Motion in Straight Line

Mechanics :- It is the study of cause and effect of motion of bodies. They are broadly two categories in which mechanics can be divided.

(a) **Kinematics** :- It is the study of the effect of motion (like velocity, acceleration, etc.) without taking into account the cause for that motion.

(b) **Dynamics** :- It is the study of the cause due to which a body is moving. The cause may be an external force or torque applied.

Motion

When an object changes its position with respect to some other object with passage of time, then that object is said to be in motion.

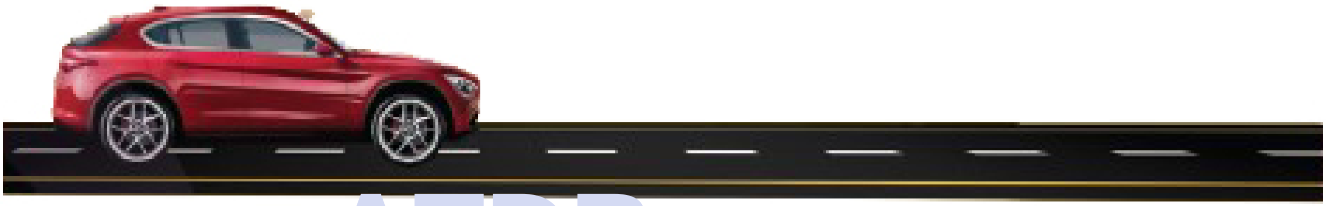
- Motion is a combined property of the object and the observer
- Motion is always relative. The same body can be at rest or moving if the observer changes
- No object in the universe is at absolute rest or in absolute motion

Dimension

The dimension of a space is the minimum number of independent information needed to accurately specify any point within it. The motion of an object can be classified according to dimension as follows :-

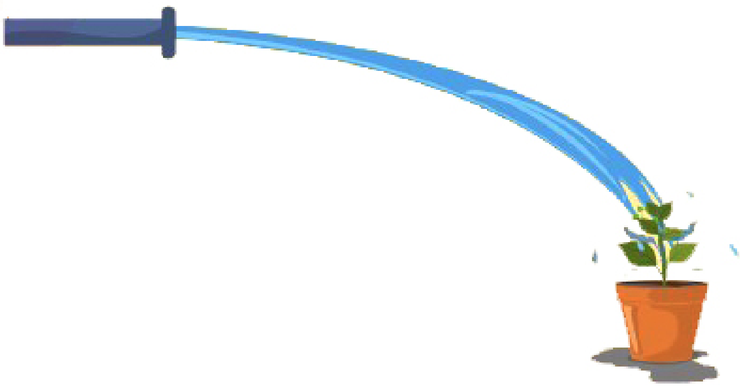
One dimensional motion:

- The body is restricted to move in a straight line
- Example :- A car moving on a straight road.



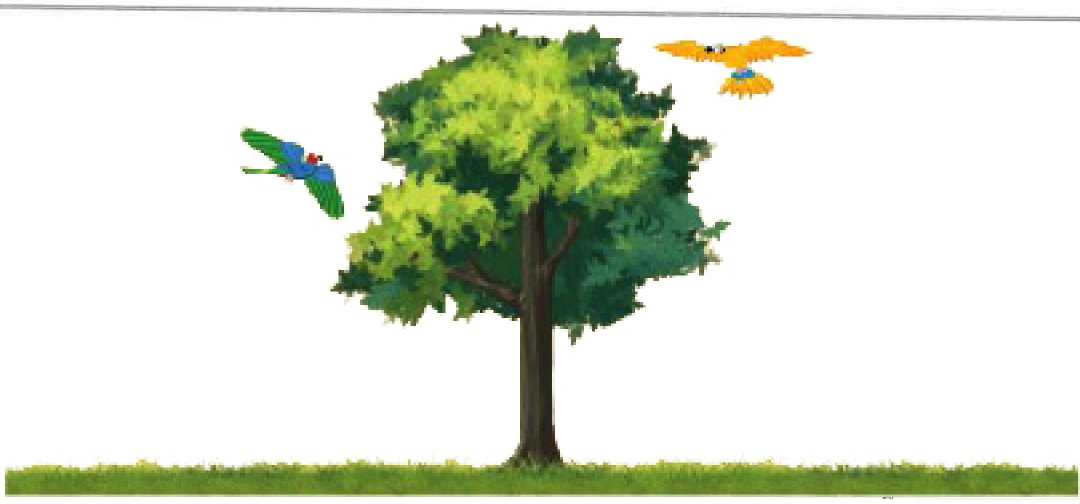
Two dimensional motion:

- The body is restricted to move in a plane
- Example :- water flowing out of a pipe.



Three dimensional motion:

- The body is free to move in space
- Example : Birds flying in the sky.



Motion in One Dimension

In one dimensional motion, the body is constrained to move in a straight line and can only change its direction of motion opposite to its original direction of motion.

Example A ball rolling on the ground

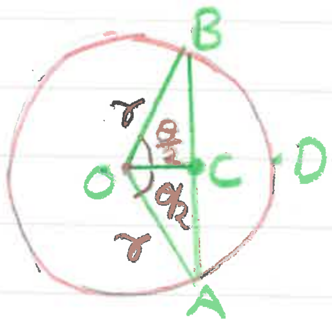
Some Basic Parameters of Motion

Position :- The position of a point is the expre-ssion of its accurate location from a pre-chosen reference point called origin.

Distance :- Distance covered by a body is the total length of the actual path covered in travelling from its initial to final position. It is a scalar quantity.

Displacement :- Displacement is the shortest distance between the initial and final position of a body. It is a vector quantity.

Displacement along circular path.



Distance covered = ADB (arc length) = $r\theta$

Linear Displacement = AB

$$AC = OA \sin\theta/2$$

$$\therefore AB = 2AC = 2r\sin\theta/2$$

Speed

Average speed :-

Average speed of a particle over a certain time interval is defined as the ratio of total distance of path travelled to the time interval

$$\langle v \rangle = \frac{\text{total distance covered}}{\text{time taken}}$$

It is a scalar quantity.

Instantaneous Speed

It is defined as the limit of the average speed when the considered time interval approaches zero

$$\text{Speed} = \frac{\Delta s}{\Delta t}$$

$$\text{Instantaneous speed} = \lim_{\Delta s \to 0} \frac{\Delta s}{\Delta t}$$

Velocity

1 Average Velocity.

The average velocity of a particle over a certain time interval is defined as the ratio of net displacement to the time interval

$$\text{Average Velocity} = \frac{\text{Displacement}}{\text{time taken}}$$

$$\langle \vec{v} \rangle = \frac{\Delta \vec{s}}{\Delta t} = \frac{\vec{r_f} - \vec{r_i}}{t_f - t_i}$$

The direction of average velocity is the direction of the net displacement for the time interval it is calculated.

$$\langle \vec{v} \rangle \parallel \Delta \vec{s}$$

2 Instantaneous Velocity

Instantaneous rate of change of position with respect

to time ie. velocity at an instant.
It is the ratio of displacement to an infinitesimally small interval of time

$$\langle \vec{v} \rangle = \frac{\Delta \vec{S}}{\Delta t}$$

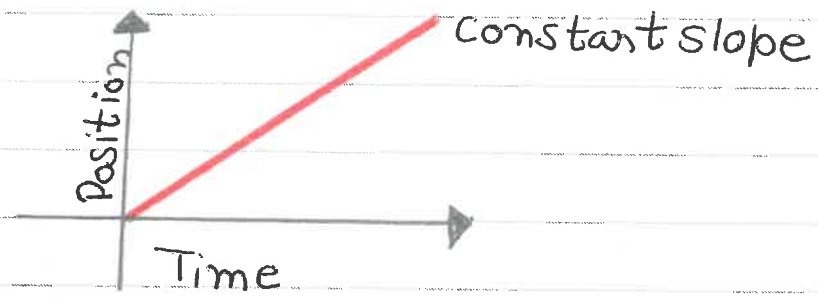
$$\langle \vec{v} \rangle = \lim_{\Delta t \to 0} \frac{\Delta \vec{S}}{\Delta t}$$

The direction of instantaneous velocity at any instant is tangential to the path at that instant

$$\therefore \quad \vec{v} \parallel d\vec{S}$$

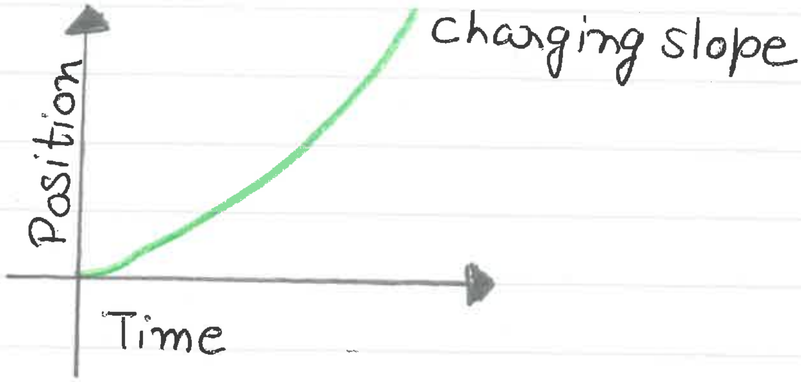
Uniform motion

If a body travels equal distance in equal interval of time, the motion is uniform



Non-Uniform motion

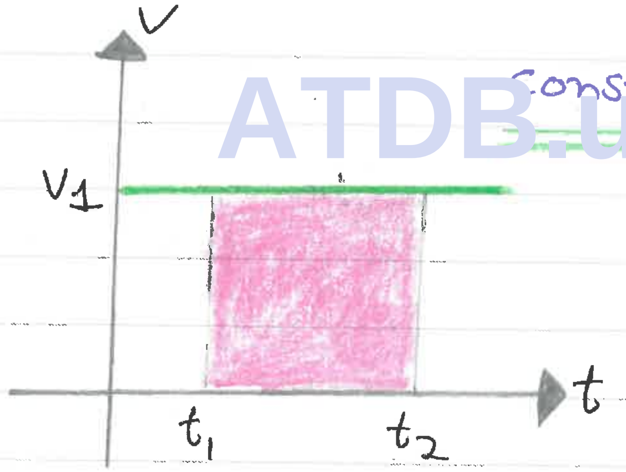
If a body travels unequal distance in equal interval of time, the motion is non uniform.



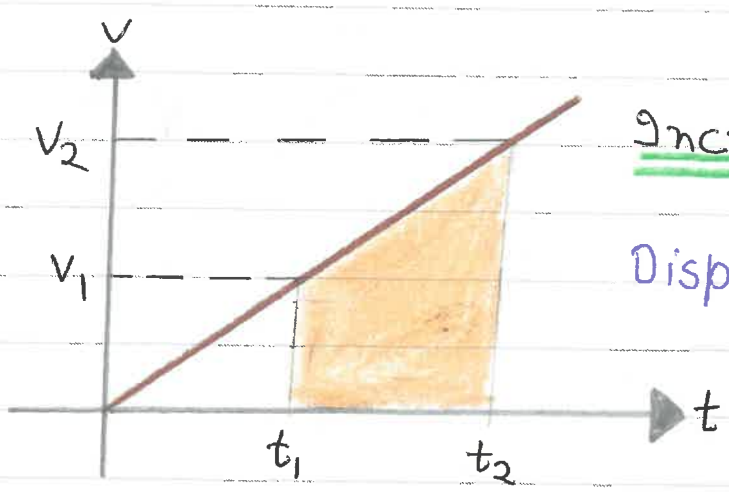
Velocity Vs Time Graph.

The area under the curve on a velocity vs time graph give us the change in position or displacement in that time interval.

Constant velocity.



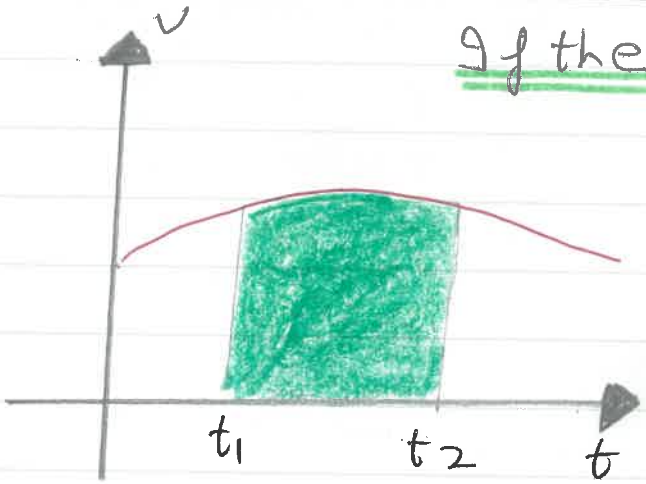
$$\text{Displacement} = v_1(t_2 - t_1)$$



Increasing velocity.

$$\text{Displacement} = \frac{1}{2}(v_1 + v_2)(t_2 - t_1)$$

If the curve is irregular



Displacement covered $= \int_{t_1}^{t_2} v\,dt.$

Acceleration

Acceleration is the rate of change of velocity of an object per unit time

Average Acceleration

Average acceleration $= \dfrac{\text{change in velocity}}{\text{time taken}}$

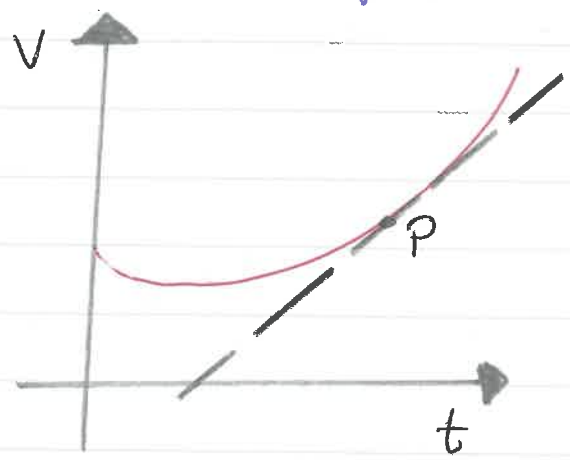
$$\langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v_f} - \vec{v_i}}{t_f - t_i}$$

It is a vector quantity. Its direction is always along the "change in velocity vector"

$$\langle \vec{a} \rangle \parallel \Delta \vec{v}$$

Instantaneous Acceleration.

It is the rate of change in velocity at a particular instant of time. It is given by the slope of the tangent to the v-t graph at that instant of time



$$\text{Slope} = \frac{dv}{dt} = \text{instantaneous acceleration}$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}.$$

★★ Uniform velocity ⇒ zero acceleration.

★★ Slope of acceleration and time graph gives jerk. The area under a-t graph gives change in velocity.

$$\int_{t_1}^{t_2} a\,dt = |\vec{v_2} - \vec{v_1}| = |\Delta \vec{v}|$$

Position–time (x–t) graph

- Slope $= \frac{dx}{dt}$ = velocity.

Velocity–time (v–t) graph

- Slope $= \frac{dv}{dt}$ = acceleration
- Area $= \int_{t_1}^{t_2} v\,dt$ = displacement.

Acceleration–time (a–t) graph.

- Slope $= \frac{da}{dt}$ = Jerk
- Area $= \int_{t_1}^{t_2} a\,dt$ = Change in velocity.

Equation of Motion for Uniform Acceleration

For uniform accelerated motion along a straight line, we have simple equation that relates

u = initial velocity (at $t=0$)
a = acceleration (constant)
v = final velocity (at time t)
s = displacement
x_f = final coordinate
x_i = initial coordinate

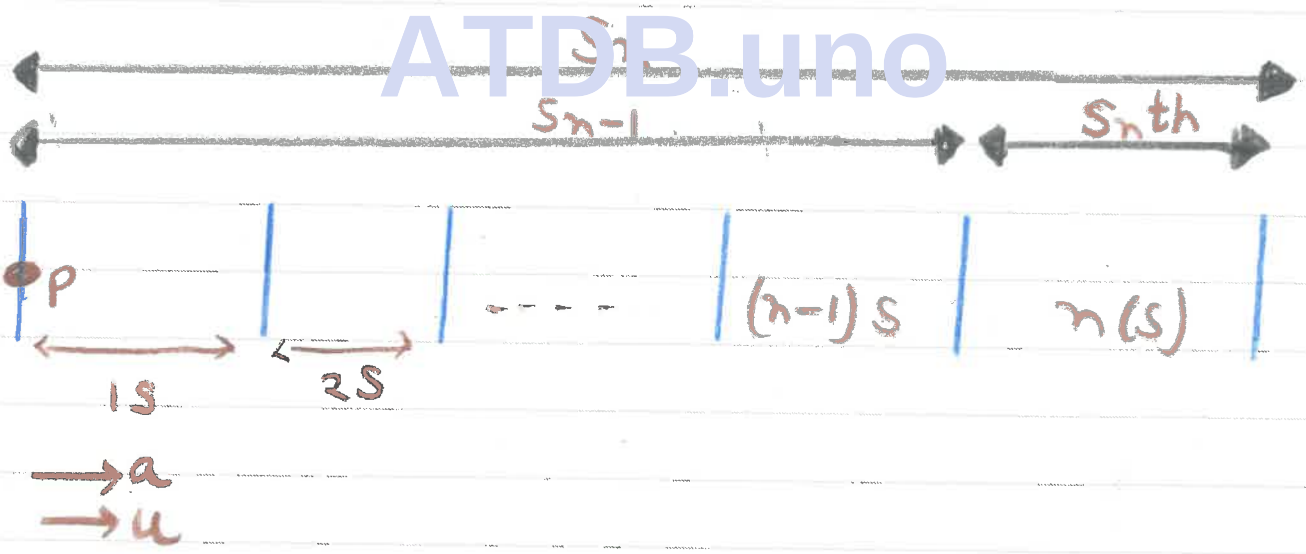
1st $\quad v = u + at$

2nd $\quad \Delta x = S = ut + \frac{1}{2}at^2$

3rd $\quad v^2 = u^2 + 2aS = u^2 + 2a(\Delta x)$

Displacement in n^{th} second

An object P with initial velocity u and constant acceleration a undergoes a displacement S_n in n seconds and S_{n-1} in $(n-1)$ sec. The displacement in n^{th} sec can be calculated like this.



Displacement in n sec is

$$S_n = un + \frac{1}{2}an^2 \quad --- (1)$$

Displacement in $(n-1)$ sec is

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2 \quad --- (2)$$

Displacement in n^{th} sec

$$S_{n^{th}} = S_n - S_{n-1}$$

$$= u(n) - u(n-1) + \frac{1}{2}an^2 - \frac{1}{2}a(n-1)^2$$

$$= u(n-(n-1)) + \frac{1}{2}a(n^2-(n-1)^2)$$

$$= u + \frac{a}{2}(n^2 - n^2 + 2n - 1)$$

$$S_{n^{th}} = u + \frac{a}{2}(2n-1).$$

Motion Under Gravity

For objects moving vertically near the surface of earth, the only force acting on it is its weight (mg) i.e the gravitational pull of the earth. The acceleration due to gravity (a=g) has a fixed value irrespective of the mass of the object.

$$a = 9.8\ ms^{-2} \text{ (in SI or MKS)}$$

$$a = 32.2\ fts^{-2} \text{ (in FPS)}$$

Equations of motion becomes (for freely falling)

(i) $v = u + gt$

(ii) $h = ut + \frac{1}{2}gt^2$

(iii) $v^2 = u^2 + 2gh$

Note: If an object is thrown upward the g is replaced by -g in above three equation

(i) Time taken to reach maximum height

$$t_A = \frac{u}{g} = \sqrt{\frac{2h}{g}}$$

(ii) Maximum height reached by the body

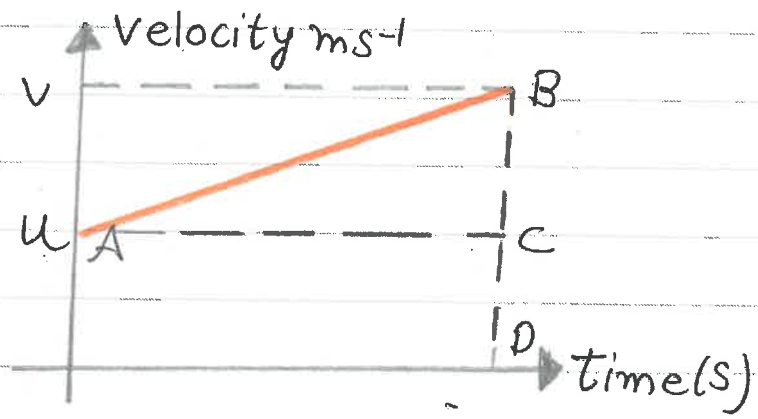
$$h_{max} = \frac{u^2}{2g}$$

(iii) when a body is dropped freely from the top of the tower and another body is projected horizontally from the same point, both will reach the ground at the same time.

(iv) In motion under gravity we usually assume "air resistance is negligible"

Equation of Motion.

First equation of motion



Slope of the velocity vs time graph represents the acceleration of the object. As the graph is a Straight line from A to B

Acceleration = Slope of graph AB

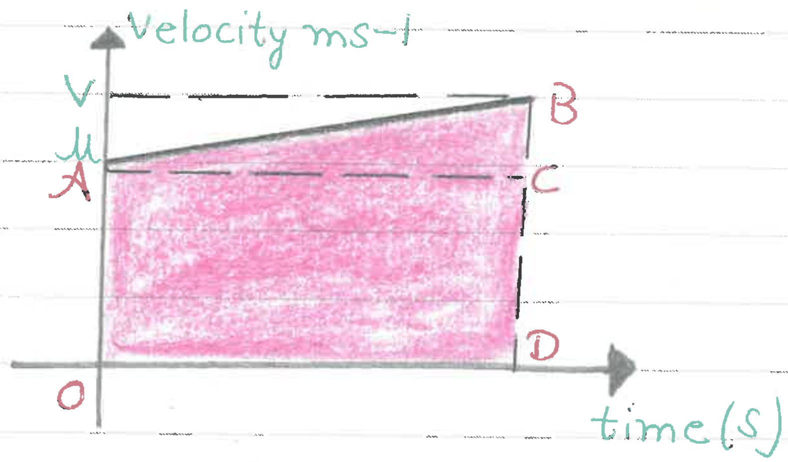
$$a = \frac{BC}{AC}$$

$$a = \frac{BD - CD}{AC}$$

$$a = \frac{v - u}{t}$$

$$v = u + at.$$

Second equation of motion.



Area included between the velocity vs time graph and time axis is the displacement covered by the object in the given time interval t.

Area of the trapezium ABCDOA is

$$S = \frac{1}{2}(OA + BD) \times OD$$

$$S = \frac{1}{2}(v + u) \times t$$

But from the first equation $v = u + at$

$$\Rightarrow S = \frac{1}{2}(u + u + at) \times t$$

$$S = ut + \frac{1}{2}at^2$$

Third equation of motion

As we know, the displacement is

$$S = \frac{1}{2}(OA + BD) \times OD$$

And acceleration is.

$$a = \frac{BC}{AC} = \frac{BD - CD}{AC}$$

$$a = \frac{v - u}{t}$$

$$t = \frac{v - u}{a}$$

$$S = \frac{1}{2}(u + v) \times \frac{(v - u)}{a}$$

$$S = \frac{v^2 - u^2}{2a}$$

$$v^2 - u^2 = 2as.$$

Relation between Motion Parameters.

Acceleration when the velocity is a function of time

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$a = \frac{d^2x}{dt^2}$$

Acceleration when the velocity is a function of position

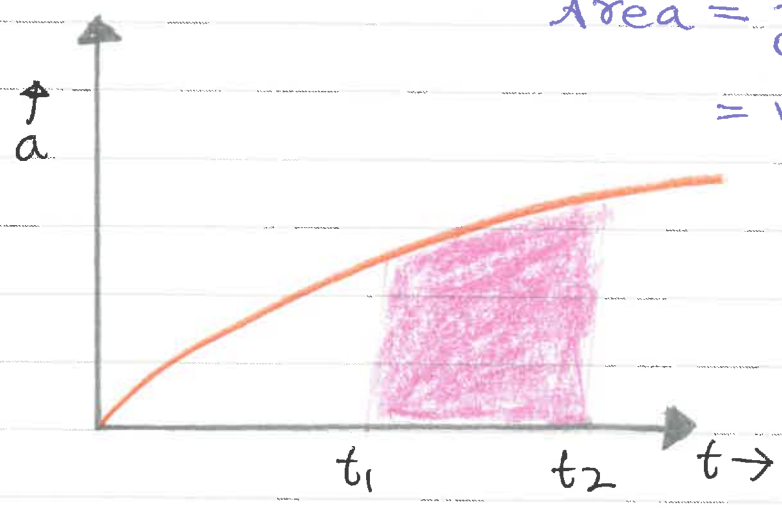
$$a = \frac{dv}{dt}\,\frac{dx}{dx}$$

$$a = \frac{dv}{dx}\left(\frac{dx}{dt}\right) = \frac{v\,dv}{dx}$$

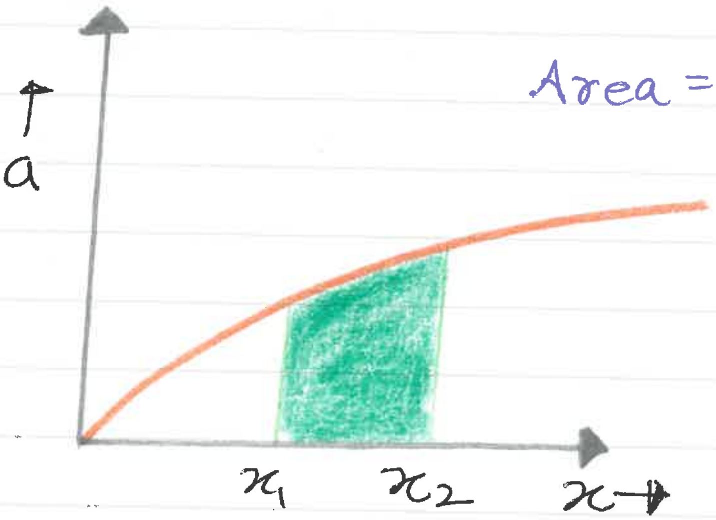
For a-t graph.

$$\text{Area} = \int_{t_1}^{t_2} a\,dt$$

$$= v_2 - v_1 = \Delta v.$$



For a-x graph



$$\text{Area} = \int_{x_1}^{x_2} a\,dx = \frac{v_2^2 - v_1^2}{2}$$

Equation of Motion Using Calculus.

first equation of motion

We know that

$$a = \frac{dv}{dt}$$

Integrating it over the time interval 0s to ts wherein the velocity charges from initial velocity u to some final velocity v, we get

$$\int_0^t a\,dt = \int_u^v dv$$

Since acceleration is constant

$$a\,[t]_0^t = [v]_u^v$$

$$a(t-0) = v-u$$

$$v = u + at.$$

Second equation of motion

Velocity is

$$\frac{dx}{dt} = v = u + at$$

Integrating it over the time interval 0s to ts, where the position changes from initial position x_i to some final position x_f we get,

$$\int_{x_i}^{x_f} dx = \int_0^t (u + at)\, dt$$

$$[x]_{x_i}^{x_f} = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

$$(x_f - x_i) = u(t - 0) + a\left(\frac{t^2}{2} - 0\right)$$

$$S = \Delta x = ut + \frac{1}{2}at^2$$

where $\Delta x = x_f - x_i$

Third equation of motion

When velocity is a function of position ie $v(x)$ the acceleration is written as follows:

$$a = v\frac{dv}{dx}$$

Cross multiplying and integrating it we get

$$\Rightarrow v\,dv = a\,dx$$

$$\int_u^v v\,dv = \int_{x_i}^{x_f} a\,dx$$

$$\left[\frac{v^2}{2}\right]_u^v = a[x]_{x_i}^{x_f}$$

$$\frac{v^2}{2} - \frac{u^2}{2} = a(x_f - x_i)$$

$$v^2 = u^2 + 2a(\Delta x)$$

Frame of reference.

It is a region or a zone from where observation are taken for motion parameters of a body. It can be at rest or it can be moving. It is a perspective that one uses to determine whether an object is at rest or in motion

Some points about motion.

(1) Motion is a combined property of the object under study as well as the observer

(2) Motion is always defined with respect to an observer or reference frame.

(3) It is always relative, there is no such thing as absolute motion or absolute rest.

✱✱ Motion can be analysed only if we know

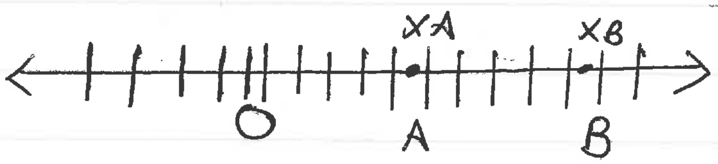
(1) what is being looked at

(2) who is looking.

Basic Parameter of Relative Motion

Relative position

A position defined with reference to another position, either fixed or moving.



Here

$$x_{BA} = \text{Position of B wrt A} = x_B - x_A \quad \text{---(i)}$$

where B is what is being looked at (object) and A is who is looking (observer).

Relative velocity.

The relative velocity of an object B with respect to another object A is the velocity with which the object B would appear to move to an observer situated on object A

Differentiating equation (i) w.r.t time (t)

$$\frac{dx_{BA}}{dt} = \frac{dx_B}{dt} - \frac{dx_A}{dt}$$

$$\Rightarrow v_{BA} = v_B - v_A = \text{Relative velocity of B wrt A} \quad \text{---(ii)}$$

Relative acceleration

The relative acceleration of an object B with respect to another object A is the acceleration with which the object B would appear to move to an observer situated on object A.

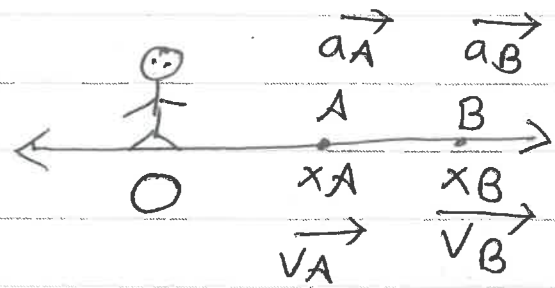
Differentiating equation(ii) wrt time(t)

$$\frac{dv_{BA}}{dt} = \frac{dv_B}{dt} - \frac{dv_A}{dt}$$

$\Rightarrow a_{BA} = a_B - a_A$ = Relative acceleration of B w.r.t A.

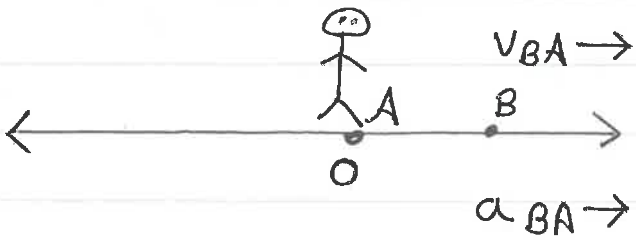
Physical Significance of Relative Motion

Ground frame



(1) observer is standing on the ground
(2) Both A and B are moving for the observer

A (particle) frame



(1) observer is present on body A
(2) A is at rest, while B and ground are moving for the observer.

* With the change in reference frame, the origin of every parameter is recalibrated i.e the new chosen reference frame is the 'new rest'

If P and C are two observers,

$$\Rightarrow V_{AB} = V_{AP} + V_{PB} = V_{AC} + V_{CB}$$