

CBSE 2025-26

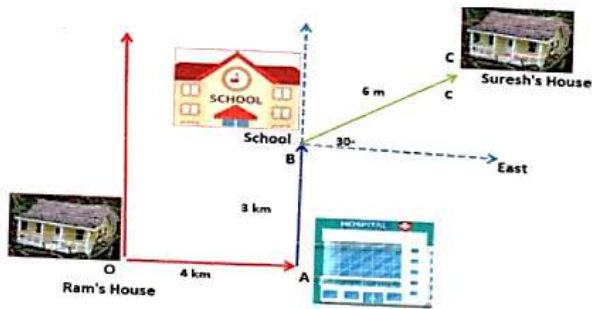
Class 12 - Mathematics

Vector Algebra

Important Question

Question No. 1 to 5 are based on the given text. Read the text carefully and answer the questions:

Ram's house is situated at Gandhi Nagar at Point O, for going to school he first travels by bus in the east. Here at Point A, a hospital is situated. From Hospital Ram takes an auto and goes 3 km in the north direction, here at point B school is situated. Suresh's house is at 30° east, 6 km from point B. (Refer image for information)



- What is vector distance between Ram's house and school?
 - 8 km
 - 4 km
 - 5 km
 - 7 km
- How many km Ram travels to reach school?
 - 5 km
 - 7 km
 - 8 km
 - 4 km
- What is the vector distance from school to Suresh's home?
 - $6\hat{i}$
 - $6\hat{j}$
 - $\sqrt{3}\hat{i} + \hat{j}$
 - $3\sqrt{3}\hat{i} + 3\hat{j}$
- What is the displacement from Ram's house to Suresh house?
 - $(4 + 3\sqrt{3})\hat{i} + 6\hat{j}$
 - $13\hat{j}$
 - $4\hat{i} + 6\hat{j}$
 - $13\hat{i}$
- What is the total distance from Ram's house to Suresh's home?
 - 13 km
 - 5 km
 - 11 km
 - 9 km

6. If
$$\begin{vmatrix} |\vec{a}|^2 & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & |\vec{b}|^2 & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & |\vec{c}|^2 \end{vmatrix} = [\vec{a} \ \vec{b} \ \vec{c}]^n$$
 where $n \in \mathbb{N}$, then n is
- 1
 - none of these
 - 2
 - 3
7. Let $A(3, 0, -1)$, $B(2, 10, 6)$ and $C(1, 2, 1)$ be the vertices of a triangle and M be the mid-point of AC . If G divides BM in the ratio $2 : 1$, then $\cos(\angle GOA)$ (O being the origin) is equal to
- $\frac{1}{6\sqrt{10}}$
 - $\frac{1}{\sqrt{30}}$
 - $\frac{1}{2\sqrt{15}}$
 - $\frac{1}{\sqrt{15}}$
8. If the vectors $4\hat{i} + 11\hat{j} + m\hat{k}$, $7\hat{i} + 2\hat{j} + 6\hat{k}$ and $\hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar, then $m =$
- 38
 - 10
 - 0
 - None of these
9. Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ be such that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$. If P_1 and P_2 are planes determined by the pairs of vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$, $\vec{d}$ respectively, then the angle between P_1 and P_2 is
- 0
 - $\frac{\pi}{3}$
 - $\frac{\pi}{2}$
 - $\frac{\pi}{4}$
10. The vector $(\cos \alpha \cos \beta)\hat{i} + (\cos \alpha \sin \beta)\hat{j} + (\sin \alpha)\hat{k}$ is a
- none of these
 - constant vector
 - null vector
 - unit vector
11. State True or False:
- The length of the vector $\vec{AB}$ is called a magnitude of $\vec{AB}$.
 - True
 - False
 - If $\vec{a}$, $\vec{b}$ are two vectors, then write the truth value of the statement: $\vec{a} = -\vec{b} \Rightarrow |\vec{a}| = |\vec{b}|$.
 - True
 - False
12. Fill in the blanks:
- A quantity that has magnitude as well as direction, is called a _____.
 - The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is _____.
13. Find the components along the coordinate axes of the position vector of the point: $S(4, -3)$.
14. Write two different vectors having same magnitude.

15. If $A(-2, 1, 2)$ and $B(2, -1, 6)$ are two given points, find a unit vector in the direction of $\overrightarrow{AB}$.
16. Find a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, where $\vec{a} = 4\hat{i} - \hat{j} + 8\hat{k}, \vec{b} = -\hat{j} + \hat{k}$.
17. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
18. If O is the circumcentre and O' the orthocentre of a triangle ABC, prove that $\overrightarrow{AO'} + \overrightarrow{O'B} + \overrightarrow{O'C} = \overrightarrow{AP}$, where $\overrightarrow{AP}$ is the diameter of the circumcircle.

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Solution

1. (c) 5 km

Explanation: 5 km

2. (b) 7 km

Explanation: 7 km

3. (d) $3\sqrt{3}\hat{i} + 3\hat{j}$

Explanation: $3\sqrt{3}\hat{i} + 3\hat{j}$

4. (a) $(4 + 3\sqrt{3})\hat{i} + 6\hat{j}$

Explanation: $(4 + 3\sqrt{3})\hat{i} + 6\hat{j}$

5. (a) 13 km

Explanation: 13 km

6. (c) 2

Explanation: Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Let $\begin{vmatrix} |\vec{a}|^2 & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & |\vec{b}|^2 & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & |\vec{c}|^2 \end{vmatrix} = \Delta$. Then

$$\Delta = \begin{vmatrix} a_1^2 + a_2^2 + a_3^2 & a_1b_1 + a_2b_2 + a_3b_3 & a_1c_1 + a_2c_2 + a_3c_3 \\ a_1b_1 + a_2b_2 + a_3b_3 & b_1^2 + b_2^2 + b_3^2 & b_1c_1 + b_2c_2 + b_3c_3 \\ a_1c_1 + a_2c_2 + a_3c_3 & b_1c_1 + b_2c_2 + b_3c_3 & c_1^2 + c_2^2 + c_3^2 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\Delta = [\vec{a} \ \vec{b} \ \vec{c}]^2 \Rightarrow n = 2$$

7. (d) $\frac{1}{\sqrt{15}}$

Explanation: Given vertices of a $\triangle ABC$ are A(3,0, -1), B(2,10,6) and C (1, 2, 1) and a point M is mid-point of AC. Another point G divides BM in ratio 2 : 1, so G is the centroid of $\triangle ABC$.

$$\therefore G \left(\frac{3+2+1}{3}, \frac{0+10+2}{3}, \frac{-1+6+1}{3} \right) = (2, 4, 2)$$

Now, $\cos (\angle GOA) = \frac{\vec{OG} \cdot \vec{OA}}{|\vec{OG}| |\vec{OA}|}$, where O is the origin.

$$\therefore \vec{OG} = 2\hat{i} + 4\hat{j} + 2\hat{k} \Rightarrow |\vec{OG}| = \sqrt{4 + 16 + 4} = \sqrt{24}$$

$$\text{and } |\vec{OA}| = 3\hat{i} - \hat{k} \Rightarrow |\vec{OA}| = \sqrt{9 + 1} = \sqrt{10}$$

$$\text{and } \vec{OG} \cdot \vec{OA} = 6 - 2 = 4$$

$$\therefore \cos(\angle GOA) = \frac{4}{\sqrt{24}\sqrt{10}} = \frac{1}{\sqrt{15}}$$

8. (d) None of these

Explanation: Given vectors $4\vec{i} + 11\vec{j} + m\vec{k}$, $7\vec{i} + 2\vec{j} + 6\vec{k}$ and $\vec{i} + 5\vec{j} + 4\vec{k}$ are coplanar then

$$\begin{vmatrix} 4 & 11 & m \\ 7 & 2 & 6 \\ 1 & 5 & 4 \end{vmatrix} = 0$$

$$\implies 4(8-30) - 11(28 - 6) + m(35 - 2) = 0$$

$$- 88 - 242 + 33 m = 0$$

$$- 330 + 33 m = 0$$

$$m = 10$$

9. (a) 0

Explanation: If θ is the angle between P_1 and P_2 , then normal to the planes are

$$\left. \begin{aligned} N_1 &= \vec{a} \times \vec{b} \\ N_2 &= \vec{c} \times \vec{d} \end{aligned} \right\}$$

$$\therefore N_1 \times N_2 = 0$$

$$\text{Then, } |N_1| \times |N_2| \sin\theta = 0$$

$$\Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$$

10. (d) unit vector

Explanation: Given vector $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\sin\beta)\hat{j} + \sin\alpha\hat{k}$

$$\text{The magnitude of the vector} = \sqrt{\cos^2\alpha\cos^2\beta + \cos^2\alpha\sin^2\beta + \sin^2\alpha}$$

$$= \sqrt{\cos^2\alpha(\cos^2\beta + \sin^2\beta) + \sin^2\alpha}$$

$$= \sqrt{\cos^2\alpha + \sin^2\alpha}$$

$$= 1$$

$\therefore$ It is unit vector

11. State True or False:

i. (a) True

Explanation: True

ii. (a) True

Explanation: True

$$\text{Let } \vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$

$$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

Given that, $a = -b$

$$a_1\hat{i} + b_1\hat{j} + c_1\hat{k} = -a_2\hat{i} - b_2\hat{j} - c_2\hat{k}$$

Comparing the coefficients of i, j, k in LHS and RHS,

$$a_1 = -a_2 \dots (i)$$

$$b_1 = -b_2 \dots (ii)$$

$$c_1 = -c_2 \dots (iii)$$

$$|\vec{a}| = \sqrt{a_1^2 + b_1^2 + c_1^2}$$

Using (i), (ii) and (iii),

$$|\vec{a}| = \sqrt{(-a_2)^2 + (-b_2)^2 + (-c_2)^2}$$

$$|\vec{a}| = \sqrt{a_2^2 + b_2^2 + c_2^2}$$

$$\therefore |\vec{a}| = |\vec{b}|$$

12. Fill in the blanks:

a. vector

b. two

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13. The position vector of the given point S(4, -3) is given by ;

$$\overrightarrow{OS} = 4\hat{i} - 3\hat{j}$$

Component of $\overrightarrow{OS}$ along x- axis = a vector of magnitude 4 having its direction along the positive direction of x-axis.

Component of $\overrightarrow{OS}$ along y-axis = a vector of magnitude 3 having its direction along the negative direction of y-axis.

14. $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$|\vec{a}| = \sqrt{1 + 4 + 9} = \sqrt{14}$$

$$\vec{b} = 3\hat{i} + 2\hat{j} + 1\hat{k}$$

$$|\vec{b}| = \sqrt{9 + 4 + 1} = \sqrt{14}$$

15. We have, A = (-2, 1, 2), and B = (2, -1, 6)

$$\begin{aligned} \therefore \overrightarrow{AB} &= \text{position vector of B} - \text{position vector of A} = \{2 - (-2)\}\hat{i} + \{(-1) - 1\}\hat{j} + \{6 - 2\}\hat{k} \\ &= 4\hat{i} - 2\hat{j} + 4\hat{k} \end{aligned}$$

We know that for any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$

$$\begin{aligned} \therefore \hat{AB} &= \frac{4\hat{i} - 2\hat{j} + 4\hat{k}}{\sqrt{4^2 + 2^2 + 4^2}} \\ &= \frac{4}{6}\hat{i} - \frac{2}{6}\hat{j} + \frac{4}{6}\hat{k} \\ &= \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k} \end{aligned}$$

16. Here we have $\vec{a} = 4\hat{i} - \hat{j} + 8\hat{k}, \vec{b} = -\hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 8 \\ 0 & -1 & 1 \end{vmatrix} = 7\hat{i} - 4\hat{j} - 4\hat{k}$$

$$\begin{aligned} \text{Required unit vector} &= \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|} \\ &= \frac{1}{9}(7\hat{i} - 4\hat{j} - 4\hat{k}) \end{aligned}$$

17. Given that ,

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\Rightarrow \vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{a} \times \vec{0} \text{ [Taking cross-product on left with } \vec{a}]$$

$$\Rightarrow \vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0} \text{ [Using distributive law]}$$

$$\Rightarrow \vec{a} \times \vec{b} - \vec{c} \times \vec{a} = \vec{0} \text{ [}\because \vec{a} \times \vec{a} = \vec{0} \text{ and } \vec{a} \times \vec{c} = -\vec{c} \times \vec{a}]$$

$$\Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a} \text{ ...(i)}$$

$$\text{Again, } \vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\Rightarrow \vec{b} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{b} \times \vec{0} \text{ [Taking cross-product on left with } \vec{b}]$$

$$\Rightarrow \vec{b} \times \vec{a} + \vec{b} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$$

$$\Rightarrow -\vec{a} \times \vec{b} + \vec{0} + \vec{b} \times \vec{c} = \vec{0} \text{ [}\because \vec{b} \times \vec{b} = \vec{0} \text{ and } \vec{b} \times \vec{a} = -\vec{a} \times \vec{b}]$$

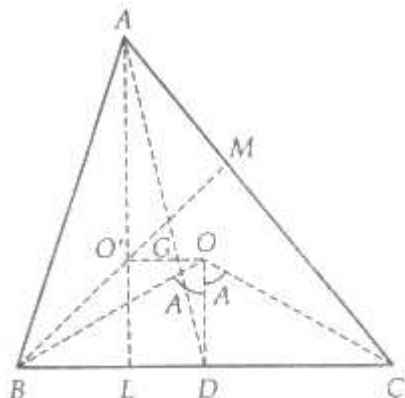
$$\Rightarrow \vec{a} \times \vec{b} = \vec{b} \times \vec{c} \text{ ...(ii)}$$

Form (i) and (ii), we obtain

$$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}. \text{ Hence proved.}$$

18. Suppose that G be the centroid of triangle ABC. First we will show that the circumcentre O, orthocentre O' and centroid G are collinear and $O'G = 2OG$

Suppose AL and BM be perpendiculars on the sides BC and CA respectively. Suppose AD be the median and OD be the perpendicular from O on side BC. If R is the circum radius of circumcircle of $\triangle ABC$, then we get $OB = OC = R$.



In $\triangle OBD$, we have

$$OD = R \cos A \dots (i)$$

In $\triangle ABM$, we have

$$AM = AB \cos A = c \cos A \dots (ii)$$

In $\triangle AO'M$, we have

$$AO' = AM \sec \angle O'AM$$

$$\Rightarrow AO' = c \cos A \sec (90^\circ - C) \text{ [Using (ii)]}$$

$$\Rightarrow AO' = c \cos A \operatorname{cosec} C$$

$$\Rightarrow AO' = \frac{c}{\sin C} \cos A = 2R \cos A \left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \right]$$

$$\triangle AO' = 2 OD \dots (iii) \text{ [Using (i)]}$$

Triangle AGO' and OGD are similar

$$\therefore \frac{OG}{O'G} = \frac{GD}{GA} = \frac{OD}{AO'} = \frac{1}{2} \text{ [Using (iii)]}$$

$$\Rightarrow 2 \cdot OG = O'G \dots (iv)$$

We have,

$$\vec{AO'} + \vec{O'B} + \vec{O'C} = 2\vec{AO'} + (\vec{O'A} + \vec{O'B} + \vec{O'C})$$

$$\Rightarrow \vec{AO'} + \vec{O'B} + \vec{O'C} = 2\vec{AO'} + 2\vec{O'O} \text{ [using (iii)]}$$

$$\Rightarrow \vec{AO'} + \vec{O'B} + \vec{O'C} = 2(\vec{AO'} + \vec{O'O}) \text{ [using (iii)]}$$

$$\Rightarrow \vec{AO'} + \vec{O'B} + \vec{O'C} = 2\vec{AO} = \vec{AP} \text{ [}\because \text{AO is the circum-radius of } \triangle ABC]$$