

PRAAYAS

JEE 2026

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Mathematics

Sequence and Series

Lecture - 3

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Topics *to be covered*



A Arithmetic Mean

B Geometric Progression

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Recap *of previous lecture*



1. K^{th} term from end in on A.P. = $(n - K + 1)^{\text{th}}$ term from beginning.

2. If $T_n = 2n + 3$ then the sequence is an A.P

3. $30 + \frac{89}{3} + \frac{88}{3} + \dots$ has largest possible sum then number of terms that should be taken is _____

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Recap *of previous lecture*



4. If a sequence has 40 terms then number of even numbered terms is 20

41 terms then no. of even numbered terms = 20
odd numbered terms = 21

5. If a sequence has 49 terms then number of even numbered terms is 24 &
number of odd numbered terms is 25

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Recap *of previous lecture*



- 6. Three numbers in A.P. $a-d, a, a+d$ — $c.d=d$
- Four numbers in A.P. $a-3d, a-d, a+d, a+3d$ — $c.d=2d$
- Five numbers in A.P. $a-2d, a-d, a, a+d, a+2d$ — $c.d=d$
- Six numbers in A.P. $a-5d, a-3d, a-d, a+d, a+3d, a+5d$ — $c.d=2d$

FIRST TERM IS NOT a

- 7. If we pick terms in an A.P. at an interval of 5 then the picked terms will form an A.P with common difference $5d$

Recap *of previous lecture*



8. If $a_1, a_2, a_3, a_4, a_5, \dots$ be an A.P. with common difference 'd' then $a_1, a_5, a_9, a_{13}, \dots$ will be in A.P with common difference $4d$. Similarly a_2, a_4, a_6, \dots will also be in A.P with common difference $2d$.

9. If $m, n, p, q \in \mathbb{N}$ & $m + n = p + q$ then in A.P. we have $T_m + T_n = T_p + T_q$.

10. If $a_1 + a_{12} = 10$ in an A.P. then value of $a_3 + a_7 + a_9 + a_4 + a_6 + a_{10}$ is $3(a_1 + a_{12}) = 30$.



Homework Discussion

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QUESTION [JEE Mains 2022 (27 July)]

TAHOI
(ADBST)

Suppose $a_1, a_2, \dots, a_n, \dots$ be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is $5 : 17$ and, $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to

- A** 290
- B** 380
- C** 460
- D** 510

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Ans. B

QUESTION [JEE Mains 2025 (29 Jan)]

TAH02

(ADBT)



Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11th term is :

- A** 108
- B** 90
- C** 122
- D** 84

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Ans. B

QUESTION [JEE Advanced 2018]

Tah03



Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11,, and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, Then, the number of elements in the set $X \cup Y$ is

(ADBST)

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QUESTION [JEE Mains 2024 (27 Jan)]

Tahoy

The number of common terms in the progressions

4, 9, 14, 19,, up to 25th term and 3, 6, 9, 12,, up to 37th term is:

A 9

B 8

C 5

D 7

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Ans. D

QUESTION [JEE Mains 2023 (1 Feb)]

Tah 05



The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15,, 399,

2, 5, 8, 11,, 359 and

2, 7, 12, 17,, 197,

is equal to _____

(ADIBST)

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Ans. 321

QUESTION [JEE Mains 2025 (4 April)]

Tah06



Let $A = \{1, 6, 11, 16, \dots\}$ and $B = \{9, 16, 23, 30, \dots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is

- A** 3814
- B** 4003
- C** 4027
- D** 3761

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Ans. D

QUESTION [JEE Mains 2021 (31 Aug)]

Tah07



Let a_1, a_2, a_3, \dots be an A.P. If $\frac{a_1+a_2+\dots+a_{10}}{a_1+a_2+\dots+a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to :

A $\frac{19}{21}$

B $\frac{100}{121}$

C $\frac{21}{19}$

D $\frac{121}{100}$

$$\frac{\frac{10}{2}(2a+9d)}{\frac{p}{2}(2a+(p-1)d)} = \frac{100}{p^2}$$

$$\frac{2a+9d}{2a+(p-1)d} = \frac{10}{p}$$

$$2ap + 9dp = 20a + 10dp - 10d$$

$$2a(p-10) = dp - 10d$$

$$2a(p-10) = d(p-10)$$

$$2a = d$$

$$\frac{a+10d}{a+9d} = \frac{a+20a}{a+18a} = \frac{21}{19}$$

Ans. C

QUESTION

KTK 1



For $a, b \in \mathbb{R} - \{0\}$, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x + \frac{7}{4}\right) = f\left(\frac{7}{4} - x\right) \forall x \in \mathbb{R}$. Also the equation $f(x) = 7x + a$ has only one real and distinct solution.

The value of $(a + b)$ is equal to

- A 4
- B 5
- C 6
- D 7

* $f\left(\frac{7}{4} + x\right) = f\left(\frac{7}{4} - x\right)$

f is symmrt about $x = \frac{7}{4}$

$f(x) = ax^2 + bx + a$

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symmrt abt $x = -\frac{b}{2a}$

$f(x) = 7x + a$ has only one real & distinct soln.

$-\frac{b}{2a} = \frac{7}{4}$

$-2b = 7a$

$ax^2 + bx + a = 7x + a$

$ax^2 + (b-7)x = 0$

$D = (b-7)^2 - 4a \cdot 0 = 0$

$b = 7$

$-14 = 7a$
 $a = -2$

$a + b = 5$

Ans. B

QUESTION

KTK 2



For $a, b \in \mathbb{R} - \{0\}$, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x + \frac{7}{4}\right) = f\left(\frac{7}{4} - x\right) \forall x \in \mathbb{R}$. Also the equation $f(x) = 7x + a$ has only one real and distinct solution.

The minimum value $f(x)$ in $\left[0, \frac{3}{2}\right]$ is equal to

A $-\frac{33}{8}$

B 0

C 4

D -2

$$\begin{aligned}
 f(x) &= -2x^2 + 7x - 2 \\
 &= -2\left(x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{49}{16}\right) - 2 \\
 &= -2\left(x - \frac{7}{4}\right)^2 + \frac{49}{8} - 2 \\
 &= \frac{33}{8} - 2\left(x - \frac{7}{4}\right)^2
 \end{aligned}$$

Ans. D



Aao Machaay Dhamaal Deh Swaal pe Deh Swaal

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Arithmetic Mean

* If a, b, c are in A.P then 'b' is called single A.M b/w a & c

$$b - a = c - b$$

$$2b = a + c \rightarrow b = \frac{a + c}{2}$$

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 a, b, c are in A.P $\Rightarrow 2b = a + c$

* If $a, A_1, A_2, A_3, \dots, A_n, b$ are in A.P then A_1, A_2, \dots, A_n are called n A.Ms b/w a & b .

$$T_1 = a$$

$$T_{n+2} = b \Rightarrow b = a + (n+1)d$$

$$d = \frac{b - a}{n+1}$$



$$A_1 = a + d$$

$$A_2 = a + 2d$$

$$A_3 = a + 3d$$

$$\vdots$$

$$A_n = a + nd$$

$$A_1 + A_2 + \dots + A_n = na + d(1 + 2 + 3 + \dots + n)$$

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$$\sum_{i=1}^n A_i = na + d \cdot \frac{n(n+1)}{2} = na + \frac{b-a}{n+1} \cdot \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n A_i = na + \frac{n(b-a)}{2} = n \left(\frac{2a+b-a}{2} \right) = n \left(\frac{a+b}{2} \right)$$



$a, A_1, A_2, \dots, A_n, b$ are in A.P

n A.Ms b/w a & b .

$$\sum_{i=1}^n A_i = n \left(\frac{a+b}{2} \right)$$

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Sum of n A.Ms b/w 2 NO.s = $n \cdot$ (Single A.M b/w the two no.s)

Ex: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

8 A.Ms b/w 2 & 20

$$\text{Sum of 8 A.Ms} = 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 = 88 = 8 \cdot \left(\frac{2+20}{2} \right)$$

QUESTION



If p arithmetic mean are inserted between 5 and 41 so that ratio $\frac{A_3}{A_{p-1}} = \frac{2}{5}$, then find the value of p .

M ① $5, A_1, A_2, \dots, A_p, 41$ are in A.P

$$T_4 = A_3 = 5 + 3d \quad T_{p+2} = 41$$

$$T_p = A_{p-1} = 5 + (p-1)d \quad 5 + (p-1)d = 41$$

$$(p+1)d = 36$$

$$\frac{A_3}{A_{p-1}} = \frac{5+3d}{5+(p-1)d} = \frac{2}{5}$$

$$2(5+3d) = 10 + 2(p-1)d$$

$$2(5 + 15 \cdot \frac{36}{p+1}) = 10 + 2(p-1) \cdot \frac{36}{p+1}$$

$$15 + \frac{15 \cdot 36}{p+1} = 72 \frac{(p-1)}{p+1} \Rightarrow 15p + 15 + 540 = 72p - 72$$

$$57p = 627$$

$$p = 11$$

QUESTION



If $A_1, A_2, A_3, \dots, A_{51}$ are arithmetic means inserted between the numbers a and b , then find the value of $\left(\frac{b+A_{51}}{b-A_{51}}\right) - \left(\frac{A_1+a}{A_1-a}\right)$.

$a, A_1, A_2, \dots, A_{51}, b$ $T_{53} = b = a + 52d$
 $b - a = 52d$

Let $E = \frac{b+A_{51}}{b-A_{51}} - \frac{(A_1+a)}{A_1-a}$ **ATDB.uno**

$$= \frac{b+A_{51}}{d} - \frac{A_1+a}{d}$$

$$= \frac{b+A_{51}-A_1-a}{d}$$

$$= \frac{b+b-d-(a+d)-a}{d}$$

$$= \frac{2b-2a-2d}{d} = \frac{104d-2d}{d} = 102 \text{ Ans}$$

QUESTION



If $a_1 a_2 a_3 \dots a_n$ are n arithmetic means inserted between 7 and 2015 whose sum is 56616 then

$$7, a_1, a_2, \dots, a_n, 2015.$$

$$a_1 + a_2 + \dots + a_n = 56616.$$

- A** n is 56
- B** n is 28
- C** $a_{19} = 2029/3$
- D** $a_{19} = 36543/57$

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QUESTION



If $S_1, S_2, S_3, \dots, S_p$ are the sums of n terms of ' p ' arithmetic series whose first term and common difference are $1, 2, 3, 4, \dots$ and whose common difference are $1, 3, 5, 7, \dots$

Prove that $S_1 + S_2 + S_3 + \dots + S_p = \frac{np}{2} (np + 1)$

$$A.P_1: T_1=1, d=1 \rightarrow S_1 = \frac{n}{2} (2 \cdot 1 + (n-1) \cdot 1)$$

$$A.P_2: T_1=2, d=3 \rightarrow S_2 = \frac{n}{2} (2 \cdot 2 + (n-1) \cdot 3)$$

$$A.P_3: T_1=3, d=5 \rightarrow S_3 = \frac{n}{2} (2 \cdot 3 + (n-1) \cdot 5)$$

$$\vdots$$

$$A.P_p: T_1=p, d=2p-1 \rightarrow S_p = \frac{n}{2} (2p + (n-1)(2p-1))$$

$$S_1 + S_2 + S_3 + \dots + S_p = \frac{n}{2} (2(1+2+3+\dots+p) + (n-1)(1+3+5+\dots+(2p-1)))$$

$$= \frac{n}{2} (2 \frac{p(p+1)}{2} + (n-1)p^2)$$



$$S_1 + S_2 + \dots + S_p = \frac{np}{2} (p+1 + p(n-1))$$

$$S_1 + S_2 + \dots + S_p = \frac{np}{2} (np+1)$$

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QUESTION [JEE Mains 2023 (Jan)]



Number of 4 digit integers less than 2800 which are either divisible by 3 or by 11 is equal to

$$n(D_3 \cup D_{11}) = n(D_3) + n(D_{11}) - n(D_3 \cap D_{11})$$

$$D_3 = \{1002, 1005, \dots, 2799\} \quad \text{---} \quad 2799 = 1002 + (n-1) \cdot 3 \quad \text{---} \quad n = 600$$

$$D_{11} = \{1001, 1012, 1023, \dots, 2794\} \quad \text{---} \quad 2794 = 1001 + (n-1) \cdot 11 \quad \text{---} \quad n = 164$$

$$D_{33} = \{1023, 1056, \dots, 2772\} \quad \text{---} \quad 2772 = 1023 + (n-1) \cdot 33 \quad \text{---} \quad n = 54$$

$$\begin{aligned} n(D_3 \cup D_{11}) &= 600 + 164 - 54 \\ &= 600 + 110 = 710 \text{ Ans} \end{aligned}$$

Ans. 710

QUESTION [JEE Advanced 2018]

Tah02



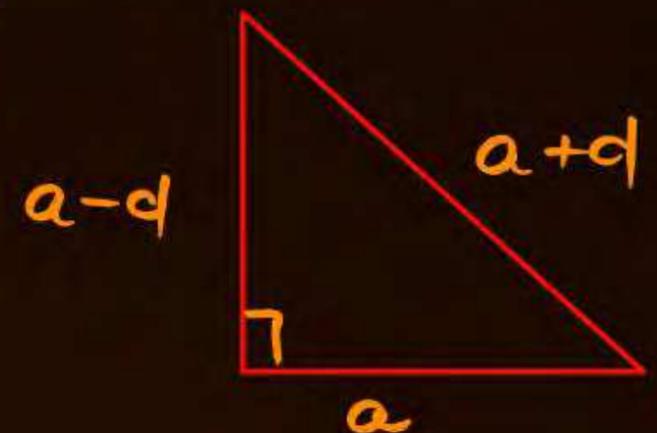
The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

$$\frac{1}{2} \cdot (a-d) \cdot a = 24$$

$$a(a-d) = 48$$

$$a^2 - ad = 48$$

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Ans. 6

QUESTION [JEE Mains 2025 (4 April)]

Tah03



Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of A.P.'s in A and B respectively such that $D = d + 3, d > 0$. If $\frac{p+q}{p-q} = \frac{19}{5}$, then p - q is equal to

- A** 540
- B** 450
- C** 600
- D** 630

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Ans. A

QUESTION [JEE Mains 2023 (31 Jan)]



Let a_1, a_2, \dots, a_n be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then

$12 \left(\frac{1}{\sqrt{a_{10}} + \sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}} + \sqrt{a_{12}}} + \dots + \frac{1}{\sqrt{a_{17}} + \sqrt{a_{18}}} \right)$ is equal to

$$12 \left(\frac{\sqrt{a_{11}} - \sqrt{a_{10}}}{a_{11} - a_{10}} + \frac{\sqrt{a_{12}} - \sqrt{a_{11}}}{a_{12} - a_{11}} + \dots + \frac{\sqrt{a_{18}} - \sqrt{a_{17}}}{a_{18} - a_{17}} \right)$$

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$$\frac{12}{d} \left((\sqrt{a_{11}} - \sqrt{a_{10}}) + (\sqrt{a_{12}} - \sqrt{a_{11}}) + (\sqrt{a_{13}} - \sqrt{a_{12}}) + \dots + (\sqrt{a_{18}} - \sqrt{a_{17}}) \right)$$

$$\frac{12}{d} (\sqrt{a_{18}} - \sqrt{a_{10}})$$

$$\frac{12}{d} (\sqrt{a_{11} + 7d} - \sqrt{a_{11} - d})$$

$$\frac{4}{3} (9 - 3) = 4 \cdot 2 = 8 \text{ Ans}$$

$$a_{11} - 6d = 2(a_{11} - 4d)$$

$$18 - 6d = 2(18 - 4d)$$

$$9 - 3d = 18 - 4d$$

$$d = 9$$

Ans. 8

QUESTION

Given $a_1, a_2, a_3, \dots, a_n$ in A.P. prove that

$$\frac{1}{a_1 a_n} + \frac{1}{a_2 a_{n-1}} + \frac{1}{a_3 a_{n-2}} + \dots + \frac{1}{a_n a_1} = \frac{2}{a_1 + a_n} \left[\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} \right]$$

$m+n = p+q$
 $a_m + a_n = a_p + a_q$
 $a_1 + a_n = a_2 + a_{n-1}$

LHS

$$\frac{1}{a_1 + a_n} \left(\frac{1}{a_1 a_n} + \frac{1}{a_2 a_{n-1}} + \frac{1}{a_3 a_{n-2}} + \dots + \frac{1}{a_n a_1} \right) = \frac{1}{a_1 + a_n} \left(\frac{a_1 + a_n}{a_1 a_n} + \frac{a_1 + a_n}{a_2 a_{n-1}} + \frac{a_1 + a_n}{a_3 a_{n-2}} + \dots + \frac{a_1 + a_n}{a_n a_1} \right)$$

$$\frac{1}{a_1 + a_n} \left(\frac{a_1 + a_n}{a_1 a_n} + \frac{a_2 + a_{n-1}}{a_2 a_{n-1}} + \frac{a_3 + a_{n-2}}{a_3 a_{n-2}} + \dots + \frac{a_1 + a_n}{a_1 a_n} \right)$$

$$\frac{1}{a_1 + a_n} \left(\left(\frac{1}{a_n} + \frac{1}{a_1} \right) + \left(\frac{1}{a_{n-1}} + \frac{1}{a_2} \right) + \left(\frac{1}{a_{n-2}} + \frac{1}{a_3} \right) + \dots + \left(\frac{1}{a_1} + \frac{1}{a_n} \right) \right)$$

$$\frac{1}{a_1 + a_n} \cdot 2 \cdot \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots + \frac{1}{a_n} \right)$$

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QUESTION [JEE Mains 2021 (August)]



Tan 04

Let a_1, a_2, \dots, a_{21} be an A.P. such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$.

If the sum of this A.P. is 189, then $a_6 \cdot a_{16}$ is equal to:

A 57

B 72

C 48

D 36

$$S = \frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \frac{1}{a_3 a_4} + \dots + \frac{1}{a_{20} a_{21}} = \frac{4}{9}$$

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QUESTION [JEE Mains 2024 (29 Jan)]



If $\log_e a, \log_e b, \log_e c$ are in an A.P. and $\log_e a - \log_e 2b, \log_e 2b - \log_e 3c, \log_e 3c - \log_e a$ are also in an A.P., then $a : b : c$ is equal to

A 6 : 3 : 2

B 9 : 6 : 4

C 25 : 10 : 4

D 16 : 4 : 1

$$2 \log_e b = \log_e a + \log_e c$$

$$\log_e b^2 = \log_e ac$$

$$b^2 = ac$$

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$$2 \log_e \frac{2b}{3c} = \log_e \frac{a}{2b} + \log_e \frac{3c}{a}$$

$$\left(\frac{2b}{3c}\right)^2 = \frac{a}{2b} \cdot \frac{3c}{a}$$

$$8b^3 = 27c^3$$

$$2b = 3c = 6\lambda$$

$$b = 3\lambda$$

$$c = 2\lambda$$

$$9\lambda^2 = a \cdot 2\lambda$$

$$a = \frac{9}{2}\lambda$$

$$a : b : c = \frac{9}{2}\lambda : 3\lambda : 2\lambda = 9 : 6 : 4$$

Ans. B

QUESTION [JEE Mains 2021 (27 July)]

Tah 05



If $\log_3 2$, $\log_3 (2^x - 5)$, $\log_3 \left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to

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Ans. 3

QUESTION



How many terms of the series $20 + 19\frac{1}{3} + 18\frac{2}{3} + \dots$ must be taken so that sum is 300.
Explain the reason of double answer.

$d = -\frac{2}{3}$

$S_n = 300$

$\frac{n}{2} (40 + (n-1)\frac{-2}{3}) = 300$

$n(20 - \frac{n-1}{3}) = 300$

$n(61-n) = 900$

$-n^2 + 61n = 900$

$n^2 - 61n + 900$

$n = 25, 36$

T_1, T_2, \dots, T_{25} $T_{26}, T_{27}, \dots, T_{31}, T_{32}, T_{33}$

$S_{25} = 300$

$S_{36} = 300$

$T_n \leq 0$

$20 + (n-1)(-\frac{2}{3}) \leq 0$

$10 \leq \frac{n-1}{3}$

$n \geq 31$

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$$\textcircled{b} S_{\text{MAX}} = S_{30} = S_{31}$$

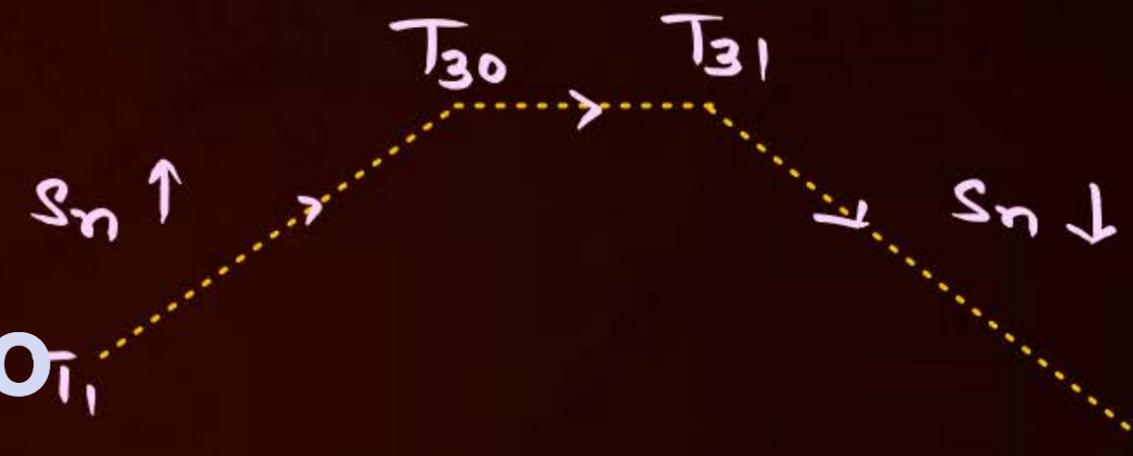
$$S_{30} = \frac{30}{2} \left(40 + 29 \cdot \left(-\frac{2}{3}\right) \right)$$

$$= \frac{15(120 - 58)}{3}$$

$$= 5 \cdot 62$$

$$S_{\text{MAX}} = 310$$

Sum inc till terms are +ve



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G.P



G.P



Geometric Progression



Definition :

G.P. is the collection of non-zero terms in which each bears the same constant ratio with its immediately preceding term the series is called a G.P. and the constant ratio is called the common ratio.

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Standard appearance of a G.P., where a is first term and r is common ratio is

aghi Term ÷ peechli Term = constant = common ratio

G.P: $a, ar, ar^2, ar^3, ar^4, \dots$

$T_1 = a$ common ratio = r

$T_n = ar^{n-1}$

None of the terms of G.P
can be zero



Sum of n terms of G.P

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

$$rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$$

$$(1-r)S_n = a - ar^n$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, \quad r \neq 1$$

$$S_n = \begin{cases} \frac{a(r^n-1)}{r-1} & r \neq 1 \\ na & r = 1 \end{cases}$$

If $r=1$

$$S = a + a + \dots + a = na$$

n terms.



* A G.P of +ve terms is inc if $r > 1$

* A G.P of +ve terms is dec if $0 < r < 1$

* A G.P of -ve terms is inc if $0 < r < 1$

* A G.P of -ve terms is dec if $r > 1$

* If $r < 0$ then G.P is alternately composed of +ve & -ve terms

Ex: $1, -2, 4, -8, 16, -32, \dots$

* If $r = 1$ then it is a constant G.P.

Sum of Infinite Terms

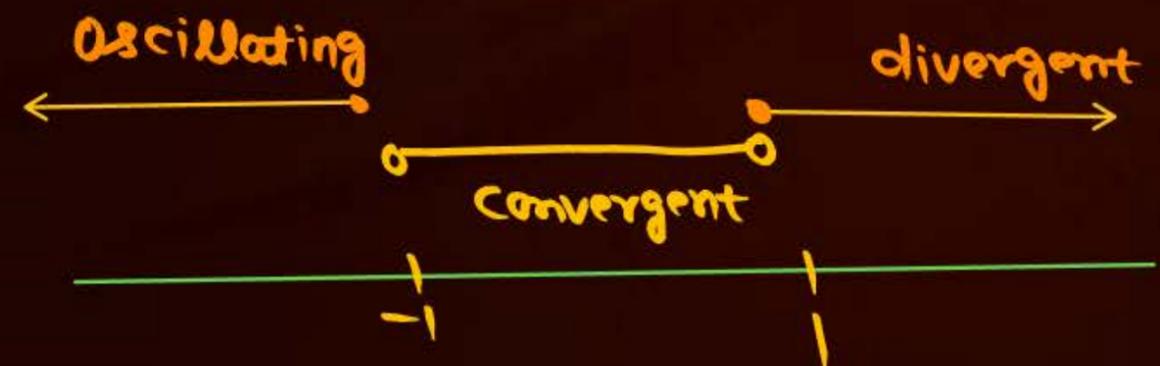


$$S_{\infty} = a + ar + ar^2 + ar^3 + \dots \rightarrow \infty \quad 0 < |r| < 1$$

$$S_{\infty} = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \frac{a(1-r^{n+1})}{1-r} = \frac{a}{1-r} \quad -1 < r < 1, r \neq 0$$

$$S_{\infty} = \frac{a}{1-r} \quad -1 < r < 1, r \neq 0$$

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* k^{th} from End in G.P of n terms = $(n-k+1)^{\text{th}}$ term from Beginning

* T_k from End = $l \cdot \left(\frac{1}{r}\right)^{k-1}$

$$\begin{array}{c} \xrightarrow{r} \qquad \qquad \qquad \xleftarrow{1/r} \\ a, ar, ar^2, ar^3, \dots, l \end{array}$$

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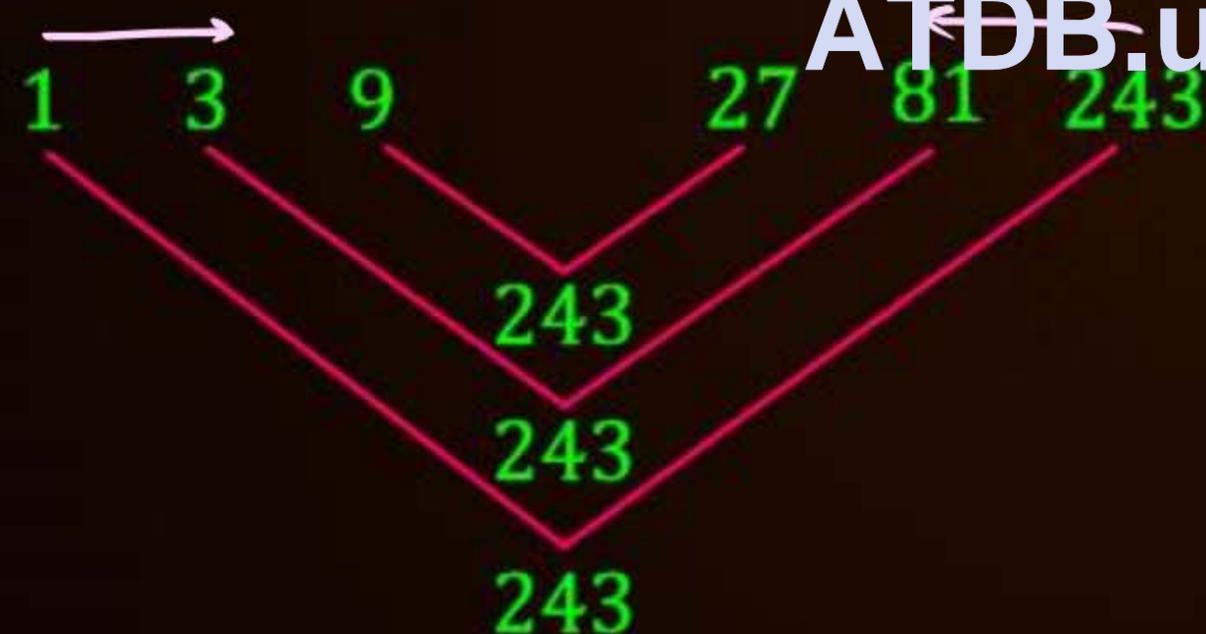
Properties of G.P.



Property 1:

In an G.P. product of k^{th} term from beginning and k^{th} term from the last is always constant which equal to product of first term and last term.

$$T_k \cdot T_{n-k+1} = \text{constant} = a \cdot l$$



$$T_k = a \cdot r^{k-1}$$

$$T_k \text{ from End} = l \cdot \left(\frac{1}{r}\right)^{k-1}$$

$$T_k \cdot (T_k \text{ from End}) = a \cdot l$$



If $m, n, p, q \in \mathbb{N}$ & $m+n = p+q$ then $T_m \cdot T_n = T_p \cdot T_q$ for a G.P

(prove yourself)

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Property 2:

Three numbers in G.P. :

$$a/r, a, ar \quad \text{---} \quad CR = r$$

Five numbers in G.P. :

$$a/r^2, a/r, a, ar, ar^2$$

Four numbers in G.P. :

$$a/r^3, a/r, ar, ar^3 \quad \text{---} \quad CR = r^2$$

Six numbers in G.P. :

$$a/r^5, a/r^3, a/r, ar, ar^3, ar^5$$

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If product of first few terms is given

first term is not a.



Sabse Important Baat



Sabhi Class Illustrations Retry Karnay hai...

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Solution to Previous TAH

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QUESTION [JEE Advanced 2015]



Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is $6 : 11$ and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

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Tan-01

Q) Suppose that all the terms of an A.P. are natural numbers. If the ratio of the sum of the first seven terms to the first eleven terms is 6:11 and the seventh term lies between 130 and 140, then the common difference of the A.P. is:

Solⁿ:

all terms of AP are natural given $\frac{\sum_{i=1}^7 (2a+6d)}{\sum_{i=1}^{11} (2a+10d)} = \frac{6}{11}$

$$\frac{2a+6d}{2a+10d} = \frac{6}{11} \quad 130 < T_7 = a+6d < 140$$

$$\frac{a+3d}{a+5d} = \frac{6}{11}$$

$$7a+21d = 6a+30d$$

$$a = 9d$$

$$T_7 = a+6d = \underline{15d}$$

factor 15's number lying between 130 and 140 is 135
so the value of 15d is 135 $\Rightarrow d = 9$

$$a = 9d \Rightarrow a = 81 \text{ and } d = 9$$

Lecture - 02 (Sequence & Series)

JEE Adv. - 2013

Tan-01] Suppose that all the terms of an arithmetic progression (A.P) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first 11 terms is 6:11 and the seventh term lies in b/w 130 and 140, then the common difference of this A.P. is:-

$$\Rightarrow \frac{\frac{7}{2}(2a+6d)}{\frac{11}{2}(2a+10d)} = \frac{6}{11} \quad \& \quad \boxed{130 < a+6d < 140}$$

$$\Rightarrow \frac{7(2a+6d)}{2a+10d} = 6$$

$$\Rightarrow \frac{7(a+3d)}{a+5d} = 6 \Rightarrow \boxed{7a+21d = 6a+30d}$$

$$\boxed{a = 9d}$$

$$\text{as, } 130 < a+6d < 140$$

$$130 < 15d < 140$$

$$\frac{130}{15} < d < \frac{140}{15}$$

$$\frac{26}{3} < d < \frac{28}{3}$$

$$\frac{26}{3} < d < \frac{28}{3}$$

as the terms of the A.P are natural numbers

$$\boxed{d = \frac{27}{3} = 9} \quad (\text{Ans})$$

Kritisha

QUESTION [JEE Mains 2022 (27 July)]

TAH



Suppose $a_1, a_2, \dots, a_n, \dots$ be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is $5 : 17$ and, $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to

A 290

B 380

C 460

D 510

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Ans. B



Tah2 $a_1, a_2, \dots, a_n \rightarrow A.P.$

$S_5 = 5$	$110 < a_{15} < 120$
$S_9 = 17$	$110 < a + 14d < 120$
$5a + 10d = 5$	$110 < a + 56d < 120$
$9a + 36d = 17$	$\frac{110}{57} < a < \frac{120}{57}$
$a + 2d = 1$	$1.9 < a < 2.1$
$a + 4d = 17$	
$17a - 9a = 36d - 34d$	$a = 2$ $d = 8$
$4a = d$	SAKSHI

Now we find: Sum of first ten terms = $S_{10} = a_1 + a_2 + a_3 + \dots + a_{10}$

$$S_{10} = \frac{10}{2} [2 \times 2 + 9 \times 8]$$

$$S_{10} = 5(4 + 72) = 380 \text{ Ans (B)}$$

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of natural numbers. If the ratio of the sum of first five numbers of terms to the sum of first nine terms of the progression is 5:17 and $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to:

$$\rightarrow \frac{S_5}{S_9} = \frac{\frac{5}{2}(2a+4d)}{\frac{9}{2}(2a+8d)} = \frac{5}{17}$$

$$\Rightarrow \frac{a+2d}{9(a+4d)} = \frac{1}{17}$$

$$\Rightarrow 17a + 34d = 9a + 36d$$

$$\Rightarrow a = 2d$$

$a_{15} = (a + 14d) \rightarrow$ acc. to the question,
 $110 < (a + 14d) < 120$
 $110 < 57a < 120$

Kritisha $\frac{110}{57} < a < \frac{120}{57}$

as $a \in \mathbb{N}$

$a = \frac{114}{57} = 2$; $d = 8$

$$S_{10} = 5(4 + 9(8)) = 5(4 + 72) = 20 + 360 = 380 \text{ (B)}$$

Ans.

QUESTION [JEE Mains 2025 (29 Jan)]

TAH



Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11th term is :

A 108

B 90

C 122

D 84

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Ans. B



TAM-03

$$S_3 = 54$$

$$\frac{3}{2}(2a + 2d) = 54$$

$$3(a + d) = 54$$

$$a + d = 18$$

$$a = 18 - d$$

$$1600 < S_{20} < 1800 \text{ --- (1)}$$

$$\therefore S_{20} = \frac{20}{2}(2a + 19d)$$

$$10(2(18 - d) + 19d)$$

$$10(36 - 2d + 19d)$$

$$360 + 170d$$

put in (1)

$$1600 < 360 + 170d < 1800$$

$$1240 < 170d < 1440$$

$$7.2 < d < 8.4$$

$$d = 8$$

$$a = 10$$

Q $T_{11} = a + 10d$
 $= 10 + 10(8)$

$$\text{Ans} = 90$$

Tam 3	$a_1 + a_2 + a_3 = 54$	$1600 < S_{20} < 1800$	$a_{11} = ?$
	$a + a + d + a + 2d = 54$	$1600 < \frac{20}{2}(2(18 - d) + 19d) < 1800$	
	$3a + 3d = 54$	$1600 < 360 + 170d < 1800$	
	$a + d = 18$	$\frac{1240}{170} < d < \frac{1440}{170}$	
	$a = 18 - d$		
	$a = 10$		Now, $a_{11} = a + 10d$
	SAKSHI	$7.2 < d < 8.4$	$a_{11} = 10 + 80$
		$d = 8$	$a_{11} = 90$



Q.6-03 Consider an A.P. of positive integers whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11th term is :-

$$\Rightarrow S_3 = \frac{3}{2}(2a + 2d) = 54 \implies a + d = 18$$

$$S_{20} = \frac{20}{2}(2a + 19d)$$

$$1600 < 10(2a + 19d) < 1800$$

$$160 < (2a + 19d) < 180$$

$$160 < 2(18 - d) + 19d < 180$$

$$160 < 36 + 17d < 180$$

$$\frac{124}{17} < d < \frac{144}{17}$$

$d = 8$
 $a = 10$
 as terms of A.P. are +ve integers.

$$t_{11} = a + 10d = 10 + 80 = 90 \text{ (B) (Ans)}$$

Let A.P. of positive integers

Given, $S_3 = \frac{3}{2}(2a + 2d) = 54 \implies a + d = 18$

and, $1600 < S_{20} < 1800$

Hint

$$2a + 19d$$

$$(a + d) + (a + d) + 17d$$

$$18 + 18 + 17d$$

$$36 + 17d$$

$$1600 < \frac{20}{2}(2a + 19d) < 1800$$

$$1600 < 10(36 + 17d) < 1800$$

$$160 - 36 < 17d < 180 - 36$$

$$\frac{124}{17} < d < \frac{144}{17}$$

tah-3
by vandana
from bihar

$$7.2 < d < 8.9$$

$$d = 8 \in \mathbb{I}, a = 18 - 8 = 10$$

Now, $t_{11} = a + 10d$
 $= 10 + 10 \times 8$
 $= 90 \text{ Ans}$

QUESTION [JEE Advanced 2018]



Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11,, and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, Then, the number of elements in the set $X \cup Y$ is

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upto 2017 terms
 $X = 1, 6, 11, 16, \dots$ $d_1 = 5$
 $Y = 9, 16, 23, \dots$ $d_2 = 7$ } $35 = D$
 $A = 16$
 $D = 35$
 $a_n = a_1 + (n-1)d_1$
 $1 + 2017(5)$
 10086

$a_n = a_2 + (n-1)d_2$
 $= 9 + 2017 \times 7$
 $= 14128$

$16 + (n-1)35 \leq 10086$
 $(n-1)35 \leq 10070$
 $(n-1) \leq 287.7$
 $n \leq 288.7$

$n \leq 288$
 $\hookrightarrow (X \cap Y)$

$n(X) = 2018$
 $n(Y) = 2018$
 $n(X \cap Y) = 288$
 $n(X \cup Y) = 2018 + 2018 - 288$
 $n(X \cup Y) = 3748$
 Ans)

Soln:
 $X: 1, 6, 11, 16, 17, \dots$ upto 2018 terms
 $Y: 9, 16, 23, \dots$ upto 2018 terms.
 $d_x = 5$
 $d_y = 7$ } $LCM = 35$

$\therefore (X \cap Y) = 16, 54, \dots$

$X_{2018} = 1 + (2017 \times 5) = 10086$

$Y_{2018} = 9 + (2017 \times 7) = 14128$

Let $(X \cap Y)$ has n terms.

$(X \cap Y)_n = a + (n-1)d \leq 10086$
 $16 + (n-1)(35) \leq 10086$

$(n-1) \leq \frac{10070}{35}$

$n-1 \leq 287.7 \dots$

$n-1 = 287$

$n = 288$

$\therefore n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$
 $= 2018 + 2018 - 288$
 $= 3748$

Ans 3748

RASIDUL

QUESTION [JEE Mains 2024 (27 Jan)]



The number of common terms in the progressions
4, 9, 14, 19,, up to 25th term and 3, 6, 9, 12,, up to 37th term is:

- A** 9
- B** 8
- C** 5
- D** 7

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Ans. D

Tak 5

$$4, \textcircled{9}, 14, 19 \dots T_n = 124$$

$$\hookrightarrow \text{upto } 25^{\text{th}} \quad d=5$$

$$\text{and } 3, 6, \textcircled{9}, 12 \dots T_n = 111$$

$$\hookrightarrow \text{upto } 37^{\text{th}} \quad d=3$$

common diff. of ^{A.P.} consecutive terms = 15

$$\text{common A.P.} \rightarrow 9, 24, 39 \dots d=15$$

$$T_n \leq 111 \quad \& \quad T_n < 124$$

↓ ATDB.uno

$$9 + (n-1)15 \leq 111$$

SAKSHI

$$9 + 15n - 15 \leq 111$$

$$15n \leq 117$$

$$n \leq 7.8$$

almost $n=7$ is

→ $T_n = 371$





lab-05

The no. of common terms in
4, 9, 14, 19 upto 25th term
3, 6, 9, 12 " 37th term is

⇒ (A.P)₁ = 4, 9, 14, 19 25th term
(A.P)₂ = 3, 6, 9, 12 37th term

Last term of (A.P)₁ ⇒ ~~4(24x5)~~
= 4 + 24(5) = 124

Last term of (A.P)₂ = 3 + 3(36) = 111

Common diff. of new A.P. ⇒ 15
1st term " " " ⇒ 9
Last " " " " ⇒ < 111

Now, 111 > 9 + 15(m-1)

$$\frac{102 + 15}{15} > m$$

$m < 7.8$ ⇒ $m = 7$ (Ans)
(D)

Kritisha

Sol For 1st AP : $T_n = 4 + (n-1)5$
= 4 + 24x5
= 4 + 120
= ~~124~~

For 2nd AP : $T_n = 3 + (n-1)7$
= 3 + 36x3
= 3 + 108
= 111

AR : 4, 9, 14, 19, 124, d = 5 > (lim = 15)
AP₂ : 3, 6, 9, 12, 111, d = 3

New AP : 9, 24, 39, T_n
⇒ T_n ≤ 124 + T_n ≤ 111
⇒ 9 + (n-1)15 ≤ 111
⇒ (n-1)15 ≤ 102
⇒ (n-1) ≤ $\frac{102}{15}$
⇒ n-1 ≤ 6.8
⇒ n ≤ 7.8
 $n = 7$ ✓

Aniket raj
From patna

Question - [JEE Mains 2024]

$$4, 9, 14, 19, \dots$$

$$n = 25$$

$$a = 4$$

$$d = 5$$

$$\begin{aligned} a_n &= a + (n-1)d \\ &= 4 + 24 \times 5 \\ &= 124 \end{aligned}$$

$$3, 6, 9, 12, \dots$$

$$n = 37$$

$$a = 3$$

$$d = 3$$

$$\begin{aligned} a_n &= 3 + 36 \times 3 \\ &= 111 \end{aligned}$$

$$4, \textcircled{9}, 14, 19, \dots, 124$$

$$3, 6, \textcircled{9}, 12, \dots, 111$$

$$d = 5$$

$$d = 3$$

$$\text{L.C.M} = 15$$

$$a = 9, d = 15 \text{ and } T_n \leq 111$$

$$a + (n-1)d \leq 111$$

$$9 + 15(n-1) \leq 111$$

$$n \leq \frac{111 - 9 + 15}{15}$$

$$n \leq \frac{117}{15}$$

$$n \leq 7$$

Option D

manik bihar



QUESTION [JEE Mains 2023 (1 Feb)]



The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15,, 399,

2, 5, 8, 11,, 359 and

2, 7, 12, 17,, 197,

is equal to _____

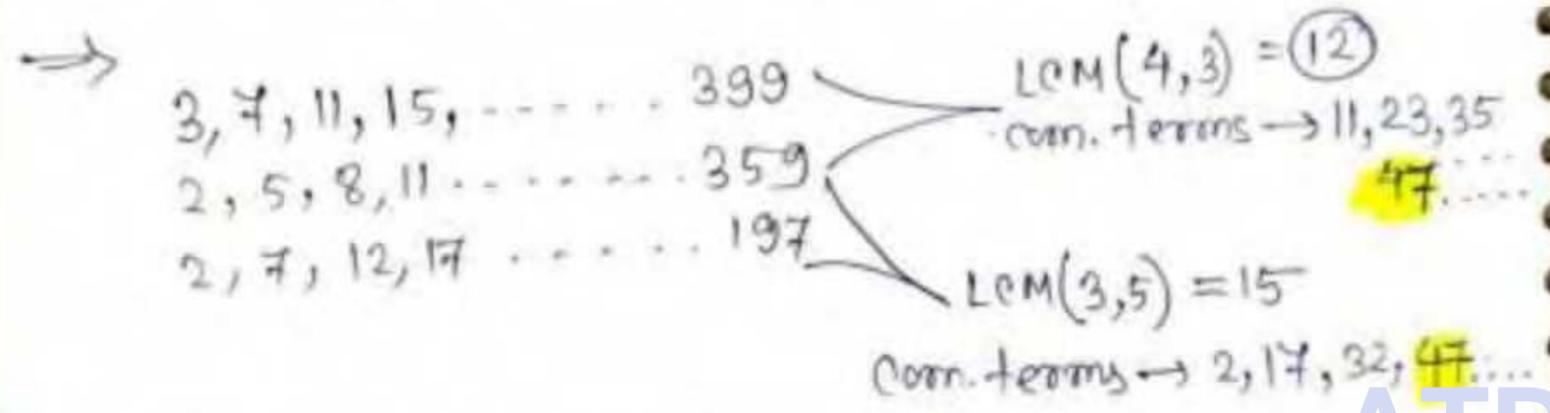
ATDB.uno

Ans. 321



following three arithmetic progressions

- 3, 7, 11, 15, ... 399
- 2, 5, 8, 11, ... 359 and
- 2, 7, 12, 17, ... 197 is equal to



common terms in all three A.P
 $\Rightarrow LCM(12, 15)$
 $\Rightarrow 60$

New A.P $\Rightarrow 47, 107, 167$ LAST TERM CAN'T EXCEED 197

Thus, sum of all the terms $\rightarrow 47 + 107 + 167 = 321$ (Ans)

Kritisha

- (d=4) $\leftarrow P = 3, 7, 11, 15, \dots 399$
- (d=3) $\leftarrow Q = 2, 5, 8, 11, \dots 359$
- (d=5) $\leftarrow R = 2, 7, 12, 17, \dots 197$

For Q & R common term = 11
 common difference = L.C.M = $4 \times 3 = 12$

(d=12) $\leftarrow PQ \rightarrow 11, 23, 35, 47, 59, 71, \dots 359$

Q or R \leftarrow common term = 2
 " difference = L.C.M = $3 \times 5 = 15$

(d=15) $\leftarrow QR \rightarrow 2, 17, 32, 47, 62, \dots 197$

(a) common terms = 47
 (d) " difference = L.C.M = $12, 15 = 60$

$$T_n \leq 197$$

$$47 + (n-1)60 \leq 197$$

$$n \leq \frac{197 - 47 + 60}{60}$$

$$n \leq \frac{210}{60}$$

$n \leq 3$

Sum of common term $S_3 = \frac{3}{2}(2a + (n-1)d)$

$$= \frac{3}{2}(2 \times 47 + 2 \times 60)$$

$$= 3(47 + 60)$$

manik bihar

QUESTION [JEE Mains 2025 (4 April)]

Let $A = \{1, 6, 11, 16, \dots\}$ and $B = \{9, 16, 23, 30, \dots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is

- A** 3814
- B** 4003
- C** 4027
- D** 3761

ATDB.uno

Ans. D



lab-07 Let $A = \{1, 6, 11, 16, \dots\}$ and $B = \{9, 16, 23, 30, \dots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is

$$\rightarrow A = \{1, 6, 11, 16, \dots, 2025^{\text{th}} \text{ term}\}$$

$$B = \{9, 16, 23, 30, \dots, 2025^{\text{th}} \text{ term}\}$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Last term of A $\Rightarrow 1 + 2024(5) = 10121$
 " " " B $\Rightarrow 9 + 2024(7) = 14177$

common diff. of $(A \cap B) \Rightarrow 35$
 1st term " $(A \cap B) \Rightarrow 16$
 Last " " $(A \cap B) < 10121$

$$35 \rightarrow 16 + 35(n-1) < 10121$$

$$35(n-1) < 10105$$

$$(n-1) < 288.71$$

$$n < 289.71$$

$n = 289$
 $n(A \cap B) = 289$

hence, $n(A \cup B) = (2025 \times 2) - 289$
 $n(A \cup B) = 3761$

Kritisha

TAH $\rightarrow 07$
 Question [JEE Mains 2025 (4 April)]

$A = 1, 6, 11, 16, \dots$ $a = 1, d = 5, n = 2025$ $a_n = 1 + (2024) \times 5$ $= 1 + 10110$ $= 10111$	$B = 9, 16, 23, 30, \dots$ $a = 9, d = 7, n = 2025$ $a_n = 9 + (2024) \times 7$ $= 9 + 14168$ $= 14177$
Now 1, 6, 11, 16, ... 10111	Now 9, 16, 23, 30, ... 14177

Common term (a) = 16
 Common difference (d) = L.C.M = $5 \times 7 = 35$

$$T_n \leq 10111$$

$$a + (n-1)d \leq 10111$$

$$16 + (n-1)35 \leq 10111$$

$$n \leq \frac{10111 - 16 + 35}{35}$$

$$n \leq \frac{10130}{35}$$

$$\begin{array}{r} 35 \overline{)10130/289} \\ \underline{70} \\ 313 \\ \underline{280} \\ \times 330 \end{array}$$

$(A \cap B) \rightarrow n \leq 289$
 $(A \cup B) = n(A) + n(B) - (A \cap B)$
 $= 2025 + 2025 - 289$
 $= 4050 - 289$
 $= 3761$



Tah-07

let $A = \{1, 6, 11, 16, \dots\}$ and $B = \{9, 16, 23, 30, \dots\}$ be the sets consisting of the first 2025 terms of two A.P.

Then $n(A \cup B)$ is

- A. 3814
- B. 4003
- C. 4027
- D. 3761

$A = \{1, 6, 11, 16, \dots\} \rightarrow d_1 = 5$

$B = \{9, 16, 23, 30, \dots\} \rightarrow d_2 = 7$

$A \cap B = \{16, 51, 86, \dots\} \quad d = 35$

$a = 16$

$d = 35$

2025th term of A

$T_{2025} = 16 + (2024)5$

2025th term of B

$T_{2025} = 10121$

$T_{2025} = 9 + (2024)7$

$T_{2025} = 14179$



Let n^{th} term of AP $A \cap B = 10191$

$$a_n = a + (n-1)d$$

$$10191 = 16 + (n-1)35$$

$$n-1 = \frac{10191 - 16}{35}$$

$$n-1 = 10105$$

tah07

Richathakur

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$$n = 289$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B) = 2025 + 2025 - 289$$

$$n(A \cup B) = 4050 - 289$$

$$n(A \cup B) = 3761 \text{ (D)}$$

Gamma 20217

QUESTION [JEE Mains 2021 (31 Aug)]

Let a_1, a_2, a_3, \dots be an A.P. If $\frac{a_1+a_2+\dots+a_{10}}{a_1+a_2+\dots+a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to :

A $\frac{19}{21}$

B $\frac{100}{121}$

C $\frac{21}{19}$

D $\frac{121}{100}$

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Ans. C



Tab-08

Let a_1, a_2, a_3, \dots be an A.P. If $\frac{a_1 + a_2 + \dots + a_{10}}{a_1 + a_2 + \dots + a_p} = \frac{100}{p^2}$

$(p \neq 10)$ then $\frac{a_{11}}{a_{10}}$ is equal to:-

$$\Rightarrow \frac{a_1 + a_2 + a_3 + \dots + a_{10}}{a_1 + a_2 + \dots + a_p} = \frac{100}{p^2} \quad (p \neq 10)$$

$$\Rightarrow \frac{\frac{10}{2} (2a_1 + 9d)}{\frac{p}{2} (2a_1 + (p-1)d)} = \frac{100}{p^2}$$

$$\Rightarrow \frac{2a_1 + 9d}{2a_1 + (p-1)d} = \frac{10}{p}$$

$$2a_1 p + 9dp = 2a_1 (p-1) + 10dp - 10d$$

$$\Rightarrow 2a_1 (10-p) + dp - 10d = 0$$

$$\Rightarrow 2a_1 (10-p) - d(10-p) = 0$$

Kritisha

$$\Rightarrow (2a_1 - d)(10-p) = 0$$

$p \neq 10$ hence, $2a_1 = d \Rightarrow a_1 = \frac{d}{2}$

$$a_{10} = a_1 + 9d$$

$$a_{11} = a_1 + 10d$$

$$\frac{a_{11}}{a_{10}} = \frac{a_1 + 10d}{a_1 + 9d} = \frac{21a_1}{19a_1} = \frac{21}{19} \quad (\text{Ans})$$

Tab-08

Let a_1, a_2, a_3, \dots be an A.P. If $\frac{a_1 + a_2 + \dots + a_{10}}{a_2 + a_3 + \dots + a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ equal to

(A) $\frac{19}{21}$

(C) $\frac{21}{19}$

(B) $\frac{100}{121}$

(D) $\frac{121}{100}$

$$\frac{a_1 + a_2 + \dots + a_{10}}{a_2 + a_3 + \dots + a_p} = \frac{100}{p^2}$$

$$\frac{\frac{10}{2} [2a_1 + 9d]}{\frac{p}{2} [2a_1 + (p-1)d]} = \frac{100}{p^2}$$

$$\Rightarrow 2a_1 p + 9dp = 20a_1 + 10dp - 10d$$

$$2a_1 (10-p) + dp - 10d = 0$$

$$\frac{20a_1 + 90d}{2a_1 + 10d} = \frac{100}{p}$$

$$2a_1 (10-p) - d(p-10) = 0$$

$$(2a_1 + d)(10-p) = 0$$

$$a_1 = \frac{d}{2} \quad ; \quad p = 10$$

$$\frac{a_1}{d} = \frac{1}{2}$$

not possible
 $p \neq 10$

$$\frac{1 + 10 \times 2}{1 + 9 \times 2} \Rightarrow \frac{21}{19} \quad (A)$$

$$\frac{d}{a_1} = 2$$

$$\frac{a_{11}}{a_{10}} = \frac{21}{19}$$



Solution to Previous KTKs

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QUESTION



KTK 1

For $a, b \in \mathbb{R} - \{0\}$, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x + \frac{7}{4}\right) = f\left(\frac{7}{4} - x\right) \forall x \in \mathbb{R}$. Also the equation $f(x) = 7x + a$ has only one real and distinct solution.

The value of $(a + b)$ is equal to

- A** 4
- B** 5
- C** 6
- D** 7

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Ans. B

QUESTION

KTK 2



For $a, b \in \mathbb{R} - \{0\}$, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x + \frac{7}{4}\right) = f\left(\frac{7}{4} - x\right) \forall x \in \mathbb{R}$. Also the equation $f(x) = 7x + a$ has only one real and distinct solution.

The minimum value $f(x)$ in $\left[0, \frac{3}{2}\right]$ is equal to

A $-\frac{33}{8}$

B 0

C 4

D -2

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Ans. D



For $f(x) = 7x + a$, also the eqⁿ $f(x) = 7x + a$ has only one real root and distinct solⁿ. Then value of $(a+b) = ?$

- (A) 4
- (B) 5
- (C) 6
- (D) 7

$f(x + \frac{7}{4}) = f(\frac{7}{4} - x)$ $\frac{7}{4} \rightarrow$ Symmetry

$P(x) = ax^2 + bx + a$ $P(x) = 7x + a$

$\frac{-b}{2a} = \frac{7}{4 \cdot 2}$

$b = \frac{-7a}{2}$

$ax^2 + bx + a = 7x + a$

$ax^2 + x(b-7) = 0$

Richathakur

$(b-7)^2 - 4(a)(0) = 0$

$b^2 - 14b + 49 = 0$ $(b-7)^2 = 0$

$b = 7$

$b = \frac{-7 \times a}{2}$

$7 = \frac{-7a}{2}$

$a = -2$

$b = 7$

$a+b \Rightarrow -2+7$

$a+b = 5$

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KTK-2

For $a, b \in \mathbb{R} - \{0\}$ - Same question
The minimum value $f(x)$ in $[0, \frac{3}{2}]$ is equal to

- (A) $-\frac{33}{8}$
- (B) 0
- (C) 4
- (D) -2

$a = -2$ means downward parabola

$f(x) = -2x^2 + 7x - 2$

$x = \frac{7}{4} \rightarrow$ vertex

$f(0) = -2$

$f(\frac{3}{2}) = -2 + 7 - 2 \Rightarrow 3$

min value = -2

KTK-01

For $a, b \in \mathbb{R} - \{0\}$, let $f(x) = ax^2 + bx + a$ satisfies;
 $f(x + \frac{7}{4}) = f(\frac{7}{4} - x) \forall x \in \mathbb{R}$, Also the equation
 $f(x) = 7x + a$ has only one real and distinct solⁿ;

The value of $(a+b)$ is equal to:

$f(x) = ax^2 + bx + a$, is satisfies: $f(x + \frac{7}{4}) = f(x - \frac{7}{4})$

To see this eqⁿ:
Clearly graph is sym about

$x = \frac{7}{4}$

$\frac{-b}{2a} = \frac{7}{4 \cdot 2}$

$7a = -2b$

$f(x) = ax^2 + bx + a$ & $f(x) = 7x + a$

$7x + a = ax^2 + bx + a$

$ax^2 + (b-7)x = 0$ (only one real & distinct solⁿ)

$(b-7)^2 - 4 \cdot a \cdot 0 = 0$

$D=0$

$b-7 = 0$

$b=7$

$7a = (-2)(7)$

$7a = -14$

$a = -2$

krish

find value of $(a+b)$.

$a+b = -2+7$

$= 5$ Ans.



THANK
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YOU