Chapter-4

:- Motion in a Plane :-

* **Physical Quantity :-** It is the property which can be measured and follow the law of physics.
  Eg = force, speed, Velocity, distance, displacement, temperature etc.

* **Vector :-** Vectors are those Physical Quantity which have magnitude as well as direction and they must follow the law of Vector addition.
  Eg = force, Velocity, displacement, acceleration, Electric field, magnetic field etc.

* **Scaler :-** Scaler are those Physical Quantity which have magnitude only and might have direction but they must not follow the law of vector addition.
  Eg :- distance, speed, work, time, power, Energy, temperature etc.

Note :- Electric current is only a physical Quantity, which have magnitude and direction both but it is a Scaler because it does not follow the law of vector addition.

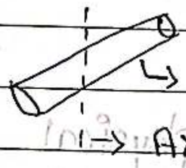
* Tensor (Not in Syllabus) :-

Tensor are those Physical Quantity, whose magnitude depends on direction.

Eg = Moment of intertia ; refractive index, relative refractive index, relative density etc.

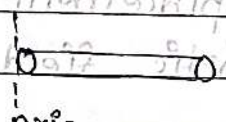
Eg = Moment of intertia

(a)



$$M O I = \frac{M l^2}{2} \quad ; \quad l = \text{length}$$

(b)



$$M.O.I = \frac{M l^2}{3}$$

(c)

$$M.O.I = 0$$

Representation of a Vector

1. Symbolic Representation :-

$\vec{A}$ = here, A is a Vector

$\vec{B}$ = here, B is a Vector

∴ $\hat{a}$ or $\vec{\alpha}$ here, $\hat{a}$ & $\vec{\alpha}$ are unit Vector.

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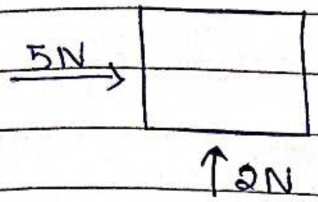
2. Digramatic representation :-



∴ (⟶), represented by arrow.
here, direction of vector is from tail to head.

Note :- Vector

↓ Magnitude ↓ Length of arrow will represent the magnitude of vector.

↓ Direction (Tail to Head) ⟶

Magnitude of vector

$\vec{F}$ = 20 N towards north
here,
20 N is magnitude & North is direction

Note :- $|\vec{F}|$ = Symbol of magnitude

So,
we can write as,

$$\boxed{|\vec{F}| = 20\ N}$$

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Simply F is also used to represent the magnitude of vector.

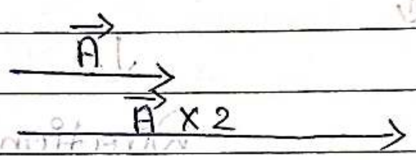
$$\boxed{F = 20\,N}$$

$\because$ F = This is representing the magnitude of $\vec{F}$

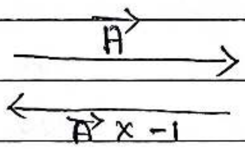
Note :-

Multiplication of a vector with Scaler.

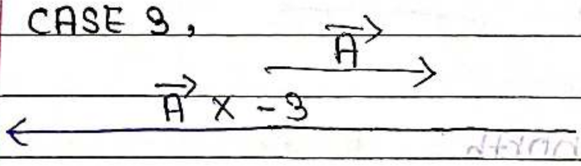
CASE 1,



CASE 2,



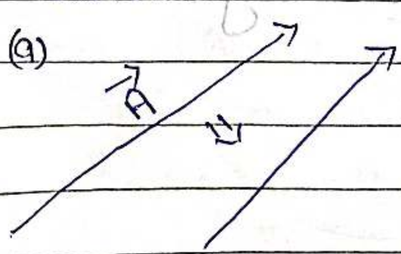
CASE 3,



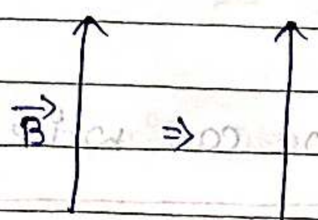
Note :-

A vector can be shifted parallel to its self.

Eg = (a)



(b)



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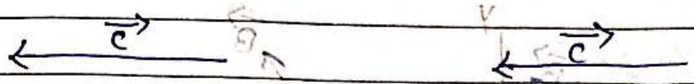
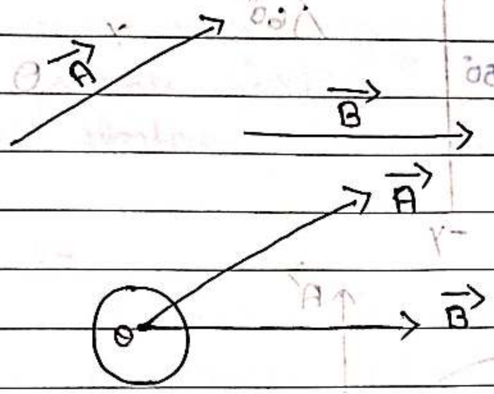
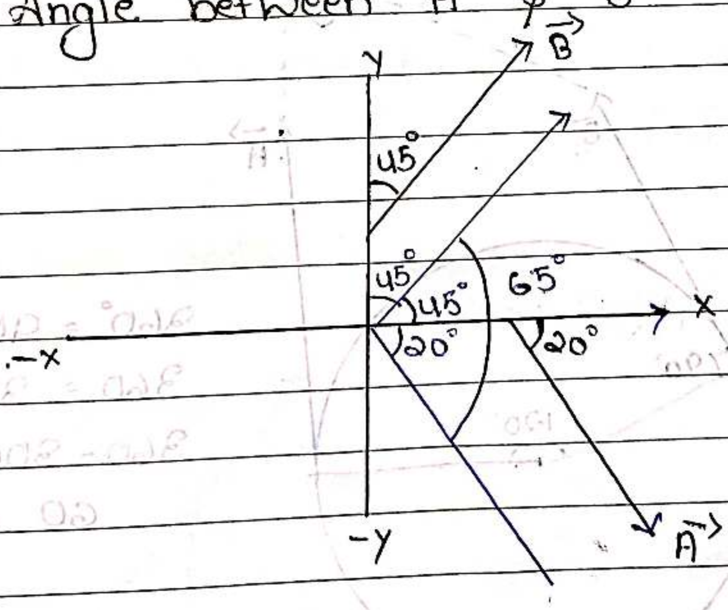
(c) 

Angle between two Vectors.



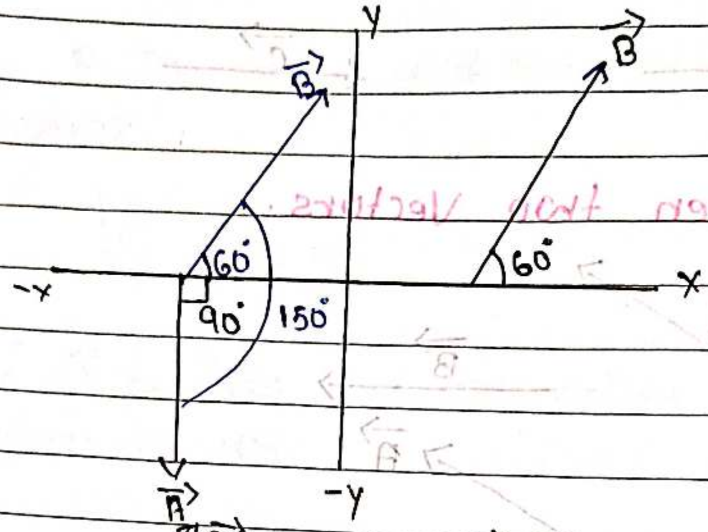
→ when we join tails of two vectors by shifting them, then there will be the formation of two angles, among them the smaller angle will be known as angle between two vectors.
here θ, is angle between $\vec{A}$ & $\vec{B}$.

Q. Find Angle between $\vec{A}$ & $\vec{B}$



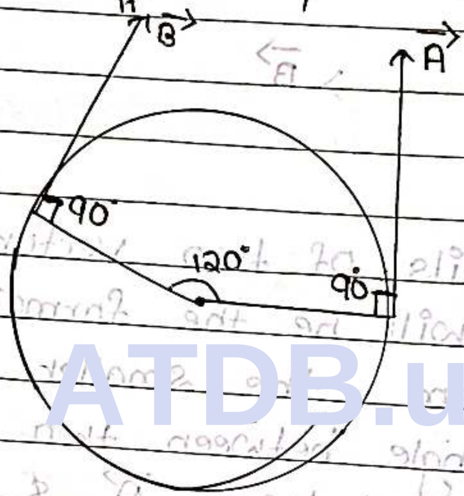
$\theta = 65^\circ$

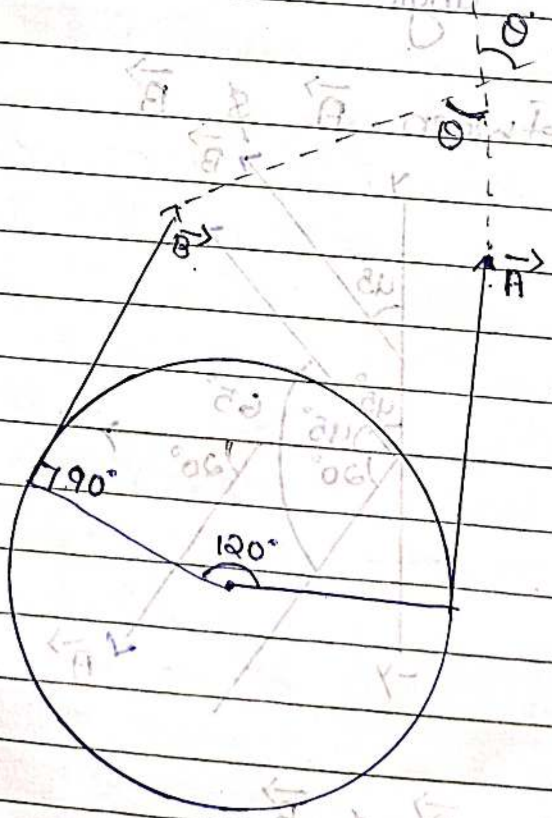
Q. Find angle b/w $\vec{A}$ & $\vec{B}$



$\theta = 60° + 90°$
$= 150°$

Q.





$360° = 90 + 90 + 120 + \theta$
$360 = 300 + \theta$
$360 - 300 = \theta$
$60 = \theta$

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* Types of vector

(i) Null Vector or zero Vector
(ii) Colinear Vector
(iii) Concurrent Vector
(iv) Co-planer Vector
(v) V.V.I Unit Vector

(i) Null Vector :-

A vector having zero magnitude is called null vector or zero vector.

Eg = $\vec{A} - \vec{A} = 0$ (X)

$\vec{A} - \vec{A} = \vec{0}$ (✓)

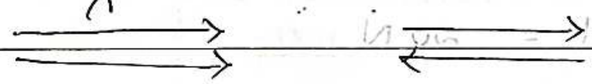
$\vec{A} \times \vec{A} = 0$ (X)

$\vec{A} \times \vec{A} = \vec{0}$ (✓)

(ii) collinear Vector :-

Vector which are along same line, parallel or antiparallel.

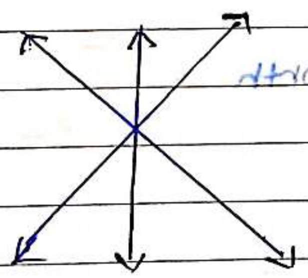
Eg :-



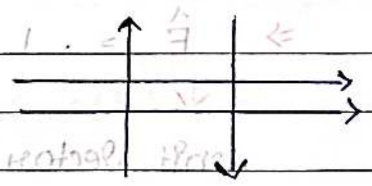
(iii) Con-current Vector :-

Vectors passing from one point.

Eg :-



concurrent



non-concurrent

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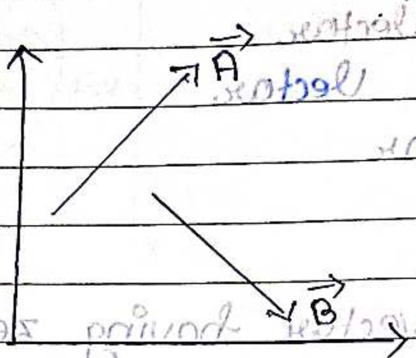
(iv) Co-planer Vector :-

Vectors which are in same plane are known as co-planer vector.

Eg :-



(v) Unit Vector :-

A vector having unit magnitude is called unit vector.

- It has no SI unit.
- Symbol $\vec{a}, \vec{b}, \vec{c}$ or $\hat{a}, \hat{b}, \hat{c}$.

Formation of unit Vector.

Eg :- $\vec{F}$ = 20 N towards north

$|\vec{F}|$ = 20 N

$\Rightarrow \frac{\vec{F}}{|\vec{F}|} = \frac{\cancel{20\,N}\text{ towards north}}{\cancel{20\,N}}$

$\Rightarrow \hat{F}$ = 1 towards north

unit vector creater

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* $\boxed{\dfrac{\vec{A}}{|\vec{A}|} = \hat{A}}$

Use of unit Vector.

→ It is used to convert a Scaler into vector.

Eg : $V = 5\ m/s$ → Scaler (speed)

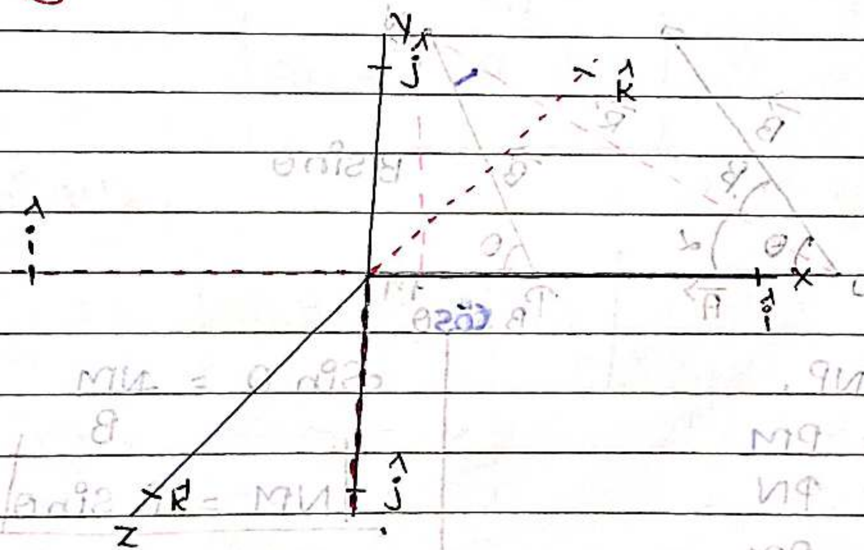
Let we have,

$\hat{V} = 1$ towards north

$V \times \hat{V} = 5\ m/s \times 1$ towards north

$\vec{V} = 5\ m/s$ towards north.

Orthogonal unit Vector (O.U.V)



Eg :- $\vec{F} = 20\hat{i}\ N$

→ It means force is 20 N towards x-axis.

Eg :- $\vec{V} = -5\hat{j}\ m/s$

→ It means Velocity is 5 m/s towards -y axis.

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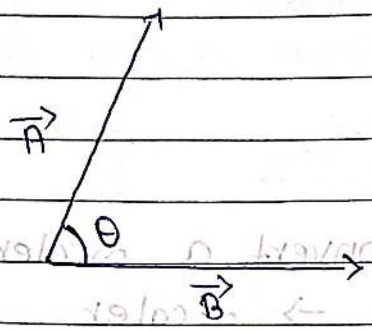
Addition of vectors.

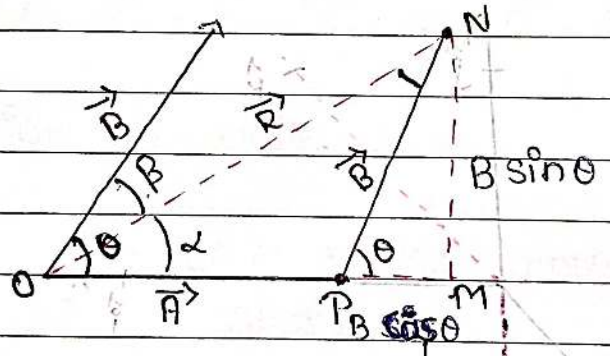


→ Two vectors can be added together only if they are co planer.

→ $\vec{R} = \vec{A} + \vec{B}$

↓

Resultant of $\vec{A}$ & $\vec{B}$



in Δ MNP,

$$\cos\theta = \frac{PM}{PN}$$

$$\cos\theta = \frac{PM}{B}$$

$$\boxed{PM = B\cos\theta}$$

$$\sin\theta = \frac{NM}{B}$$

$$\boxed{NM = B\sin\theta}$$

Again,

in same Δ MNP,

$$\sin\theta = \frac{NM}{}$$

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in Big Δ OMN:

$\because \quad ON^2 = NM^2 + OM^2$

$$R^2 = (B\sin\theta)^2 + (A + B\cos\theta)^2$$
$$R^2 = B^2\sin^2\theta + A^2 + B^2\cos^2\theta + 2AB\cos\theta$$
$$R^2 = B^2\sin^2\theta + B^2\cos^2\theta + A^2 + 2AB\cos\theta$$
$$R^2 = B^2(\sin^2\theta + \cos^2\theta) + A^2 + 2AB\cos\theta$$
$$R^2 = B^2 + A^2 + 2AB\cos\theta$$

$$\boxed{R = \sqrt{B^2 + A^2 + 2AB\cos\theta}}$$

* in Δ ONM,

$$\sin\alpha = \frac{NM}{ON} = \frac{B\sin\theta}{R}$$

$$\boxed{\sin\alpha = \frac{B\sin\theta}{R}}$$

Similarly,

$$\boxed{\sin\beta = \frac{A\sin\theta}{R}}$$

* $\tan\alpha = \dfrac{NM}{OM}$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$$

Similarly,

$$\boxed{\tan\beta = \frac{A\sin\theta}{B + A\cos\theta}}$$

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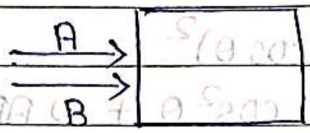
$$* \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

CASE I.



$\boxed{\theta = 0}$

$$\boxed{R = A + B}$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

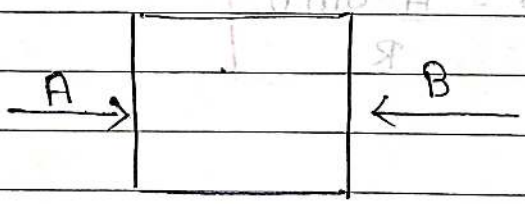
$$= \sqrt{A^2 + B^2 + 2AB \times 1}$$

$$= \sqrt{A^2 + B^2 + 2AB}$$

$$= \sqrt{(A+B)^2}$$

$$= A + B$$

CASE II.



$\boxed{\theta = 180}$

$$\boxed{R = A - B}$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos 180^\circ} \qquad \because \cos 180^\circ = -1$$

$$= \sqrt{A^2 + B^2 + 2AB \times (-1)}$$

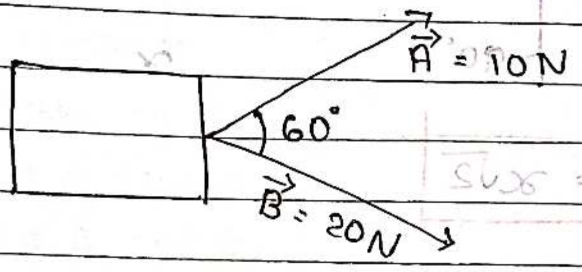
$= \sqrt{A^2 + B^2 - 2AB}$

$= \sqrt{(A-B)^2}$

$= R = A - B$

Maximum Resultant = A + B
Minimum Resultant = A − B

Q.



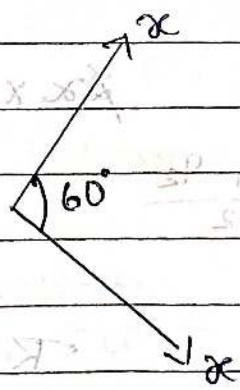
$\rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{10^2 + 20^2 + \cancel{2} \times 10 \times 20 \times \frac{1}{\cancel{2}}}$

$= \sqrt{100 + 400 + 200}$

$= \sqrt{700}$

Trick ① :-



$R = \sqrt{x^2 + x^2 + 2 \times x \times x \times \cos 60°}$

$= \sqrt{2x^2 + \cancel{2}x^2 \times \frac{1}{\cancel{2}}}$

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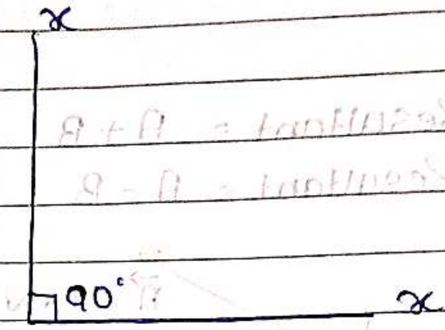
$= \sqrt{2x^2 + x^2}$

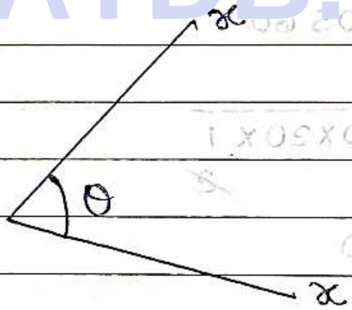
$= \sqrt{3x^2} \quad = x\sqrt{3}$

Trick 2.



$$\boxed{R = x\sqrt{2}}$$

Main trick

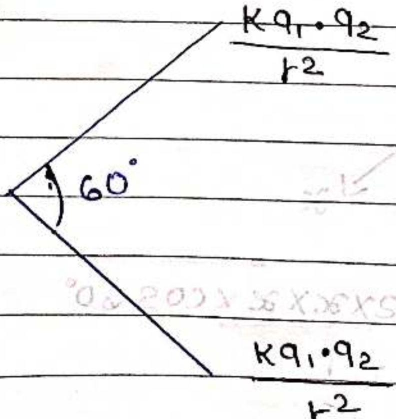


$R = 2x \cos\left(\frac{\theta}{2}\right) \qquad \because \theta = 60$

$2x \cos \frac{60}{2}$

$\cancel{2}x \times \frac{\sqrt{3}}{\cancel{2}} = x\sqrt{3}$

*



$R = \frac{Kq_1 \cdot q_2}{r^2} \sqrt{3}$

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Q. If Resultant of two unit vector is also a unit vector then find the angle between that two unit vectors.

⇒ $A = 1$, $B = 1$, $R = 1$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$1 = \sqrt{1 + 1 + 2\cos\theta}$$

$$1 = \sqrt{4\cos\theta}$$

$$1 = \sqrt{2 * 2\cos\theta}$$

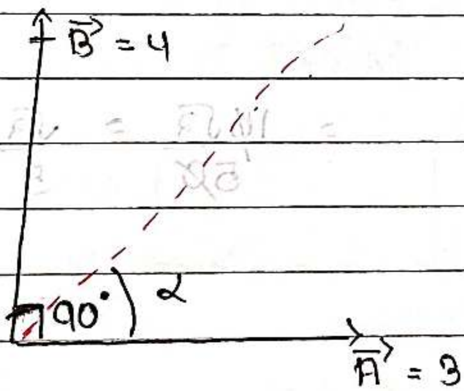
$$1^2 = 2 * 2\cos\theta$$

$$1 = 2 * 2\cos\theta$$

$$1 - 2 = 2\cos\theta$$

$$\frac{-1}{2} = \cos\theta$$

$$\theta = 120^\circ \quad \text{Ans}$$



find angle between $\vec{A}$ \$ resultant

⇒ $$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\tan\alpha = \frac{4\sin 90^\circ}{3 + 4\cos 90^\circ}$$

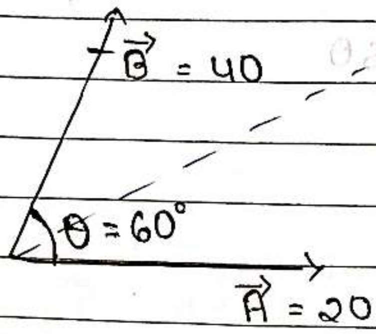
$\tan\alpha = \frac{4 \times 1}{3 + 4 \times 0}$

$= \frac{4}{3}$

$\tan\alpha = \frac{4}{3}$

$\boxed{\alpha = 53^\circ}$

Q.



Find angle between Resultant & $\vec{B}$.

→ $\tan\beta = \frac{A \sin\theta}{B + A\cos\theta}$

$= \frac{20 \sin 60}{40 + 20 \cos 60}$

$= \frac{\cancel{20}^{10} \times \frac{\sqrt{3}}{\cancel{2}}}{40 + \cancel{20}^{10} \times \frac{1}{\cancel{2}}} = \frac{1\cancel{0}\sqrt{3}}{5\cancel{0}} = \frac{\sqrt{3}}{5}$

$\tan\beta = \frac{\sqrt{3}}{5}$

$\beta = \tan^{-1}\left(\frac{\sqrt{3}}{5}\right)$

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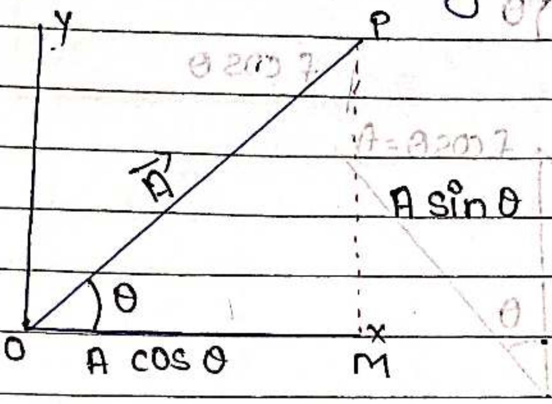
Component of Vector

→ It is the process of spiliting a vector.



in Δ OMP,

$$\sin\theta = \frac{PM}{OP}$$

$$\sin\theta = \frac{PM}{A}$$

$$\boxed{A\sin\theta = PM}$$

in Δ OMP,

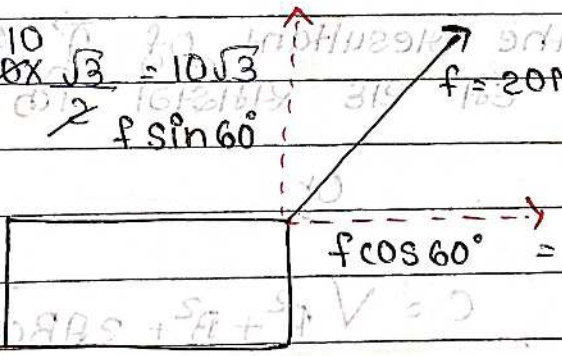
$$\cos\theta = \frac{OM}{OP}$$

$$\cos\theta = \frac{OM}{A}$$

$$\boxed{A\cos\theta = OM}$$

Eg :- $\cancel{20}^{10} \times \frac{\sqrt{3}}{\cancel{2}} = 10\sqrt{3}$

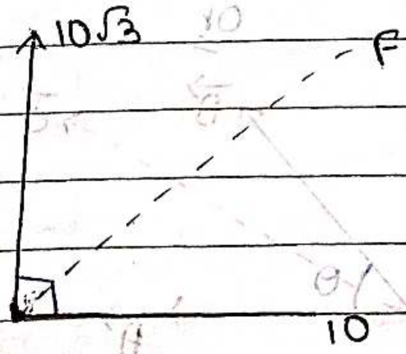
f sin 60°



$10\sqrt{3} = 10 \times 1.732$
$= 17.32$

$f\cos 60° = \cancel{20}^{10} \times \frac{1}{\cancel{2}} = 10\,N$

check,



$$C = \sqrt{(10)^2 + (10\sqrt{3})^2 + 2\times 10\times 10\sqrt{3}\cos 90°}$$

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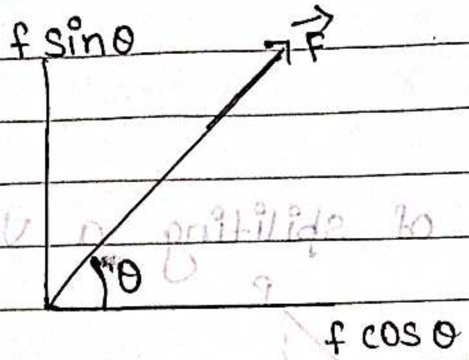
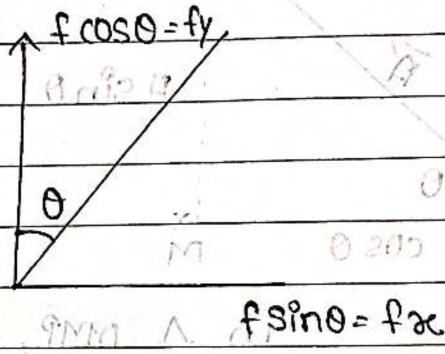




Note :- $\vec{f} = f\sin\theta\,\hat{i} + f\cos\theta\,\hat{j}$

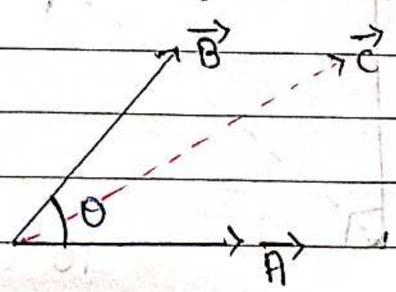
$\vec{f} = \vec{f_x} + \vec{f_y}$

Note :- अगर किसी Question में ऐसा लिखा हो कि $\vec{C}$ is the resultant of $\vec{A}$ & $\vec{B}$ तो इसका मतलब हम यह समझेंगे कि $\vec{C} = \vec{A} + \vec{B}$

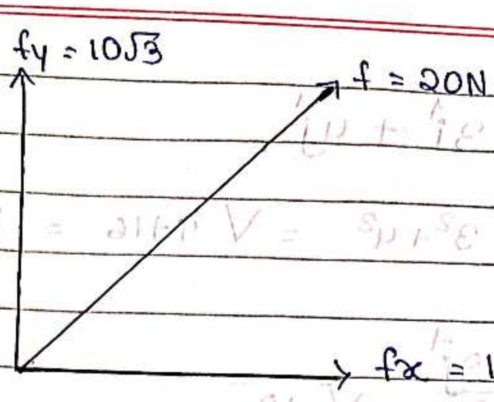
Or

$$C = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Or



Note :-



$\vec{f} = f_x\hat{i} + f_y\hat{j}$

$$\boxed{\vec{f} = 10\hat{i} + 10\sqrt{3}\hat{j}}$$

$f = \sqrt{10^2 + (10\sqrt{3})^2 + 2 \times 10 \times 10\sqrt{3}\cos 90^\circ}$ ↳ 0

$f = \sqrt{10^2 + (10\sqrt{3})^2}$

$f = \sqrt{f_x^2 + f_y^2}$

It means

* Magnitude of vector, given in the form of i, j.

$\vec{f} = 10\hat{i} + 10\sqrt{3}\hat{j}$

$f = \sqrt{10^2 + (10\sqrt{3})^2}$

$$\boxed{f = 20}$$

Eg :- $\vec{A} = 2\hat{i} + 3\hat{j}$

$A = \sqrt{2^2 + 3^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

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Eg :- $\vec{B} = 3\hat{i} + 4\hat{j}$

$B = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

* $\vec{A} = 2\hat{i} + 3\hat{j}$

  $A = \sqrt{2^2 + 3^2} = \sqrt{13}$

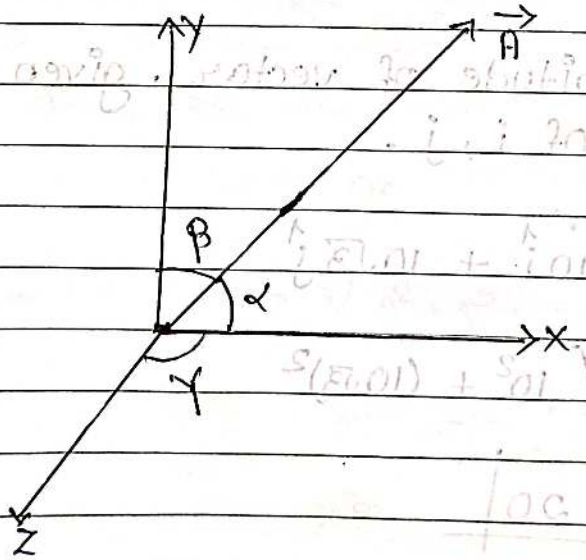
* $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$

  $= \sqrt{2^2+3^2+1^2} = \sqrt{4+9+1} = \sqrt{14}$

* $\vec{B} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

  $= \sqrt{2^2 + (-3)^2 + 4^2} = \sqrt{4+9+16} = \sqrt{29}$

Component of vector in 3-D



$A_x = A \cos\alpha$

$A_y = A \cos\beta$

$A_z = A \cos\gamma$

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$\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$A = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$

* Addition of vectors given in the form of $\hat{i}, \hat{j}, \hat{k}$.

→ $\vec{A} = 2\hat{i} + 3\hat{j} - 5\hat{k}$
$\vec{B} = \hat{i} - 2\hat{j} + 3\hat{k}$

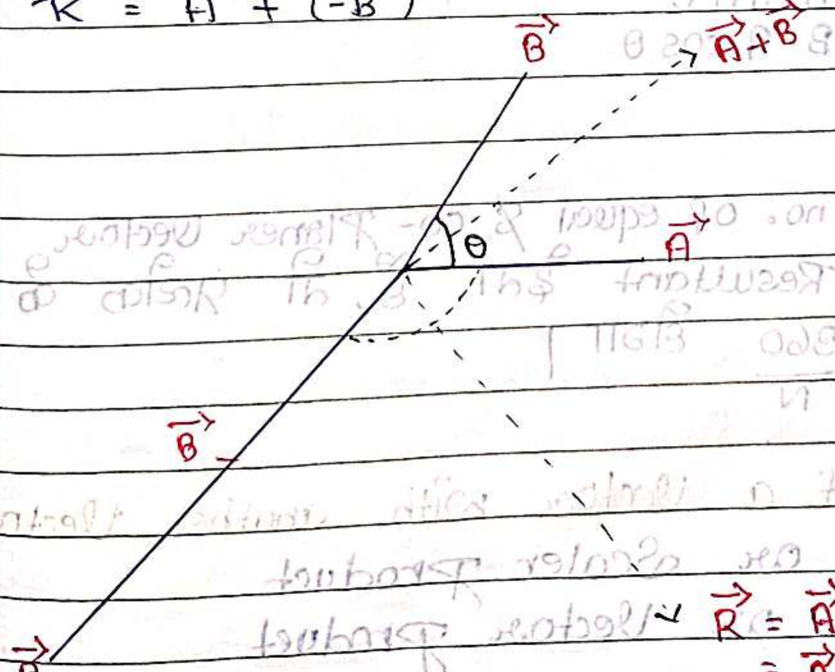
$\vec{R} = \vec{A} + \vec{B} = 3\hat{i} - \hat{j} - 2\hat{k}$

* $\vec{A} = 2\hat{i} + 3\hat{j}$
$\vec{B} = -\hat{i} + \hat{k}$

$\vec{A} + \vec{B} = \hat{i} + 3\hat{j} + \hat{k}$

* Subtraction of vector

$\vec{R} = \vec{A} - \vec{B}$
$\vec{R} = \vec{A} + (-\vec{B})$



$\vec{R} = \vec{A} + (-\vec{B})$
$= \vec{A} - \vec{B}$

Saathi

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$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$
$$R = \sqrt{A^2 + B^2 + 2AB\cos(180-\theta)}$$
$$R = \sqrt{A^2 + B^2 + 2AB\cos(90+90-\theta)}$$
$$R = \sqrt{A^2 + B^2 + 2AB \times -\sin(90-\theta)}$$
$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

Note :- $\vec{R} = \vec{A} - \vec{B}$

it means,

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$\therefore \boxed{\sin\alpha = \frac{B\sin\theta}{R}}$$

$$\therefore \boxed{\sin\beta = \frac{A\sin\theta}{R}}$$

$$\therefore \tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$

$$\therefore \tan\beta = \frac{A\sin\theta}{B - A\cos\theta}$$

Note :-

अगर N no. of equal & co-Planer Vector मिलकर 0 Resultant देता है, तो प्रत्येक के बीच का Angle $\frac{360}{N}$ होगा।

* : Multiplication of a Vector with another Vector.
→ Dot Product or Scaler Product
→ cross Product or Vector Product

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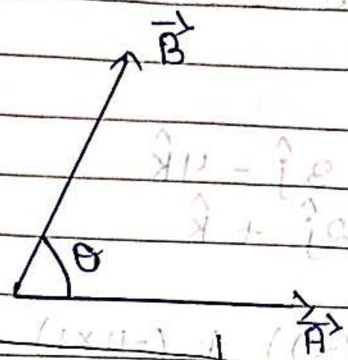
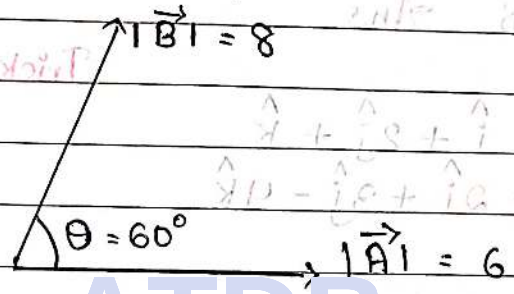
Saathi

- Dot product :-



$$\boxed{\vec{A} \cdot \vec{B} = AB\cos\theta}$$

Eg :-



$\vec{A} \cdot \vec{B} = AB\cos\theta$

$= \cancel{8}^{4} \times 6 \times \frac{1}{\cancel{2}}$

$= 24$

Note :- If, $\theta = 0$

$\vec{A} \cdot \vec{B} = AB\cos 0° = AB$ (maximum)

If, $\theta = 90°$

$\vec{A} \cdot \vec{B} = AB\cos 90° = 0$

$\hookrightarrow 0$

If, $\theta = 180°$

$\vec{A} \cdot \vec{B} = AB\underbrace{\cos 180°}_{\hookrightarrow -1}$

$= -AB$

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* Dot Product of two Vectors Given in the form of $\hat{i}, \hat{j}, \hat{k}$:-

→ Eg :- $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$
$\vec{B} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{A} \cdot \vec{B} = (2\times1) + (3\times(-2)) + (-4\times1)$
$= 2 - 6 - 4$
$= -8$ Ans

Trick :- if $\vec{A} = n\vec{B}$
then, $\theta = 0°$ Always
if, $\vec{A} = -n\vec{B}$
then, $\theta = 180°$

→ Eg :- $\vec{A} = \hat{i} + 2\hat{j} + \hat{k}$
$\vec{B} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$\vec{A} \cdot \vec{B} = 2 + 6 - 4$
$= 4$ Ans

- Cross Product (Vector Product)

→ $\vec{A} \times \vec{B}$ = Vector

Magnitude ↓ AB sinθ

Direction ↓ Right hand thumb rule ↓

A/c to this rule if we Put our finger of right hands on the Vector which is written first & curl our fingers towards another Vector then, thumb will give the direction

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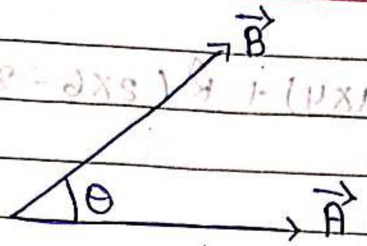
Eg :-



$$\boxed{|\vec{A} \times \vec{B}| = AB\sin\theta}$$

Direction of $\vec{A} \times \vec{B}$ = $\odot$ (outward)

Direction of $\vec{B} \times \vec{A}$ = $\otimes$ (inward)

Note :- जब दो vector का cross product किया जाता है, तो जो उसका result होता है वह हमेशा दोनों vector पर Perpendicular होता है।

or

साथ ही साथ उस पूरे Plane पर Perpendicular होता है जिसमें दोनों vector मौजूद है।

cross Product of two vectors given in the form of $\hat{i}$, $\hat{j}$, $\hat{k}$.

Determinant :-

$$\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$
$$\vec{B} = 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \end{vmatrix}$$

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$= +\hat{i}(3\times8-6\times4)-\hat{j}(2\times8-4\times4)+\hat{k}(2\times6-3\times4)$

$= +\hat{i}\times0-\hat{j}\times0+\hat{k}\times0$

$= 0$

* Dot & cross Product

Dot :- $\hat{i}\cdot\hat{j}=0$ $(\because \theta = 90^\circ)$

$-\hat{i}\cdot\hat{k}=0$

$\hat{k}\cdot\hat{j}=0$

$\hat{i}\cdot\hat{i}=1\times1\times\cos0^\circ=1\times1\times1=1$

$\hat{j}\cdot\hat{j}=1$

$\hat{k}\cdot\hat{k}=1$

cross :- $\hat{i}\times\hat{i}=1\times1\times\sin0^\circ=0$

$\hat{j}\times\hat{j}=0$

$\hat{k}\times\hat{k}=0$

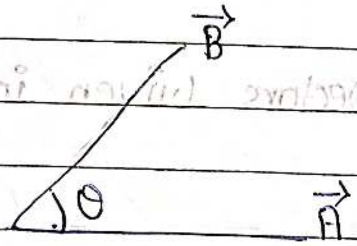
$\hat{M}\times\hat{M}=0$

$\hat{S}\times\hat{S}=0$

$\hat{i}\times\hat{j}=1\times1\times\sin90^\circ=1$

$-\hat{i}\times\hat{j}=1\times1\times\sin90^\circ=1$

V.V.I

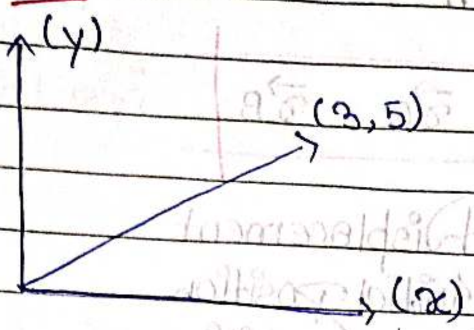


$|\vec{A}\times\vec{B}| = |\vec{B}\times\vec{A}|$
(मान) (मान)

$\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$

but $\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$ (✓) , $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$ (✓)

Position Vector



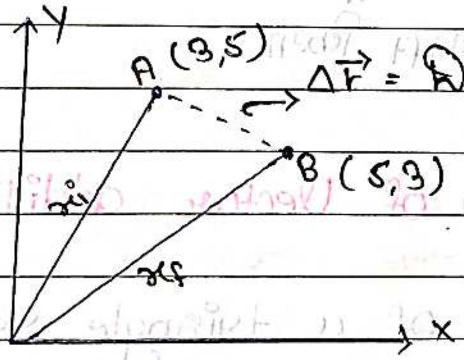
$\vec{r} = 3\hat{i} + 5\hat{j}$

→ Complete in figure formation of location.

Displacement :-

change in Position.

Displacement = final position - Initial position.



$\Delta\vec{r}$ = Displacement Vector.

Q. Find displacement ?-

→ $\vec{r}_i = 3\hat{i} + 5\hat{j}$, $\vec{r}_f = 5\hat{i} + 3\hat{j}$

$\boxed{\Delta\vec{r} = \vec{r}_f - \vec{r}_i}$

$\Delta\vec{r} = (5\hat{i} + 3\hat{j}) - (3\hat{i} + 5\hat{j})$

$= 5\hat{i} + 3\hat{j} - 3\hat{i} - 5\hat{j}$

$\vec{r}_{BA} = \Delta\vec{r} = 2\hat{i} - 2\hat{j}$

2nd Method

$\vec{r}_{BA} = \vec{r}_B - \vec{r}_A$

⇓

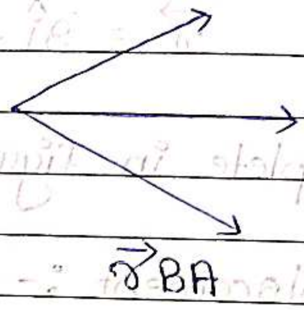
Position of $\vec{B}$

w.r.t. $\vec{A}$

* $\vec{r}_{BA} = \vec{r}_B - \vec{r}_A$

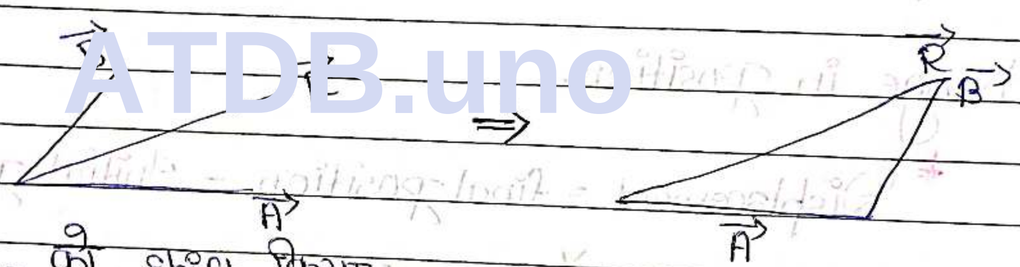
$$\boxed{\vec{r}_{BA} + \vec{r}_A = \vec{r}_B}$$

∵ $\vec{r}_{BA}$ = Displacement
$\vec{r}_A$ = Initial Position
$\vec{r}_B$ = Final Position



V.V.I

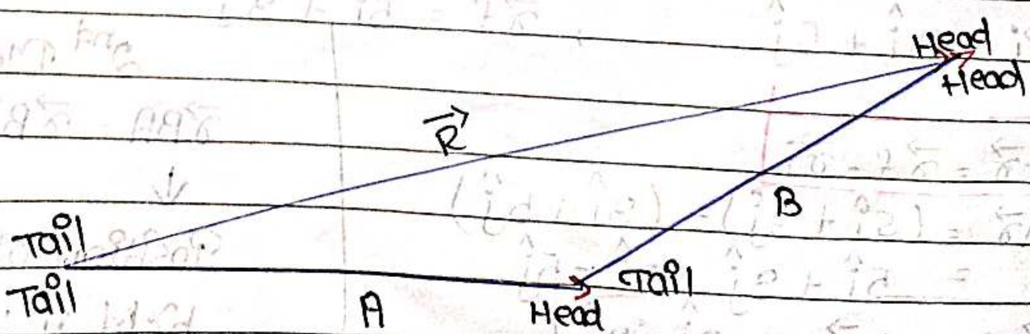
Baat अलग से :-



B को shift किया

✲ Triangle law of Vector addition :-

→ If two sides of a triangle represent the Vectors in particular order, then the 3rd remaining sides will represent the resultant that two Vectors.



Saathi

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Velocity

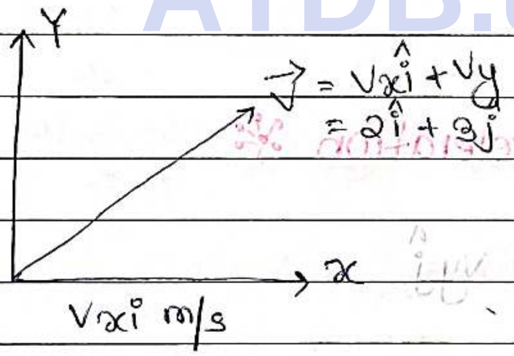
$\vec{r} = r_x\hat{i} + r_y\hat{i}$
↓
Position Vector

Now, Differentiating w.r.t time,

$$\frac{d\vec{r}}{dt} = \frac{dr_x}{dt}\hat{i} + \frac{dr_y}{dt}\hat{j}$$

$$\boxed{\vec{v} = V_x\hat{i} + V_y\hat{j}}$$

Eg:-



→ It means Velocity is 2 m/s towards x-axis & 3 m/s towards y-axis.

$$\boxed{|\vec{v}| = \sqrt{V_x^2 + V_y^2}}$$

$\Rightarrow \quad v = \sqrt{2^2 + 3^2}$

Note:- $\tan\theta$ = slope

$\tan\theta = \frac{V_y}{V_x}$

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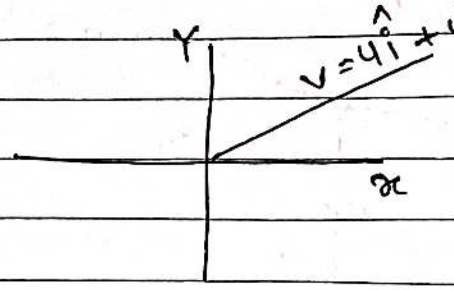
Q. Find angle between Velocity $\vec{v}$ & x-axis ?



Sol^n :- $\tan\theta = \frac{v_y}{v_x}$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{\cancel{4}\sqrt{3}}{\cancel{4}}\right)$$

$$\theta = \tan^{-1}\sqrt{3}$$

$$\theta = 60^\circ$$

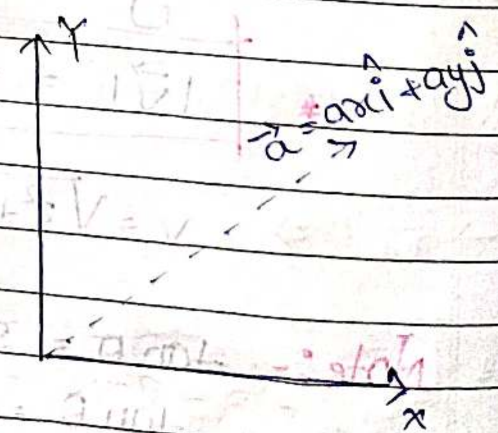
Acceleration

$\because$ $\vec{v} = v_x\hat{i} + v_y\hat{j}$

Now, differentiating w.r.t time

$$\frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$\boxed{\vec{a} = a_x\hat{i} + a_y\hat{j}}$$



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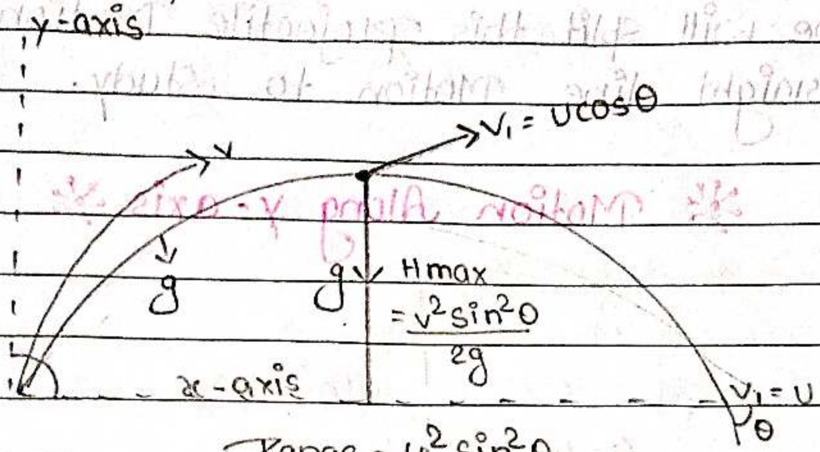
Projectile Motion

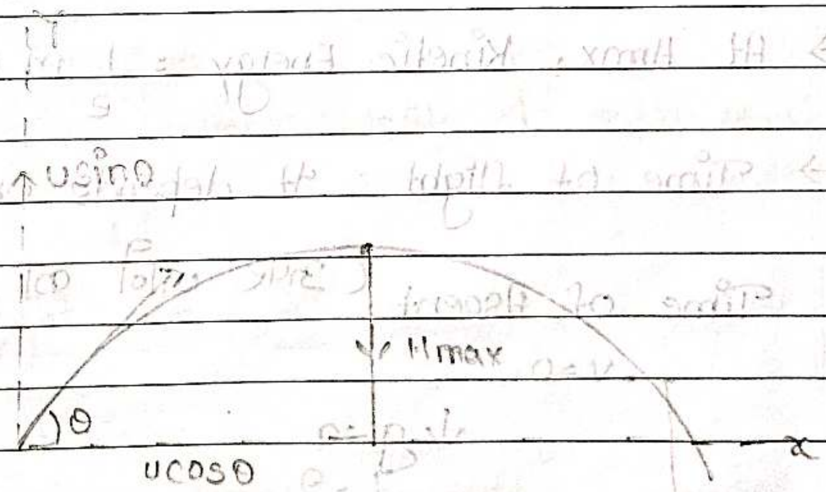


$$\text{Range} = \frac{u^2 \sin 2\theta}{g}$$

→ It is a two dimensional accⁿ motion in which particles moves on parabolic path.

$t_f = \frac{2u\sin\theta}{g}$
↓
time of fight

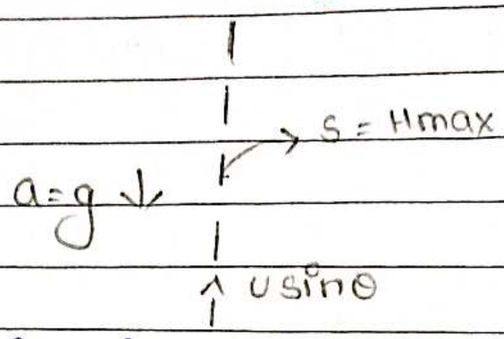
Derivation :-



- $u\cos\theta$ → Responsible for Motion along x-axis.
- $u\sin\theta$ → Responsible for Motion along y-axis.
- Hmax → It depends on only $u\sin\theta$

→ We will split this Projectile Motion in two Straight line Motion to Study.

Motion Along y-axis



$$v^2 = u^2 - 2as$$

$$0 = (u\sin\theta)^2 - 2 \times g \times H_{max}$$

$$2g H_{max} = u^2 \sin^2\theta$$

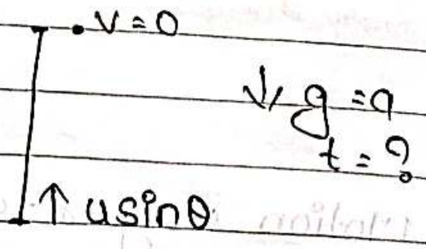
$$\boxed{H_{max} = \frac{u^2 \sin^2\theta}{2g}}$$

→ At Hmax, Velocity = $u\cos\theta$ along x-axis or range.

→ At Hmax, Kinetic Energy $= \frac{1}{2} m (u\cos)^2$

→ Time of flight = 9t depends on $u\sin\theta$.

- Time of Ascent (ऊपर जाने का time ta) :-



$\because$ $v = u - at$

$$0 = u\sin\theta - g t_a$$

$$g t_a = u\sin\theta$$

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- Time of ascent = Time of descent
  (ta) (td)

→ ∴ total time of flight $= t_a + t_d$

$$= \frac{u\sin\theta}{g} + \frac{u\sin\theta}{g}$$

→ $$\boxed{t_f = \frac{2u\sin\theta}{g}}$$

* Distance along x- axis = Range = Speed × Time

$$= u\cos\theta \times t_f$$

$$= u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$R = \frac{2\sin\theta \times \cos\theta \times u^2}{g}$$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

Q. If a ball is projected at an angle 30° with Surface of earth with Velocity 80 m/s than Calculate. (m = 1 Kg)

i) Hmax
ii) Time of flight
iii) Range
iv) Velocity at Hmax
v) Kinetic energy at Hmax
vi) Velocity of landing

i) $H_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{80^2 \times \sin^2 30}{2g} = \cancel{6400} \times \frac{1}{4}$

80 ~~1600~~ ~~3200~~

$= 80$

iii) $t_f = \frac{2u\sin\theta}{g} = \frac{\cancel{2}\times 8\cancel{0}\times\frac{1}{\cancel{2}}}{1\cancel{0}} = 8$ sec

iii) $R = \frac{u^2\sin2\theta}{g} = \frac{\overset{320}{\cancel{6400}}\times\frac{\sqrt{3}}{\cancel{2}}}{1\cancel{0}} = 320\sqrt{3}$

iv) Velocity of $H_{max} = u\cos\theta$

$= \overset{40}{\cancel{80}}\times\frac{\sqrt{3}}{\cancel{2}} = 40\sqrt{3}$

v) K.E of $H_{max} = \frac{1}{2}\times m\,(u\cos\theta)^2$

$= \frac{1}{2}\times 1\left(\overset{40}{\cancel{80}}\times\frac{\sqrt{3}}{\cancel{2}}\right)^2$

$= \frac{1}{2}(40\sqrt{3})^2$

$= \frac{1}{\cancel{2}}\times\overset{2400}{\cancel{4800}}$

$= 2400$ J

vi) Velocity of landing $= t_a = t_d = 80$ m/s

Q. Find the angle for which range & max^m height are equal.

⇒ Range = H_{max}

$\frac{\cancel{u^2}\sin2\theta}{\cancel{g}} = \frac{\cancel{u^2}\sin^2\theta}{2\cancel{g}}$

Saathi

$\Rightarrow 2\cancel{\sin\theta}\cdot\cos\theta = \frac{\cancel{\sin\theta}\times\sin\theta}{2}$

$\Rightarrow 4\cos\theta = \sin\theta$

$\Rightarrow 4 = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow 4 = \tan\theta$

$\Rightarrow \theta = \tan^{-1}4$

$\Rightarrow \theta = 76^\circ$

Q. Find the angle for which range of projtile will be max^m.

$\Rightarrow R = \frac{u^2\sin 2\theta}{g}$

$\because$ Value of sin will be maximum at $\sin 90^\circ$

$\because$ In $\sin 2\theta$; $\theta = 45^\circ$

$\rightarrow$ Range will be maximum at $\theta = 45^\circ$

Q. Find the angle for which H_{max} will be max^m.

$\Rightarrow \theta = 90^\circ$; H_{max} will be max^m.

Q. If a ball is projected at an angle 60° with Velocity 80 m/s then find H_{max} & range.

$\Rightarrow H_{max} = \frac{u^2\sin^2\theta}{2g}$

$\because u = 80$

$\theta = 60^\circ$

$= \frac{\cancel{6400}\times\frac{3}{\cancel{4}2}}{\cancel{2}\times 1\cancel{0}}$ (80, 160, 320)

$= 80\times 3 = 2400$

$R = \frac{u^2\sin 2\theta}{g}$

$= \frac{6400\times\sin 2\times 60^\circ}{10} = \frac{\cancel{6400}\times\frac{\sqrt{3}}{\cancel{2}}}{}$ (320)

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Note :-

(i) अगर किसी object को θ angle या 90° - θ angle पर फेंका जाए तो Range बराबर होगा मगर, Hmax अलग - अलग होगा।

Or,

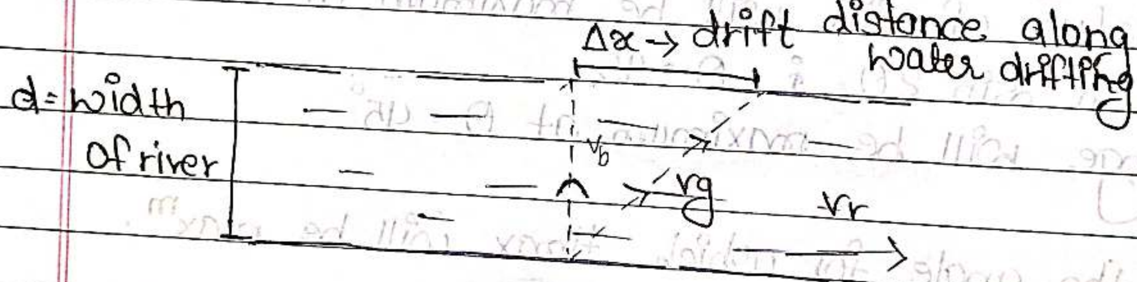
इस situation में Range बराबर $R = 4\sqrt{H_1 H_2}$

(ii) जब Range max^m होता है ($\theta = 45°$) तो

$H_{max} = \frac{R_{max}}{4}$ or; $R_{max} = 4 \times H_{max} \times \cot\theta$

भी होता है।

River Boat Problem



V_r = Velocity of river.

V_b = Velocity of boat.

This is the situation of minimum possible time of crossing the river.

$t_c = \frac{\text{distance}}{\text{speed}} = \frac{d}{V_b}$

↓

Time of crossing

$\therefore \boxed{t_c = d}$

Now,

drift distance $= \Delta x =$ Speed $\times$ time

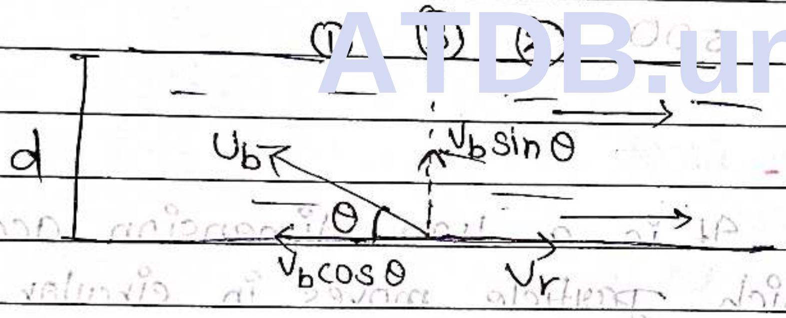
$= V_r \times t_c$

$$\boxed{\Delta x = V_r \times \frac{d}{V_b}}$$

$\because V_g =$ Velocity of boat w.r.t ground.

CASE II

→ To reach just opposite side of river.



$V_b \sin\theta =$ responsible for crossing the river.

$t_c = \frac{\text{dist}}{\text{speed}}$

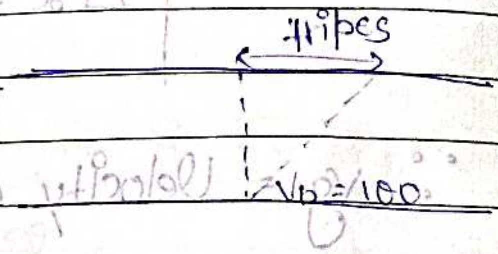
$$\boxed{t_c = \frac{d}{V_b \sin\theta}}$$

① $V_b \cos\theta > V_r$

② $V_r > V_b \cos\theta$

③ $V_b \cos\theta = V_r$

Q. If a boat can moves with a maximum speed of 100 m/s then calculate minimum possible time to cross the river of width 5 km and also find the tripe distance if Velocity of river is 10 m/s.



$\Rightarrow$ $t_c = \frac{d}{V_b}$

$= \frac{5000}{100}$

$= 50\ sec$

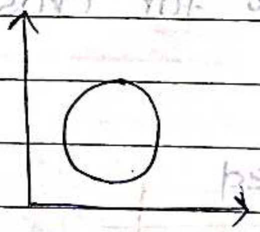
Distance = speed x time

$= 10 \times 50$

$= 500$

Circular motion :-

It is a two dimension accelerated motion in which particle moves in circular motion.



* Types of circular motion

(i) Uniform circular motion

(ii) non-uniform circular motion

(i) Uniform circular motion :-

In this case speed of object constant but direction always change so it Velocity

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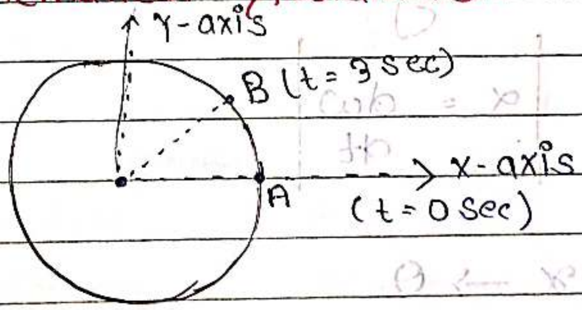
(ii) non- uniform circular motion :- In this case speed & direction both change with time.

* Velocity
→ Speed (change)
→ Direction (change)

(i) * Velocity → change accl^n is present
→ Speed (constant)
→ Direction (change)

Circular Kinematical Parameters



(1) Angular position (θ) :-

at $t = 0$, $\theta = 0°$

at $t = 3$ sec , $\theta = 60°$

(2) Angular Displacement :-

$\Delta\theta = \theta_f - \theta_i$

$= 60 - 0$

$= 60°$

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(3) Average Angular Velocity ($\langle \omega \rangle$)

$$\boxed{\langle \omega \rangle = \frac{\theta_f - \theta_i}{t_f - t_i}}$$

(4) Instantaneous Angular Velocity (ω)

$$\boxed{\omega = \frac{d\theta}{dt}}$$

(5) Average Angular acceleration ($\langle \alpha \rangle$)

$$\boxed{\langle \alpha \rangle = \frac{\omega_f - \omega_i}{t_f - t_i}}$$

(6) Instantaneous Angular acceleration

$$\boxed{\alpha = \frac{d\omega}{dt}}$$

Note :- $x \rightarrow \theta$
$v \rightarrow \omega$
$a \rightarrow \alpha$

Time Period :- It is the time taken in one circulation.

$$T = \frac{360}{\omega} \Rightarrow \boxed{T = \frac{2\pi}{\omega}}$$

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Saathi

Frequency :-

No. of circulation in 1 sec.

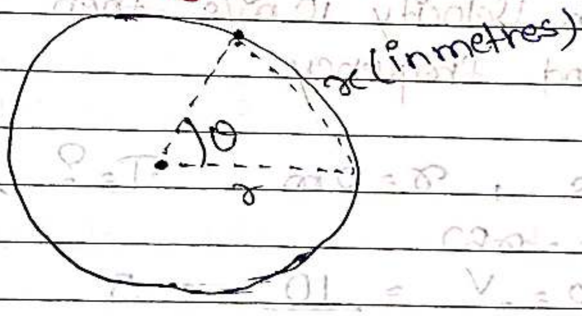
$$\nu = \frac{1}{\text{Time Period}}$$

$$\nu = \frac{1}{\frac{2\pi}{\omega}}$$

$$\boxed{\nu = \frac{\omega}{2\pi}}$$

V.V.I

Relation b/w Angular & linear quantity.



r = radius
x = linear distance
θ = angular distance

$\because$ Angle = $\frac{\text{Arc}}{\text{radius}}$

$\therefore$ Arc = Angle × radius

$$x = \theta \times r \qquad (r = \text{constant})$$

differentiating both side w.r.t time

$$\frac{dx}{dt} = \frac{d\theta}{dt} \times r$$

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$$\boxed{v = r \times \omega}$$

∵ v = linear Velocity (linear)
r = radius
ω = angular Velocity in (rad/sec)

* We have $v = r\omega$
differentiating both side w.r.t time

$$\frac{dv}{dt} = r \times \frac{d\omega}{dt}$$

$$\boxed{a = r \times \alpha}$$

∵ a = linear acceleration
α = angular acceleration

Q. If an object is moving in circular path of radius 2m with Velocity 10 m/s then Calculate its time period and frequency.

→ $v = 10$ m/s , $r = 2$ m, $T = ?$, $F = ?$

∵ $v = r\omega$

$$\omega = \frac{v}{r} = \frac{10}{2} = 5$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5}$$

$$F = \frac{\omega}{2\pi} = \frac{5}{2\pi}$$

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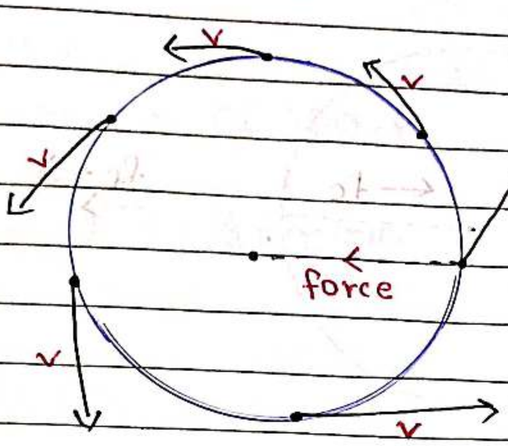
※ Centripetal force & centripetal accl^n



Velocity
- Speed
- Direction.

→ force acting towards centre is called centripetal force.

$$\boxed{f_c = \frac{mv^2}{r}}$$

or,

$$\boxed{f_c = m\omega^2 r}$$

→ Acceleration towards centre is called Centripetal ~~fo~~ accl^n.

$$\boxed{a_c = \frac{v^2}{r}}$$

or,

$$\boxed{a_c = \omega^2 r}$$

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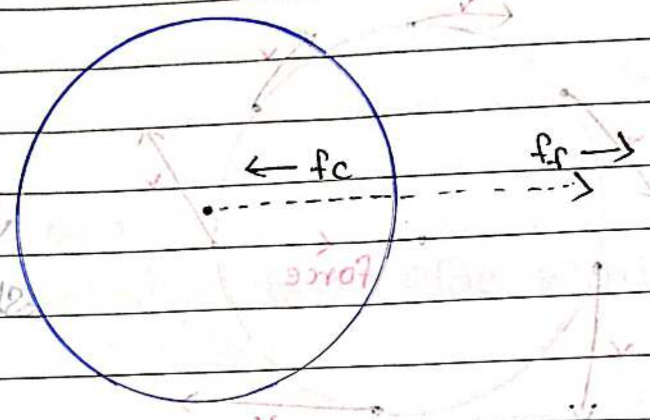
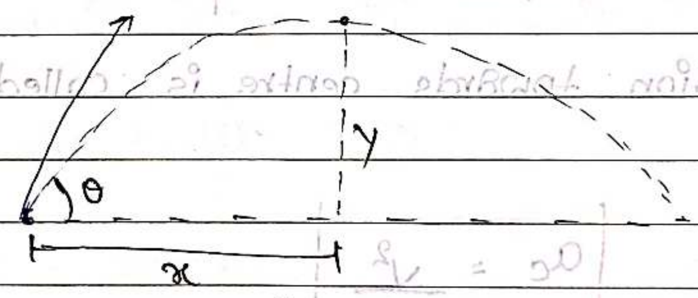
Centrifugal force:



Magnitude of centrifugal force $= \frac{mv^2}{r} =$ Magnitude of centrifugal force.

→ Both have opposite direction.

Equation of Trajactory



$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Q. If a boat can move with maximum speed of 360 km/h and speed of river is 5 m/s then calculate minimum time of crossing and also calculated drift distance, in this case. (width = 1 km)

$V_b = 360\ km/h = \frac{\cancel{360}^{20} \times 5}{\cancel{18}} = 100\ m/s$

$V_r = 5\ m/s$

$d = 100\ Km = 100000\ m$

(a) $t_c = \frac{d}{V_b} = \frac{10000\cancel{0}}{10\cancel{0}} = 1000\ sec$

(b) $\Delta x = V_r \times \frac{d}{V_b}$

$= V_r \times t_c$

$= 5 \times 1000$

$= 5000\ m$

$= 5\ km$

Q. If a boat reaches at just opposite side of the river at angle $\theta = 30°$ with surface and velocity of river is 5 m/s then calculated velocity of boat and also calculated time of crossing in this case. (width = 100 km)

⇒ Given,

$\theta = 30°$

$V_r = 5\ m/s$

$d = 100\ Km = 100000\ m$

(a) $V_b \cos\theta = V_r$

$V_b \cos 30° = 5$

$V_b \times \frac{\sqrt{3}}{2} = 5$

$V_b = 10\ m/s$

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(b) $t_c = \dfrac{d}{v_b \sin\theta}$

$= \dfrac{100000}{\frac{10}{\sqrt{3}} \times \sin 30^\circ} = \dfrac{100000}{\frac{10}{\sqrt{3}} \times \frac{1}{2}}$

$= \dfrac{10000\cancel{0}}{1\cancel{0}} \times 2\sqrt{3}$

$= 10000 \times 2\sqrt{3}$

$= 20000\sqrt{3}$ sec (d)