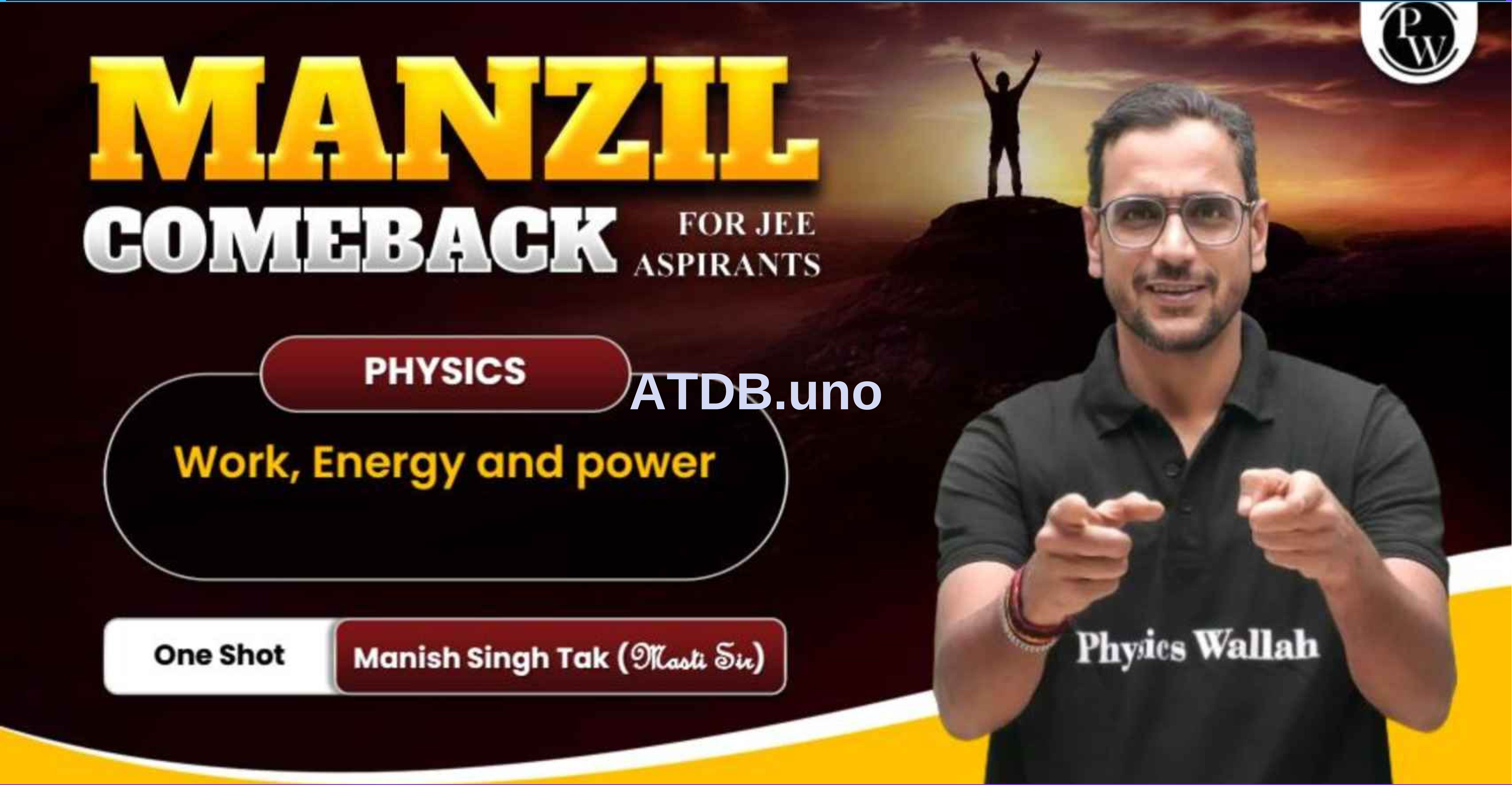
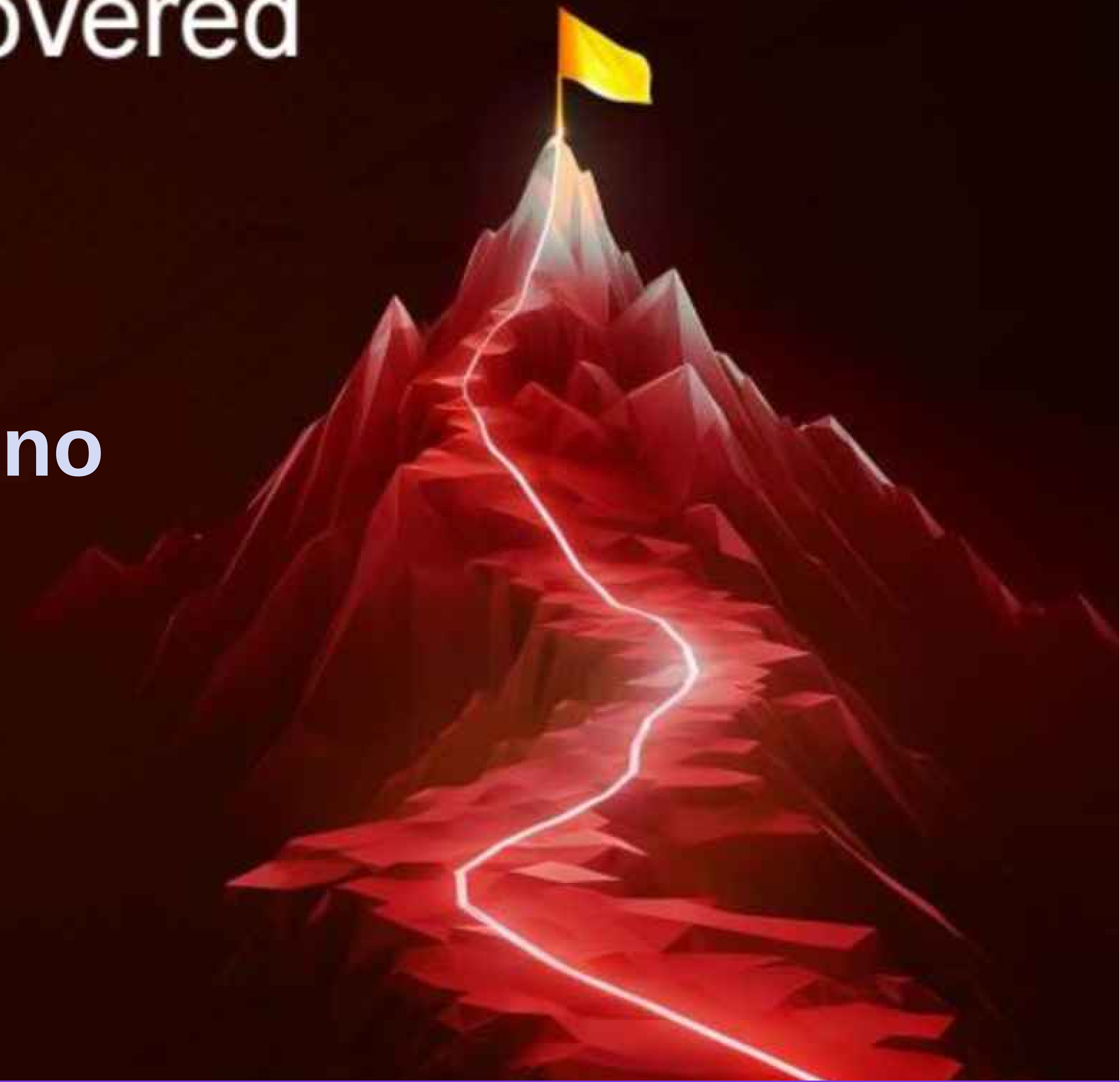
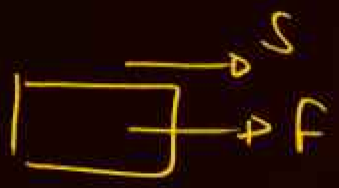

MANZIL
COMEBACK
FOR JEE ASPIRANTS
PHYSICS
Work, Energy and power
One Shot
Manish Singh Tak (Masti Sir)
Physics Wallah
ATDB.uno

Topics to be covered

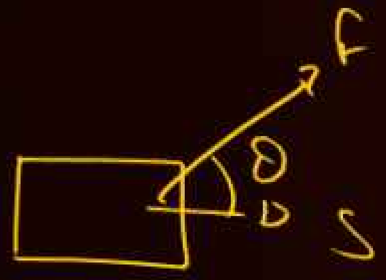
1. # Work, Kinetic Energy
2. # Work Energy Theorem
3. # Conservative Force
4. # Potential Energy And Mechanical Energy
5. # Power



Concept of Work done and work Energy Theorem



(FS)



$(F\cos\theta)S$

$\vec{F}\cdot\vec{S}$

in 1-D

$F = ma = mv\frac{dv}{dx}$

$\int F\,dx = \int mv\,dv$

$\int F\,dx = \frac{1}{2}m(v_f^2 - v_i^2)$

Work done by force = Change in K.E.

$$\boxed{\vec{F} = \frac{d\vec{P}}{dt}}$$

inertial frame

$\int d\vec{P} = \int \vec{F}\,dt$

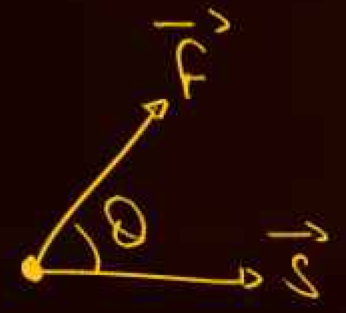
$\vec{P}_f - \vec{P}_i = \int \vec{F}\,dt = \text{Impulse}$

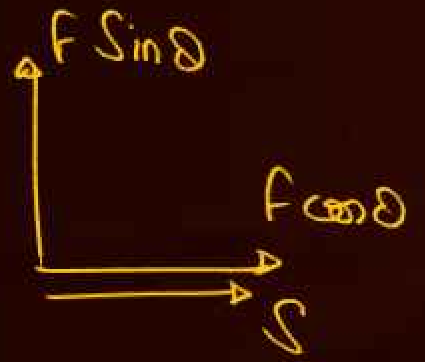
momentum

Work done by any force $\vec{F}$ on particle is define as

** $\boxed{W = \vec{F} \cdot \vec{S} = FS\cos\theta}$

$\vec{S} \rightarrow$ displacement of particle





$\vec{F} \perp \vec{S}$ $\qquad W = 0$

for variable force

** $\boxed{W = \int \vec{F} \cdot d\vec{S}}$

Unit $\rightarrow$ J $\qquad ML^2T^{-2}$

Scalar $\rightarrow$ Sign रखता जनाब है

$(W = 80J \qquad W = -80J)$

uniform F

$\longrightarrow \quad W = \int \vec{F} \cdot d\vec{s} = \vec{F} \cdot \vec{S}$

$\longrightarrow \quad \boxed{W = \Delta K.E.}$

Work-Energy Theorem

Kinetic Energy

$$\boxed{m} \longrightarrow V$$

→ $K.E. = \frac{1}{2} m v^2$

→ Scalar (non-ive)

→ frame dependent

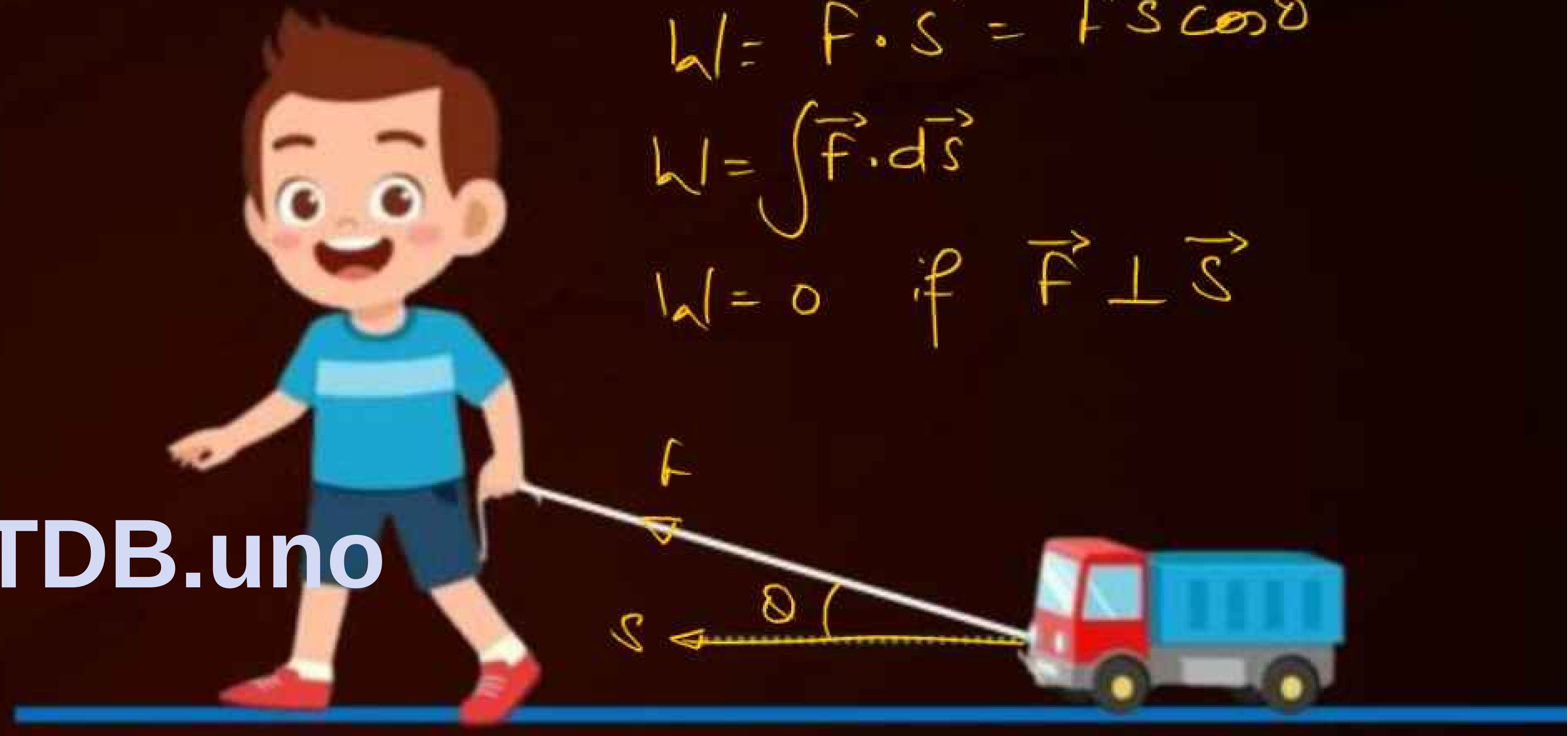
→ $V^2 = V_x^2 + V_y^2 + V_z^2$

Work Done by Force

$$W = \vec{F} \cdot \vec{S} = FS\cos\theta$$

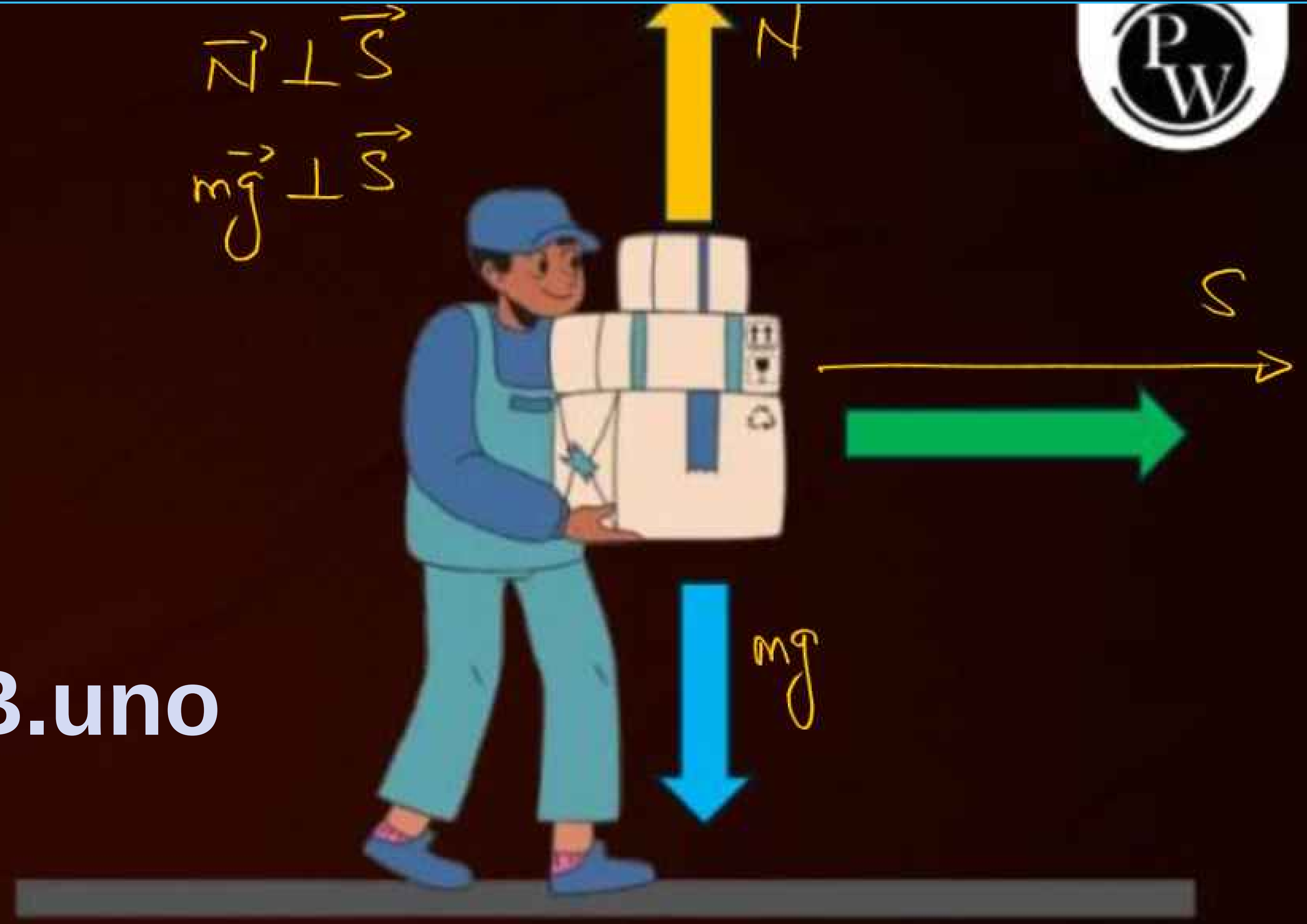
$$W = \int \vec{F} \cdot d\vec{S}$$

$$W = 0 \text{ if } \vec{F} \perp \vec{S}$$



$\vec{N} \perp \vec{S}$

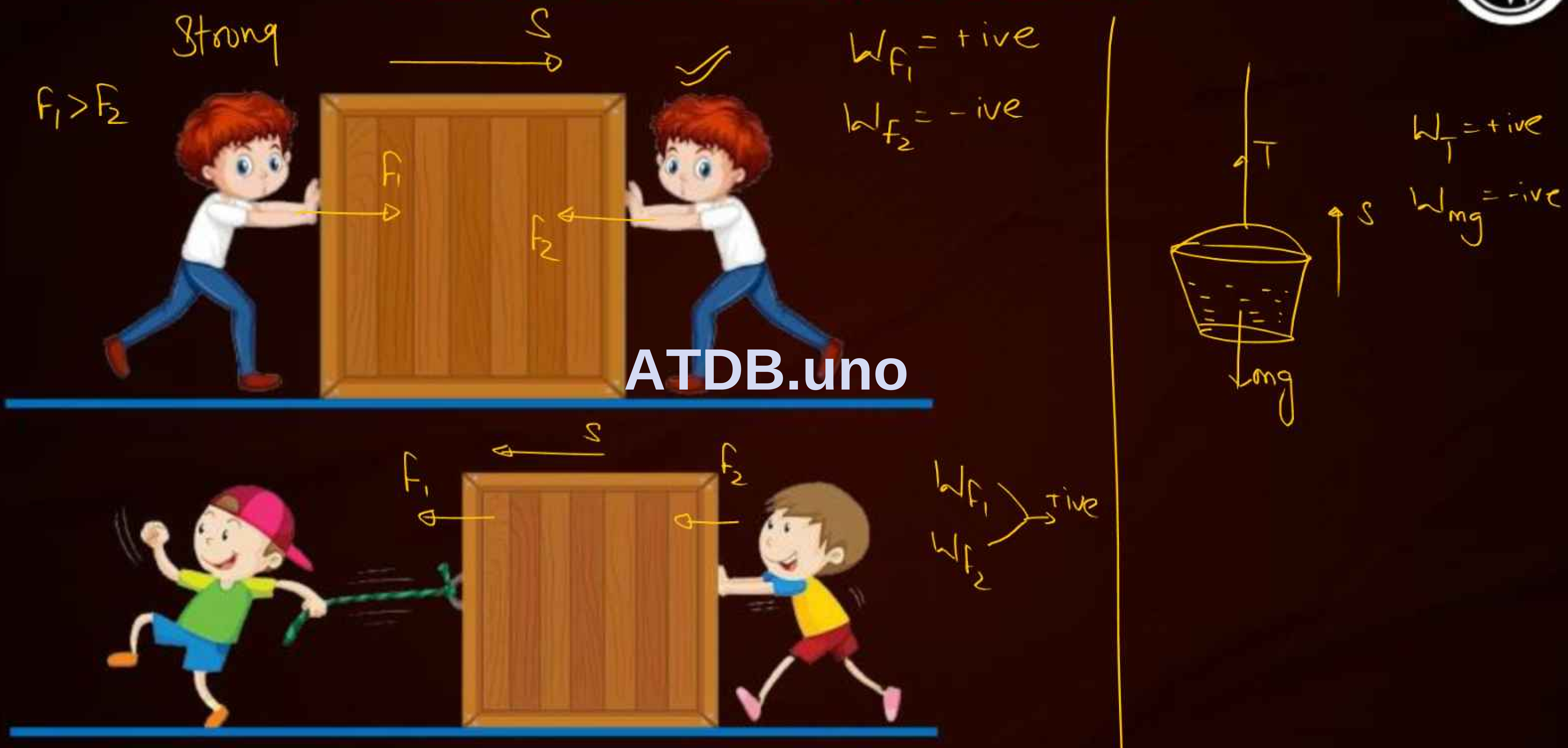
$\vec{mg} \perp \vec{S}$

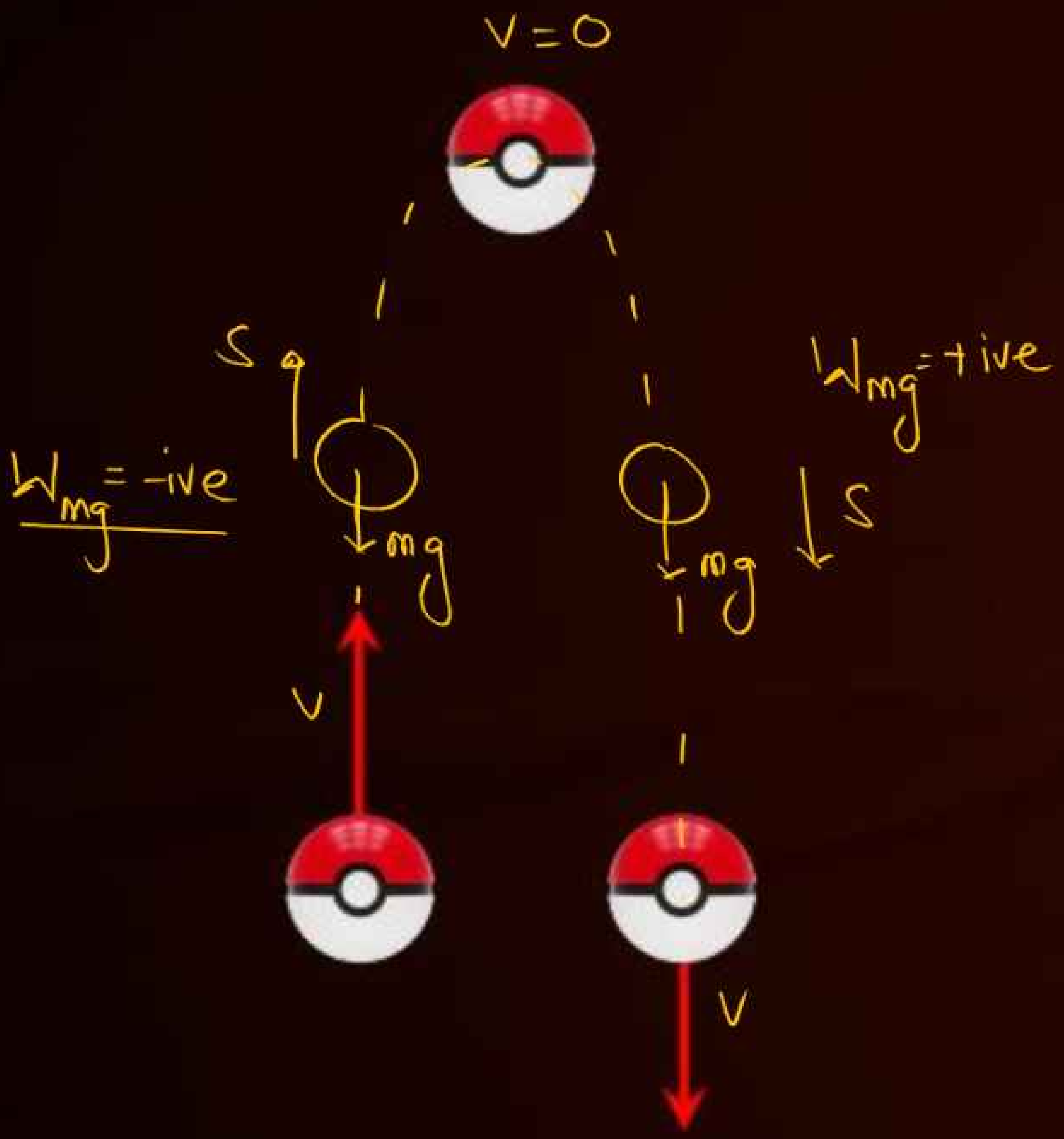


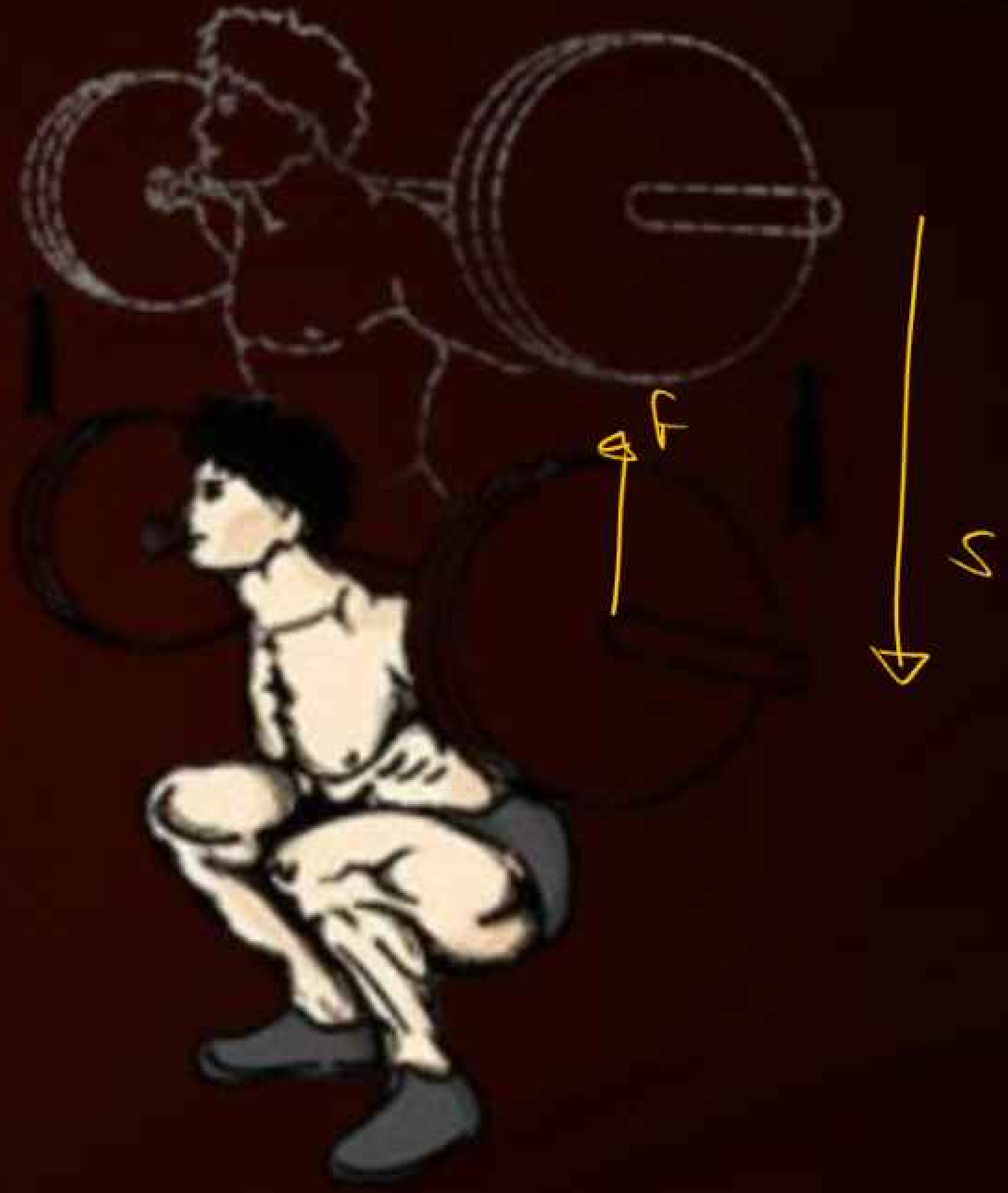
$|a|_{Normal} = 0$

$|a|_{mg} = 0$

+ive and –ive sign of work

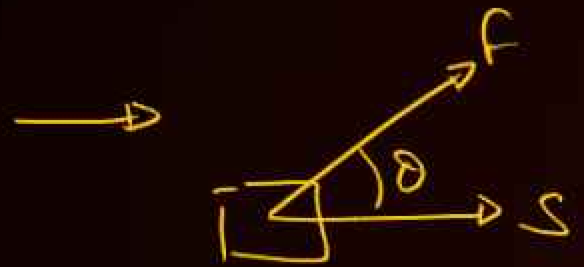


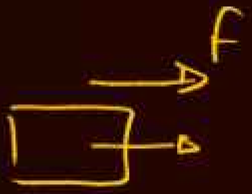

V=0
S
W_{mg} = -ive
mg
mg
S
W_{mg} = +ive
V
V


F
S

Calculation of Work

$W = \vec{F} \cdot \vec{S} = FS\cos\theta$





Fx

$\rightarrow \vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\vec{S} = S_x\hat{i} + S_y\hat{j} + S_z\hat{k}$

$\underline{F_x S_x} + F_y S_y + F_z S_z$

$\vec{S} = \vec{r}_f - \vec{r}_i$

Ex. $\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k} \Rightarrow W = \vec{F}\cdot\vec{S}$

Constant

$\vec{F} \rightarrow$ variable

$W = \int \vec{F} \cdot d\vec{s}$

$\rightarrow$ force is in x-dir^n $\quad W = \int F\,dx$

Ex. $F = 3x^2 + 4x$ in x-dir^n $(x = 1 \text{ to } x = 2)$

$W = \int_1^2 (3x^2 + 4x)\,dx$

$\rightarrow$ force is in $\hat{i}, \hat{j}, \hat{k}$ $\quad (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})$

$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$ **

$W = \int F_x\,dx + F_y\,dy + F_z\,dz$

Ex. $\vec{F} = 6x\hat{i} - 3y^2\hat{j}$

$A(1,1) \longrightarrow B(3,2)$

$W = \int_1^3 6x\,dx - \int_1^2 3y^2\,dy$

$W = 3[3^2 - 1^2] - [8-1]$

$W = 3(8) - 7 = 17\,J$

Ex. $\vec{F} = 6y\hat{i} + 6x\hat{j}$

$A(1,1) \rightarrow B(3,2)$

$W = \int \cancel{6y\,dy + 6x\,dx}$

$W = \int 6y\,dx + 6x\,dy$

$\left\{ \begin{array}{l} \int x\,dx = \frac{x^2}{2} + C \\ \int y\,dx \rightarrow ?? \end{array} \right\}$

$W = 6\int (y\,dx + x\,dy)$

क्या मैं किसी का derivative हूँ

$W = 6\int d(xy) = 6\left[xy\right]_{1,1}^{3,2}$

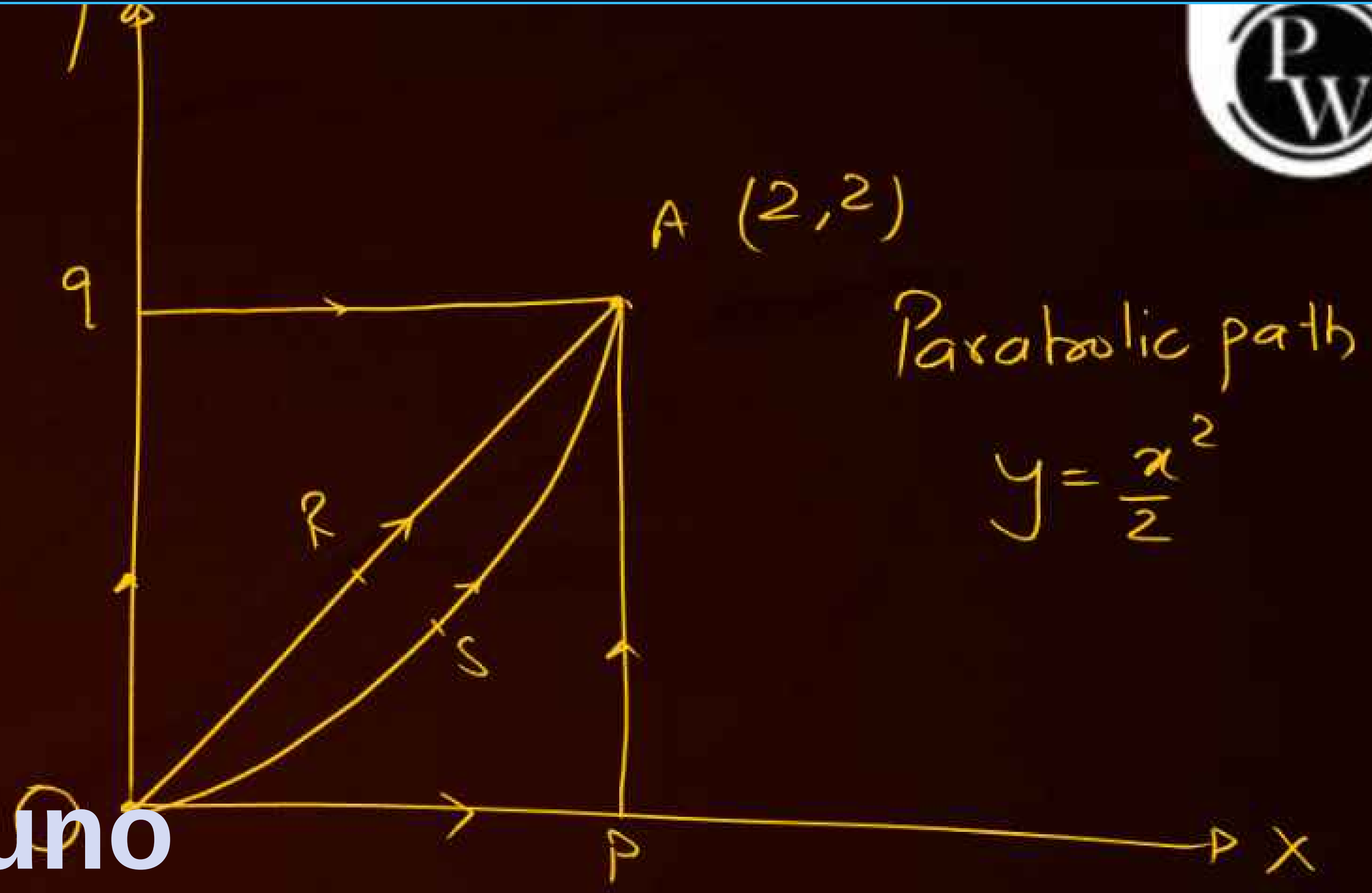
$6[3\times2 - 1\times1]$

$= 30\,J$

$\cancel{6((3-1)(2-1)) = 12\,J}$

Ex. 3 $\vec{F} = 6y\hat{i} - 6x\hat{j}$

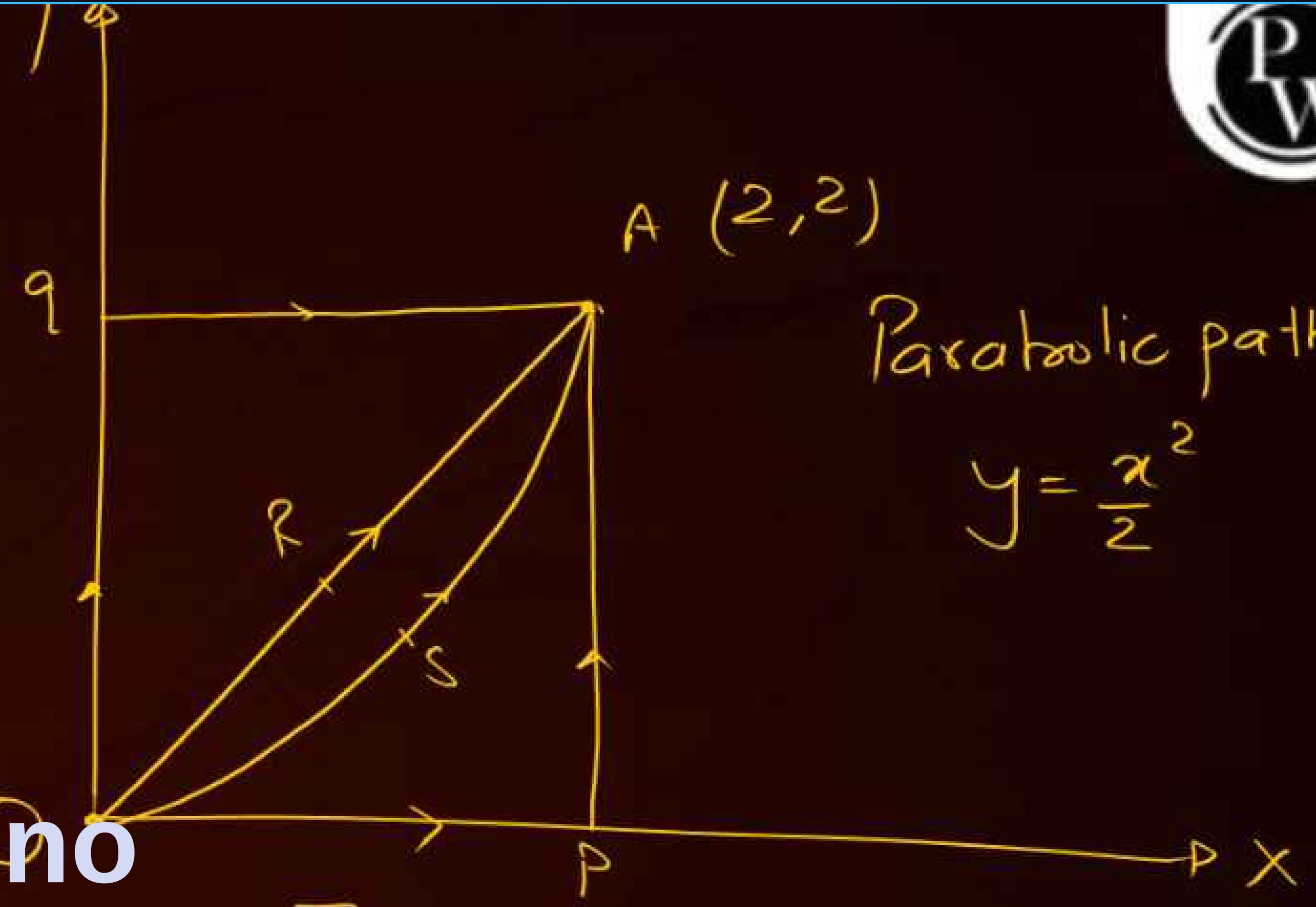
Particle move from O (0,0) to point A (2,2) in 4 different paths as shown find work done by this force in each path



$\vec{F} = 6y\hat{i} - 6x\hat{j}$

$|a| = \int \vec{F} \cdot \vec{ds} = \int 6y\,dx - 6x\,dy$

मैं किसी का derivative नहीं हूँ
(Single term)



Parabolic path

$y = \frac{x^2}{2}$

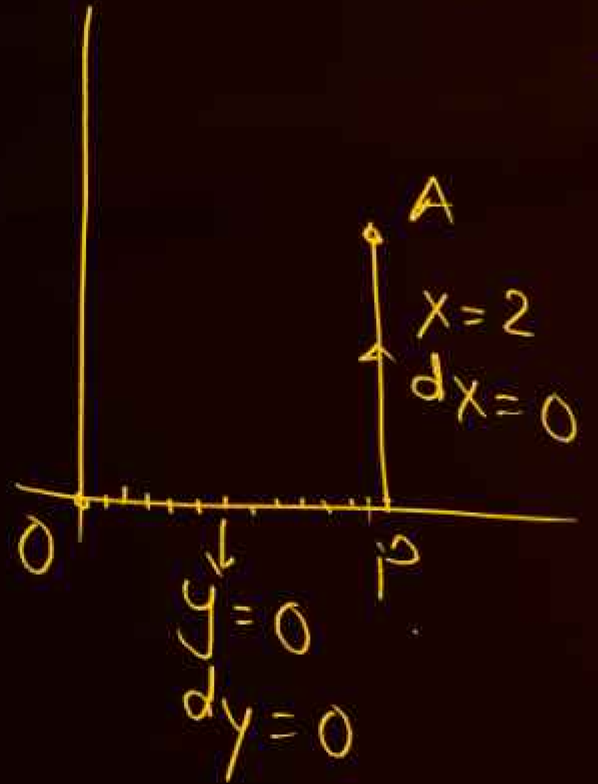
now i need information of path

OPA

$W_{OPA} = W_{OP} + W_{PA}$

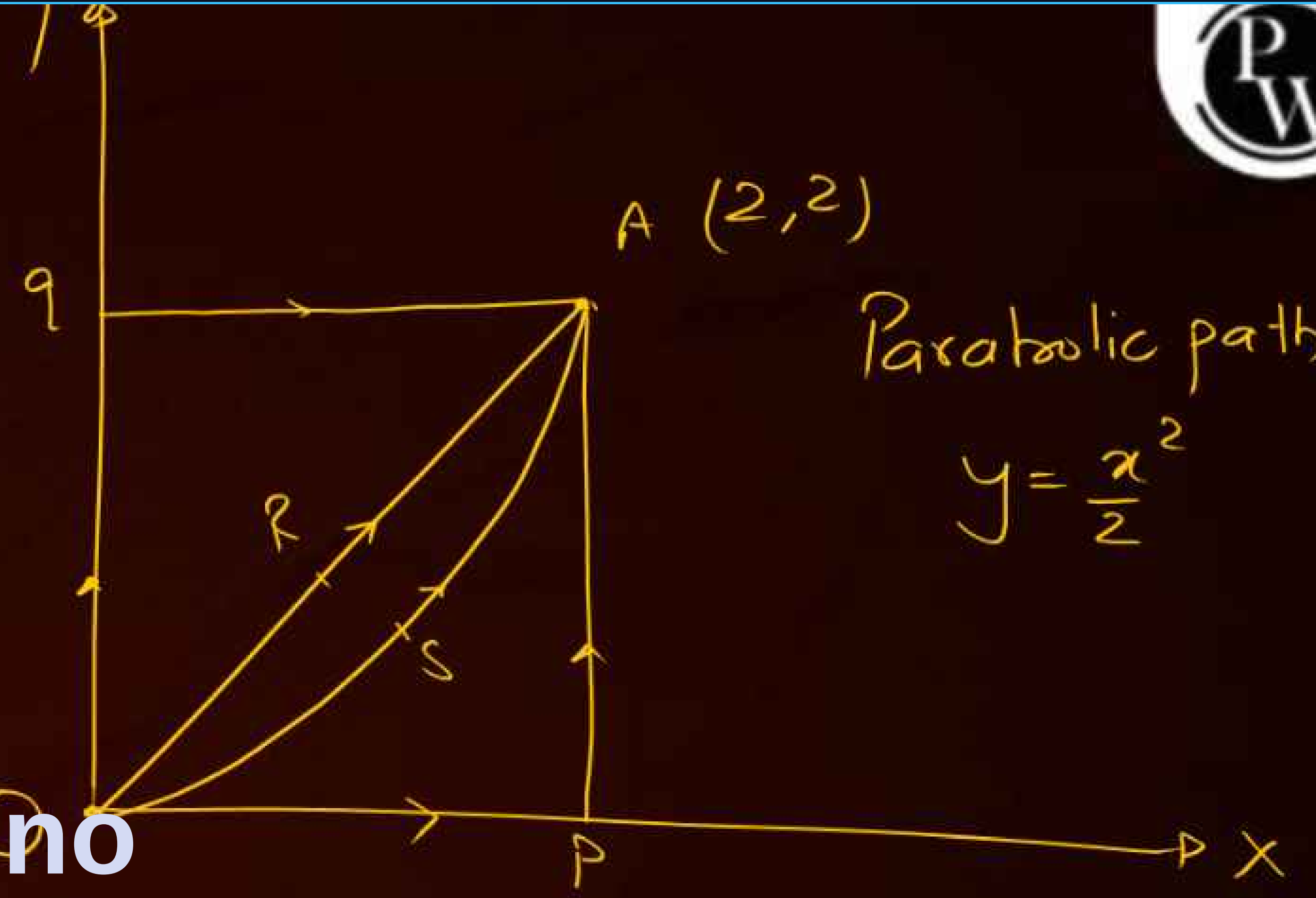
$= \left[\int 6(0)dx - 6x(0)\right] + \left[\int 6y(0) - 6(2)dy\right]$

$= 0 + -12\int_0^2 dy = -24\,J$



$$\vec{F} = 6y\hat{i} - 6x\hat{j}$$

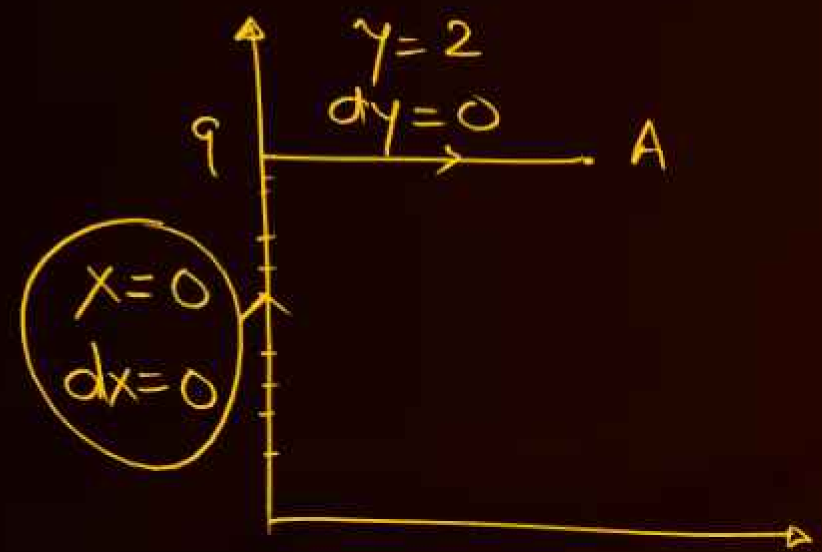
$$W = \int \vec{F}\cdot d\vec{s} = \int 6y\,dx - 6x\,dy$$



Parabolic path

$$y = \frac{x^2}{2}$$

OQA: → OQ + QA

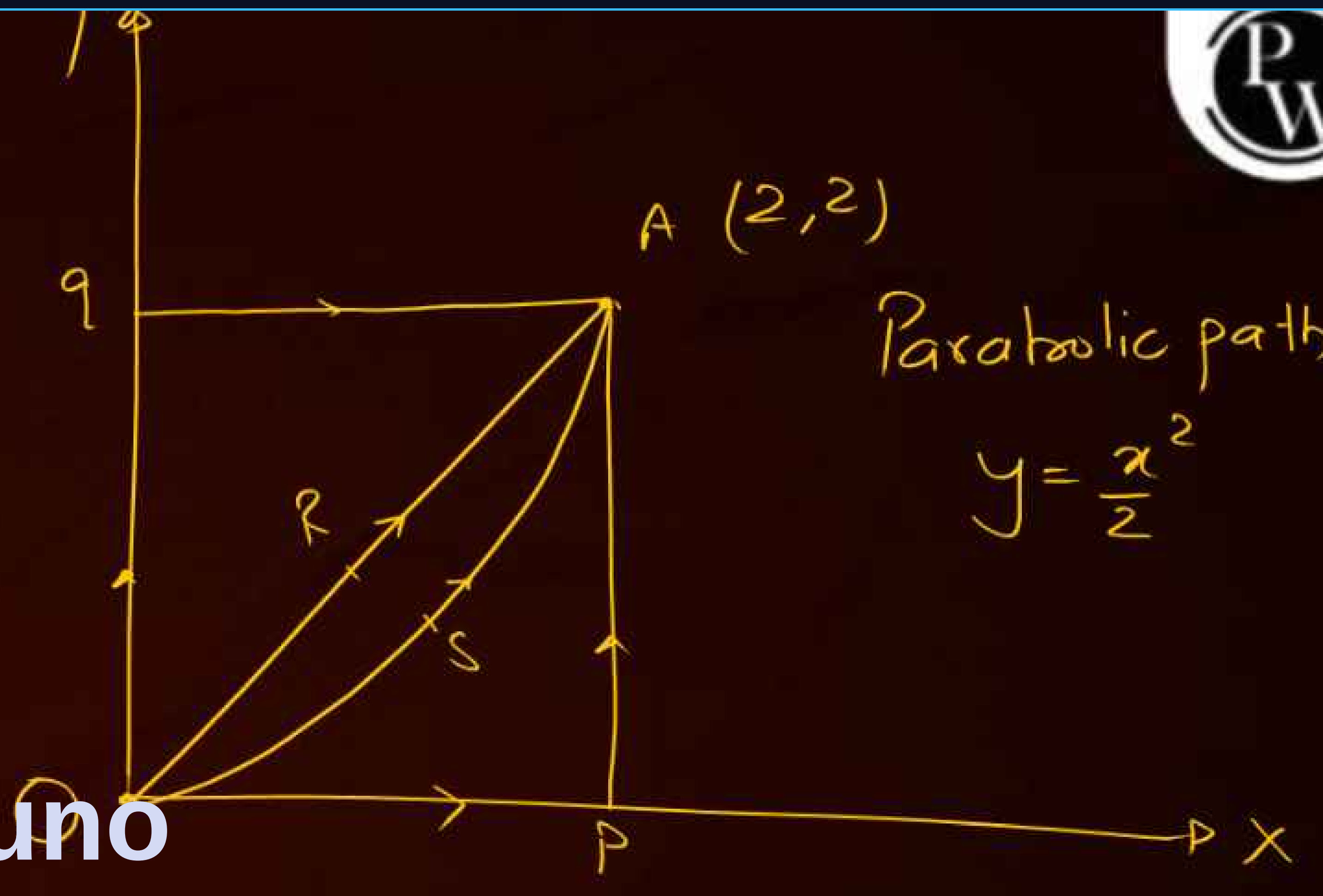


$$W = \left[\int 6y(0) - 6(0)dy\right] + \left[\int 6(2)dx - 6x(0)\right]$$

$$W = 0 + 12\int_0^2 dx = 12(2-0) = 24\,J$$

$\vec{F} = 6y\hat{i} - 6x\hat{j}$

$W = \int \vec{F}\cdot\vec{ds} = \int 6y\,dx - 6x\,dy$



Parabolic path

$y = \frac{x^2}{2}$

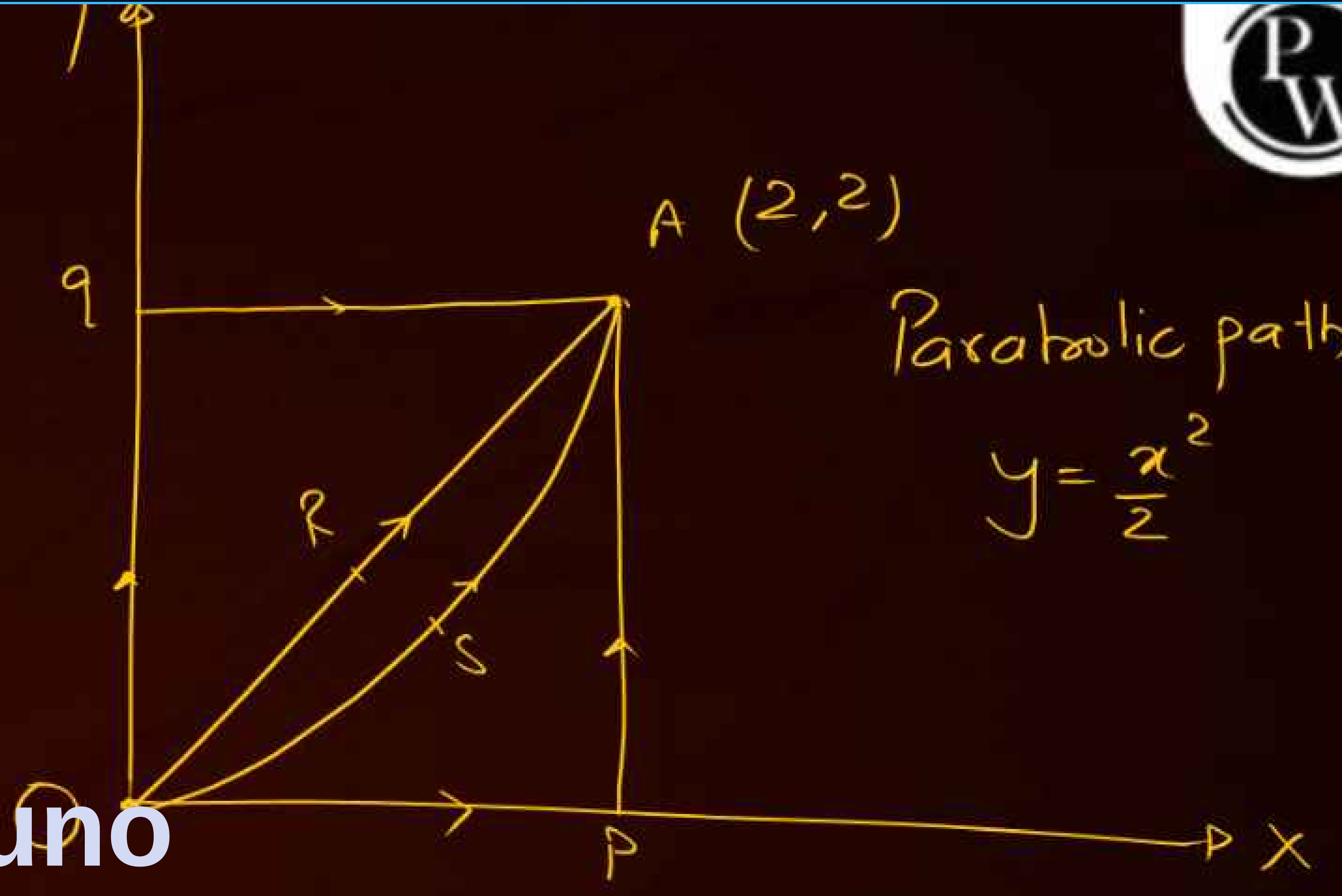
<u>ORA</u> St. line $y = x \Rightarrow dy = dx$

$$W = \int 6x\,dx - 6x\,dx = 0$$

<u>OSA</u> $\rightarrow$ $y = \frac{x^2}{2} \Rightarrow dy = x\,dx$

$$W = \int 6\left(\frac{x^2}{2}\right)dx - 6x(x\,dx) = \int_0^2 -3x^2\,dx$$

$$W = -\left[2^3 - 0^3\right] = -8\,J$$



Parabolic path

$y = \frac{x^2}{2}$

$W_{OPA} = -24J$

$W_{OqA} = 24J$

$W_{ORA} = 0J$

$W_{OSA} = 8J$

$\longrightarrow \vec{F} = \frac{8}{r^2}\hat{r}$ (Centeral force)

$\vec{ds} \rightarrow \vec{dr}$

$W = \int \vec{F} \cdot \vec{dr} = \int \frac{8}{r^2} dr$

$$\left(\begin{array}{l} \vec{F} = \frac{kq_1q_2}{r^2}\hat{r} \\ \vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r} \end{array} \right) \begin{array}{l} \rightarrow W = \int \frac{kq_1q_2}{r^2} dr \\ \rightarrow W = -\int \frac{Gm_1m_2}{r^2} dr \end{array}$$

$\int \frac{8}{r^2} \underline{\hat{r} \cdot \vec{dr}} = \int \frac{8}{r^2}(1)(dr)\cos 0$

$\longrightarrow \hat{r}$

$\longrightarrow \vec{dr}$

Conservative force :→

force → for which work done does not depend on path
→ for which work done in a closed loop is zero

$W = \int \vec{F} \cdot \vec{ds}$ → integrate हो गया बिना Path की information के तो Conservative है
(Start initial and final position)

non-Conservative → work done depend on path

Conservative → Central force → Gravitation $\frac{\;}{r^2}$, g
→ electrostatic $\frac{kq_1q_2}{r^2}$

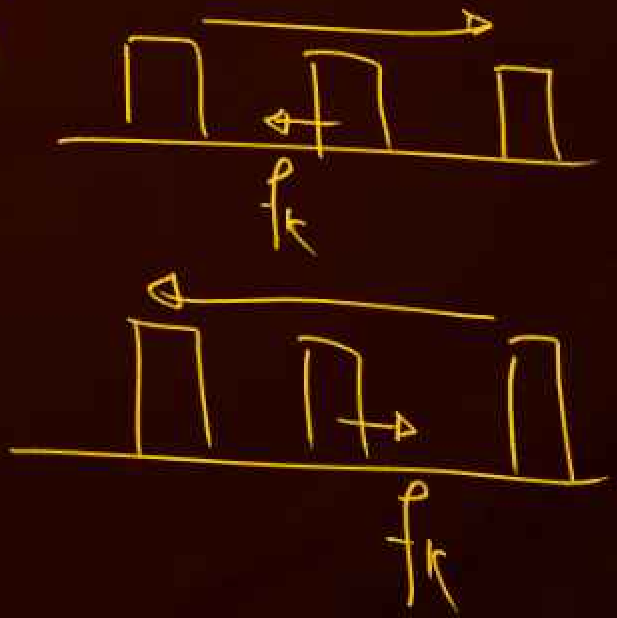
→ Constant force $(\vec{F}\cdot\vec{S})$

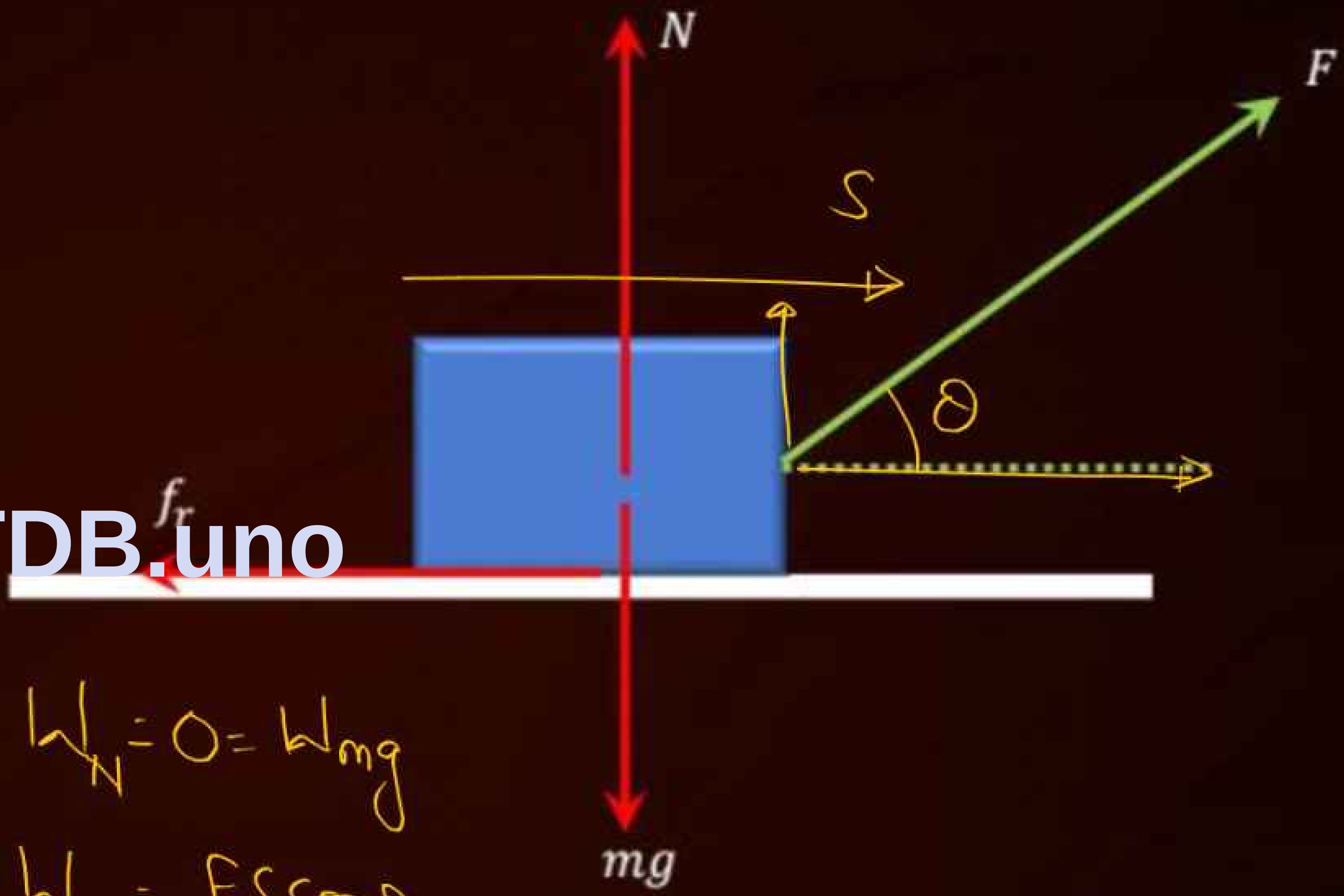
→ $\vec{F} = 3x^2\hat{i}$, $\vec{F} = 6y^2\hat{j}$, $\vec{F} = 3x^2\hat{i} + 6y^2\hat{j}$

→ $\vec{F} = 6y\hat{i} + 6x\hat{j}$

non Conservative → $\vec{F} = 6y\hat{i} - 6x\hat{j}$

→ kinetic friction





$W_N = 0 = W_{mg}$

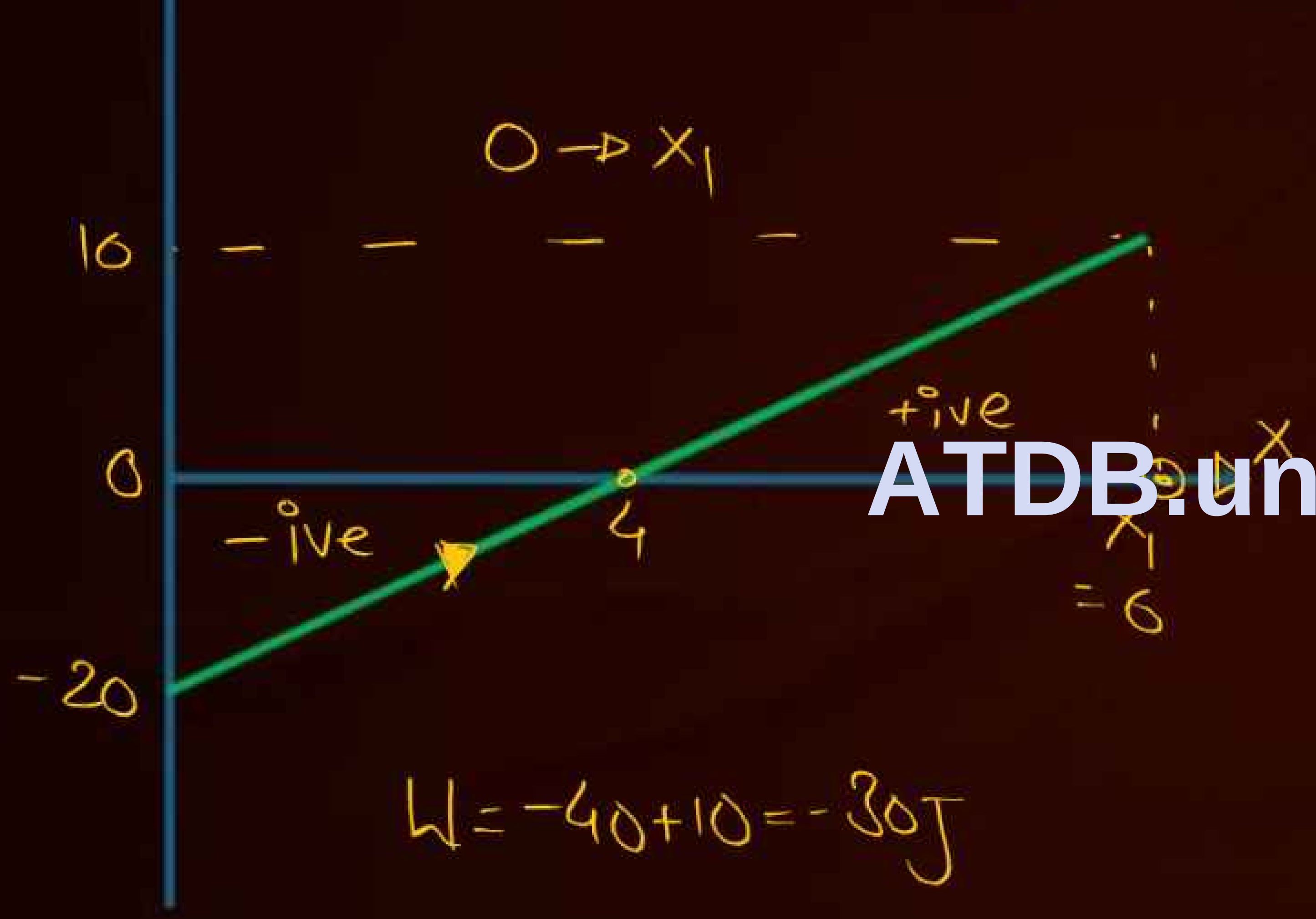
$W_F = FS\cos\theta$

$W_{f_r} = -f_r S$

F-X Graph

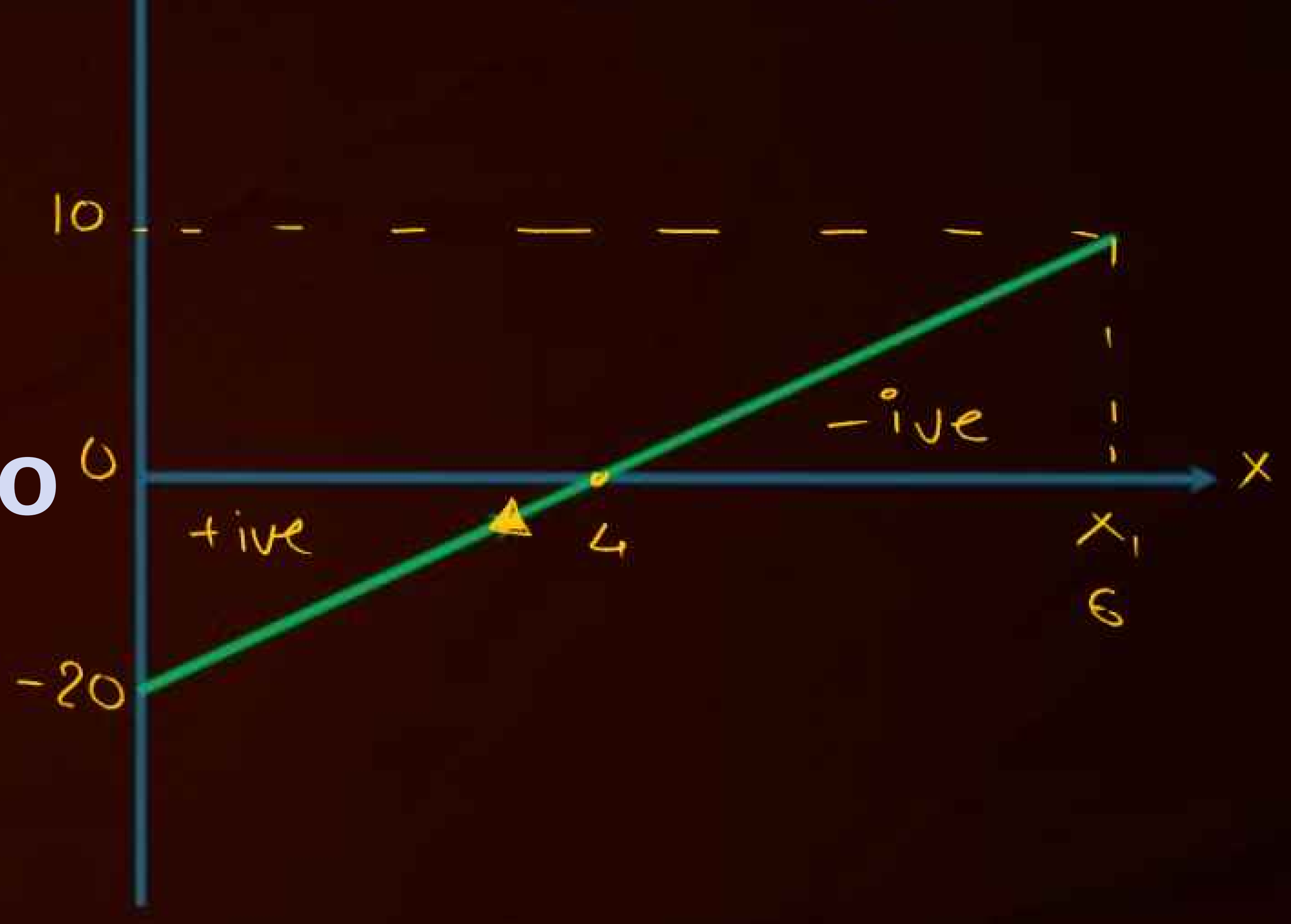
$W = \int F\,dx$ = Area under curve in f-x Graph

(x=6 to x=0)



$W = -40 + 10 = -30J$

$W = 40 - 10 = 30J$



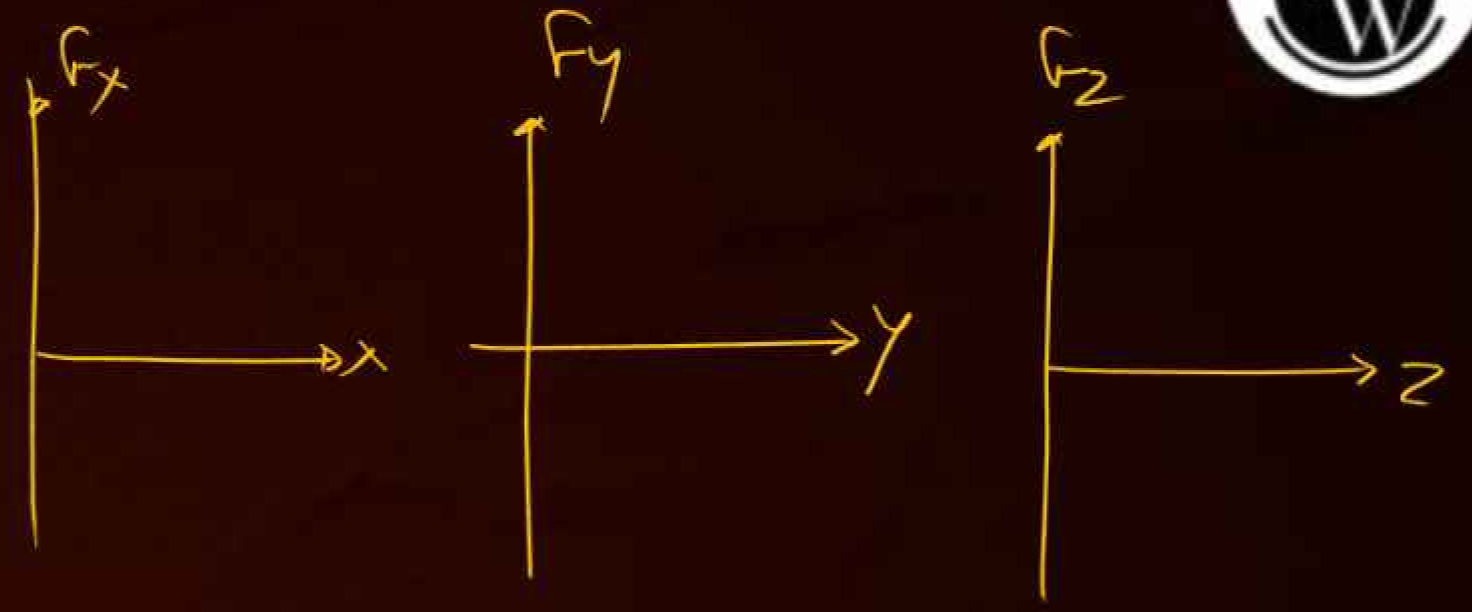
$x_1 \rightarrow 0$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$W = \int F_x\,dx + \int F_y\,dy + \int F_z\,dz$$

Area in

$F_x - x$ Graph



Work done by Variable force

Gravitational Force

Normal Force

Tension Force

Friction Force

Spring Force

Variable Force

Work-Energy Theorem :→

$$\boxed{W_{net} = \Delta K.E.}$$

→ net work done by all forces

→ $W_{\vec{F_1}} + W_{\vec{F_2}} + \ldots = \Delta K.E$

$\left(\vec{F}_{net} = \vec{F_1} + \vec{F_2} + \ldots\right)$ (Sign का ध्यान रखें)

$\left(W_{net} = \int \vec{F}_{net} \cdot \vec{ds} = \int \vec{F_1} \cdot \vec{ds} + \ldots\right)$

→ Valid from inertial frame

non-inertial (linear acc.)

$W_{net} + W_{Pseudo} = \Delta K.E.$

Question

A particle of mass 3 kg is moving along x-axis and its position at time t is given by equation $x = (2t^2 + 5)\ m$. Work done by all the forces acting on it in time interval $t = 0$ to $t = 3$ is

a. 144 J

b. 72 J

c. 108 J

d. 216 J

$V = 4t$

$t = 0$ $t = 3$

$V = 0$ $V = 12$

$\frac{1}{2}m(0)^2 = 0$ $\frac{1}{2} \times 3 \times 12^2$

$= 216$

$W_{net} = \Delta K.E$

$216 - 0$

Ans: (d)

Question

A particle of mass 2 kg travels along a straight line with velocity $v = a\sqrt{x}$, where a is a constant. The work done by net Force during the displacement of particle from $x = 0$ to 4 m is

a. a^2

b. $2a^2$

c. $4a^2$

d. $\sqrt{2}a^2$

$x = 0$, $V = 0$, $k \cdot \mathcal{E} = 0$

$x = 4$, $v = 2a$

$k \cdot \mathcal{E} = \frac{1}{2} \times 2(2a)^2 = \underline{4a^2}$

$v = a\sqrt{x}$

$acc \cdot a = v\frac{dv}{dx} = (a\sqrt{x})\left(\frac{a}{2\sqrt{x}}\right) = \frac{a^2}{2}$

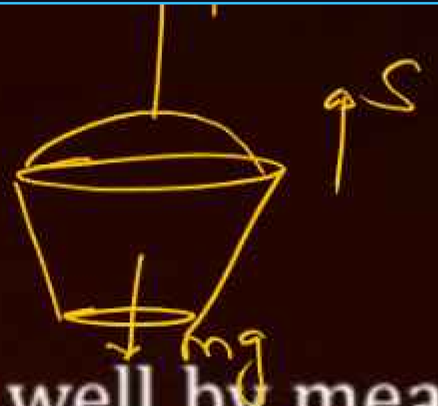
$F = 2\left(\frac{a^2}{2}\right) = \underline{a^2} = \text{Const}$

$W = \overline{(a^2)(4)}$

$\int_0^4 a^2 dx$

Ans: (c)

Question

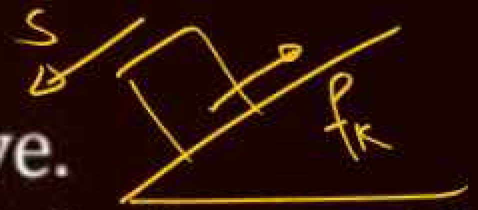


Identify the correct statements from the following:

A. Work done by a person in lifting a bucket out of a well by means of a rope tied to the bucket is negative.

B. Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative.

C. Work done by friction on a body sliding down an inclined plane is positive.



D. Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity is zero.

E. Work done by the air resistance on an oscillating pendulum is negative.

Choose the correct answer from the options given below: **[29 Jan, 2023 (Shift-II)]**

(1) B and E only

(2) A and C only

(3) B, D and E only

(4) B and D only

Ans : (1)

Question

Arrange the four graphs in descending order of total work done; where W_1, W_2, W_3, and W_4, are the work done corresponding to figure A, B, C and d respectively.

[26 June, 2022 (Shift-II)]

1. $W_3 > W_2 > W_1 > W_4$
2. $W_3 > W_2 > W_4 > W_1$
3. $W_2 > W_3 > W_4 > W_1$
4. $W_2 > W_3 > W_1 > W_4$

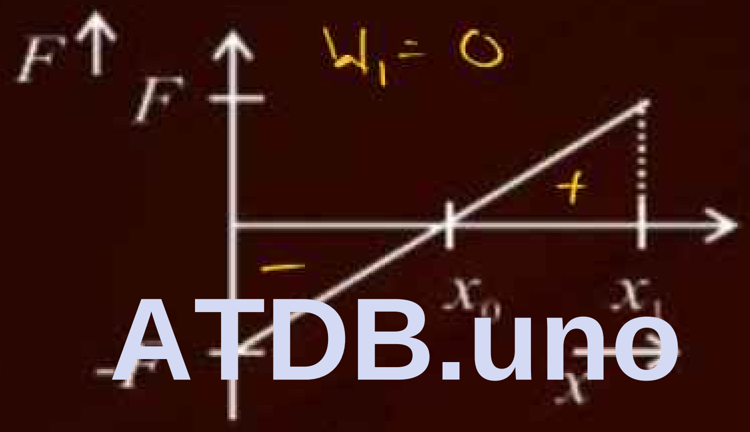


Figure - a

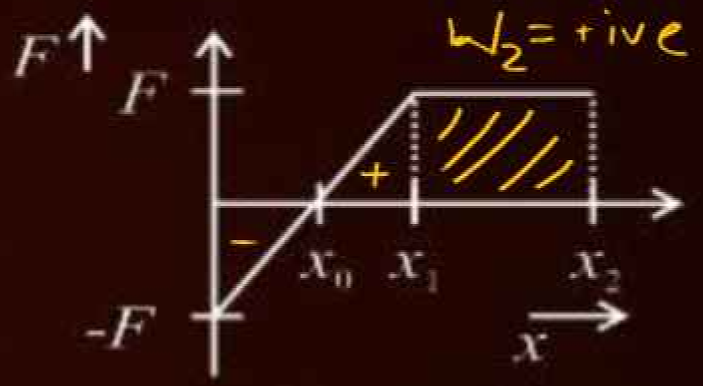


Figure - b

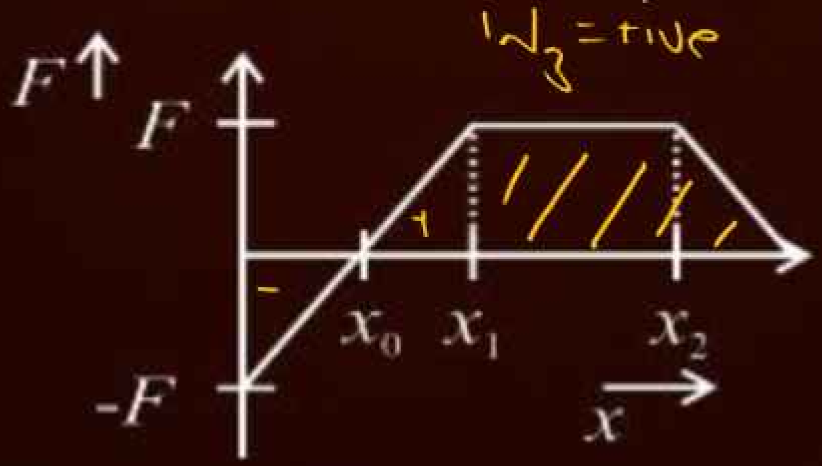


Figure - c

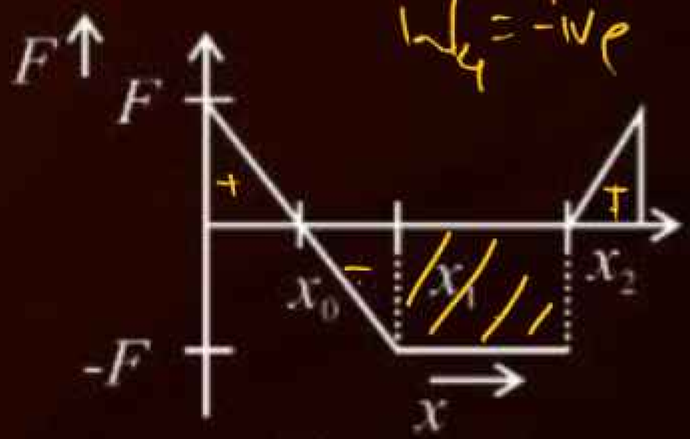


Figure - d

Ans : (1)

Question

A block of mass m is kept on a platform which starts from rest with constant acceleration $g/2$ upward, as shown in figure. Work done by normal reaction on block in time t is:

[10 Jan, 2019 (Shift-I)]

(1) $-\dfrac{mg^2t^2}{8}$

(2) $\dfrac{mg^2t^2}{8}$

(3) 0

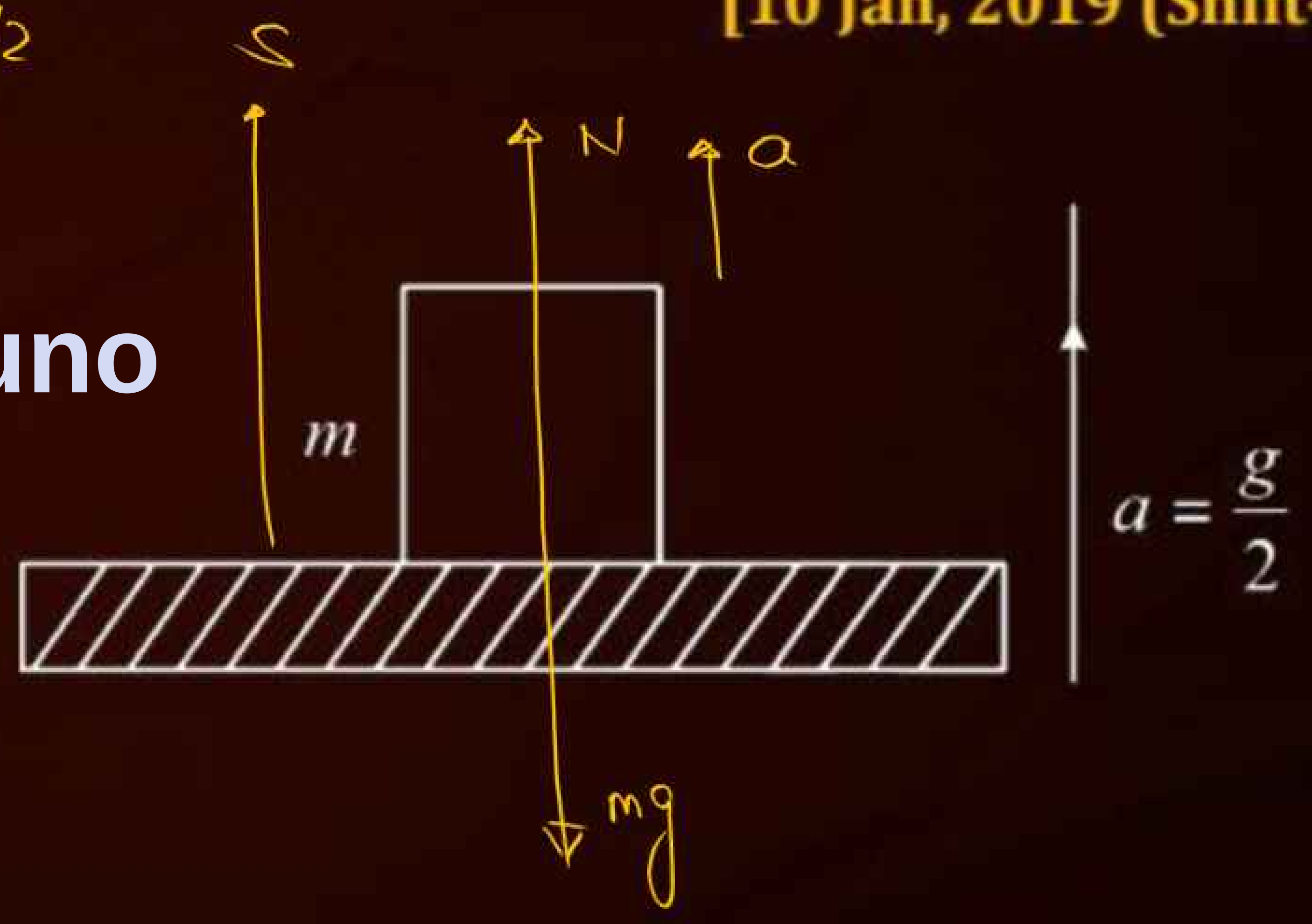
(4) $\dfrac{3mg^2t^2}{8}$ ✓

$N - mg = ma = mg/2$

$N = \dfrac{3mg}{2}$

$S = \cancel{u}^{0}t + \dfrac{1}{2}\dfrac{g}{2}t^2 = \dfrac{gt^2}{4}$

$W_N = NS = \dfrac{3mg^2t^2}{8}$

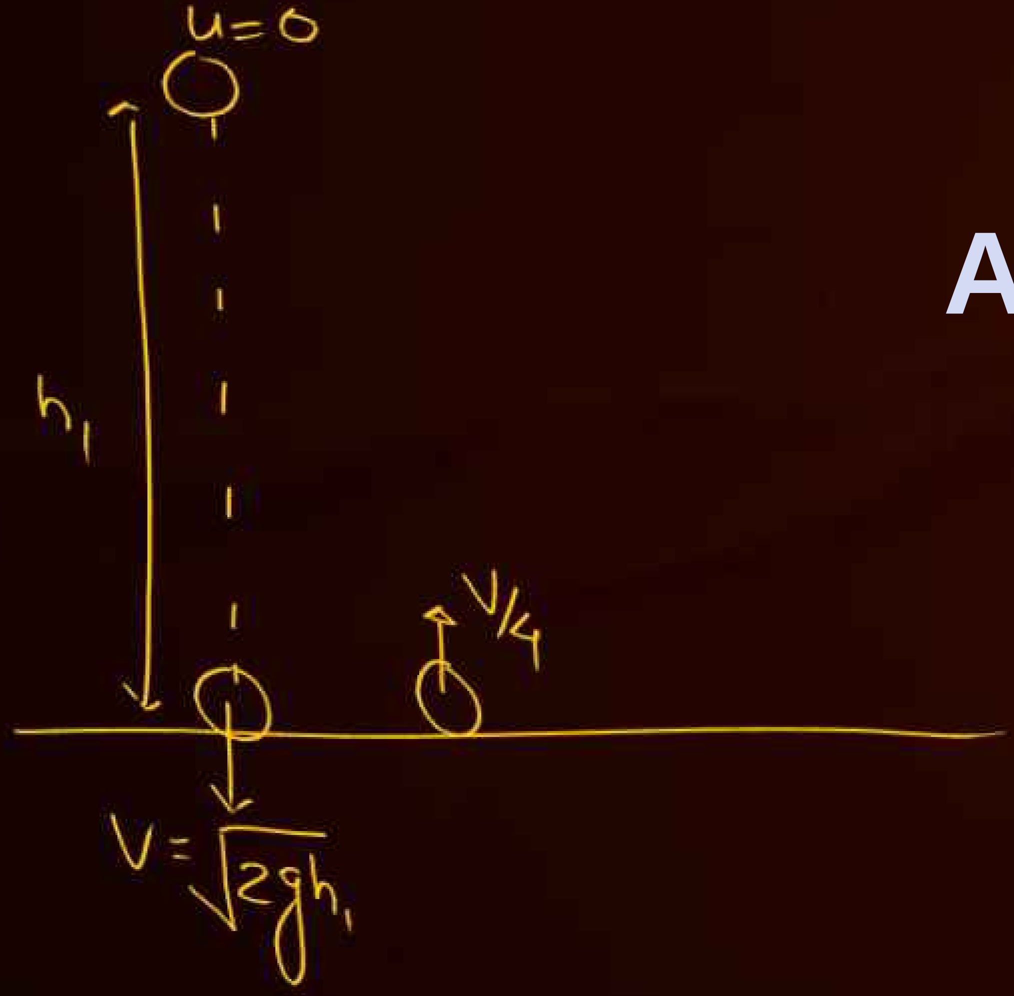


Ans : (4)

Question

A body is dropped on ground from a height 'h_1' and after hitting the ground, it rebounds to a height 'h_2' If the ratio of velocities of the body just before and after hitting ground is 4, then percentage loss in kinetic energy of the body is $\frac{x}{4}$. The value of x is

[6 April, 2023 (Shift-II)]



loss in K.E. = $\frac{1}{2}m(V)^2 - \frac{1}{2}m\left(\frac{V}{4}\right)^2$

$= \frac{1}{2}mv^2\left[1 - \frac{1}{16}\right] = \frac{15}{32}mv^2$

% $\frac{\frac{15}{32}mv^2}{\frac{1}{2}mv^2} \times 100\% = \frac{1500}{16} = \frac{x}{4} \Rightarrow$ $\underline{x = 375}$

Ans : (375)

Question

$F = (5 + 3y^2)$

A force $F = (5 + 3y^2)$ acts on a particle in the y direction, where F is newton and y is in meter. The work done by the force during a displacement from $y = 2m$ to $y = 5m$ is:

[1 Feb, 2023 (Shift-II)]

$$W = \int_2^5 (5 + 3y^2)\, dy$$

Ans : (132)

Question

final

A small particle moves to position $5\hat{i} - 2\hat{j} + \hat{k}m$ from its initial position $2\hat{i} + 3\hat{j} - 4\hat{k}m$ under the action of force $5\hat{i} + 2\hat{j} + 7\hat{k}$ N. The value of work done will be ______ J.

[1 Feb, 2023 (Shift-I)]

$\vec{S} = 3\hat{i} - 5\hat{j} + 5\hat{k}$

$\vec{F} \cdot \vec{S} = 15 - 10 + 35 = 40$

Ans : (40)

Question

A force $\vec{F} = (2 + 3x)\hat{i}$ acts on a particle in the x direction where F is in newton and x is in meter. The work done by this force during a displacement from $x = 0$ to $x = 4$ m, is ______ J. [11 April, 2023 (Shift-I)]

$$\int_0^4 (2+3x)\,dx$$

Ans : (32)

Question

Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is: (all quantities are in SI units)

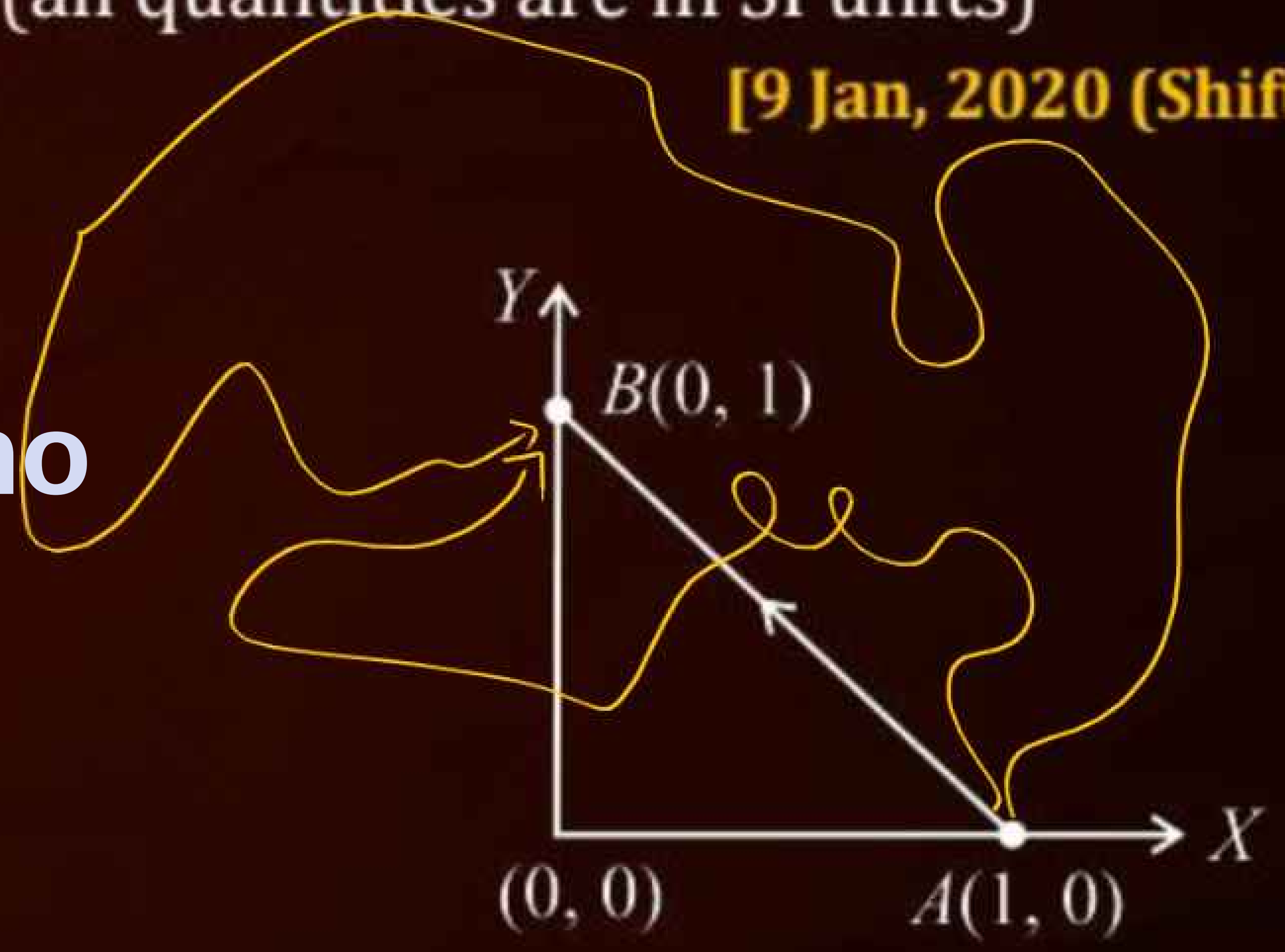
[9 Jan, 2020 (Shift-I)]

1. 1/2
2. 3/2
3. 2
4. 1

$$W = \int_1^0 -x\,dx + \int_0^1 y\,dy$$

$$W = -\frac{1}{2}\left[0^2 - 1^2\right] + \frac{1}{2}\left[1^2 - 0^2\right]$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$



Ans : (4)

Question

A particle is moved along a path $AB - BC - CD - DE - EF - FA$, as shown in figure, in presence of a force $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})$ N where x and y are in meter and $a = -1\ \text{N/m}^{-1}$. The work done on the particle by this force $\vec{F}$ will be _____ Joule

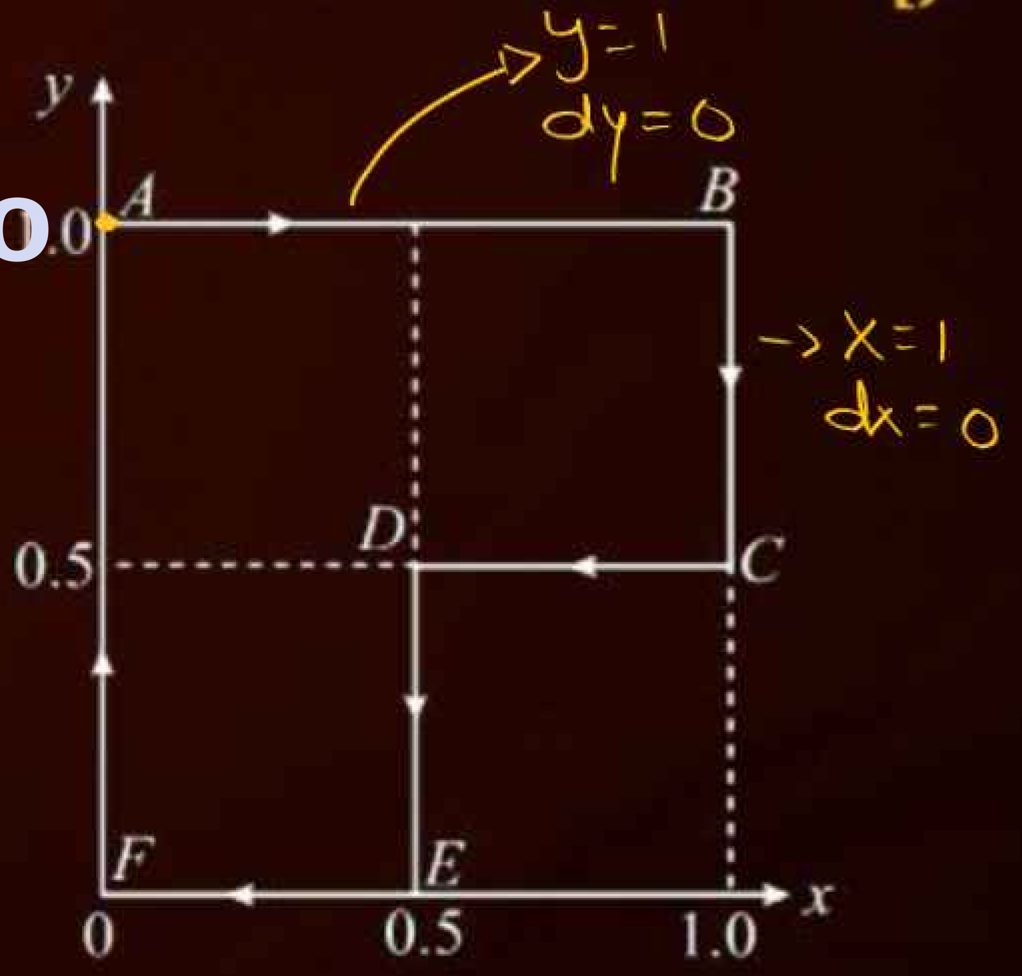
[JEE Adv, 2019]

$$W = \int \alpha y\,dx + 2\alpha x\,dy$$

$$W = \alpha\int(y\,dx + x\,dy) + \alpha\int x\,dy$$

$$\alpha\left[xy\right]_A^A + \underline{\alpha\int x\,dy}$$

$$= 0$$



Ans : (0.75)

Question

A particle of mass 500 gm is moving in a straight line with velocity $\upsilon = bx^{5/2}$. The work done by the net force during its displacement from $x = 0$ to $x = 4$ m is:
(Take $b = 0.25\ m^{-3}/2s^{-1}$).

[29 June, 2022 (Shift-I)]

$V_i = 0$ $\quad V_f = b(4)^{5/2} = 32b$

$K.E_i = 0$ $\quad K.E_f = \frac{1}{2} \times \frac{1}{2}(32b)^2$

$= 32 \times 8b^2$

$= 32 \times 8 \times \frac{1}{16}$

(1) 2 J

(2) 4 J

(3) 8 J

(4) 16 J

Ans : (4)

Question

A block of mass 2 kg moving on a horizontal surface with speed of 4 ms^{-1} enters a rough surface ranging from $x = 0.5$ m to $x = 1.5$ m. The retarding force in this range of rough surface is related to distance by $F = -kx$, where $k = 12\ Nm^{-1}$. The speed of the block as it just crosses the rough surface will be:

[28 June, 2022 (Shift-II)]

(1) Zero

(2) 1.5 ms^{-1}

(3) 2.0 ms^{-1}

(4) 2.5 ms^{-1}

$$W = \int_{1/2}^{3/2} -kx\,dx$$

$$= -\frac{12}{2}\left[\left(\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right]$$

$$= -6\left[\frac{8}{4}\right] = -12\,J$$

$$W_{net} = \Delta K.E$$

$$-12 = \frac{1}{2}\times 2\times v^2 - \frac{1}{2}\times 2\times 4^2$$

$$v^2 = 16 - 12 \Rightarrow \underline{v = 2\,m/s}$$

Ans : (3)

Question

A particle experiences a variable force $\vec{F} = (4x\hat{i} + 3y^2\hat{j})$ in a horizontal $x - y$ plane. Assume distance in meters and force is Newton. If the particle moves from point $(1, 2)$ to point $(2, 3)$ in the $x - y$ plane, then kinetic Energy changes by: **[24 June, 2022 (Shift-I)]**

Work

$$= \int_1^2 4x\,dx + \int_2^3 3y^2\,dy$$

1. 50.0 J
2. 12.5 J
3. 25.0 J
4. 0 J

Ans : (3)

Question

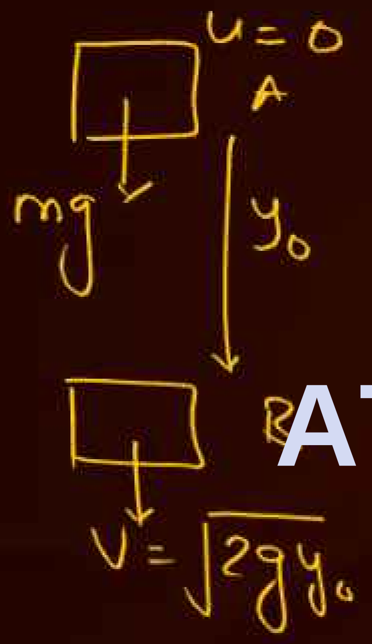
In the given figure, the block of mass m is dropped from the point 'A'. The expression for kinetic energy of block when it reaches point 'B' is: **[29 June, 2022 (Shift-II)]**

(1) $\frac{1}{2}mgy_0^2$

(2) $\frac{1}{2}mgy^2$

(3) $mg(y - y_0)$

(4) mgy_0

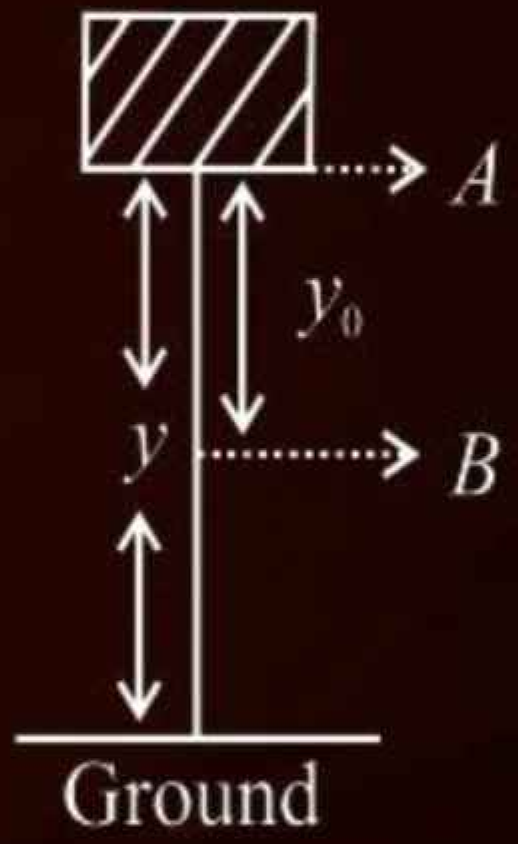


$K.E = \frac{1}{2}m(2gy_0)$

$= \underline{mgy_0}$

$W = \Delta K.E$

$mgy_0 = K.E_B - 0$



Ans : $\cancel{(3)}$ 4

Question

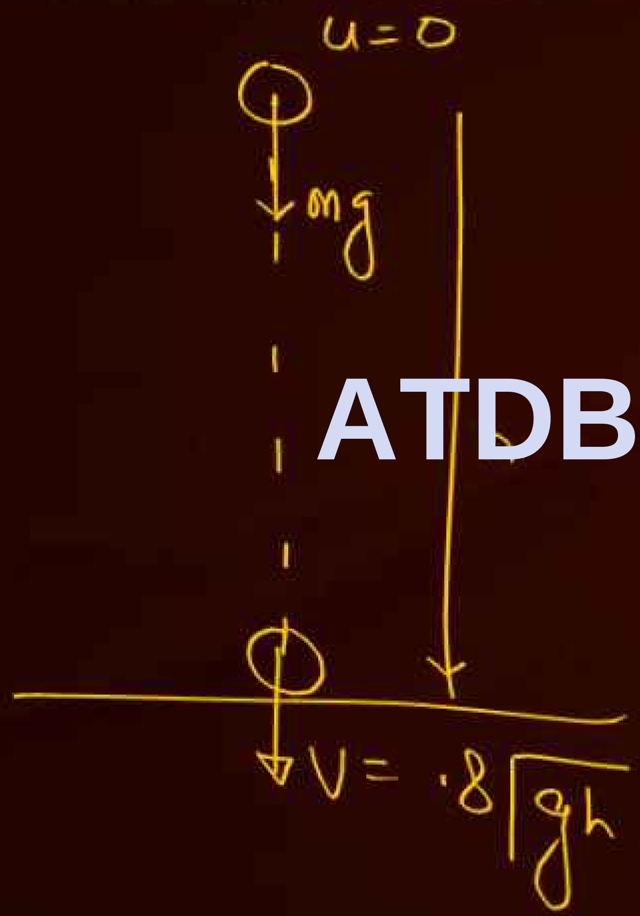
A body of mass 'm' dropped from a height 'h' reaches the ground with a speed of $0.8\sqrt{gh}$. The value of work done by the air-friction is: [1 Sep, 2021 (Shift-II)]

(1) –0.68 mgh

(2) 0.64 mgh

(3) mgh

(4) 1.64 mgh



$$W_{mg} + W_{aR} = \Delta K.E$$

$$mgh + W_{aR} = \frac{1}{2}m\left(.8\sqrt{gh}\right)^2 - 0$$

$$W_{aR} = -mgh + .32\,mgh$$

$$= -.68\,mgh$$

Ans : (1)

Question

A particle which is experiencing a force, given by $\vec{F} = 3\hat{i} - 12\hat{j}$, undergoes a displacement of $\vec{d} = 4\hat{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement?

[10 Jan, 2019 (Shift-II)]

$W = \vec{F} \cdot \vec{d} = K.E_f - 3$

$\downarrow$

12

(1) 9 J

(2) 12 J

(3) 10 J

(4) 15 J

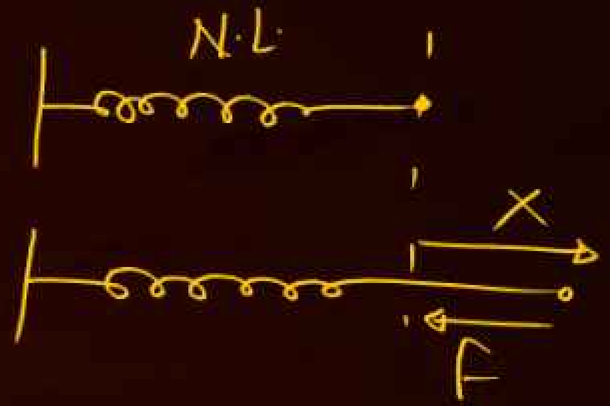
Ans : (4)

Work done of gravity

$W = mgh$ $\qquad$ $W = -mgh$

Work done by Spring force

$F = -kx$



$$W = -\int_{x_1}^{x_2} kx\,dx$$

** $\boxed{W = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)}$ $\qquad$ $x_2, x_1 \rightarrow$ Elongation / Compression

Work done by Spring

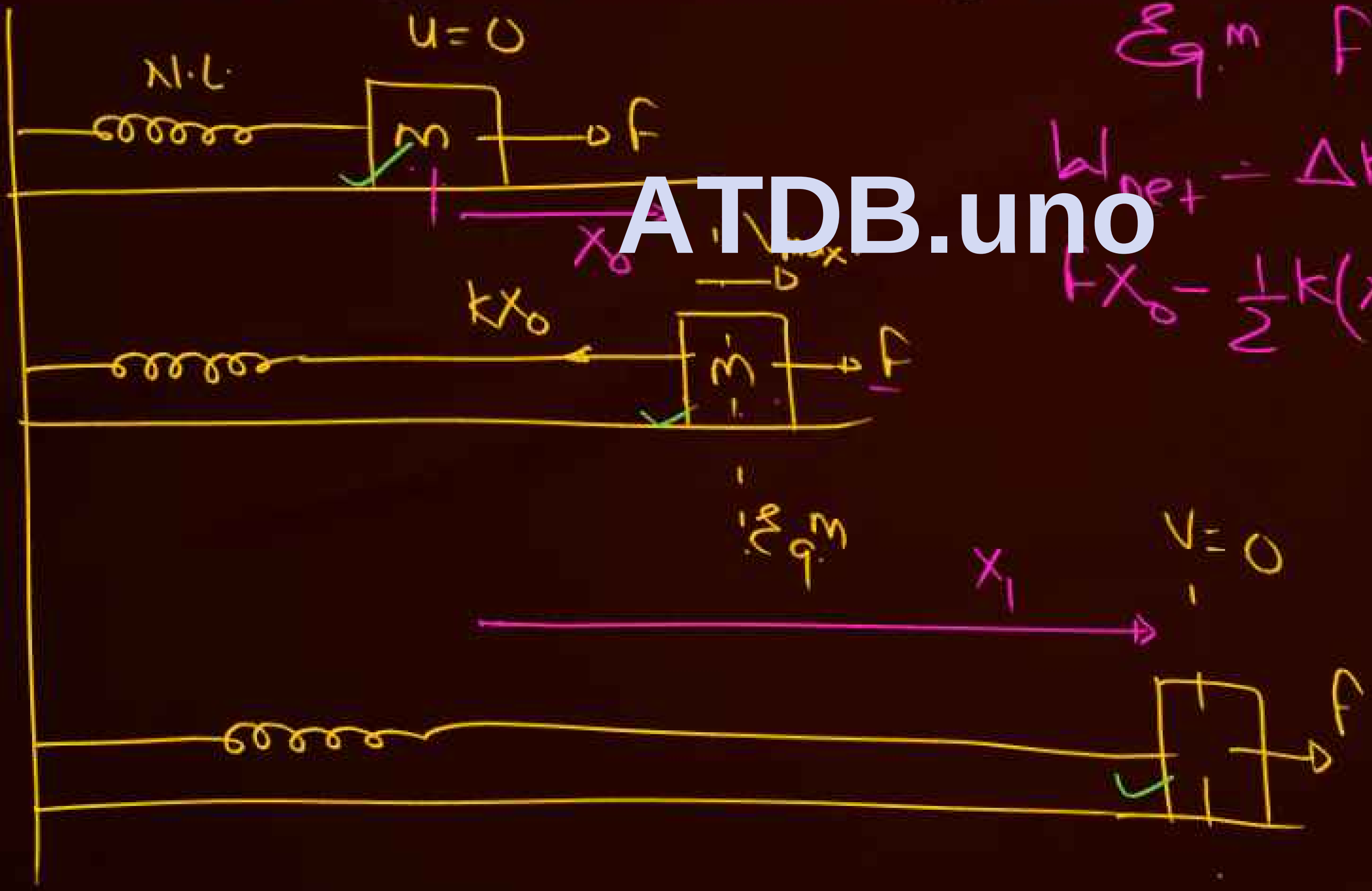
Question

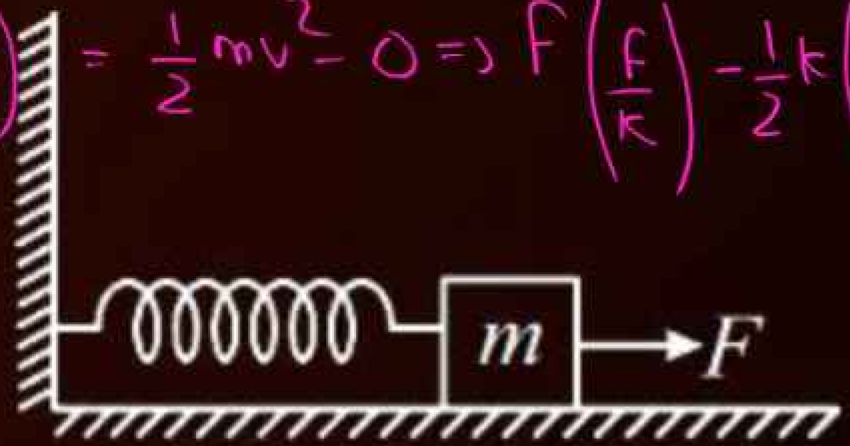
$W = \Delta K \cdot E$

$Fx_1 - \frac{1}{2}k(x_1^2 - 0^2) = 0 - 0 \Rightarrow x_1 = \frac{2F}{K}$

N.L. V_{max} $v = 0$ Eq^m $x_0 = \frac{mg}{K}$

A block of mass *m*, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant *k*. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in an equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is: **[9 Jan, 2019 (Shift-I)]**





(1) $\frac{2F}{\sqrt{mk}}$

(2) $\frac{F}{\pi\sqrt{mk}}$

(3) $\frac{\pi F}{\sqrt{mk}}$

(4) $\frac{F}{\sqrt{mk}}$

Eq^m $F = kx_0 \Rightarrow x_0 = F/K$

$W_{net} = \Delta K \cdot E$

$Fx_0 - \frac{1}{2}k(x_0^2 - 0^2) = \frac{1}{2}mv^2 - 0 \Rightarrow F\left(\frac{F}{K}\right) - \frac{1}{2}k\left(\frac{F^2}{K^2}\right)$

$V = \sqrt{\frac{F}{mk}}$

Ans : (4)

Question

A closed circular tube of average radius 15 cm, whose inner walls are rough, is kept in vertical plane. A block of mass 1 kg just fit inside the tube. The speed of block is 22 m/s, when it is introduced at the top of tube. After completing five oscillations, the block stops at the bottom region of tube. The work done by the tube on the block ______ is J. [Given $g = 10\ m/s^2$]

[10 April, 2023 (Shift-I)]

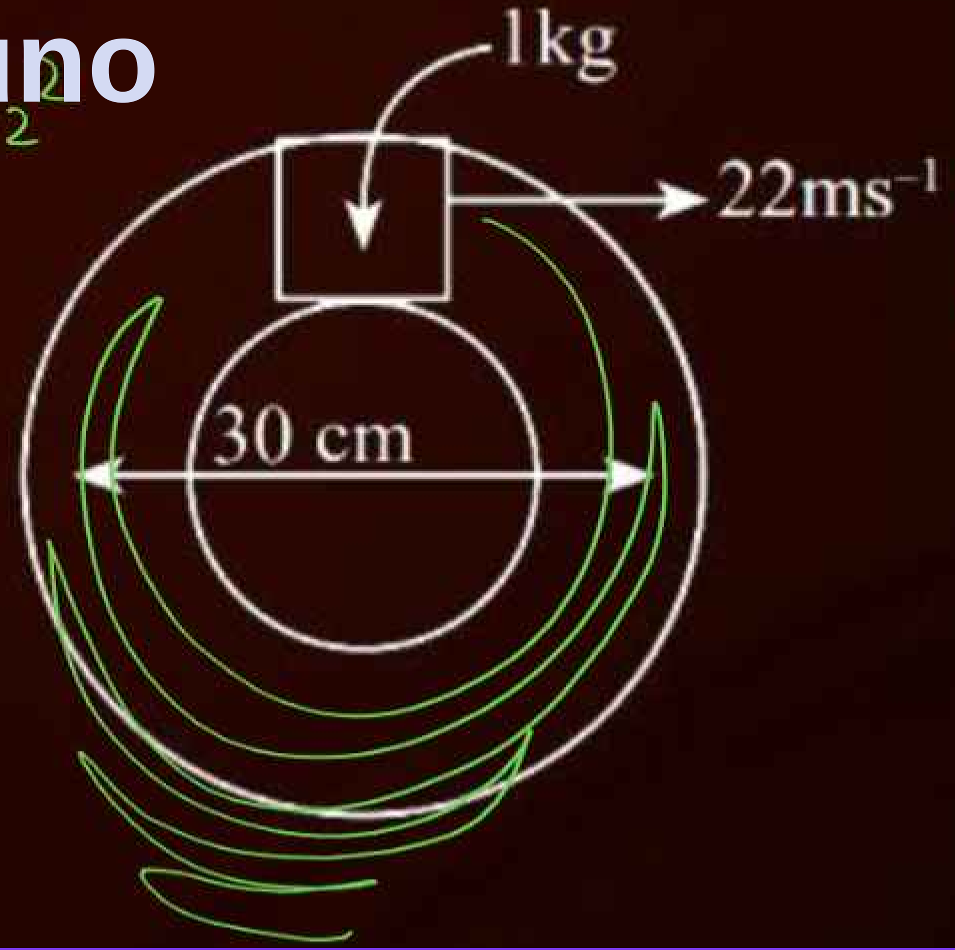
$$W_{mg} + W_{tube} = \Delta K.E.$$

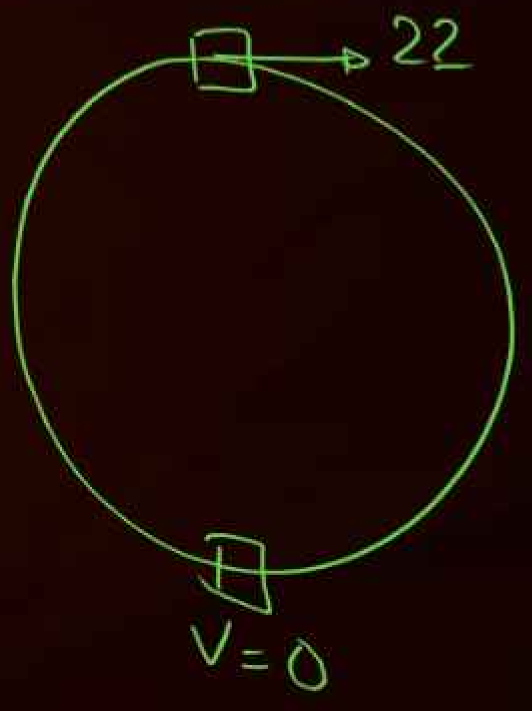
121

$$mg(.3) + W_{tube} = 0 - \frac{1}{2} \times 1 \times 22^2$$

$$3 + W_t = -242$$

$$W_t = -245$$





Ans : (–245)

Question

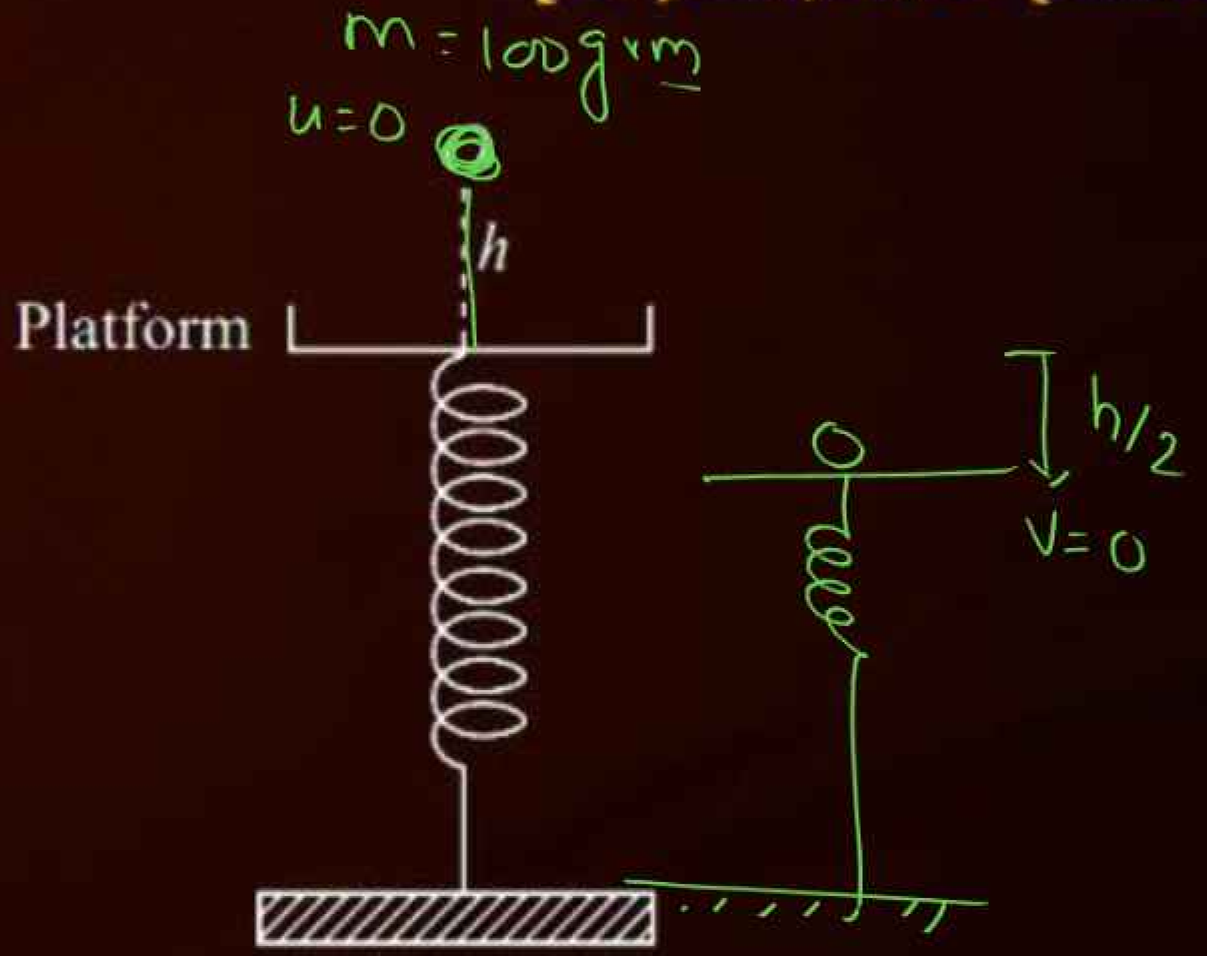
massless

A ball of mass 100 g is dropped from a height $h = 10$ cm on a platform fixed at the top of a vertical spring (as shown in figure). The ball stays on the platform and the platform is depressed by a distance $h/2$. The spring constant is ______ Nm^{-1}. (Use $g = 10\ \mathrm{ms}^{-2}$)

[24 June, 2022 (Shift-I)]

$$W_{mg} + W_{spring} = \Delta K.E.$$

$$mg\left(\frac{3h}{2}\right) - \frac{1}{2}k\left(\left(\frac{h}{2}\right)^2 - 0^2\right) = 0 - 0$$



Ans : (120)

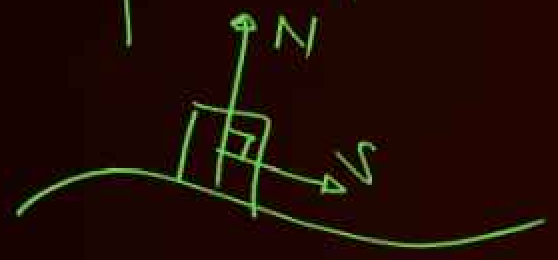
Question

As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s. The value of 'x' to the nearest integer is (Take $g = 10\ m/s^2$)

[18 March, 2021 (Shift-I)]

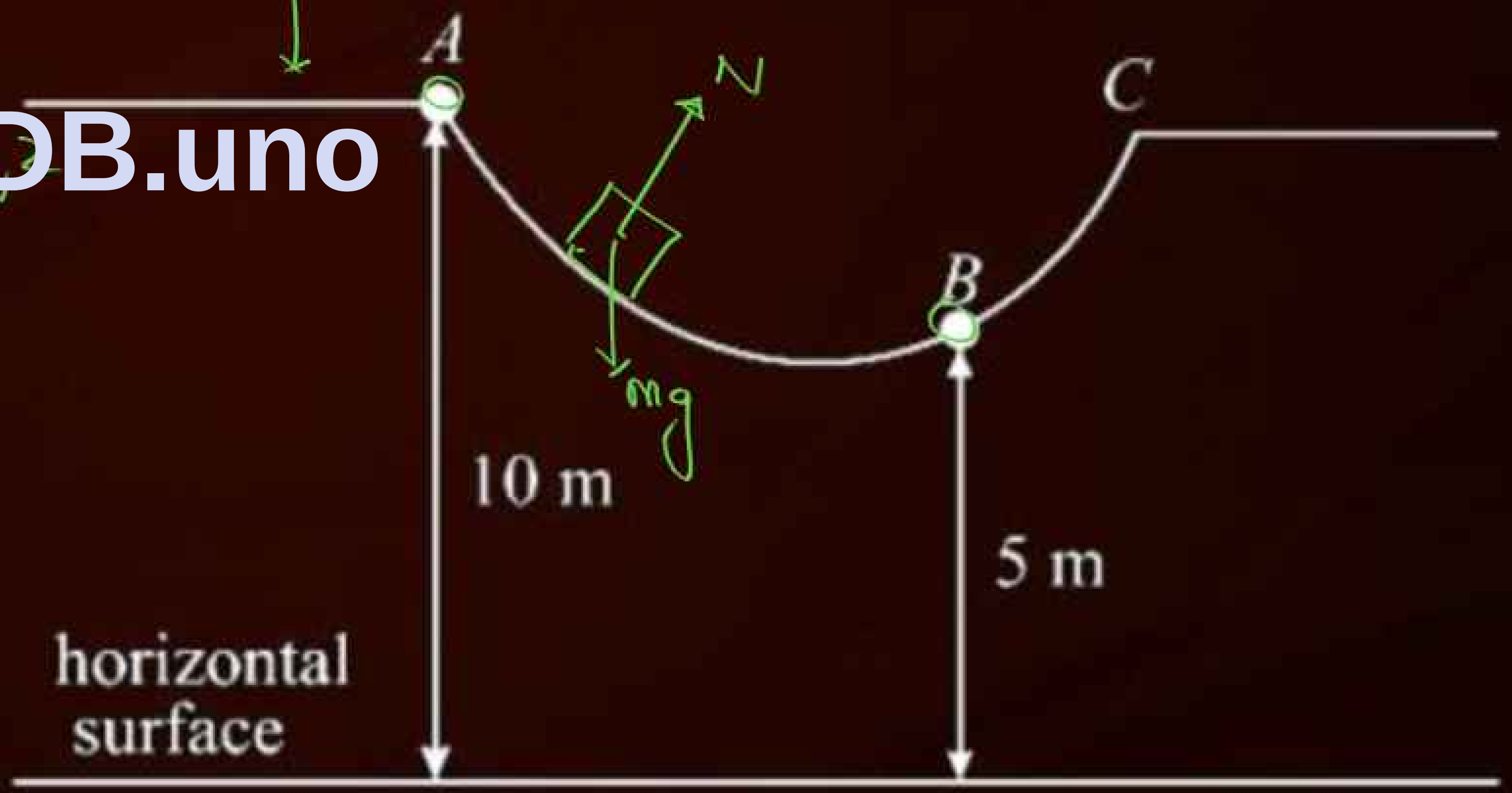
on fixed path $W_N = 0$

Smooth



$\cancel{m}g(5) = \frac{1}{2}\cancel{m}v^2$

$\Rightarrow V = 10\ m/s$

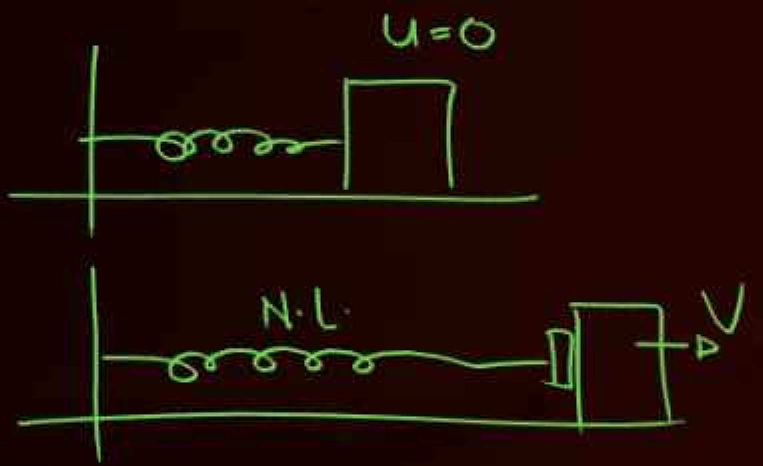


Ans : (10)

Question

In a spring gun having spring constant 100N/m a small ball 'B' of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05 m. There should be a box placed at a distance 'd' on the ground so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2 m above the ground . The value of d is ______m.

[20 July, 2021 (Shift-I)]



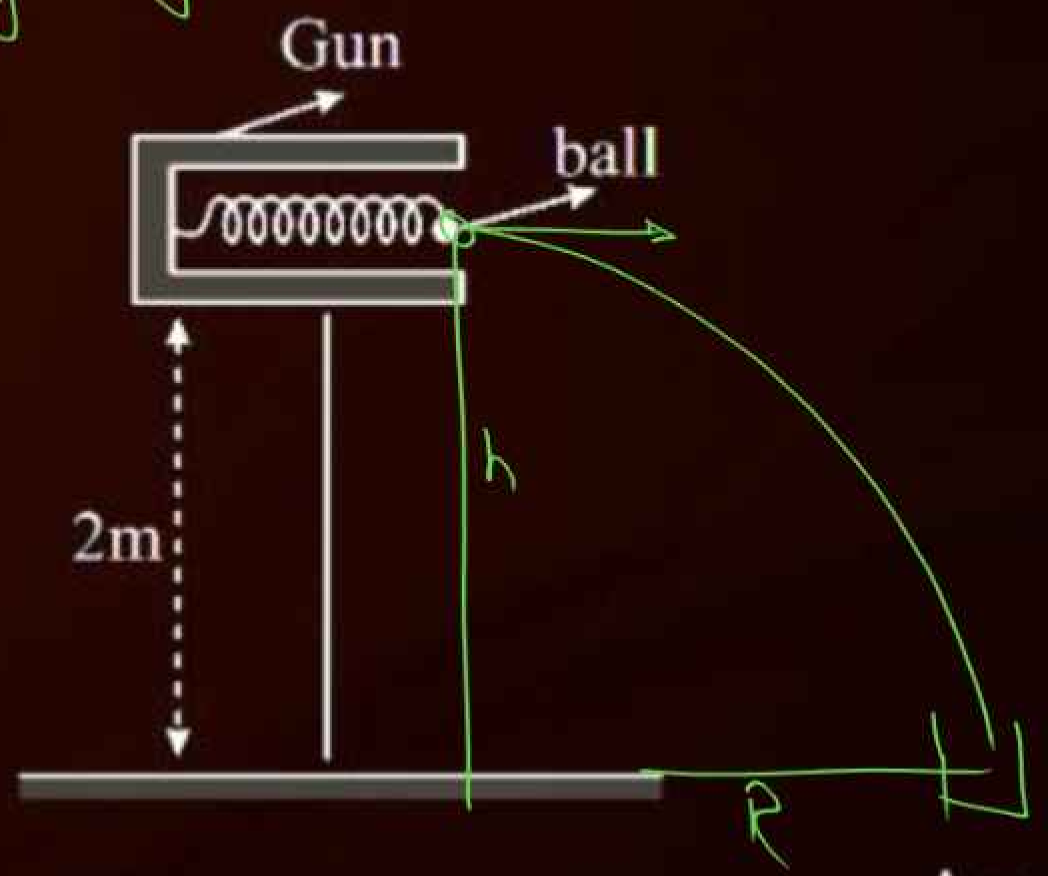
$T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{4}{g}} = \sqrt{\frac{2}{5}}$

$d = vT$

$= \sqrt{\frac{5}{2}}\sqrt{\frac{2}{5}} = 1$

$-\frac{1}{2}100\left(0^2 - \left(\frac{5}{100}\right)^2\right) = \frac{1}{2}mv^2 - 0$

$\frac{25}{100} = \frac{1}{10}v^2 \Rightarrow v^2 = 2.5$

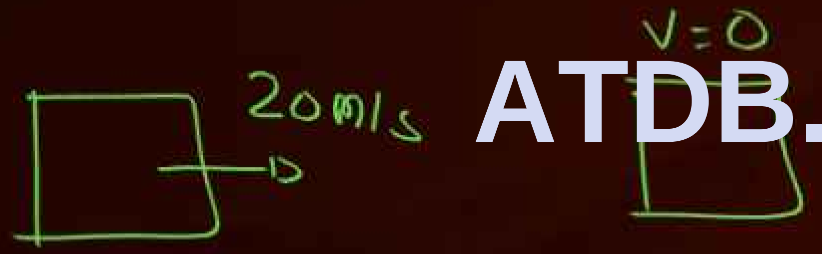


Ans : (1)

Question

An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 40,000 kg is moving with a speed of 72 kmh^{-1} when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, the spring constant is __16__ $\times 10^5$ N/m. **[1 Sep, 2021 (Shift-II)]**

N.L → 1m



$$10\% \text{ of } K.E = -\left(-\frac{1}{2}k(1^2-0^2)\right)$$

$$\frac{\cancel{10}}{\cancel{100}_{10}}\times\frac{1}{\cancel{2}}\times(40000)(400)=\frac{1}{\cancel{2}}k$$

Ans : (16)

Conservative Forces

electrostatic force

Gravitational Force

Spring Force

Constant Force

Central Force

$\vec{F} = x\hat{i} + y\hat{j}$

$\vec{F} = y\hat{i} + x\hat{j}$

Non Conservative Forces

Gravitational P.E

Potential Energy (U)

Elastic P.E (Spring)

★★ $\boxed{W_{net} = \Delta K.E.}$

$W_C + W_{NC} = \Delta K.E.$

W_{NC} → Path

W_C, $\Delta K.E.$ → initial and final

$W_{NC} = \Delta K.E. - W_C$

$W_{NC} = \Delta K.E. + \Delta U$

→ Change in P.E. is −ive of work done by Conservative force

★★ $\boxed{\Delta U = -W_C}$

→ K.E + U → Total mechanical energy (TME)

★★ $\boxed{W_{NC} = \Delta K.E. + \Delta U = \Delta T.M.E.}$

Total Mechanical energy Conservation

if $\underline{W_{NC} = 0}$ (if net work done by Conservative forces is Zero then T.M.E. is conserved)

$\Rightarrow \Delta T.M.E = 0$

$T.M.E._i = T.M.E_f$

$$\boxed{K.E_i + U_i = K.E_f + U_f}$$

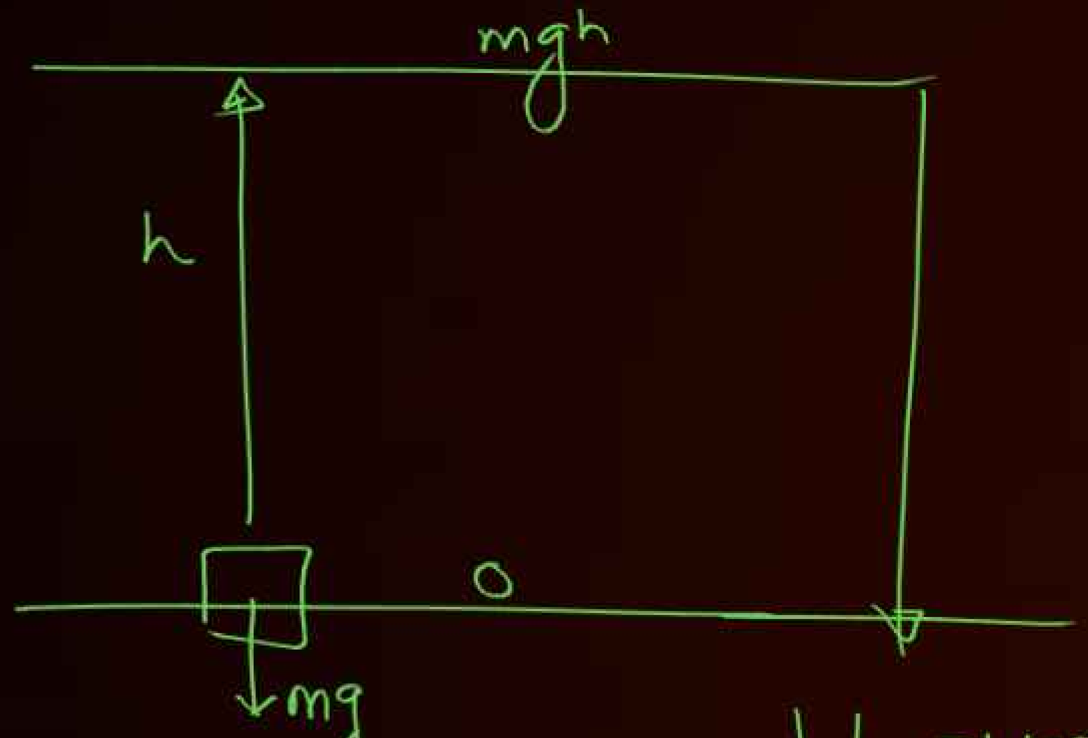
$$\boxed{W_{net} = \Delta K.E}$$

$$\Delta U = -W_C$$

$$K.E + U = T.M.E$$

$$W_{NC} = \Delta T.M.E$$

Gravitation P.E



$W_{mg} = -mgh$

$\Delta U = mgh$

$W_{mg} = +mgh$

$\Delta U = -mgh$

Elastic P.E

$$\Delta U = -W_{spring} = -\left(-\frac{1}{2}k(x^2 - x_1^2)\right)$$

$$\Delta U = \frac{1}{2}k(x_2^2 - x_1^2) = U_2 - U_1$$

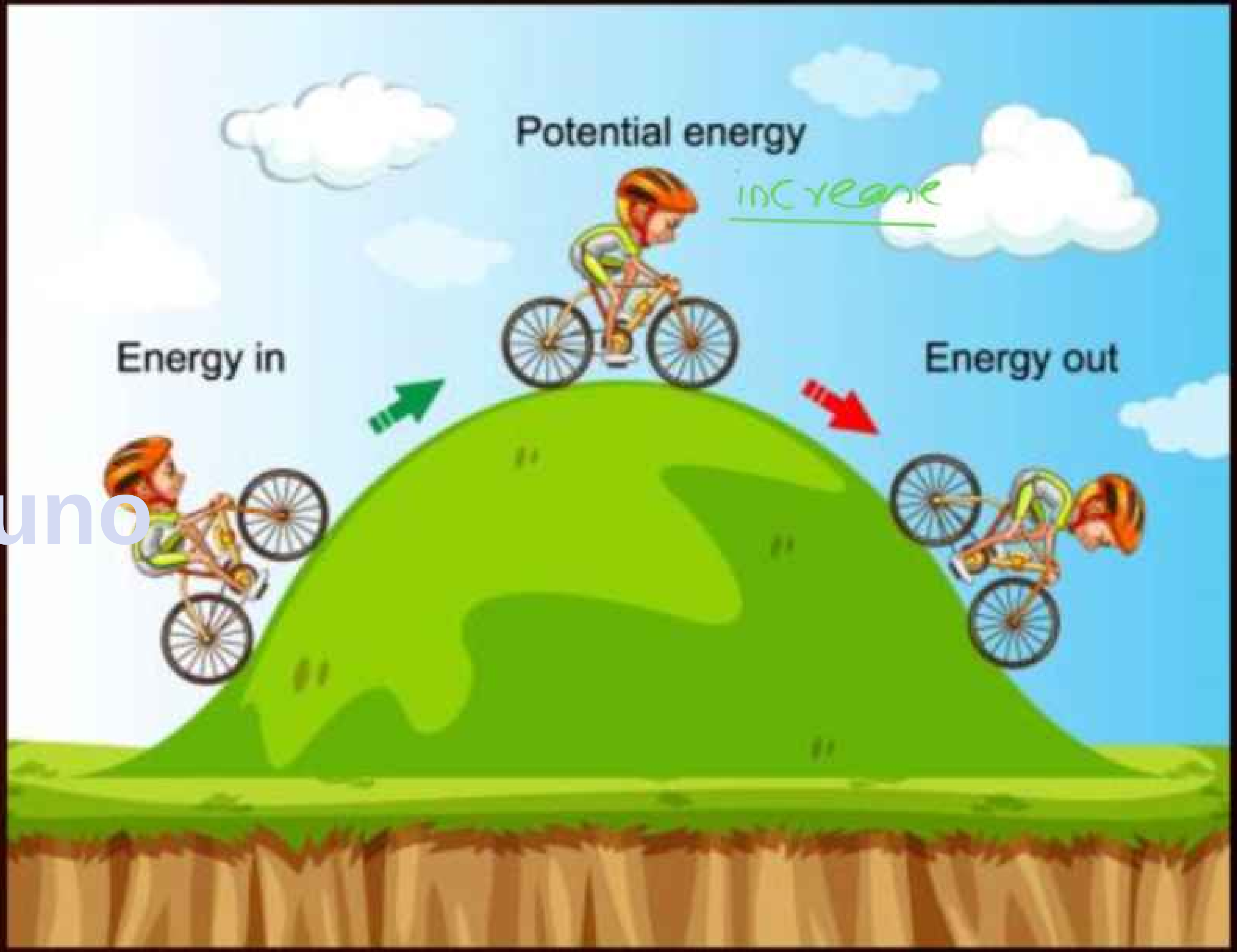
Let $U_1 = 0$ at natural length $(x_1 = 0)$

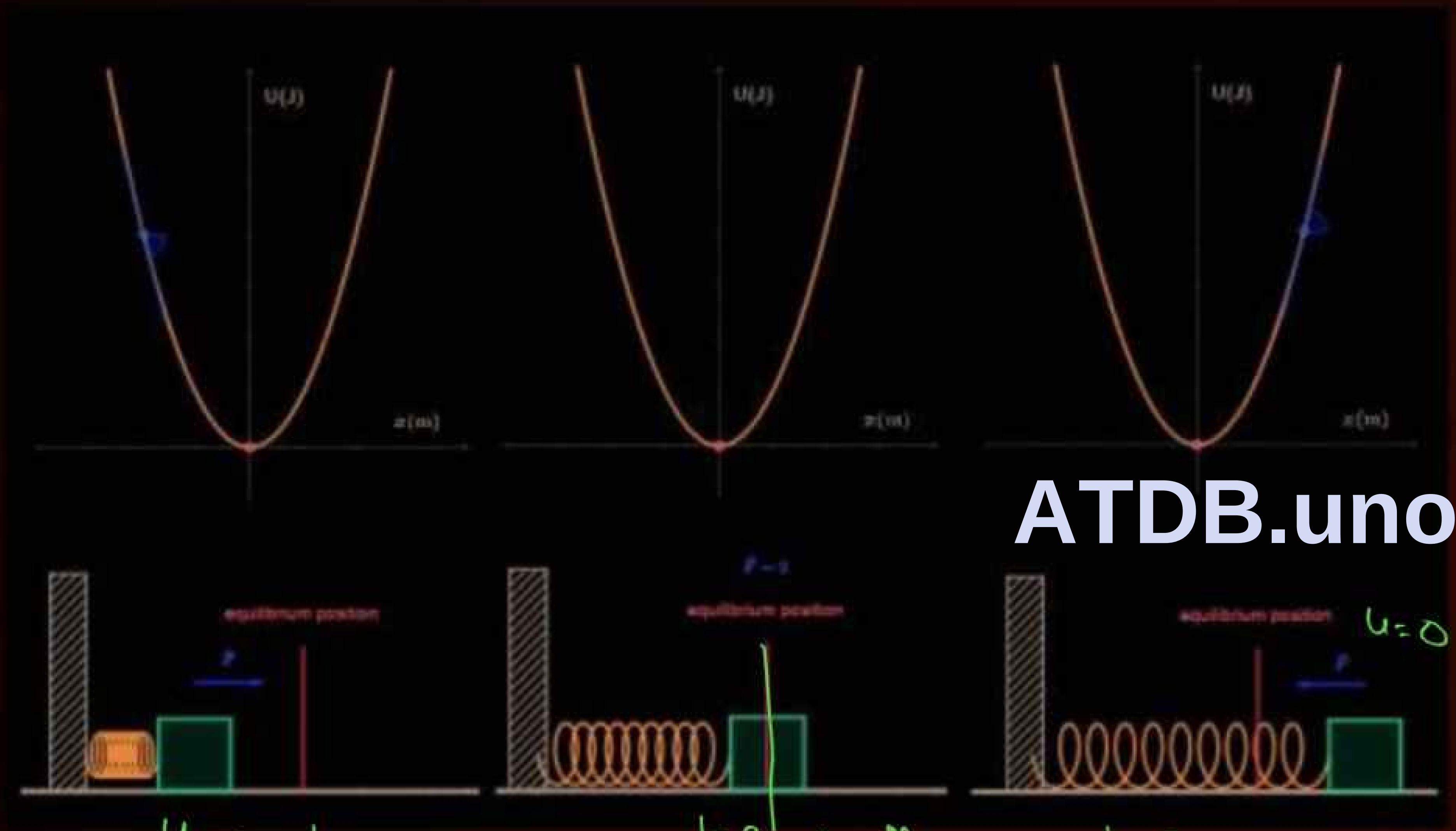
** $\boxed{U = \frac{1}{2}kx^2}$

$$\left\{ U = \frac{1}{2}kx^2 + U_0 \right)$$

P.E. at N.L.

Mechanical Energy Conservation





$u = 0$

$U = 0$ $\quad k.E = 0$

$U \to max^m$

$k.E \to max^m$

$U \to 0$

$k.E \Rightarrow 0$

$U \to max^m$

Question

When a conservative force does positive work on a body then the

a. Potential energy of body increases

b. Potential energy of body decreases

c. Total mechanical energy of body increases

Constant

d. Total mechanical energy of body decreases

$\Delta U = -W_c$

+ive

$\Delta U = -\text{ive} = U_2 - U_1$

U decreases

Ans: (b)

Question

Force constants K_1 and K_2 of two springs are in the ratio 5 : 4. They are stretched by same length. If potential energy stored in 1st spring is 25 J then potential energy stored in second spring is

a. 25 J

b. 16 J

c. 100 J

d. 20 J

$\frac{k_1}{k_2} = \frac{5}{4}$ $\quad$ $\underline{x_1 = x_2}$

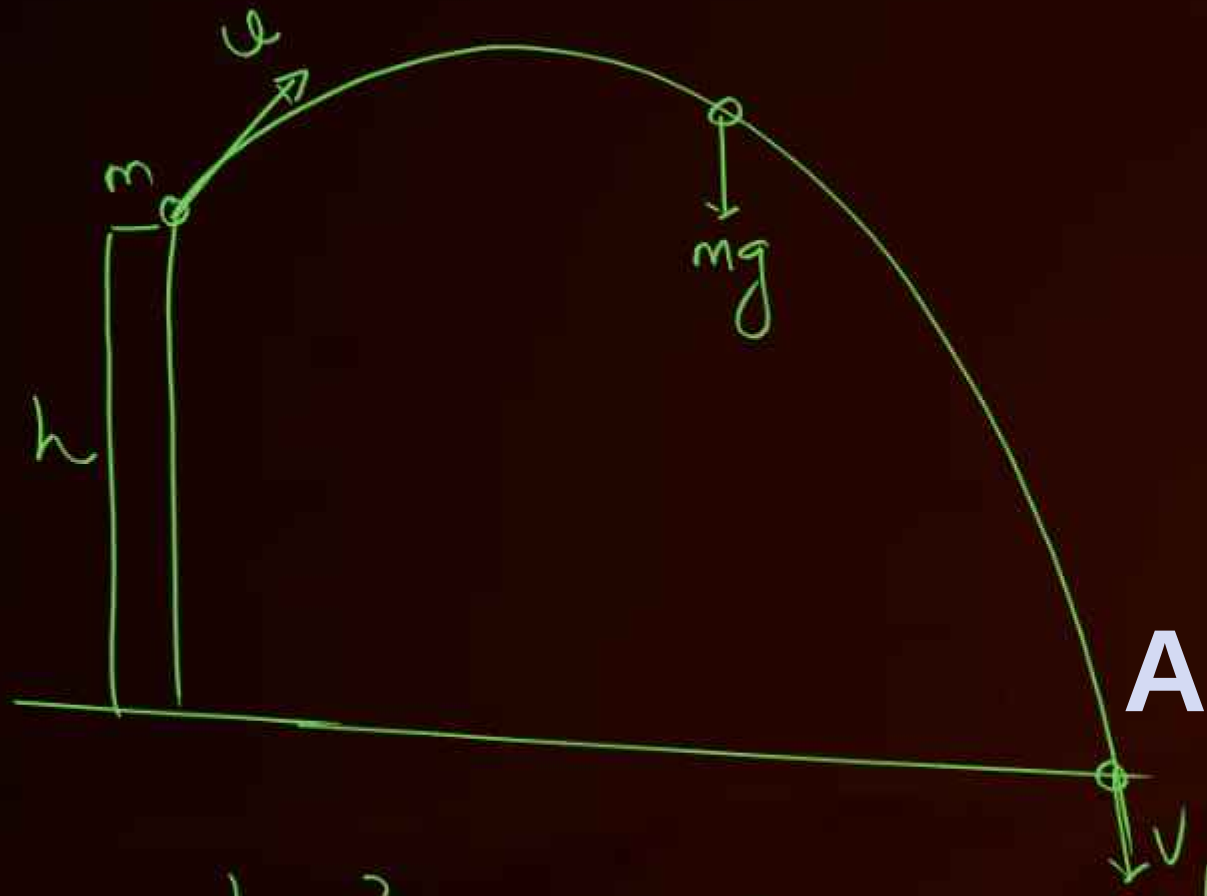
$\frac{\frac{1}{2}k_1 x^2}{\frac{1}{2}k_2 x^2} = \frac{25}{N}$

$\frac{5}{4} = \frac{25}{N} \Rightarrow \underline{N = 20J}$

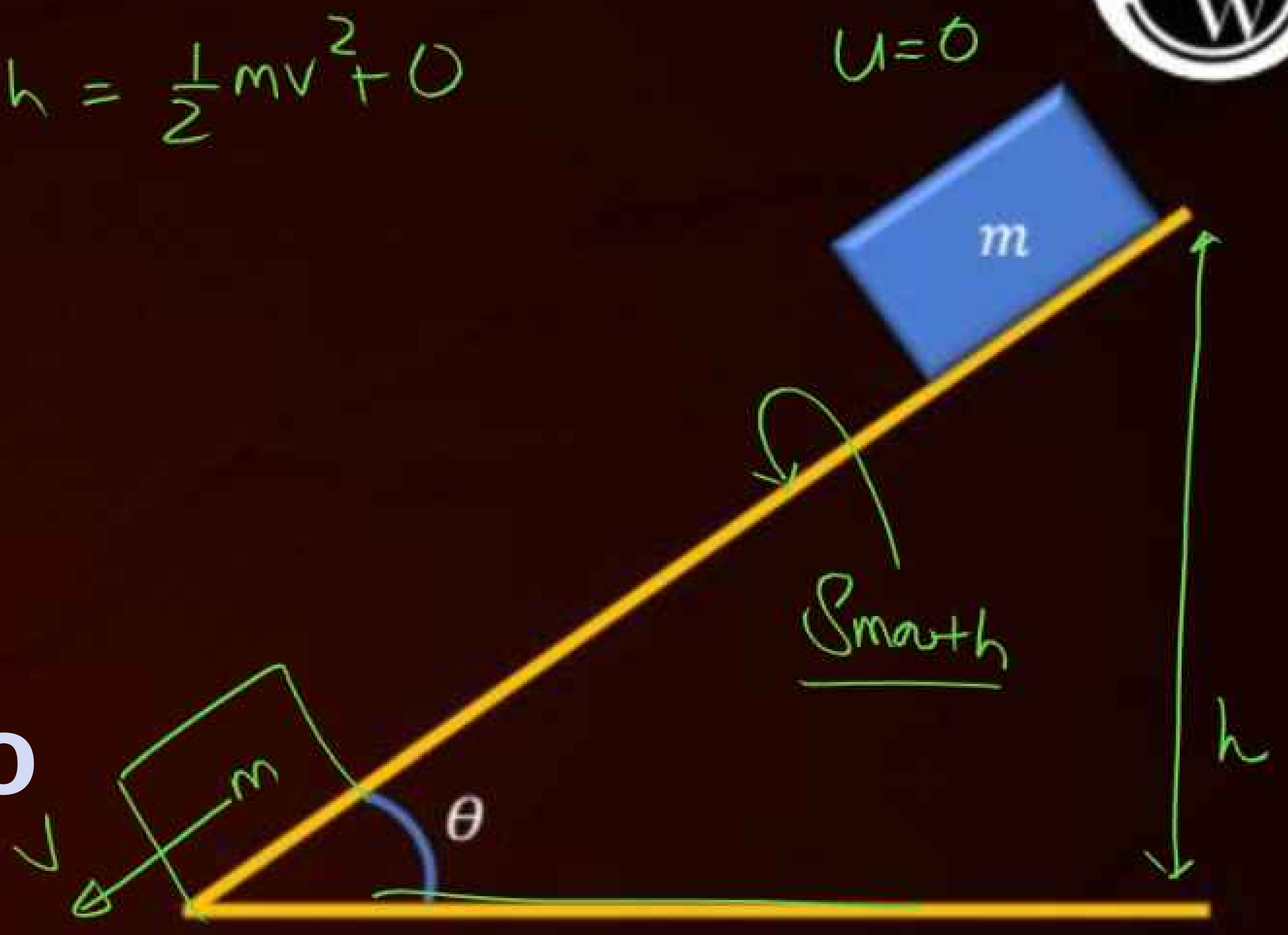
Ans: (d)

$$K_1 + U_1 = K_2 + U_2$$

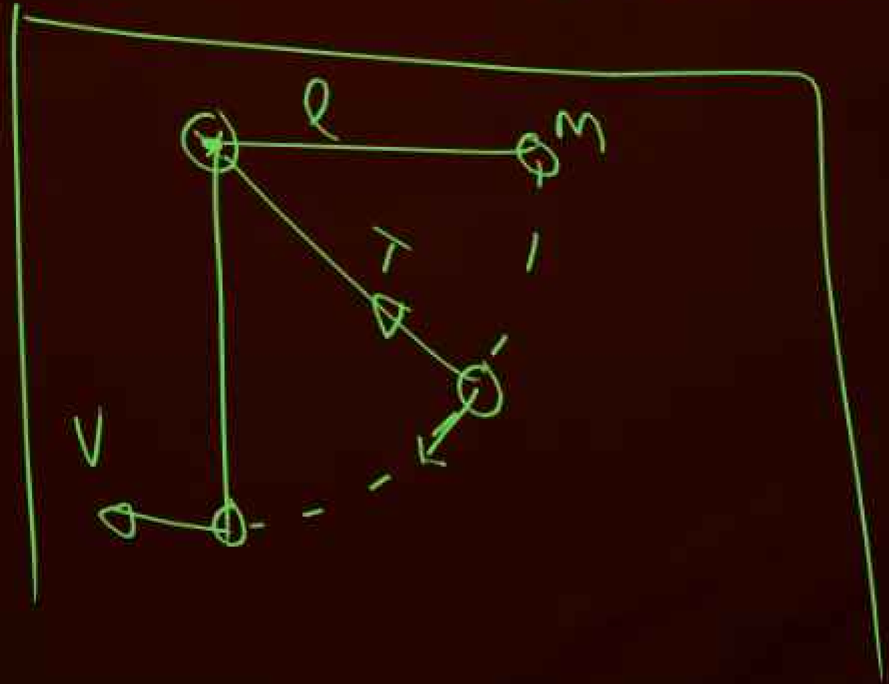
$$0 + mgh = \frac{1}{2}mv^2 + 0$$



$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mu^2 + mgh$$



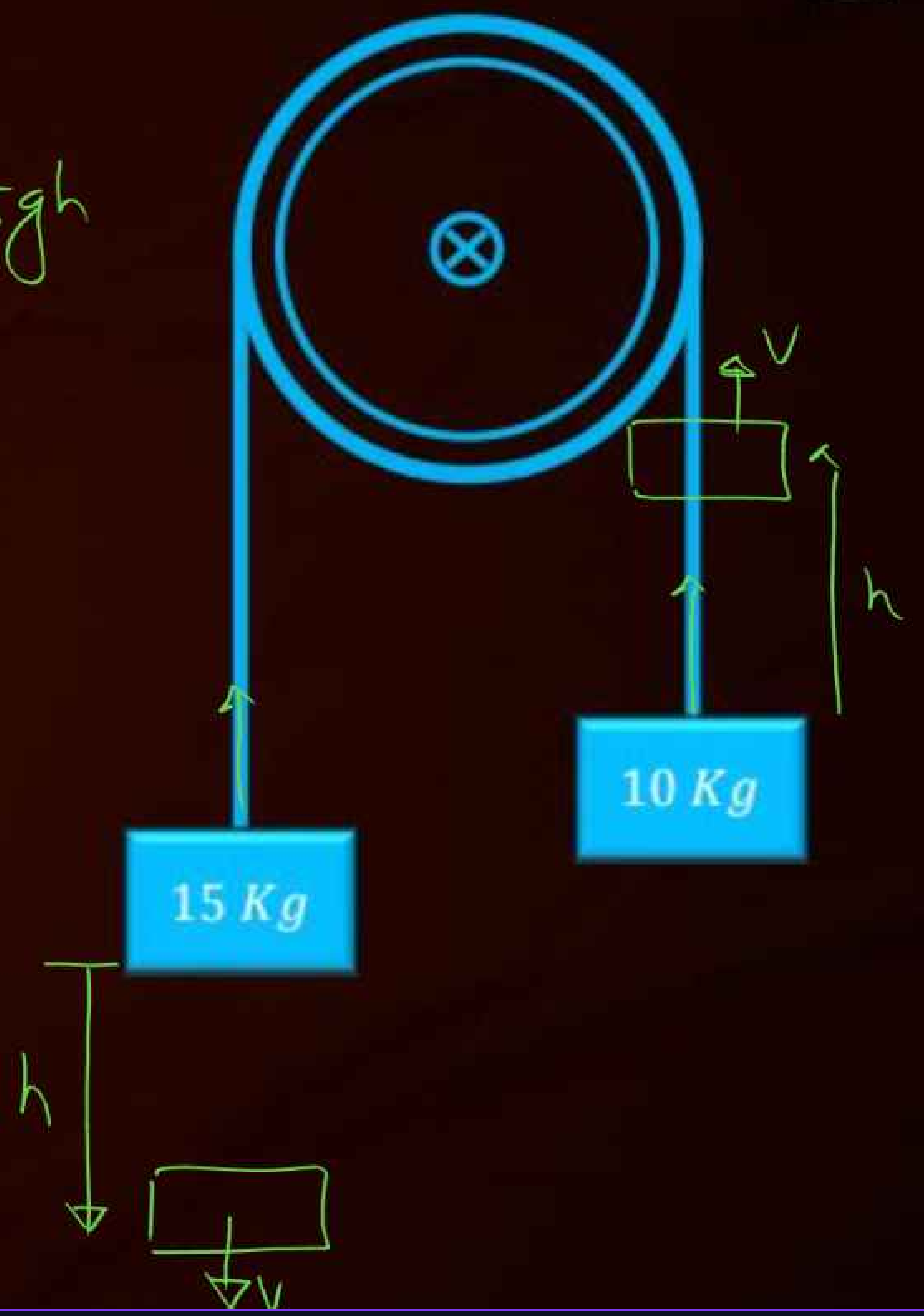
Let U=0



$$k_1 + U_1 = k_2 + U_2$$

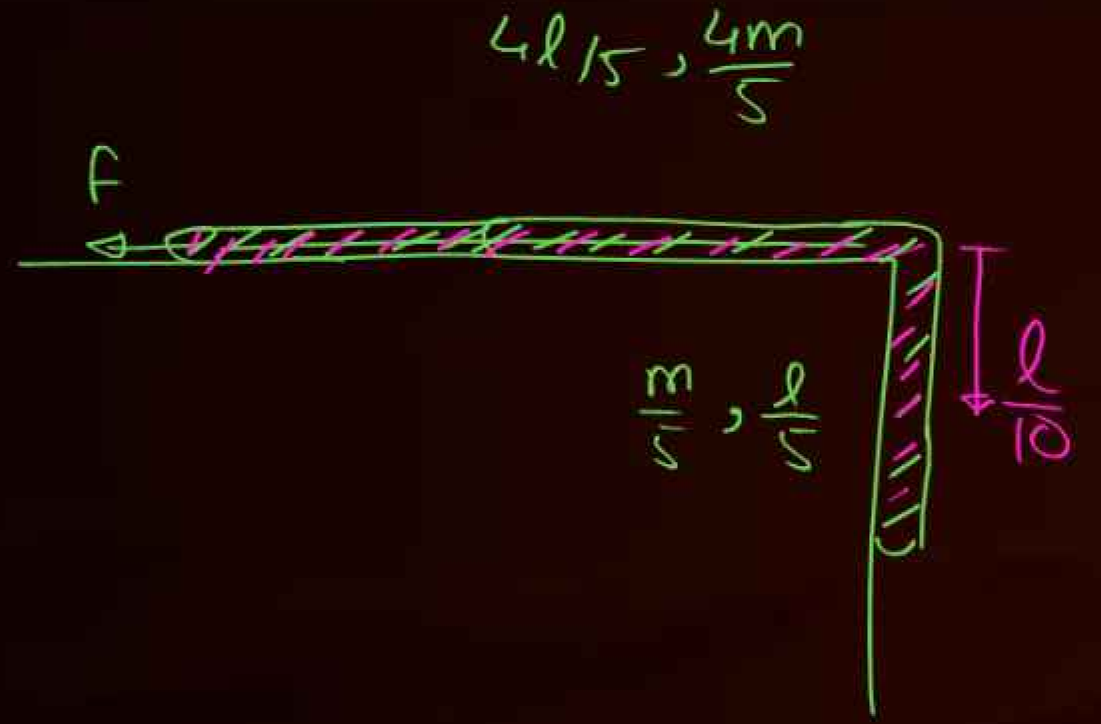
$$0 + 0 = \frac{1}{2}10v^2 + \frac{1}{2}15v^2$$

$$+ 10gh - 15gh$$



Question

A chain is on a frictionless table with one fifth of its length hanging over the edge. If the chain has length L and mass M, the work required to be done to pull the hanging part back onto the table is



Let U = 0

$$W_{NC} = \Delta TME = \cancel{\Delta KE}^{\,0} + \Delta U$$

$$W_{NC} = 0 - \left(0 - \frac{m}{5} g \frac{l}{10}\right)$$

$$= \frac{mgl}{50}$$

Ans. $+\dfrac{MgL}{50}$

Relation in P.E and Conservative Force

$$\Delta U = -W_c = -\int \vec{F_c} \cdot \vec{ds}$$

★★

$$\boxed{\Delta U = -\int \vec{F} \cdot \vec{ds}} \Rightarrow \boxed{\vec{F} = -\vec{\nabla} U}$$

$$\rightarrow \vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$$

if $U \to x, y, z$

$\rightarrow$ if $U \to r$ $\quad \vec{F} = -\frac{dU}{dr}\hat{r}$

$\rightarrow$ if $U \to x$ $\quad \vec{F} = -\frac{dU}{dx}\hat{i} \Rightarrow$

P.E. is only define for Conservative force

$F = -\frac{dU}{dx}$ = -ive of Slope in U-x Graph

Types of Equilibrium

(net force = 0)

Stable Equilibrium

A particle is initially in equilibrium position and is displaced by a small distance. If the particle tries to return back to the same equilibrium position, then this equilibrium is called position of stable equilibrium.

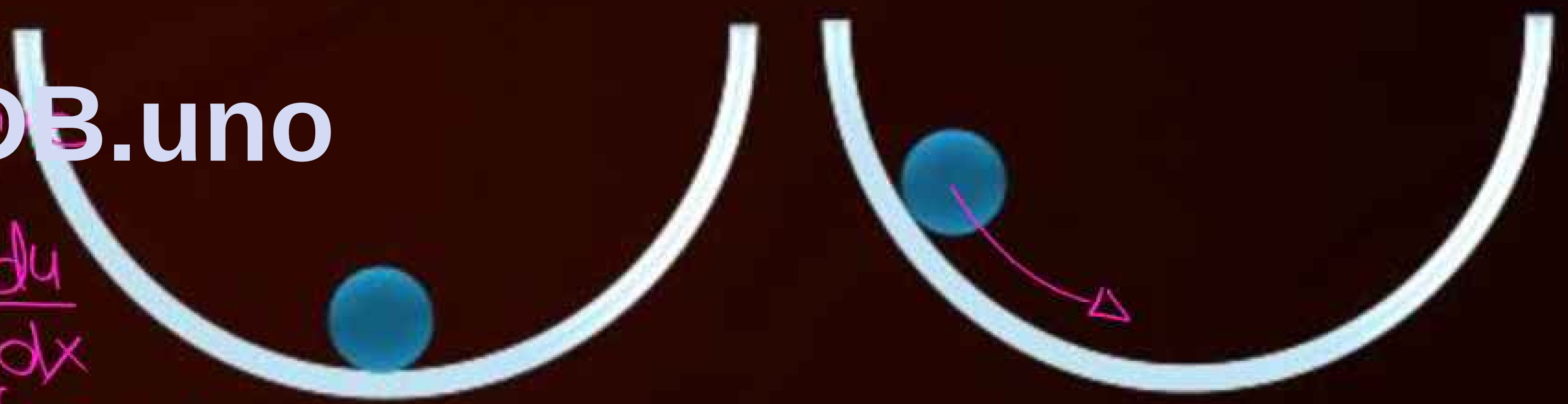
if only Conservative forces are acting on particle $f_{net} \rightarrow$ Conservative

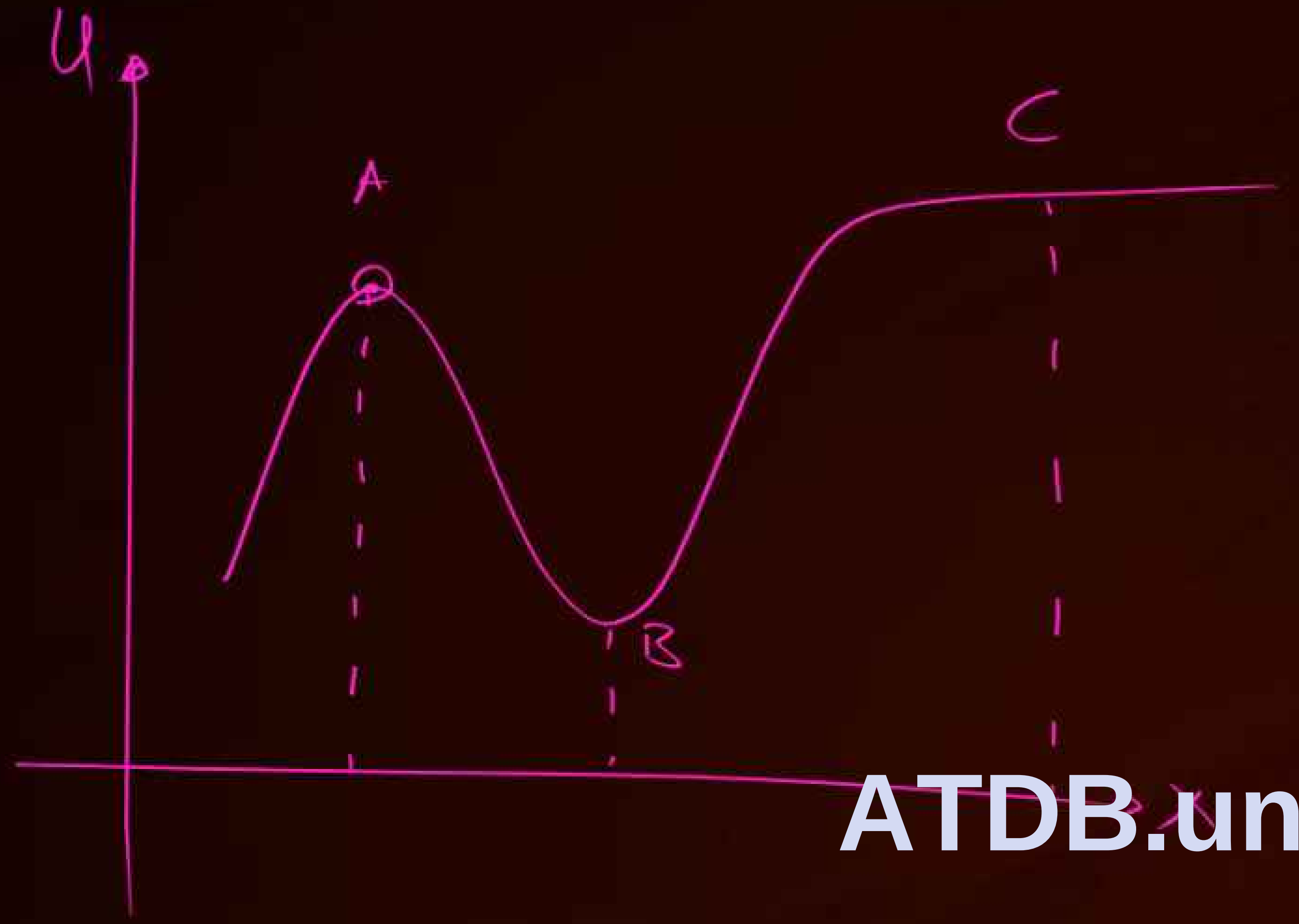
** $\boxed{F = -\frac{du}{dx}}$

Equilibrium $F = 0 = \frac{du}{dx}$

(Slope in u-x Graph)

in u-x Graph → Slope = 0 ⇒ Equilibrium





A, B, C → Equilibrium

$$\frac{du}{dx} = 0 = F$$

A → Unstable (maxima)

$$\frac{d^2u}{dx^2} = -ive$$

B → Stable (minima)

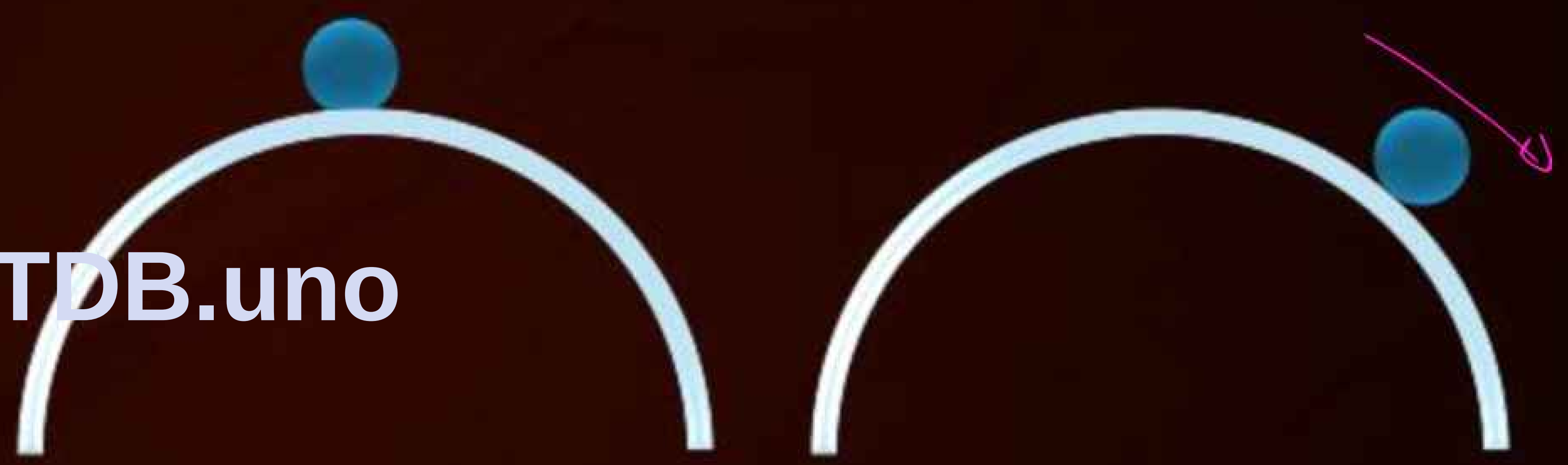
$$\frac{d^2u}{dx^2} = +ive$$

C → neutral (St. line)

$$\frac{d^2u}{dx^2} \ldots = 0$$

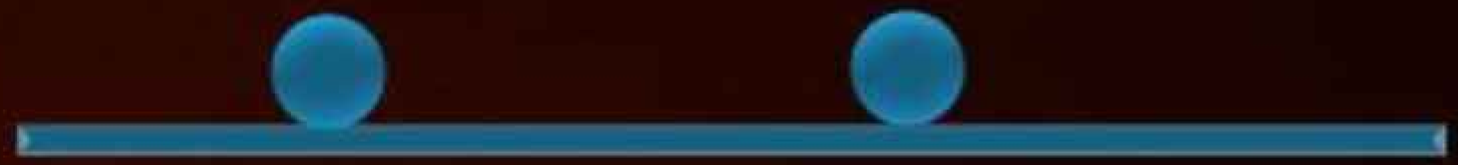
Unstable Equilibrium

A particle is initially in equilibrium position and is displaced by a small distance. If the particle not tries to return back to the same equilibrium position, then this equilibrium is called position of unstable equilibrium.

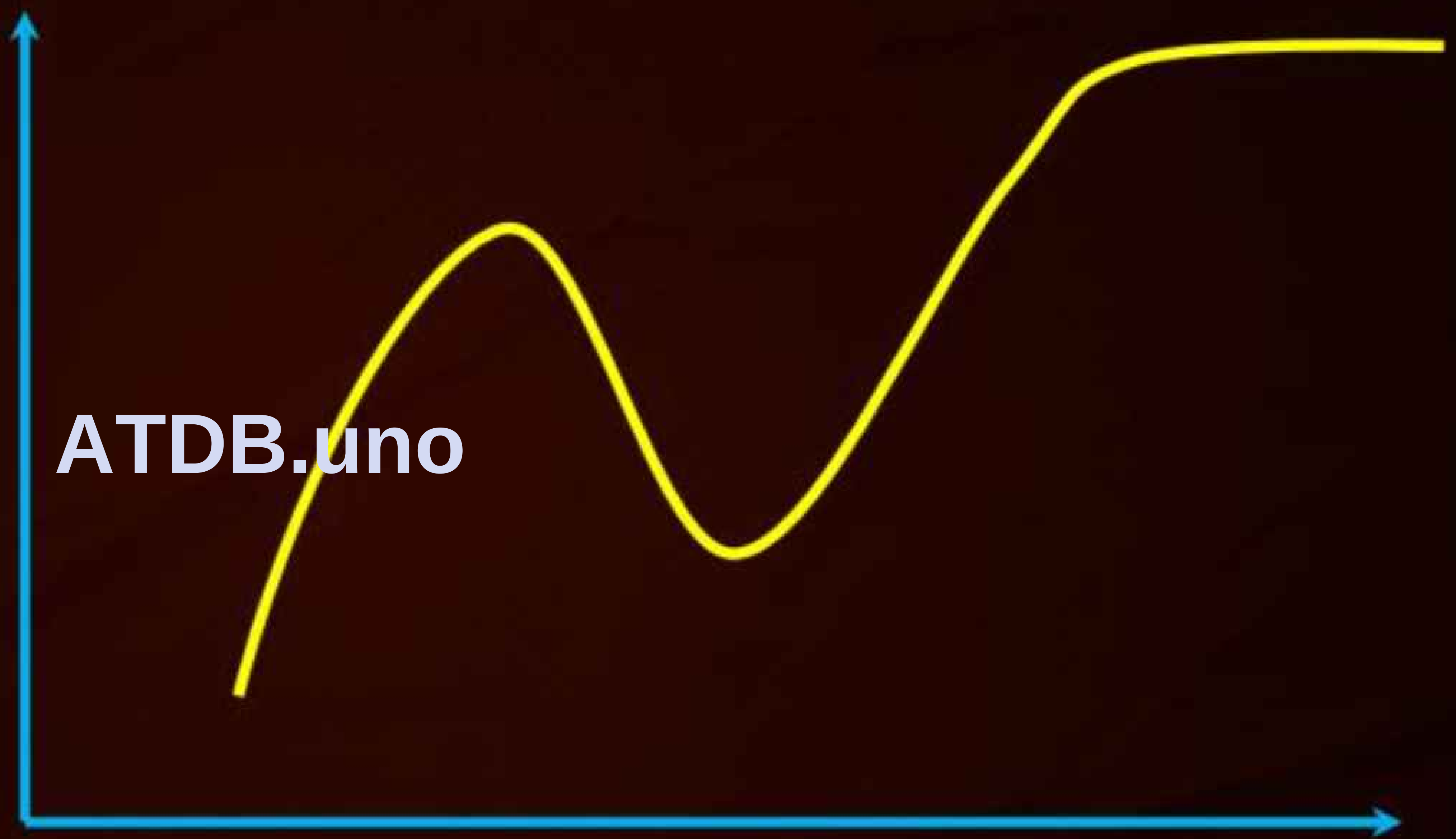


Neutral Equilibrium

If particle is displaced by a small distance and it is still in equilibrium condition, then it is called neutral equilibrium.



Graph in P.E and Conservative Force



Question

In a region, potential energy of a particle of mass 4 kg depends on its x-coordinate as $U = 5x^3 + 4x + 7$ Joule.
Find the acceleration of particle at position (2, 4).

U → Given acc. ??

$$\boxed{F = -\frac{du}{dx}}$$

$$\vec{F} = -\vec{\nabla} U$$

$$\vec{F} = -(15x^2 + 4)\hat{i} = -(15 \times 4 + 4)\hat{i}$$

$$acc. = \frac{-64\hat{i}}{4} =$$

Ans: $-16\ m/s^2$

Question

In a region potential energy of a particle of mass 2 kg is given by

$U = (3x^2 + 4y^2)$ J

Find the acceleration of particle when it is released from (1, 1).

$$\vec{F} = -\left[6x\hat{i} + 8y\hat{j}\right] = -\left(6\hat{i} + 8\hat{j}\right)$$

$$\underline{\vec{a} = -3\hat{i} - 4\hat{j}}$$

Ans: $5\ m/s^2$

Question

$Ar^{-10} - Br^{-5}$

Potential energy as a function of r is given by, $U = \frac{A}{r^{10}} - \frac{B}{r^5}$, where r is the interatomic distance, A and B are positive constant. The equilibrium distance between the two atoms will be: **[24 June, 2022 (Shift-II)]**

$F = \frac{du}{dr} = 0 = \frac{-10A}{r^{11}} + \frac{5B}{r^6}$

$\Rightarrow r^5 = \frac{2A}{B}$

1. $\left(\frac{A}{B}\right)^{\frac{1}{5}}$
2. $\left(\frac{B}{A}\right)^{\frac{1}{5}}$
3. $\left(\frac{2A}{B}\right)^{\frac{1}{5}}$
4. $\left(\frac{B}{2A}\right)^{\frac{1}{5}}$

Ans : (3)

Question

$k.\mathcal{E} + U = 8$

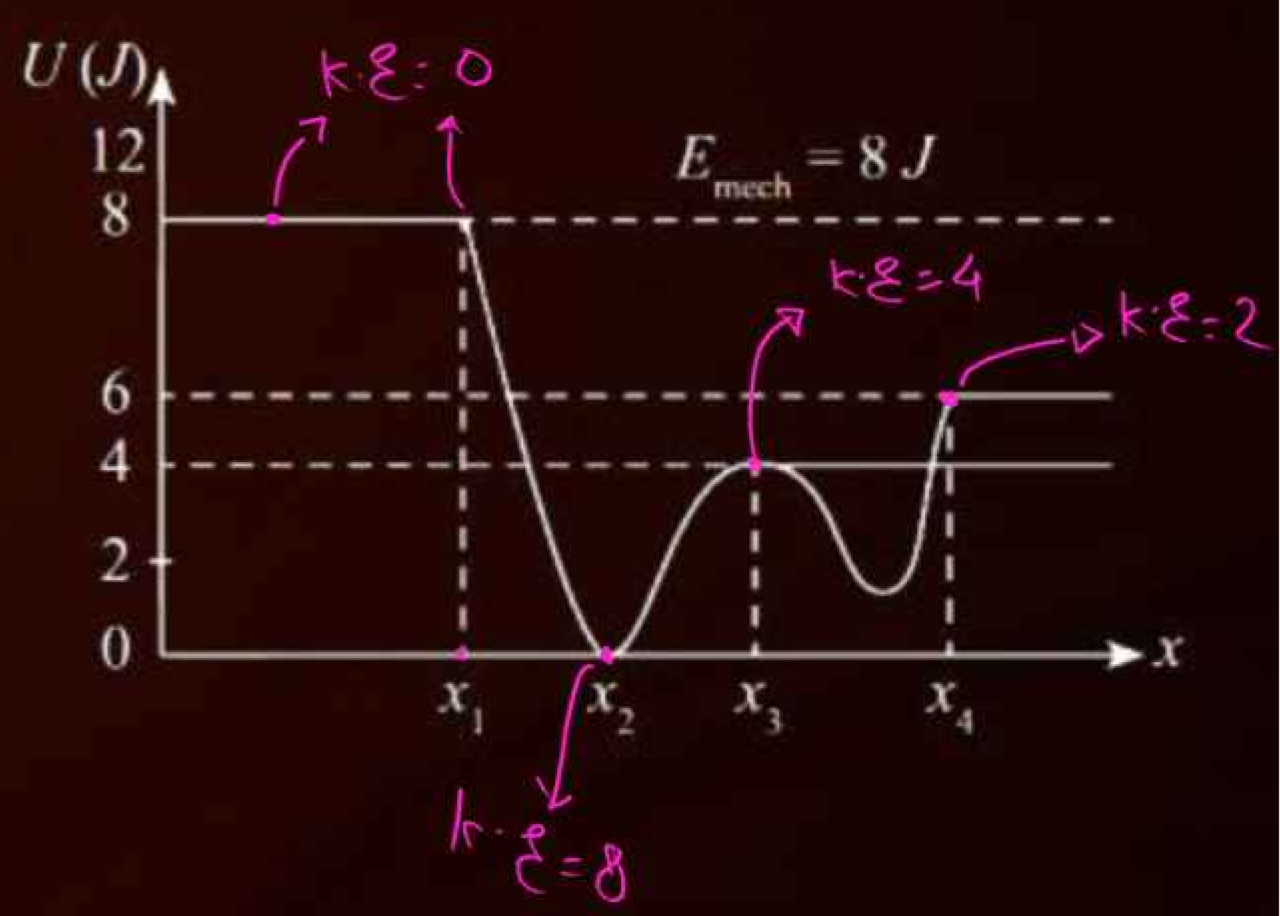
Given below is the plot of a potential energy function $U(x)$ for a system, in which a particle is in one dimensional motion, while a conservation force $F(x)$ acts on it. Suppose that $E_{mech} = 8$ J, the incorrect statement for this system is:
[where K.E. = kinetic energy] **[27 July, 2021 (Shift-II)]**

(1) at $x = x_3, K.E. = 4$ J.

(2) at $x > x_4, K.E.$ is constant throughout the region.

(3) at $x < x_1, K.E.$ is smallest and the particle is moving at the slowest speed. (zero)

(4) at $x = x_2, K.E.$ is greatest and the particle is moving at the fastest speed.



Ans : (3)

Power

insta Power → rate of change of energy or rate of work done

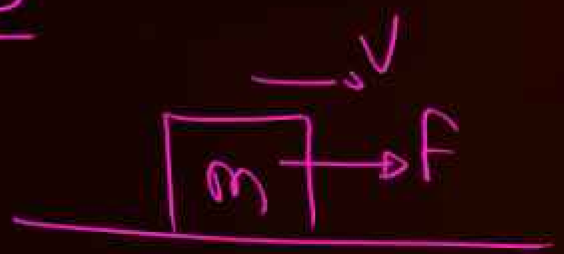
→ $P = \frac{dW}{dt} = \frac{\vec{F}\cdot d\vec{s}}{dt} = \vec{F}\cdot\frac{d\vec{s}}{dt} \Rightarrow$ ** $\boxed{P = \vec{F}\cdot\vec{V}} = FV\cos\theta$

→ $\vec{F} \perp \vec{V}$ $\quad P = 0$, $\quad P = +ive$, $\quad P = -ive$

→ Avg. Power ** $\boxed{P = \frac{W}{time}}$

Unit → Watt (W)
1 BHP → 746 W

1-D



$P = FV = maV$

$P = mV\frac{dV}{dt}$ $\quad (P \to time)$

$P = mV^2\frac{dV}{dx}$ $\quad (P \to x)$

Question

$P = Fv = mav$

A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time 't' is proportional to:

[18 March, 2021 (Shift-I)]

$P = \text{Constant} = mv\frac{dv}{dt}$

$\int_0^v mv\,dv = \int_0^t P\,dt \Rightarrow \frac{mv^2}{2} = Pt \Rightarrow v = \sqrt{\frac{2Pt}{m}} = \frac{dx}{dt}$

$\int_0^x dx = \int_0^t \sqrt{\frac{2P}{m}}\sqrt{t}\,dt \Rightarrow x = \sqrt{\frac{2P}{m}}\left(\frac{t^{3/2}}{3/2}\right)$

1. $t^{1/2}$
2. t
3. $t^{3/2}$
4. $t^{2/3}$

Ans : (3)

Question

An automobile of mass 'm' accelerates starting from origin and initially at rest, while the engine supplies constant power P. The position is given as a function of time by:

[27 July, 2021 (Shift-II)]

$P = mav = mv\frac{dv}{dt}$

$\frac{mv^2}{2} = Pt \Rightarrow v = \sqrt{\frac{2Pt}{m}}$

$x = \frac{2}{3}\sqrt{\frac{2P}{m}}\, t^{3/2}$

(1) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

(2) $\left(\frac{9P}{8m}\right)^{\frac{1}{2}} t^{\frac{2}{3}}$

(3) $\left(\frac{9P}{8P}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

(4) $\left(\frac{8P}{9P}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

Ans : (1)

Question

A body of mass 2 kg is initially at rest. It starts moving unidirectionally under the influence of a source of constant power P. Its displacement in 4 s is $\frac{1}{3}\alpha^2\sqrt{P}$m. The value of α will be:

[30 Jan, 2023 (Shift-II)]

Ans : (4)

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Question

A force $\vec{F} = 2\hat{i} + b\hat{j} + \hat{k}$ is applied on a particle and it undergoes a displacement $\hat{i} - 2\hat{j} - \hat{k}$. What will be the value of b, if work done on the particle is zero.

(January 2025)/22-01-2025/Evening Shift)

(1) 0

(2) $\frac{1}{2}$

(3) $\frac{1}{3}$

(4) 2

$2 - 2b - 1 = 0$

$b = \frac{1}{2}$

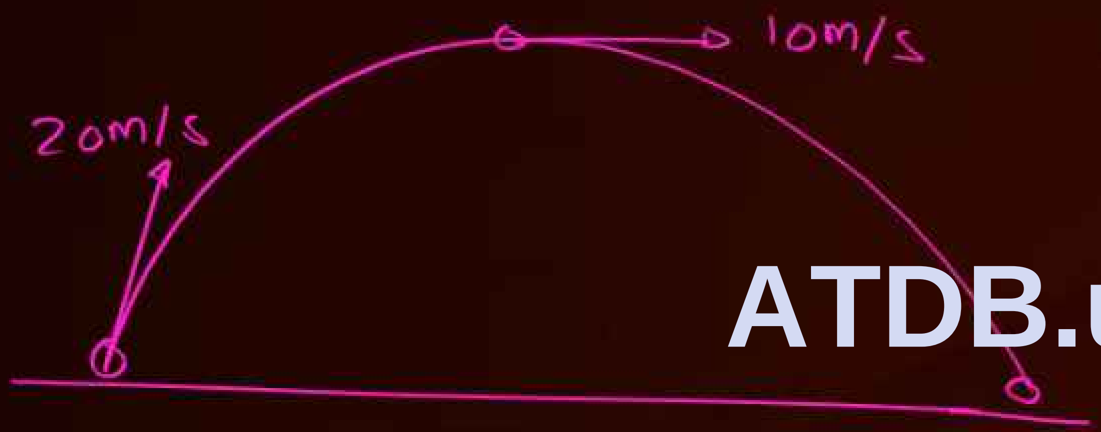
Ans : (2)

FOR NOTES & DPP BATTLEGROUND CHECK PW APP

Question

A ball of mass 100 g is projected with velocity 20 m/s at 60° with horizontal. The decrease in kinetic energy of the ball during the motion from point of projection to highest point is: (January 2025)/22-01-2025/Evening Shift)

1. 20 J
2. 15 J
3. zero
4. 5 J



$\frac{1}{2}\times .1\left(20^2-10^2\right)$

$=$

Ans : (2)

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Question

$x^2 y\hat{i} + y^2\hat{j}$

$y = 10 - x$

$dy = -dx$

$\frac{64 \times 3}{192}$

A force $f = x^2 y\hat{i} + y^2\hat{j}$ acts on a particle in a plane x + y = 10. The work done by this force during a displacement from (0, 0) to (4m, 2m) is ____ Joule (round off to the nearest integer)

(January 2025)/ 23-01-2025/Morning Shift

$W = \int x^2 y\,dx + \int y^2 dy$

$W = \int_0^4 x^2(10-x)dx + \frac{1}{3}\left[2^3 - 0^3\right]$

$\frac{10}{3}(4^3 - 0^3) - \frac{1}{4}(4^4 - 0^4) + \frac{8}{3}$

$\frac{640}{3} - \frac{64 \times 3}{3} + \frac{8}{3}$

$648 - 192 = 456$

$\frac{456}{3} = 152$

Ans. (152)

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Question

A ball having kinetic energy KE, is projected at an angle 60° from the horizontal. What will be the kinetic energy of the ball at the highest point of its flight

(January 2025)/23-01-2025/Evening Shift)

1. $\frac{(KE)}{8}$
2. $\frac{(KE)}{4}$
3. $\frac{(KE)}{16}$
4. $\frac{(KE)}{2}$

Ans : (2)

Question

A force $F = \alpha + \beta x^2$ acts on an object in the x-direction. The work done by the force is 5 J when the object is displaced by 1 m. If the constant $\alpha = 1N$ then β will be

(January 2025)/24-01-2025/Morning Shift)

$$\int_0^1 (1 + \beta x^2)\, dx = 5$$

1. $15\ N/m^2$
2. $10\ N/m^2$
3. $12\ N/m^2$
4. $8\ N/m^2$

Ans : (3)

Question

C

Assertion A: In a central force field, the work done is independent of the path chosen

Reason R: Every force encountered in mechanics does not have an associated potential energy.

NC

In the light of the above statements, choose the most appropriate answer from the options given below. (January 2025)/28-01-2025/Morning Shift)

1. A is true but R is false
2. Both A and R are true but R is NOT the correct explanation of A
3. Both A and R are true and R is the correct explanation of A
4. A is false but R is true

Ans : (2)

Question

$\vec{S} = 3\hat{i} + 6\hat{j}$

A body of mass 4 kg is placed on a plane at a point P having coordinate (3, 4) m. Under the action of force $\vec{F} = (2\hat{\imath} + 3\hat{\jmath})$ N, it moves to a new point Q having coordinates (6, 10)m in 4 sec. The average power and instantaneous power at the end of 4 sec are in the ratio of :

(January 2025)/28-01-2025/Evening Shift)

(1) 13 : 6

(2) 6 : 13

(3) 1 : 2

(4) 4 : 3

$W = 6 + 18 = 24$

$P_{avg} = \frac{24}{4} = 6$

$\vec{a} = \frac{2\hat{i} + 3\hat{j}}{4}$

$\vec{v} = 0 + \left(\frac{2\hat{i} + 3\hat{j}}{4}\right)4$

$= 2\hat{i} + 3\hat{j}$

$P_{avg} = \vec{F} \cdot \vec{v} = 4 + 9 = 13W$

Ans : (2)

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Question

$\frac{dm}{dt} = \alpha\sqrt{v}$

A sand dropper drops sand of mass $m(t)$ on a conveyer belt at a rate proportional to the square root of speed (v) of the belt, i.e. $\frac{dm}{dt} \propto \sqrt{v}$. If P is the power delivered to run the belt at constant speed then which of the following relationship is true ?

(January 2025)/29-01-2025/Evening Shift)

1. $P^2 \propto v^2$
2. $P \propto \sqrt{v}$
3. $P \propto v$
4. $P^2 \propto v^5$

$P^2 \propto v^5$

$F_{th} = v_{rel.}\frac{dm}{dt} = v(\alpha\sqrt{v})$

$= \alpha v^{3/2}$

$P = Fv = \alpha v^{5/2}$

Ans : (4)

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Extra

Question

Two bodies of masses m_1 and m_2 are moving with same kinetic energy. If P_1 and P_2 are their respective momentum, the ratio $\frac{P_1}{P_2}$ is equal to ?

a. $\frac{m_1}{m_2}$

b. $\sqrt{\frac{m_2}{m_1}}$

c. $\sqrt{\frac{m_1}{m_2}}$

d. $\frac{m_1^2}{m_2^2}$

Ans: (c)

Question

KE of a body is increased by 44%. What is the percent increase in the momentum ?

a. 10%

b. 20%

c. 30%

d. 44%

Ans: (b)

Question

Under the action of a force, a 2 kg body moves such that its position x as a function of time t is given by, $x = t^2/3$ where x is in metres and t in seconds. The work done by the force in first two seconds is :

a. $1600\ J$

b. $160\ J$

c. $16\ J$

d. $\frac{16}{9}\ J$

Ans: (d)

Question

Two springs have their force constants in the ratio of 3 : 4. Both the springs are stretched by applying equal force F. If elongation in first spring is x then elongation in second spring is

a. $3x$

b. $4x$

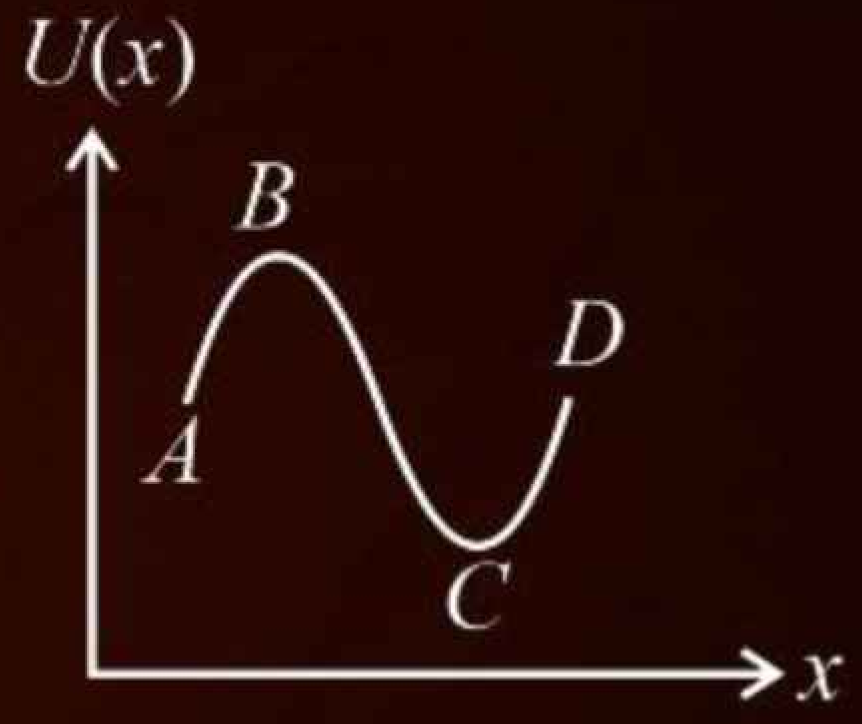
c. $\frac{4}{3}x$

d. $\frac{3}{4}x$

Ans: (d)

Question

The potential energy of a particle varies with distance x as shown in the graph. The force acting on the particle is zero at [AIEEE 2008]



a. C

b. B

c. B and C

d. A and D

Ans: (c)

Question

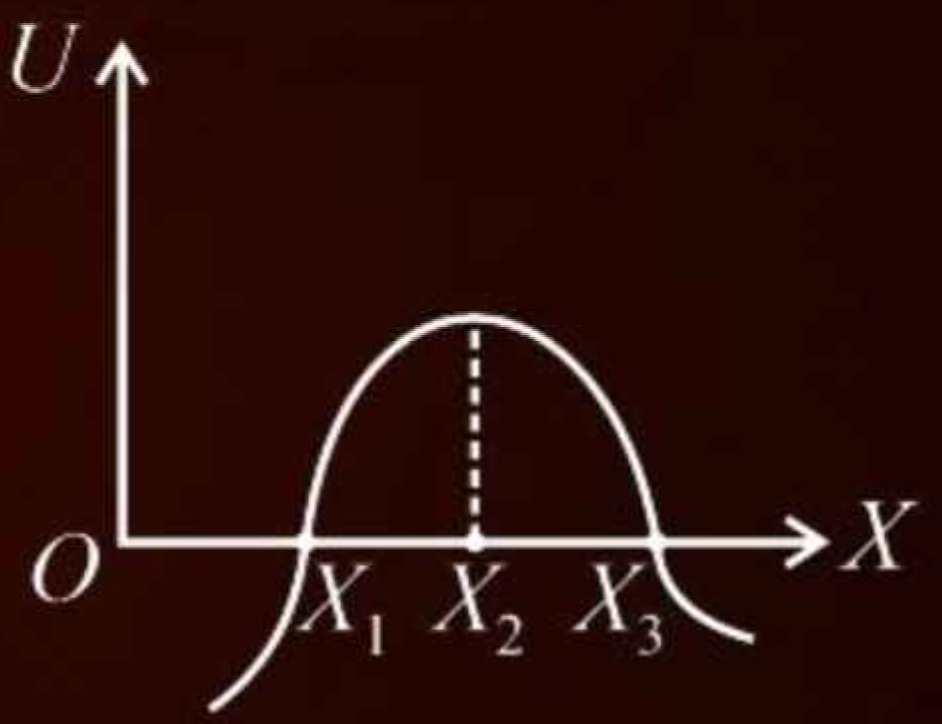
In the figure shown the potential energy (U) of a particle is plotted against its position 'X' from origin. Then which of the following statement is correct. A particle at: [2009]

a. X_1 is in stable equilibrium

b. X_2 is in stable equilibrium

c. X_3 is in stable equilibrium

d. None of these



Ans: (d)

Question

If the potential energy of two molecules is given by $U = \frac{A}{r^{12}} - \frac{B}{r^6}$, then at equilibrium position, its potential energy is equal to:

a. $\frac{A^2}{4B}$

b. $-\frac{B^2}{4A}$

c. $\frac{2B}{A}$

d. $3A$

Ans: (b)

Question

The potential energy for a force field $\vec{F}$ is given by $U(x, y) = \sin(x + y)$. The force acting on the particle of mass m at $\left(0, \frac{\pi}{4}\right)$ is [2020]

a. 1

b. $\sqrt{2}$

c. $1\sqrt{2}$

d. 0

Ans: (a)

Question

$F = 2x^2 - 3x - 2$. Choose correct option

a. $x = -1/2$ is position of stable equilibrium

b. $x = 2$ is position of stable equilibrium

c. $x = -1/2$ is position of unstable equilibrium

d. $x = 2$ is position of neutral equilibrium

Ans: (a)

Question

A uniform cable of mass M and length L is placed on a horizontal surface that its $\left(\frac{1}{n}\right)$th part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be [2019 Main, 9 April (I)]

a. $\frac{2MgL}{n^2}$

b. $nMgL$

c. $\frac{MgL}{n^2}$

d. $\frac{MgL}{2n^2}$

Ans: (d)

Question

A force acts on a 2 kg object, so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds ?

[2019] Main, 9 Jan II

a. 850 J

b. 900 J

c. 950 J

d. 875 J

Ans: (b)


THANK YOU