
MANZIL
FOR JEE ASPIRANTS
Mathematics
Determinants
In One Shot
By – Tarun Khandelwal Sir
B-Tech IIT Delhi, (A.I.R. 289)
Physics Wallah

Topics *to be covered*

1. Minor & Cofactors
2. Area of Triangle
3. Properties of Determinants
4. Some Important Determinants to Remember
5. Summation & Multiplication of Determinants
6. Differentiation of Determinants
7. Cramer's Rule

FOR NOTES & DPP CHECK DESCRIPTION

Determinant

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}_{(2\times 3)}$$

$$A = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}_{(2\times 2)}$$

$$|A| = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

$$|A| = \begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix} = 7\times 1 - 5\times 3 = 7 - 15 = -8$$

FOR NOTES & DPP CHECK DESCRIPTION

Minor of an Element

The **determinant obtained by deleting** the row and the column in which the given element lies.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$M_a = d$

$M_b = c$

$M_c = b$

$M_d = a$

$i=1$, $j=1$

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\boxed{M_{a_1} = b_2c_3 - c_2b_3} = \text{Cofactor}$$

$M_{b_2} = a_1c_3 - a_3c_1 = \text{Cofactor}$

FOR NOTES & DPP CHECK DESCRIPTION

Cofactor of an Element a_{ij}

C_{ij}

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$C_{21} = -M_{21} = -(a_{12}a_{33} - a_{32}a_{13})$

Cofactor of the element a_{ij} is denoted by C_{ij} and is given by
$\boxed{C_{ij} = (-1)^{i+j} M_{ij}}$
Where i and j denotes the row and column in which the particular element lies.

If $i+j = \text{Even} \Rightarrow C_{ij} = M_{ij}$

If $i+j = \text{odd} \Rightarrow C_{ij} = -M_{ij}$

FOR NOTES & DPP CHECK DESCRIPTION

$A = (-4)(-3) + 3(27) + 2(11)$

QUESTION

If $A = \begin{vmatrix} 5 & 6 & 3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{vmatrix}$, then cofactors of the elements of 2^{nd} row are

Minor $= 18 + 21 = 39$

Cofactor $= -39$

$5(-39) + 6 \times 27 + 3 \times 11$

$= -39 \times 5 + 162 + 33$

$-\cancel{195} + \cancel{195}$

$= 0$

Minor $= 15 + 12 = 27 =$ Cofactor

Minor $= -35 + 24 = -11$

Cofactor $= 11$

(A) 39, −3, 11

(B) −39, 3, 11

(C) −39, 27, 11

(D) −39, −3, 11

FOR NOTES & DPP CHECK DESCRIPTION

Expansion of a Determinant

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\boxed{a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = 0}$$

$$a_{12}C_{13} + a_{22}C_{23} + a_{32}C_{33} = 0$$

Expansion along R_1

$$\Delta = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

Expansion along R_3 $\quad \Delta = a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$

Exp. along C_1 : $\quad \Delta = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$

FOR NOTES & DPP CHECK DESCRIPTION

Note

(i) The sum of the product of elements of any row (or column) with their corresponding cofactors is always equal to the **value of the determinant.**

(ii) The sum of the product of elements of any row (or column) with the cofactors of other row (or column) is **always equal to zero.**

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

If in the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, A_i, B_i, C_i etc. be the co-factors of a_i, b_i, c_i etc., then which of the following relations is incorrect

(A) $a_1A_1 + b_1B_1 + c_1C_1 = \Delta$ True

(B) $a_2A_2 + b_2B_2 + c_2C_2 = \Delta$ → True

(C) $a_3A_3 + b_3B_3 + c_3C_3 = 0$ → False

(D) $a_1B_1 + a_2B_2 + a_3B_3 = 0$ → True.

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [1981, 2]

[Ans. 0]

Let $\boxed{p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t} = \begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda+1 & -2\lambda & \lambda-4 \\ \lambda-3 & \lambda+4 & 3\lambda \end{vmatrix}$

be an identity in λ, where p, q, r, s and t are constants. Then, the value of t is

$p + q + r + s + t = ?$

Put $\lambda = 1$

Put $\boxed{\lambda = 0}$

$t = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix}$ → direct 0

→ skew symm

$t = 0\times \; + 1[0-12] + 3[4]$

$t = -12 + 12 = 0$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2023 (11 Apr Shift 2)]

[Ans. D]

If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$, for all $x \in R$

then $\lambda, \frac{\lambda}{3}$ are the roots of the equation

(A) $4x^2+24x-27=0$

(B) $4x^2-24x-27=0$

(C) $4x^2+24x+27=0$

(D) $4x^2-24x+27=0$

Put $\boxed{x=0}$

$\rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9\times 81}{8}$

$1[\lambda^3-0] = \frac{9\times 81}{8}$

$\lambda^3 = \frac{9^3}{2^3} \Rightarrow \boxed{\lambda = 9/2}$

$\lambda = 9/2 = \alpha$

$\lambda/3 = \frac{9/2}{3} = 3/2 = \beta$

$\boxed{\begin{array}{l} \alpha = 9/2 \\ \beta = 3/2 \end{array}}$

$\boxed{\begin{array}{l} \alpha+\beta = 6 \\ \alpha\beta = 27/4 \end{array}}$

$x^2 - (\text{sum})x + \alpha\beta = 0$

$x^2 - 6x + 27/4 = 0$

$4x^2 - 24x + 27 = 0$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

The cofactor of the element '4' in the determinant $\begin{vmatrix} 1 & 3 & 5 & 1 \\ 2 & 3 & 4 & 2 \\ 8 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \end{vmatrix}$ is

$i = 2$
$j = 3$
odd

(A) 4

(B) 10

(C) −10

(D) −4

$$M_4 = \begin{vmatrix} 1 & 3 & 1 \\ 8 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= 1 \times [0-2] - 3[8-0] + 1[16-0]$$

$$= -2 - 24 + 16.$$

$$-26 + 16$$

$$= -10$$

$$\boxed{C_4 = 10}$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

[Ans. 4]

If $x, y, z \in R$ & $\begin{vmatrix} x & y^2 & z^3 \\ x^4 & y^5 & z^6 \\ x^7 & y^8 & z^9 \end{vmatrix} = 2$, find the value of

$$\begin{array}{ccc} C_{11} & C_{12} & \\ \end{array}$$

$$\begin{vmatrix} y^5z^6(z^3-y^3) & x^4z^6(x^3-z^3) & x^4y^5(y^3-x^3) \\ y^2z^3(y^6-z^6) & xz^3(z^6-x^6) & xy^2(x^6-y^6) \\ y^2z^3(z^3-y^3) & xz^3(x^3-z^3) & xy^2(y^3-x^3) \end{vmatrix}$$

$$C_x = y^5z^9 - y^8z^6$$
$$= y^5z^6[z^3 - y^3]$$

$$C_{y^2} = -[x^4z^9 - x^7z^6]$$
$$= -x^4z^6[z^3 - x^3]$$
$$= x^4z^6[x^3 - z^3]$$

$$\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$$

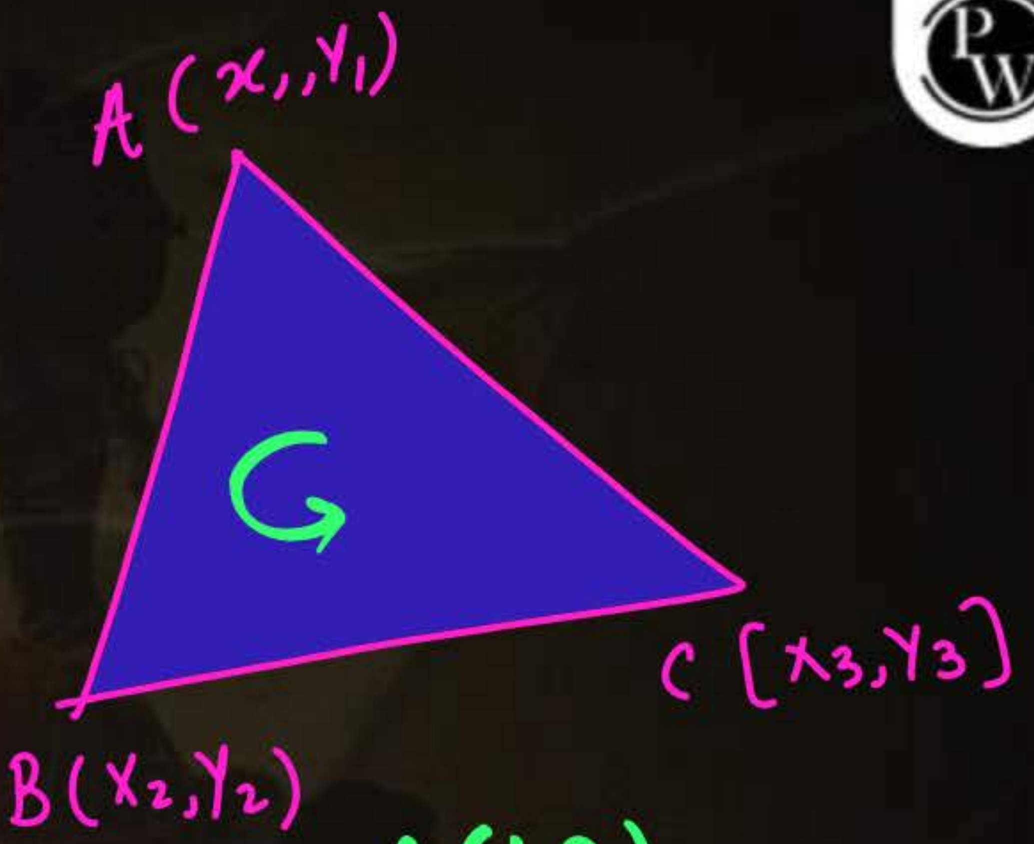
$$|\operatorname{adj} A| = |A|^{n-1} = |A|^2$$
$$= 2^2 = 4$$

FOR NOTES & DPP CHECK DESCRIPTION

Area of a Triangle

$$\text{Area} = \left| \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right|$$

Mod det



If $\boxed{\text{Area} = 0}$

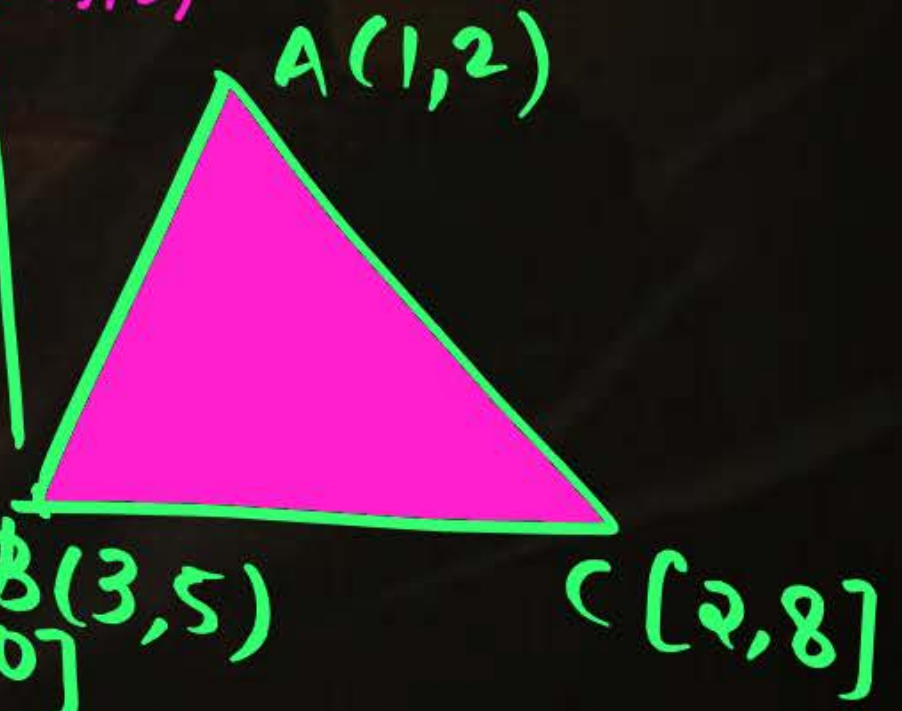
$\Rightarrow$ A, B, C are collinear points.

Ex →

$$A = \frac{1}{2} \begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 1 \\ 2 & 8 & 1 \end{vmatrix}$$

$$\frac{1}{2}\left[(5-8) - 2(3-2) + 24 - 10\right]$$

$$\frac{1}{2}\left[-3-2+14\right] = 9/2$$



FOR NOTES & DPP CHECK DESCRIPTION

Properties of Determinants

$$= \begin{vmatrix} 0 & 0 & 0 \\ 2 & 3 & 5 \\ 8 & 7 & 10 \end{vmatrix} = 0$$

Property 1 If all the elements of a row (or column) are zero, then the value of the determinant is zero.

Property 2 If all the rows and columns of a determinant are inter-changed, the value of determinant remains unchanged.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\boxed{|A| = |A^T|}$$

FOR NOTES & DPP CHECK DESCRIPTION

Properties of Determinants

Property 3

If any two rows (or any two columns) of a determinant are **interchanged**, the value of determinant **changes sign** only.

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \Rightarrow R_1 \leftrightarrow R_2 \quad \begin{vmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = -\Delta$$

$R_1 \leftrightarrow R_3$

$$\begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} = -\Delta$$

FOR NOTES & DPP CHECK DESCRIPTION

Properties of Determinants

Property 4

If any two pair of rows (or any two pair of columns) of a determinant are interchanged, the value of determinant remains same.

$$\Delta \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \xrightarrow{R_1 \leftrightarrow R_2 \;\&\; R_2 \leftrightarrow R_3} \begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{vmatrix} = \Delta$$

4

Property 5

If any two rows (or any two columns) of a determinant are identical, then the value of determinant is always zero.

Ex →
$$\begin{vmatrix} 1 & 3 & 10 \\ 1 & 3 & 10 \\ 5 & 8 & 12 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ a & b & c \end{vmatrix} \underset{R_1 \leftrightarrow R_3}{=} - \underbrace{\begin{vmatrix} a & b & c \\ d & e & f \\ a & b & c \end{vmatrix}}_{\Delta}$$

$\Delta = -\Delta$

$\Rightarrow 2\Delta = 0 \Rightarrow \Delta = 0$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

$$\begin{vmatrix} 3\sin 90^\circ & 3 & \pi \\ \ln e^3 & 3 & \sqrt{5} \\ \log_{10}1000 & 3 & e \end{vmatrix}$$ equals

$\ln e^3 = 3(\ln e) = 3\times 1 = 3$

$\log_{10}(10)^3 = 3\log_{10}10 = 3$

(A) 1

(B) e

(C) $\sqrt{\pi}$

(D) 0 ✓

$$\Delta = \begin{vmatrix} 3 & 3 & \pi \\ 3 & 3 & \sqrt{5} \\ 3 & 3 & e \end{vmatrix}$$

$C_1 = C_2$

$\Rightarrow \Delta = 0$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

Without expanding the determinants, prove that

$$\begin{vmatrix} 103 & 115 & 114 \\ 111 & 108 & 106 \\ 104 & 113 & 116 \end{vmatrix} + \begin{vmatrix} 113 & 116 & 104 \\ 108 & 106 & 111 \\ 115 & 114 & 103 \end{vmatrix} = 0$$

$R_1 \leftrightarrow R_3$

$$-\begin{vmatrix} 104 & 113 & 116 \\ 111 & 108 & 106 \\ 103 & 115 & 114 \end{vmatrix}$$

$C_1 \leftrightarrow C_2$

$$\begin{vmatrix} 113 & 104 & 116 \\ 108 & 111 & 106 \\ 115 & 103 & 114 \end{vmatrix} = -\begin{vmatrix} 113 & 116 & 104 \\ 108 & 106 & 111 \\ 115 & 114 & 103 \end{vmatrix}$$

Ans = 0 ✓

— FOR NOTES & DPP CHECK DESCRIPTION —

Properties of Determinants

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} a+p & b+q \\ c+r & d+s \end{bmatrix}$$

Property 6

If each element of any row (or column) is expressed as a **sum of two** (or more) terms, then the determinant can be expressed as the sum of two (or more) determinants.

$$\begin{vmatrix} a_1+\alpha & b_1+\beta & c_1+\gamma \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha & \beta & \gamma \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Ex:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 7 & 5 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 3 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 10 & 10 \end{vmatrix}$$

FOR NOTES & DPP CHECK DESCRIPTION

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a & b \\ p & q \end{vmatrix} = \begin{vmatrix} a & b \\ c+p & d+q \end{vmatrix} \checkmark$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} p & q \\ r & s \end{vmatrix} = (ad-bc)+(ps-qr)$$

Ex→ $$\begin{vmatrix} a & c \\ b & d \end{vmatrix} + \begin{vmatrix} a & p \\ b & q \end{vmatrix} = \begin{vmatrix} a & c+p \\ b & d+q \end{vmatrix}$$

$$\checkmark \quad \begin{vmatrix} a & c \\ b & d \end{vmatrix} + \begin{vmatrix} a & c \\ b & d \end{vmatrix} = 2\begin{vmatrix} a & c \\ b & d \end{vmatrix}$$

Properties of Determinants

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$2A = \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$$

Property 7

If all the elements of any row (or column) are multiplied by the a scalar number "K", then the value of determinant is multiplied by that number "K".

Example:

If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D_1 = \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$.

Then $D_1 = KD$.

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$2\Delta = \begin{vmatrix} 2a & 2b \\ c & d \end{vmatrix}$$

$$K\Delta = \begin{vmatrix} Ka & b \\ Kc & d \end{vmatrix}$$

FOR NOTES & DPP CHECK DESCRIPTION

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$3\Delta = \begin{vmatrix} a_1 & 3b_1 & c_1 \\ a_2 & 3b_2 & c_2 \\ a_3 & 3b_3 & c_3 \end{vmatrix}$$

Ex $\rightarrow$ $\begin{vmatrix} 2a & b \\ 2c & d \end{vmatrix}$

$$2\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\begin{vmatrix} 3 & 5 & 8 \\ 6 & 4 & 10 \\ 9 & 2 & 12 \end{vmatrix}$$

$$2 \times 3 \begin{vmatrix} 1 & 5 & 4 \\ 2 & 4 & 5 \\ 3 & 2 & 6 \end{vmatrix}$$

QUESTION

The value of $\begin{vmatrix} 5^{200} & 5^{300} & 5^{400} \\ 5^{300} & 5^{400} & 5^{500} \\ 5^{400} & 5^{500} & 5^{700} \end{vmatrix}$ is

$$5^{400} \cdot 5^{300} \cdot 5^{200} \begin{vmatrix} 1 & 5^{100} & 5^{200} \\ 1 & 5^{100} & 5^{200} \\ 1 & 5^{100} & 5^{300} \end{vmatrix} = \underline{\underline{0}}$$

(A) 5^{200}

(B) 0 ✓

(C) 5^{1300}

(D) 5^{900}

— FOR NOTES & DPP CHECK DESCRIPTION —

QUESTION

Prove that $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix} = 0$

LHS $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha\cos\delta + \cos\alpha\sin\delta \\ \sin\beta & \cos\beta & \sin\beta\cos\delta + \cos\beta\sin\delta \\ \sin\gamma & \cos\gamma & \sin\gamma\cos\delta + \cos\gamma\sin\delta \end{vmatrix}$

$\cos\delta \begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha \\ \sin\beta & \cos\beta & \sin\beta \\ \sin\gamma & \cos\gamma & \sin\gamma \end{vmatrix} + \sin\delta \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha \\ \sin\beta & \cos\beta & \cos\beta \\ \sin\gamma & \cos\gamma & \cos\gamma \end{vmatrix}$

$0 + 0 = \underline{0}$

QUESTION

If x, y, z are respectively the p^{th} q^{th} r^{th} terms of an AP, then without expanding at any stage show that $\begin{vmatrix} x & y & z \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = 0$

$$\boxed{T_n = a + (n-1)d} \checkmark$$

$$x = a + (p-1)d$$
$$y = a + (q-1)d$$
$$z = a + (r-1)d$$

$$\begin{vmatrix} a+(p-1)d & a+(q-1)d & a+(r-1)d \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} a & a & a \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} (p-1)d & (q-1)d & (r-1)d \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$$a\begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} + d\begin{vmatrix} p-1 & q-1 & r-1 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} p & q & r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$$0 \qquad 0$$

ATDB.uno

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

[Ans. 3]

Let $\Delta_1 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 6a_1 & 2a_2 & 2a_3 \\ 3b_1 & b_2 & b_3 \\ 12c_1 & 4c_2 & 4c_3 \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} 3a_1+b_1 & 3a_2+b_2 & 3a_3+b_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix}$ &

$\Delta_3 - \Delta_2 = k\Delta_1$, find k.

$$\Delta_2 = 24 \begin{vmatrix} 3a_1 & a_2 & a_3 \\ 3b_1 & b_2 & b_3 \\ 3c_1 & c_2 & c_3 \end{vmatrix}$$

$$\Delta_2 = 8 \cdot 3 \underbrace{\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}}_{\Delta_1}$$

$$\boxed{\Delta_2 = 24\Delta_1}$$

$$\Delta_3 = \begin{vmatrix} 3a_1 & 3a_2 & 3a_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix} + 3\cancel{\begin{vmatrix} b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}$$

$$\boxed{\Delta_3 = 27\Delta_1}$$

$$\boxed{\Delta_3 - \Delta_2 = 3\Delta_1}$$

$$K = 3$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE MAIN 2020 (7 Jan. II)]

HW ✓

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)} a_{ij}$, If the determinant of B is 81, then $|A|$ is:

$$\begin{matrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{matrix}$$

(A) 1/3

(B) 3

(C) 1/81

(D) 1/9

FOR NOTES & DPP CHECK DESCRIPTION

Properties of Determinants

Property 8

Row-column operation : If any row (or any one column) of a determinant is added to or subtracted with equi-multiples of another Row (or another column), the value of determinant remains unchanged.

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$\boxed{R_1 \to R_1 + KR_2}$ ✓, $\boxed{K \in R}$ ✓

$R_1 \to R_1 + 2R_2$

$$\Delta' = \begin{vmatrix} a_1+2b_1 & a_2+2b_2 & a_3+2b_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \Delta$$

$$\Delta' = \begin{vmatrix} a_1+Kb_1 & a_2+Kb_2 & a_3+Kb_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\Delta' = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} Kb_1 & Kb_2 & Kb_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

0

$\Delta' = \Delta + 0$

FOR NOTES & DPP CHECK DESCRIPTION

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\boxed{C_1 \longrightarrow C_1 + 3C_3}$$

$$\Delta = \begin{vmatrix} a_1+3c_1 & b_1 & c_1 \\ a_2+3c_2 & b_2 & c_2 \\ a_3+3c_3 & b_3 & c_3 \end{vmatrix} \checkmark$$

$$\Delta = \begin{vmatrix} 3 & 5 & 7 \\ 2 & 1 & 3 \\ 8 & 1 & 0 \end{vmatrix}$$

$$R_1 \longrightarrow R_1 - R_2$$

$$\Delta = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 3 \\ 8 & 1 & 0 \end{vmatrix} \checkmark$$

Valid operations :→

1) $R_1 \longrightarrow R_1 \pm KR_2$ ✓

2) $\boxed{R_1 \longrightarrow R_2 + KR_3}$ (X)

3) $\boxed{R_1 \longrightarrow \underbrace{R_1 + 3R_1}}$ (X)

$R_1 \rightarrow 4R_1$

(4) $R_1 \longrightarrow \underbrace{R_1 + K_1R_2} + K_2R_3$ ✓

(5) $C_1 \longrightarrow C_1 + 5R_2$ (x)

(6) $C_1 \longrightarrow C_1 + C_2 + C_3$ ✓

Some Important Determinants to Remember

1. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Proof:

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix}$

$(b-c)(a-b)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a+b & b+c & c^2 \end{vmatrix}$

$(b-c)(a-b)\left[(\cancel{b}+c)-(a+\cancel{b})\right]$

$(b-c)(a-b)(c-a)$

$= \underline{\underline{RHS}}$

FOR NOTES & DPP CHECK DESCRIPTION

Some Important Determinants to Remember

2.

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = \boxed{(a-b)(b-c)(c-a)(a+b+c)}$$

Proof:

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a-b)(a^2+b^2+ab) & (b-c)(b^2+c^2+bc) & c^3 \end{vmatrix}$$

$$(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+b^2+ab & b^2+c^2+bc & c^3 \end{vmatrix}$$

$$(a-b)(b-c)\left[(\cancel{b^2}+c^2+bc)-(a^2+\cancel{b^2}+ab)\right]$$

$$(a-b)(b-c)\left[\underbrace{(c^2-a^2)}+b(c-a)\right]$$

$$(a-b)(b-c)(c-a)\left[c+a+b\right]$$

FOR NOTES & DPP CHECK DESCRIPTION

Recap Some Important Determinants to Remember

$bc^2 \rightarrow 3^\circ$ term

1. $$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \boxed{(a-b)(b-c)(c-a)}$$

$bc^3 \rightarrow 4^\circ$ term

2. $$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = \boxed{(a-b)(b-c)(c-a)(a+b+c)}$$

5° term

3. $$\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = \boxed{(a-b)(b-c)(c-a)(ab+bc+ca)}$$

2°

4. *Cyclic or Circulant Determinant* $$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3+b^3+c^3-3abc)$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

If $x \neq y \neq z$ and $\begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix} = 0$, then the value of $xy + yz + zx$ is

(A) $x + y + z$

(B) xyz

(C) $\dfrac{xyz}{x+y+z}$

(D) $\dfrac{x+y+z}{xyz}$ ✓

$$\begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

$C_1 \leftrightarrow C_3$
$C_2 \leftrightarrow C_3$

$$xyz\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix}$$

$$xyz\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} - \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$$

$$xyz\,\cancel{(x-y)}\cancel{(y-z)}\cancel{(z-x)}\,\underbrace{(xy+yz+zx)} = \cancel{(x-y)}\cancel{(y-z)}\cancel{(z-x)}\,(x+y+z)$$

$$xy+yz+zx = \frac{x+y+z}{xyz}$$

FOR NOTES & DPP CHECK DESCRIPTION

Some Important Determinants to Remember

4. Cyclic or Circulant Determinant

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(\underbrace{a^3+b^3+c^3-3abc})$$

Proof:

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} a+b+c & b+c+a & c+a+b \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix}$$

$$(a+b+c)\left[(a-b)(b-c) - (c-a)^2\right]$$

$$(a+b+c)\left[ab - \cancel{ac} + bc - b^2 - c^2 - a^2 + \cancel{2}ac\right]$$

$$(a+b+c)\left[ab + bc + ca - a^2 - b^2 - c^2\right]$$

$$-\underbrace{(a+b+c)}\underbrace{\left[a^2+b^2+c^2-ab-bc-ca\right]}$$

$$-(a^3+b^3+c^3-3abc)$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

If α, β and γ are the roots of the equations

$\boxed{x^3 + px + q = 0}$ then value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

α, β, γ

$\boxed{\alpha + \beta + \gamma = 0}$

$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$

$-(\cancel{\alpha^3 + \beta^3 + \gamma^3} \cancel{- 3\alpha\beta\gamma})$

$= 0$ ✓

A) p

B) q

C) $p^2 - 2q$

D) 0 ✓

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

If ω is the cube root of unity, then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} =$

$R_1 \to R_1 + R_2 + R_3$

$\boxed{1 + \omega + \omega^2 = 0}$ ✓

$\boxed{\omega^3 = 1}$ ✓

$\begin{vmatrix} 0 & 0 & 0 \\ - & - & - \\ - & - & - \end{vmatrix}$

(A) 1

(B) 0 ✓

(C) ω

(D) ω^2

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE MAIN 2019 (Apr.)]

HW

[Ans. A]

Let α and β be the roots of the equation $\boxed{x^2 + x + 1 = 0}$.

ω, ω^2

Then for $y \neq 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to

$\alpha = \omega$
$\beta = \omega^2$

$1 + \omega + \omega^2 = 0$
$\omega^3 = 1$

(A) y^3

(B) $y^3 - 1$

(C) $y(y^2 - 1)$

(D) $y(y^2 - 3)$

$$\begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & \omega^2+y & \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

HW

If ω, ω^2 are imaginary cube roots of unity and

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} \text{ and } \Delta_2 = \begin{vmatrix} 1 & 1 & \omega \\ 1 & 1 & \omega^2 \\ \omega^2 & \omega & 1 \end{vmatrix}, \text{ then } \frac{\Delta_1}{\Delta_2} \text{ is equal to}$$

(A) $\sqrt{3}$

(B) $\sqrt{3}i$

(C) 1

(D) -1

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE MAIN 2019 (Jan.)]

[Ans. D]

$x^2 = 2$

If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)(x+a+b+c)^2$, $\boxed{x \neq 0}$ and $a+b+c \neq 0$, then x is equal to.

(A) abc

(B) $-(a+b+c)$

(C) $2(a+b+c)$

(D) $-2(a+b+c)$ ✓

$R_1 \to R_1 + R_2 + R_3$

$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b+c+a & -(b+c+a) & 2b \\ 0 & c+a+b & c-a-b \end{vmatrix}$

$(a+b+c)\left[(a+b+c)^2 - 0\right]$

$= (a+b+c)^3$

$(a+b+c)^{\cancel{3}} = (x+a+b+c)^2 \cancel{(a+b+c)}$

$(a+b+c)^2 = (x+a+b+c)^2$

$x+a+b+c = \pm(a+b+c)$

$(+)$ $\boxed{x=0}$ ✗

$(-)$ $x+a+b+c = -a-b-c$

$\boxed{x = -2a-2b-2c}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

Show that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

$$\frac{1}{\cancel{a}bc}\begin{vmatrix} \cancel{a}(a^2+1) & a^2\cancel{b} & a^2\cancel{c} \\ \cancel{a}b^2 & \cancel{b}(b^2+1) & b^2\cancel{c} \\ \cancel{a}c^2 & \cancel{b}c^2 & \cancel{c}(c^2+1) \end{vmatrix}$$

$$\begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$R_1 \to R_1+R_2+R_3$

$$(a^2+b^2+c^2+1)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$$(a^2+b^2+c^2+1)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$C_1 \to C_1 - C_2$
$C_2 \to C_2 - C_3$

$$(a^2+b^2+c^2+1)\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & b^2 \\ 0 & -1 & c^2+1 \end{vmatrix}$$

$$(a^2+b^2+c^2+1)\{1-0\}$$

BRAIN TEASER

If $\boxed{a^2+b^2+c^2=1}$ then show that the value of the determinant

$$\begin{vmatrix} a^2+(b^2+c^2)\cos\theta & ba(1-\cos\theta) & ca(1-\cos\theta) \\ ab(1-\cos\theta) & b^2+(c^2+a^2)\cos\theta & cb(1-\cos\theta) \\ ac(1-\cos\theta) & bc(1-\cos\theta) & c^2+(a^2+b^2)\cos\theta \end{vmatrix}$$

simplifies to $\boxed{\cos^2\theta}$

$$\frac{1}{abc}$$

$$\begin{vmatrix} a^2+(b^2+c^2)\cos\theta & a^2(1-\cos\theta) & a^2(1-\cos\theta) \\ b^2(1-\cos\theta) & b^2+(c^2+a^2)\cos\theta & b^2(1-\cos\theta) \\ c^2(1-\cos\theta) & c^2(1-\cos\theta) & c^2+(a^2+b^2)\cos\theta \end{vmatrix}$$

$R_1 \to R_1+R_2+R_3$

$$\begin{vmatrix} 1 & 1 & 1 \\ b^2(1-\cos\theta) & b^2+(c^2+a^2)\cos\theta & b^2(1-\cos\theta) \\ c^2(1-\cos\theta) & c^2(1-\cos\theta) & c^2+(a^2+b^2)\cos\theta \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ -\cos\theta & \cos\theta & b^2(1-\cos\theta) \\ 0 & -\cos\theta & \end{vmatrix}$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$(\cos^2\theta - 0) = \cos^2\theta$ ✓

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

Prove that

$$\begin{vmatrix} (b+c)^2 & ba & ac \\ ba & (c+a)^2 & cb \\ ca & cb & (a+b)^2 \end{vmatrix} = \underbrace{\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}}_{} = 2abc(a+b+c)^3.$$

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\begin{vmatrix} (b+c)^2-a^2 & 0 & a^2 \\ b^2-(c+a)^2 & (c+a)^2-b^2 & b^2 \\ 0 & c^2-(a+b)^2 & (a+b)^2 \end{vmatrix}$$

$$(a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ b-c-a & c+a-b & b^2 \\ 0 & c-a-b & (a+b)^2 \end{vmatrix}.$$

HW

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

If in a triangle, s denotes the semi-perimeter and a, b, c denote the lengths of sides, then prove that

$$\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c)$$

$$s = \frac{a+b+c}{2}$$

$$\begin{vmatrix} a^2 & \alpha^2 & \alpha^2 \\ \beta^2 & b^2 & \beta^2 \\ \gamma^2 & \gamma^2 & c^2 \end{vmatrix}$$

Let $s-a=\alpha$

$s-b=\beta$

Add $s-c=\gamma$

$$3s-(a+b+c) = \alpha+\beta+\gamma$$

$$3s-2s = \alpha+\beta+\gamma$$

$$\boxed{\alpha+\beta+\gamma = s}$$

$a = s-\alpha = \beta+\gamma$

$b = s-\beta = \alpha+\gamma$

$c = s-\gamma = \alpha+\beta$

$$\begin{vmatrix} (\beta+\gamma)^2 & \alpha^2 & \alpha^2 \\ \beta^2 & (\alpha+\gamma)^2 & \beta^2 \\ \gamma^2 & \gamma^2 & (\alpha+\beta)^2 \end{vmatrix}$$

$$= 2\alpha\beta\gamma(\alpha+\beta+\gamma)^3 = 2(s-a)(s-b)(s-c)(s)^3$$

Use from previous Ques.

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

$$\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix}$$ is equal to

(A) abc

(B) 2 abc

(C) 4 abc

(D) 0

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE MAIN 2020 (Jan.)]

Let $\boxed{a - 2b + c = 1}$. If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then:

(A) $f(-50) = -1$

(B) $f(50) = 1$

(C) $f(50) = -501$

(D) $f(-50) = 501$

$a_{11} = x + a + x + c - 2(x+b)$

$2\cancel{x} + a + c - \cancel{2x} - 2b$

$a + c - 2b = 1$

$R_1 \to R_1 + R_3 - 2R_2$

a_{12}

$(x+2)+(x+4)-2(x+3)$

$\cancel{2x} + \cancel{6} - \cancel{2x} - \cancel{6} = 0$

$\begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

$(x+3)^2 - (x+4)(x+2)$

$\cancel{x^2} + \cancel{6x} + 9 - (\cancel{x^2} + \cancel{6x} + 8)$

$\boxed{f(x) = 1}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE MAIN 2024]

[Ans. B]

The values of α, for which $\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$, lie in the interval

(A) $(-2, 1)$

(B) $(-3, 0)$

(C) $\left(-\frac{3}{2}, \frac{3}{2}\right)$

(D) $(0, 3)$

$R_1 \to R_1 - R_2$

$$\begin{vmatrix} 0 & \left(\frac{3}{2}-\frac{1}{3}\right) & \left(\frac{3}{2}-\frac{1}{3}\right) \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$$

$$\cancel{\left(\frac{3}{2}-\frac{1}{3}\right)} \begin{vmatrix} 0 & 1 & 1 \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$$

$C_2 \to C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & -\alpha & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$$

$(3\alpha+1) + \alpha(2\alpha+3) = 0$

$3\alpha + 1 + 2\alpha^2 + 3\alpha = 0$

$\boxed{2\alpha^2 + 6\alpha + 1 = 0}$

$\alpha = \dfrac{-6 \pm \sqrt{36-8}}{2 \times 2}$

$= \dfrac{-6 \pm \sqrt{28}}{2 \cdot 2} = \dfrac{-6 \pm 2\sqrt{7}}{2 \times 2}$

$\boxed{\alpha = \dfrac{-3 \pm \sqrt{7}}{2}}$

ATDB.uno

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2024 (08 Apr Shift 2)]

[Ans. B]

If $\alpha \neq a, \beta \neq b, \gamma \neq c$ and $\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0$, then $\frac{a}{\alpha-a} + \frac{b}{\beta-b} + \frac{\gamma}{\gamma-c}$ is equal to:

(A) 3

(B) 0

(C) 1

(D) 2

$\frac{a}{\alpha-a} + \frac{b}{\beta-b} + \frac{c}{\gamma-c}$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$\begin{vmatrix} \alpha-a & b-\beta & 0 \\ 0 & \beta-b & c-\gamma \\ a & b & \gamma \end{vmatrix} = 0$$

$$(\alpha-a)\left[(\beta-b)\gamma - b(c-\gamma)\right] + (b-\beta)\left[a(c-\gamma)\right] = 0$$

$$(\alpha-a)(\beta-b)\gamma - b(c-\gamma)(\alpha-a) + a(b-\beta)(c-\gamma) = 0$$

$$(\alpha-a)(\beta-b)\gamma + b(\gamma-c)(\alpha-a) + a(\beta-b)(\gamma-c) = 0$$

$(\alpha-a)(\beta-b)(\gamma-c)$

$$\frac{\gamma-c+c}{\gamma-c} + \frac{b}{\beta-b} + \frac{a}{\alpha-a} = 0$$

$$1 + \frac{c}{\gamma-c} + \frac{b}{\beta-b} + \frac{a}{\alpha-a} = 0$$

$$\boxed{\frac{a}{\alpha-a} + \frac{b}{\beta-b} + \frac{c}{\gamma-c} = -1}$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE MAIN 2023]

HW

[Ans. 495]

Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as $A, A+1, A+2$, respectively. Let **a, b, c** be the 7th, 9th, 17th terms of A_1, A_2, A_3, respectively such that $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0$. If a = 29, then the sum of first 20 terms of an AP whose first term is $c - a - b$ and common difference is $\frac{d}{12}$, is equal

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE MAIN 2019 (Jan.)]

HW

[Ans. A]

Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is

(A) $2\sqrt{3}$

(B) $-2\sqrt{3}$

(C) $-\sqrt{3}$

(D) $\sqrt{3}$

FOR NOTES & DPP CHECK DESCRIPTION

Summation of Determinants

$\sum \rightarrow$ add

$$\Delta_r = \begin{vmatrix} r & 1 & 7 \\ 2r & 3 & 8 \\ 3r & 5 & 9 \end{vmatrix} \quad \text{Const}$$

$$\sum_{r=1}^{5} \Delta_r = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \Delta_5 = ?$$

$$\Sigma \Delta_r = \begin{vmatrix} \Sigma r & 1 & 7 \\ \Sigma 2r & 3 & 8 \\ \Sigma 3r & 5 & 9 \end{vmatrix}$$

Special Series_Remember

1. Sum of the first n natural numbers $\sum_{r=1}^{n} r = \boxed{\dfrac{n(n+1)}{2}}$

2. Sum of the squares of the first n natural numbers $\boxed{\sum_{r=1}^{n} r^2 = \dfrac{n(n+1)(2n+1)}{6}}$

3. Sum of the cubes of the first n natural numbers $\boxed{\sum_{r=1}^{n} r^3 = \dfrac{n^2(n+1)^2}{4} = \left[\sum_{r=1}^{n} r\right]^2}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

The value of $\sum_{n=1}^{N} U_n$ if $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N \end{vmatrix}$ is

(A) 0

(B) 1

(C) −1

(D) 2

n → Variable

N → const

$$\sum U_n = \begin{vmatrix} \sum n & - & - \\ \sum n^2 & - & - \\ \sum n^3 & - & - \end{vmatrix}$$

Const

$$= \begin{vmatrix} \frac{N(N+1)}{2} & 1 & 5 \\ \frac{N(N+1)(2N+1)}{6} & 2N+1 & 2N+1 \\ \left(\frac{N(N+1)}{2}\right)^2 & 3N^2 & 3N \end{vmatrix}$$

$$\frac{N(N+1)}{2}\begin{vmatrix} 1 & 1 & 5 \\ \frac{2N+1}{3} & 2N+1 & 2N+1 \\ \frac{N(N+1)}{2} & 3N^2 & 3N \end{vmatrix}$$

$$\frac{N^2(N+1)(2N+1)}{2}\begin{vmatrix} 1 & 1 & 5 \\ \frac{1}{3} & 1 & 1 \\ \frac{N+1}{2} & 3N & 3 \end{vmatrix}$$

R_3

$\frac{N+1}{2}(-4) - 3N(1-5/3) + 3\cdot\frac{2}{3}$

$-2(N+1) - 3N\cdot(-2/3) + 2$

$-\cancel{2N} - \cancel{2} + \cancel{2N} + \cancel{2} = 0$

FOR NOTES & DPP CHECK DESCRIPTION

$r \to$ variable

$S_n = \frac{a(r^n-1)}{r-1}$

QUESTION [1993]

[Ans. A]

If $\Delta_r = \begin{vmatrix} 2^r & 2.3^r & 4.5^r \\ \alpha & \beta & \gamma \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$, then the value of $\sum_{r=0}^{n-1} \Delta_r$ is

(A) 0

(B) $\alpha\beta\gamma$

(C) $\alpha + \beta + \gamma$

(D) $\alpha \cdot \alpha^n + \beta \cdot 3^n + \gamma \cdot 4^n$

$$\begin{vmatrix} \sum 2^r & \sum 2.3^r & \sum 4.5^r \\ - & - & - \\ - & - & \end{vmatrix}$$

$$\begin{vmatrix} 2^n-1 & 3^n-1 & 5^n-1 \\ \alpha & \beta & \gamma \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$$

0

$$\sum_{r=0}^{n-1} 2^r = 2^0 + 2^1 + 2^2 + \cdots + 2^{n-1}$$
$$= 1\frac{[2^n-1]}{2-1} = (2^n-1)$$

$$\sum_{r=0}^{n-1} 2.3^r = 2[3^0+3^1+3^2+\cdots+3^{n-1}]$$
$$= \frac{\cancel{2}[3^n-1]}{\cancel{3-1}} = 3^n-1$$

$$\sum_{r=0}^{n-1} 4.5^r = 4[5^0+5^1+5^2+\cdots]$$
$$= \frac{\cancel{4}(5^n-1)}{\cancel{(5-1)}} = (5^n-1)$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE MAIN 2023]

[Ans. 6]

Let $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$. If $\sum_{k=1}^{n} D_k = 96$, then n is equal to

Const

$$\sum D_K = \begin{vmatrix} \sum_{K=1}^{n} 1 & \sum_{K=1}^{n} 2K & \sum_{K=1}^{n} 2K-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$

$$= \begin{vmatrix} n & n^2+n & n^2 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = \begin{vmatrix} 0 & -2 & 0 \\ 0 & 2 & -n-2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$2[n(n+2)] = 96$

$\Sigma 1 = 1+1+1+\dots 1 = n$

$\Sigma 2K = 2[1+2+\dots+n]$

$= \cancel{2}\frac{(n)(n+1)}{\cancel{2}}$

$= n^2+n$

$\sum_{K=1}^{n}(2K-1) = \Sigma 2K - \Sigma 1$

$= n^2 + \cancel{n} - \cancel{n}$

$= n^2$

$\boxed{n(n+2) = 48}$

6×8

$\boxed{n=6}$

ATDB.uno

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2024 (06 Apr Shift 1)]

[Ans. B]

For $\alpha, \beta \in \mathbb{R}$ and $n \in N$, let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$. Then $\underline{2A_{10} - A_8} =$

(A) 0

(B) $4\alpha + 2\beta$

(C) $2\alpha + 4\beta$

(D) $2n$

$$2A_{10} = \begin{vmatrix} 2\times10 & - & - \\ 4\times10 & - & - \\ 2(28) & - & - \end{vmatrix}$$

$$A_8 = \begin{vmatrix} 8 & - & - \\ 16 & - & - \\ 22 & - & - \end{vmatrix}$$

$$\Rightarrow 2A_{10} - A_8 = \begin{vmatrix} 12 & 1 & \frac{n^2}{2}+\alpha \\ 24 & 2 & n^2-\beta \\ 34 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$\boxed{C_1 \to C_1 - 12C_2}$$

$$\begin{vmatrix} 0 & 1 & \frac{n^2}{2}+\alpha \\ 0 & 2 & n^2-\beta \\ -2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$-2\left[n^2-\beta-2\left(\frac{n^2}{2}+\alpha\right)\right] = -2[-\beta-2\alpha] = 2(\beta+2\alpha)$$

FOR NOTES & DPP CHECK DESCRIPTION

HW_QUESTION

HW

If 'm' is a positive integer $\Delta_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(2m) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$

then value of $\sum_{r=0}^{m} \Delta_r$ is:

(A) 1

(B) 0

(C) 2

(D) −1

FOR NOTES & DPP CHECK DESCRIPTION

HW_QUESTION

HW

Ans = 0

If $f(x) = \begin{vmatrix} \int \sin^5 x & \int \ln \sin x & \int \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} \\ n & \sum_{k=1}^{n} k & \prod_{k=1}^{n} k \\ \frac{8}{15} & \frac{\pi}{2}\ln\left(\frac{1}{2}\right) & \frac{\pi}{4} \end{vmatrix}$ then find the value of $\int_0^{\pi/2} f(x)\,dx$

Const.

FOR NOTES & DPP CHECK DESCRIPTION

Differentiation of Determinant

$$\frac{d}{dx}(fg) = fg' + gf'$$

$$\Delta(x) = \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix}$$

$$\Delta'(x) = \begin{vmatrix} f_1' & f_2' & f_3' \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix} + \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1' & g_2' & g_3' \\ h_1 & h_2 & h_3 \end{vmatrix} + \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \\ h_1' & h_2' & h_3' \end{vmatrix}$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

If $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} \equiv px^5+qx^4+rx^3+sx^2+tx+w$

then find the value of t & w.

Identity.

for w

put $\boxed{x=0}$

$$\begin{vmatrix} 1 & 1 & 2x \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} + \begin{vmatrix} 1+x & x & x^2 \\ 1 & 1 & 2x \\ x^2 & x & 1+x \end{vmatrix} + \begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ 2x & 1 & 1 \end{vmatrix} = \cancel{5px^4} + \cancel{4qx^3} + \cancel{3rx^2} + \cancel{2sx} + t$$

Put $x=0$

$$\underbrace{\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}}_{1} + \underbrace{\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}}_{1} + \underbrace{\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}}_{1} = t = \begin{vmatrix} 3 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 1 & 3 \end{vmatrix}$$

$\Rightarrow \boxed{t=3}$ ✓

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE MAIN 2024]

HW

[Ans. C]

If $f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all $x \in \mathbb{R}$, then $\underbrace{2f(0)} + \underbrace{f'(0)}$ is equal to

(A) 48

(B) 24

(C) 42

(D) 18

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE MAIN 2024]

[Ans. A]

If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5}f'(0)$ is equal to ________

(A) 0

(B) 1

(C) 2

(D) 6

$\frac{d}{dx}(\cos^4 x) = 4\cos^3 x(-\sin x) = 0$

$\frac{d}{dx}(\sin^2 2x) = 2\sin 2x \cos 2x \cdot 2$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$f(x) = \begin{vmatrix} -3 & 0 & 3 \\ 3 & -3 & 0 \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$$

$$f(x) = 9\begin{vmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$$

$$= 9\left[\sin^2 2x + 3 + \underbrace{2\sin^4 x + 2\cos^4 x}\right]$$

$$f(x) = 9\left[\underbrace{\sin^2 2x} + 3 + 2(1 - 2\sin^2 x\cos^2 x)\right]$$

$$9\left[4\sin^2 x\cancel{\cos^2 x} + 5 - 4\cancel{\sin^2 x\cos^2 x}\right]$$

$$\boxed{f(x) = 45}$$

$\Rightarrow f'(x) = 0$

M-2

$$\underbrace{\begin{vmatrix} 0 & - & - \\ 0 & - & - \\ 0 & - & - \end{vmatrix}}_{0} + \underbrace{\begin{vmatrix} - & 0 & - \\ - & 0 & - \\ - & 0 & - \end{vmatrix}}_{0} + \underbrace{\begin{vmatrix} - & - & 0 \\ - & - & 0 \\ - & - & 0 \end{vmatrix}}_{0}$$

FOR NOTES & DPP CHECK DESCRIPTION

Multiplication of 2 Determinants

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix} = \begin{vmatrix} a_1\alpha_1 + a_2\beta_1 + a_3\gamma_1 & - & - \\ - & - & - \\ - & - & - \end{vmatrix}$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

For all real values of A, B, C and P, Q, R show that

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$$

$$\begin{vmatrix} \cos A\cos P + \sin A\sin P & \cos A\cos Q + \sin A\sin Q & \cos A\cos R + \sin A\sin R \\ \cos B\cos P + \sin B\sin P & \cos B\cos Q + \sin B\sin Q & - \\ \cos C\cos P + \sin C\sin P & \cos C\cos Q + \sin C\sin Q & - \end{vmatrix}$$

$$= \underbrace{\begin{vmatrix} \cos A & \sin A & 0 \\ \cos B & \sin B & 0 \\ \cos C & \sin C & 0 \end{vmatrix}}_{0} \cdot \begin{vmatrix} \cos P & \cos Q & \cos R \\ \sin P & \sin Q & \sin R \\ 0 & 0 & 0 \end{vmatrix}$$

$0 \times 0 = 0$ ✓

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

$(c+d)(a+b) + (a+b)(c+d) + 0$

$2(a+b)(c+d)$

Without expanding at any stage show that:

$$\begin{vmatrix} 2 & a+b+c+d & ab+cd \\ a+b+c+d & 2(a+b)(c+d) & ab(c+d)+cd(a+b) \\ ab+cd & ab(c+d)+cd(a+b) & 2abcd \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 1 & 0 \\ c+d & a+b & 0 \\ cd & ab & 0 \end{vmatrix} \begin{vmatrix} 1 & a+b & ab \\ 1 & c+d & cd \\ 0 & 0 & 0 \end{vmatrix}$$

$0 \times 0 = 0$ ✓

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

If α, β, γ are real numbers, then without expanding at any stage, show that

$$\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix} = 0$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

Prove that

$$\boxed{\begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-z)^2 \end{vmatrix}} = \begin{vmatrix} (1+ax)^2 & (1+bx)^2 & (1+cx)^2 \\ (1+ay)^2 & (1+by)^2 & (1+cy)^2 \\ (1+az)^2 & (1+bz)^2 & (1+cz)^2 \end{vmatrix}$$

$$= \underbrace{2(b-c)(c-a)(a-b) \times (y-z)(z-x)(x-y)}$$

LHS

$$\begin{vmatrix} a^2+x^2-2ax & a^2+y^2-2ay & a^2+z^2-2az \\ b^2+x^2-2bx & b^2+y^2-2by & b^2+z^2-2bz \end{vmatrix} = \begin{vmatrix} a^2 & 1 & -2a \\ b^2 & 1 & -2b \\ c^2 & 1 & -2c \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x & y & z \end{vmatrix}.$$

$$2\begin{vmatrix} a^2 & 1 & a \\ b^2 & 1 & b \\ c^2 & 1 & c \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$C_1 \leftrightarrow C_2$ & $C_2 \leftrightarrow C_3$

$$2\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

Rhs: $2(a-b)(b-c)(c-a)(x-y)(y-z)(z-x)$

QUESTION

Prove that

$$\begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-z)^2 \end{vmatrix} = \boxed{\begin{vmatrix} (1+ax)^2 & (1+bx)^2 & (1+cx)^2 \\ (1+ay)^2 & (1+by)^2 & (1+cy)^2 \\ (1+az)^2 & (1+bz)^2 & (1+cz)^2 \end{vmatrix}}$$

$$= 2(b-c)(c-a)(a-b) \times (y-z)(z-x)(x-y)$$

$$\begin{vmatrix} 1+a^2x^2+2ax & 1+b^2x^2+2bx & 1+c^2x^2+2cx \\ 1+a^2y^2+2ay & & \\ & & \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ 2a & 2b & 2c \end{vmatrix}$$

$$2\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$= 2(x-y)(y-z)(z-x)(a-b)(b-c)(c-a)$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Adv.-2015]

[Ans. B,C]

Which of the following values of α satisfy the equation $\begin{vmatrix}(1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2\\(2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2\\(3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2\end{vmatrix} = -648\alpha$

(A) -4

(B) 9

(C) -9

(D) 4

$$\begin{vmatrix}1+\alpha^2+2\alpha & 1+4\alpha^2+4\alpha & 1+9\alpha^2+6\alpha\\4+\alpha^2+4\alpha & 4+4\alpha^2+8\alpha & \\9+\alpha^2+6\alpha & & \end{vmatrix} = \begin{vmatrix}1 & \alpha^2 & \alpha\\4 & \alpha^2 & 2\alpha\\9 & \alpha^2 & 3\alpha\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\1 & 4 & 9\\2 & 4 & 6\end{vmatrix}$$

$$= \alpha^3\begin{vmatrix}1 & 1 & 1\\4 & 1 & 2\\9 & 1 & 3\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\1 & 4 & 9\\2 & 4 & 6\end{vmatrix}$$

$$= 2\alpha^3\begin{vmatrix}1 & 1 & 1\\4 & 1 & 2\\9 & 1 & 3\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\1 & 4 & 9\\1 & 2 & 3\end{vmatrix}$$

FOR NOTES & DPP CHECK DESCRIPTION

HW_QUESTION

[Ans. C]

If $\alpha, \beta \neq 0$ and $\boxed{f(n) = \alpha^n + \beta^n}$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = k(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$ then 'k' is equal to:

(A) $\alpha\beta$

(B) $1/\alpha\beta$

(C) 1

(D) -1

$$\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

$C_1 \to C_1 - C_2$

FOR NOTES & DPP CHECK DESCRIPTION

Solving System of Linear Equation in 3 Variable

Case 1: Non Homogenous System of Equation

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Case 2: Homogenous System of Equation

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

FOR NOTES & DPP CHECK DESCRIPTION

Cramer's Rule for Non Homogeneous System of Equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$$\boxed{\begin{aligned} x &= D_1/D \\ y &= D_2/D \\ z &= D_3/D \end{aligned}}$$

Proof →

$$D_1 = \begin{vmatrix} a_1x+b_1y+c_1z & b_1 & c_1 \\ a_2x+b_2y+c_2z & b_2 & c_2 \\ a_3x+b_3y+c_3z & b_3 & c_3 \end{vmatrix}$$

$$C_1 \rightarrow C_1 - yC_2 - zC_3$$

$$D_1 = \begin{vmatrix} a_1x & b_1 & c_1 \\ a_2x & b_2 & c_2 \\ a_3x & b_3 & c_3 \end{vmatrix}$$

$$D_1 = x \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D_1 = xD \Rightarrow \boxed{x = D_1/D}$$

FOR NOTES & DPP CHECK DESCRIPTION

Nature of Solutions

Case→1 If $\boxed{D_1 = D_2 = D_3 = 0 \text{ \& } D \neq 0}$ ⇒ Unique trivial solⁿ $(x = y = z = 0)$ → all "zero"

Case→2 If at least one of D_1, D_2, D_3 is non zero & $D \neq 0$

Ex→ $\left[\begin{array}{l} \underbrace{D_1 = 10,\ D_2 = 20,\ D_3 = 5}_{x = \frac{10}{7},\ y = \frac{20}{7},\ z = 5/7},\ D = 7 \end{array} \right]$ ⇒ Unique non trivial solⁿ

Case→3 If $\boxed{D_1 = D_2 = D_3 = 0 \text{ \& } D = 0}$ ⇒ x, y & z will have infinite solⁿ

Case→4 If at least one of D_1, D_2, D_3 is non zero but $D = 0$ ⇒ no solⁿ → system is inconsistent

$\boxed{\text{☆☆ Unique sol}^n}$ ⇒ $\boxed{\text{Case→1 \& 2 Union}}$ ⇒ $\boxed{D \neq 0}$ ☆☆

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

Investigate for what values of λ, μ the simultaneous equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have;

(a) A unique solution.
(b) An infinite number of solutions.
(c) No solution.

a) Unique sol^n

$$D=\begin{vmatrix}1&1&1\\1&2&3\\1&2&\lambda\end{vmatrix}\neq 0$$

$$\boxed{\lambda\neq 3}$$

$$D_2=\begin{vmatrix}1&6&1\\1&10&3\\1&\mu&3\end{vmatrix}=0 \Rightarrow \mu=10$$

(b) $D=\begin{vmatrix}1&1&1\\1&2&3\\1&2&\lambda\end{vmatrix}=0 \Rightarrow \boxed{\lambda=3}$

&

$$D_3=\begin{vmatrix}1&1&6\\1&2&10\\1&2&\mu\end{vmatrix}=0$$

$$\boxed{\mu=10}$$

$$D_1=\begin{vmatrix}6&1&1\\10&2&3\\\mu&2&3\end{vmatrix}=0 \Rightarrow \mu=10$$

(b) $\lambda=3$ & $\mu=10$

(c) No sol^n: $\lambda=3,\ \mu\in R-\{10\}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2024 (08 Apr Shift 2)]

[Ans. B]

If the system of equations $x + 4y - z = \lambda, 7x + 9y + \mu z = -3, 5x + y + 2z = -1$ has infinitely many solutions, then $(2\mu + 3\lambda)$ is equal to :

(A) 3

(B) −3

(C) −2

(D) 2

$$D = \begin{vmatrix} 1 & 4 & -1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{vmatrix} = 0$$

$$1[18-\mu] - 4[14-5\mu] - 1[7-45] = 0$$

$$\cancel{18} - \mu - \cancel{56} + 20\mu + \cancel{38} = 0$$

$$19\mu = 0$$

$$\Rightarrow \boxed{\mu = 0}$$

$$D_3 = \begin{vmatrix} 1 & 4 & \lambda \\ 7 & 9 & -3 \\ 5 & 1 & -1 \end{vmatrix} = 0$$

$$1[-9+3] - 4[-7+15] + \lambda[7-45] = 0$$

$$-6 - 32 - 38\lambda = 0$$

$$\cancel{38} = -\cancel{38}\lambda$$

$$\boxed{\lambda = -1}$$

Check $D_2 = D_1 = 0$ for $\boxed{\lambda = -1}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2024 (Feb.-I)]

[Ans. B]

If the system of equations

$$2x + 3y - z = 5$$
$$x + \alpha y + 3z = -4$$
$$3x - y + \beta z = 7$$

has infinitely many solutions, then $13\alpha\beta =$

A 1110

B 1120

C 1210

D 1220

$$D = \begin{vmatrix} 2 & 3 & -1 \\ 1 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 2 & 3 & 5 \\ 1 & \alpha & -4 \\ 3 & -1 & 7 \end{vmatrix} = 0$$

$$= 2[7\alpha - 4] - 3[7 + 12] + 5[-1 - 3\alpha] = 0$$

$$= 14\alpha - 8 - 57 - 5 - 15\alpha = 0$$

$$\boxed{\alpha = -70}$$

$$D_2 = \begin{vmatrix} 2 & 5 & -1 \\ 1 & -4 & 3 \\ 3 & 7 & \beta \end{vmatrix} = 0$$

$$= 2[-4\beta - 21] - 5[\beta - 9] - 1[7 + 12] = 0$$

$$-8\beta - 42 - 5\beta + 45 - 19 = 0$$

$$-13\beta + 3 - 19 = 0$$

$$13\beta = -16$$

$$\boxed{\beta = \frac{-16}{13}}$$

$$13\alpha\beta = \cancel{13} \cdot 70 \times \frac{16}{\cancel{13}} = 1120$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2023]

[Ans. C]

For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations

$$x - y + z = 5$$
$$2x + 2y + \alpha z = 8$$
$$3x - y + 4z = \beta$$

has infinitely many solutions. Then α and β are the roots of

(A) $x^2 - 10x + 16 = 0$

(B) $x^2 + 18x + 56 = 0$

(C) $x^2 - 18x + 56 = 0$

(D) $x^2 - 14x + 24 = 0$

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0$$

— FOR NOTES & DPP CHECK DESCRIPTION —

QUESTION [JEE Main 2024 (05 Apr Shift 1)]

[Ans. C]

If the system of equations

$11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

has infinitely many solutions, then $\lambda^4 - \mu$ is equal to :

(A) 51

(B) 45

(C) 47

(D) 49

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2024 (Jan.-II)]

HW

[Ans. 113]

Let for any three distinct consecutive terms a, b, c of an A.P., the lines $ax + by + c = 0$ be concurrent at the point P and $Q(\alpha, \beta)$ be a point such that the system of equations

$x + y + z = 6$

$2x + 5y + \alpha z = \beta$ and

$x + 2y + 3z = 4$,

has infinitely many solutions. Then $(PQ)^2$ is equal to ?

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2023 (01 Feb. Shift-I)]

[Ans. D]

Let S denote the set of all real values of λ such that the system of equations

$$\left.\begin{aligned}\lambda x + y + z &= 1\\ x + \lambda y + z &= 1\\ x + y + \lambda z &= 1\end{aligned}\right\}$$

Put $\lambda = 1 \Rightarrow \left.\begin{aligned}x+y+z&=1\\x+y+z&=1\\x+y+z&=1\end{aligned}\right\}$ Infinite sol^n

is inconsistent, then $\sum_{\lambda \in S}(|\lambda|^2 + |\lambda|)$ is equal to

$\Rightarrow \lambda = -2$ gives no sol^n

$S = \{-2\}$

$(-2)^2 + |-2| = 4 + 2 = 6$

no sol^n

$\Rightarrow D = 0$

$\boxed{D_1, D_2, D_3 \neq 0}$

(A) 2

(B) 12

(C) 4

(D) 6

$$D = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$D = \lambda[\lambda^2 - 1] - 1[\lambda - 1] + 1[1 - \lambda] = 0$

$\lambda^3 - \lambda - \lambda + 1 + 1 - \lambda = 0$

$\boxed{\lambda^3 - 3\lambda + 2 = 0}$

$\lambda^3 - 3\lambda + 3 - 1 = 0$

$(\lambda^3 - 1) - 3(\lambda - 1)$

$(\lambda - 1)(\lambda^2 + 1 + \lambda) - 3(\lambda - 1) = 0$

$(\lambda - 1)[\lambda^2 + 1 + \lambda - 3] = 0$

$(\lambda - 1)(\lambda^2 + \lambda - 2) = 0$

$(\lambda - 1)(\lambda + 2)(\lambda - 1) = 0 \Rightarrow \lambda = 1, -2$

Infinite

ATDB.uno

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2024 (30 Jan Shift 2)]

[Ans. D]

Consider the system of linear equations
$x+y+z=5, x+2y+\lambda^2 z=9, x+3y+\lambda z=\mu$, where $\lambda, \mu \in R$.
Then, which of the following statement is NOT correct?

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow 1[2\lambda - 3\lambda^2] - 1[\lambda - \lambda^2] + 1(3-2) = 0$$

$$2\lambda - 3\lambda^2 - \lambda + \lambda^2 + 1 = 0 \Rightarrow -2\lambda^2 + \lambda + 1 = 0$$

$$2\lambda^2 - \lambda - 1 = 0$$

$$2\lambda^2 - 2\lambda + \lambda - 1 = 0$$

$$(2\lambda+1)(\lambda-1) = 0$$

$$\lambda = -\tfrac{1}{2}, \lambda = 1$$

(A) System has infinite number of solution if $\lambda = 1$ and $\mu = 13$ → True

(B) System is inconsistent if $\lambda = 1$ and $\mu \neq 13$
True   no solⁿ

(C) System is consistent if $\lambda \neq 1$ and $\mu = 13$

(D) System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$
False

$\lambda = 1$

$$D_2 = \begin{vmatrix} 1 & 0 & 1 \\ 1 & & 1 \\ 1 & & 1 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 1 \\ \mu & 3 & 1 \end{vmatrix} = 0$$

at $\lambda = 1 \Rightarrow \boxed{D = 0}$

$$D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & \mu \end{vmatrix}$$

$$= 1[2\mu - 27] - 1[\mu - 9] + 5(3-2) = 0$$

$$= 2\mu - 27 - \mu + 9 + 5 = 0$$

$$\boxed{\mu = 13}$$

ATDB.uno

FOR NOTES & DPP CHECK DESCRIPTION

Cramer's Rule for Homogeneous System of Equations

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

here $\boxed{D_1 = D_2 = D_3 = 0}$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Nature of solⁿ : →

1) If $D \neq 0 \Rightarrow$ Unique trivial sol^n

2) If $D = 0 \Rightarrow$ Infinite sol^n.

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2023 (08 Apr Shift 2)]

[Ans. A]

Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations

$$\boxed{\begin{aligned} x + y + \sqrt{3}z &= 0 \\ -x + (\tan\theta)y + \sqrt{7}z &= 0 \\ x + y + (\tan\theta)z &= 0 \end{aligned}}$$

→ Homogenous system.

has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta \in S} \theta$ is equal to

(A) 20 ✓

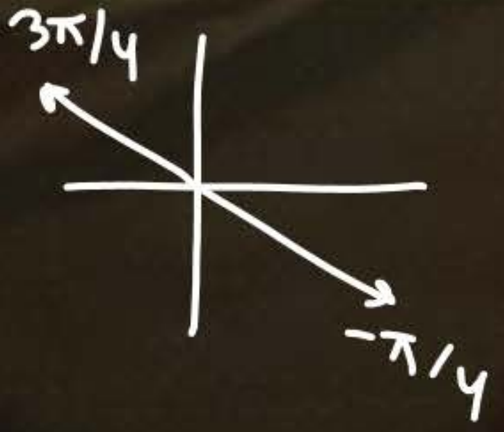
(B) 40

(C) 30

(D) 10

Infinite solⁿ

$\boxed{D = 0}$



$$\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_3$

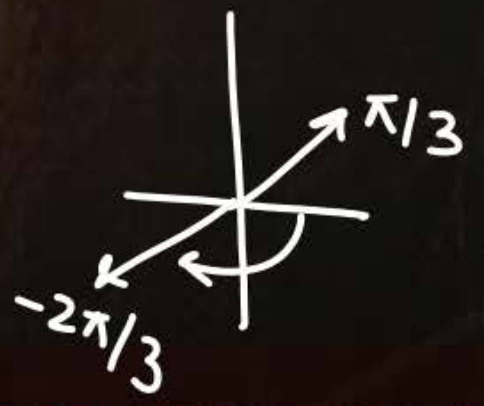
$$\begin{vmatrix} 0 & 0 & \sqrt{3} - \tan\theta \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix}$$

$(\sqrt{3} - \tan\theta)[-1 - \tan\theta] = 0$

$(\sqrt{3} - \tan\theta)(1 + \tan\theta) = 0$

$\tan\theta = \sqrt{3}$ or $\tan\theta = -1$

$\theta = \pi/3$ or $-2\pi/3$ $\quad$ $\theta = -\pi/4, \frac{3\pi}{4}$



$\frac{120}{\pi}\left[-\pi/3 + \pi/2\right] = \frac{120}{\pi} \cdot \pi/6 = 20$

FOR NOTES & DPP CHECK DESCRIPTION

Note

If the homogenous system of equations

$$\begin{cases} a_1x + b_1y + c_1 = 0; \\ a_2x + b_2y + c_2 = 0; \\ a_3x + b_3y + c_3 = 0 \end{cases}$$

have some Non trivial Solution then this implies that the given system must have Infinite Solutions

$$\Rightarrow D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2024 (04 Apr Shift 1)]

[Ans. C]

If the system of equations

$x + (\sqrt{2}\sin\alpha)y + (\sqrt{2}\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

$x + (\sin\alpha)y - (\cos\alpha)z = 0$

has a non-trivial solution, then $\alpha \in \left(0, \frac{\pi}{2}\right)$ is

$D = 0$

(A) $\frac{11\pi}{24}$

(B) $\frac{7\pi}{24}$

(C) $\frac{5\pi}{24}$

(D) $\frac{3\pi}{4}$

$$\begin{vmatrix} 1 & \sqrt{2}\sin\alpha & \sqrt{2}\cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$1[-\cos^2\alpha - \sin^2\alpha] - \sqrt{2}\sin\alpha[-\cos\alpha - \sin\alpha] + \sqrt{2}\cos\alpha[\sin\alpha - \cos\alpha]$$

$$= -1 + \sqrt{2}\sin\alpha\cos\alpha + \sqrt{2}\sin^2\alpha + \sqrt{2}\sin\alpha\cos\alpha - \sqrt{2}\cos^2\alpha = 0$$

$$= -1 + \sqrt{2}\sin 2\alpha - \sqrt{2}\cos 2\alpha = 0$$

$$\sqrt{2}(\sin 2\alpha - \cos 2\alpha) = 1$$

$$\boxed{\sin 2\alpha - \cos 2\alpha = \frac{1}{\sqrt{2}}}$$

dont S.B.S

divide by $\sqrt{2}$

$$\frac{\sin 2\alpha}{\sqrt{2}} - \frac{\cos 2\alpha}{\sqrt{2}} = \frac{1}{2}$$

$$\sin(2\alpha - \pi/4) = \frac{1}{2}$$

$$2\alpha - \pi/4 = \pi/6$$

$$2\alpha = \pi/4 + \pi/6$$

$$2\alpha = \frac{3\pi + 2\pi}{12}$$

$$2\alpha = 5\pi/12$$

$$\alpha = 5\pi/24$$

ATDB.uno

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2019 (April)]

[Ans. C]

If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trivial solution (x, y, z), then $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to

(A) $3/4$

(B) -4

(C) $1/2$

(D) $-1/4$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2018]

[Ans. A]

If the system of linear

$x + ky + 3z = 0;$

$3x + ky - 2z = 0;$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z) then $\frac{xz}{y^2} = ?$

(A) 10

(B) −30

(C) 30

(D) −10

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Adv. 2023]

$D = \begin{vmatrix} 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix} = 0$

Let α, β and γ be real numbers. Consider the following system of linear equations

$$\boxed{\begin{aligned} x + 2y + z &= 7 \\ x + \alpha z &= 11 \\ 2x - 3y + \beta z &= \gamma \end{aligned}}$$

$1[3\alpha] - 2[\beta - 2\alpha] + 1(-3) = 0$

$3\alpha - 2\beta + 4\alpha - 3 = 0$

$7\alpha = 2\beta + 3$

$\boxed{\beta = \frac{7\alpha - 3}{2}}$

Match each entry in List-I to the correct entries in List-II.

List – I

(P) If $\boxed{\beta = \frac{1}{2}(7\alpha - 3)}$ and $\boxed{\gamma = 28}$, then the system has

(Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has

(R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\underline{\alpha = 1}$ and $\underline{\gamma \neq 28}$, then the system has

$\boxed{D_3 \neq 0}$

$\Rightarrow z = D_3/D \neq 0$

(S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has

List-II

(1) A unique solution

(2) No solution

(3) Infinitely many solution

(4) $x = 11, y = -2$ and $z = 0$ as a solution

(5) $x = -15, y = 4$ and $\boxed{z = 0}$ as a solution

— FOR NOTES & DPP CHECK DESCRIPTION —

$$D_3 = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix} = 0$$

$$D_3 = 1[33] - 2[\gamma - 22] + 7(-3) = 0$$

$$33 - 2\gamma + 44 - 21 = 0$$

$$2\gamma = 77 - 21$$

$$2\gamma = 56$$

$$\boxed{\gamma = 28}$$

Check D_1 & D_2 also

QUESTION [JEE Adv. 2023]

[Ans. A]

The correct option is:

(A) $(P) \rightarrow (3)$, $(Q) \rightarrow (2)$, $(R) \rightarrow (1)$, $(S) \rightarrow (4)$

(B) $(P) \rightarrow (3)$, $(Q) \rightarrow (2)$, $(R) \rightarrow (5)$, $(S) \rightarrow (4)$

(C) $(P) \rightarrow (2)$, $(Q) \rightarrow (1)$, $(R) \rightarrow (4)$, $(S) \rightarrow (5)$

(D) $(P) \rightarrow (2)$, $(Q) \rightarrow (1)$, $(R) \rightarrow (1)$, $(S) \rightarrow (3)$

— FOR NOTES & DPP CHECK DESCRIPTION —

QUESTION [JEE Main 2019 (Jan.)]

[Ans. B]

The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$x + 3y + 7z = 0;$

$-x + 4y + 7z = 0;$

$x \sin 3\theta + y \cos 2\theta + 2z = 0$

has non-trivial solutions, is

(A) three

(B) two

(C) four

(D) one

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2023 (01 Feb. Shift-II)]

[Ans. A]

For the system of linear equations
$\alpha x + y + z = 1, x + \alpha y + z = 1, x + y + \alpha z = \beta$,
which one of the following statements is NOT correct?

(A) It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$

(B) It has no solution if $\alpha = -2$ and $\beta = 1$

(C) $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$

(D) It has infinitely many solutions if $\alpha = 1$ and $\beta = 1$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2019 (Jan.)]

[Ans. D]

The system of linear equation

$x + y + z = 2,$

$2x + 3y + 2z = 5,$

$2x + 3y + (a^2 - 1)z = a + 1$

(A) is inconsistent when $a = 4$

(B) has a unique solution for $|a| = \sqrt{3}$

(C) has infinitely many solutions for $a = 4$

(D) inconsistent when $|a| = \sqrt{3}$

FOR NOTES & DPP CHECK DESCRIPTION

BRAIN TEASER

$Y_2 \to \pm 2$ $\quad Y_3 \to \pm 2$ $\quad \to 2, -2$

If every element of a (3 × 3) determinant is from the set {1, –1} then show that the value of such determinant can only be 0, –4 or 4.

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= a_1[b_2c_3 - c_2b_3] - a_2[b_1c_3 - b_3c_1] + a_3[b_1c_2 - c_1b_2]$$

$$\Delta = \underbrace{(a_1b_2c_3 - a_1b_3c_2)}_{Y_1} + (a_2b_3c_1 - a_2b_1c_3) + \underbrace{(a_3b_1c_2 - a_3b_2c_1)}_{Y_3}$$

$$\Delta = Y_1 + Y_2 + Y_3$$

$\Delta \to \pm 6, \pm 4, \pm 2, 0$

$\pm 6, \pm 2 \to$ Reject ?

FOR NOTES & DPP CHECK DESCRIPTION

Consider $\boxed{\Delta = 6}$

$\Rightarrow \boxed{y_1 = 2 = y_2 = y_3}$

$\Rightarrow \left\{ \boxed{\begin{matrix} a_1 b_2 c_3 = 1 \\ a_2 b_3 c_1 = 1 \\ a_3 b_1 c_2 = 1 \end{matrix}} \begin{matrix} \& \\ \& \\ \& \end{matrix} \boxed{\begin{matrix} a_1 b_3 c_2 = -1 \\ a_2 b_1 c_3 = -1 \\ a_3 b_2 c_1 = -1 \end{matrix}} \right\}$ not possible

multiply

$\boxed{a_1 a_2 a_3 b_1 b_2 b_3 c_1 c_2 c_3 = 1}$

$a_1 a_2 a_3 b_1 b_2 b_3 c_1 c_2 c_3 = -1$

Contradict

$\boxed{\Delta = -6} \rightarrow$ Rejected

$\Delta = 2$

$y_1 = 2, \; y_2 = y_3 = 0$

HW

$y_1 = 2, \; y_2 = 2, \; y_3 = -2$

BRAIN TEASER

[Ans 0, 1, −1, 2, −2]

If every element of a (3×3) determinant is from the set $\{0, 1\}$ then find the possible number of values of such determinant.

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Adv. 2024]

HW

[Ans. 16]

Let $S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : \underbrace{a, b, c, d, e} \in \underbrace{\{0, 1\}} \text{ and } \underbrace{|A| \in \{-1, 1\}} \right\}$,

where $|A|$ denotes the determinant of A.

Then the number of elements in S is __________ .

FOR NOTES & DPP CHECK DESCRIPTION

Homework

REDO ALL QUES

+ H.W.

FOR NOTES & DPP CHECK DESCRIPTION


Thank
YOU
Keep Hustling!