Work, Energy and Power

Work

When a force act on an object such that it displaces through some distance in the direction of applied force, then the work is said to be done by the force

Work done by the force is equal to the product of the force and the displacement of the object in the direction of force.

If under a constant force F the object is displa--ced through a distance S then work done by the force

$$W = \vec{F} \cdot \vec{S} = FS\cos\theta$$

where θ is the smaller angle between F and S

Work is a scalar quantity. Its SI unit is joule and CGS unit is erg.

$$1 \text{ Joule} = 10^7 \text{ erg.}$$

Its dimensional formula is $[ML^3T^{-2}]$

Work done by a force is zero if

(a) body is not displaced actually i.e $S=0$
(b) body is displaced perpendicular to the direction of force, i.e $\theta = 90°$

Work done by force is positive, if angle between F and S is acute angle

work done by force is negative, if angle between F and S is obtuse angle.

work done by a constant force depends only on the initial and final positions of the object and not on the actual path followed between initial and final positions.

Work done in different conditions.

(i) work done by a variable force

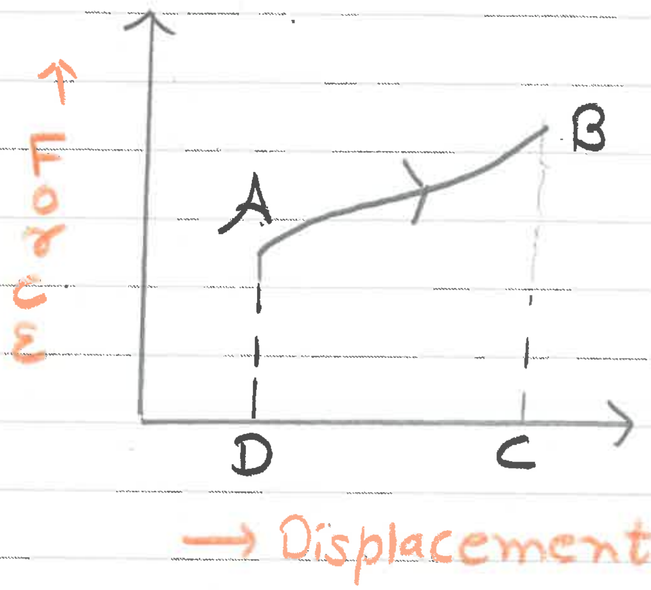
The total work done will be the sum of ~~inde~~ infinitesimal work over infinitesimal displacements.

$$W = \int dW$$

Work done by a force in moving a particle from A to B is given by following:

$$W = \int_A^B \vec{F} \cdot d\vec{r} = \int_A^B F\cos\theta\, dr.$$

It is also equal to the area under the force displacement graph, along with proper sign.



work done = Area ABCDA

(ii) work done in displacing any body under the action of a number of forces is equal to the work done by the resultant force.

(iii) In equilibrium (static or dynamic), the resultant force is zero, therefore resultant work done is zero.

(iv) work done by the force of gravity on a particle of mass m is given by $W = mgh$

where g is acceleration due to gravity and h is height through which the particle is displaced

(v) work done is compressing or streching a spring is given by

$$W = -\frac{1}{2}kx^2$$

where k is spring constant and x is displacement from mean position.

(vi) When one end of a spring is attached to a fixed vertical support and a block attached to the free end moves on a horizontal table from $x = x_1$ to $x = x_2$ then

$$W = \frac{1}{2}k(x_2^2 - x_1^2)$$

(vii) Work done by the couple for an angular displacement θ is given by $W = \tau \cdot \theta$

where τ is the torque of the couple.

Energy

Energy of a body is it capacity of doing work. It is a scalar quantity.

Its SI unit is Joule and CGS unit is erg. Its dimensional formula is $[ML^2T^{-2}]$

There are several types of energies, such as mechanical energy (kinetic energy and potential energy), chemical energy, light energy, heat energy

Sound energy, nuclear energy and electric energy etc.

Mechanical Energy.

The sum of kinetic and potential energy is known as mechanical energy.

Mechanical energy is of two types.

Kinetic Energy

The energy possessed by any object by virtue of its motion is called its kinetic energy.

Kinetic energy of an object is given by $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

where m = mass of the object
v = velocity of the object
and $p = mv$ = momentum of the object.

Potential Energy.

The energy possessed by an object by virtue of its position or configuration is called its potential energy.

In one dimensional motion, potential energy $U(x)$ is defined if force $F(x)$ can be written as

$$F(x) = -\frac{dU}{dx}$$

$$F(x) \cdot dx = -dU$$

$$\int_{x_i}^{x_f} F(x) \cdot dx = -\int_{U_i}^{U_f} dU = U_i - U_f$$

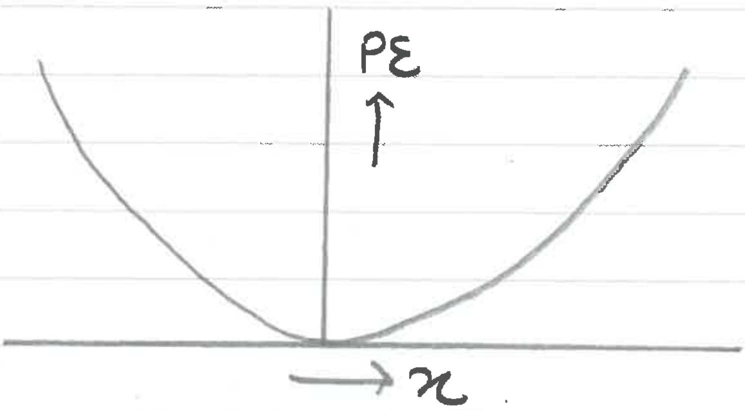
Potential energy depends upon frame of reference

There are three important types of potential energies.

(i) Gravitational Potential Energy If a body of mass m is raised through height h against gravity, then its gravitational potential energy = mgh

(ii) Elastic Potential Energy If a spring of constant K is streched through a distance x, then elastic potential energy of the spring $= \frac{1}{2}kx^2$

The variation of potential energy with distance is shown in figure.



(iii) Electric Potential Energy The electric potential energy of two point charges q_1 and q_2 seperated by a distance r in vaccum is given by.

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

Here $\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9 \frac{N\text{-}m^2}{C^2}$ = constant.

Conservative Force

If the work done by a force is independent of the path travelled by the body, then the force is said to be a conservative force.

Example :- Gravitational force.

In general form, a force is conservative if the work it does around any closed path is zero

Non conservative Force.

If the work done by a force depends on the path travelled by the body, then the force is said to be a non-conservative force.

Example : Friction force

Non-conservative forces are also known as dissipative forces, as they dissipate mechanical energy in other forms.

✶ Note

1 conservative forces are state/point function which means that they depend only on the initial and final states.

2. Non conservative forces are path function which means that they depend on the path travelled.

Central Forces.

Those forces whose magnitude is a function of the position vector $\vec{r}$ and direction is also along the position vector $\vec{r}$ are known as central forces.

$$\vec{F} = f(r)\,\vec{r}$$

Example

1. Electrostatic force $\vec{F} = \frac{Kq_1q_2}{r^2}$

2) Gravitational force $\vec{F} = \frac{Gm_1m_2}{r^2}\hat{r}$

3) Spring force $\vec{F} = -kx\hat{x}$

Equilibrium

If the forces acting on the object are conservative and it is in equilibrium then

$$F_{net} = 0 \Rightarrow \frac{-\partial U}{\partial r} = 0 \text{ or } \frac{\partial U}{\partial r} = 0$$

Equilibrium of an object or a system can be divided into three types

(i) Stable equilibrium :- An object is said to be in stable equilibrium, if on slight displacement from equilibrium position, it has the tendency to come back.

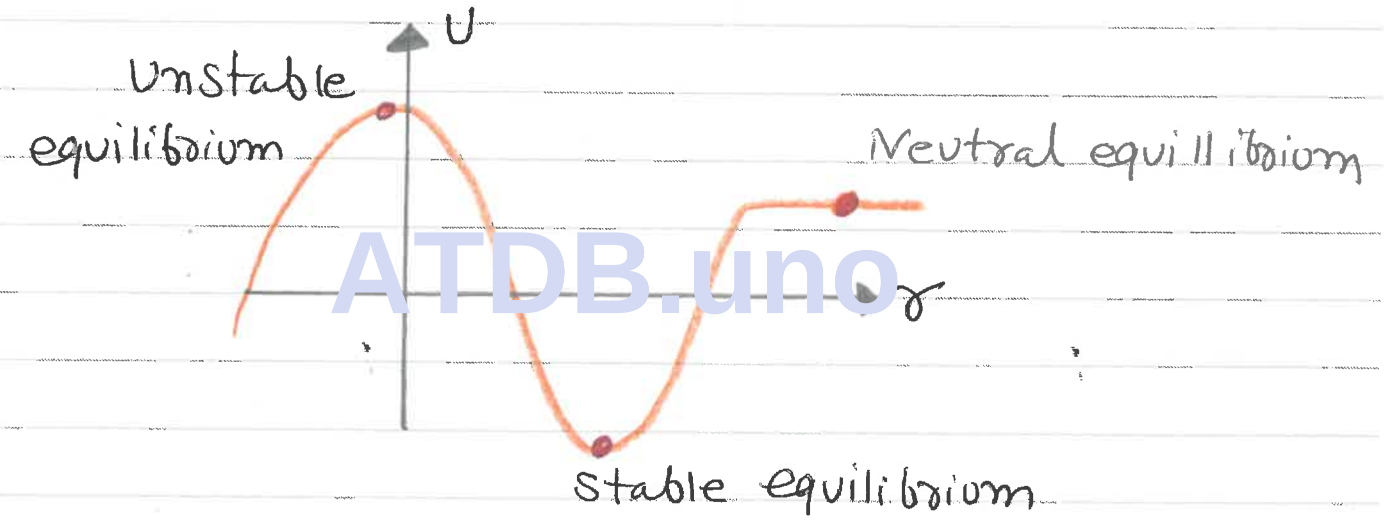
Here $\frac{\partial^2 U}{\partial r^2}$ = positive

(ii) Unstable equilibrium An object is said to be in unstable equilibrium, if on slight displacement from equilibrium position, it moves in the direction of displacement.

Here $\partial^2 U$ = negative

(iii) **Neutral equilibrium** : An object is said to be in neutral equilibrium if on displacement from its equilibrium position, it has neither the tendency to move in direction of displacement nor to come back to equilibrium position.

Here $\frac{d^2U}{dr^2} = 0$.



Work - Energy theorem

Work done by a force in displacing a body is equal to change in its kinetic energy.

$$W = \int F \cdot ds = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$K_f - K_i = \Delta KE$$

where K_i = initial kinetic energy
K_f = final kinetic energy

Regarding the work-energy theorem, it is worth noting that.

(i) if W_{net} is positive, then $K_f - K_i$ = positive ie $K_f > K_i$ or kinetic energy will increase and vice-versa

(ii) This theorem can be applied to non-inertial frames also. In a non-inertial frame it can be written as.

work done by all the forces (including the Pseudo force ~~is~~ is = change in kinetic energy in non-inertial frame.)-

Principle of conservation of Energy.

Energy can neither be created nor be destroyed it can only be transferred from one form to another form.

For conservative forces, the total mechanical energy (som of kinetic and potential energies) of any object remains constant.

Power:-

The rate at which work is done by a body or energy is transferred is called it power

$$\text{Power} = \text{Rate of doing work} = \frac{\text{work done}}{\text{Time taken}}$$

If under a constant force F a body is displaced through a distance S in time t then the power $P = \frac{W}{t} = \frac{F.S}{t}$

But $\frac{S}{t} = v$, uniform velocity with which body is displaced

$$\therefore \quad P = F \cdot v = Fv\cos\theta$$

where, θ is the Smaller angle between F and v

Power is a scalar quantity. Its SI unit is Watt and its dimensional formula is $[ML^2T^{-3}]$

Its other units are kilowatt and horse power

1 Kilo watt = 1000 watt
1 horse power = 746 watt
1 KWh = 3.6×10^6 J.

Other forms of Energy.

Heat Energy :— A body posses heat energy due to the disorderly motion of its molecules.

Heat energy is also related to the internal energy of the body.

Chemical Energy :- Chemical energy is stored in the chemical bonds of atoms and molecules. If the total energy of the reactant is more than the product of the reaction, the heat is released and the reaction is said to be an exothermic reaction. If the reverse is true then heat is absorbed and the reaction is endothermic.

Electrical Energy It is the energy which is associated with the flow of electric current or with charging or discharching of a body.

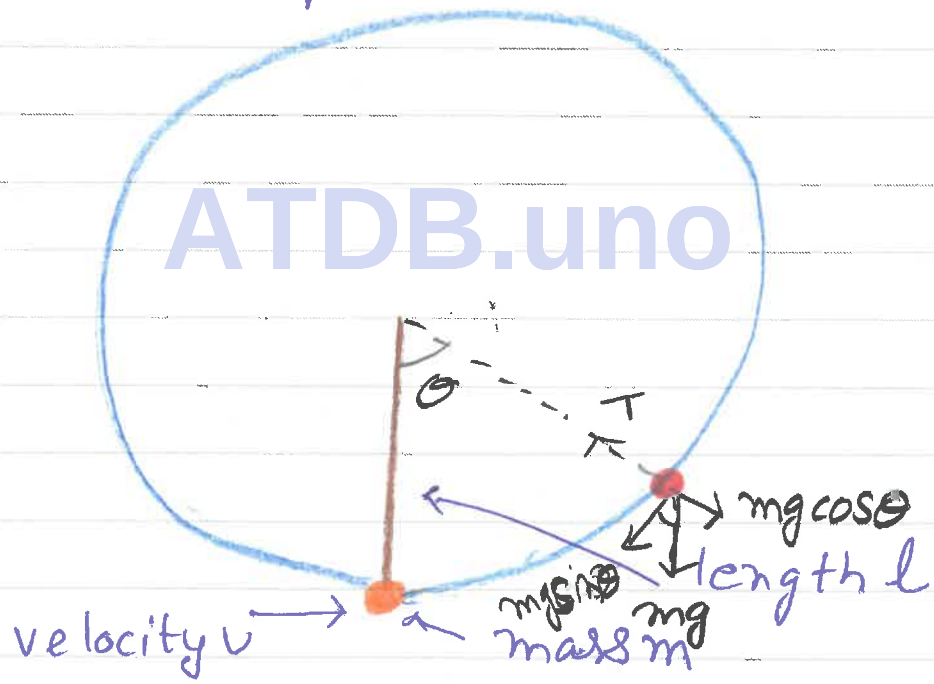
Nuclear Energy It is the binding energy of the nucleus of an atom. It is used in nuclear reactors, nuclear fission etc.

Mass-Energy Equivalance According to Einstein, the mass can be trasformed into energy and vice-versa. When Δm mass disappears, then produced Energy, $E = \Delta m c^2$ where c is the speed of light in vaccum.

Vertical circular motion :-

Circular motion in a vertical plane is known as vertical circular motion.
consider a small bob of mass m tied to an ideal string of length l constrained to move in a vertical plane.

Let the bob be given a velocity v at the bottom most point.



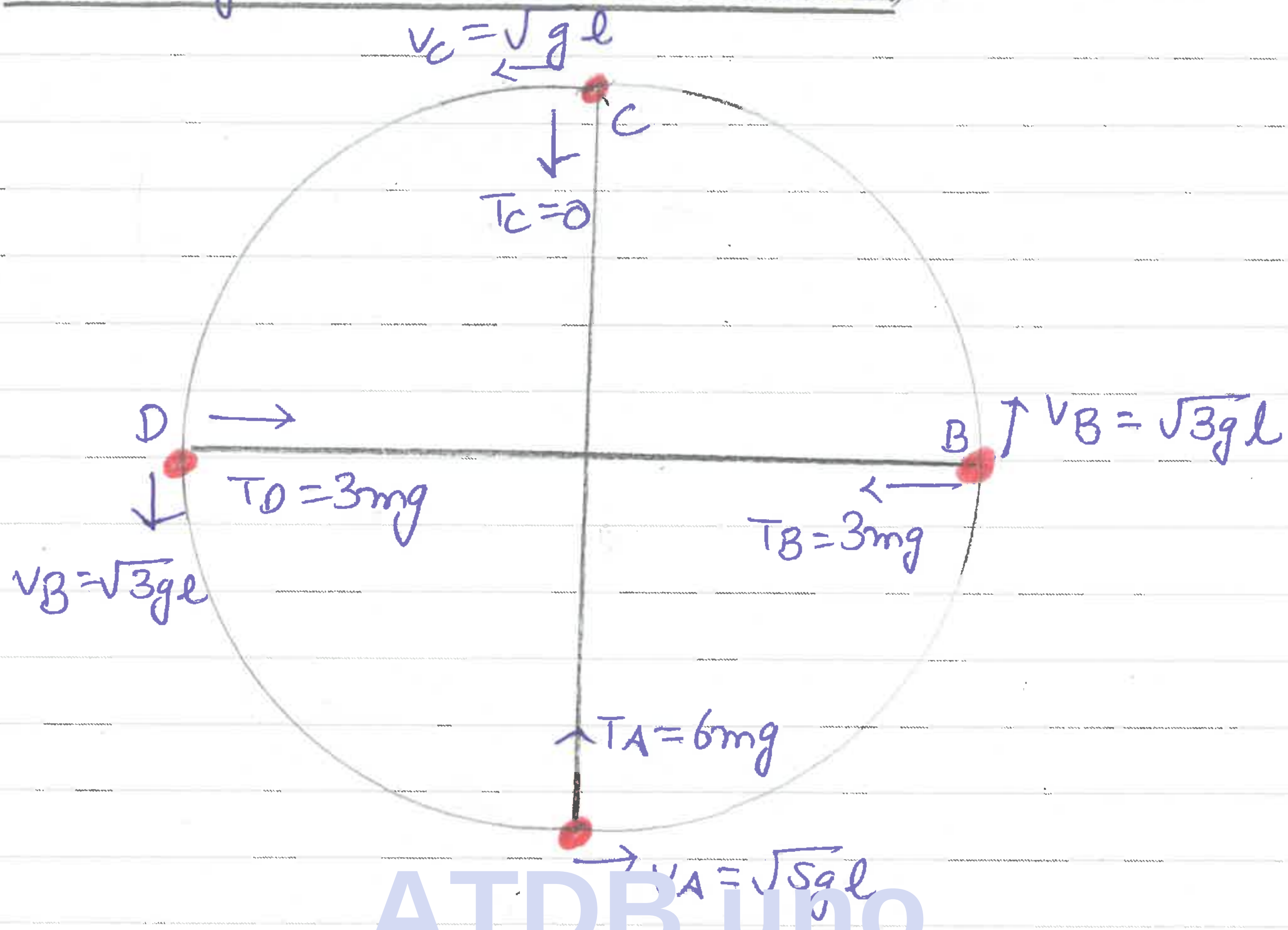
Lower quadrant

$$T = \frac{mv^2}{l} + mg\cos\theta$$

Upper quadrant

$$T = -mg\cos\theta + \frac{mv^2}{l}$$

In case of critical circular motion



Position	Velocity	Tension
A	$\sqrt{5gl}$	6mg
B	$\sqrt{3gl}$	3mg
C	$\sqrt{gl}$	0
D	$\sqrt{3gl}$	3mg.

Inequalities

Complete circle	$v_A \geq \sqrt{5gl}$
projectile	$\sqrt{2gl} < v_A < \sqrt{5gl}$
Oscillation	$v_A \leq \sqrt{2gl}$.