

CBSE 2025-26

Class 12 - Mathematics

Determinants

Important Question

Question No. 1 to 5 are based on the given text. Read the text carefully and answer the questions:

Pankaj purchased 5 pens, 3 bags and 1 instrument box and pays ₹16. From the same shop, Dinesh purchased 2 pens, 1 bag and 3 instrument boxes and pays ₹19, while Ankit purchased 1 pen, 2 bags and 4 instrument boxes and pays ₹25.



1. The cost of one pen is
- a. ₹2
- b. ₹3
- c. ₹5
- d. ₹1
2. What is the cost of one pen and one bag?
- a. ₹5
- b. ₹8
- c. ₹3
- d. ₹7
3. What are the cost of one pen and one instrument box?
- a. ₹6
- b. ₹7
- c. ₹8
- d. ₹9
4. Which of the following is correct?
- a. A determinant is a number associated to a square matrix.
- b. A determinant is a number associated to a matrix.
- c. A determinant is a square matrix.
- d. All of these
5. From the matrix equation $AB = AC$, it can be concluded that $B = C$ provided
- a. A is non-singular
- b. A is singular
- c. A is symmetric
- d. A is square
6. Let M and N be two 3×3 non-singular skew-symmetric matrices such that $MN = NM$. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to
- a. $-M^2$

- b. MN
- c. -N²
- d. M²

7. Let a, b, c, d, u, v be integers. If the system of equations, ax + by = u, cx + dy = v, has a unique solution in integers, then

- a. ad - bc need not be equal to ±1.
- b. ad - bc = - 1
- c. ad - bc = 1
- d. ad - bc = ±1

8. The number of values of x for which the matrix $A = \begin{bmatrix} x - 1 & x & x + 1 \\ 2 & -1 & 3 \\ x + 3 & x - 2 & x + 7 \end{bmatrix}$ has no inverse is

- a. 2
- b. 1
- c. 3
- d. infinitely many

9. Adj.(KA) = _____

- a. Kⁿ⁻¹ Adj. A
- b. None of these
- c. K Adj. A
- d. Kⁿ Adj.A

10. State True or False:

i. If $A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & x \\ 2 & 3 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} \frac{1}{2} & 4 & \frac{5}{2} \\ 2 & 2 & -\frac{3}{2} \\ \frac{1}{2} & y & \frac{1}{2} \end{bmatrix}$ then $x = 1, y = -1$

- a. True
- b. False

ii. For a square matrix A in matrix equation AX = B if |A| = 0 and (adj A)B ≠ 0, then there exists no solution.

- a. True
- b. False

11. Fill in the blanks:

- a. If A and B are square matrices each of order 3 and |A| = 5, |B| = 3, then the value of |3AB| is _____.
- b. If all the elements of a determinant above or below the main diagonal consists of zeros, then the value of the determinant is equal to the product of _____ elements.

12. Match the column:

(a) The determinant	$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 31 & 11 & 38 \end{vmatrix} =$	(i) 1
(b) The determinant	$\begin{vmatrix} 5 & 3 & 2 \\ 0 & 4 & 1 \\ 0 & 0 & 3 \end{vmatrix} =$	(ii) 18
(c) The determinant	$\begin{vmatrix} \sin 10^\circ & \cos 10^\circ \\ \sin 80^\circ & \cos 80^\circ \end{vmatrix} =$	(iii) 0
		(iv) 60

(d) The determinant $\begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix} =$	
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13. Evaluate the determinant: $\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$

14. Write A^{-1} for $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

15. Using matrix method, solve the system of equations

$4x + 3y + 2z = 60;$

$x + 2y + 3z = 45;$

$6x + 2y + 3z = 70.$

16. Use determinants to show that the following points are collinear. A (3, 8), B (-4,2) and C(10,14).

17. If the co-ordinates of the vertices of an equilateral triangle with sides of length 'a' are $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, then

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{3a^4}{4}.$$

18. Find adjoint of the matrix $\begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{vmatrix}$

Solution

1. (d) ₹1
- Explanation: ₹1
2. (c) ₹3
- Explanation: ₹3
3. (a) ₹6
- Explanation: ₹6
4. (a) A determinant is a number associated to a square matrix.
- Explanation: A determinant is a number associated to a square matrix.

5. (a) A is non-singular
- Explanation: A is non-singular

6. (a) $-M^2$
- Explanation: Given, $M^T = -M$, $N^T = -N$ and $MN = NM \dots(i)$
- $$\therefore M^2N^2 (M^TN)^{-1} (MN^{-1})^T$$
- $$\Rightarrow M^2N^2N^{-1} (M^T)^{-1} (N^{-1})^T .M^T$$
- $$\Rightarrow M^2N (NN^{-1}) (-M)^{-1} (N^T)^{-1} (-M)$$
- $$\Rightarrow M^2N I (-M^{-1}) (-N)^{-1} (-M)$$
- $$\Rightarrow -M^2NM^{-1} N^{-1} M$$
- $$\Rightarrow -M \cdot (MN) M^{-1} N^{-1} M = -M (NM) M^{-1} N^{-1} M$$
- $$\Rightarrow -MN (NM^{-1}) N^{-1} M = -M (NN^{-1}) M \Rightarrow -M^2$$

7. (a) $ad - bc$ need not be equal to ± 1 .
- Explanation: $ax + by = u$, $cx +dy = v$,
- $$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc;$$
- $$\Delta_1 = \begin{vmatrix} u & b \\ v & d \end{vmatrix} = ud - bv;$$
- $$\Delta_2 = \begin{vmatrix} a & u \\ c & v \end{vmatrix} = av - cu;$$
- $$\Rightarrow x = \frac{\Delta_1}{\Delta} = \frac{ud-bv}{ad-bc}, y = \frac{\Delta_2}{\Delta} = \frac{av-cu}{ad-bc}$$
- since the solution is unique in integers. $\Delta = \pm 1$, $ad - bc = \pm 1$

8. (d) infinitely many
- Explanation: Given matrix has no inverse if $|A| = 0$
- $$|A| = \begin{vmatrix} x-1 & x & x+1 \\ 2 & -1 & 3 \\ x+3 & x-2 & x+7 \end{vmatrix}$$
- $$= (x-1) \{ -1 (x+7) - 3 (x-2) \} - x \{ 2 (x+7) - 3 (x+3) \} + (x+1) \{ 2 (x-2) + (x+3) \}$$
- $$= (x-1) (-4x-1) - x (-x+5) + (x+1) (3x-1)$$

$$= -4x^2 - x + 4x + 1 + x^2 - 5x + 3x^2 - x + 3x - 1$$
$$= 0$$

⇒ For infinitely many values of x, the given matrix has no inverse.

9. (a) $K^{n-1} \text{Adj. } A$

Explanation: $\text{Adj. } (KA) = K^{n-1} \text{Adj. } A$, where K is a scalar and A is a $n \times n$ matrix.

10. State True or False:

i. (a) True

Explanation: True

ii. (a) True

Explanation: True

11. Fill in the blanks:

a. 405

b. Diagonal

12. (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)

13. Let $\Delta = \begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$

$$\Rightarrow \Delta = 2 \begin{vmatrix} 1 & -3 & 1 \\ 4 & -1 & 1 \\ 3 & 5 & 1 \end{vmatrix}$$

Applying, $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get

$$\Rightarrow \Delta = 2 \begin{vmatrix} 1 & -3 & 1 \\ 3 & 2 & 0 \\ 2 & 8 & 0 \end{vmatrix}$$

$$= 2[1(24 - 4)] = 40$$

So, $\Delta = 40$.

14. We have, $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Clearly, $\text{adj } A = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

and $|A| = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 6 - 5 = 1$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } (A) = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

15. Given set of equations are:-

$$4x + 3y + 2z = 60$$

$$x + 2y + 3z = 45$$

$$6x + 2y + 3z = 70$$

Converting the following set of equations in matrix form, we have

$$AX = B$$

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$$R_2 \rightarrow 4R_2 - R_1$$

$$R_3 \rightarrow 2R_3 - 3R_1$$

$$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 5 & 10 \\ 0 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 120 \\ -40 \end{bmatrix}$$

Again converting into equations, we get

$$4x + 3y + 2z = 60$$

$$5y + 10z = 120$$

$$-5y = -40$$

$$Y = 8$$

$$5 \times 8 + 10z = 120$$

$$10z = 120 - 40$$

$$10z = 80$$

$$z = 8$$

$$4x + 3 \times 8 + 2 \times 8 = 60$$

$$4x = 60 - 24 - 16$$

$$4x = 20$$

$$x = 5$$

$$\therefore x = 5, y = 8, z = 8$$

16. If area of triangle = 0, then points are collinear.

$$\begin{aligned} \text{Now Area of a triangle} &= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \\ &= \frac{1}{2} \begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 10 & 14 & 1 \end{vmatrix} \end{aligned}$$

Expanding with C_3

$$= \frac{1}{2} [(-56 - 20) - (42 - 80) + (6 + 32)]$$

$$= \frac{1}{2} [-76 + 38 + 38]$$

$$= 0$$

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

17. Since, we know that area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, is given by $\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$\Rightarrow \Delta^2 = \frac{1}{4} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 \dots(i)$$

We know that, area of an equilateral triangle with side a ,

$$\Delta = \frac{1}{2} \left(\frac{\sqrt{3}}{2} \right) a^2 = \frac{\sqrt{3}}{4} a^2$$

$$\Rightarrow \Delta^2 = \frac{3}{16} a^4 \dots(ii)$$

$$\text{from eq. (i) and (ii), } \frac{3}{16} a^4 = \frac{1}{4} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$$

$$= \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{3}{4}a^4$$

Hence proved.

18. Here $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{vmatrix}$$

$$A_{11} = + \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3$$

$$A_{12} = - \begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -(2 + 10) = -12$$

$$A_{13} = + \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 6$$

$$A_{21} = - \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -(-1) = 1$$

$$A_{22} = + \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 1 + 4 = 5$$

$$A_{23} = - \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -(-2) = 2$$

$$A_{31} = + \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5 - 6 = -11$$

$$A_{32} = + \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -(5 - 4) = -1$$

$$A_{33} = + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 + 2 = 5$$

$$\therefore Adj. A = \begin{bmatrix} 3 & -12 & 6 \\ 1 & 5 & 2 \\ -11 & -1 & 5 \end{bmatrix}'$$

$$= \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$$