	$10^2 = 100$
$11^2 = 121$	$12^2 = 144$	$13^2 = 169$	$14^2 = 196$	$15^2 = 225$
$16^2 = 256$	$17^2 = 289$	$18^2 = 324$	$19^2 = 361$	$20^2 = 400$
$21^2 = 441$	$22^2 = 484$	$23^2 = 529$	$24^2 = 576$	$25^2 = 625$
$26^2 = 676$	$27^2 = 729$	$28^2 = 784$	$29^2 = 841$	$30^2 = 900$

Yaad Karnay hai

$31^2 = 961$	$32^2 = 1024$	$33^2 = 1089$	$34^2 = 1156$	$35^2 = 1225$
$36^2 = 1296$	$37^2 = 1369$	$38^2 = 1444$	$39^2 = 1521$	$40^2 = 1600$
$41^2 = 1681$	$42^2 = 1764$	$43^2 = 1849$	$44^2 = 1936$	$45^2 = 2025$
$46^2 = 2116$	$47^2 = 2209$	$48^2 = 2304$	$49^2 = 2401$	$50^2 = 2500$
$51^2 = 2601$	$52^2 = 2704$	$53^2 = 2809$	$54^2 = 2916$	$55^2 = 3025$
$56^2 = 3136$	$57^2 = 3249$	$58^2 = 3364$	$59^2 = 3481$	$60^2 = 3600$

Don’t Forget to
ATDB.uno
Retry all the class illustrations

Today's KTK

No Selection —TRISHUL Apnao IIT Jao→ Selection with Good Rank

QUESTION

KTK 1

The equation $\frac{2x^2}{x-1} - \frac{2x+7}{3} + \frac{4-6x}{x-1} + 1 = 0$ has the roots-

A 4 and 1

B only 1

C only 4

D neither 4 nor 1

Ans. C

QUESTION

KTK 2

Which one of the following does not reduce to sin x for every x, wherever defined, is

A $\frac{\tan x}{\sec x}$

B $\frac{\sin x}{\sec^2 x - \tan^2 x}$

C $\frac{\sin^2 x \sec x}{\tan x}$

D All reduce to sin x

Ans. D

QUESTION

KTK 3

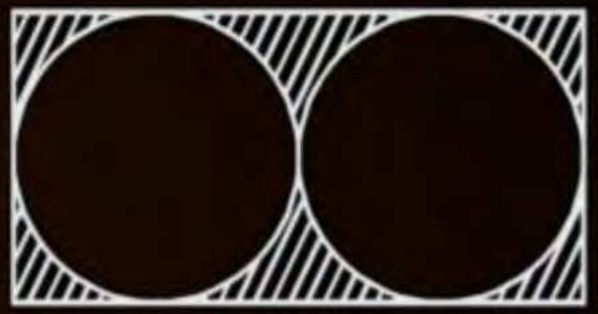
Column-I		Column-II	
(A)	A rectangular box has volume 48, and the sum of the length of the twelve edges of the box is 48. The largest integer that could be the length of an edge of the box, is	(P)	1
(B)	The number of zeroes at the end in the product of first 20 prime numbers, is	(Q)	2
(C)	The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is	(R)	3
		(S)	4
		(T)	6

Ans. (A) T; (B) P; (C) P

QUESTION

KTK 4

The ratio of total area of the rectangle to the total shaded area



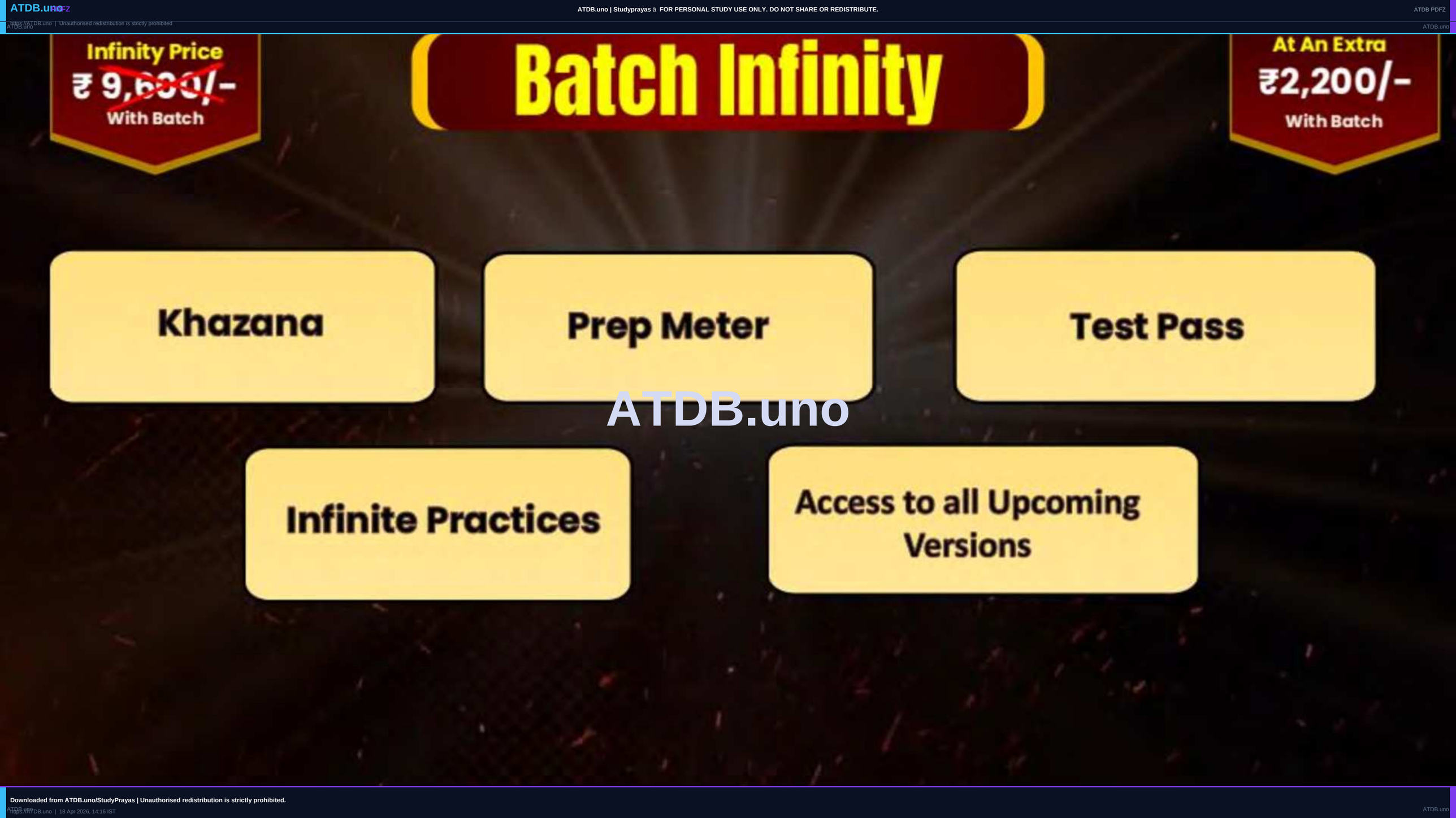
A $\frac{2}{\pi}$

B $\frac{4}{4-\pi}$

C $\frac{4-\pi}{\pi}$

D $\frac{\pi}{4}$

Ans. B



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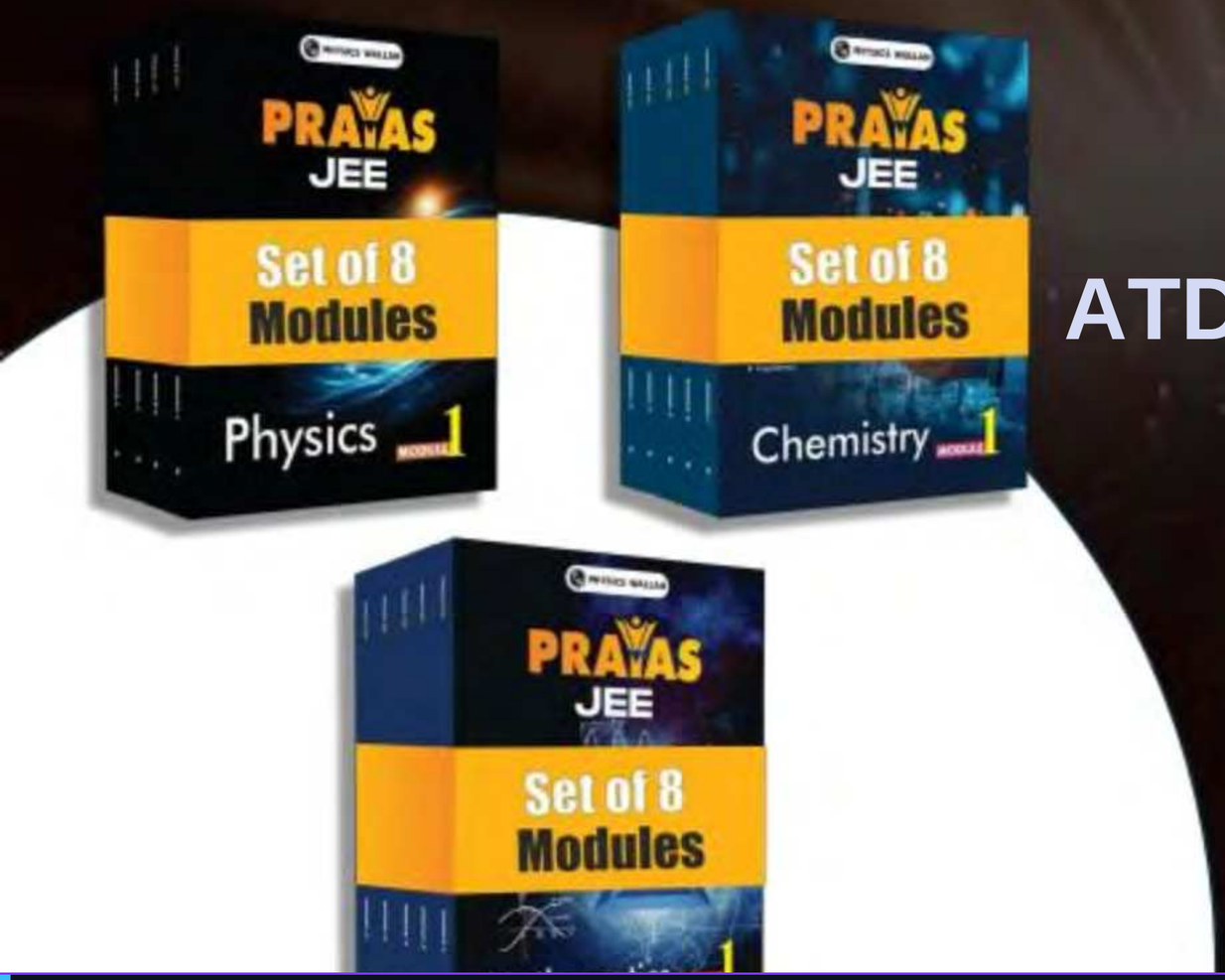

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