

CBSE 2025-26

Class 12 - Mathematics

Integrals

Important Question

1. $\int \frac{\sin x}{\sin(x-\alpha)} dx = ?$
a. $x \sin \alpha - (\sin \alpha) \log |\sin(x - \alpha)| + C$
b. $x \cos \alpha - (\sin \alpha) \log |\sin(x - \alpha)| + C$
c. $x \sin \alpha + (\sin \alpha) \log |\sin(x - \alpha)| + C$
d. $x \cos \alpha + (\sin \alpha) \log |\sin(x - \alpha)| + C$
2. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx = A(x) \cos 2\alpha + B(x) \sin 2\alpha + C$, where C is a constant of integration, then the functions A(x) and B (x) are respectively
a. $x + \alpha$ and $\log_e |\sin(x - \alpha)|$
b. $x + \alpha$ and $\log_e |\sin(x + \alpha)|$
c. $x - \alpha$ and $\log_e |\cos(x - \alpha)|$
d. $x - \alpha$ and $\log_e |\sin(x - \alpha)|$
3. $\int \frac{\sqrt{x}}{\sqrt{x} + \sqrt[3]{x}} dx = f(x) + c$. If $f(1) = \frac{m+n \log 2}{10}$, then $|m - n|$ equals
a. 97
b. 34
c. 91
d. 38
4. $\int \sqrt{ax + b} dx = ?$
a. $\frac{2(ax+b)^{3/2}}{3a} + C$
b. $\frac{1}{2\sqrt{ax+b}} + c$
c. None of these
d. $\frac{3(ax+b)^{3/2}}{2a} + C$
5. The value of $\int_0^2 (|x - 2| + [x]) dx$, where $[x]$ denotes the greatest integer less than or equal to x, is
a. 4
b. 1
c. 2
d. 3
6. State True or False:
i. The value of integral $P_7 : \int_{-a}^a f(x) dx$ is $2 \int_0^a f(x) dx$.
a. True
b. False
ii. Two indefinite integrals with the same derivative lead to the same family of curves and so they are equivalent.
a. True
b. False
7. Fill in the blanks:
a. The value of $\int_{-\pi}^{\pi} \sin^3 x \cos^2 x dx$ is _____.

- b. $\int_0^{\frac{\pi}{2}} \cos x e^{\sin x} dx$ is equal to _____
8. Evaluate $\int \frac{x^2}{1+x^3} dx$.
9. Write the value of $\int \frac{1-\sin x}{\cos^2 x} dx$.

10. Evaluate: $\int \frac{2x+1}{(x+1)(x-2)} dx$
11. Evaluate: $\int_0^{\pi/2} \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)}$
12. Evaluate $\int_0^{\pi/2} x^2 \sin x dx$.
13. Evaluate: $\int \frac{x^3-3x}{x^4+2x^2-4} dx$

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Solution

1. (d) $x \cos \alpha + (\sin \alpha) \log |\sin(x - \alpha)| + C$

Explanation: Given:

$$\int \frac{\sin x}{\sin(x-\alpha)} dx$$

Let $x - \alpha = t$

$dx = dt$

$$I = \int \frac{\sin(t+\alpha)}{\sin t} dx$$

$$= \int \frac{\sin t \cos \alpha + \cos t \sin \alpha}{\sin t} dt$$

$$= \int \cos \alpha + \sin \alpha \cot t \, dt$$

$$= t \cos \alpha + \sin \alpha \ln |\sin t| + c$$

$$= (x - \alpha) \cos \alpha + (\sin \alpha) \log |\sin(x - \alpha)| + c$$

$$= x \cos \alpha + (\sin \alpha) \log |\sin(x - \alpha)| + c$$

2. (d) $x - \alpha$ and $\log_e |\sin(x - \alpha)|$

Explanation: Let $I = \int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx, \alpha \in (0, \frac{\pi}{2})$

$$= \int \frac{\frac{\sin x}{\cos x} + \frac{\sin \alpha}{\cos \alpha}}{\frac{\sin x}{\cos x} - \frac{\sin \alpha}{\cos \alpha}} dx$$

$$= \int \frac{\sin x \cos \alpha + \sin \alpha \cos x}{\sin x \cos \alpha - \sin \alpha \cos x} dx$$

$$= \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)} dx$$

Now, put $x - \alpha = t \Rightarrow dx = dt$, so

$$I = \int \frac{\sin(t+2\alpha)}{\sin t} dt$$

$$= \int \frac{\sin t \cos 2\alpha + \sin 2\alpha \cos t}{\sin t} dt$$

$$= \int (\cos 2\alpha + \sin 2\alpha \frac{\cos t}{\sin t}) dt$$

$$= t (\cos 2\alpha) + (\sin 2\alpha) \log_e |\sin t| + C$$

$$= (x - \alpha) \cos 2\alpha + (\sin 2\alpha) \log_e |\sin(x - \alpha)| + C$$

$$= A(x) \cos 2\alpha + B(x) \sin \alpha + C \text{ (given)}$$

Now on comparing, we get

$$A(x) = x - \alpha \text{ and } B(x) = \log_e |\sin(x - \alpha)|$$

3. (a) 97

Explanation: Let $I = \int \frac{\sqrt{x}}{\sqrt{x} + \sqrt[3]{x}} dx$

Let $x = t^6 \Rightarrow dx = 6t^5 dt$

$$I = \int \frac{t^3}{t^2 + t^3} (6t^5) dt$$

$$= \int \frac{6t^6}{1+t} dt$$

$$= 6 \int \frac{t^6 - 1 + 1}{1+t} dt$$

$$= 6 \int \left(\frac{(t+1)(t^5 - t^4 + t^3 - t^2 + t - 1)}{t+1} + \frac{1}{t+1} \right) dt$$

$$= 6 \int \left(t^5 - t^4 + t^3 - t^2 + t - 1 + \frac{1}{t+1} \right) dt$$
$$= 6 \left[\frac{t^6}{6} - \frac{t^5}{5} + \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} - t + \log|1+t| \right] + c,$$

where $t = x^{\frac{1}{6}}$

$$\Rightarrow f(1) = 6 \left[\frac{1}{6} - \frac{1}{5} + \frac{1}{4} - \frac{1}{3} + \frac{1}{2} - 1 + \log 2 \right]$$
$$= \frac{-37+60\log 2}{10}$$

$$\Rightarrow m = -37, n = 60$$

$$\Rightarrow |m - n| = |-37 - 60| = 97$$

4. (a) $\frac{2(ax+b)^{3/2}}{3a} + C$

Explanation: Given integral is $\int \sqrt{ax+b}$

Let, $ax + b = z^2$

$$\Rightarrow adx = 2zdz$$

So,

$$\int \sqrt{ax+b} dx$$
$$= \int z \frac{2zdz}{a}$$
$$= \frac{2}{a} \int z^2 dz \quad \text{where, c is the integrating constant.}$$
$$= \frac{2}{a} \frac{z^3}{3} + c$$
$$= \frac{2}{3a} z^3 + c$$
$$= \frac{2(ax+b)^{3/2}}{3a} + c$$

Hence, $\int \sqrt{ax+b} = \frac{2(ax+b)^{3/2}}{3a} + c.$

Which is the required solution.

5. (d) 3

Explanation: $\int_0^2 (|x-2| + [x]) dx = \int_0^2 |x-2| dx + \int_0^2 [x] dx$

$$= - \int_0^2 (x-2) dx + \int_0^1 [x] dx + \int_1^2 [x] dx$$
$$= \int_0^2 (2-x) dx + \int_0^1 0 \cdot dx + \int_1^2 1 \cdot dx$$
$$= \left[2x - \frac{x^2}{2} \right]_0^2 + 0 + [x]_1^2 = 3$$

6. State True or False:

i. (a) True

Explanation: We have,

$$I = \int_{-a}^a f(x) dx$$

$$I = \int_{-a}^0 f(x) dx + \int_0^a f(x) dx$$

Let $z = -x \Rightarrow dz = -dx$

When $x = -a, z = a$ and $x = 0, z = 0$ and $x = -z$

$$I = - \int_a^0 f(-z) dz + \int_0^a f(x) dx$$

Since $f(x)$ is even, $f(-x) = f(x)$

Since, $\int_a^b f(z) dz = \int_a^b f(x) dx$

$$I = 2 \int_0^a f(x) dx$$

which is the required solution.

ii. (a) True

Explanation: True

7. Fill in the blanks:

- a. 0
b. e - 1

8. Let $I = \int \frac{x^2}{1+x^3} dx$
Put $1 + x^3 = t \Rightarrow 3x^2 dx = dt \Rightarrow x^2 dx = \frac{dt}{3}$
 $\therefore I = \int \frac{x^2}{1+x^3} dx = \frac{1}{3} \int \frac{dt}{t}$
 $= \frac{1}{3} \log|t| + C \quad \left[\because \int \frac{dx}{x} = \log|x| + C \right]$
 $= \frac{1}{3} \log|1 + x^3| + C \quad [\text{put } t = 1 + x^3].$

9. Let $I = \int \frac{1-\sin x}{\cos^2 x} dx$
 $= \int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \right) dx$
 $= \int \sec^2 x dx - \int \sec x \tan x dx$
 $= \tan x - \sec x + C$

10. Let, $I = \int \frac{2x+1}{(x+1)(x-2)} dx$
Using partial fractions,
 $\frac{2x+1}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$
 $\Rightarrow 2x + 1 = A(x - 2) + B(x + 1)$
Put $x = 2$
 $\Rightarrow 5 = 3B \Rightarrow B = \frac{5}{3}$
Put $x = -1$
 $\Rightarrow -1 = -3A \Rightarrow A = \frac{1}{3}$

So, we have
 $I = \int \frac{2x+1}{(x+1)(x-2)} dx = \frac{1}{3} \int \frac{dx}{x+1} + \frac{5}{3} \int \frac{dx}{x-2}$
 $\Rightarrow I = \frac{1}{3} \log|x + 1| + \frac{5}{3} \log|x - 2| + c$
Thus,
 $I = \frac{1}{3} \log|x + 1| + \frac{5}{3} \log|x - 2| + c.$

11. Let $I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2 \cos^2 x + b^2 \sin^2 x} dx$
Dividing by $\cos^2 x$ in numerator and denominator, we have
 $I = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx$
Let $\tan x = t$ then $\sec^2 x dx = dt$
 $I = \int_0^{\frac{\pi}{2}} \frac{1}{a^2 + b^2 t^2} dt = \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{a^2}{b^2} + t^2} dt$
Let $t = \frac{a}{b} \tan \theta = \tan x$
 $I = \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{\frac{a}{b} \sec^2 \theta}{\frac{a^2}{b^2} + \frac{a^2}{b^2} \tan^2 \theta} d\theta$
 $= \frac{1}{ab} \theta$
 $\frac{1}{ab} \tan^{-1} \left(\frac{b}{a} \tan x \right) \Big|_0^{\frac{\pi}{2}}$
 $= \frac{\pi}{2ab}$

12. Given, $I = \int_0^{\pi/2} x^2 \sin x dx$
 $I = \int_0^{\pi/2} x^2 \sin x dx$
 $I = \int_0^{\pi/2} x^2 \sin x dx$

By using integration by parts we get ,

$$\begin{aligned}
 I &= [-x^2 \cos x]_0^{\pi/2} + 2 \int_0^{\pi/2} x \cos x \, dx \\
 &= [-x^2 \cos x]_0^{\pi/2} + 2 \left[[x(\sin x)]_0^{\pi/2} - \int_0^{\pi/2} 1 \cdot (\sin x) dx \right] \\
 &= [-x^2 \cos x]_0^{\pi/2} + 2 \left[[x(\sin x)]_0^{\pi/2} + [\cos x]_0^{\pi/2} \right] \\
 \therefore I &= \int_0^{\pi/2} x^2 \sin x \, dx = [-x^2 \cos x + 2(x \sin x + \cos x)]_0^{\pi/2} \\
 &= \left[-\left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2} \sin \frac{\pi}{2} + \cos \frac{\pi}{2}\right) - 2(0 + \cos 0) \right] \\
 &= -\frac{\pi^2}{4} \times 0 + 2\left(\frac{\pi}{2} + 0\right) - 2(0 + 1) \\
 \therefore I &= \pi - 2
 \end{aligned}$$

13. Let the given integral be,

$$\begin{aligned}
 I &= \int \frac{x^3 - 3x}{x^4 + 2x^2 - 4} dx \\
 &= \int \frac{x(x^2 - 3)}{x^4 + 2x^2 - 4} dx
 \end{aligned}$$

Let $x^2 = t$ or,

$$2x \, dx = dt$$

$$\begin{aligned}
 \Rightarrow I &= \frac{1}{2} \int \frac{(t-3)}{t^2 + 2t - 4} dt \\
 &= \frac{1}{4} \int \frac{2t-6}{t^2 + 2t - 4} dt \\
 &= \frac{1}{4} \int \frac{2t+2-8}{t^2 + 2t - 4} dt \\
 &= \frac{1}{4} \int \left(\frac{2t+2}{t^2 + 2t - 4} - \frac{8}{t^2 + 2t - 4} \right) dt \\
 &= \frac{1}{4} \left(\int \frac{2t+2}{t^2 + 2t - 4} dt - \int \frac{8}{t^2 + 2t - 4} dt \right) \\
 \Rightarrow I &= \frac{1}{4} (I_1 + I_2) \dots (i)
 \end{aligned}$$

Now,

$$I_1 = \int \frac{2t+2}{t^2 + 2t - 4} dt$$

$$t^2 + 2t - 4 = u$$

$$\text{or, } (2t + 2) \, dt = du$$

$$\Rightarrow I_1 = \int \frac{1}{u} \, du = \ln |u| + c_1$$

$$\Rightarrow I_1 = \ln |t + 2t - 4| + c_1$$

$$\therefore I_1 = \ln |x^4 + 2x^2 - 4| + c_1$$

Now,

$$I_2 = \int \frac{-8}{(t+1)^2 - 5} dt$$

$$\Rightarrow I_2 = \int \frac{8}{(\sqrt{5})^2 - (t+1)^2} dt$$

$$\therefore I_2 = \frac{8}{2\sqrt{5}} \ln \left| \frac{\sqrt{5} + x^2 + 1}{\sqrt{5} - x^2 - 1} \right| + c_2$$

So, from (i), we get

$$I = \frac{1}{4} \left[\ln |x^4 + 2x^2 - 4| + \frac{4}{\sqrt{5}} \ln \left| \frac{\sqrt{5} + x^2 + 1}{\sqrt{5} - x^2 - 1} \right| \right] + C$$

$$\therefore I = \frac{1}{4} \ln |x^2 + 2x^2 - 4| + \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5} + x^2 + 1}{\sqrt{5} - x^2 - 1} \right| + C$$